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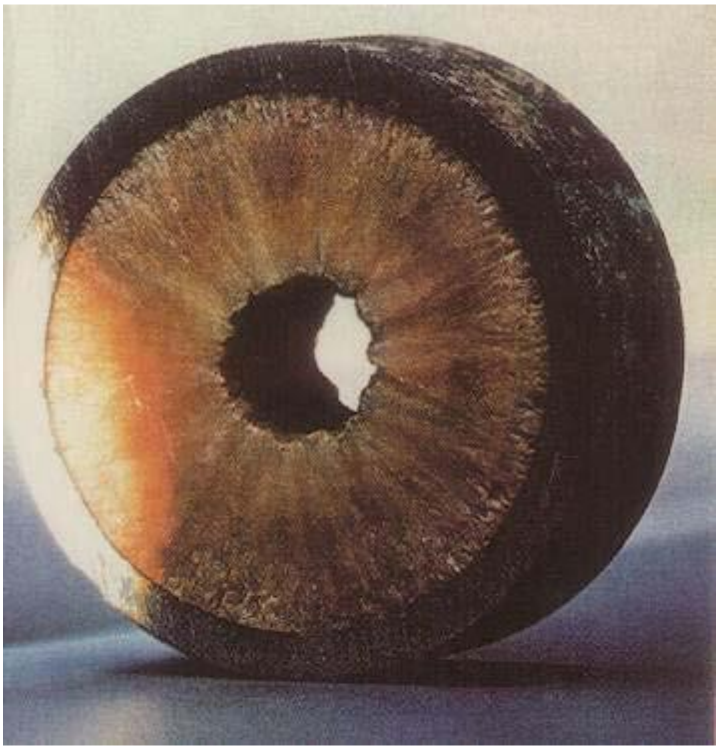
Modeling wax deposition with deposition-release models



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Preface

This project is carried out in TPG4510 Petroleum Production, Specialization Project. This subject is compulsory in the 9^{th} semester at NTNU for Petroleum Production. The subject counts for 15 points, half of one normal semester. I would like to thank my supervisor, Professor Jon Steinar Gudmundsson, for the topic and guidance through the semester.

Abstract

Deposition models are used to understand and predict deposition of solids. A major flow issue in the oil industry is related to deposition of paraffin wax. In this project two wax deposition models are evaluated. They are similar mathematical models called the exponential and logarithmic model. The rate of deposition in each model is decided trough thickness of wax layer and two correlations k_1 and k_2. Both models were matched against experimental data from a single phase wax deposition experiment. The experiment consists of one series with varying flow rate and one with varying temperature. The exponential model was a poor match to these data and was not investigated further. The logarithmic model proved to be a good way of modeling these experiments. Values of k_1, controlling initial deposition rate, seem to be a function of 1/Re for varying flow rates. When the temperature series is investigated k_1 seem to be proportional to ΔT^+. Values of k_2, limiting deposition over time, seem to be proportional to Re^2 when varying flow rate series is evaluated. When the temperature series is considered k_2 seem to be proportional with 1/ ΔT^+. Two new equations based on these dependencies were made. Limited data made it hard to give dependencies on both Re and ΔT^+ in the same equation.

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1. Introduction

Wax deposition in pipelines is one of the biggest flow assurance challenges in the oil industry. The need for understanding deposition becomes greater as hydrocarbons are being transported over increasingly greater distances. Wax precipitates from oil when it is cooled and may deposit on pipeline walls. Wall deposits can lead to severe problems and need to be removed in an efficient way. It is difficult to perform accurate deposition measurements on real pipelines. That is why laboratory experiments are performed, and models based on experiments are developed. Such a model could be scaled up to field data and used to predict deposition of solids.

In this project I investigate two versions of the deposition-release model. The exponential model and the logarithmic model are evaluated against experimental data presented by Rosvold (2008). The first task is to find out which model that best suits the experimental data. The second part is to relate model constants to physical parameters that are important mechanisms in deposition. In this project the two mechanisms investigated is varying flow rate and varying flow temperature.

2. Paraffin wax and wax related problems

Deposition of paraffin wax is a major issue in the oil industry. Wax precipitates from both crude oil and condensate when temperature falls below a certain value. As oil temperature decreases further, more wax will precipitate. If wax precipitates it may deposit in pipes and equipment causing flow problems. Wax deposition is mainly a problem in pipelines, production equipment and in wells. Deposited wax in pipelines may cause increased pressure drop, decreased production and clogged pipes. It may also damage production equipment or make it less efficient.

Wax molecules are mostly long chain n-alkanes, and weight% of 1-15 is considered typical in both crude oil and condensates (Aske 2011). These n-alkanes normally have a carbon number between C_{20} and C_{40} (Gudmundsson 2010). When oil is cooled below a certain temperature, wax will start to precipitate. This temperature is called the wax appearance temperature (WAT), and is normally found around 30 - 40°C (Gudmundsson 2010). The term cloud point temperature is another term used to describe the WAT. Below the cloud point there is another temperature called the pour point temperature. When the pour point is reached the paraffin wax will become a soft solid (a gel). This solid loses the ability to flow and may deposit.

Normal ways of preventing wax deposition in pipelines includes heating or insulation, pigging and chemical injection. Active heating is expensive and limited by distance, especially for subsea pipelines. Insulation is common on long land pipelines (Gudmundsson 2010). Pigging is also common and removes deposited wax mechanically by scraping it of the wall. In a startup phase pigging is usually performed when wax thickness reaches 2-3 mm (Labes-Carrier 2002). This criterion is set to avoid incidents with a stuck pig. Chemical additives may prevent agglomeration of wax molecules and prevent wax from depositing on the wall (Gudmundsson 2010).

3. Wax deposition experiments

The experimental data used in this report was primarily taken from Karianne Rosvold's master thesis from 2008. The experiments were performed in a test rig at Statoil's multiphase flow loop laboratory in Porsgrunn. The test fluid was a waxy gas condensate and it was operated at atmospheric pressure (Rosvold 2008). Detailed information about the rig can be found in Table 1. The same experimental data are also presented in a paper by Rainer Hoffmann and Lene Amundsen (2010).

3.1 Experimental data

The experimental data available to me was the graphs from Rosvold's thesis. These data was presented as plots with measured wax thickness (mm) vs. time (h). In order to utilize the experimental data and make new plots, the plots were digitized. This procedure is explained in Appendix A. Some additional data were taken from Hoffman and Amundsen (2009). This data was used in order to perform calculations.

Experiments were performed by flowing condensate trough a horizontal steel pipe with length 5.55 m. The fluid is assumed to be single phased. The outside of the pipe was cooled by water at 10°C in all experiments. The wax thickness in the pipe was estimated using measurement of pressure drop. Two different flow series experiments were performed. One series varied flow rate and kept condensate temperature constant. The other one did the opposite and varied condensate temperature while keeping flow rate constant. The total of eight experiments were given by Rosvold (2008).

3.1.1 Experiments with varying flow rate

Five runs with flow rate ranging from 5 m^3/h to 25 m^3/h are given by Rosvold (2008). The condensate temperature T_{cond} = 20°C in all five runs. Experiments were run between 65 and 160 hours. The digitized version of these experiments can be seen in Figure 1. Specific values from this flow series are found in Table 2, and constant values in Table 3. The fluid velocity, u, in Table 2 ranges from 0.66 m/s to 3.31 m/s. A typical fluid velocity for oil would be from 2 m/s to 4 m/s (Gudmundsson 2009). Reynolds number ranges from 13836 to 69179, all in the turbulent flow area.

3.1.2 Experiments with varying condensate temperature

Four experiments with condensate temperature between 15°C and 40°C are given. Flow rate is kept constant at 21 m^3/h. These experiments were run between 100 and 335 h. The experiment with T_{cond} = 20°C and Q = 21 m^3/h is part of both series. The digitized experiments can be seen in Figure 2. Specific values from these runs are given in Table 4, and constant values in Table 5. The oil density is given as a constant for temperature because it is only given at 20°C (Hoffmann and Amundsen 2010).

3.2 Additional data

Very little data about the condensate was given by Rosvold (2008). The only fluid data given was the wax appearance temperature of 45°C. A WAT of about 30°C was reported by Hoffman and Amundsen (2009). This value was determined through several tests that all came up with a WAT of about 30°C. The fluid used in the experiment was a North Sea condensate with wax content of about 4.5 % (Hoffman and Amundsen 2010). A WAT of 26°C at atmospheric conditions was given in another study of a North Sea gas condensate by Labes-Carrier et al. (2002). The wax content of this condensate was similar at 4.4 %. Using this information it was decided to base calculations on a WAT of 30°C.

Fluid density (at 20°C), viscosity and wall temperatures during some experiments were given by Hoffman and Amundsen (2010). Viscosities are given in a figure plotting viscosity against temperature. The viscosities are taken from this plot as accurate as possible. Wall temperatures are given by Hoffmann and Amundsen (2010) for all runs in the varying temperature series. For the varying flow rate series wall temperature was only given for two out of five runs. In order to obtain a value a linear relationship between wall temperature and flow rate was assumed. Estimated values for the three other runs were obtained using this assumption. They may not be completely accurate but they are given in Table 2. Reynolds number given in Table 4 may also be inaccurate since oil density at 20°C is used in all calculations.

4. Wax deposition models

Deposition models can be used to predict how wax builds up with time. A figure plotting deposition thickness as a function of time is an effective way of showing deposit buildup. Figure 3 show some of the different trends that are possible outcomes of a deposition experiment. These trends are linear buildup trend, an exponential (asymptotic) trend and a logarithmic trend. Of course it may be other possibilities or mixtures of the three trends given. In Figure 4 we can see how the rate of deposition is influenced by deposition thickness. A linear trend might cause a plugged pipe. In the other cases deposition may stop when a certain wax thickness is reached. Many models exist and some are even a part of simulation software. The focus here is on the deposition release model.

4.1 Deposition-release model

When experimental results are obtained we try to match them with a model. If data can be matched with a model, the model can be developed in order to predict wax buildup. This project focuses on two simple mathematical models and tries to match them with experimental data. These models are the exponential model and the logarithmic model. Both of these models are based on the deposition release principle. One part of the model concerns the buildup of deposition with time. The other part of the model concerns deposits breaking down. When these two mechanisms are equally big the deposition will stop. This is seen in Figure 4 when rate of deposition goes towards zero. The deposition release model is typical for deposition caused by changes in pressure or temperature. Like wax deposition which occurs when oil is cooled.

4.1.1 Exponential model

This model is based on the exponential line in Figure 3. It has been showed to work well for different types of deposition situations by Gudmundsson (1981). Rate of wax deposition is described by the equation

$$\frac{dx}{dt} = k_1 - k_2 x \ .$$

Both k_1 and k_2 are constants or functions, x is wax thickness [mm] and t is time [hour]. When integrated the deposition thickness as a function of time is described by

$$x = \frac{k_1}{k_2}\,[1 - \exp(-k_2 t)].$$

See Appendix B for the full derivation. The initial rate of deposition is

$$\frac{dx}{dt}_{x=0} = k_1.$$

When time is increased the deposition thickness will reach its maximum asymptotic level at

$$x_\infty = \frac{k_1}{k_2}.$$

4.1.2 Logarithmic model

In some cases the exponential deposition-release model does not match the data. A similar model with logarithmic increase has been proposed by Gudmundsson (2010). This model resembles the logarithmic line in Figure 3. Rate of wax deposition is described by

$$\frac{dx}{dt} = k_1 \, k_2{}^{-x}.$$

Similar to the exponential model both k_1 and k_2 are constants or functions, x is wax thickness [mm] and t is time [hour]. If the equation is integrated and solved for deposition thickness we get

$$x = \frac{1}{\ln(k_2)} \, ln[1 + (k_1 \ln k_2)t].$$

See the full derivation in Appendix C. As for the exponential model the initial rate of deposition is

$$\frac{dx}{dt}_{x=0} = k_1, \qquad (k_2{}^{-0} = 1).$$

When using this model the deposition thickness will not stop increasing, but the rate of deposition will decline. In practical cases an asymptotic level is likely to be reached, if not this deposition trend leads to a clogged pipe. An example is k_2 = 500 and x = 2mm: 500^{-2} = 4E-6 ≈ 0 => dx/dt = 0. The bigger the value for k_2, the faster $k_2{}^{-x}$ goes towards zero.

5. Matching the experimental data with a model

Both the exponential and the logarithmic deposition model were matched to the experimental data. In order to do this, values for k_1 and k_2 had to be identified. The detailed procedure for estimating k_1 and k_2 is explained in Appendix D. Parameter k_1, which is initial wax buildup rate, is the same parameter for both models. Parameter k_2 has a different value for each of the two models, these are called $k_{2,exp}$ and $k_{2,log}$. For the best possible match both models were adjusted by trial and error. The eight experiments and the best fit with each model are given in Figure 5-12. The values for k_1 and k_2 from each model is given in Table 6 and 7, for all eight experiments. The values given in the tables are the values which gave the best fit for each model.

5.1 Evaluating k_1

Figure 5-12 makes it easy to evaluate the two models against the experimental data. From the figures we observe that the initial deposition rates for both models match the experiments well. That means that estimates of k_1 are good in most cases. The different k_1 values matched to variable rate experiments are found in Table 6. There seems to be no real consistency in this value related to the increase in condensate rate. The k_1 values estimated from the variable temperature experiments are given in Table 7. One k_1 value in this Table, T_{cond} = 20°C, is much higher than the others. If we exclude this value the other three k_1's decreases with increasing condensate temperature.

5.2 Exponential model and $k_{2,exp}$

From Figure 5-12 we observe that the exponential model does not match most experiments. The only experiment with a good match to the exponential model is T_{cond} 40 = °C and Q = 21 m^3/h. All other experiment does not reach an asymptotic value. At least not in the timeframe used in these experiments. Another issue is that the asymptotic value of the exponential model is reached to fast. A lower value of k_1 and adjusted $k_{2,exp}$ makes it possible to increase the time it takes to reach this asymptotic value. But then we are faced with the problem that the initial deposition rate is predicted as too low. Because of the poor match between experiments and this model it is hard to determine the best $k_{2,exp}$ values. This poor match also makes it hard to comment on trends in $k_{2,exp}$ values given in Table 6 and 7.

5.3 Logarithmic model and $k_{2,log}$

When evaluating Figure 5-12 we observe that the logarithmic model gives a good match in most cases. The only bad match is the T_{cond} = 40 °C and Q = 21 m^3/h, which was the best match to the exponential model. Because this model seems to be more compatible to the data it is easier to comment on the $k_{2,log}$ values. In Table 6 we can see that $k_{2,log}$ increases with increasing condensate rate. When comparing $k_{2,log}$ values for the varying temperature in Table 7 one value stands out. The $k_{2,log}$ for T_{cond} = 20°C is higher than the rest, the same trend as for the k_1 value. If this value is excluded an increasing $k_{2,log}$ with increasing temperature is observed.

6. Determine dependencies of physical parameters

Changes in k_1 and k_2 suggests that changes in both flow rate and temperature influence the wax deposition. The same trend is observed in Figure 1 and 2. After evaluating models and determining k_1 and k_2 values in each experiment the focus is now on determining physical parameters. These physical parameters are based on flow rate changes or temperature changes. The focus will now be on the logarithmic model. The exponential model was a poor fit to the experiment and will not be investigated further.

6.1 Varying rate

When condensate flow rate is increased values for both k_1 and $k_{2,log}$ change. Parameter $k_{2,log}$ increases with increasing flow rate as seen in Table 6. It is hard to say anything about k_1 because it shows no clear trend. Reynolds number (Re) is a function of fluid velocity and increase with increasing velocity. In Figure 13 k_1 is plotted against 1/Re. In this figure we observe a possible linear relationship between rates 10 m^3/h, 15 m^3/h and 25 m^3/h. That could indicate that k_1 is a linear function of 1/Re. That means that in this case $k_1 = k_3 /Re + k_4$, where k_3 and k_4 are new constants. Another linear trend is observed when $k_{2,log}$ is plotted against Re^2 as seen in Figure 14. This trend indicates that $k_{2,log}$ is proportional to Re^2. That means that $k_2 = k_5 * Re^2$. The points furthest away from this linear trend have the rate 21 m^3/h and 25 m^3/h. If the correlations for k_1 and k_2 are used the result becomes

$$\frac{dx}{dt} = \left({k_3}/{Re} + k_4\right)(k_5 Re^2)^{-x} = \frac{{k_3}/{Re} + k_4}{{k_5}^x Re^{2x}}.\ (Eq. 6.1)$$

Where constants in this case are $k_3 = 1.75*10^4$, $k_4 = -0.186$ and $k_5 = 4*10^{-7}$. This equation is only dependent on Re and wax thickness, the temperature is not included. In Figure 17 k_1 is plotted against ΔT^+/Re. The same linear trend seen in Figure 13 is observed. That could give a possible correlation of k_1 with both ΔT^+ and Re.

6.2 Varying temperature

When temperature of condensate increases values k_1 and $k_{2,log}$ change. Previously it was pointed out from Table 7 that k_1 seems to decrease and $k_{2,log}$ increase with increasing temperature. ΔT^+ is a dimensionless temperature driving force parameter and a function of wall, fluid and cloud point temperature. The equation is given in Nomenclature and Variables and was given by Gudmundsson (2010). If k_1 is plotted against ΔT^+ a linear relationship between three out of four points can be seen in Figure 15. This linear relationship implies that k_1 is proportional to ΔT^+. That means that $k_1 = k_3 * \Delta T^+$. The one not in line with the others is T_{cond} = 20°C. If constant $k_{2,log}$ is plotted against $1/\Delta T^+$ a linear relationship between three points can be seen in Figure 16. This relationship implies that $k_{2,log}$ might be proportional to $1/\Delta T^+$. That means $k_2 = k_4 / \Delta T^+$. The same point, T_{cond} = 20°C, does not appear on this line. If the new correlations of k_1 and k_2 are used in the model the result becomes

$$\frac{dx}{dt} = k_3 \Delta T^+ \left({k_4}/{\Delta T^+} \right)^{-x} = \frac{k_3 \Delta T^+}{\left({k_4}/{\Delta T^+} \right)^{x}} = \frac{k_3 (\Delta T^+)^{1+x}}{{k_4}^{x}}. (Eq. 6.2)$$

Where constants in this case have the values $k_3 = 5.72*10^{-2}$ and $k_4 = 1.31*10^{2}$. In this equation the only parameters are the temperature parameter and wax thickness. The ideal case would be if the data made it possible to make an equation of both Re and ΔT^+. The linear relationship between k_1 and ΔT^+/Re was worse than with k_1 and ΔT^+. That means that the correlation found for the flow rate series was not a good match. The correlations presented here in the equations were the best correlations with the data available.

7. Discussion

A claim stated by Gudmundsson (2010) suggests that “The initial rate of deposition and the asymptotic deposition both decrease with increased flow rate”. Figure 1 shows the digitized version of the experiments with varying rate presented by Rosvold (2008). The initial rate of deposition seen in Figure 1 is not easy to evaluate. But it seems like three series Q = 10, 15 and 25 m^3/h show the expected trend. The other two, Q = 5 and 21 m^3/h, deviates from this trend. In the model k_1 shows the initial deposition rate. Values of k_1 in Table 6 indicate the same as Figure 1. That Q = 5 and 21 m^3/h deviates from the trend that would be expected. Three out of five series confirms the claim that the initial rate of deposition decrease with increasing flow rate. The asymptotic deposition can also be evaluated from Figure 1. The asymptotic level seems to decrease with increasing flow rate. One exception is Q = 21 m^3/h which have a lower asymptotic deposition than expected. Values of $k_{2,log}$ in table 6 increase with increasing flow rate. An increased $k_{2,log}$ gives a decreased asymptotic level. Both Figure 1 and $k_{2,log}$ values seem to confirm that asymptotic level decrease with increasing flow rate. Experiment Q = 5 m^3/h had a big spread in data and initial deposition rate was hard to digitize correctly. This is shown in Figure 19. That might be a reason for why it does not behave as expected.

Another claim by Gudmundsson (2010) states that “The initial rate of deposition and the asymptotic deposition both increase with increased difference between solution cloud point and wall temperature”. When condensate temperature is decreased the difference between solution cloud point (WAT) and wall temperature is increased. That means that with lower condensate temperature a higher rate of deposition and asymptotic deposition is expected. From the experiments in Figure 2 it is clear that three out of four experiments behave as expected. The run with T_{cond} = 20°C and Q = 21 m^3/h shows a deviation. The same trend is observed for k_1 and $k_{2,log}$ values in Table 7. The k_1 value decrease with increasing flow rate and $k_{2,log}$ values increase with increasing flow rate. This expected trend is seen in the same three runs as in Figure 1. That means that the claim is confirmed for three runs in both Figure 2 and values from Table 7. The T_{cond} = 20°C and Q = 21 m^3/h does not show the expected behavior. This run is part of both the rate and the temperature series. It does not really behave as expected in either of the series, with the biggest deviation the temperature series. One possible explanation for this is that the wax thickness is estimated trough measurement of pressure loss. This may cause inaccuracies, especially since wax roughness is not known.

When observing Figure 5 - 12 it is clear that the logarithmic model matches the experiments far better than the exponential model. The exponential model was a poor fit to seven out of eight experiments. The constant $k_{2,exp}$ for the exponential model was therefore not evaluated against physical parameters. The logarithmic model was a good fit to the experiments and constant $k_{2,log}$ was evaluated against physical parameters. Because of limited data available the constants were only evaluated against simple rate dependent and simple temperature dependent parameters.

The statement discussed above is that rate of deposition, k_1, decreases with increasing flow rate. This is only clear for 3 out of 5 runs in the flow rate series. It was shown in Figure 13 that k_1 for these three runs form a linear relationship with 1/Re. The same linear trend was also observed for other rate dependent parameters (eg. u, u*). The wall temperature is also dependent on flow rate, although the difference between runs is small. Since only two wall temperatures are given for the flow rate series the estimated ones may be slightly inaccurate. The temperature driving force have been introduced along with Re in Figure 17. Plotting k_1 against ΔT^+/Re does not change the linear relationship between the three points. When it comes to the $k_{2,log}$ values all flow runs show the same trend, decreased asymptotic deposition with increasing flow rate. A relationship that suggest that $k_{2,log}$ is proportional to Re^2 is observed in Figure 14. The run furthest from this trend is the run mentioned before, T_{cond} 20°C and Q 21 m^3/h. Using the relationship between k_1 and $k_{2,log}$ with Re we get equation 6.1. This equation considers Re and three new constants given as k_3, k_4 and k_5. The $k_{2,log}$ values in this series show a better consistency than k_1. That might indicate that $k_{2,log}$ is more stable and easier to estimate than k_1. Or it could mean that initial deposition is harder to calculate correctly using the pressure loss method.

The other statement discussed above states that k_1, initial rate of deposition, should decrease with increasing temperature. The $k_{2,log}$ should increase with increasing temperature, indicating a decrease in asymptotic deposition level. As mentioned both of these are true for three out of four runs, all except T_{cond} = 20°C. As seen in Figure 15 an almost linear relationship between these three is observed when k_1 is plotted against ΔT^+. This indicates that k_1 is proportional to ΔT^+. Plotting k_1 against ΔT^+/Re gave a good trend for the rate series seen in Figure 17. If k_1 for temperature series are plotted against ΔT^+/Re a linear relationship is harder to see. But change in Re with temperature is only based on change in viscosity and not in density. That may cause Re to be inaccurate. When considering $k_{2,log}$ the best linear relationship between these three points appear when $k_{2,log}$ is plotted against $1/\Delta T^+$. This indicates that $k_{2,log}$ is proportional to $1/\Delta T^+$. The same relationship appears if $k_{2,log}$ is plotted against $Re/\Delta T^+$. Considering temperature dependence a new equation is formed, equation 6.2. This equation introduces two new constants k_3 and k_4. As stated before the T_{cond} = 20°C does not match the other runs. Considering the other three runs it seems like $k_1 \alpha \Delta T^+$ while $k_{2,log} \alpha 1/\Delta T^+$. Limited data and possible inaccuracies made it hard to evaluate dependence on both temperature and Re at the same time. Some possible correlations with both was discovered but was not consistent for both series. That is why the equations 6.1 and 6.2 only use either Re or ΔT^+, not both.

8. Conclusion

- The logarithmic deposition release model does a much better job modeling these experimental data than the exponential model. This is observed in Figure 5-12.
- For the varying rate series only three out of five runs show the expected trend in initial deposition rate. Values for k_1 on these three runs seems to be a linear function of 1/Re. The linear function is equally good when compared to ΔT^+/Re. The asymptotic deposition level is more consistent, and values for $k_{2,log}$ appears to be proportional with Re^2. The dependencies of Re are used to form equation 6.1.
- When varying temperature three out of four series show the expected trend related to initial deposition and asymptotic level. If focusing on these three the initial rate k_1 seems proportional to ΔT^+. The asymptotic level controlled by $k_{2,log}$ seems to be proportional with 1/ ΔT^+. This help form equation 6.2.

9. Nomenclature and Formulas

Q [m^3/h] – Flow rate

ID [m] – Inner diameter of flow pipe

A [m^2] – Inner area of flow pipe, $A = \frac{\pi}{4} ID^2$

u [m/s] – Flow velocity, $u = \frac{Q}{A}$

ρ [kg/m^3] – Density of gas condensate

μ [Pas] – Viscosity if gas condensate

Re – Reynolds number, dimensionless, $Re = \frac{\rho u ID}{\mu}$

T_c [°C] – Cloud point temperature, also known as WAT

T_w [°C] – Wall temperature

T_{cond} [°C] – Condensate temperature

ΔT^+ – Dimensionless deposition temperature driving force (Gudmundsson 2010), $\Delta T^+ = \frac{T_c - T_w}{T_{cond}}$

10. References

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Tables

Table 1: Porsgrunn wax rig data. Ref Rosvold (2008)

Parameter	Value
Oil Pipe, ID	52,58 mm
Oil Pipe, OD	60,56 mm
Water Pipe, ID	131,33 mm
Water Pipe, OD	0,1397 m
Tank Volume	300 l
Epoxy coating pipe diameter	51,7 mm
Differential pressure length	5,55 m
Water jacket length	5,31 m
Fluid	A waxy gas- condensate

Table 2: Specific values for varying rate experiments.

Rate of condensate [m^3/h], Q	**5**	**10**	**15**	**21**	**25**
Fluid velocity [m/s], u	0,66	1,32	1,98	2,78	3,31
Reynolds Number, Re	13836	27672	41508	58111	69179
Wall temperature[°C], Tw	11,9	12,7	13,5	14,4	15,0
Temperature driving force, ΔT+	0,91	0,87	0,83	0,78	0,75

Table 3: Constant values for varying rate experiments. Ref. Hoffmann and Amundsen (2010).

Constant rate parameters	Value
Condensate temperature, Tcond	20 °C
Oil density, ρ	809 kg/m3
Viscosity, μ	2,00E-03 Pas
WAT (Cloud point temperature),Tc	30 °C

Table 4: Specific values for varying temperature experiments. Wall temperature and viscosity from Hoffman and Amundsen (2010).

Condensate temperature [°C], Tcond	**15**	**20**	**30**	**40**
Wall temperature[°C], Tw	12,1	14,4	19,4	24,7
Temperature driving force, ΔT+	1,19	0,78	0,35	0,13
Viscosity [Pa s], μ	3,00E-03	2,00E-03	1,00E-03	9,00E-04
Reynolds Number, Re	38740	58111	116221	129135

Table 5: Constant values for varying temperature experiments. Ref. Hoffmann and Amundsen (2010).

Constant temperatur parameters	Value
Rate of condensate, Q	21 m3/h
Oil density, ρ	809 kg/m3
WAT (Cloud point temperature),Tc	30 °C

Table 6: Model constants for varying rate experiments.

Rate of condensate [m^3/h], Q	5	10	15	21	25
k_1	0,21	0,45	0,23	0,29	0,07
k_2 exponential model	0,22	0,50	0,34	0,48	0,18
k_2 logarithmic model	40	210	580	1050	2000

Table 7: Model constants for varying temperature experimenst.

Condensate temperature [°C], Tcond	15	20	30	40
k_1	0,07	0,29	0,03	0,01
k_2 exponential model	0,10	0,48	0,06	0,03
k_2 logarithmic model	80	1050	360	1000

Figures

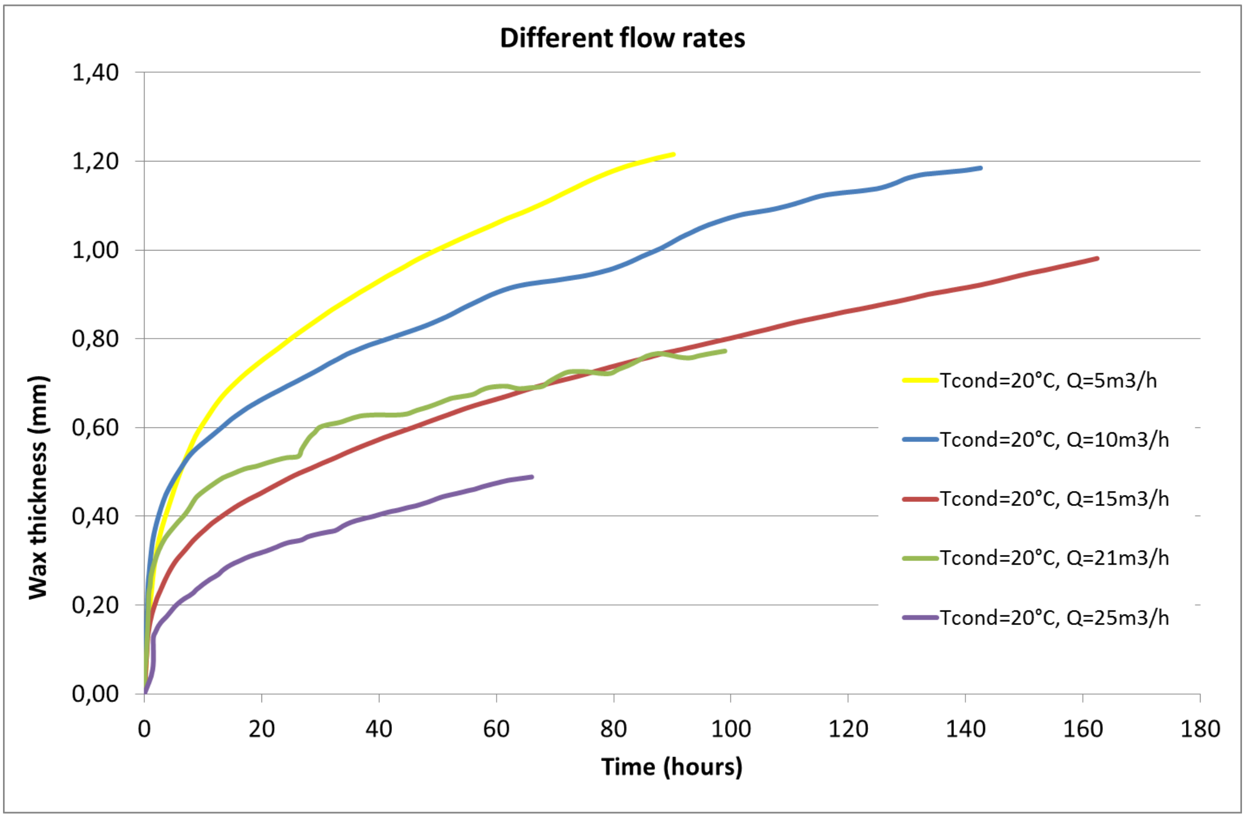

Figure 1: Wax thickness vs. time for digitized flow rate experiments.

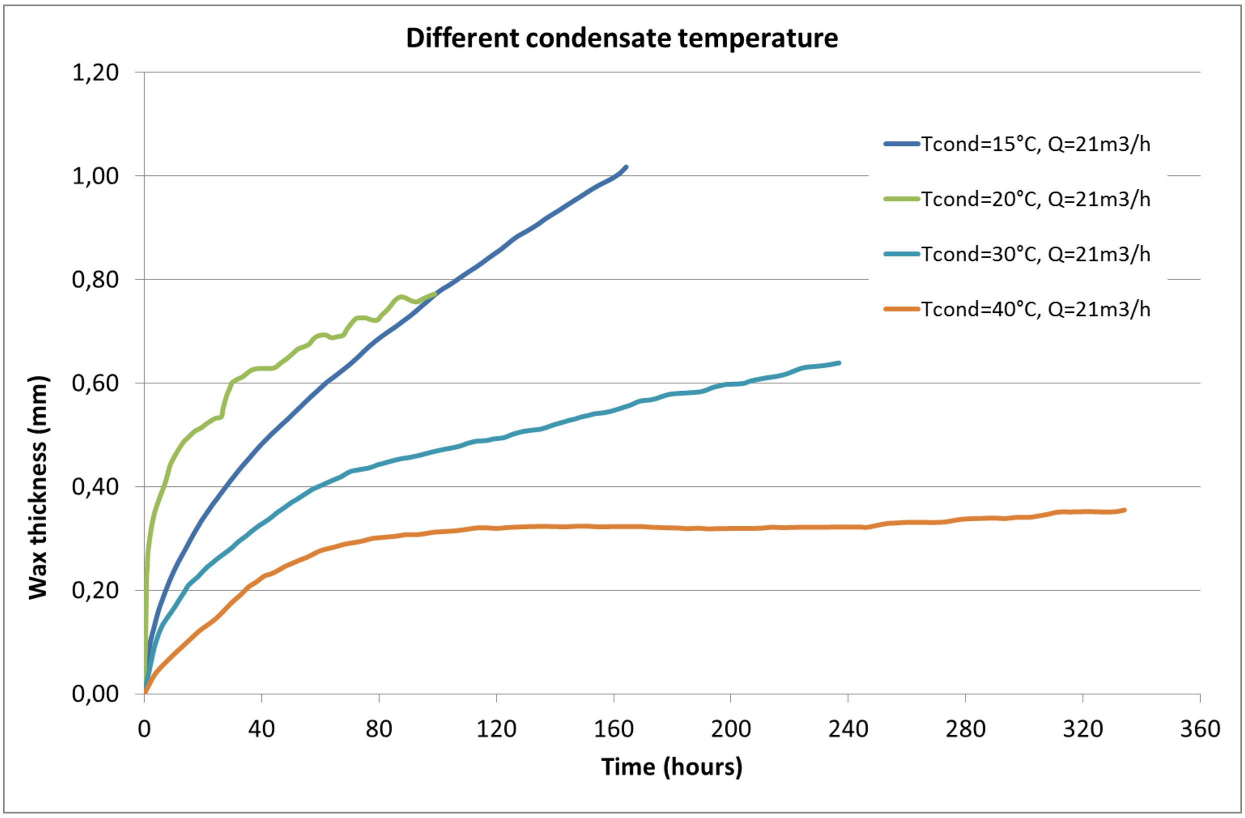

Figure 2: Wax thickness vs. time for digitized temperature experiments.

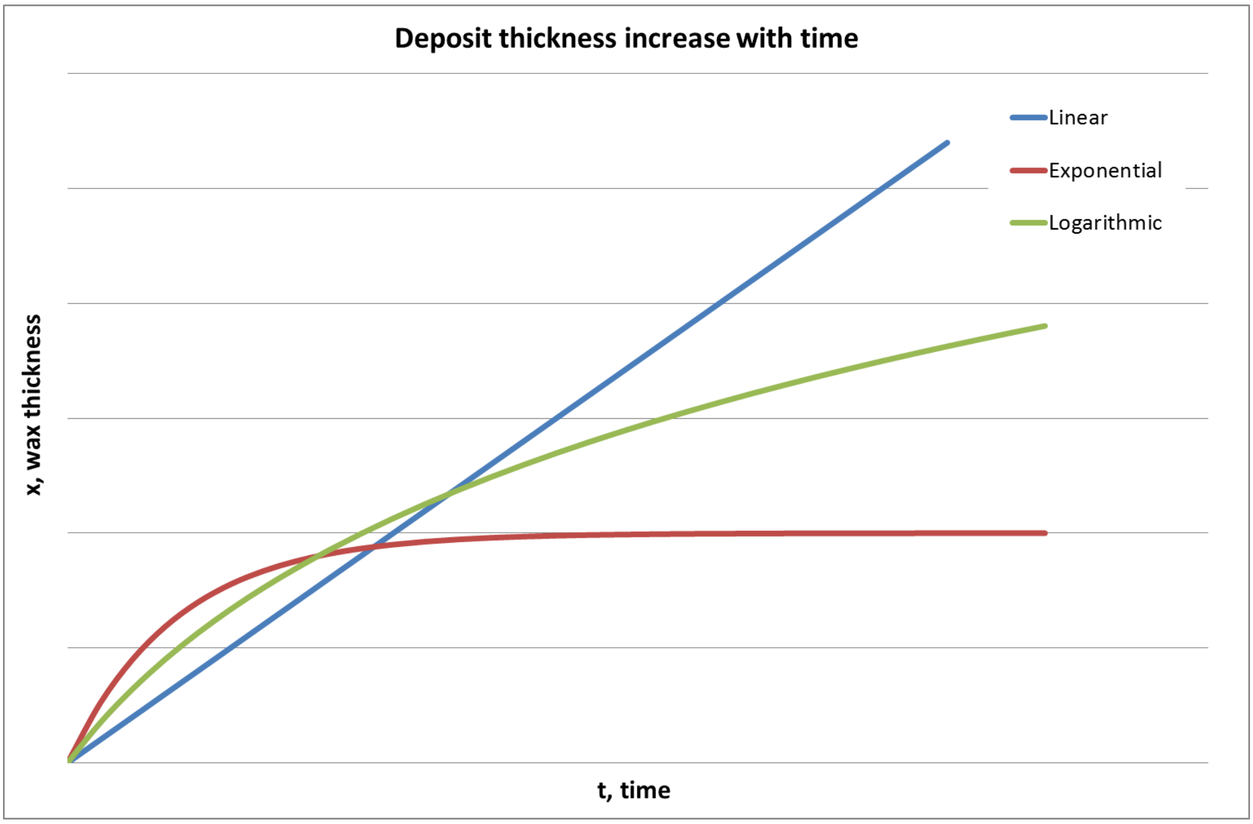

Figure 3: Possible deposition buildup trends.

Figure 4: Possible rate of deposition trends.

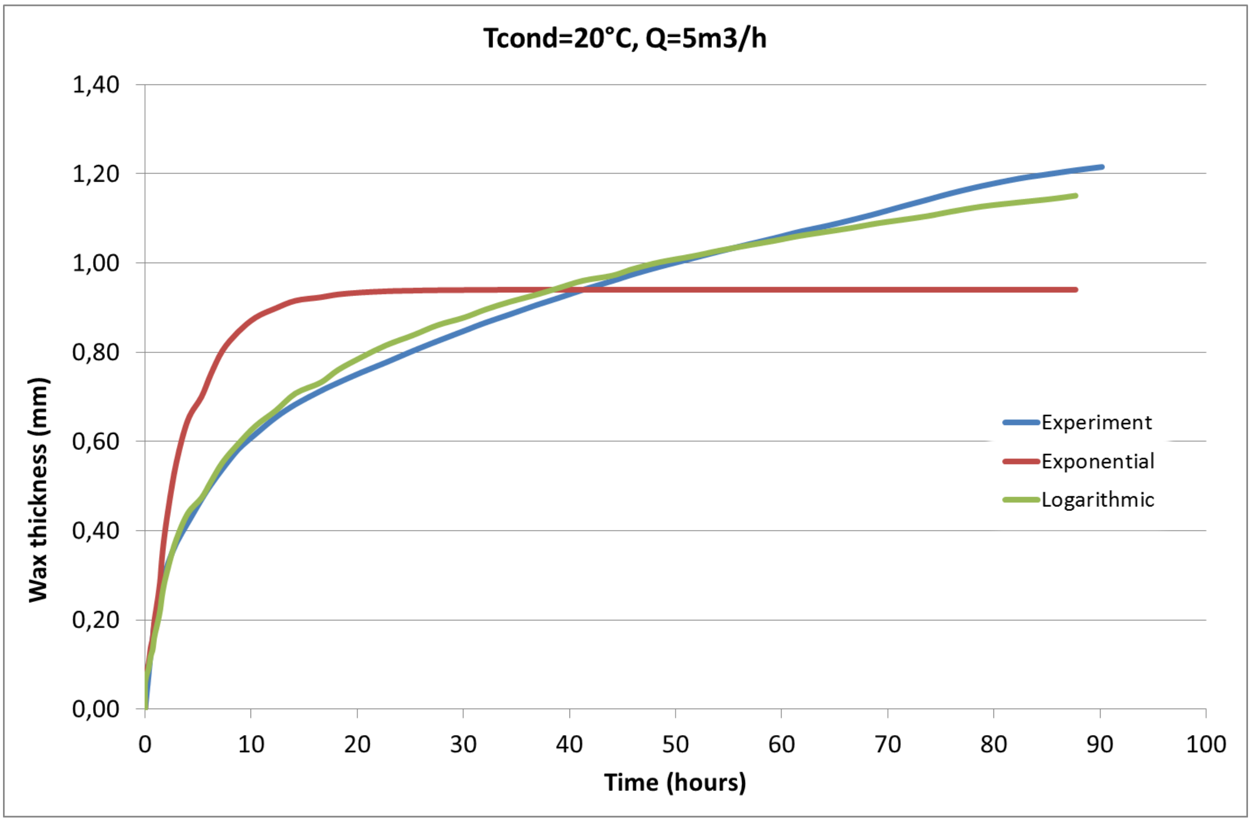

Figure 5: Matching models to experimental data.

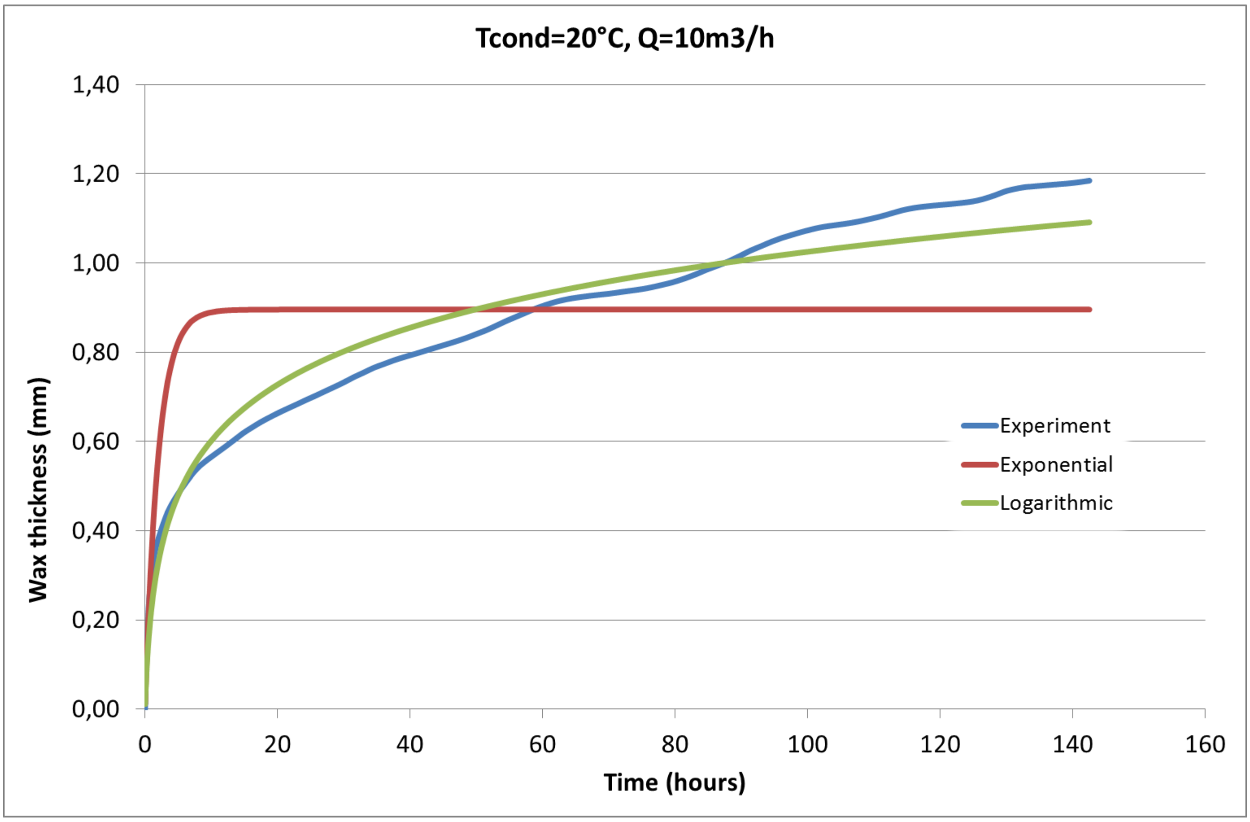

Figure 6: Matching models to experimental data.

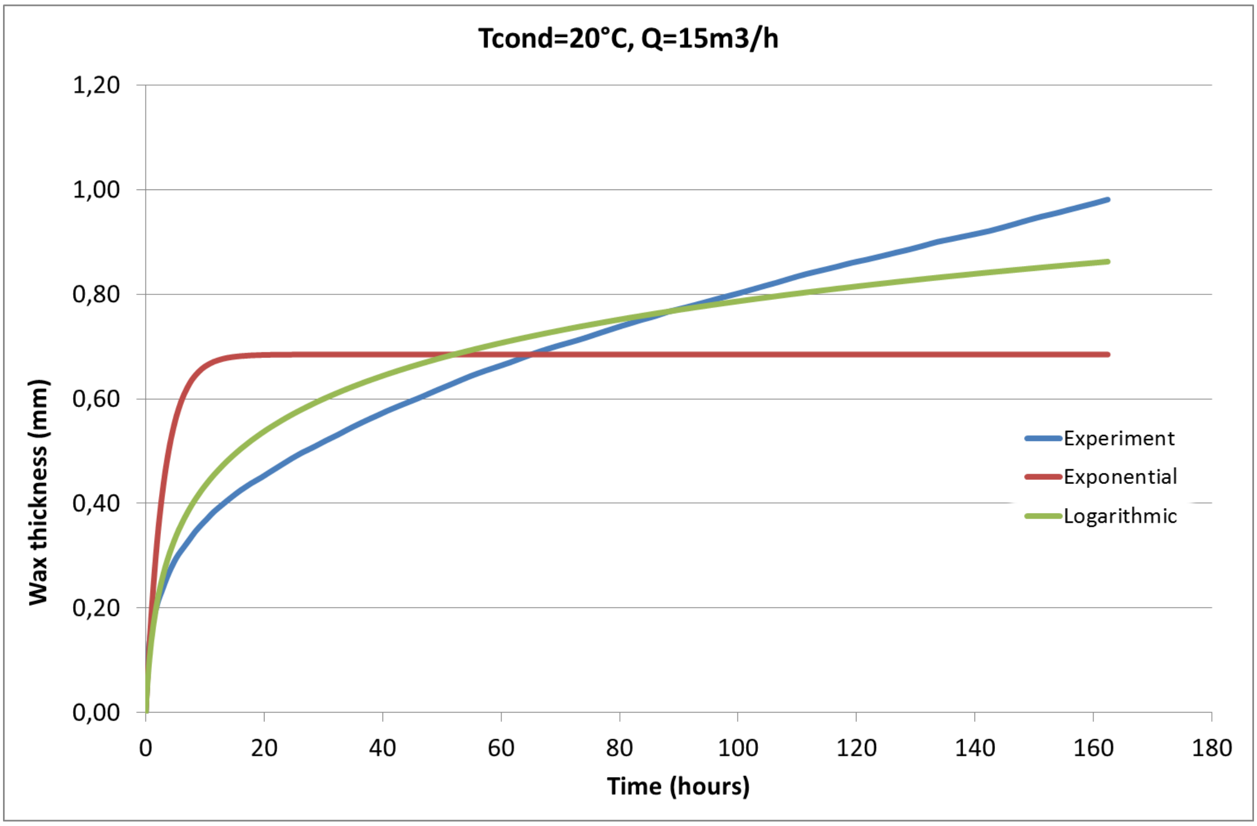

Figure 7: Matching models to experimental data.

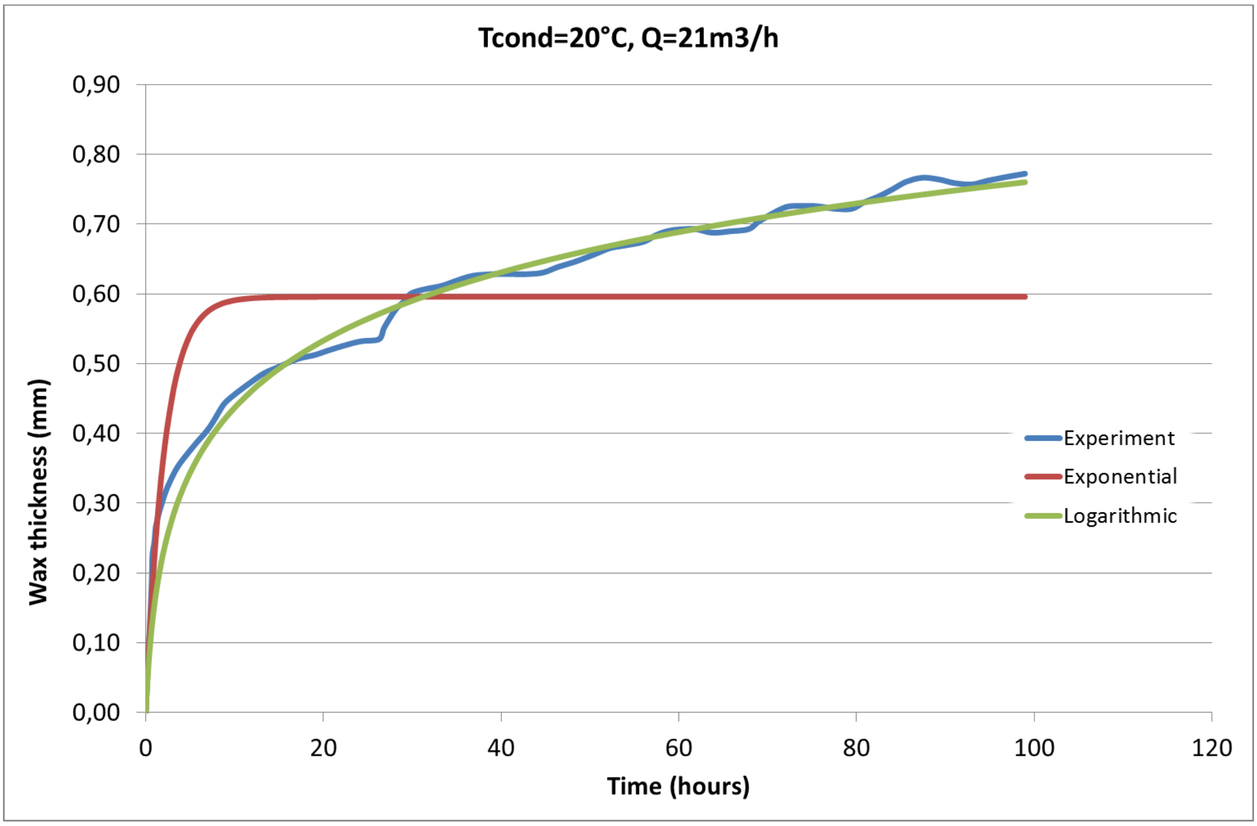

Figure 8: Matching models to experimental data.

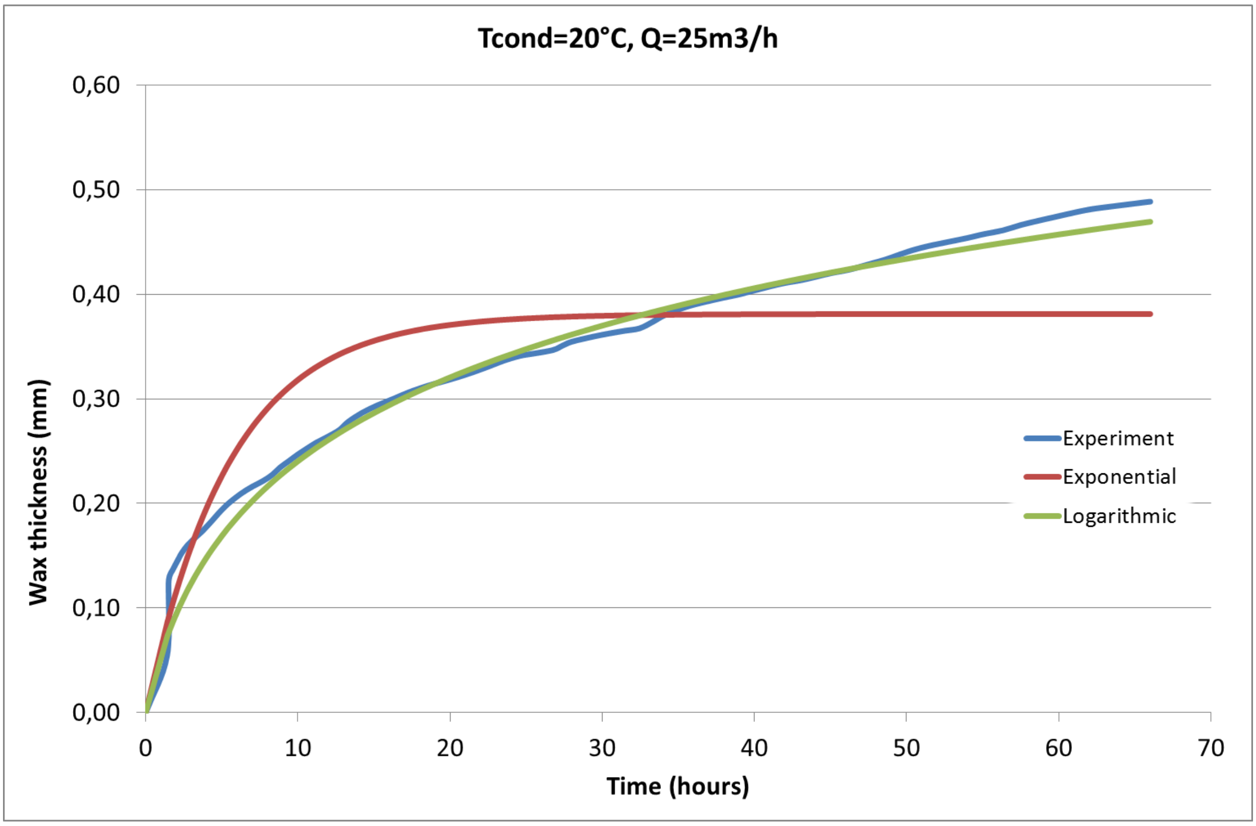

Figure 9: Matching models to experimental data.

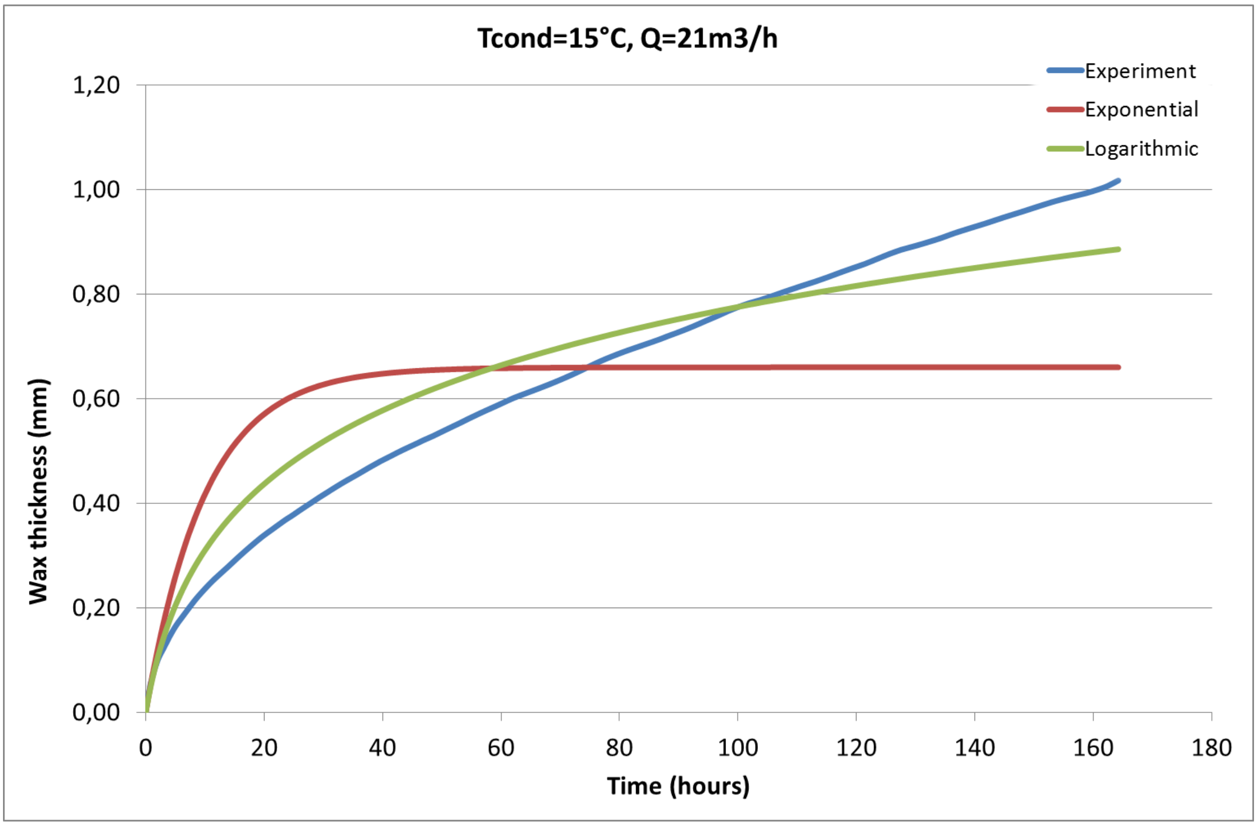

Figure 10: Matching models to experimental data.

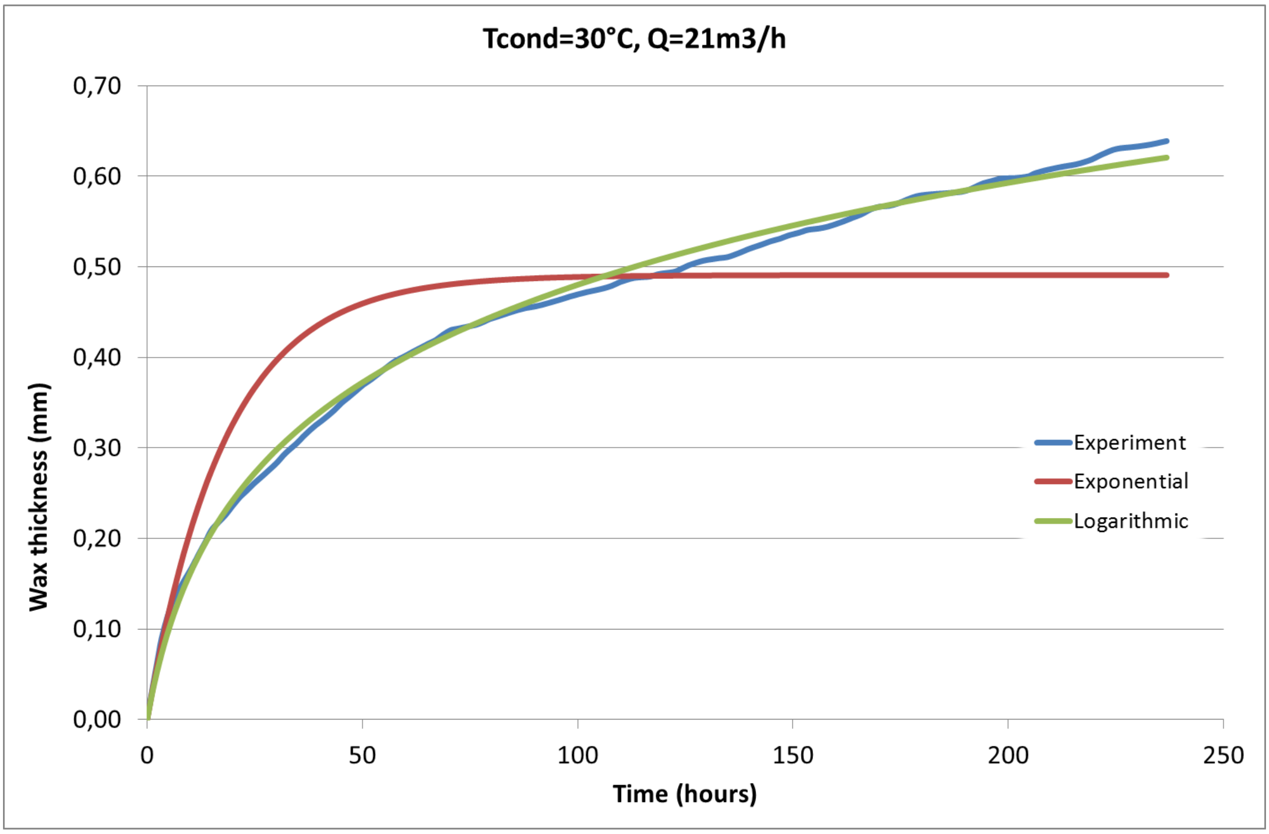

Figure 11: Matching models to experimental data.

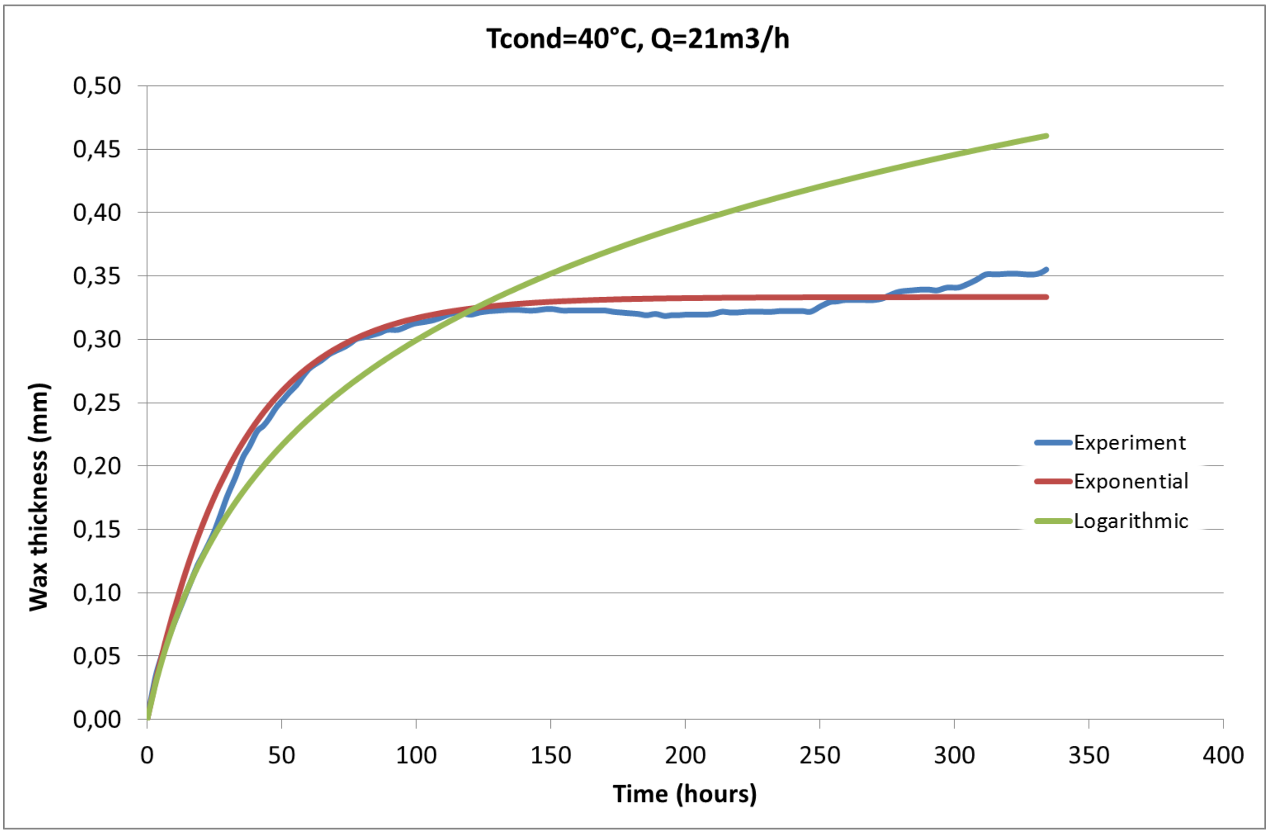

Figure 12: Matching models to experimental data.

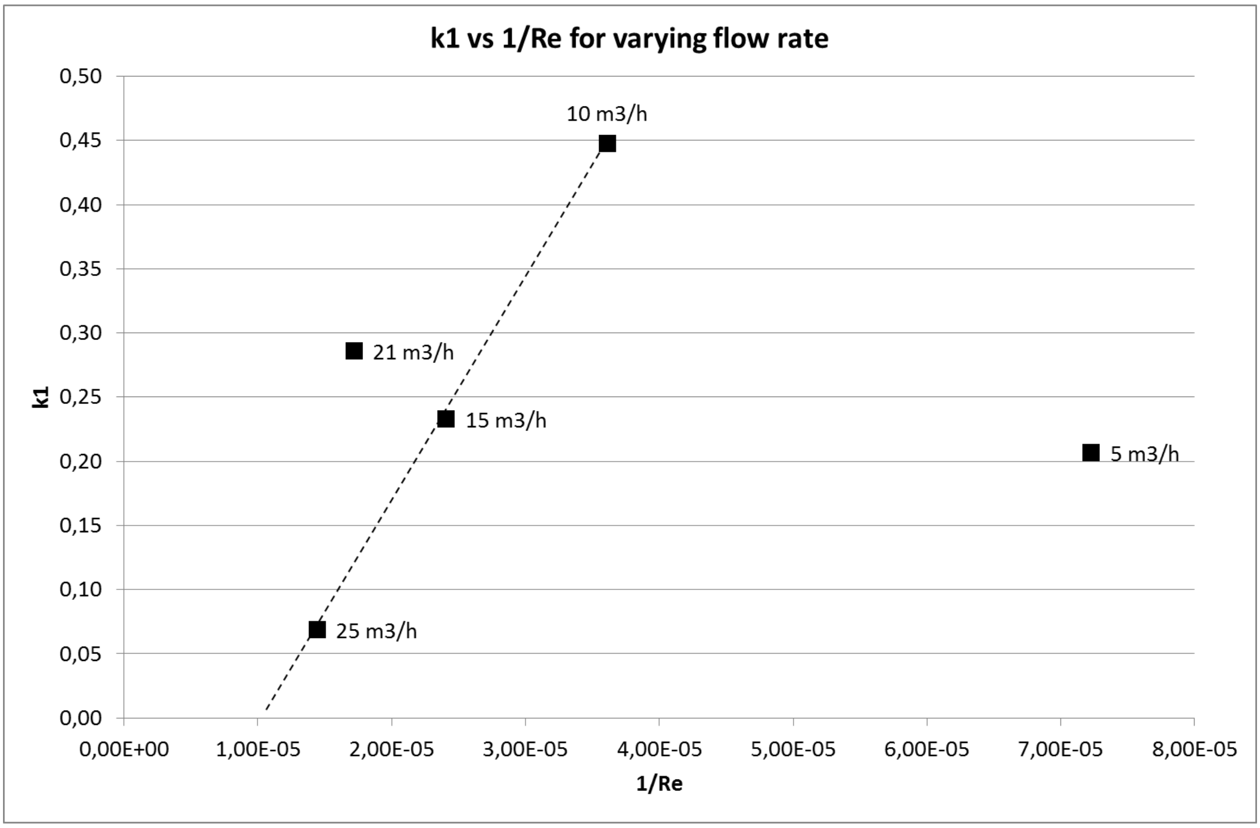

Figure 13: Constant k1 vs. 1/Re for varying flow rate series.

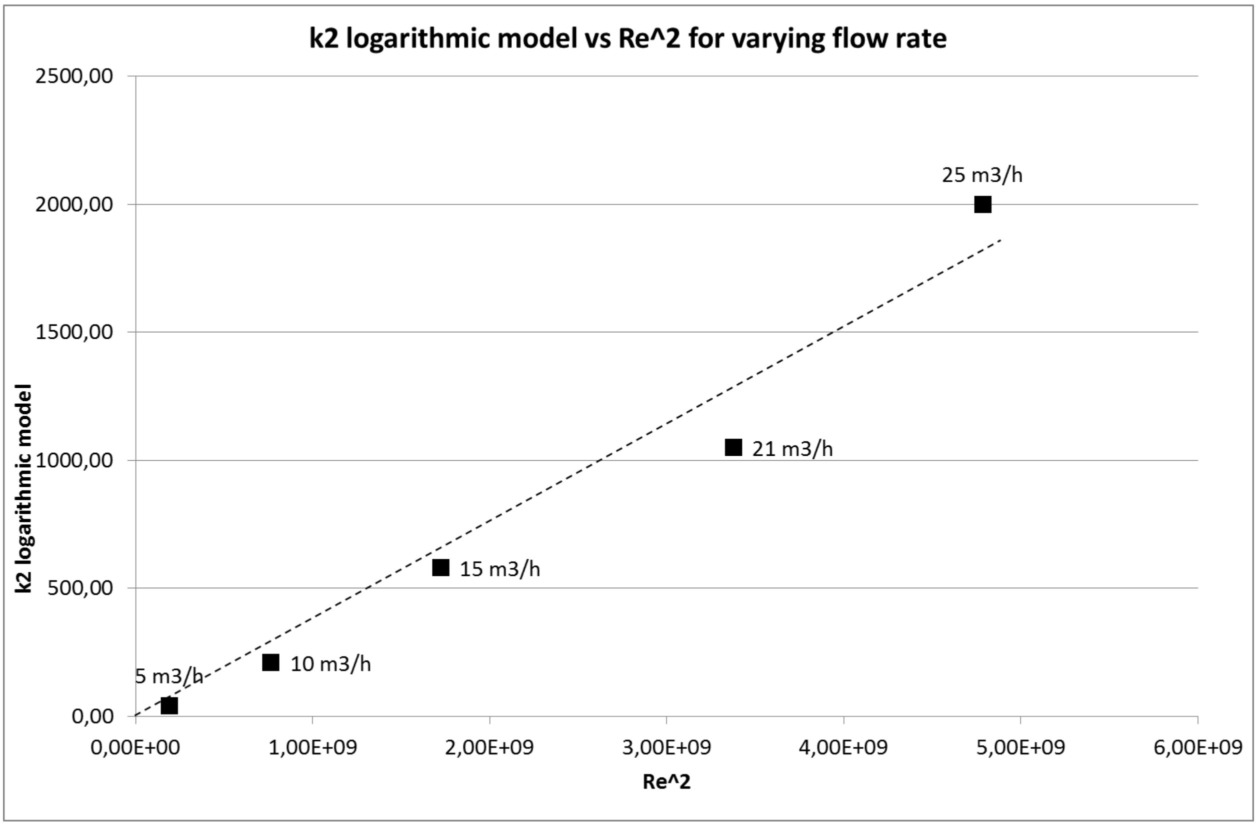

Figure 14: $k_{2,log}$ vs. Re^2 for varying flow rate.

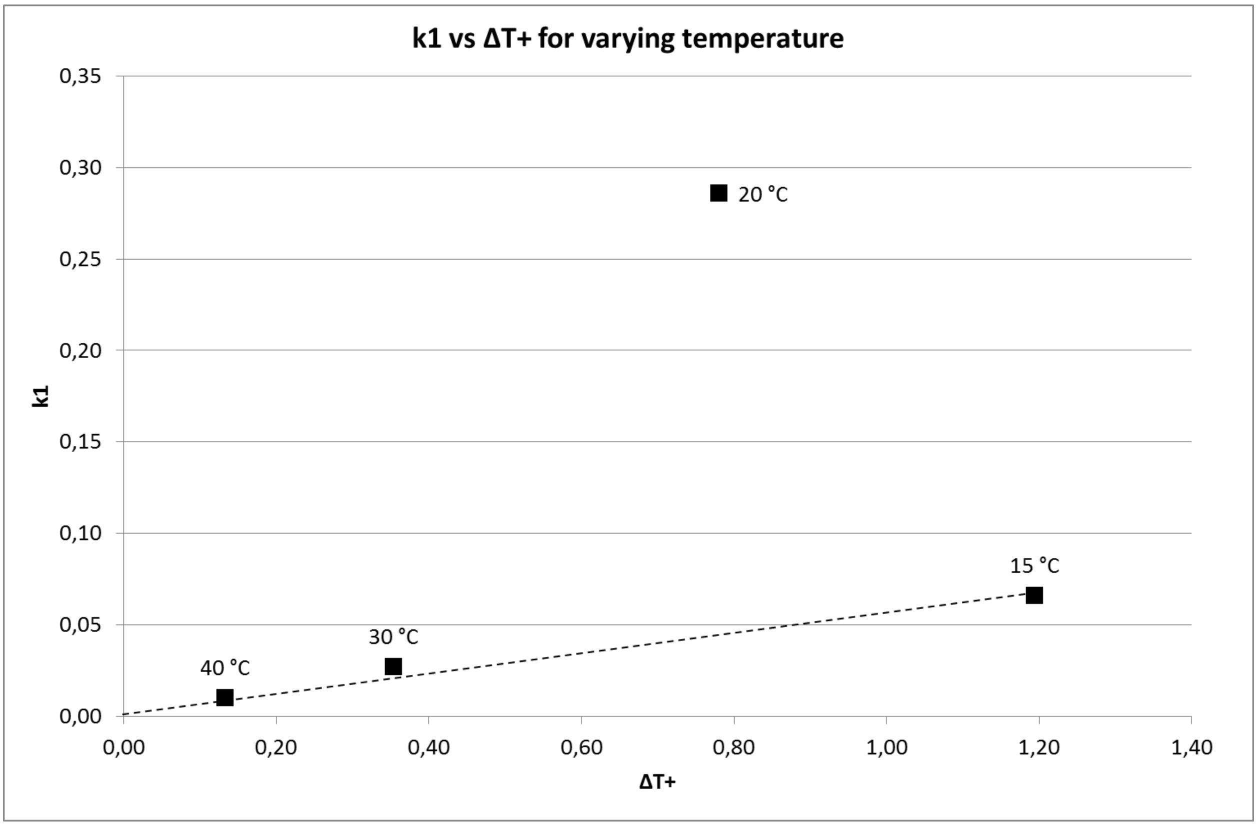

Figure 15: k_1 vs. ΔT^+ for varying temperature series.

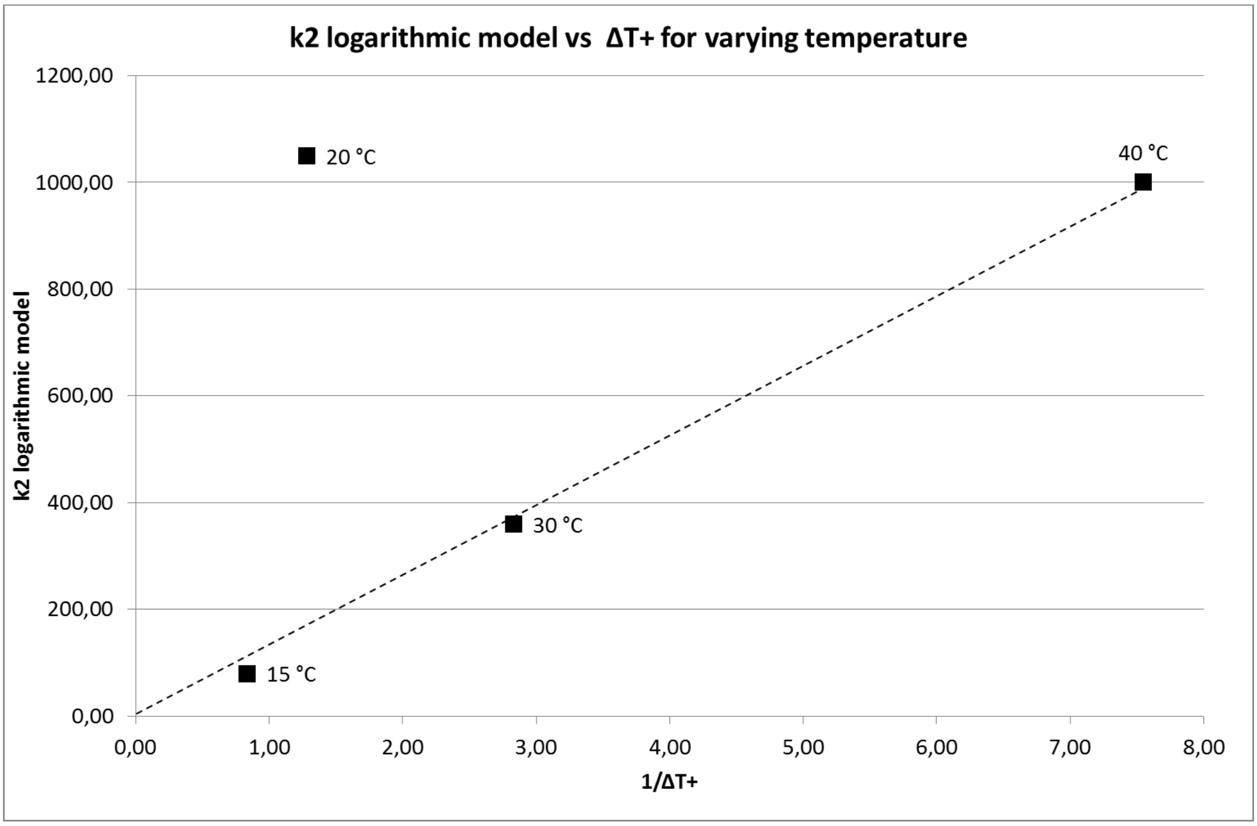

Figure 16: $k_{2,log}$ vs. $1/\Delta T^+$ for varying temperature series.

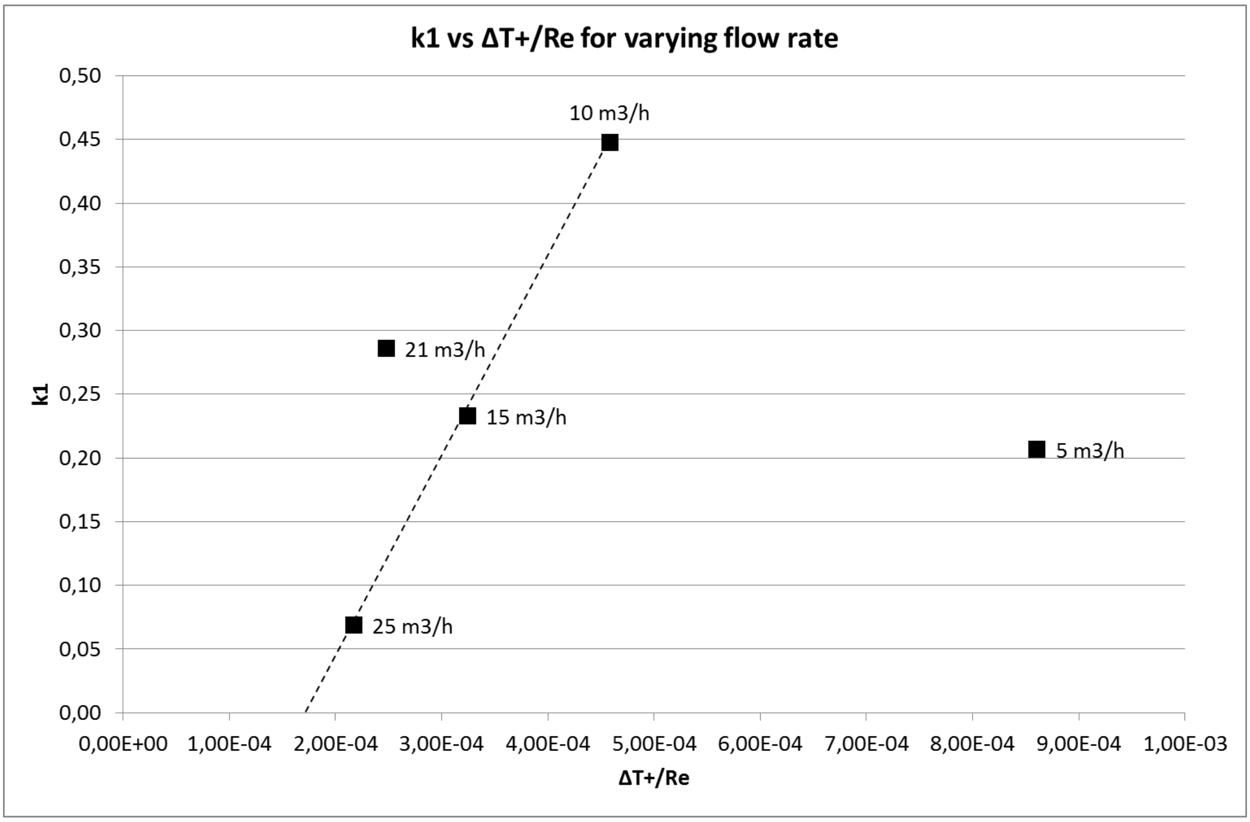

Figure 17: k1 vs. $\Delta T^{+}/Re$ for varying flow rate series.

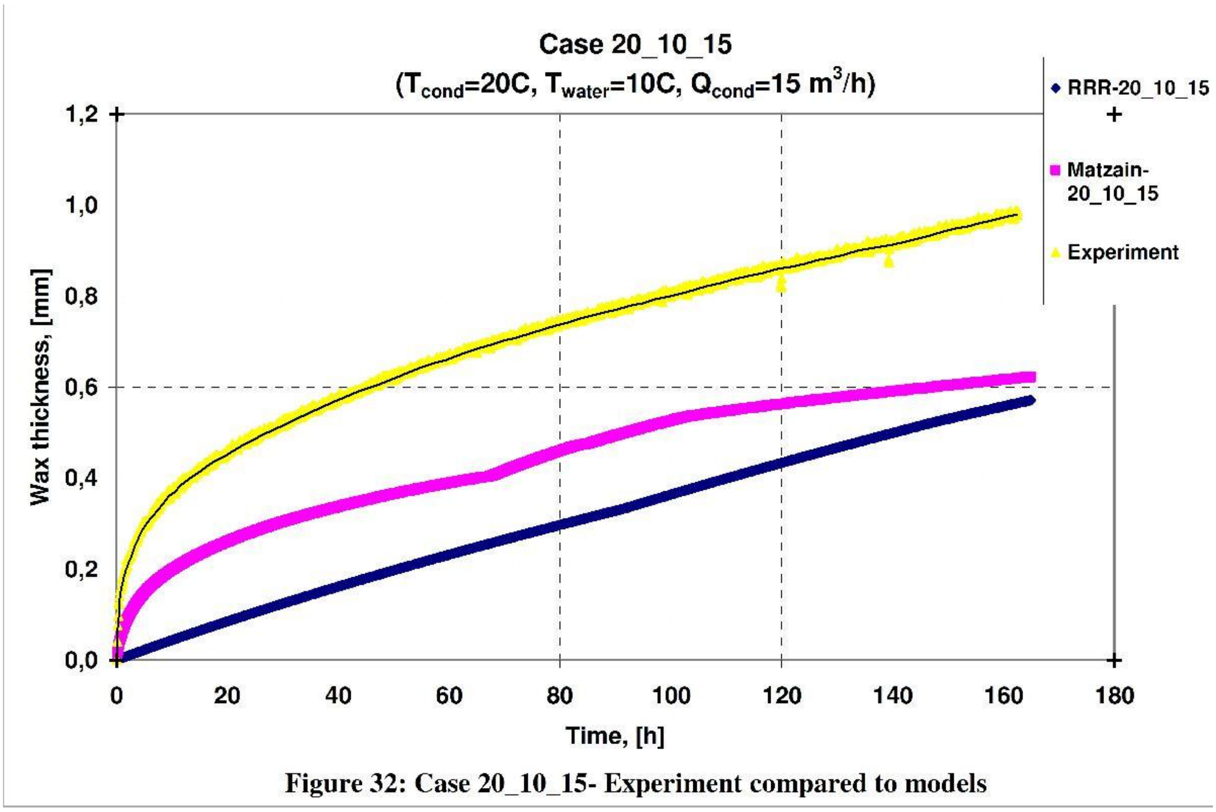

Figure 18: Successful digitizing of experiment (yellow and black line).

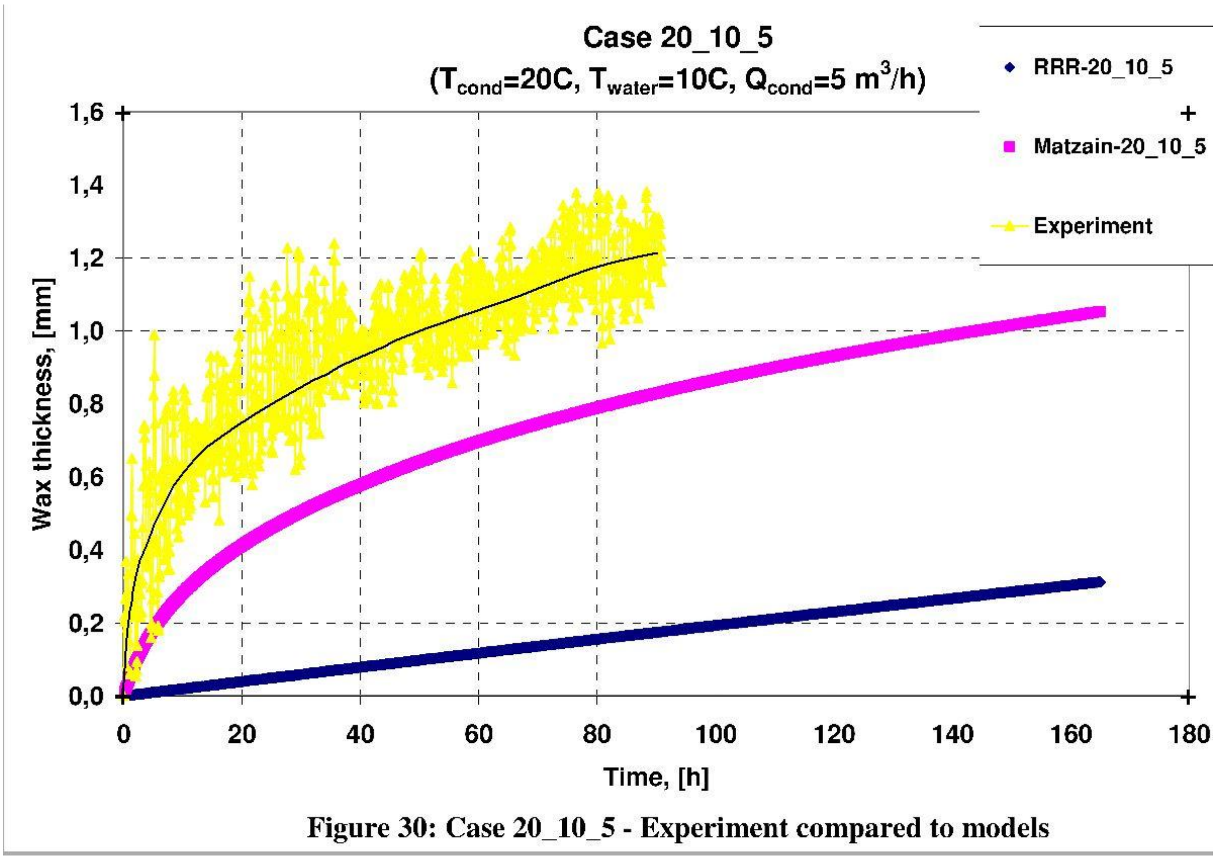

Figure 19: Experiment hard to digitize with one line (yellow and black line).

Appendix A: Digitizing graphical data

As mentioned before the experimental data was presented as graphs in Rosvold's (2008) MSc thesis. For me to utilize these data properly I needed to digitize them. To do this I used a software called Didger (ver. 2). Here is a short presentation on how this was done.

1. The thesis exists as a PDF file and plots from this file was saved as a picture file. The picture was then loaded into the Didger software.
2. First four points on each graph was manually selected as accurate as possible (using zoom). The coordinates for the selected point was typed in. For simplicity I used the four points (0, 0), (max x, 0), (0, max y) and (max x, max y). The value of max x and max y varied from graph to graph.
3. Using zoom the entire graph was identified as accurate as possible. Each graph was identified with 80 – 140 points. Any graphs with a big spread were digitized as good as possible by staying in the middle.
4. And output file was saved as a data file and opened in Microsoft Excel. This data file contained the x and y coordinates to all digitized points on a graph. The data was plotted using Excel and the plot was compared to the original from the thesis.

In total 8 plots were digitized. A very good match with 7 graphs was accomplished. One of the graphs (Q = 5 m3/h) had a big spread. The only thing I could do was to make a sort of an average line. A graph digitized accurately can be seen in Figure 18 and the difficult case can be seen in Figure 19.

In order to use this digitized data I had no other option than to trust the digitizing software. But the visual comparison between my plots and plots from the thesis was good.

Appendix B: Derivation of the exponential model:

$$\frac{dx}{dt} = k_1 - k_2 x$$

Rewrite

$$\frac{1}{k_1 - k_2 x} dx = dt$$

Integrate

$$\int_0^x \frac{1}{k_1 - k_2 x} dx = \int_0^t dt$$

Fill in

$$\left[\frac{-\ln(k_1 - k_2 x)}{k_2}\right]_0^x = [t]_0^t$$

Rewrite

$$\frac{-\ln(k_1 - k_2 x) + ln k_1}{k_2} = t$$

Rewrite

$$-\ln(k_1 - k_2 x) + ln\, k_1 = k_2 t$$

Rewrite

$$\ln(k_1 - k_2 x) = ln k_1 - k_2 t$$

Use exponential function

$$k_1 - k_2 x = \exp(ln\, k_1 - k_2 t)$$

Rewrite

$$k_2 x = k_1 - \exp(ln\, k_1 - k_2 t)$$

Rewrite

$$x = \frac{1}{k_2}[k_1 - \exp(ln\, k_1 - k_2 t)]$$

Rewrite

$$x = \frac{1}{k_2}\left[k_1 - \frac{\exp(ln\, k_1)}{\exp(k_2 t)}\right]$$

Rewrite

$$x = \frac{1}{k_2}\left[k_1 - \frac{k_1}{\exp(k_2 t)}\right]$$

Rewrite

$$x = \frac{k_1}{k_2}\left[1 - \frac{1}{\exp(k_2 t)}\right]$$

The solution becomes

$$x = \frac{k_1}{k_2}[1 - \exp(-k_2 t)]$$

Considering both x and dx/dt the following is obtained

$$x_\infty = \frac{k_1}{k_2} \; and \; \frac{dx}{dt}_{x=0} = k_1$$

Appendix C: Derivation of the logarithmic model:

$$\frac{dx}{dt} = k_1 {k_2}^{-x} \quad \left(= \frac{k_1}{{k_2}^x}\right)$$

Rewrite

$$\frac{{k_2}^x dx}{k_1} = dt$$

Integrate

$$\int_0^x \frac{{k_2}^x dx}{k_1} = \int_0^t dt$$

Fill in

$$\frac{1}{k_1}\left[\frac{{k_2}^x}{\ln k_2}\right]_0^x = [t]_0^t$$

Rewrite

$$\frac{{k_2}^x}{\ln k_2} - \frac{1}{\ln k_2} = k_1 t$$

Rewrite

$${k_2}^x - 1 = k_1 \ln k_2 \, t$$

Rewrite

$${k_2}^x = 1 + k_1 \ln k_2 \, t$$

Use natural logarithm (ln)

$$\ln {k_2}^x = \ln[1 + (k_1 \ln k_2)t]$$

Rewrite

$$x ln\, k_2 = \ln[1 + (k_1 \ln k_2)t]$$

The solution becomes

$$x = \frac{1}{ln\, k_2} \ln[1 + (k_1 \ln k_2)t]$$

Initial rate is given by

$$\frac{dx}{dt}_{x=0} = k_1$$

Appendix D: Estimating k_1 and k_2 in the models

This is a brief introduction to how I estimated the values for parameters k_1 and k_2 in each model. Both models have one parameter in common. The k_1 is the same for both models as shown in Appendix B and C.

$$\frac{dx}{dt}_{x=0} = k_1$$

Estimating k_1

Initial wax buildup is determined by k_1. It can be identified by drawing a tangent line to the start of a graph. I used Microsoft Excel and plotted the first 4-8 points as wax thickness vs. time. Then I used the trend line function through the origin and got an equation for the line. This equation was a linear function like y = ax. Then a [mm/h] is the k_1 value we are looking for. To ensure a good match the tangent was plotted against the entire graph. All tangents found had a good match to the experimental data (digitized data).

Estimating k_2

Exponential model

Values for k_2 in each model was found using tables from digitizing and values for k_1. The exponential function derived in Appendix B was used to estimate k_2.

$$x = \frac{k_1}{k_2}[1 - \exp(-k_2 t)]$$

This function or equation is not solvable for k_2 in a regular way. I used the solver function in Excel to help me with this. Paramter k_2 was set equal to 1 and wax thickness (x_{calc}) was calculated using time (t) and already estimated values of k_1. The solver function was then used in order to find the k_2 that would give $x_{calc} = x_{real}$.

In order to match k_2 with all points in the experimental table this method was applied to all points. A macro was used so that Excel would perform this procedure automatically on all points. This produced a different value for k_2 in each point. A small range of values was selected from this table and used to make a wax thickness plot. This plot was then compared to the experimental plot. The k_2 value with the best fit to the experimental plot was selected as the best match.

Logarithmic model

The logarithmic k_2 was estimated using the same procedure as for the exponential model. The only thing that was different in this case was the equation for wax thickness as a function of time.

$$x = \frac{1}{ln\, k_2} \ln[1 + (k_1 \ln k_2)t]$$

Other than that the procedure is exactly the same. A simple try and fail method from the start would probably produce the same result. But in order to save time I choose the procedure described here.