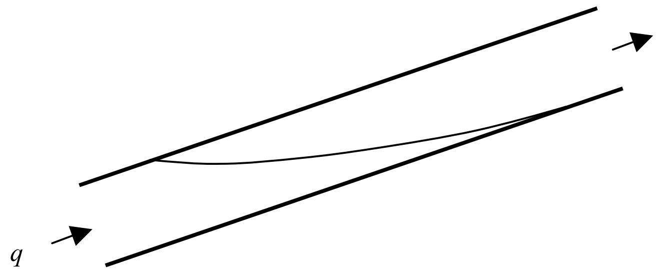
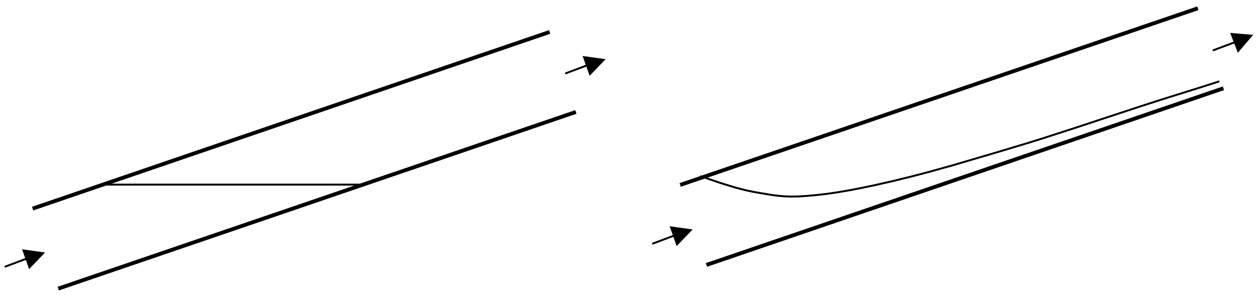
Dietz Stability Analysis

Oil-Water Systems

Consider displacement of oil by water in an inclined layer, consisting of a homogeneous porous media, as shown below, where gravity plays an important role:



In this system, where the oil-water interface per definition is strongly influenced by gravity, we may identify two limiting cases; one where the injection rate is so low that the interface is horizontal (a), and one where the injection rate is so high that the interface is becoming parallel to the layer (b).



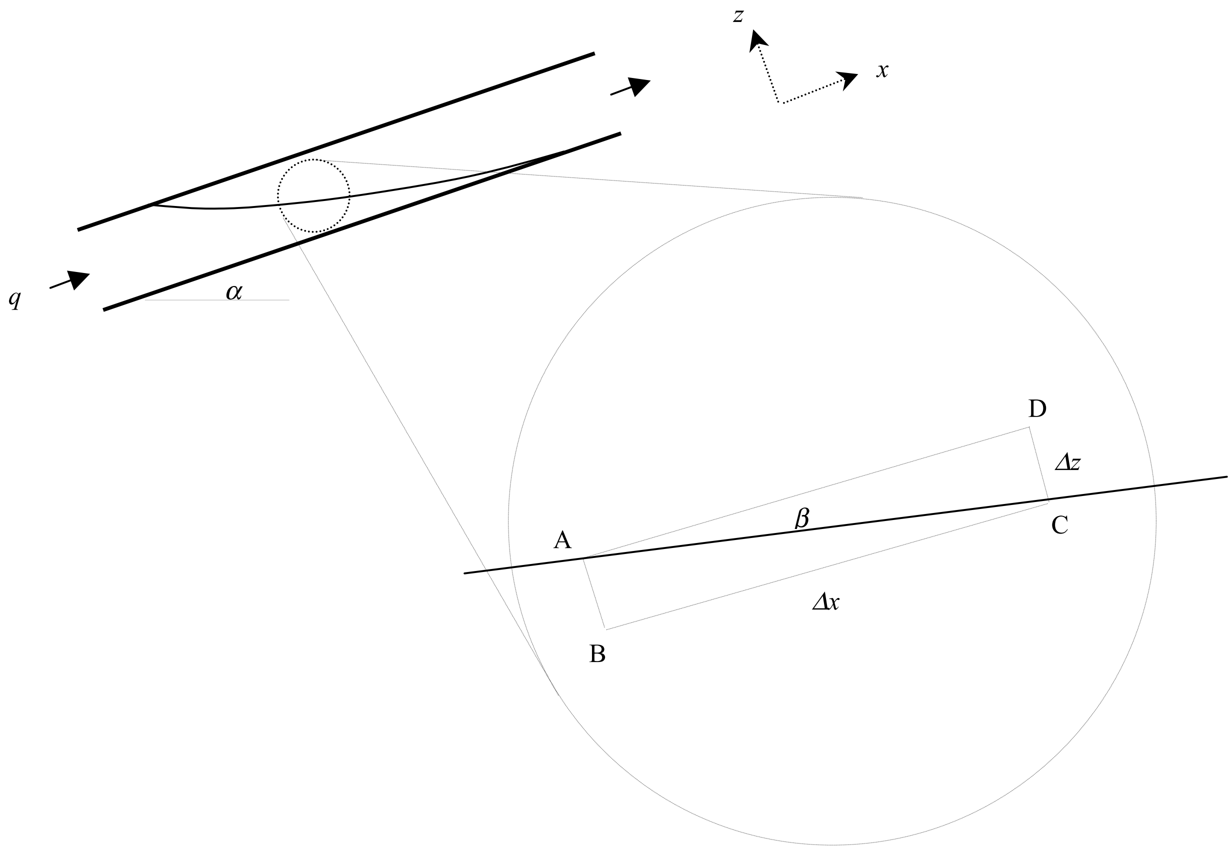
a) very low water injection rate b) very high water injection rate (but still gravity dominated)

For the low rate, the displacement is completely gravity stable, while the displacement in the high-rate case clearly is unstable, since the water is advancing along the bottom of the layer, bypassing the oil. In a paper in 1955, Dietz[1] proposed a method for analyzing the stability of this particular system. He made the following simplifying assumptions:

1. vertical equilibrium of oil and water
2. piston displacement of oil by water
3. oil-water capillary pressure may be neglected
4. compressibility effects of rock and fluids may be neglected

The next step is to make a pressure balance on the interface between oil and water. Enlarging a portion of the interface, and drawing a parallelogram with corners labeled A, B, C and D, we have the following schematic representation of the system:

[1] Dietz, D. N.: "A Theoretical Approach to the Problem of Encroaching and By-Passing Edge Water", Akad. van Wetenschappen, Amsterdam, *Proc. V.56-B*:83, 1953



Here, the oil-water interface is represented by a straight line, having an angle β with the layer, and an angle α with the horizontal plane. Next, Dietz made a pressure balance at the interface:

- Vertically, assuming pressure equilibrium, the following expressions for pressures may be written:

$$P_B = P_A + (AB)\rho_w g\cos\alpha = P_A + \Delta z\rho_w g\cos\alpha$$
$$P_C = P_D + (CD)\rho_o g\cos\alpha = P_D + \Delta z\rho_o g\cos\alpha$$

- Along the direction of flow, Darcy´s equation may be applied, assuming only water flowing behind the oil-water interface, and only oil flowing ahead of the interface:

$$q = -\frac{kk'_{ro}A}{\mu_o}\left(\frac{dP}{dx} + \rho_o g\sin\alpha\right) \qquad \text{(only oil flowing)}$$
$$q = -\frac{kk'_{rw}A}{\mu_w}\left(\frac{dP}{dx} + \rho_w g\sin\alpha\right) \qquad \text{(only water flowing)}$$

or

$$q = -\frac{kk'_{ro}A}{\mu_o}\left(\frac{P_D - P_A}{\Delta x} + \rho_o g\sin\alpha\right)$$
$$q = -\frac{kk'_{rw}A}{\mu_w}\left(\frac{P_C - P_B}{\Delta x} + \rho_o g\sin\alpha\right)$$

Since $\tan\beta = \frac{\Delta z}{\Delta x}$, we may combine these 4 equations, so that:

$$\tan\beta = \frac{1 - M_e}{M_e N_{ge} \cos\alpha} + \tan\alpha,$$

which is the Dietz stability equation.

Here, the end point gravity number for an oil-water system is defined as

$$N_{ge} = \frac{k'_{ro}}{\mu_o} \frac{Ak(\rho_w - \rho_o)g}{q_{inj}}$$

and the end-point mobility ratio is

$$M_e = \left(\frac{k'_{rw}}{\mu_w}\right) \bigg/ \left(\frac{k'_{ro}}{\mu_o}\right)$$

The Dietz stability equation relates the angle of the oil-water interface to the angle of the layer. Clearly, if the interface is horizontal, the displacement is stable, and in fact, as long as the interface remains at an angle greater than zero with the layer, one may claim that the displacement is stable. The Dietz stability criterion is therefore defined as:

$$\beta > 0$$

Applying this criterion to the equation above, we get the following requirement for stability:

$$\frac{1 - M_e}{M_e N_{ge} \cos\alpha} + \tan\alpha > 0$$

As may be seen, if $M_e < 1$, the stability requirement is always fulfilled. However, if $M_e > 1$, the stability is conditional, ie. the displacement is stable for only a range of the displacement velocity. Rewriting the expression, we get:

$$\tan\alpha > \frac{M_e - 1}{M_e N_{ge} \cos\alpha}.$$

Substituting and rearranging, we obtain the following expression for the maximum fluid velocity allowed for stable displacement when $M_e > 1$:

$$u_{winj} < \frac{k(k'_{rw}/\mu_w)\Delta\rho_{ow} g \sin\alpha}{M_e - 1}$$

Thus, if the velocity exceeds the critical velocity, the oil-water interface becomes parallel to the layer.

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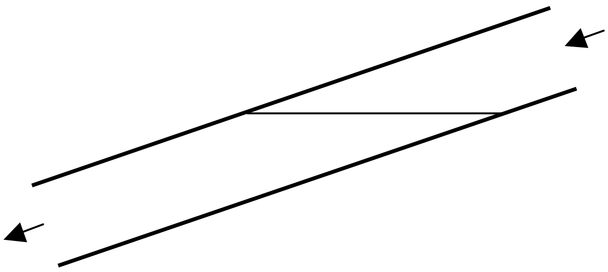
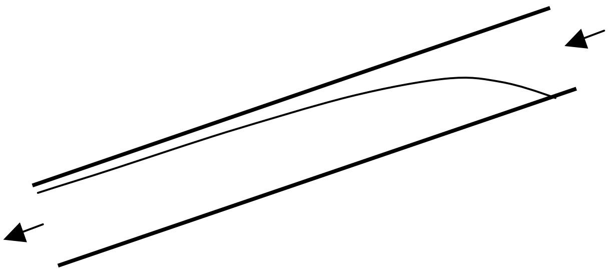
Gas-Oil Systems

In displacement of oil by gas injection at the top, we may make a similar discussion. The gas-oil interface will be strongly influenced by gravity, and we may again identify the two limiting cases, with very low and very high injection rates:



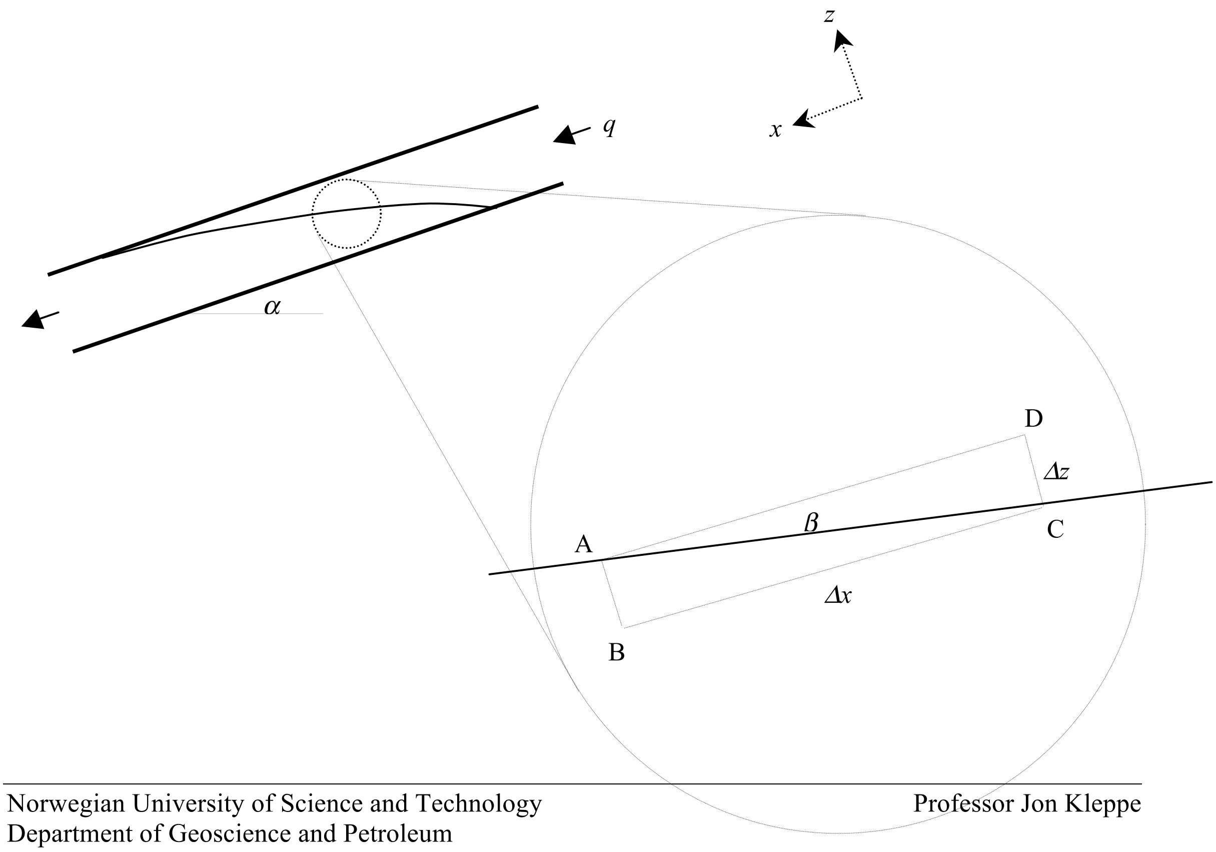


a) very low gas injection rate b) very high gas injection rate

For the low rate, gas-oil contact remains horizontal, and for the high rate case gas will advance along the top of the reservoir. The assumptions are similar to the oil-water case:

1) vertical equilibrium of oil and gas
2) piston displacement of oil by gas
3) gas-oil capillary pressure may be neglected
4) compressibility effects of rock and fluids may be neglected

The schematic representation of the system now becomes:



The pressure balance at the interface becomes:

- Vertically:

$$P_B = P_A + (AB)\rho_o g\cos\alpha = P_A + \Delta z\rho_o g\cos\alpha$$
$$P_C = P_D + (CD)\rho_g g\cos\alpha = P_D + \Delta z\rho_g g\cos\alpha$$

- Along the direction of flow:

$$q = -\frac{kk'_{ro}A}{\mu_o}\left(\frac{dP}{dx} - \rho_o g\sin\alpha\right) \qquad \text{(only oil flowing)}$$
$$q = -\frac{kk'_{rg}A}{\mu_g}\left(\frac{dP}{dx} - \rho_g g\sin\alpha\right) \qquad \text{(only gas flowing)}$$

or

$$q = -\frac{kk'_{rg}A}{\mu_g}\left(\frac{P_A - P_D}{\Delta x} - \rho_g g\sin\alpha\right)$$
$$q = -\frac{kk'_{ro}A}{\mu_o}\left(\frac{P_B - P_C}{\Delta x} - \rho_o g\sin\alpha\right)$$

Combining the 4 equations, the Dietz stability equation for gas displacement of oil becomes:

$$\tan\beta = \frac{1 - M_e}{M_e N_{ge}\cos\alpha} + \tan\alpha,$$

which is identical to the one for the oil-water system. However, the parameters involved have different definitions.

End-point gravity number for oil-gas is defined as

$$N_{ge} = \frac{k'_{ro}}{\mu_o}\frac{Ak(\rho_o - \rho_g)g}{q}$$

and the end point mobility ratio is now

$$M_e = \left(\frac{k'_{rg}}{\mu_g}\right)\Bigg/\left(\frac{k'_{ro}}{\mu_o}\right)$$

Again applying the Dietz stability criterion, $\beta > 0$, to the equation above, we get the requirement for stability:

$$\frac{1 - M_e}{M_e N_{ge}\cos\alpha} + \tan\alpha > 0$$

However, in the case of a gas-oil system, the mobility ratio is never less than 1. Thus, the stability of the gas-oil interface is always only conditionally stable; the stability requirement is always fulfilled. For $M_e > 1$:

$$\tan\alpha > \frac{M_e - 1}{M_e N_{ge} \cos\alpha},$$

and the expression for the maximum fluid velocity becomes:

$$u_{ginj} < \frac{k(k'_{rg}/\mu_g)\Delta\rho_{og} g \sin\alpha}{M_e - 1}$$

Stability of water displacement relative to gas displacement

Clearly, since the gas mobility is much higher than the water mobilty, the maximum displacement velocity of gas injection by the Dietz formula is less than the maximum displacement velocity of water:

$$u_{ginj} < \frac{k(k'_{rg}/\mu_g)\Delta\rho_{og} g \sin\alpha}{M_{eog} - 1}$$

$$u_{winj} < \frac{k(k'_{rw}/\mu_w)\Delta\rho_{ow} g \sin\alpha}{M_{eow} - 1}$$

Taking the ratio of the two maximum velocities (for $M_{eow} > 1$):

$$\frac{u_{ginj}}{u_{winj}} = \frac{\Delta\rho_{og}}{\Delta\rho_{ow}} \frac{M_{eog}}{M_{eow}} \frac{(M_{eow} - 1)}{(M_{eog} - 1)}$$

For a typical North Sea field, the parameters involved may be:

$$\begin{array}{lll} \rho_o = 0.7 & \mu_o = 1 & k'_{ro} = 0.8 \\ \rho_w = 1 & \mu_w = 0.5 & k'_{rw} = 0.5 \\ \rho_g = 0.2 & \mu_g = 0.02 & k'_{rg} = 0.8 \\ \alpha = 7^o & k = 1 & \end{array}$$

Thus,

$$\frac{\Delta\rho_{og}}{\Delta\rho_{ow}} = 1.67$$
$$M_{eog} = 50$$
$$M_{eow} = 1.25$$

and the critical velocities are:

$$u_{ginj} \approx 4.2\ cm/day$$
$$u_{winj} \approx 12.4\ cm/day$$

The ratio of these velocities is:

$$\frac{u_{ginj}}{u_{winj}} = 0.34$$

Based on this calculation, water injection is a better alternative than gas injection, since it is stable for higher velocities. However, experience from gas injection projects in the North Sea indicates that gas injection projects are successful at injection velocities higher than the calculated maximum Dietz velocity. Probably, one reason for this is that there are miscibility effects at the oil-gas interface that make the displacement front more stable. Another reason may be that the assumptions behind the Dietz analysis are not valid for the fields in question.

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