M.B. Standing (~74)

RESERVOARTEKNIKK III

DYKSTRA - PARSONS METHOD OF CALCULATING LINEAR DISPLACEMENTS IN LAYERED SYSTEMS

In this method an oil reservoir is characterized as a layered system and recovery is calculated as a function of the permeability variation of this layered system and the mobility ratio.

Assumptions:

1. The reservoir consists of isolated layers of uniform permeability with no cross flow between layers.
2. Piston-like displacement; that is only one phase is flowing in any given volume element.
3. Flow is linear.
4. The fluids are incompressible; that is there are no transient pressure effects.
5. The pressure drop across every layer is the same.
6. Mobility ratio, porosity, and fluid saturation are the same in each layer. (This is not a necessary assumption for the method but was made in the interest of simplifying the calculations of coverage charts.)

Theory

It will be assumed that the reservoir may be thought of as a series of layers piled one on top of the other. It may be true that two adjacent layers have the same permeability. In general, however, the absolute permeability will vary from one layer to the next. In any given layer the permeability is taken to be constant. In each layer it will be assumed that there is piston-like displacement, i.e., only oil is flowing ahead of the front and only water behind the front. This means that in any layer all the oil is produced at breakthrough that will be produced.

Consider first the determination of the velocity of the front in any layer. By Darcy's law:

$$q_o = -\frac{k_o}{\mu_o}\frac{dP}{dx} \qquad (1a)$$

$$q_w = -\frac{k_w}{\mu_w}\frac{dP}{dx} \qquad (1b)$$

Suppose the flood front is located at x_1, and let ΔP_1 be the difference in pressure between the point x_1 and the influx end of the layer. Then

$$q_w = -\frac{k_w}{\mu_w}\frac{\Delta P_1}{x_1} \qquad (2)$$

The difference in pressure between the efflux end of the layer and x_1 is

$$\Delta P - \Delta P_1 \qquad (3)$$

where P is the difference in pressure between the efflux end of the layer and the influx end. Hence

$$q_o = -\frac{k_o}{\mu_o}\frac{(\Delta P - \Delta P_1)}{L-x_1} \qquad (4)$$

When equations (2) and (3) are added:

$$\frac{\mu_w q_w x_1}{k_w} + \frac{\mu_o q_o (L-x_1)}{k_o} = -\Delta P \qquad (5)$$

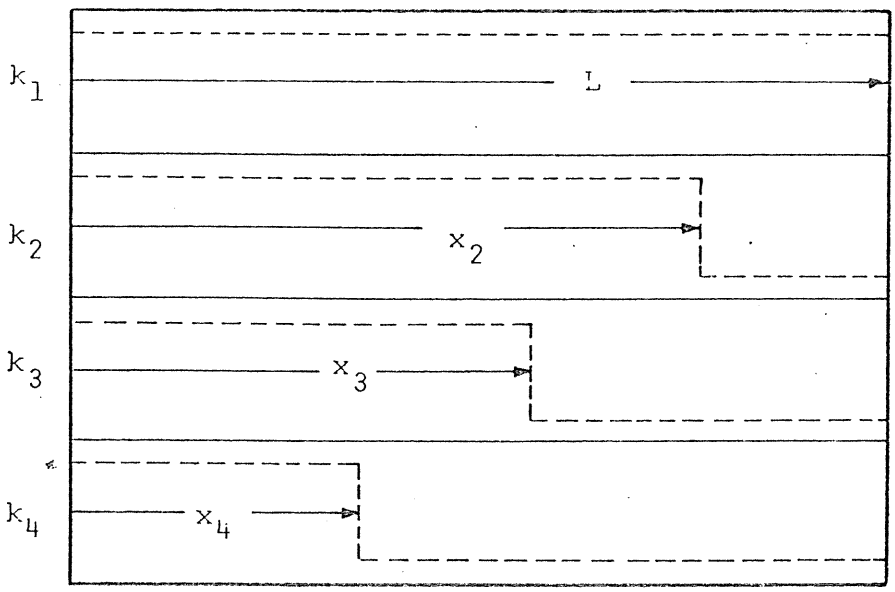
However, inasmuch as the fluids are incompressible and only all oil or all water is flowing,

$$q_w = q_o = q \qquad (6)$$

It follows that:

$$q_o = q = \frac{-k\ \Delta P}{\frac{\mu_w}{k_{rw}} x_1 + \frac{\mu_o}{k_{ro}} (L-x_1)} \qquad (7)$$

It is now desired to find the ratio of the distance of advance in one layer where $x_1 = L$, i.e., where breakthrough has just occurred, to the interface position x_1 in any other layer with a smaller permeability (see sketch). The pressure drop will be



assumed to be the same across all the layers. The ratio of rates is given by

$$\frac{q_i}{q_1} = + \frac{k_i}{k_1} \frac{\frac{\mu_w}{k_{rw}} x_1 + \frac{\mu_o}{k_{ro}} (L-x_1)}{\frac{\mu_w}{k_{rw}} x_i + \frac{\mu_o}{k_{ro}} (L-x_i)} \qquad (8)$$

where k_{rw} and k_{ro} are taken to be the same in each layer. But

$$q_i = \frac{dx_i}{dt} \; ; \quad q_1 = \frac{dx_1}{dt} \qquad (9)$$

Hence

$$\frac{dx_i}{dx_1} = + \frac{k_i}{k_1} \frac{\frac{\mu_w}{k_{rw}} x_1 + \frac{\mu_o}{k_{ro}} (L-x_1)}{\frac{\mu_w}{k_{rw}} x_1 + \frac{\mu_o}{k_{ro}} (L-x_i)} \qquad (10)$$

To find x_i it is necessary to integrate the above equation:

$$\int_0^L \left[\frac{\mu_w}{k_{rw}} x_1 + \frac{\mu_o}{k_{ro}}(L-x_1)\right] dx_1 = + \frac{k_1}{k_i} \int_0^{x_i} \left[\frac{\mu_w}{k_{rw}} x_i + \frac{\mu_o}{k_{ro}}(L-x_i)\right] dx_i \qquad (11)$$

The limits are chosen so that both interfaces start off at the inlet at the same time. It is desired to find the ratio x_i/x_1 when $x_1 = L$.

Integration gives:

$$(1 + M)L^2 = + \frac{k_1}{k_i} [x_i^2 + 2M(Lx_i) - Mx_i^2] \qquad (12)$$

where

$$M = \frac{k_{rw}}{k_{ro}} \frac{\mu_o}{\mu_w}$$

On rearrangement:

$$(1-M)\left[\frac{x_i}{L}\right]^2 + 2M\left[\frac{x_i}{L}\right] - \frac{k_i}{k_1}(1 + M) = 0 \tag{13}$$

Solving this quadratic equation gives

$$\frac{x_i}{L} = \frac{M \pm \sqrt{M^2 + \frac{k_i}{k_1}(1 - M^2)}}{(M - 1)} \tag{14}$$

When x_i also refers to the first layer $k_i = k_1$ and $x_i = L$, or

$$1 = \frac{M \pm 1}{M - 1} \tag{15}$$

Hence the minus sign must be chosen in equation (14). Thus when the jth layer has broken through,

$$\frac{x_i}{x_j} = \frac{x_i}{L} = \frac{M - \sqrt{M^2 + \frac{k_i}{k_j}(1 - M^2)}}{(M - 1)} \tag{16}$$

which gives the distance of advance of the front in layer i having a permeability less than j when layer j has broken through.

If in equation (13) the mobility ratio M is set equal to one, then:

$$\frac{x_i}{x_1} = \frac{k_i}{k_1} \tag{17}$$

This is just an expression for the basic assumption of the Stiles method, that the ratio of the distances of advance in the various layers is the same as the corresponding permeability ratio. Thus the Stiles method will give the same answers as the Dykstra-Parsons method when the mobility ratio is unity.

It is now of interest to find an expression for the coverage when the nth layer has broken through. The coverage is defined as the fraction of the reservoir which has been invaded by water. Let N be the total number of layers in the system. Number the layers in order of decreasing permeability.

When the nth layer has broken through, all the layers with permeability greater than that of the nth layer will also have broken through. Hence the fraction of the reservoir for which the layers have been completely flooded out is n/N. The remaining layers, which have permeabilities less than the nth layer, will be only partially swept out. The distances the flood has advanced in the jth layer (j>n) when the nth layer has just broken through is

$$\frac{x_j}{x_n} = \frac{M - \sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}}{M - 1} \qquad (18)$$

$$\frac{x_j}{x_n} = \frac{x_j}{L} \qquad (19)$$

is just the fraction of the jth layer which has been swept out. The complete coverage is then just:

$$\text{COVERAGE} = \frac{n + \sum_{j>n} \left[\frac{x_j}{x_n}\right]}{N} \qquad (20)$$

or

$$\text{COVERAGE} = \frac{n + \sum_{j>n} \left[\frac{M - \sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}}{(M - 1)}\right]}{N} \qquad (21)$$

but

$$\sum_{j>n} M = (N - n)\,M. \qquad (22)$$

Hence

$$\text{COVERAGE} = \frac{n + \frac{(N-n)M}{M-1} - \frac{1}{M-1} \sum_{j>n} \sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}}{N} \qquad (23)$$

The above formula makes it possible to calculate the coverage, or fraction of the reservoir which has been invaded by water, when the nth layer has broken through.

An expression for computing the water-oil ratio when the nth layer has broken through will now be derived. When the nth layer has broken through, only water is flowing in the layers with permeability greater than that of the nth layer. The total flow rate of water per unit breadth is:

$$Q_{w_n} = \sum_{j<n} H \Delta Z q_{w_j} = \Delta Z \sum_{j<n} H \left[\frac{-k_j k_{rw}}{\mu_w} \frac{\Delta P}{L} \right] \qquad (24)$$

Only oil is flowing out in the layers with permeability less than that in the nth layer. The total flow rate of oil per unit breadth is:

$$Q_{o_n} = \Delta Z \sum_{j>n} H q_{o_j} \qquad (25)$$

Equation (7) must be used for q_{oi}, since there is a front moving along in the layers where oil is flowing out. Thus

$$Q_{o_n} = \Delta Z \sum_{j>n} \left[- \frac{H k_j \Delta P}{\frac{\mu_w}{k_{rw}} x_j + \frac{\mu_o}{k_{ro}} (L - x_j)} \right] \qquad (26)$$

$$Q_{o_n} = \Delta Z \sum_{j>n} \left[- \frac{H k_j \frac{\Delta P}{L}}{\frac{\mu_w}{k_{rw}} \frac{x_j}{L} + \frac{\mu_o}{k_{ro}} (1 - \frac{x_j}{L})} \right] \qquad (27)$$

But equation (18) may be used for x_j/L so that:

$$Q_{o_n} = \Delta Z \sum \left[- \frac{H k_j \frac{\Delta P}{L}}{\frac{\mu_w}{k_{rw}} \left\{ \frac{M - \sqrt{M^2 + \frac{k_j}{k_n}(1-M^2)}}{M-1} \right\} + \frac{\mu_o}{k_{ro}} \left\{ \frac{-1 + \sqrt{M^2 + \frac{k_j}{k_n}(1-M^2)}}{M-1} \right\}} \right] \qquad (28)$$

$$Q_{o_n} = \Delta Z \sum \left[- \frac{H k_j \frac{k_{rw}}{\mu_w} \frac{\Delta P}{L}}{\frac{M - \sqrt{M^2 + \frac{k_j}{k_n}(1-M^2)}}{M-1} + \frac{-M + M\sqrt{M^2 + \frac{k_j}{k_n}(1-M^2)}}{M-1}} \right] \qquad (29)$$

$$Q_{o_n} = \Delta Z \sum_{j>n} \left[- \frac{H k_j \frac{k_{rw}}{\mu_w} \frac{\Delta P}{L}}{\sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}} \right] \qquad (30)$$

The water-oil ratio when the nth layer has broken through and for layers of equal thickness is then:

$$WOR_n = \frac{Q_{w_n}}{Q_{o_n}} = \frac{\sum_{j<n} k_j}{\sum_{j>n} \frac{k_j}{\sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}}} \quad (31)$$

This expression can be used to calculate the water-oil ratio when the nth layer has just broken through. The producing water-oil ratio F_{wo} is then given by B_o times the value given by equation (31), or

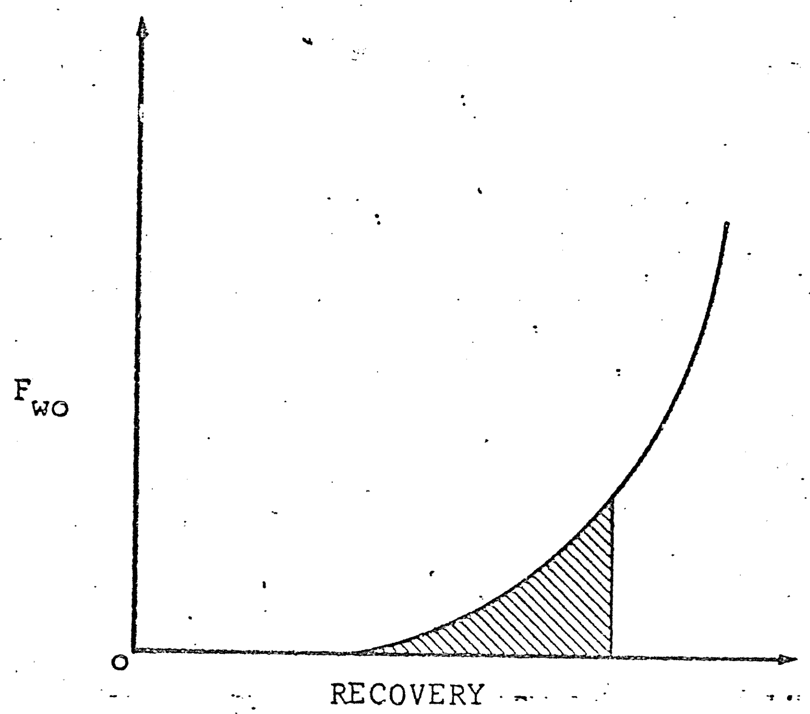
$$F_{wo} = B_o \times WOR$$

The cumulative oil recovery, Np, is calculated from the following equation:

$$Np = \frac{7758\ Ah\phi C\ (S_{oi} - S_{or})\ Ea}{B_o} \quad (32)$$

where Ah is area thickness product in acre-feet
ϕ is fractional porosity
C is coverage from equation (23), or from charts
S_{oi} and S_{or} are initial and residual oil saturations
Ea is areal sweep efficiency
B_o is oil formation volume factor.

The calculation of the recovery as a function of time will now be considered. If the F_{wo} is plotted against the recovery on rectangular coordinates, a curve would be obtained which looks something like that in the following sketch.



Now the water-oil ratio, F_{wo}, is given by

$$F_{wo} = \frac{q_w}{q_o} = \frac{\frac{dW_p}{dt}}{\frac{dN_p}{dt}} = \frac{dW_p}{dN_p} \qquad (33)$$

where W_p is the cumulative water produced. The question can then be asked, what does the area under the F_{wo} vs recovery curve represent? The area is just:

$$\text{AREA} = \int_0^{N_p} (F_{wo})dN_p = \int_0^{N_p} \frac{dW_p}{dN_p} = W_p \qquad (34)$$

Thus the area under the curve is just the water produced up to the given recovery N_p. The water injected W_i when the recovery is N_p is just:

$$W_i = W_p + B_o N_p + W_F \qquad (35)$$

W_F being the volume of water required for fill-up and is equal to $7758\ Ah\phi(S_{gi}-S_{gr})$. The time required to reach a given recovery is just:

$$\text{TIME, } t = \frac{W_i}{i_w} \qquad (36)$$

where i_w is the water injection rate (assumed to be constant). Thus by finding the area under the F_{wo}-N_p curve up to a given N_p, it is possible to obtain curves for the cumulative water injected as a function of F_{wo} and the cumulative production as a function of time. To find the production rate it is only necessary to divide the differences in recovery by the corresponding differences in time.

Coverage Charts

The above equations hold for an arbitrary permeability distribution. Dykstra and Parsons wanted to obtain generalized curves in which the permeability distribution could be characterized by a single number, so that engineers would not need to perform the rather long calculations necessary to compute the F_{wo}-N_p curve. To obtain such generalized curves, they assumed that if the percentage of the permeabilities greater than a given value was plotted against that permeability on log probability paper, a straight line results. They characterized this straight line by the permeability variation which was defined to be the median permeability minus the permeability at 84.1 cumulative percent, this difference divided by the median permeability. The permeability variation essentially measures the slope of the straight line. Only this permeability variation is necessary to characterize the distribution as far as the calculations are concerned. The reason for this is that the magnitudes of the permeability are not important, inasmuch as only ratios of permeabilities appear in the calculations. Thus it was possible to calculate curves giving the coverage as a function of the permeability variation and mobility ratio for any given water-oil ratio. These calculations were made and put in the form of two charts in the original paper as Figures 9 and 10 for water-oil ratios of 1 and 25. Since the time, computers have become available and made it possible to make calculations covering a wide range of water-oil ratios. New coverage charts have been prepared recently and are now available covering a WOR range of 0.1 to 100. A set of 10 charts are included with this write-up.

Correlations of Recovery with Coverage

In the original paper, a correlation was presented (Figure 11) of a "recovery modulus" and coverage, C. The recovery modulus is defined as $R\left[1 - S_w\,(WOR)^{-0.2}\right]$, where R is the fractional recovery of the original oil. Coverage, C, is a function of WOR and also of permeability variation and mobility ratio. The correlation was based on results of laboratory core floods and gave best results for initial oil saturations lying between 45 and 60 per cent. The correlation was particularly useful in estimating a recovery factor when oil saturation were assumed to lie close to the range mentioned above.

C. E. Johnson (Trans AIME 207 (1956) 345) was able to simplify use of the correlation by constructing charts for WOR's of 1, 2, 5, and 100. On these charts, lines of constant recovery modulus values are plotted as functions of mobility ratio, M, and permeability variation, V. These charts are shown on pages 100 and 101. The procedure is to find the value of recovery modulus from the appropriate Johnson chart and, knowing the water saturation, S_w, calculate the fractional recovery, R. Stocktank oil recovery can then be calculated from the relationship:

$$N_p = \frac{7758\, Ah\phi\, S_{oi}\, E_A \cdot R}{B_o}$$

Proceaure for Using the Dykstra-Parsons Method

The steps for calculating recovery with the aid of the coverage charts is as follows:

1. Assemble permeability data in descending order. Calculate "percentage equal to or greater than" for each entry.

2. Plot percentage against log permeability on probability paper. Calculate permeability variation from

$$V = \frac{k_{50} - k_{84.1}}{K_{50}}$$

3. Calculate mobility ratio,

$$M = \frac{k_w \mu_o}{\mu_w k_o}$$

4. From charts get coverage, C.

5. Calculate recovery from the following equation:

$$N_p = \frac{7758\ AhØC\ (S_{oi} - S_{or})\ E_a}{B_o}$$

6. Plot N_p vs F_{wo}.

7. Integrate $N_p - F_{wo}$ curve graphically to get W_p.

8. Calculate $W_i = W_F + N_p B_o + W_p$.

9. Time in years is given by

$$\frac{W_t}{i_w \times 365}$$

10. Calculate oil and water rates from

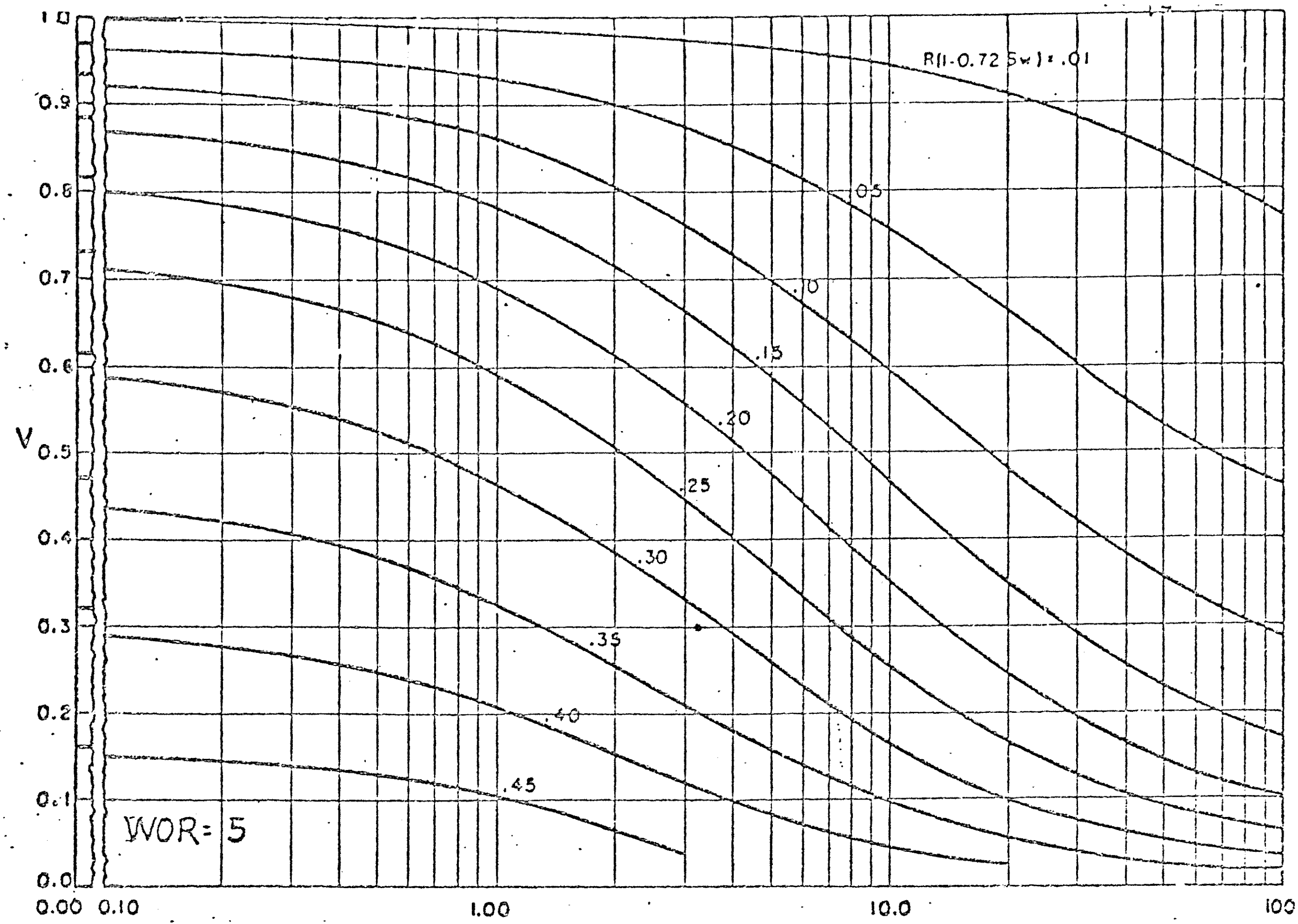
$$q_o = \frac{i_w}{B_o + F_{wo}}$$

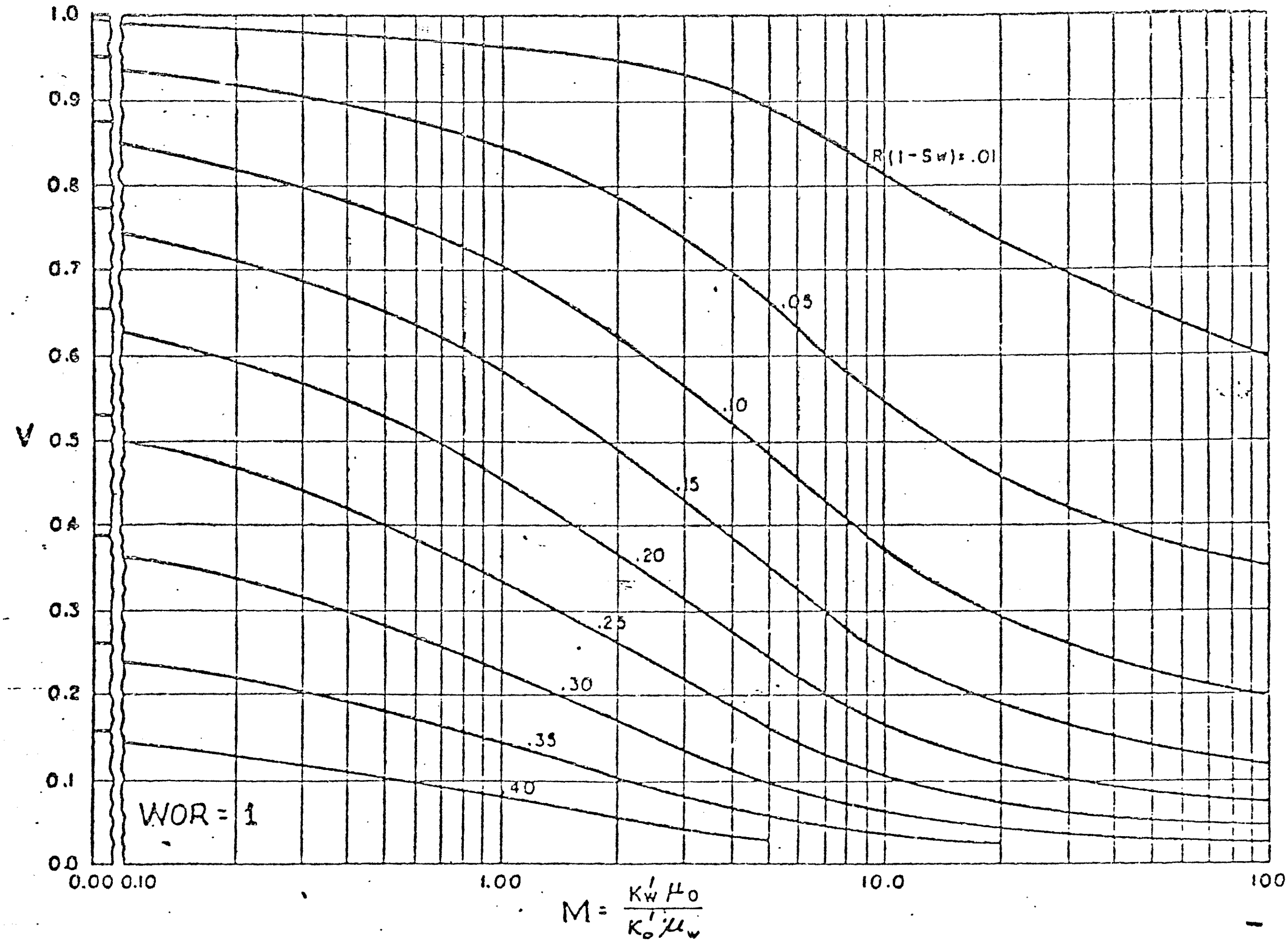
$$q_w = q_o F_{wo} \quad \text{or} \quad q_w = i_w - B_o q_o$$

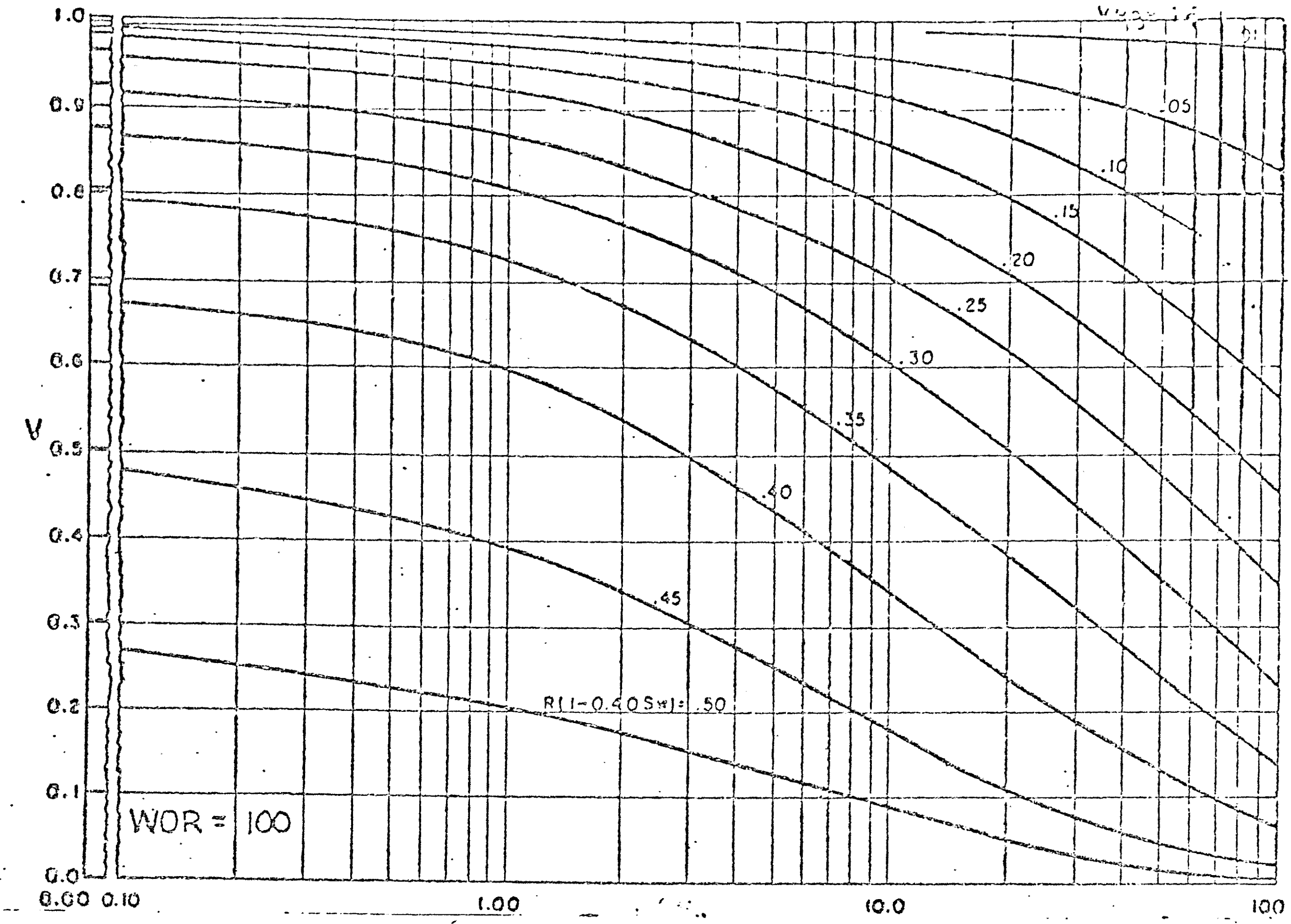
REFERENCES

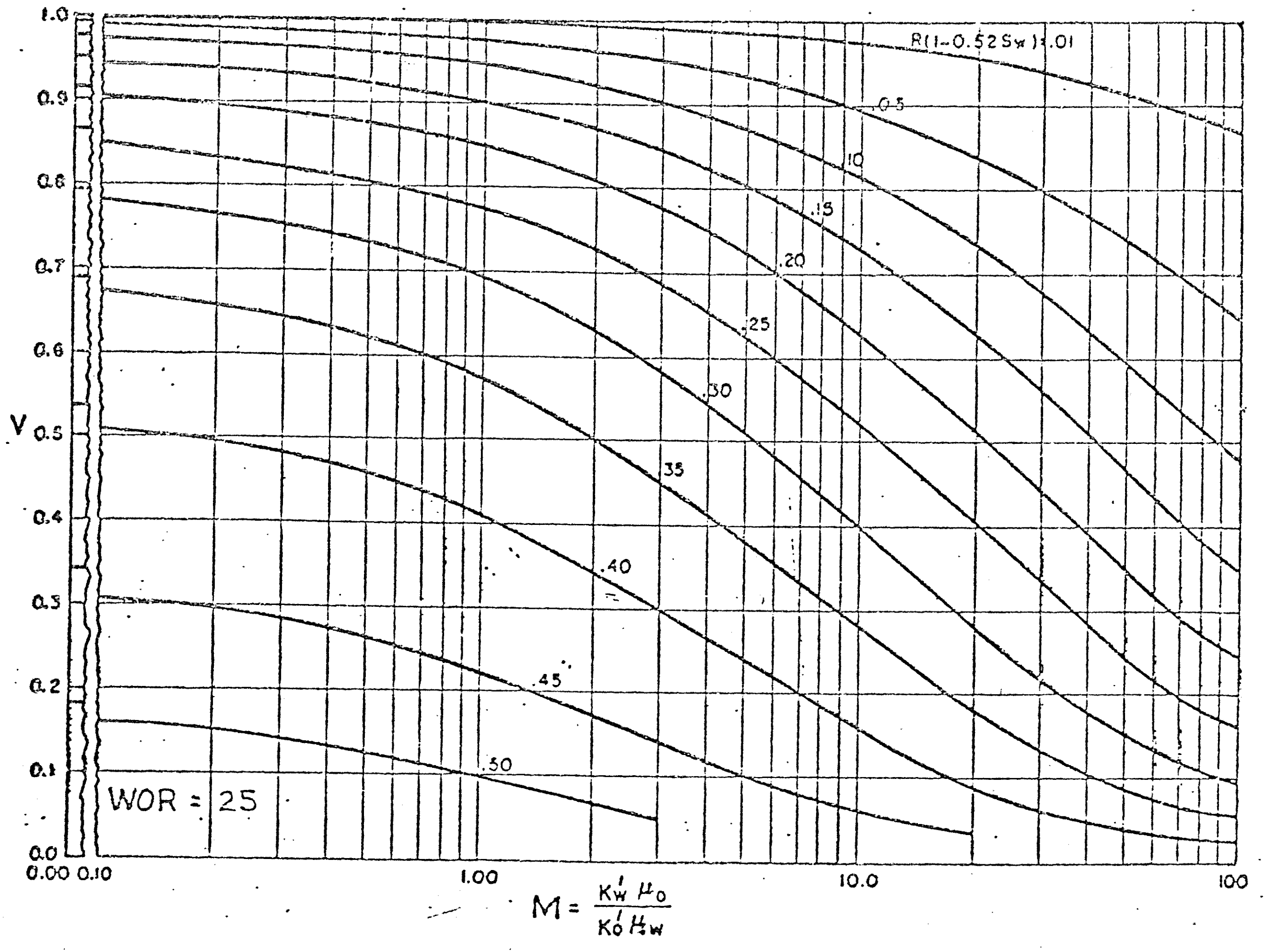
1. Dykstra and Parsons, API Sec. Rec. of Oil in the U.S., p.160 2nd Ed, 1950.

2. Johnson, Trans AIME 207, 345 (1956).


R(1-0.72 Sw) = .01
.05
.10
.15
.20
.25
.30
.35
.40
.45
WOR = 5
V
0.00
0.10
1.00
10.0
100


R(1-Sw) = .01
.05
.10
.15
.20
.25
.30
.35
.40
WOR = 1
V
0.00
0.10
1.00
10.0
100
M = Kw' μo / Ko' μw


V
WOR = 100
R(1-0.40Sw) = .50
.45
.40
.35
.30
.25
.20
.15
.10
.05
0.00 0.10
1.00
10.0
100


V
WOR = 25
R(1-0.52Sw) = .01
.05
.10
.15
.20
.25
.30
.35
.40
.45
.50
0.00 0.10
1.00
10.0
100
M = Kw' μo / Ko' μw

PREDICTION of OIL RECOVERY by WATER FLOOD — A SIMPLIFIED GRAPHICAL TREATMENT of the DYKSTRA-PARSONS METHOD

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INTRODUCTION

A method for predicting waterflood oil recovery was reported by H. Dykstra and R. L. Parsons[1] in 1950. It is now generally known as the Dykstra-Parsons method and is widely used by petroleum engineers.

The method is semiempirical and consists of a correlation of four fundamental variables. These are: V, vertical permeability variation;[2] α, mobility ratio; S_w, initial water saturation; and R, fractional recovery of oil in place at a given producing water-oil ratio. The correlation extends over a wide range in each of these variables, and may be applied to all formations with initial oil saturations of 45 per cent or greater.

The method of calculation originally outlined by Dykstra and Parsons closely follows their derivation. And this is a convenient way of illustrating the development of some rather complicated ideas. However, in actual use it is more cumbersome than necessary. The purpose of this note, therefore, is to provide a simplified method for making Dykstra-Parsons predictions.

A SIMPLIFIED GRAPHICAL SOLUTION

The correlation between V, α, S_w, and R, corresponding to a given producing water-oil ratio, can be shown on a single graph. This is best done by plotting V against α to show lines of constant $R(1 - S_w \cdot WOR^{-0.2})$. Figs. 1, 2, 3, and 4 show these plots

Manuscript received in Petroleum Branch office on May 28, 1956.
[1]References given at end of paper.

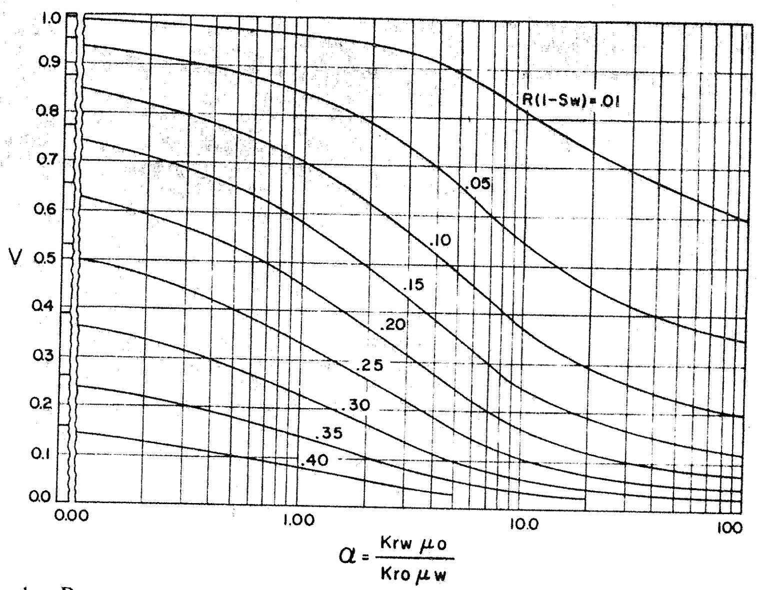

FIG. 1—PERMEABILITY VARIATION PLOTTED AGAINST MOBILITY RATIO SHOWING LINES OF CONSTANT $R(1 - S_w)$ FOR A PRODUCING WATER-OIL RATIO OF 1.

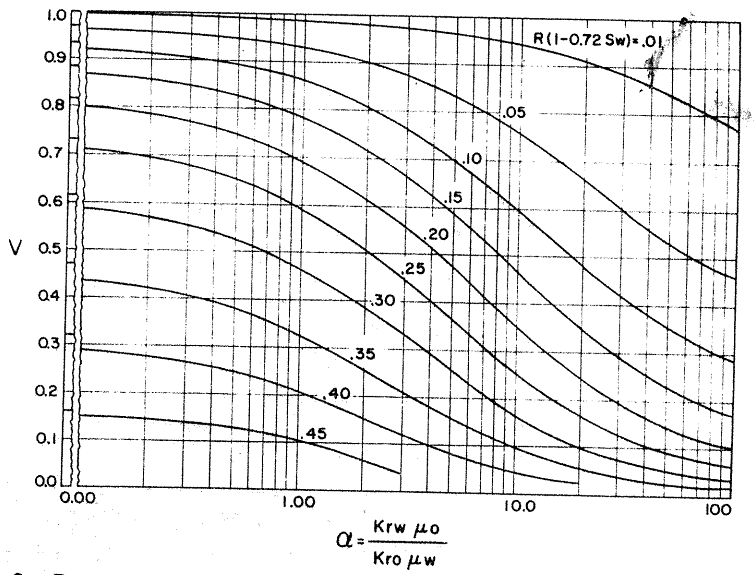

FIG. 2—PERMEABILITY VARIATION PLOTTED AGAINST MOBILITY RATIO SHOWING LINES OF CONSTANT $R(1 - 0.72\,S_w)$ FOR A PRODUCING WATER-OIL RATIO OF 5.

for water-oil ratios (WOR) of 1, 5, 25, and 100, respectively. In each figure $WOR^{-0.2}$ has been reduced to its numerical value.

Use of the figures requires assignment of definite values to V, α, and S_w. Both V and S_w can be obtained from core analysis data.[2] The mobility ratio, α, depends upon four subsidiary variables. It is given by the expression:

$$\alpha = \frac{k_{rw}}{k_{ro}} \frac{\mu_o}{\mu_w}$$

where μ_o and μ_w are the viscosities of the oil and the flood water under reservoir conditions, k_{rw} is the relative permeability to water in the reservoir when only water is flowing, and k_{ro} is the relative permeability to oil when only oil is flowing. Measurement of the two viscosities is seldom a problem. And the two relative permeabilities can be measured experimentally or estimated from the data of Leverett and Lewis[3] for unconsolidated sand packs.

To find R the appropriate figure is entered at V and α. For example, suppose we wish to predict the fractional oil recovery when the producing water-oil ratio reaches five. Fig. 2, which shows the correlation at a water-oil ratio of five, must be used. Suppose V and α are 0.50 and 2.0, respectively. Enter Fig. 2 at these values. The point of intersection shows that $R(1 - 0.72\ S_w)$ is 0.25. If the water saturation, S_w, at the beginning of the flood is 21 per cent (0.21), then R is found to be 0.29. This means that 29 per cent of the oil in place will have been recovered when the producing water-oil ratio reaches five.

The R value obtained above applies to a linear flood. When dealing with pattern floods, R must be multiplied by a correction factor which takes into account the fact the areal sweep efficiency of the pattern is less than one. Furthermore, to convert R to barrels of oil recovered, the volume of oil in place must be known. If this number is expressed in stock tank barrels, then its product with R will be the recovery in stock tank barrels.

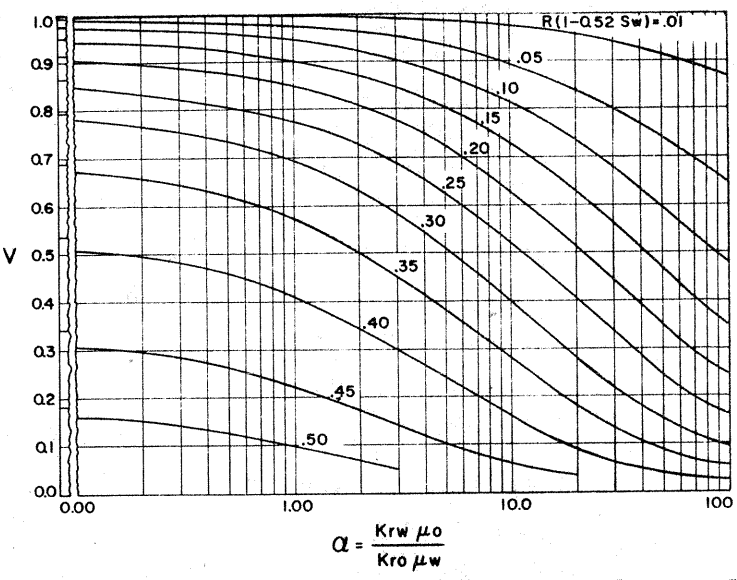

FIG. 3—PERMEABILITY VARIATION PLOTTED AGAINST MOBILITY RATIO SHOWING LINES OF CONSTANT $R(1 - 0.52\ S_w)$ FOR A PRODUCING WATER-OIL RATIO OF 25.

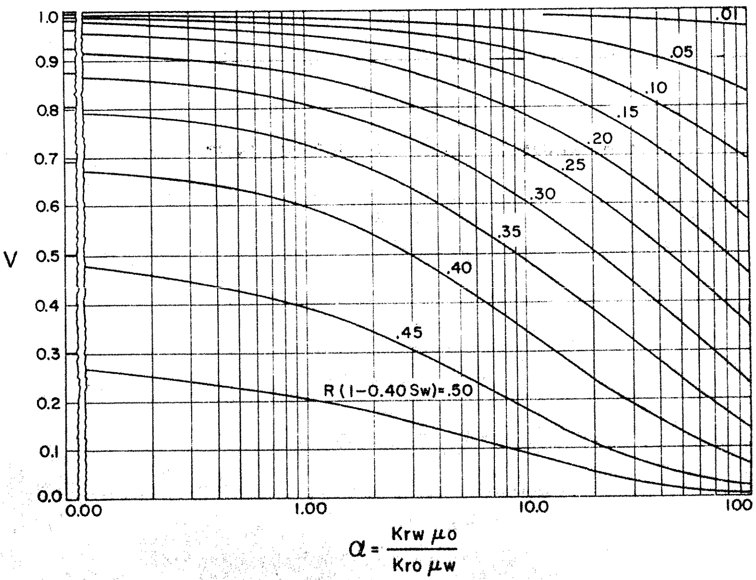

FIG. 4—PERMEABILITY VARIATION PLOTTED AGAINST MOBILITY RATIO SHOWING LINES OF CONSTANT $R(1 - 0.40\ S_w)$ FOR A PRODUCING WATER-OIL RATIO OF 100.

ACKNOWLEDGMENT

The author expresses his appreciation to Herman Dykstra who provided the basic information necessary to the construction of Figs. 2 and 3.

REFERENCES

1. Dykstra, H., and Parsons, R. L.: "The Prediction of Oil Recovery by Water Flood" *Secondary Recovery of Oil in the United States*, Second Edition, API, New York, N. Y. (1950).
2. For an explanation of the determination of V from core sample data, see Ref. 1, p. 171. Single copies of Ref. 1 may be obtained by writing to the author at California Research Corp., P. O. Box 446, La Habra, Calif.
3. Leverett, M. C., and Lewis, W. B.: *Trans.* AIME (1941), **142**, 107.

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