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An Iterative Matrix Solution Technique for Steamflood Simulation

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ABSTRACT

Iterative techniques, such as line successive overrelaxation (LSOR), have generally proven to be ineffective in solving linear systems of equations arising in thermal reservoir simulators. This is partly due to the occurrence of negative effective compressibilities or transmissibilities which can occur when cold fluids move into blocks having steam saturations. This paper describes an effective iterative method for solving steamflood equations using partial elimination followed by block successive overrelaxation on the reduced matrix. This scheme has been implemented in a steamflood model allowing any number N_c of components. The model's implicit formulation requires the solution of a linear system of equations in which each grid block has $N_c + 1$ unknowns.

A cyclic steam stimulation process provides a severe test for the iterative technique. During the production portion of the cycle, cold fluids can flow into the steam zone causing the negative compressibility/transmissibility relationship mentioned above. The flow of cold fluids into a steam block is a physically real, dominant feature of both cyclic stimulation and steam-bank flooding. This paper presents data and model results for two such test problems, including comparisons of slice (planar) SOR, direct solution using D-4 ordering and the proposed method. These comparisons indicate that for three-dimensional problems involving a large number of blocks and/or components, this iterative approach can reduce storage by factors of two to four and CPU time by factors of three or more. In comparison with standard SOR techniques, the proposed method has proven to be significantly more reliable for steamflood problems.

INTRODUCTION

Simulation of thermally enhanced oil recovery processes using a fully implicit treatment of component concentrations, phase saturations, pressure and temperature requires the simultaneous solution of large systems of linear equations. These equations result from finite difference approximations to a set of mass and energy balance equations and constraint equations for each reservoir grid block. Solution of these linear systems is accomplished by direct or iterative methods. This paper describes an iterative technique which compares favorably with the alternate diagonal (D-4) method[1] and line[4,5] or slice successive overrelaxation[6] (SSOR) for steamflood problems.

References and illustrations at end of paper.

For large three dimensional multi-component problems, the work and storage requirements of direct solution may make simulation impractical due to the lack of economic and/or computer resources. Iterative techniques have the potential to reduce both work and storage, but they sometimes suffer from a lack of reliability. When they do work, uncertainties involving the selection of convergence criteria and optimum acceleration parameters remain.

For years iterative methods have been widely applied in black-oil type reservoir simulation. Often, however, they have proven ineffective in solving steamflood equations. This is, in part, due to the negative compressibility/transmissibility ratio for a block pair where cold fluid flows into a hotter block containing a free steam phase. From a mathematical point of view, the corresponding coefficient matrix may possess one or more eigenvalues with negative real parts, a characteristic which inhibits the convergence of many iterative schemes, including successive overrelaxation.

Several variations of cyclic reduction followed by block successive overrelaxation have been found to be effective in steamflood simulation. In this paper, we discuss these methods, their efficient implementation, and their application. Results for two steamflood problems are then presented to compare several solution methods.

GENERAL PROBLEM

The set of mass and energy balance equations and constraint equations required to simulate the steamflood process are as follows:

Component Mass Balances $I = 1, 2, \ldots, N_C$

$$\frac{V}{\Delta t}\bar{\delta}\left(\phi \sum_{J=1}^{N_p} \rho_J S_J x_{IJ}\right) = \sum_{J=1}^{N_p} \Delta\left[\tau \rho_J x_{IJ} \frac{k_{rJ}}{\mu_J} \cdot(\Delta p + \Delta P_{cJ} - \gamma_J \Delta Z)\right] - q_I \quad (1)$$

Energy Balance

$$\frac{V}{\Delta t}\bar{\delta}\left[\phi \sum_{J=1}^{N_p} \rho_J S_J U_J + (1-\phi) M_f (T - T_i)\right] = \sum_{J=1}^{N_p} \Delta\left[\tau\rho_J \frac{k_{rJ}}{\mu_J} H_J (\Delta p + \Delta P_{cJ} - \gamma_J \Delta Z)\right] + \Delta(\tau_c \Delta T) - \dot{Q}_H - \dot{Q}_{HL} \quad (2)$$

Saturation Constraint

$$\sum_{J=1}^{N_p} \bar{\delta}\, S_J = 0 \quad (3)$$

Mole-Fraction Constraints $J = 1, 2, \ldots, N_p$

$$\sum_{I=1}^{N_c} x_{IJ} = 1.0 \quad (4)$$

An implicit finite difference formulation of these equations has been given by Coats.[2]

The finite difference approximations result in a system of linear equatons of the form

$$Ax = b \quad (5)$$

where A is the coefficient matrix and x is the unknown vector.

The implicit treatment requires the solution of $n_e = N_c + 1$ unknowns per block where N_c is the number of components in the process description. Moreover, the unknowns may vary by type (mole fractions, saturations, temperature, pressure) from block to block. The matrix A with the natural ordering has either five (two-dimensional problems) or seven (three-dimensional problems) diagonals and each element of A is an $n_e \times n_e$ submatrix. A is generally asymmetric and may not be diagonally dominant.

ALGORITHM DESCRIPTION

For a rectangular grid the blocks can be divided into sets of "red" and "black" blocks, such that blocks of one color are coupled through the five point finite difference approximation only to blocks of the other color. An ordering in which we first number all red blocks and then all black blocks is termed a two-cyclic, red/black or checkerboard ordering. Various red/black orderings may be chosen for a particular grid. In the case of the proposed method, we have found it best to use a red/black ordering in which each of the two sets is ordered by lines or planes in the direction(s) of largest transmissibilities. Figure 1 shows a two-line red/black ordering on a 9x5 grid. Blocks numbered from 1 to 23 are considered red blocks while blocks 24 through 45 are considered black. Figure 2 shows the same type of ordering on a grid containing zero pore volume blocks.

With red/black ordering, the coefficient matrix A of equation 5 is in two-cyclic form. Partitioning A according to the two sets of blocks, Equation 5 becomes

$$\begin{bmatrix} D_1 & B_2 \\ B_1 & D_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad (6)$$

where D_1 and D_2 are $n_r \times n_r$ and $n_b \times n_b$ block diagonal matrices.

Equation 6 may be simplified to

$$\begin{bmatrix} I_1 & B_4 \\ B_3 & I_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \quad (7)$$

where I_1 and I_2 are $n_r n_e \times n_r n_e$ and $n_b n_e \times n_b n_e$ identity matrices, respectively, and

$$B_3 = D_2^{-1} B_1, \quad B_4 = D_1^{-1} B_2, \quad g_1 = D_1^{-1} b_1$$

and $g_2 = D_2^{-1} b_2$.

We can reduce the order of this $(n_r + n_b)n_e \times (n_r + n_b)n_e$ matrix problem by eliminating the lower left block in Equation 7. The resulting equation is

$$\begin{bmatrix} I_1 & B_4 \\ 0 & M \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} g_1 \\ h_2 \end{bmatrix} \quad (8)$$

where $M = I_2 - B_3 B_4$ and $h_2 = g_2 - B_3 g_1$

Thus, we may obtain x_2 by solving

$$M x_2 = h_2 \quad (9)$$

and obtain x_1 by back substitution

$$x_1 = g_1 - B_4 x_2 \quad (10)$$

The motivation for solving Equations 9 and 10 instead of Equation 5 is two-fold. We first note that the equations generated by a steamflood simulator are elliptic in character. Hageman[3] has shown that n-line block SOR ($n \geq 2$) applied to the cyclically reduced matrix equation is asymtotically iteratively faster than when applied to Equation 7 for the numerical solution of five point difference approximations of certain second order self-adjoint elliptic partial differential equations. The second motivation is somewhat intuitive. Since the reduced matrix M represents couplings of blocks more distant from each other than in the original coefficient matrix A, a troublesome negative compressibility/transmissibity ratio which affects A may not be evidenced in M. For steamflood problems our experience has been that SOR techniques are more effective in solving Equation 9.

IMPLEMENTATION

For two-dimensional problems cyclic reduction followed by n-line block successive overrelaxation is most effective for $n \geq 2$ since M is two-cyclic. The implementation is straightforward and details for one-equation formulation may be found in Hageman[3].

In order to significantly reduce storage requirements in the three dimensional case, a somewhat different formulation can be used. To develop the appropriate notation, partition the $n_b n_e \times n_b n_e$ identity matrix I_2 as follows:

$$I_2 = \begin{bmatrix} P_1^T \\ P_2^T \\ \vdots \\ P_\ell^T \end{bmatrix} \tag{11}$$

where P_i is a $n_b n_e \times q_i n_e$ submatrix, q_i is the number of black blocks in the ith n-line, and ℓ is the number of n-lines.

Define the vector $v_m^{(k)}$ by

$$v_m(k) = \begin{bmatrix} y_1^{(k+1)} \\ \vdots \\ y_{m-1}^{(k+1)} \\ y_m^{(k)} \\ \vdots \\ y_\ell^{(k)} \end{bmatrix} \tag{12}$$

where $y_i^{(k)}$ is the kth iterate for n-line i.

Let M_i, i = 1 to ℓ, denote the explicitly formed blocks in M representing the coupling among the black blocks in the ith n-line. We note that the M_i are tridiagonal for one-line SOR and pentadiagonal for two-line SOR. In addition, let c_i denote the right hand side subvector corresponding to n-line i.

Then the "semi-implicit" cyclic n-line block SOR iteration with relaxation parameter ω is given by

$$y_i^{(k+1)} = y_i^{(k)} + \omega \cdot M_i^{-1} \cdot \left[c_i - y_i^{(k)} + P_i^T B_3 (B_4 v_i^{(k)}) \right] \tag{13}$$

Blocking M by planes rather than n-lines Equation 5 may be written in the following partitioned form:

$$\begin{bmatrix} W_1 & Q_1 & R_1 & & & & \\ T_2 & W_2 & Q_2 & R_2 & & & \\ S_3 & T_3 & W_3 & Q_3 & R_3 & & \\ & \cdot & \cdot & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot & \cdot & R_{p-2} \\ & & & \cdot & \cdot & \cdot & Q_{p-1} \\ & & & & S_p & T_p & W_p \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \cdot \\ \cdot \\ \cdot \\ y_p \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \cdot \\ \cdot \\ \cdot \\ c_p \end{bmatrix} \tag{14}$$

Using the notation in Equations 11 and 12 and replacing n-lines by planes (slices), the explicit form of the cyclic slice SOR (CSSOR) iteration is

$$y_i^{(k+1)} = (1-\omega) y_i^{(k)} + \omega W_i^{-1} \cdot \left[c_i - S_i y_i^{(k+1)} - T_i y_{i-1}^{(k+1)} - Q_i y_{i+1}^{(k)} - R_i y_{i+2}^{(k)} \right] \tag{15}$$

The semi-implicit form is the same as in Equation 13 replacing M_i by W_i and letting n be the number of lines per plane.

CALCULATION OF ω

We compute a single relaxation parameter ω for all equations. The method is described as follows:

1. Initially set $\omega = 1$ and the first iterate $x^{(o)}$ to all zeros.
2. Make SOR sweeps for k = 1, 2.
3. Make SOR sweep k + 1.
4. Compute $d_k = \max_i |x_i^{(k)} - x_i^{(k-1)}|$ and

   $d_{k+1} = \max_i |x_i^{(k+1)} - x_i^{(k)}|$
5. Compute $\mu_{k+1} = d_{k+1}/d_k$
6. If $|\mu_{k+1} - \mu_k|$ is sufficiently small, continue to step 7, otherwise go to step 3.
7. Compute

   $$\omega = 2/\left(1 + \sqrt{1 - \mu_{k+1}}\right)$$

MULTI-LEVEL ITERATION

Consider the two dimensional case where M is in mxm block tridiagonal form (m > 2) and the unknown vector x is partitioned into m groups corresponding to n-lines. In block SOR, after groups 1 to j have been relaxed in ascending order, then for a fixed integer s $\geq$ 1 groups j-1 to j-s may be relaxed in descending order. Upon completion of this process for j=1 to m, group i is at iteration level 1+min (m-i,s). The aforementioned procedure forms the first part of the two part scheme. The process is then repeated in the opposite direction beginning with group m, and we have a single iteration in the multi-level scheme.

Suppose q is a positive integer such that q < m + 1 - s. Then only data corresponding to, at most, s + q - 1 distinct n-lines need occupy real memory over the course of q's successive block relaxations. This can significantly reduce paging for out-of-core solution and thereby result in overall improvements in CPU time as compared with the standard single-level SOR implementations. In the three dimensional case when M is block pentadiagonal, this type of multi-level iteration yields similar advantages.

The multi-level cyclic n-line SOR (n $\geq$ 2) for the two dimensional case is described as follows:

1. Relax in order n-lines k, k-1, k-2, ..., k-s for k=1, 2, ..., ℓ where ℓ is the number of n-lines (ℓ > 2)
2. Relax in order n-lines k, k+1, k+2, ..., k+s for k=ℓ, ℓ-1, ..., 1
3. Repeat steps 1 and 2.

For the three dimensional case, the multi-level cyclic slice SOR (MCSSOR) is described by:

1. Relax in order planes k, k-2, k-4, ...k-2w for k=1, 2, ..., p where p is the number of planes (p > 2)
2. Relax in order planes k, k+2, k+4, ...k+2w for k=p to 1
3. Repeat steps 1 and 2.

STORAGE REQUIREMENTS

Defining i, j, and k as the number of grid blocks in the x, y and z directions respectively and N_w as the product of the smaller two of i, j, and k, Table 1 gives the storage requirements for five of the methods considered. For example, in three dimensional problems with N_w=40, cyclic two-line SOR requires about 53% of the storage needed by the alternate diagonal (D-4) method. With $N_w \geq 80$, the requirement drops to 28%. For CSSOR with k=4, the requirements are 78% and 42% respectively.

APPLICATION

SSOR, CSSOR, MCSSOR and the alternate diagonal method (D-4) have been implemented in INTERCOMP's Implicit Steamflood Model developed by Coats[2]. This model tracks the flow of any number of components involved in the thermal recovery process. Each component may partition among any of three phases (water, oil and gas) as dictated by user-specified pressure-and temperature-dependent equilibrium constants (K-values). The density, viscosity and enthalpy of each phase is treated as a function of pressure, temperature and composition. Mass transport is described by Darcy flow, accounting for viscous, gravitational and capillary forces, in three spatial directions. The heat transport includes the mechanisms of conduction and convection within the reservoir, and conductive heat loss to the overburden and underburden strata.

The linear solution methods were tested by simulating a combination steam stimulation-steam drive operation in a three-dimensional element of a reservoir. The geometric configuration for the sample problem is illustrated in Figure 3. The 15-foot, 4.925-acre reservoir element is modeled with 3 layers and an 11 x 11 block grid in the areal plane. Note that the cross-hatched blocks in Figure 3 represent zero pore volume grid blocks which are ignored by the simulator. Rock properties are given in Table 2.

The reservoir oil is characterised as a two-component system. The heavy oil component has an API gravity of 16° and a viscosity of 137 cp at the reservoir temperature of 200°F. The light oil component has an API gravity of 74° and viscosity of 1.7 cp. Other oil component properties are given in Table 3. The properties of the water component are input via a steam table.

The heavy component in the oil is assumed to be non-distillable, i.e., it does not vaporize in the temperature range expected during the steam-flooding. The equilibrium distribution of the light component between the gas and liquid phases is specified by the following K-value relationship:

$$K = \frac{97749}{p} \text{ EXP } \left(- \frac{5193}{T} \right)$$

where K = mole fraction in gas phase/mole fraction in oil phase,

p = pressure, psia,

and T = temperature, °R.

The water-oil and gas-oil relative permeability data used for this problem are given in Table 4. The effect of temperature on relative permeability is included by specifying partial derivatives of the saturation functions with respect to temperature.

The locations of the wells are indicted in Figure 3. Well 1 in the center serves both as an injector and producer, while wells 2 through 4 are injectors. All wells are perforated in layers 2 and 3 only.

Steam of 75% quality at 450°F is injected at the maximum rate of 1000 STB/D (cold water equivalent) in well 1 and 500 STB/D in wells 2 through 4. The flowing bottomhole pressure constraint during injection is 1000 psia. Steam tion is terminated after 10 days, when the ge reservoir pressure reaches 997 psia. After a period of 10 days, steam injection is resumed in 2-4 and production is initiated from well 1 at owing bottomhole pressure of 150 psi. The oil water production rates are plotted in Figures 4 5. A summary of the results of the simulation is n in Table 5.

Simulation runs were made on a Harris-800 uter using direct solution with D-4 ordering, , CSSOR and MCSSOR with w=5. All slices were n in the x-z plane. Linear solution CPU time and step cuts due to convergence failure of the ix solution scheme are given in Table 6. The convergence failures of SSOR illustrate the iculty in applying SOR techniques directly to the ficient matrix A. In contrast, MCSSOR untered only one failure and obtained the best time, (50% faster than the alternate diagonal od). In a similar study with a 11 x 10 x 5 grid active blocks), MCSSOR was three times faster D-4.

The second example is a cyclic steam stimulation lem. The 6 x 6 areal grid configuration for this e layer problem is shown in Figure 6. This esents one-fourth of the full element of etry. Data on the grid block dimensions and rock erties are given in Table 7. The fluid erties and relative permeability data are the as those listed in Tables 3 and 4 for the first .

Steam of 75% quality is injected at 450°F at the of 150 STB/D for a period of 10 days. The age reservoir pressure at the end of the steam ction is 505 psia. Production at a flowing omhole pressure of 100 psia is initiated immedi- y following the steam injection without an rvening soak period. The average reservoir sure declines to 108 psia after 10 days. The ulative oil and water produced over the 10-day od are 364 and 1185 STB, respectively.

Table 8 shows the solution method summary for lem 2. As in problem 1, all slices were taken in x-z direction. MCSSOR was almost three times ter than SSOR and 34% faster than the alternate gonal method. No convergence difficulties were ountered by MCSSOR or CSSOR; SSOR, however, failed converge on two occasions. It should be noted t in similar studies involving cylindrical grids, three iterative schemes performed effectively. ng r-z slices CSSOR, MCSSOR and SSOR generally verged in several iterations due to the relatively ll crossflow in the θ-direction.

CLUSIONS

1. For large three-dimensional problems, the proposed method can offer significant savings in CPU time (factors of three or more) and storage (factors of 2 or more).

2. Convergence difficulties due to negative compressibility transmissibility ratios are substantially reduced by solving the reduced problem.

NOMENCLATURE

A	Cross-section area normal to flow, ft^2
H_J	Enthalpy of phase J, Btu/mole
I	Component index
J	Phase index
k	Absolute permeability, md x 0.00633
k_{rJ}	Relative permeability to phase J, fraction
L	Distance between adjacent grid block centers, ft
M_f	Reservoir rock heat capacity, Btu/cu. ft rock- °F
N_c	Number of components
N_p	Number of phases
p	Gas phase pressure, psia
P_{cJ}	Phase J capillary pressure, $p_J - p$, psi
$\dot{q}_I$	Production rate of component I from grid block, mole/D
$\dot{Q}_H$	Production rate of enthalpy from grid block associated with fluid production, Btu/D
Q_{HL}	Heat loss rate to overburden and under-burden from grid block, Btu/D
S_J	Saturation of phase J, fraction
T	Temperature, °R
T_i	Initial reservoir temperature, °R
U_J	Internal energy of phase J, Btu/mole
V	Grid-block volume, $\Delta x \Delta y \Delta z$, cu ft
x_{IJ}	Mole fraction of component I in phase J
Z	Subsea depth measured positively vertically downward, ft
γ_J	Specific weight of phase J, psi/ft
λ	Thermal conductivity, Btu/D-ft-°F
μ_J	Viscosity of phase J, cp
ρ_J	Density of phase J, mole/cu ft
τ	Fluid-flow transmissibility, kA/L, res. cu ft-cp/D-psi
τ_c	Heat-conduction transmissibility, $\lambda A/L$
ϕ	porosity, fraction
$\bar{\delta}$	Difference operator with respect to time

REFERENCES

1. Price, H. S. and Coats, K. H.: "Direct Methods in Reservoir Simulation", Soc. Pet. Engr. J., (June, 1974), p. 295.

2. Coats, K. H.: "In Situ Combustion Model", Soc. Pet. Engr. J., (Dec. 1980), p. 533-554.

3. Hageman, L. A. and Varga, R. S.: "Block Itera-tive Methods for Cyclically Reduced Matrix Equations", Numer. Math., (1964) Vol. 6, p. 106-119.

4. Varga, R. S.: Matrix Iterative Analysis, Prentice-Hall, Inc., Englewood, 1962.

5. Young, D. M.: Iterative Solution of Large Linear Systems, Academic Press, New York, 1971.

6. Wattenbarger, R. A. and Thurnau, D. H.: "Application of SSOR to Three-Dimensional Reservoir Problems", Fourth SPE Symposium on Numerical Simulation of Reservoir Performance, (1976), p. 135-147.

TABLE 1 - STORAGE REQUIREMENTS ($x\ n_e^2$)

METHOD	2-D	3-D
Two-Line SOR	5 IJ	7 IJK
Cyclic Two-Line SOR	6.5 IJ	8.5 IJK
SSOR	---	(2K+3) IJK
CSSOR	---	(K+8.5) IJK
D4 - GAUSS	$\frac{J^2}{2}\left(I - \frac{J}{3}\right)$	0.4 N_w IJK

TABLE 2 - RESERVOIR DATA

Gross thickness, ft	15.0
x-direction length, ft	476.7
y-direction length, ft	450.0
Number of layers	3
Number of grid blocks in x-direction	11
Number of grid blocks in y-direction	11
Horizontal permeability, md	4000
Vertical permeability, md	2000
Porosity, %	36
Rock compressibility, $10^{-6}psi^{-1}$	150
Rock thermal conductivity, Btu/ft-D-^oF	38.4
Rock heat capacity, Btu/ft^3-^oF	35
Average initial pressure, psi	100
Water saturation, %	17
Oil saturation %	83
Active pore volume, Mbbl	138.1

TABLE 3 - OIL COMPONENT PROPERTIES

	Oil Component	
	Heavy	Light
Tank oil gravity, °API	16	74
Molecular weight	300	100
Compressibility, 10^{-6} psi^{-1}	15	31
Thermal expansion coefficient, 10^{-6} °F	410	690
Specific heat, Btu/lb-°F	.5	.528
Mole % in oil	85	15

	Viscosity (cp)	
Temperature (°F)	Heavy	Light
200	137.0	1.7
300	15.1	0.75
400	4.7	0.58
500	2.3	0.53
600	1.5	0.51

TABLE 4 - RELATIVE PERMEABILITY DATA

Water-Oil Relative Permeability

Water Saturation	Relative Permeability Water	Relative Permeability Oil	Capillary pressure (psi)
0.17	0	1.00	4.0
0.20	.01	.75	2.5
0.25	.03	.50	1.55
0.30	.05	.35	1.00
0.40	.09	.20	.55
0.50	.13	.12	.35
0.60	.18	.06	.22
0.70	.25	0	.15
0.80	.35	0	0
1.00	.35	0	0

Gas-Oil Relative Permeability

Liquid Saturation	Relative Permeability Oil	Relative Permeability Gas	Capillary pressure (psi)
.20	0	.60	5
.30	.06	.40	4
.40	.12	.28	3
.50	.20	.20	2
.60	.30	.15	1
.70	.45	.09	0
.80	.65	.05	-1
.90	.85	.03	-2
1.00	1.00	0	-3

TABLE 4 - RELATIVE PERMEABILITY DATA (Continued)

Temperature Dependence

Quantity	Derivative with Respect to Temperature, %/100°F
Irreducible water saturation	4.000
Residual oil to water	-6.154
Residual oil to gas	-1.231
Water relative permeability at residual oil saturation	0.246

TABLE 5 - SUMMARY OF SIMULATION RESULTS

Stimulation and Soak Periods

Steam temperature, °F	450
Steam quality	0.75
Steam injected in well 1, MSTB	7.4
Steam injected in wells 2-4, MSTB	10.5
Cumulative steam injected, MSTB	17.9
Injection time, days	10
Average reservoir pressure at 10 days, psi	997
Soak time, days	10
Cumulative heat loss at 20 days, MMBtu	499.3

Production/Steam Drive Periods

Steam temperature, °F (wells 2-4)	450
Steam quality	0.75
Total elapsed time, days	150
Cumulative steam injected, MSTB	213.1
Cumulative water production, MSTB	191.6
Cumulative oil production, MSTB	63.3
Cumulative heat loss, MMBtu	7538
Average reservoir pressure, psi	242

TABLE 6 - SOLUTION METHOD SUMMARY FOR EXAMPLE 1

Solution Method	CPU Time (sec)	Time Steps Cuts
D-4	4633	0
SSOR	7256	9
CSSOR	3901	3
MCSSOR (w=6)	3153	1

TABLE 7 - RESERVOIR DATA FOR CASE 2

Gross thickness, ft	36
x-direction length, ft	100
y-direction length, ft	115
Number of layers	3
Number of grid blocks in x-direction	6
Number of grid blocks in y-direction	6
Porosity, %	36
Rock compressibility, 10^{-6}psi^{-1}	150
Rock thermal conductivity, Btu/ft-D-^{o}F	38.4
Rock heat capacity, Btu/ft^{3}-^{o}F	35
Average initial pressure, psi	100
Reservoir temperature, ^{o}F	200
Water saturation, %	17
Oil saturation, %	83
Pore volume, Mbbl	26.54

GRID BLOCK SIZES AND PEREMABILITIES IN X AND Y DIRECTIONS

Block index	Block size, ft in direction		Permeability, md in direction	
	x	y	x	y
1	3.5	4.0	4000	3500
2	4.5	6.0	3500	3100
3	8.0	10.0	3100	2800
4	14.0	18.0	2700	2500
5	25.0	29.0	2500	2200
6	45.0	48.0	2000	1800

GRID BLOCK SIZES AND PERMEABILITIES IN Z DIRECTION

Layer No.	Thickness, ft	Vertical Permeability, md
1	14	1000
2	12	800
3	10	500

TABLE 8 - SOLUTION METHOD SUMMARY FOR EXAMPLE 2

Solution Method	CPU Time (sec)	Time Steps Cuts
D-4	1325	0
SSOR	2851	2
CSSOR	1091	0
MCSSOR (r=3)	990	0

1	24	6	29	11	34	16	39	21
25	2	30	7	35	12	40	17	44
3	26	8	31	13	36	18	41	22
27	4	32	9	37	14	42	19	45
5	28	10	33	15	38	20	43	23

Figure 1-Red/Black Ordering

			18	7	23			
		19	3	24	8	28		
	16	4	20	9	25	12	29	
	1	21	5	26	10	30	13	
2	17	6	22	11	27	14	31	15

Figure 2-Red/Black Ordering With Zero Pore Volume Blocks

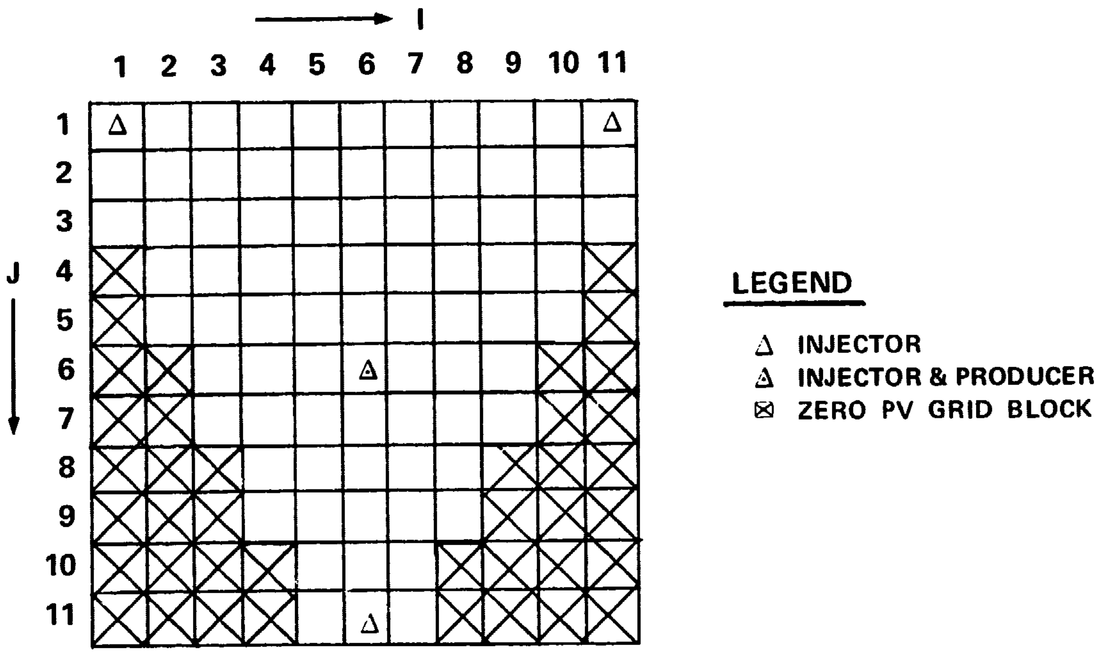

Figure 3–Areal View of Reservoir Element

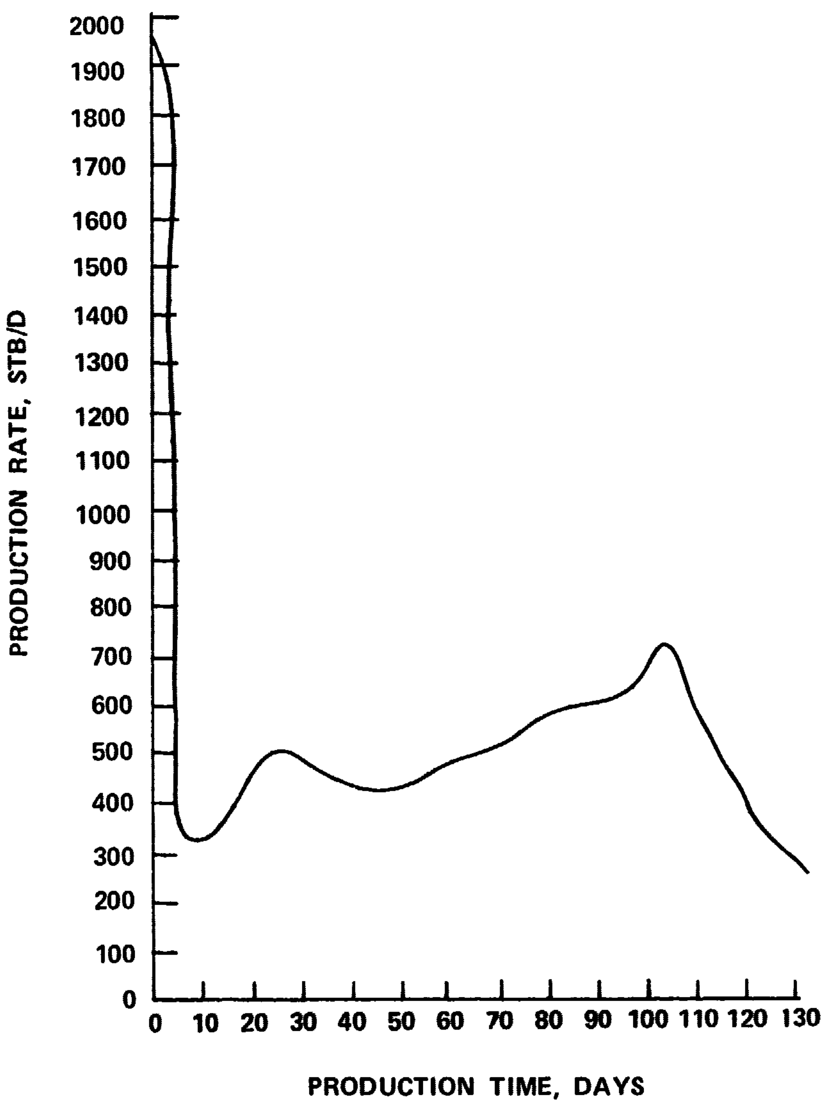

Figure 4–Oil Production Rato

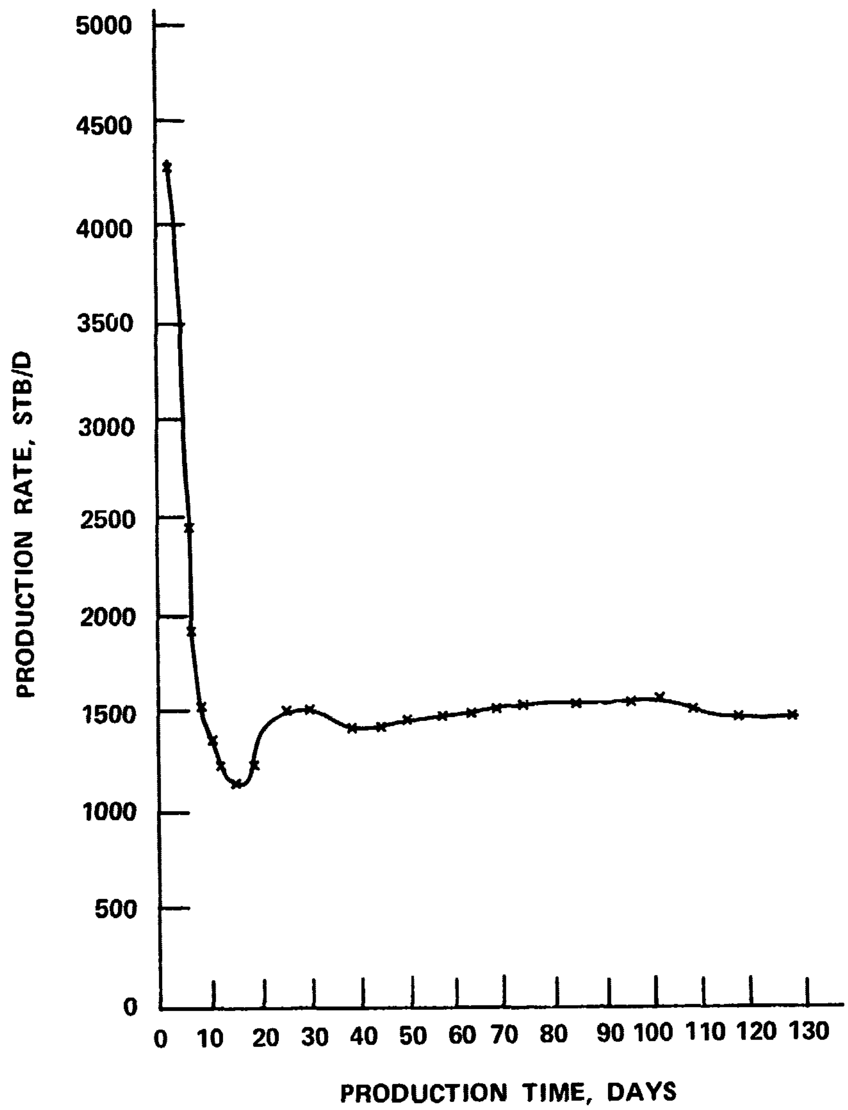

Figure 5-Water Production Rate

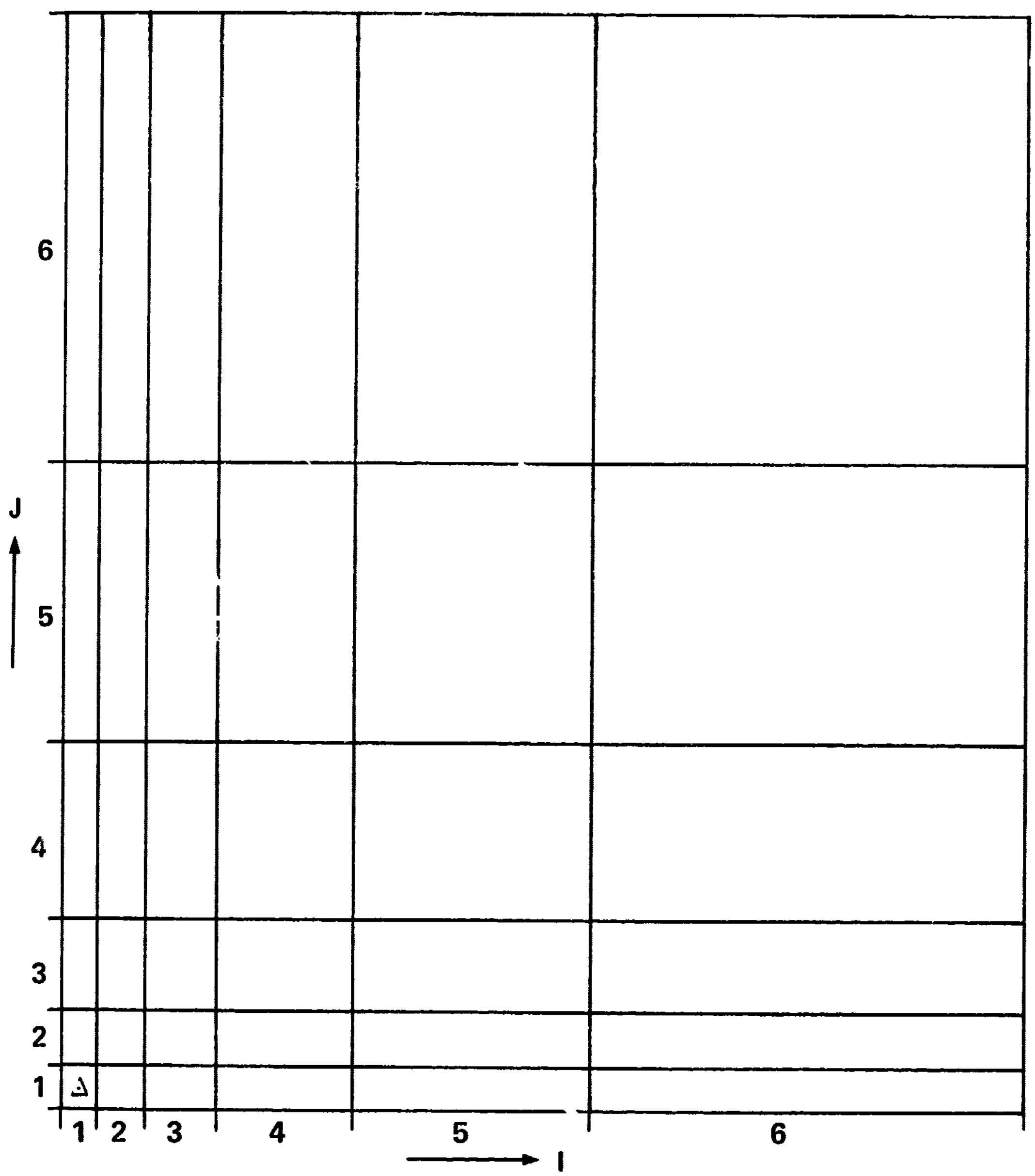

Figure 6–Areal Grid for Second Case