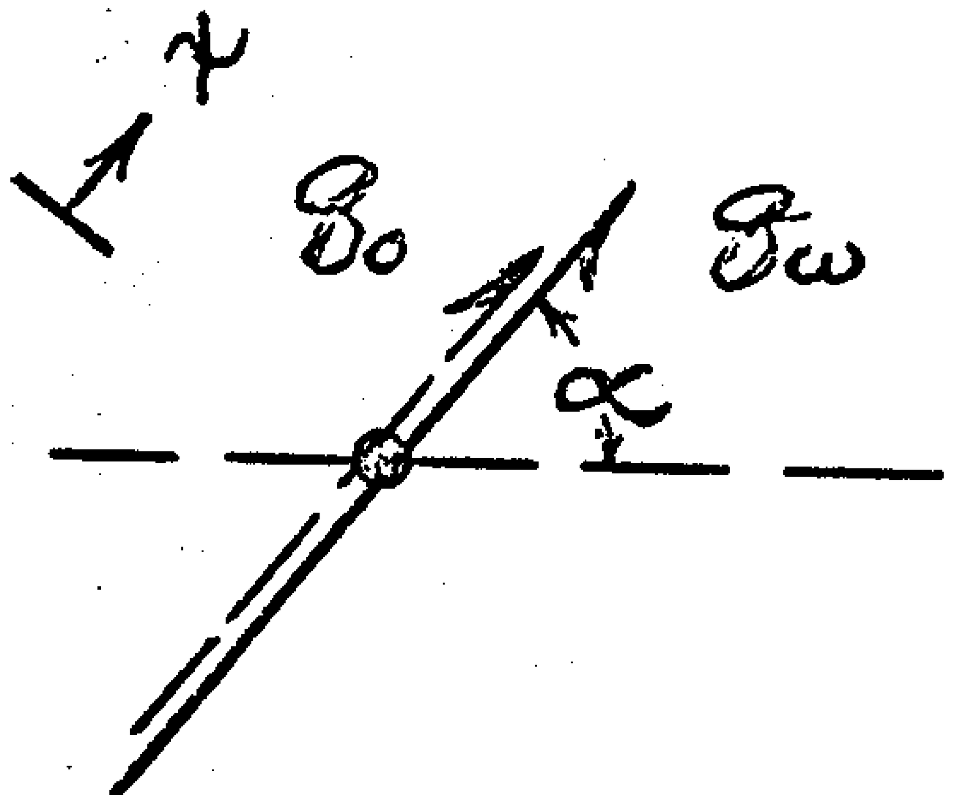
FRACTIONAL FLOW EQUATION, I



$$q_o = -\frac{k_o}{\mu_o}\left[\frac{\partial p_o}{\partial x} + \rho_o g \sin\alpha\right]$$

$$q_w = -\frac{k_w}{\mu_w}\left[\frac{\partial p_w}{dx} + \rho_w g \sin\alpha\right]$$

TOTAL FLOW, $q = q_o + q_w$

CAPILLARY PRESSURE, $P_c = p_o - p_w$

$$\frac{\partial P_c}{\partial x} = \frac{\partial p_o}{\partial x} - \frac{\partial p_w}{\partial x}$$

$$\frac{\partial P_c}{\partial x} = -\frac{(q - q_w)\mu_o}{k_o} - \rho_o g \sin\alpha + \frac{q_w \mu_w}{k_w} + \rho_w g \sin\alpha$$

LET $\Delta\rho = \rho_w - \rho_o$

$$q_w\left[\frac{\mu_o}{k_o} + \frac{\mu_w}{k_w}\right] = \frac{q\mu_o}{k_o} + \frac{\partial P_c}{\partial x} - \Delta\rho g \sin\alpha$$

FRACTIONAL FLOW EQUATION, II

DIVIDING BY $\frac{q\mu_o}{k_o}$;

$$\frac{q_w}{q}\left[1+\frac{k_o}{\mu_o}\cdot\frac{\mu_w}{k_w}\right]=1+\frac{k_o}{q\mu_o}\left[\frac{\partial P_c}{\partial x}-\Delta\rho g\sin\alpha\right]$$

$$\frac{q_w}{q}=f_w=\frac{1+\frac{k_o}{q\mu_o}\left[\frac{\partial P_c}{\partial x}-\Delta\rho g\sin\alpha\right]}{1+\frac{k_o}{k_w}\cdot\frac{\mu_w}{\mu_o}}$$

AS $k_o = k\cdot k_{ro}$ AND IGNORING $\frac{\partial P_c}{\partial x}$ BECAUSE SMALL

$$f_w=\frac{1-a k_{ro}}{1+\frac{k_{ro}}{k_{rw}}\cdot\frac{\mu_w}{\mu_o}}$$ WHERE

$$a=\frac{0.488\,k\,\Delta\rho\sin\alpha}{q\mu_o}$$ IF

$\Delta\rho$ = gm/cc
k = DARCY
q = BBL/DAY/FT^2
μ_o = cp

FRACTIONAL FLOW EQUATION, III

$$a = \frac{7.84(10^{-6})\,k\,\Delta\rho \sin\alpha}{q\,\mu_o}$$

IF $\Delta\rho = \frac{lbs}{ft^3}$

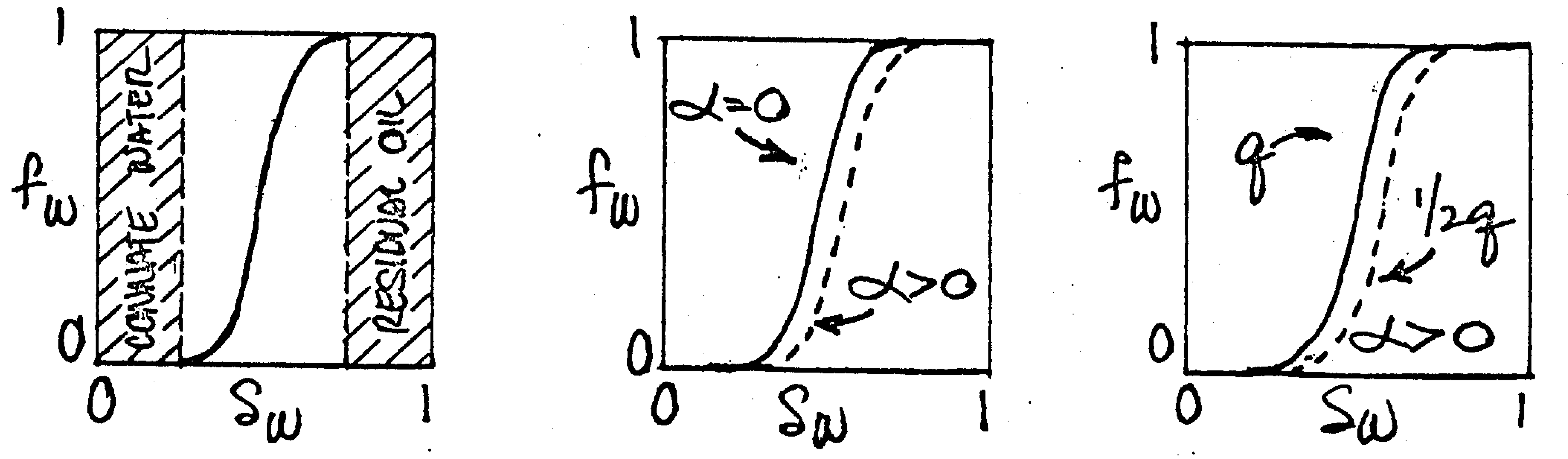
k = md.

q = BBL/DAY/FT^2

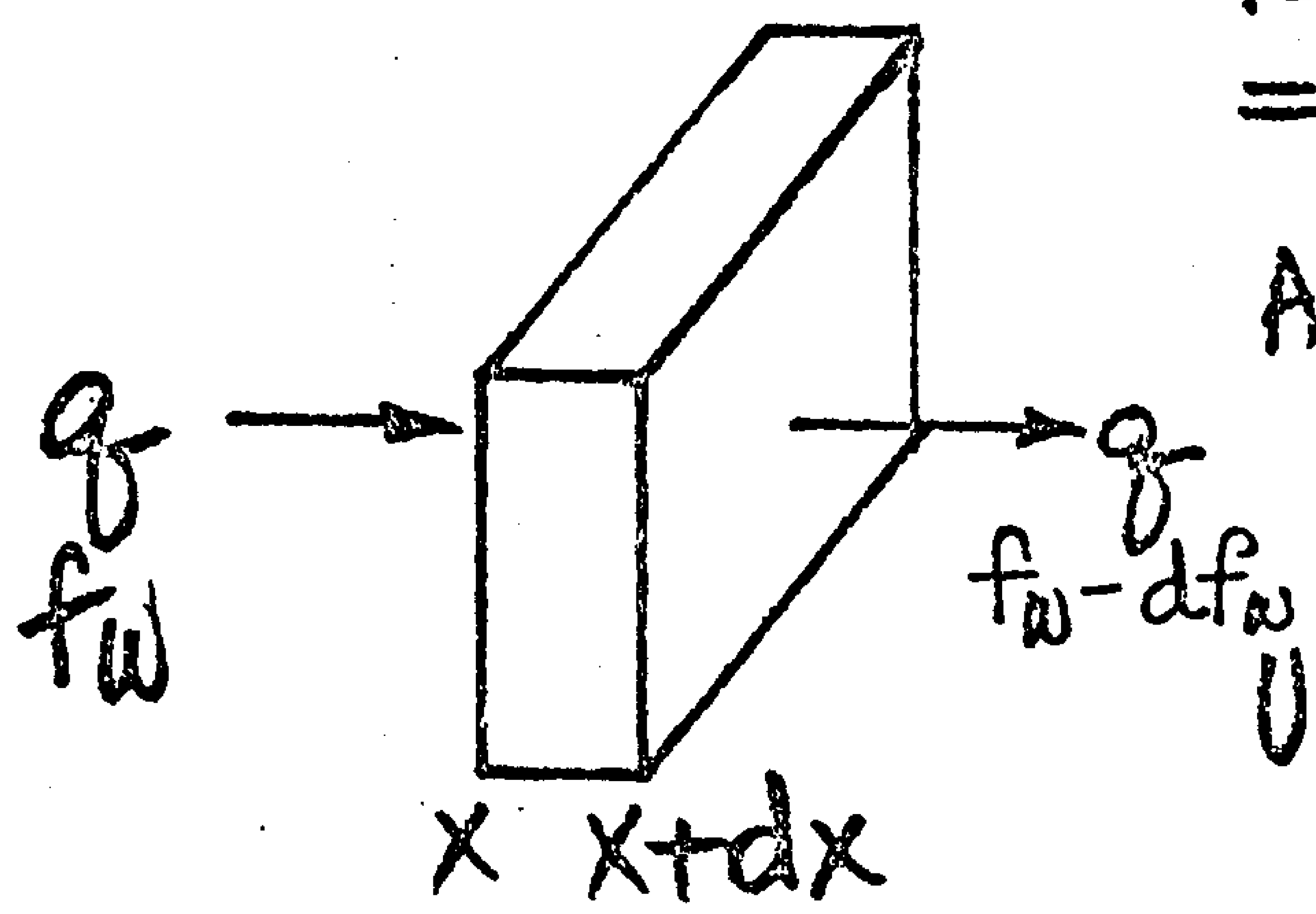
μ_o = cp.

WHEN $\alpha = 0$

$$f_w = \frac{1}{1 + \frac{k_{ro}}{k_{rw}} \cdot \frac{\mu_w}{\mu_o}}$$



FRONTAL ADVANCE EQUATION, I



RATE OF WATER ACCUMULATION
= WATER IN MINUS WATER OUT

$$\text{ACCUM. RATE} = q f_w - q(f_w - df_w)$$
$$= q\, df_w$$

$$\text{UNIT PORE VOL.} = \frac{A \cdot dx \cdot \phi}{5.615}$$

RATE OF WATER SATURATION CHANGE, $\frac{dS_w}{dt}$, IS RATE OF WATER ACCUMULATION DIVIDED BY PORE VOLUME.

$$\frac{dS_w}{dt} = \frac{q\, df_w}{A\, dx\, \phi} \cdot 5{,}615$$

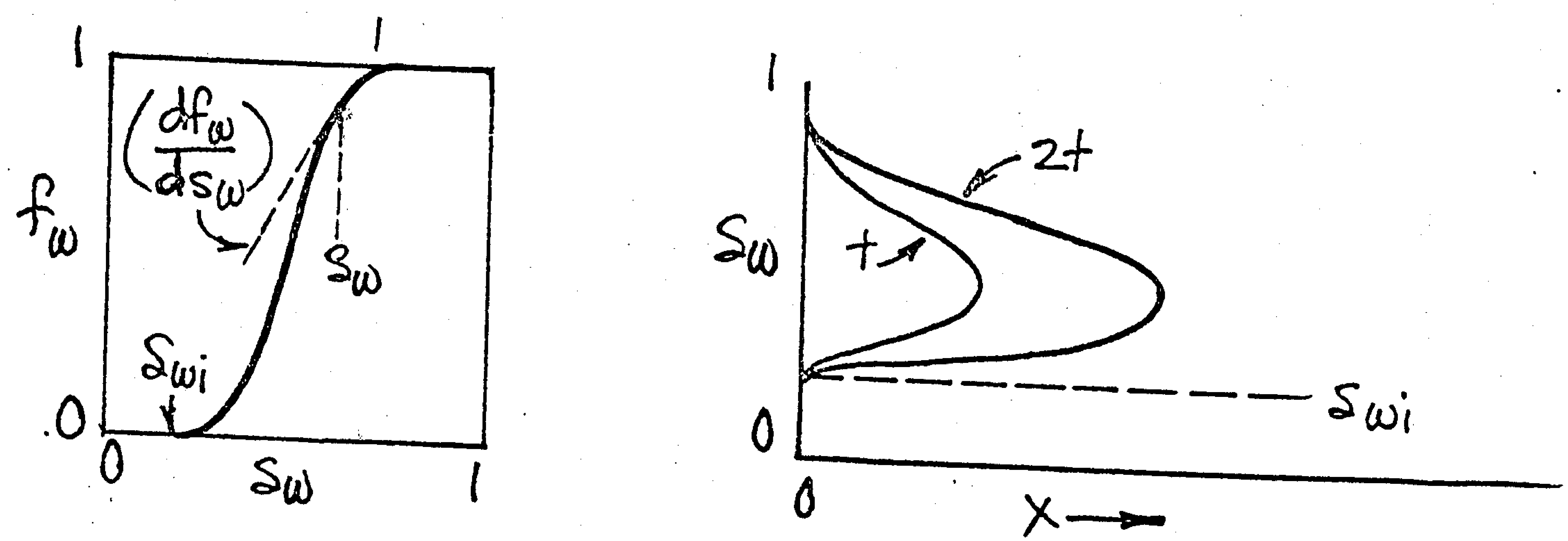
$$dx = \frac{5.615\, q}{A \phi} \left(\frac{df_w}{dS_w}\right) dt$$

$$\int_{x_0}^{x_{S_w}} dx = 5.615 \cdot \frac{q}{A\phi} \left(\frac{df_w}{dS_w}\right) \int_{t=0}^{t} dt$$

FRONTAL ADVANCE EQUATION, II

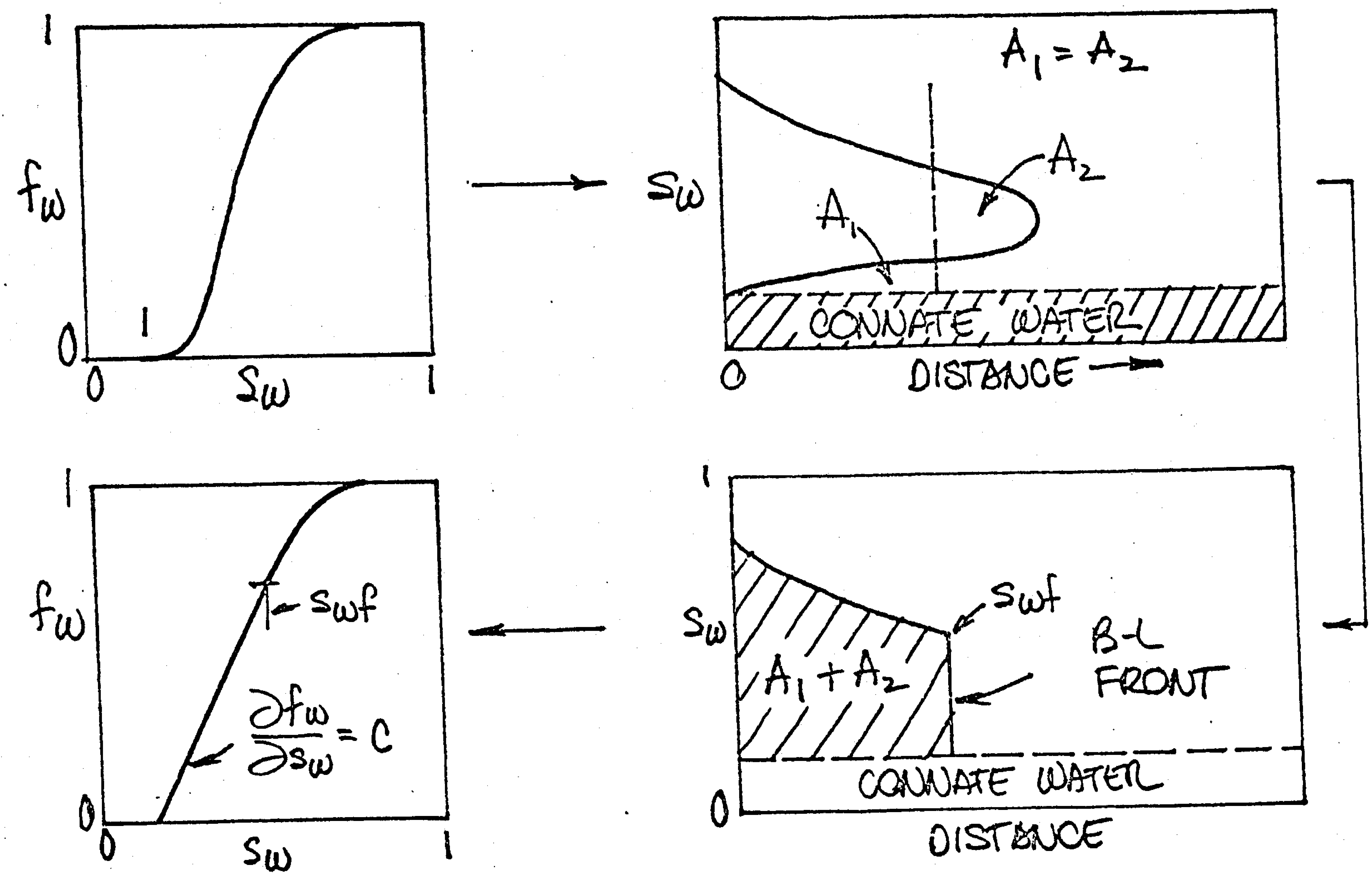
$$(x)_{sw} - X_o = \frac{5.615}{A\phi}\, qt \left(\frac{df_w}{ds_w}\right)$$

THIS IS THE BUCKLEY-LEVERETT FRONTAL ADVANCE EQUATION. IT GIVES THE DISTANCE, $[(x)_{sw} - X_o]$, THAT A GIVEN SATURATION, s_w, MOVES IN TIME t.



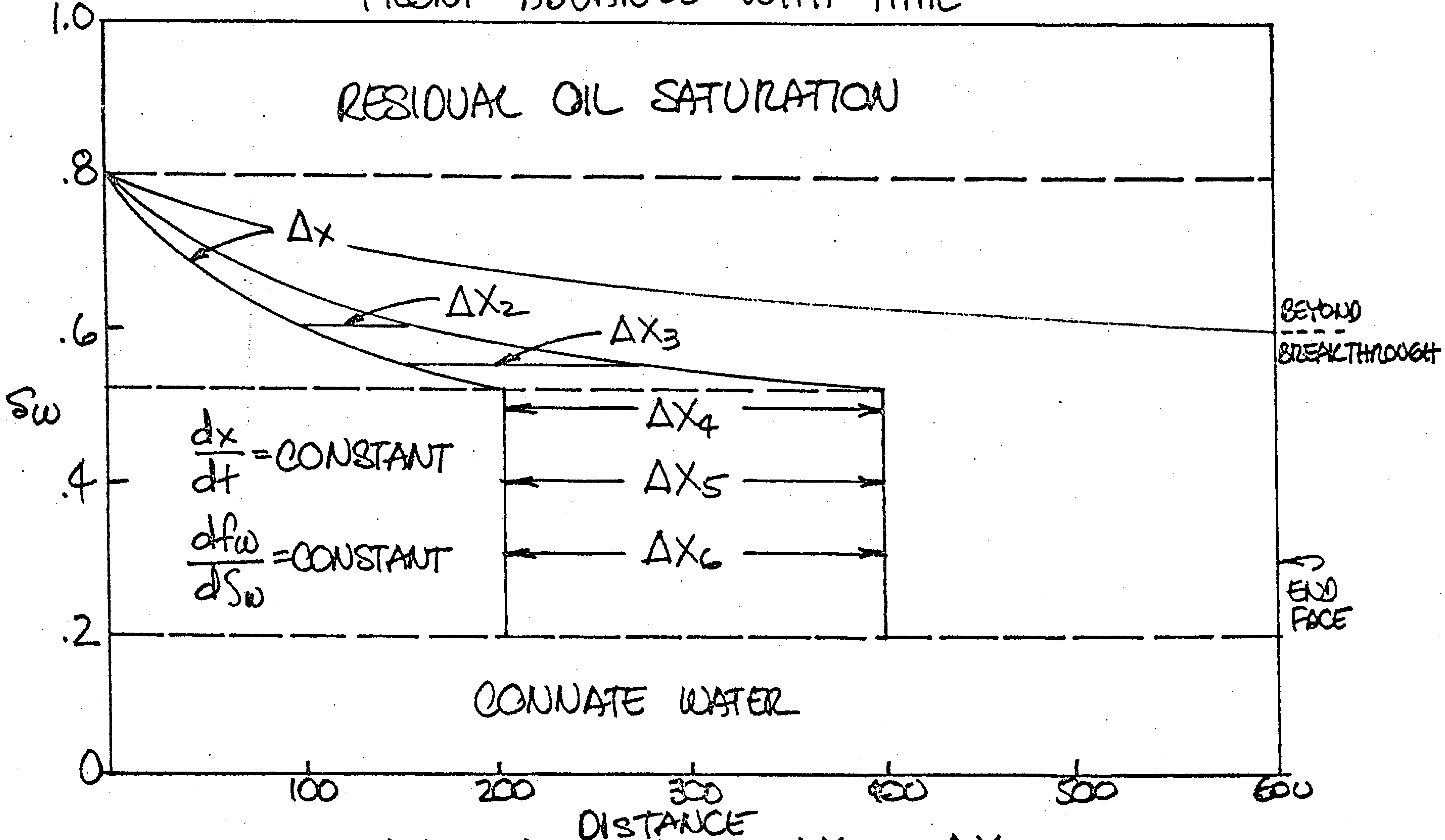
FRONTAL ADVANCE EQUATION, III

MODIFICATION OF B-L RELATIONSHIP TO AVOID TRIPLE SATURATION VALUES.



FRONTAL ADVANCE EQUATION, IV

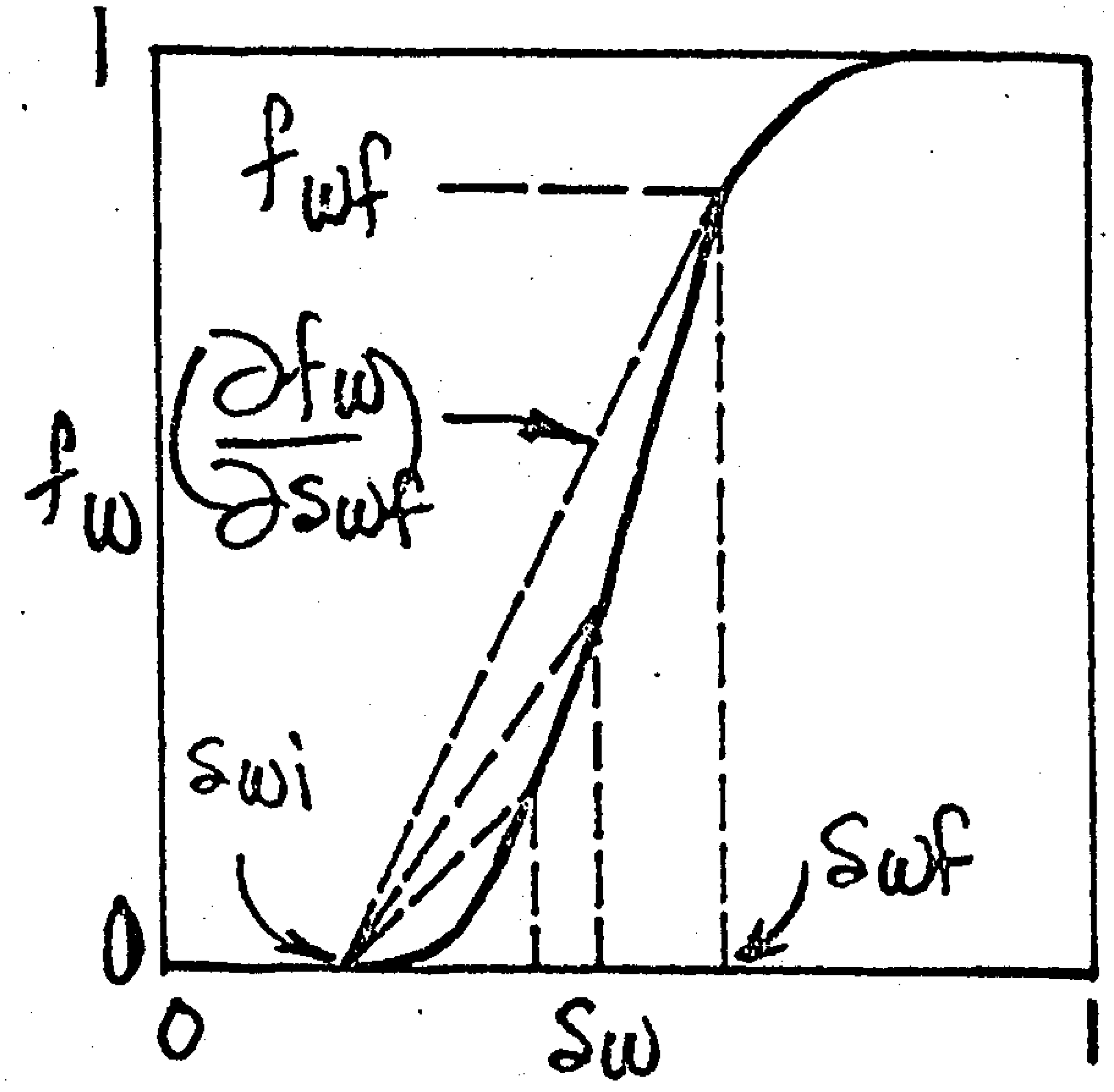
FRONT ADVANCE WITH TIME



$$\Delta X_1 < \Delta X_2 < \Delta X_3 < \Delta X_4 = \Delta X_5 = \Delta X_6$$

FRONTAL ADVANCE EQUATION V

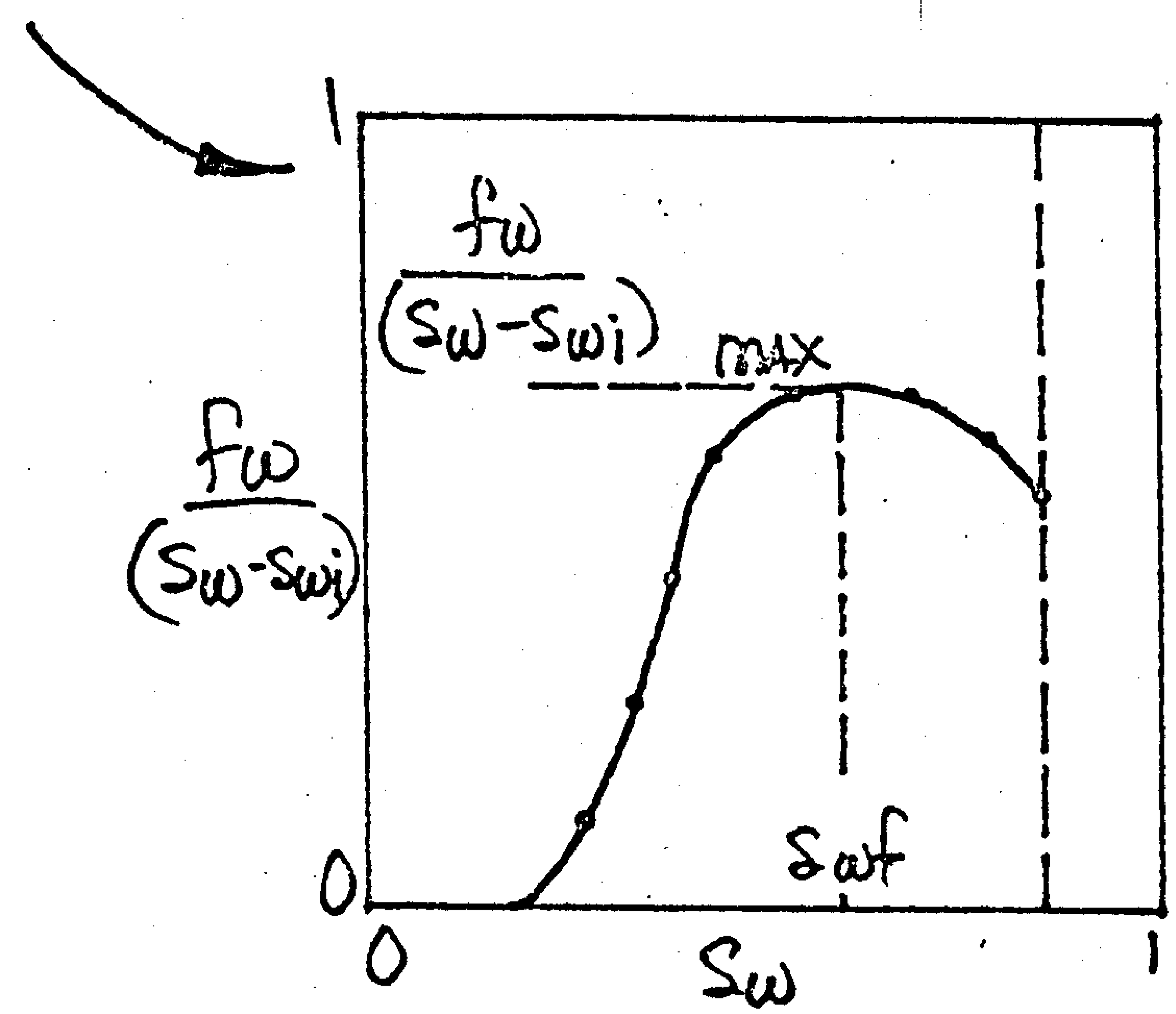
DETERMINATION OF SATURATION AND FRACTIONAL FLOW AT THE FRONT.



CALCULATE $\frac{f_w}{(S_w - S_{wi})}$ AND PLOT AGAINST S_w.

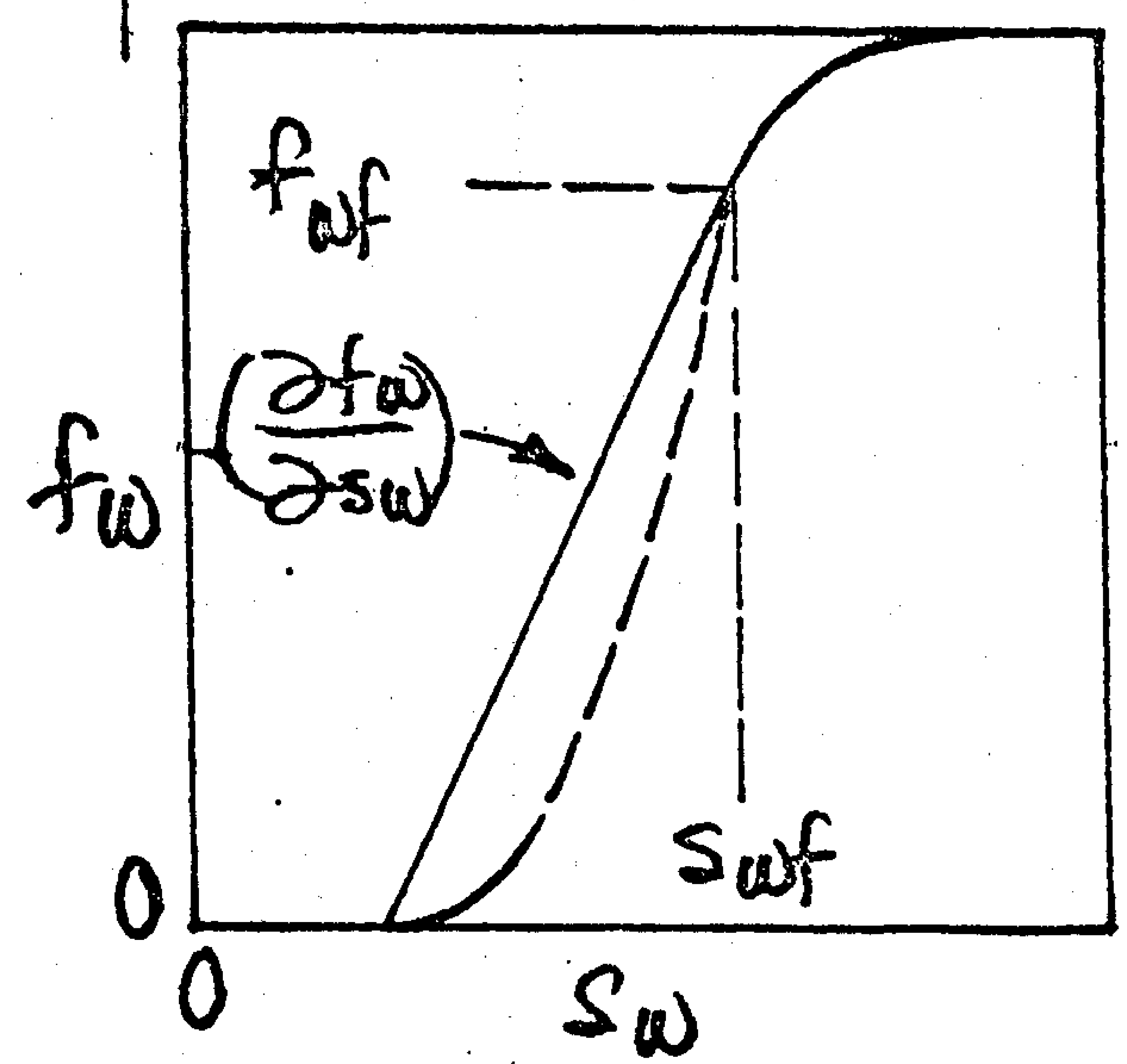
FRONT CONDITIONS ARE WHERE THIS HAS THE MAXIMUM VALUE.

ALSO CAN BE OBTAINED GRAPHICALLY



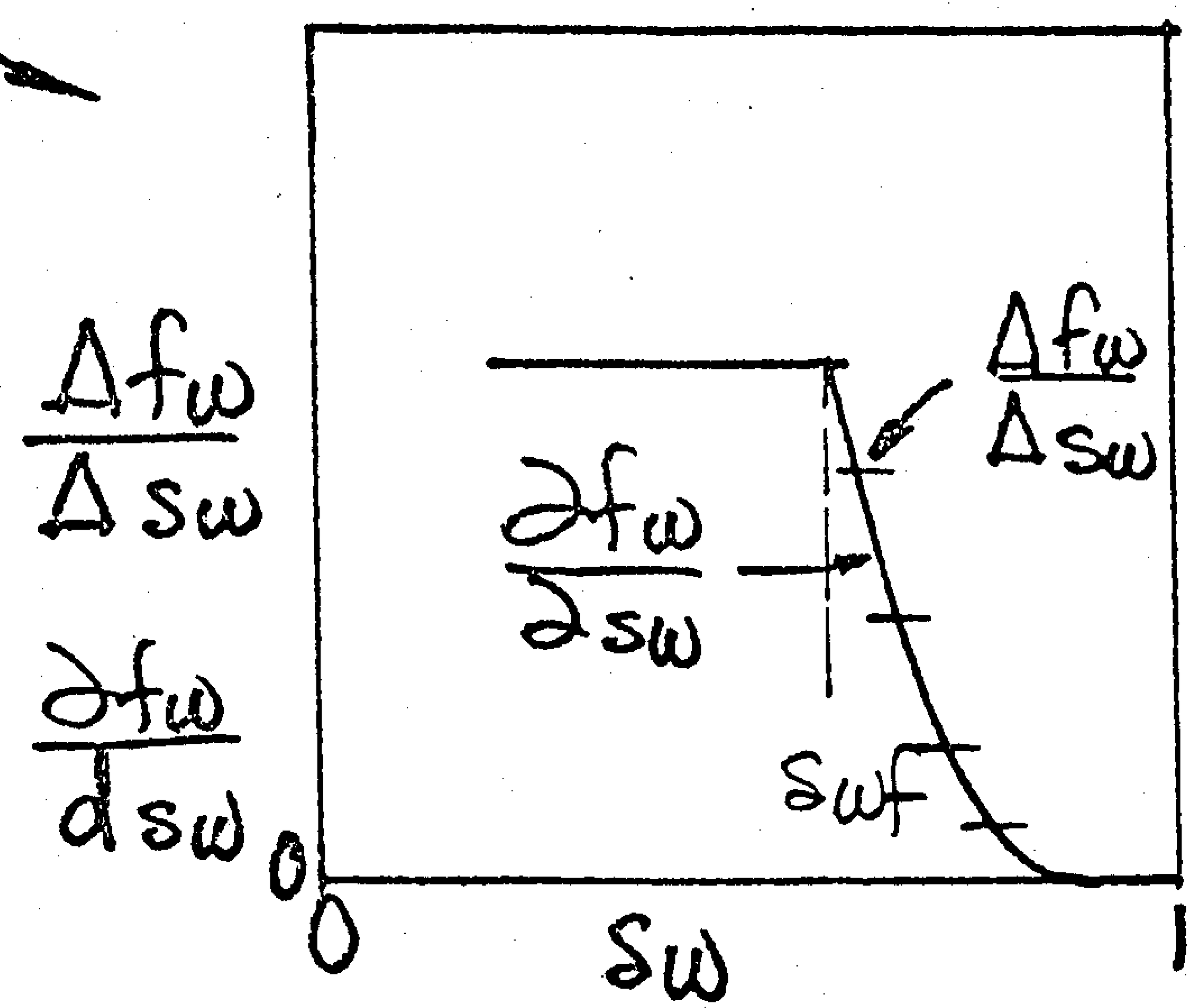
FRONTAL ADVANCE EQUATION VI

DETERMINATION OF $\frac{\partial f_w}{\partial s_w}$

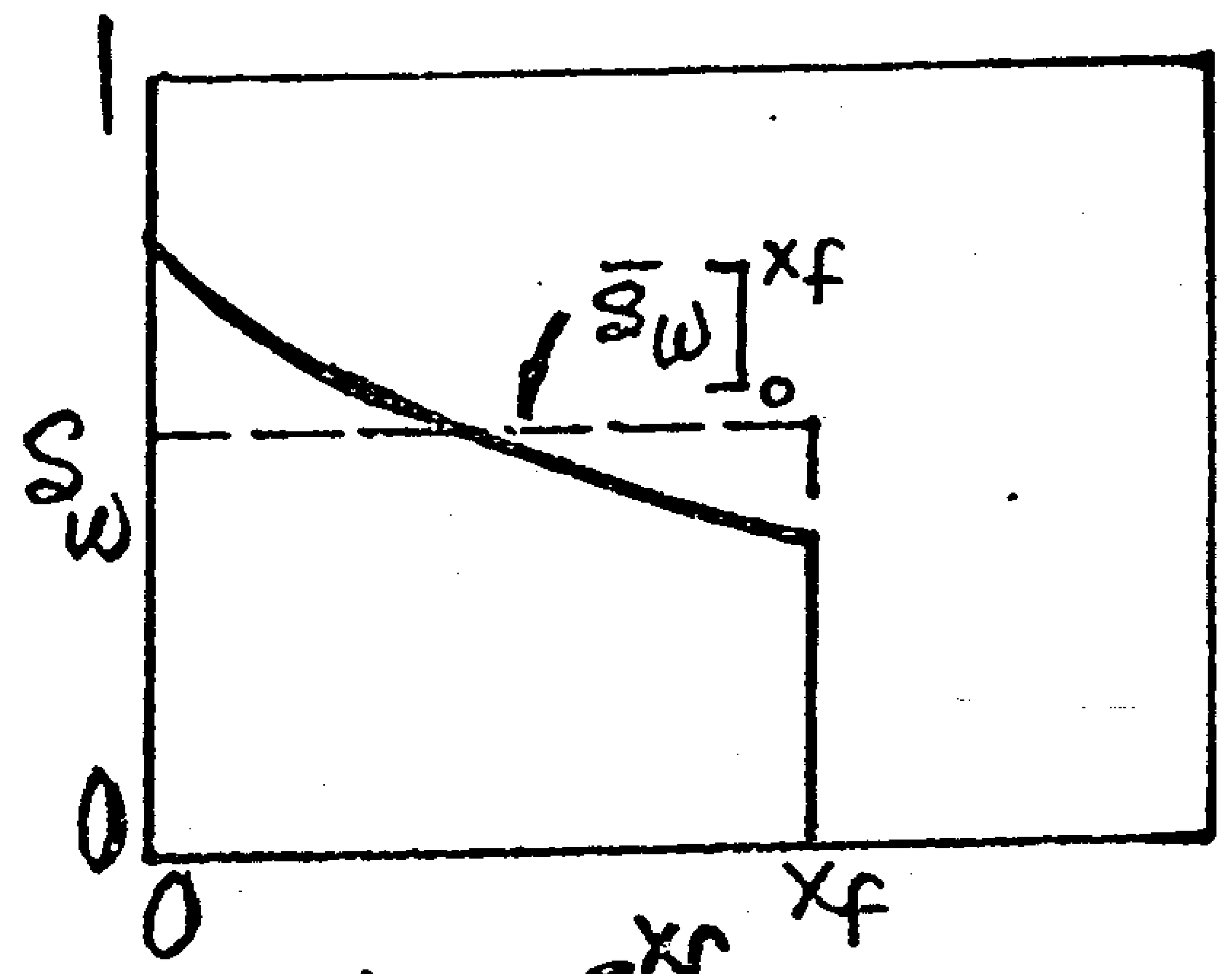


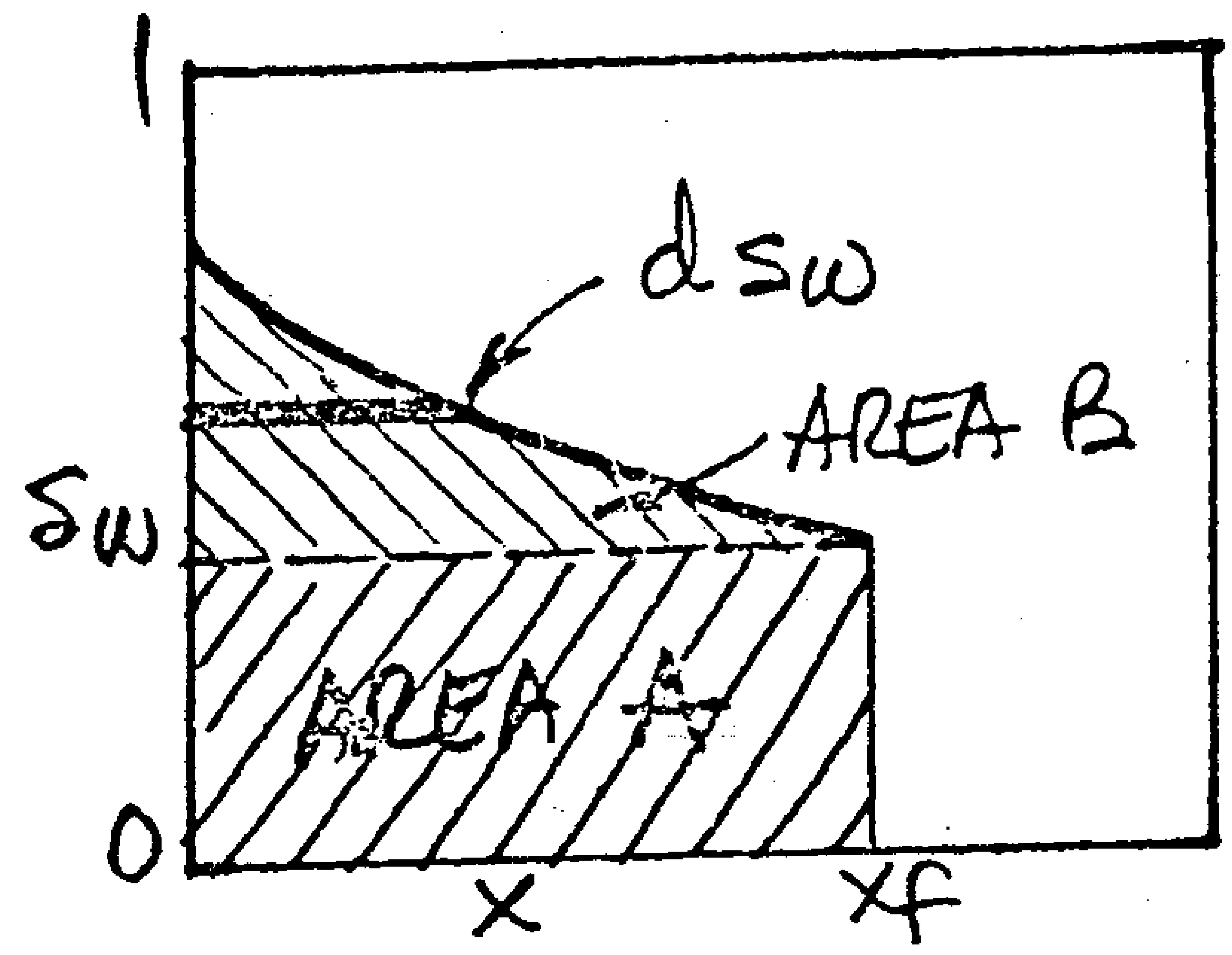
PERFORM GRAPHICAL DIFFERENTIATION OF $f_w - s_w$ DATA AT $S_w > S_{wf}$ · to OBTAIN VALUES OF $\frac{\partial f_w}{\partial s_w}$ VS S_w

AND



AVERAGE SATURATION BEHIND FRONT - I





- $$\bar{S}_w\Big]_0^{x_f} = \frac{\int_0^{x_f} S_w \, dx}{x_f} = \frac{\text{AREA A} + \text{AREA B}}{x_f}$$

FROM BUCKLEY-LEVERETT

$$x_f = \frac{5.615}{A\phi}\left(\frac{\partial f_w}{\partial s_w}\right)_f \int_0^t q \, dt$$

$$= \frac{5.615 \; Q}{A\phi}\left(\frac{\partial f_w}{\partial s_w}\right)_f$$

- AREA A $= S_{wf} \cdot x_f$

AREA B $= \int_{s_{wf}}^{1} x \, ds_w$

AS $\quad x = \dfrac{5.615 Q}{A\phi}\left(\dfrac{\partial f_w}{\partial s_w}\right)_x$

AREA B $= \dfrac{5.615 \; Q}{A\phi} \int_{s_{wf}}^{1} \dfrac{df_w}{ds_w} \cdot ds_w$

AVERAGE SATURATION BEHIND FRONT, II

$$\text{AREA } B = \frac{5.615\,Q}{A\phi} \int_{f_w @ s_{wf}}^{f_w @ s_w = 1} df_w$$

$$= \frac{5.615\,Q}{A\phi} \left[(f_w @ s_w = 1) - (f_w @ s_{wf}) \right]$$

BUT: $(f_w @ s_w = 1) = 1$; $(f_w @ s_{wf}) = f_{wf}$

THEREFORE

$$\text{AREA } B = \frac{5.615\,Q}{A\phi} \left[1 - f_{wf} \right]$$

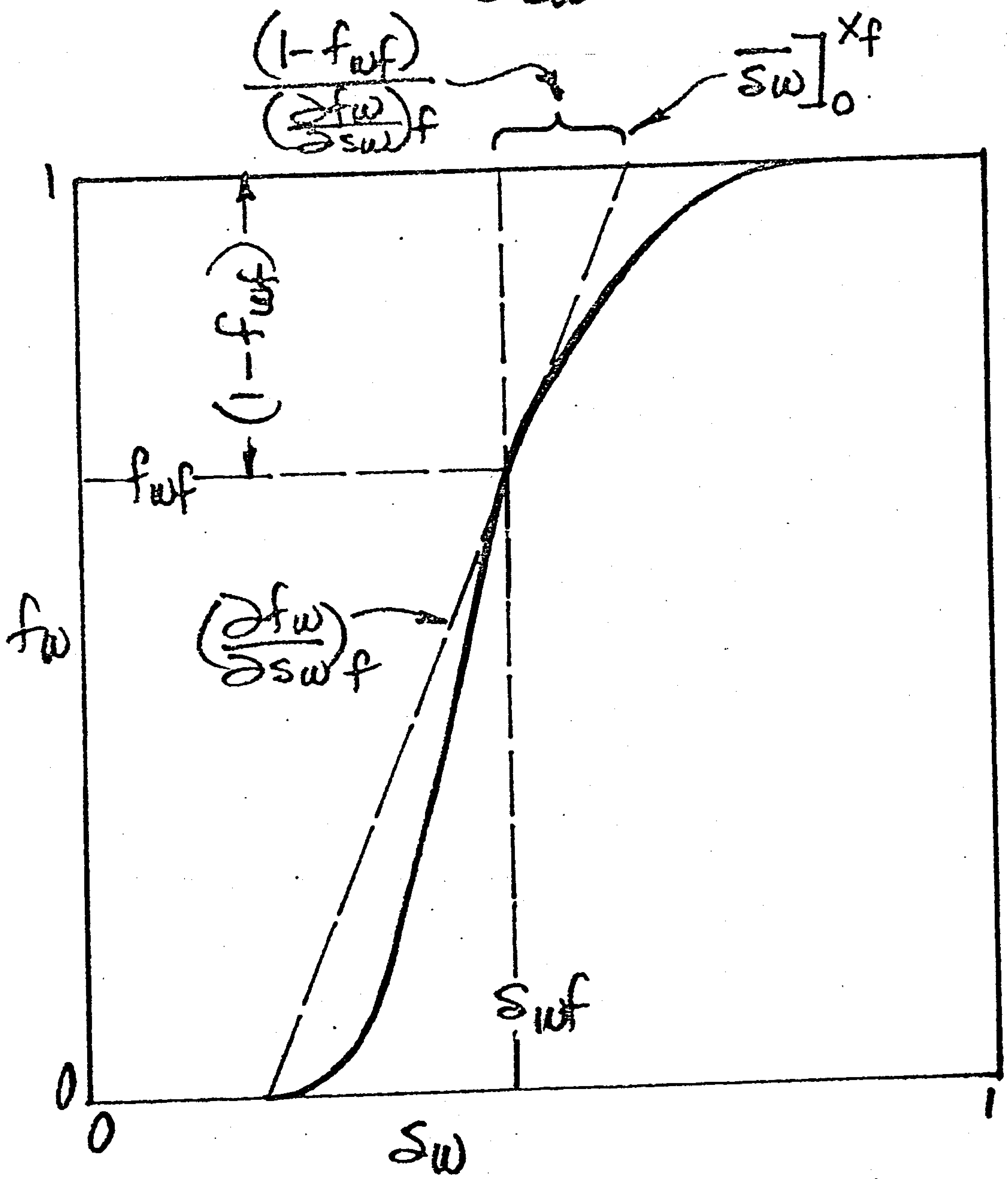
AS $\bar{S}_w \Big]_0^{x_f} = \dfrac{\text{AREA A} + \text{AREA B}}{x_f}$

$$\bar{s}_w \Big]_0^{x_f} = \frac{\dfrac{5.615\,Q}{A\phi} \left[s_{wf} \left(\dfrac{\partial f_w}{\partial s_w} \right)_f + (1 - f_{wf}) \right]}{\dfrac{5.615\,Q}{A\phi} \left(\dfrac{\partial f_w}{\partial s_w} \right)_f}$$

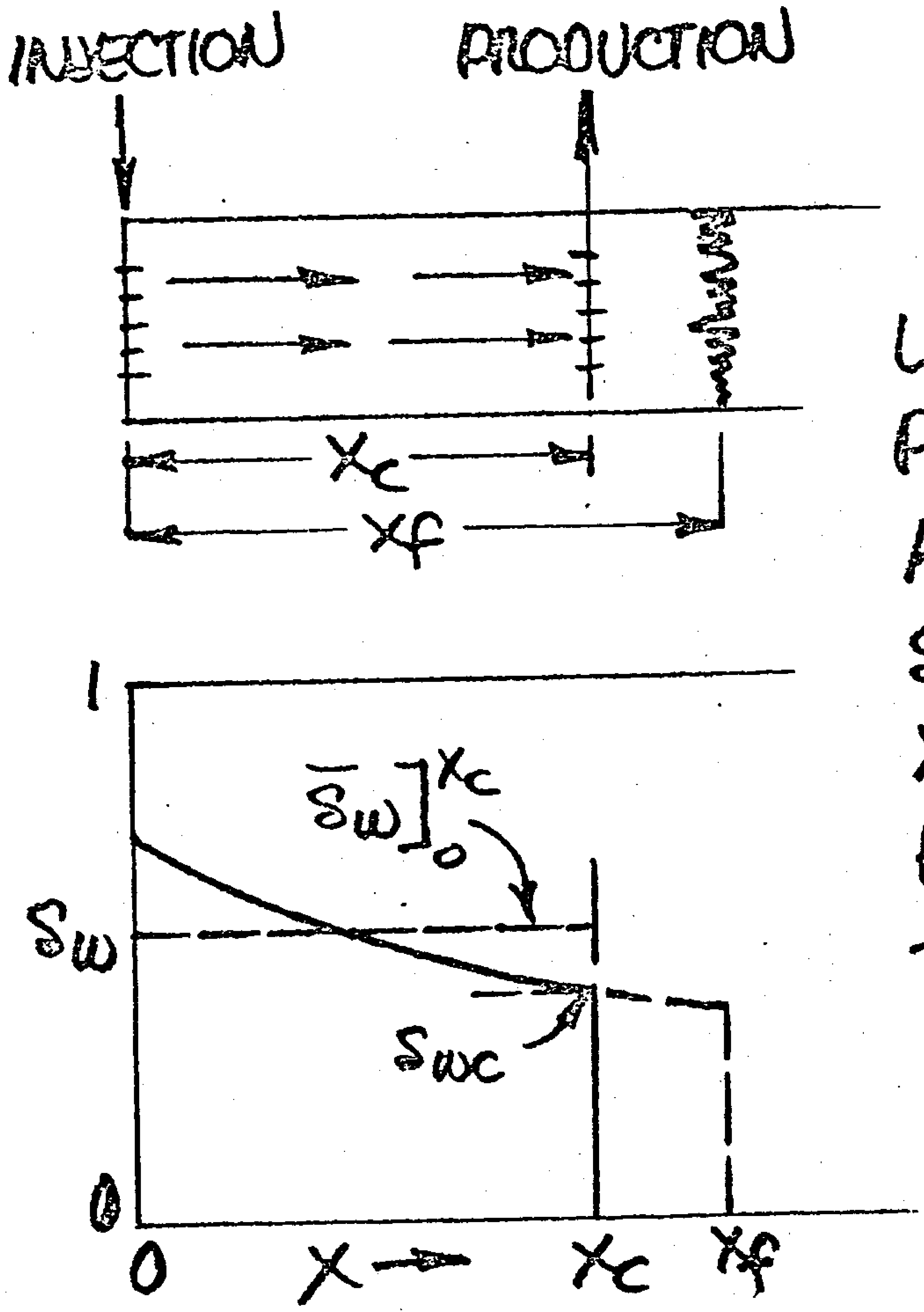
$$\bar{s}_w \Big]_0^{x_f} = s_{wf} + \frac{(1 - f_{wf})}{\left(\dfrac{\partial f_w}{\partial s_w} \right)_f}$$

AVERAGE SATURATION BEHIND FRONT III

$$\bar{S}_w\Big]_0^{x_f} = S_{wf} + \frac{(1 - f_{wf})}{\left(\frac{\partial f_w}{\partial s_w}\right)_f}$$



AVERAGE SATURATION BEHIND FRONT - IV

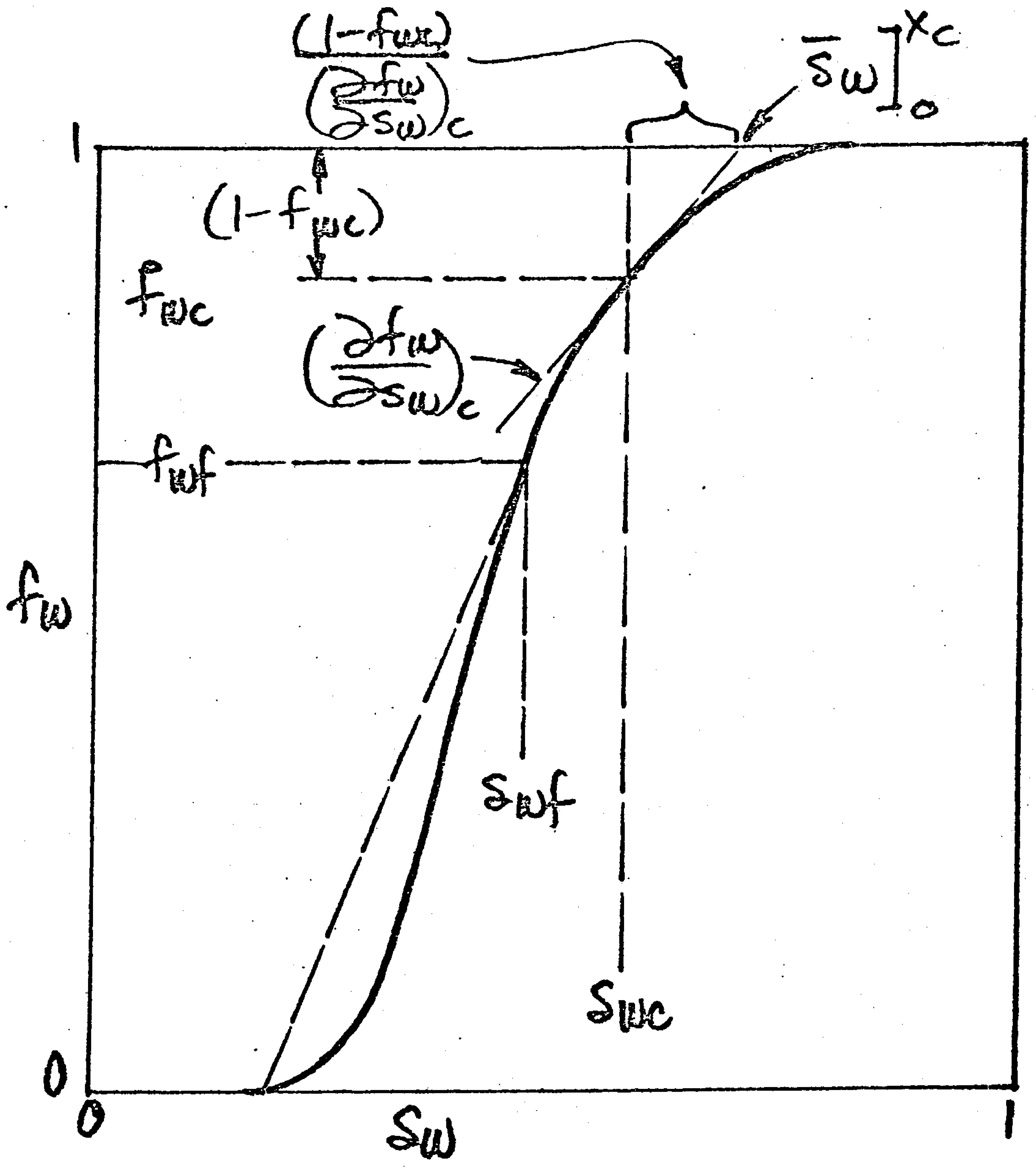


WHEN THE FRONT LINE HAS PASSED POSITION x_c, THE AVERAGE WATER SATURATION BEHIND x_c CAN BE DEVELOPED BY A SIMILAR APPROACH. THE FINAL EQUATION IS

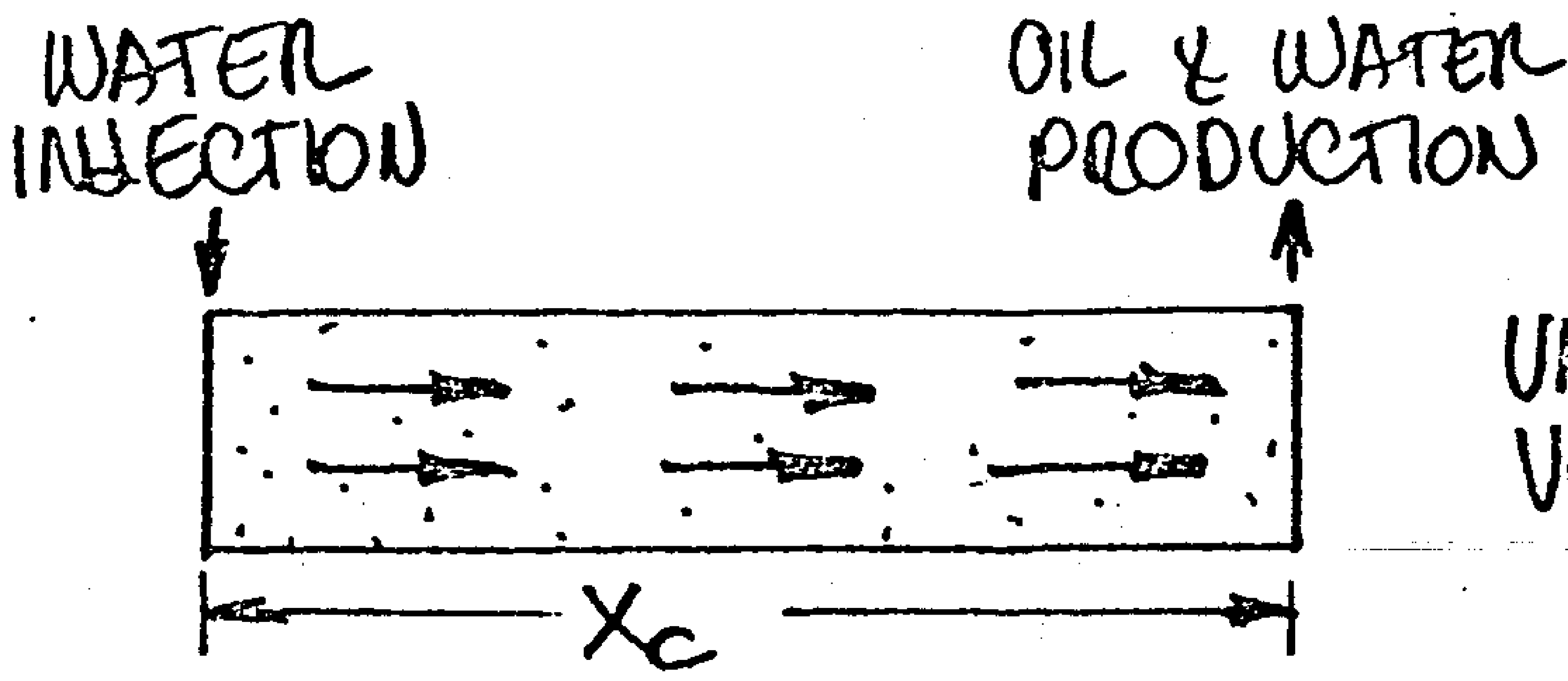
$$\bar{S}_w \Big]_0^{x_c} = S_{wc} + \frac{(1 - f_{wc})}{\left(\frac{\partial f_w}{\partial S_w}\right)_c}$$

AVERAGE SATURATION BEHIND FRONT - II

$$\bar{S}_w\Big]_0^{x_c} = S_{wc} + \frac{(1 - f_{wc})}{\left(\frac{\partial f_w}{\partial S_w}\right)_c}$$



INJECTION VOLUME VS. RECOVERY RELATIONS, I



UNIT PORE VOLUME, $V_p = \dfrac{A\phi x_c}{5.615}$

DIMENSIONLESS PORE VOLUMES INJECTED

$$V_{iD} = \frac{W_i}{V_p} = \frac{W_i}{\frac{A\phi x_c}{5.615}} = \frac{5.615\,W_i}{A\phi x_c}$$

FROM BUCKLEY - LEVERETT

$$x_c = \frac{5.615\,W_i}{A\phi}\left(\frac{\partial f_w}{\partial s_w}\right)_c$$

OR

$$\frac{5.615\,W_i}{A\phi x_c} = 1\Big/\left(\frac{\partial f_w}{\partial s_w}\right)_c$$

THEREFORE, THE PORE VOLUMES INJECTED TO REACH A GIVEN WATER SATURATION AT THE OUTFLOW FACE, s_{wc} IS,

$$V_{iD} = 1\Big/\left(\frac{\partial f_w}{\partial s_w}\right)_c$$

INJECTION VOLUME VS. RECOVERY RELATIONS II

RESERVOIR OIL DISPLACED $= V_p (\Delta \bar{S}_w)_o^{x_c}$

$$= \frac{A \phi x_c}{5.615} \left[(\bar{S}_w)_o^{x_c} - S_{wi} \right]$$

STOCK TANK OIL PRODUCED $= \dfrac{\text{RES. OIL DISP.}}{B_o}$

$$N_p = \frac{A \phi x_c}{5.615 B_o} \left[(\bar{S}_w)_o^{x_c} - S_{wi} \right]$$

FLOWING WATER-OIL RATIO AT OUTFLOW FACE;

$$WOR = \frac{f_{wc}}{1 - f_{wc}}$$

SURFACE PRODUCING WATER-OIL RATIO,

$$F_{wo} = \frac{f_{wc} \cdot B_o}{(1 - f_{wc}) B_w}$$

SURFACE WATER CUT $= \dfrac{\dfrac{f_{wc}}{B_w}}{\dfrac{f_{wc}}{B_w} + \dfrac{(1-f_{wc})}{B_o}} = \dfrac{1}{1 + F_{wo}}$

TIME, $t = \dfrac{W_i}{i_w}$

$$= \frac{V_{iD} \cdot A \phi x_c}{5.615 i_w} \text{ (DAYS)}$$

WHERE i_w IS BBL PER DAY