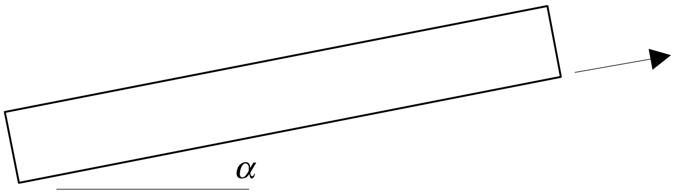
BUCKLEY-LEVERETT ANALYSIS

Derivation of the fractional flow equation for a one-dimensional oil-water system

Consider displacement of oil by water in a system of dip angle α



We start with Darcy´s equations

$$q_o = -\frac{kk_{ro}A}{\mu_o}\left(\frac{\partial P_o}{\partial x} + \rho_o g \sin\alpha\right)$$
$$q_w = -\frac{kk_{rw}A}{\mu_w}\left(\frac{\partial P_w}{\partial x} + \rho_w g \sin\alpha\right),$$

and replace the water pressure by $P_w = P_o - P_{cow}$, so that

$$q_w = -\frac{kk_{rw}A}{\mu_w}\left(\frac{\partial (P_o - P_{cow})}{\partial x} + \rho_w g \sin\alpha\right).$$

After rearranging, the equations may be written as:

$$-q_o \frac{\mu_o}{kk_{ro}A} = \frac{\partial P_o}{\partial x} + \rho_o g \sin\alpha$$
$$-q_w \frac{\mu_w}{kk_{rw}A} = \frac{\partial P_o}{\partial x} - \frac{\partial P_{cow}}{\partial x} + \rho_w g \sin\alpha$$

Subtracting the first equation from the second one, we get

$$-\frac{1}{kA}\left(q_w \frac{\mu_w}{k_{rw}} - q_o \frac{\mu_o}{k_{ro}}\right) = -\frac{\partial P_{cow}}{\partial x} + \Delta\rho g \sin\alpha$$

Substituting for

$$q = q_w + q_o$$

and

$$f_w = \frac{q_w}{q},$$

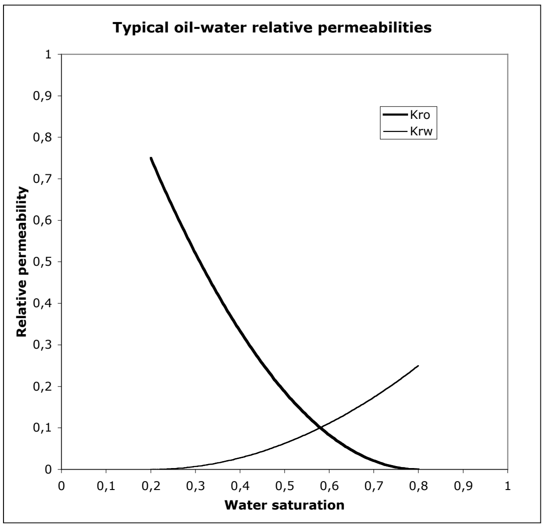
and solving for the fraction of water flowing, we obtain the following expression for the fraction of water flowing:

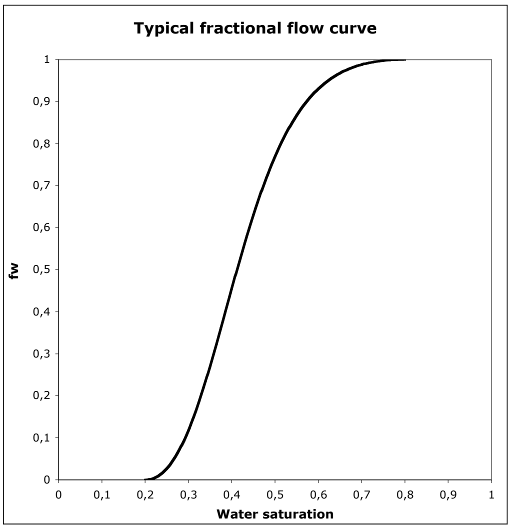
$$f_w = \frac{1 + \dfrac{kk_{ro}A}{q\mu_o}\left(\dfrac{\partial P_{cow}}{\partial x} - \Delta\rho g \sin\alpha\right)}{1 + \dfrac{k_{ro}}{\mu_o}\dfrac{\mu_w}{k_{rw}}}$$

For the simplest case of horizontal flow, with negligible capillary pressure, the expression reduces to:

$$f_w = \frac{1}{1 + \dfrac{k_{ro}}{\mu_o}\dfrac{\mu_w}{k_{rw}}}$$

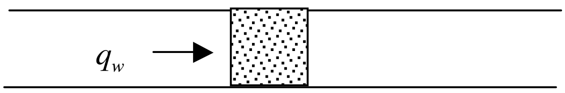
Typical plots of relative permeabilities and the corresponding fractional flow curve are:





Derivation of the Buckley-Leverett equation

For a displacement process where water displaces oil, we start the derivation with the application of a mass balance of water around a control volume of length Δx of in the following system for a time period of Δt:



The mass balance may be written:

$$\left[(q_w\rho_w)_x - (q_w\rho_w)_{x+\Delta x}\right]\Delta t = A\Delta x\phi\left[(S_w\rho_w)^{t+\Delta t} - (S_w\rho_w)^t\right]$$

which, when $\Delta x \to 0$ and $\Delta t \to 0$, reduces to the continuity equation:

$$-\frac{\partial}{\partial x}(q_w\rho_w) = A\phi\frac{\partial}{\partial t}(S_w\rho_w)$$

Let us assume that the fluid compressibility may be neglected, ie.

$$\rho_w = \text{constant}$$

Also, we have that

$$q_w = f_w q$$

Therefore

$$-\frac{\partial f_w}{\partial x} = \frac{A\phi}{q}\frac{\partial S_w}{\partial t}$$

Since

$$f_w(S_w),$$

the equation may be rewritten as

$$-\frac{df_w}{dS_w}\frac{\partial S_w}{\partial x} = \frac{A\phi}{q}\frac{\partial S_w}{\partial t}$$

This equation is known as the Buckley-Leverett equation above, after the famous paper by Buckley and Leverett[1] in 1942.

Derivation of the frontal advance equation

Since

$$S_w(x,t)$$

we can write the following expression for saturation change

$$dS_w = \frac{\partial S_w}{\partial x}dx + \frac{\partial S_w}{\partial t}dt$$

In the Buckley-Leverett solution, we follow a fluid front of constant saturation during the displacement process; thus:

$$0 = \frac{\partial S_w}{\partial x}dx + \frac{\partial S_w}{\partial t}dt$$

Substituting into the Buckley-Leverett equation, we get

$$\frac{dx}{dt} = \frac{q}{A\phi}\frac{df_w}{dS_w}$$

Integration in time

[1] Buckley, S. E. and Leverett, M. C.: “Mechanism of fluid displacement in sands”, *Trans. AIME*, **146**, 1942, 107-116

$$\int_t \frac{dx}{dt} dt = \int_t \frac{q}{A\phi} \frac{df_w}{dS_w} dt$$

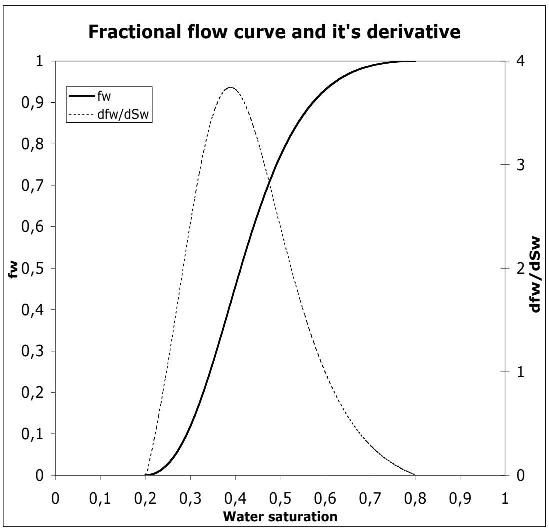
yields an expression for the position of the fluid front:

$$x_f = \frac{qt}{A\phi} \left(\frac{df_w}{dS_w}\right)_f$$

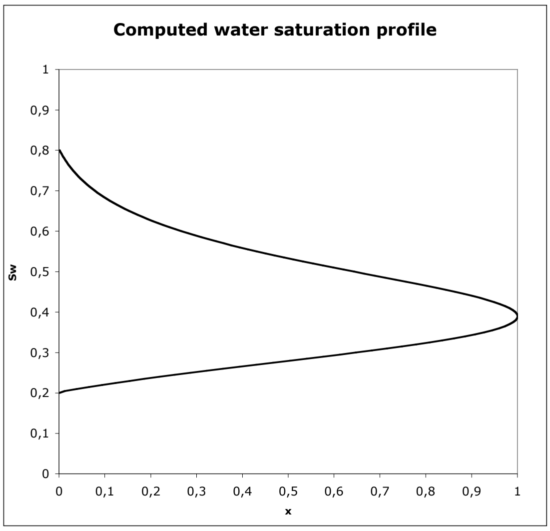
which often is called the frontal advance equation.

The Buckley-Leverett solution

A typical plot of the fractional flow curve and it´s derivative is shown below:

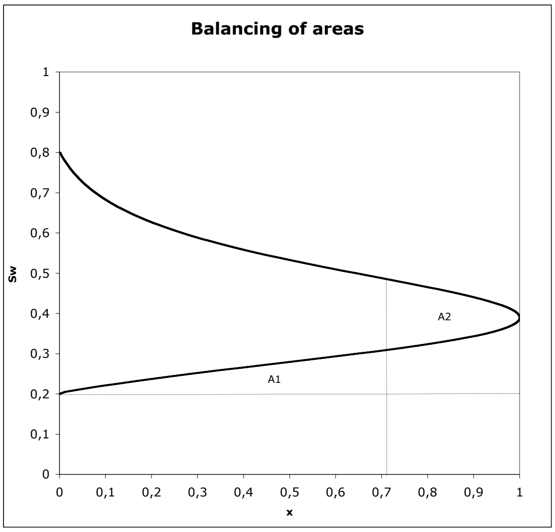


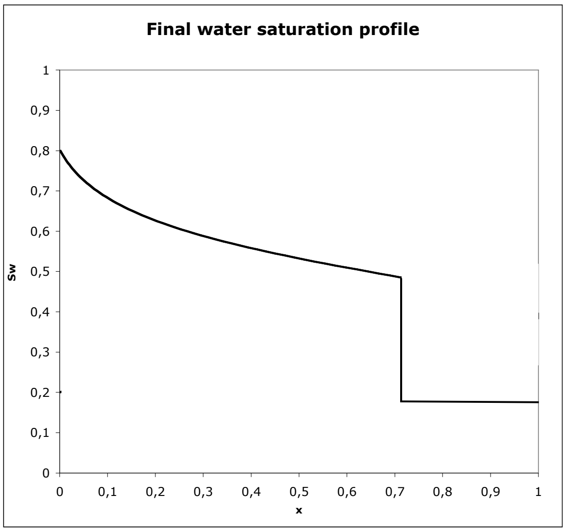
Using the expression for the front position, and plotting water saturation vs. distance, we get the following figure:
Clearly, the plot of saturations is showing an impossible physical situation, since we have two saturations at each x-position. However, this is a result of the discontinuity in the saturation function, and the Buckley-Leverett solution to this problem is to modify the plot by defining a



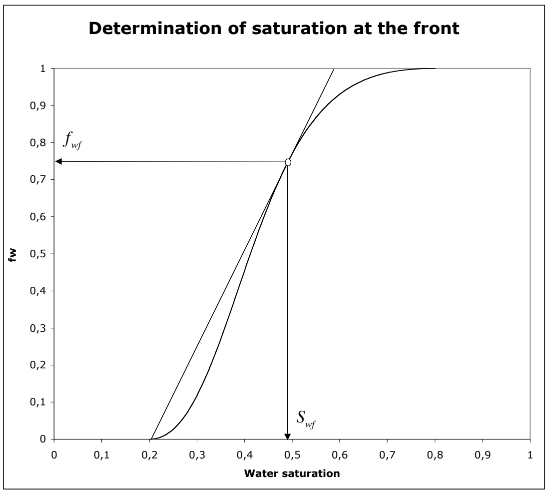
saturation discontinuity at x_f and balancing of the areas ahead of the front and below the curve, as shown:

The final saturation profile thus becomes:

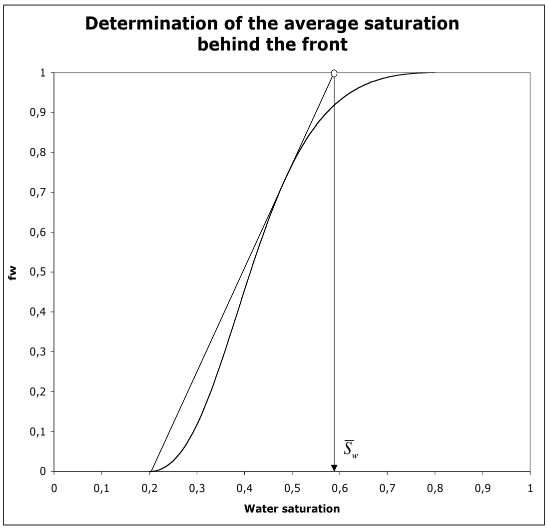




The determination of the water saturation at the front is shown graphically in the figure below:



The average saturation behind the fluid front is determined by the intersection between the tangent line and $f_w = 1$:



At time of water break-through, the oil recovery factor may be computed as

$$RF = \frac{\overline{S}_w - S_{wir}}{1 - S_{wir}}$$

The water-cut at water break-through is

$$WC_R = f_{wf} \quad \text{(in reservoir units)}$$

Since $q_S = q_R / B$, and $f_{wS} = \dfrac{\mathrm{q_{wS}}}{\mathrm{q_{wS}} + q_{oS}}$ we may derive $f_{\mathrm{wS}} = \dfrac{1}{1 + \dfrac{1 - f_w}{f_w} \dfrac{B_w}{B_o}}$

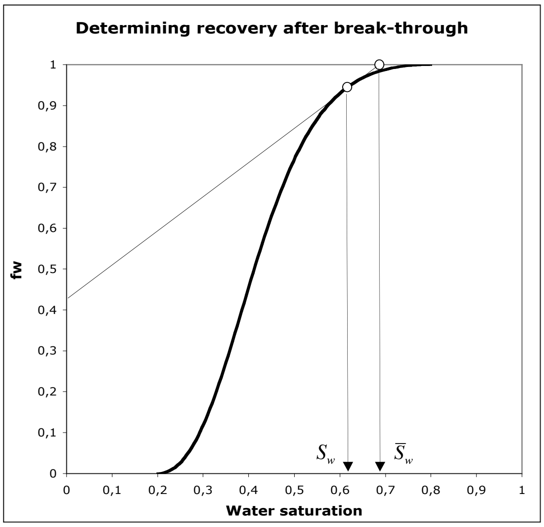
or

$$WC_S = \frac{1}{1+\frac{1-f_w}{f_w}\frac{B_w}{B_o}} \quad \text{(in surface units)}$$

For the determination of recovery and water-cut after break-through, we again apply the frontal advance equation:

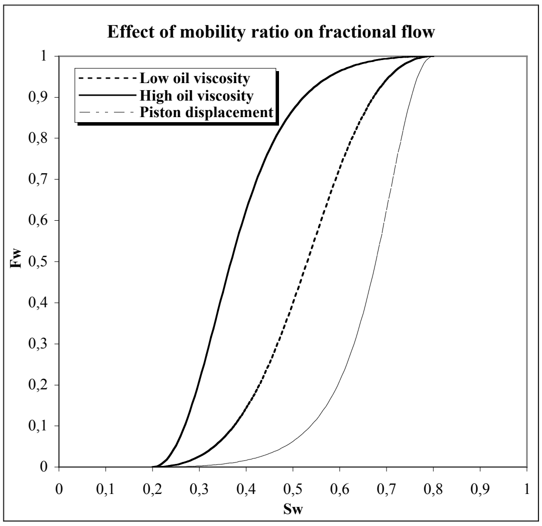
$$x_{S_w} = \frac{qt}{A\phi}\left(\frac{df_w}{dS_w}\right)_{S_w}$$

At any water saturation, S_w, we may draw a tangent to the f_w −curve in order to determine saturations and corresponding water fraction flowing.



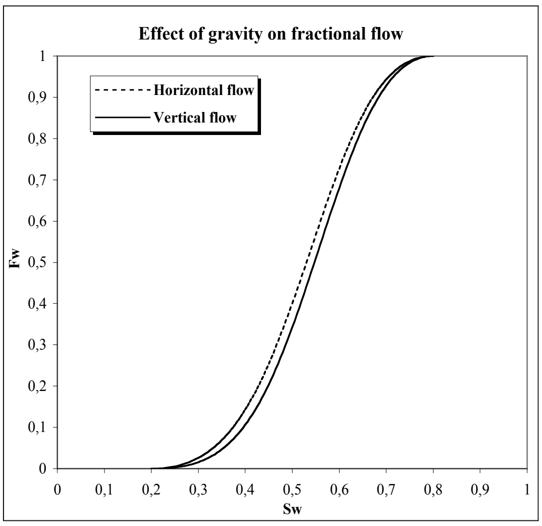
The effect of mobility ratio on the fractional flow curve

The efficiency of a water flood depends greatly on the mobility ratio of the displacing fluid to the displaced fluid, $\frac{k_{rw}}{\mu_w}/\frac{k_{ro}}{\mu_o}$. The lower this ratio, the more efficient displacement, and the curve is shifted right. Ulimate recovery efficiency is obtained if the ratio is so low that the fractional flow curve has no inflection point, ie. no S-shape. Typical fractional flow curves for high and low oil viscosities, and thus high or low mobility ratios, are shown in the figure below. In addition to the two curves, an extreme curve for perfect displacement efficiency, so-called piston-like displacement, is included.



Effect of gravity on fractional flow curve

In a non-horizontal system, with water injection at the bottom and production at the top, gravity forces will contribute to a higher recovery efficiency. Typical curves for horizontal and vertical flow are shown below.



Effect of capillary pressure on fractional flow curve

As may be observed from the fractional flow expression

$$f_w = \frac{1 + \dfrac{kk_{ro}A}{q\mu_o}\left(\dfrac{\partial P_{cow}}{\partial x} - \Delta\rho g \sin\alpha\right)}{1 + \dfrac{k_{ro}}{\mu_o}\dfrac{\mu_w}{k_{rw}}},$$

capillary pressure will contribute to a higher f_w (since $\frac{\partial P_{cow}}{\partial x} > 0$), and thus to a less efficient displacement. However, this argument alone is not really valid, since the Buckley-Leverett solution assumes a discontinuous water-oil displacement front. If capillary pressure is included in the analysis, such a front will not exist, since capillary dispersion (ie. imbibition) will take place at the front. Thus, in addition to a less favorable fractional flow curve, the dispersion will also lead to an earlier water break-through at the production well.