

# SHEAR DISPERSION OF PARTICLES IN PIPE FLOW

A misunderstood mechanism in solid deposition in pipelines?

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# Area of concern

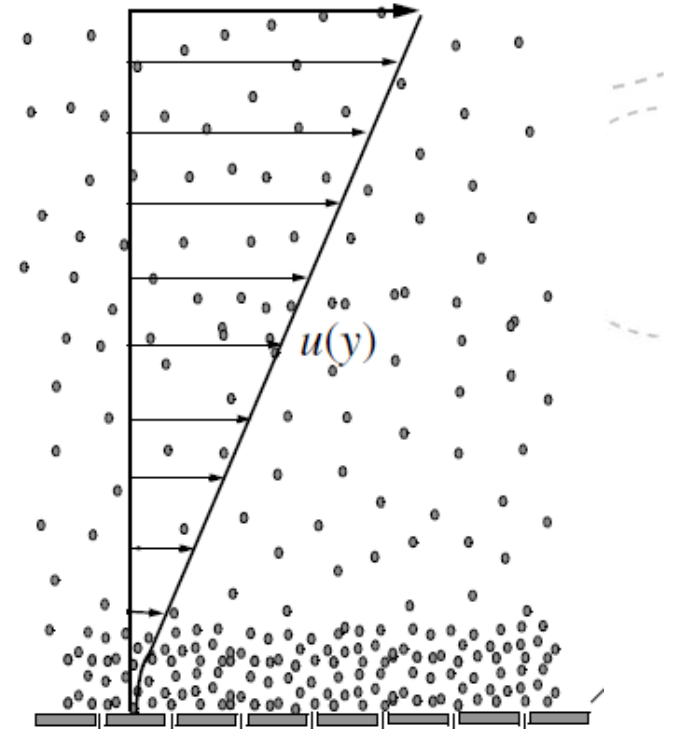
- Pipeline transportation
- Shear dispersion is mentioned as a contributing mechanism to particle deposition.
  - SPE-literature.
  - Olga's wax module.
  - Matzain- model, RRR- model, Rygg, O.B.(1998)
- Other literature implies the opposite.

# Agenda

- Description of phenomena
- SPE-literature
- Quantitative comparison

# The shear dispersion effects

- Migration of particles across streamlines due to shear effects.
- Relevant phenomena:
  - Self-diffusion
  - Shear lift
    - Saffman lift force
    - Rotational induced lift
- Assumptions
  - Neutrally buoyant spherical particles
  - Linear shear



# Concept of self-diffusion

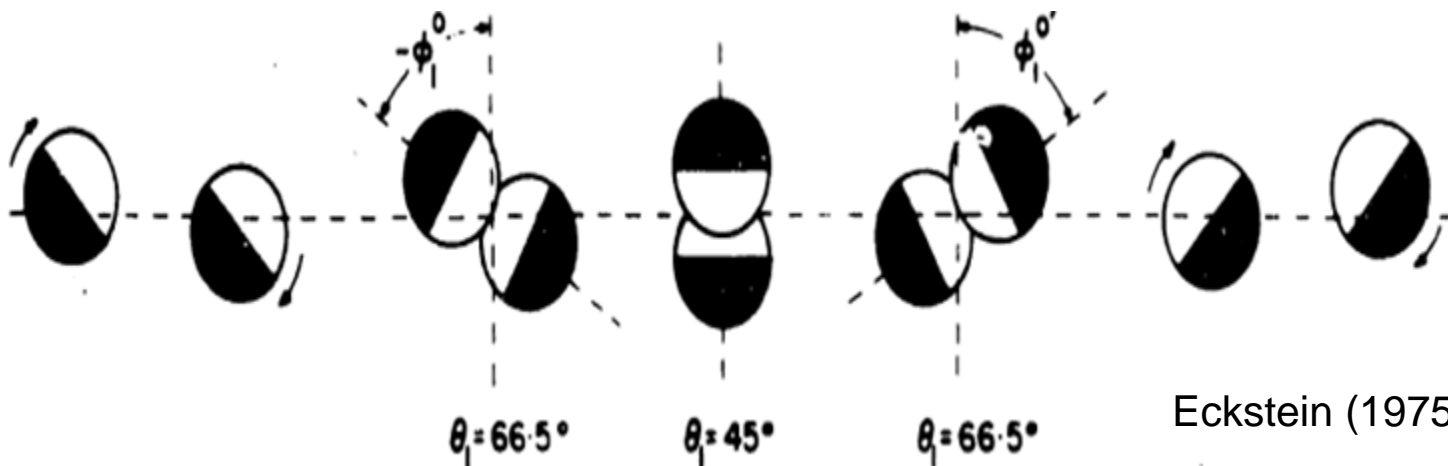
- Particles with zero particle Reynolds number rotates half the shear rate, Jeffery(1922)

$$\omega = \frac{1}{2} \dot{\gamma}$$

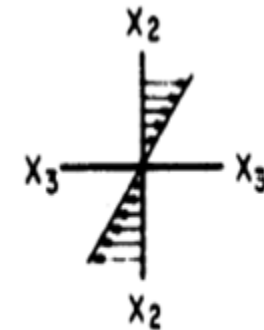
$$\dot{\gamma} = \frac{du}{dy} - \text{shear rate}$$

$\omega$ - angular velocity (rad/s)

- «Collides» with faster/slower particles
- Particles rotates together
- Two particles: No net vertical displacement
- Three or more particles: Vertical displacement



Eckstein (1975)



# Self diffusion: Quantitative description

- Fick's law:  $J = -D_{SI} \frac{\partial C}{\partial r}$
- Dimensionless diffusion coefficient for low particle Reynolds number shear flow (Eckstein et al. 1976):
  - $\frac{D_{SI}}{r_p^2 \dot{\gamma}} = 0.020\alpha$  for  $\alpha < 0.2$
  - $\alpha > 0.2$  : *No clear trend*
- More accurate equation proposed by Leighton & Acrivos (1987):
  - $\frac{D_{SI}}{r_p^2 \dot{\gamma}} = 0.33\alpha_p^2 (1 + 0.5e^{8.8\alpha_p})$

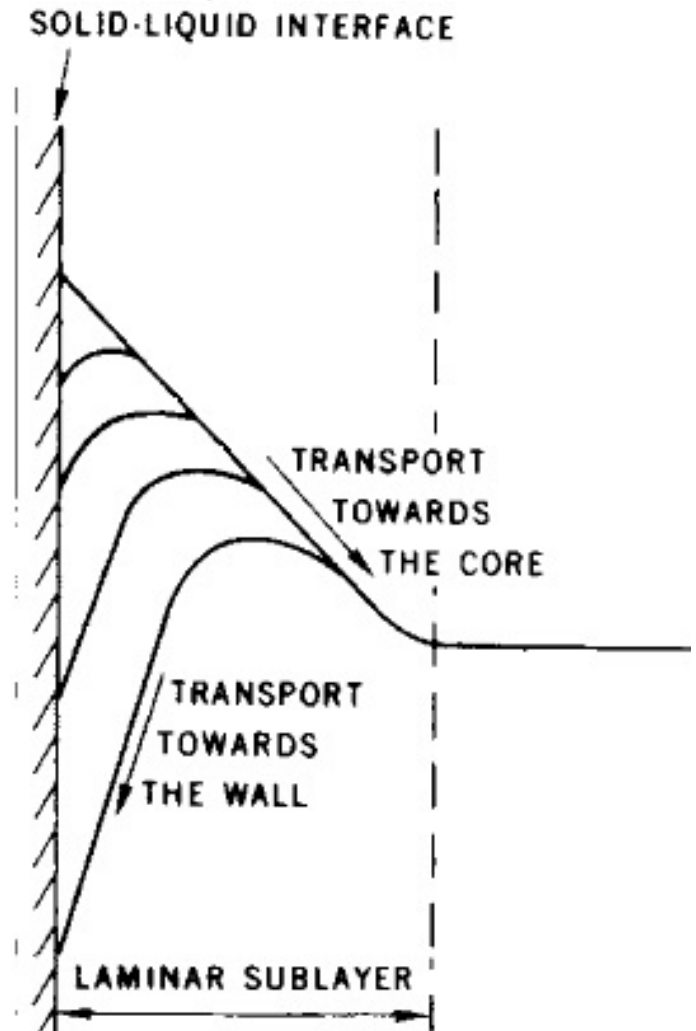
$\dot{\gamma}$  -shear rate

$r_p$  - particle radius

$\alpha$  - volume fraction

# Conclusion self-diffusion

- Concentration gradient decides amount and direction of migration.
- The concentration gradient is a result of:
  - Temperature gradient
  - Diffusional effects
- Segre&Silberberg(1962) showed that particles tend to aggregate at  $0.6r$  for laminar flow.
- Volumetric fraction  $\alpha$  is important



# Lift

- Different types of lift due to shear rate:
  - Saffman lift force
  - Rotational induced lift
- Migrational velocity:  $u_m = \frac{F_D}{6\pi r\mu}$ , Stokes' law (1851)

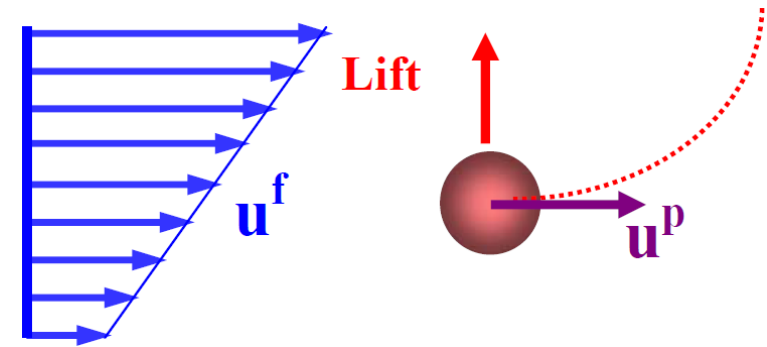
$u_l$  – lateral speed



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# 9 Saffman lift force

- Due to slip velocity and shear.



- Saffman (1965/1968):  $F_L = 1.615\mu V r^2 \left(\frac{du}{dy}\right)^{\frac{1}{2}} \nu^{-\frac{1}{2}}$

- Valid for  $\varepsilon \gg 1 \Rightarrow R_{eG}^{1/2} \gg R_{es} \Rightarrow \sqrt{\frac{\dot{\gamma} d_p^2}{\nu}} \gg \frac{V d_p}{\nu}$

- $\dot{\gamma} \gg \frac{V^2}{\nu} \Rightarrow \dot{\gamma} \gg 4785$

$$\varepsilon = \frac{R_{eG}^{1/2}}{R_{es}}$$

$\nu$  - Kinematic viscosity

$V = u_f - u_p$  - slip velocity: Difference between fluid and particle

$R_{es} = \frac{V d_p}{\nu}$  - Reynolds number based on particle slip velocity

$R_{eG} = \frac{\dot{\gamma} d_p^2}{\nu}$  Reynolds number based on particle shear rate

Particle size:  $6\mu\text{m}$

Viscosity:  $2\text{cp}$

$\nu = 2.09 \cdot 10^{-6}$

Slip velocity(10%):  $0,1\text{m/s}$



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# Safmann lift force

- McLaughlin(1991):

- For  $\varepsilon = \frac{Re_G^{1/2}}{Re_s} \ll 1$ :

$$\frac{F_L}{F_{L(Saff)}} = 14.2\pi^2 \varepsilon^5 \ln\left(\frac{1}{\varepsilon^2}\right)$$

- And for  $0.025 \leq \varepsilon \leq 20 \Rightarrow 120 \leq \dot{\gamma} \leq 95693$

$$\frac{F_L}{F_{L(Saff)}} = 0.3\{1 + \tanh[2.5\log_{10}(\varepsilon + 0.191)]\}\{0.667 + \tan[6(\varepsilon - 0.32)]\}$$

Type equation here.

$Re_s$  - Reynolds number based on particle slip velocity

$Re_G$  - Reynolds number based on particle shear rate

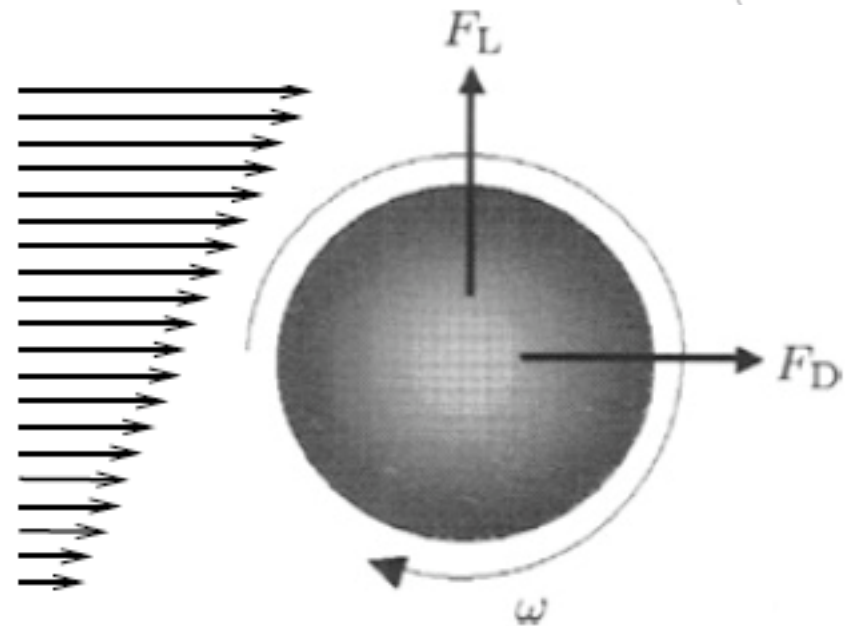
# Rotation-induced lift

- Rubinow&Keller:  $F_L = \frac{\pi}{8} d_p^3 \rho_f \omega V$
- Valid for  $Re_p \ll 1$
- One order of magnitude less than shear-induced lift when particle Reynolds number is small.

$\omega$ - Angular velocity (rad/s)

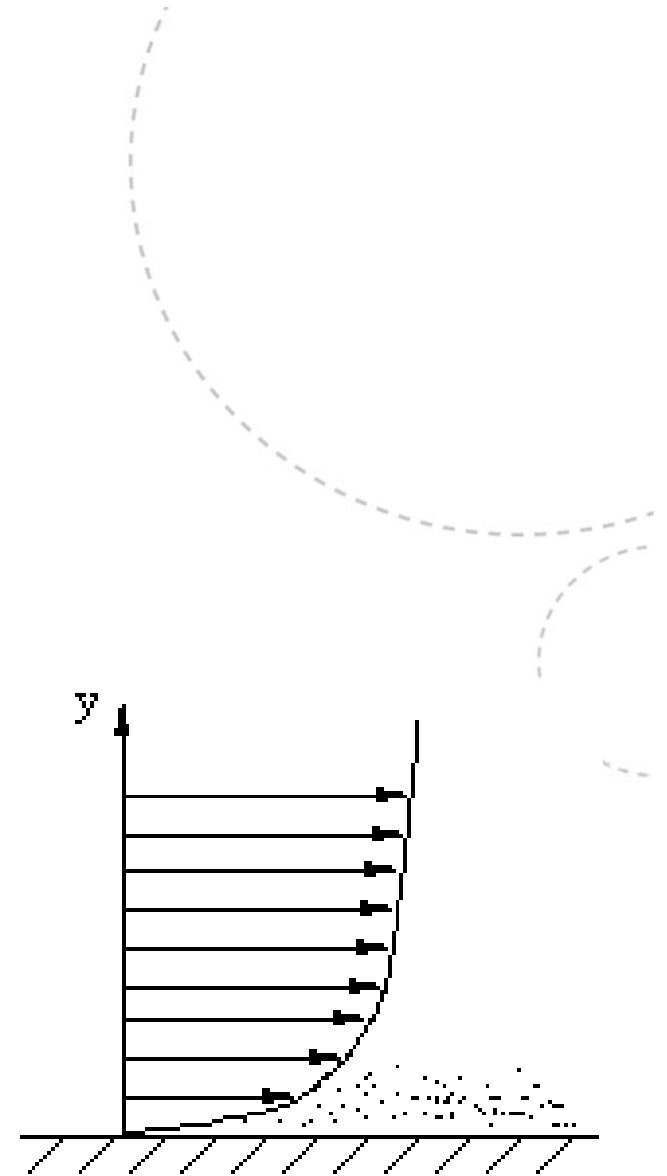
$V = u_f - u_p$  - difference between fluid and particle

$$\omega = \frac{1}{2} \dot{\gamma}$$



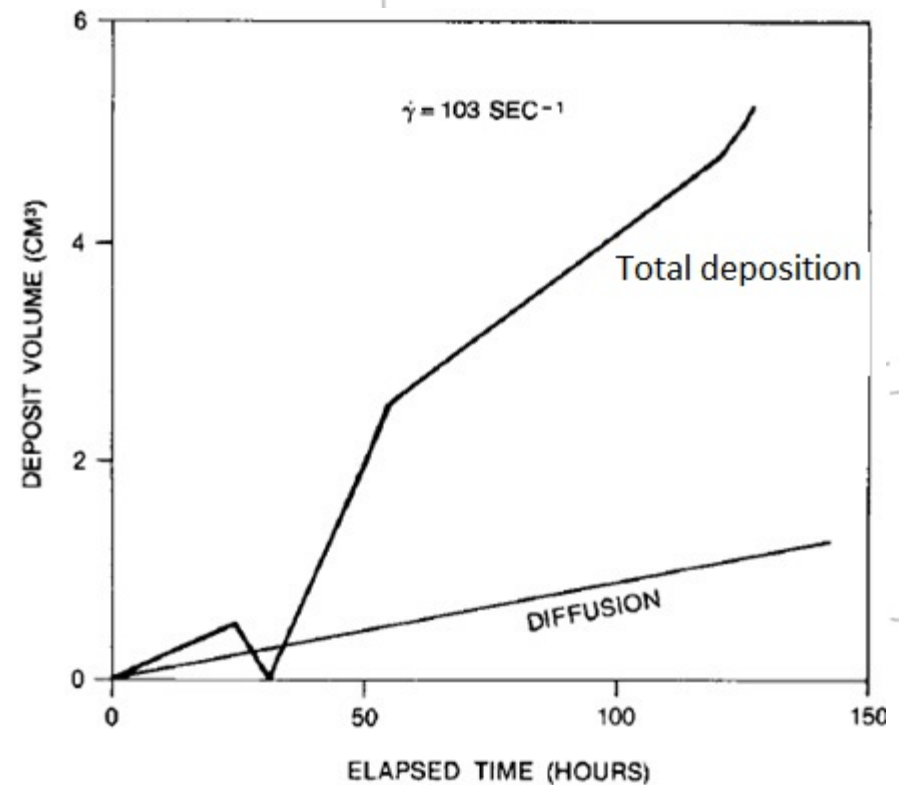
# Conclusions, lift

- More investigation about slip velocity.
- More investigation about shear rate
  - SPE-literature:  $10\text{-}5000\text{ s}^{-1}$
  - Law of the wall
- Induces migration away from the wall



# SPE-literature

- Burger et al. (1981): «Shear dispersion is a major deposition mechanism»:
  - $D_{SI} = \frac{r^2 \gamma \alpha}{10}$
- Weingarten et al. (1988):  
Shear dispersion=Total-molecular diffusion
- Brown et al.(1993): Pressure drop in an operating pipeline.



Weingarten et al. (1988)



# 14 Quantitative comparison

- Shear induced self-diffusion  $J = 6.61 \cdot 10^{-6} \frac{kg}{m^2s}$  Eckstein et al.(1976)
- Shear induced self-diffusion  $J = 3.53 \cdot 10^{-9} \frac{kg}{m^2s}$  Leighton & Acrivos(1987)
- Saffman lift force,  $J = 5.81 \cdot 10^{-4} \frac{kg}{m^2s}$ , McLaughlin (1991)
- Lift force, rotational induced:  $J = 6.34 \cdot 10^{-5} \frac{kg}{m^2s}$ , McLaughlin (1991)
- Molecular diffusion:  $J = 6,35 \cdot 10^{-5} \frac{kg}{m^2s}$ , Venkatesan (2004)
- Brownian diffusion:  $J = 5.12 \cdot 10^{-7} \frac{kg}{m^2s}$ , Green, H.L et al. (1957)

Laminar sublayer size: 3,47mm (law of the wall)

$T_{wall} = 5 \text{ C}$

$T_{bulk} = 43 \text{ C}$

Concentration gradient (from solubility curve, Norne crude):  $12533 \frac{kg}{m^2}$

Particle size:  $6\mu\text{m}$ , Burger et al. (1981)

Shear rate:  $120s^{-1}$ , Weingarten et al. (1988)



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# Conclusion

- Migration due to lift seems to be quantitatively bigger than due to self diffusion.
- Lift causes migrational motion away from the wall.
- Self diffusion may cause migration in both direction
- A more accurate equation should be considered for self diffusion:
  - Replacing  $D_{SI} = \frac{r^2 \gamma \alpha}{10}$  with  $\frac{D_{SI}}{r_p^2 \dot{\gamma}} = 0.33 \alpha_p^2 (1 + 0.5 e^{8.8 \alpha_p})$
- Need more knowledge about:
  - Particle distribution and amount
  - Shear rates
  - Slip velocity

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