

# A Highly Implicit Steamflood Model

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## ABSTRACT

This paper describes a three-dimensional, highly implicit numerical model for simulating steamflooding with distillation or solution gas. The model uses direct solution to solve simultaneously three and four equations for the dead oil and two-component oil cases, respectively. The model is compared in stability and computing time with a model reported earlier. The paper includes comparative discussion of alternate steamflood model formulations, one of which we have adopted as a highly stable, isothermal, black-oil model formulation.

## INTRODUCTION

A brief review of published descriptions of steamflood models is given in an earlier paper.<sup>1</sup> That paper described a partially compositional, three-dimensional model that solves first a single-variable pressure equation, then two simultaneous saturation equations. In our experience with dead-oil steamflood problems, that model exhibits adequate stability in most cases and marginal stability in some cases. In some compositional problems, the formulation of that model leads to deteriorating material balances for light hydrocarbon components.

The model described here was developed to gain improved stability for all types of steamflood problems and to eliminate the material balance shortcoming of the earlier model formulation in compositional problems. This highly implicit, three-dimensional model treats oil as a two-component mixture to accommodate problems involving solution or inert gas or distillation. The model simultaneously solves three equations for the dead-oil case and four equations for the compositional case. Transmissibilities, capillary pressures, and production terms are treated implicitly in saturations and composition; they also are treated implicitly in temperature in grid blocks where no free gas is present.

The term "implicit" refers to evaluation of

interblock flow terms and production rates at the new time level,  $n + 1$ . We have found insensitivity to explicit or implicit dating of molar densities and viscosities in these terms and therefore simply evaluate them explicitly. We evaluate relative permeabilities at time level  $n + 1$  by the first-order approximation,  $k_r(S_{n+1}) = k_r(S_n) + (\partial k_r / \partial S)_n \delta S$ , which ignores second- and higher-order Taylor series terms  $(\partial^2 k_r / \partial S^2)_n (\delta S)^2 / 2 + \dots$ . Temperature dependence of relative permeability, if present, is treated explicitly.

We present the model equations, and describe the highly implicit formulation and method for solution. This model is compared with the earlier steamflood model in stability and efficiency through discussion and example field problems.

## MODEL DESCRIPTION

### BASIC EQUATIONS

The model consists of five equations expressing conservation of energy, conservation of mass, and phase equilibrium. The mass conservation equations apply to water and to two hydrocarbon components. In finite-difference form, these equations are

Energy Balance

$$\frac{V}{\Delta t} \delta [\phi (\rho_w S_w U_w + \rho_g S_g Y_s U_s + (\rho_o S_o x_1 + \rho_g S_g y_1) U_1 + \rho_o S_o x_2 U_2) + (1 - \phi) (\rho C_p)_R T] = \Delta (T_H \Delta p) + \Delta (T_C \Delta T) - q_L - q_H \dots (1)$$

Mass Balance on H<sub>2</sub>O

$$\frac{V}{\Delta t} \delta (\phi \rho_w S_w + \phi \rho_g S_g Y_s) = \Delta [T_w (\Delta p_w - \gamma_w \Delta Z) + T_g Y_s (\Delta p_g - \gamma_g \Delta Z)] - q_w \rho_w - q_g \rho_g Y_s \dots (2)$$

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Mass Balances on Hydrocarbon Components

$$\frac{V}{\Delta t} \delta (\phi \rho_o S_o x_i + \phi \rho_g S_g y_i) = \Delta [T_o x_i (\Delta p_o - \gamma_o \Delta Z) + T_g y_i (\Delta p_g - \gamma_g \Delta Z)] - q_o \rho_o x_i - q_g \rho_g y_i, \quad i = 1, 2 \dots \dots (3,4)$$

Gas Mol Fraction Constraint

$$\delta y_1 + \delta y_s = 1 - y_{1n} - y_{sn} \dots \dots \dots (5)$$

Hydrocarbon Component 2 is nonvolatile so that  $y_2$  in Eq. 4 is 0. Component 1 is volatile with  $y_1 = K_1 x_1$ , where  $K_1$  is a specified function of temperature and pressure, independent of composition. Depending on the  $K_1$  function specified, the model can treat Component 1 as inert gas, solution gas, a distillable component, or a pseudo light end lying between solution gas and a distillable component.

These model equations (Eqs. 1 through 5) are identical to those in Ref. 1, except that two rather than three hydrocarbon components are treated. This reduction was made because (1) very few of our steamflood applications have required both solution gas and a distillable component, and (2) treatment of three rather than two hydrocarbon components using the highly implicit method described here would roughly double the model's computing time requirements.

PVT AND RELATIVE PERMEABILITY TREATMENT

Ref. 1 describes the PVT dependence of molar densities, viscosities, and internal energies. The only altered or additional treatment here is as follows.

Water and steam internal energies,  $U_w$  and  $U_s$ , are calculated as their steam table values less the internal energy of water at initial reservoir temperature,  $T_i$ . Hydrocarbon component internal energies are calculated as

$$U_1 = C_{p1} (T - T_i), \dots \dots (6a)$$

$$U_2 = C_{p2} (T - T_i), \dots \dots (6b)$$

where the specific heats are assumed constant. Oil-phase molar density is entered as a single-value function of Component 1 mol fraction at original reservoir pressure and temperature. Denoting this value by  $\rho_o(x_1)$ , the value of  $\rho_o$  is then calculated as

$$\rho_o(p, T, x_1) = \rho_o(x_1) (1 - C_{To} (T - T_i) + c_o (p - p_i)) \dots \dots \dots (7)$$

Oil viscosity is calculated as the product of a compositional-dependent factor  $\mu_o(x_1)$  and a temperature-dependent factor  $\mu_o(T)$ :

$$\mu_o(T, x_1) = \mu_o(x_1) \mu_o(T) \dots (8)$$

The factor  $\mu_o(x_1)$  is 1.0 at  $x_1 =$  initial  $x_1$  and varies with  $x_1$  as specified in tabular form.  $\mu_o(T)$  is thus the viscosity of original reservoir oil as a function of temperature. Stock-tank oil composition is calculated using wellstream composition and a specified value of  $K_1$  at stock-tank conditions.

Relative permeabilities with temperature dependence are calculated using Stone's method<sup>2</sup> as described in Ref. 1. The implicit treatment of interblock flow terms described below requires relative permeabilities and capillary pressures evaluated at time level  $n + 1$ . These are approximated by retaining only the first-order terms in Taylor series at time level  $n$  saturations. Oil relative permeability is given by<sup>2</sup>

$$k_{ro} = (k_{row} + k_{rw}) \cdot (k_{rog} + k_{rg}) - k_{rw} - k_{rg} \dots \dots \dots (9)$$

Partial derivatives with respect to  $S_w$  and  $S_g$  are evaluated as

$$\frac{\partial k_{ro}}{\partial S_w} = k_{row} = (dk_{row}/dS_w + k'_{rw}) (k_{rog} + k_{rg}) - k'_{rw} \dots \dots \dots (10)$$

$$\frac{\partial k_{ro}}{\partial S_g} = k_{rog} = (k_{row} + k_{rw}) (dk_{rog}/dS_g + k'_{rg}) - k'_{rg} \dots \dots \dots (11)$$

LEFT-HAND-SIDE EXPANSIONS

The expansions described here and in later sections involve variables dated at the new time level  $n + 1$ . Since these new time level values are unknown until convergence, all such  $n + 1$  level variables are approximated in the calculations by their latest iterative values.

As described in Ref. 1 and further illustrated here, the left-hand sides of Eqs. 1 through 5 are expanded in terms of the time differences  $\delta T$ ,  $\delta S_g$ ,  $\delta S_w$ ,  $\delta x_1$ , and  $\delta p$ . To illustrate, the left-hand side of the Component 1 mass balance, Eq. 3, can be expressed as

$$\begin{aligned} & \delta(\phi \rho_o S_o x_1 + \phi \rho_g S_g y_1) = \\ & (\phi \rho_o S_o)_n \delta x_1 + x_{1n+1} (S_{on} (\phi_{n+1} \delta \rho_o \\ & + \rho_{on} \delta \phi) - (\phi \rho_o)_{n+1} (\delta S_w + \delta S_g)) \\ & + (\phi \rho_g S_g)_n \delta y_1 + y_{1n+1} (S_{gn} (\phi_{n+1} \delta \rho_g \\ & + \rho_{gn} \delta \phi) + (\phi \rho_g)_{n+1} \delta S_g) \dots (12) \end{aligned}$$

This expansion is incomplete in that  $\delta \rho_o$ ,  $\delta \phi$ ,  $\delta y_1$ , and  $\delta \rho_g$  must be expressed in terms of the unknowns  $\delta T$ ,  $\delta x_1$ , and  $\delta p$ . As developed in the Appendix,

$$\begin{aligned} \delta \rho_o & \cong \rho_{oT} \delta T + \rho_{ox1} \delta x_1 \\ & + \rho_{op} \delta p + \rho_o^\ell - \rho_{on} - \rho_{oT} (T^\ell - T_n) \\ & - \rho_{ox1} (x_1^\ell - x_{1n}) - \rho_{op} (p^\ell - p_n) \dots (13) \end{aligned}$$

$$\delta \phi = \phi_i c_r \delta p \dots (14)$$

$$\begin{aligned} \delta y_1 & \cong x_{1n} K_{1T} \delta T + x_{1n} K_{1p} \delta p \\ & + K_{1n+1} \delta x_1 + x_{1n} (K_1^\ell - K_{1n} \\ & - K_{1T} (T^\ell - T_n) - K_{1p} (p^\ell - p_n)) \dots (15) \end{aligned}$$

$$\begin{aligned} \delta \rho_g & \cong \rho_{gT} \delta T + \rho_{gp} \delta p \\ & + \rho_g^\ell - \rho_{gn} - \rho_{gT} (T^\ell - T_n) \\ & - \rho_{gp} (p^\ell - p_n) \dots (16) \end{aligned}$$

where superscript  $\ell$  denotes latest iterative values. Insertion of Eqs. 13 through 16 into Eq. 12 gives an expression of the form

$$\delta(\phi \rho_o S_o x_1 + \phi \rho_g S_g y_1) =$$

$$\begin{aligned} & C_{30} + C_{31} \delta T + C_{32} \delta S_g + C_{33} \delta S_w \\ & + C_{34} \delta x_1 + C_{35} \delta p \dots (17) \end{aligned}$$

Expansions for the other equations lead to the

general form  $C_{io} + \sum_{j=1}^5 C_{ij} p_j$  for the left-hand side of the  $i$ th equation,  $i = 1, 2, \dots, 5$ . We have introduced here the notation  $p_1 = \delta T$ ,  $p_2 = \delta S_g$ ,  $p_3 = \delta S_w$ ,  $p_4 = \delta x_1$ , and  $p_5 = \delta p$ .

#### IMPLICIT TREATMENT OF FLOW TERMS

For brevity, we will discuss the term in Eq. 3 for interblock flow of Component 1 in the  $x$  direction from grid block  $i - 1, j, k$  to  $i, j, k$ :

$$\begin{aligned} & T_o x_1 (\Delta p - \gamma_o \Delta Z) + T_g y_1 (\Delta p_g \\ & - \gamma_g \Delta Z) \dots (18) \end{aligned}$$

The notation  $\Delta X$  is  $X_{i-1} - X_i$  and the transmissibilities are

$$T_o = T_x k_{ro} \rho_o / \mu_o \dots (19a)$$

$$T_g = T_x k_{rg} \rho_g / \mu_g \dots (19b)$$

where  $T_x$  is the interblock absolute transmissibility  $(kA/\ell)_{i-1/2, j, k}$ . We have found insensitivity to implicit vs explicit dating of the PVT terms,  $\rho_o/\mu_o$ ,  $\rho_g/\mu_g$ ,  $\gamma_o$ , and  $\gamma_g$  and simply express them at time level  $n$ . Viscosities and  $\rho_g$  are calculated as upstream grid block values;  $\rho_o$ ,  $\gamma_o$ , and  $\gamma_g$  are calculated as arithmetic averages of their respective adjacent grid block values.

As introduced by Blair and Weinaug<sup>3</sup> and discussed by Nolen and Berry,<sup>4</sup> a fully implicit treatment requires dating of all terms in Eq. 18 at time level  $n+1$ . Considering the explicit PVT terms just discussed, we then write a highly implicit expression for the oil-phase flow term as

$$\begin{aligned} & (T_o x_1 (\Delta p - \gamma_o \Delta Z))_{n+1} = \\ & (T_o x_1 (\Delta p - \gamma_o \Delta Z))_n + T_{on} x_{1n} \Delta \delta p \\ & + (\Delta p_{n+1} - \gamma_o \Delta Z) (T_{on} \delta x_1 \\ & + x_{1n+1} \delta T_o) \dots (20) \end{aligned}$$

where

$$\begin{aligned} \delta T_o & = T_x (\rho_o / \mu_o)_n (k_{rosw} \delta S_w \\ & + k_{rosg} \delta S_g) \dots (21) \end{aligned}$$

Relative permeability ( $k_{ro}$ ) is evaluated at the oil-phase upstream grid block saturations. The  $\delta S_w$

and  $\delta S_g$  values in Eq. 21 thus apply at the oil-phase upstream block. This upstream block is determined at the start of iteration  $\ell + 1$  by the value of  $\Delta p^\ell - \gamma_o \Delta Z$ . Thus, the upstream block and corresponding values of  $k_{rgn}$  and  $T_{on}$  can change from iteration to iteration within a time step.

The highly implicit expression for the gas-phase flow term in Eq. 18 is

$$\begin{aligned} (T_g y_1 (\Delta p_g - \gamma_g \Delta Z))_{n+1} &= \\ & (T_g y_1 (\Delta p_g - \gamma_g \Delta Z))_n \\ & + T_{gn} y_{1n} (\Delta \delta p + \Delta \delta P_{cgo}) \\ & + (\Delta p_{n+1} + \Delta P_{cgon+1} - \gamma_g \Delta Z) \\ & (T_{gn} \delta y_1 + y_{1n+1} \delta T_g), \dots \dots (22) \end{aligned}$$

where

$$\delta T_g = T_x (\rho_g / \mu_g)_n k'_{rg} \delta S_g, \dots \dots (23)$$

$$\delta P_{cgo} = P'_{cgo} \delta S_g, \dots \dots (24)$$

and  $k_{rgn}$  and  $k'_{rg}$  are evaluated at the gas-phase upstream block determined by the sign of  $(\Delta p^\ell + P_{cgo}^\ell - \gamma_g \Delta Z)$ .  $\delta S_g$  and  $k'_{rg}$  in Eq. 23 apply at the gas-phase upstream block. The time difference  $\delta y_1$  used in Eq. 22 is

$$\begin{aligned} \delta y_1 &= K_{1n+1} x_{1n+1} - K_{1n} x_{1n} \\ &= K_{1n+1} \delta x_1 + (K_{1n+1} - K_{1n}) x_{1n} \\ & \dots \dots \dots (25) \end{aligned}$$

We found that the expansion of Eq. 22 led to significant stability in many cases and that more stable performance resulted from replacing  $\Delta P_{cgon+1}$  in Eq. 22 by  $\Delta P_{cgon}$  and determining flow direction from the sign of  $(\Delta p^\ell + \Delta P_{cgon} - \gamma_g \Delta Z)$ .

The right-hand side of the energy balance, Eq. 1, includes the convective energy flow term  $\Delta(T_H \Delta p)$  and the conductive term  $\Delta(T_c \Delta T)$ . For  $x$ -direction flow between grid blocks  $i-1, j, k$  and  $i, j, k$ , the convective term is

$$\begin{aligned} T_H \Delta p &= \sum_{m=w, o, g} \{ T_m H_m (\Delta p_m \\ & - \gamma_m \Delta Z) \} \dots \dots (26) \end{aligned}$$

The phase enthalpies are calculated as  $H_m = U_m + p/\rho_m$ , where

$$U_o = (x_1 C_{p1} + x_2 C_{p2}) (T - T_i) \dots \dots (27a)$$

$$U_g = y_1 C_{p1} (T - T_i) + y_s U_s \dots \dots (27b)$$

A conversion factor is necessary on the  $p/\rho$  term. The enthalpies are calculated as their upstream grid block values. The mol fractions,  $x_1, x_2, y_1$ , and  $y_2$ , and the  $p/\rho$  values are fixed at their time level  $n$  values. For a given phase, the convective term is expressed implicitly in  $p, S_w$ , and  $S_g$  and explicitly in temperature if free gas is present in the upstream grid block. For example, the oil-phase convective term in this case is

$$\begin{aligned} (T_o H_o (\Delta p - \gamma_o \Delta Z))_{n+1} &= \\ (T_o H_o (\Delta p - \gamma_o \Delta Z))_n \\ & + H_{on} \delta (T_o (\Delta p - \gamma_o \Delta Z)) \dots (28) \end{aligned}$$

If the upstream block for a given phase has no free gas, then the convective flow term for that phase is expressed implicitly in  $p, S_w$ , and  $T$ . For example, the oil-phase convective term in this case is

$$\begin{aligned} (T_o H_o (\Delta p - \gamma_o \Delta Z))_{n+1} &= \\ (T_o H_o (\Delta p - \gamma_o \Delta Z))_n \\ & + T_{on} H_{on} \Delta \delta p + (\Delta p_{n+1} - \gamma_o \Delta Z) \\ & (T_{on} H_{on} \delta T + H_{on+1} \delta T_o) \dots (29) \end{aligned}$$

where the temperature derivative  $H_o T$  is  $x_{1p} C_{p1} + x_{2n} C_{p2}$ . The term  $\delta [T_o (\Delta p - \gamma_o \Delta Z)]$  in Eq. 28 follows from a development similar to that leading to Eq. 20,

$$\begin{aligned} \delta (T_o (\Delta p - \gamma_o \Delta Z)) &= \\ T_{on} \Delta \delta p + (\Delta p_{n+1} - \gamma_o \Delta Z) \delta T_o, \\ & \dots \dots \dots (30) \end{aligned}$$

where for Eq. 30,  $\delta T_o$  is given by Eq. 21 and for Eq. 29,

$$\delta T_o = T_x (\rho_o / \mu_o) k_{ros w} \delta S_w \dots \dots (31)$$

The conductive heat flow between grid blocks  $i-1, j, k$  and  $i, j, k$  is explicit or implicit in temperature depending on whether free gas is present in the grid block. Thus, the conductive energy flow is expressed as

$$T_c \Delta T = T_c (T_{i-1,n} + (p_{n+1} - p_{wb}) \lambda_{gs} \delta S_g) \dots (34c)$$

$$\alpha_{i-1} (T_{i-1} - T_{i,n} - \alpha_i \delta T_i) \dots (32)$$

where  $\alpha_i$  is 0 if free gas is present and is 1 otherwise.

Using implicit expressions of the type of Eqs. 20 and 22 for the interblock flow terms in Eqs. 1 through 4 leads to  $R'_i + \sum_{j=1}^5 \Delta(T_{ij} \Delta p_j)$  as a representation of these right-hand side flow terms. The notation  $\Delta(T_{ij} \Delta p_j)$  is approximate; it is not a true Laplacian or second-order difference due to the upstream dependence of implicit saturation, composition, and convective energy transmissibilities.

The heat loss term  $q_L$  in Eq. 1 is represented implicitly by an expression  $q_{Ln} + a \delta T$  as described in Ref. 5.

#### IMPLICIT TREATMENT OF PRODUCTION TERMS

Production-well rates in the model are specified as reservoir barrels per day of total liquid. A productivity index and limiting flowing bottom-hole pressure are specified in addition to the desired rate. The model calculates bottom-hole wellbore pressure necessary to produce the desired liquid production rate. If this pressure is less than the limiting pressure read in, then the well is put on deliverability against the limiting pressure. If the well is completed in more than one layer, then the total well production is allocated among the layers on the basis of layer mobilities and pressures.

For brevity we describe only the case of a well on deliverability producing from a single layer. As described previously,<sup>1</sup> the production rate of phase  $m$  ( $m = w, o, g$ ) in reservoir barrels per day is

$$q_m = Pl \cdot \lambda_m (p - p_{wb}) \dots (33)$$

where  $p$  is grid-block oil pressure and  $p_{wb}$  is the specified limiting flowing bottom-hole pressure.

The three-phase production rates are expressed implicitly as

$$q_{wn+1} = q_{wn} + Pl (\lambda_{wn} \delta p + (p_{n+1} - p_{wb}) \lambda_{wsw} \delta S_w) \dots (34a)$$

$$q_{on+1} = q_{on} + Pl (\lambda_{on} \delta p + (p_{n+1} - p_{wb}) (\lambda_{osw} \delta S_w + \lambda_{osg} \delta S_g)) \dots (34b)$$

$$q_{gn+1} = q_{gn} + Pl (\lambda_{gn} \delta p$$

Viscosities in the mobilities and their derivatives are expressed at time level  $n$ .

The production rate of Component 1 is given by Eq. 3 as

$$q_o \rho_o x_1 + q_g \rho_g y_1 \dots (35)$$

The molar densities  $\rho_o$  and  $\rho_g$  are expressed at time level  $n$ . The remaining terms in Eq. 35 are expressed implicitly as

$$(q_o x_1 + q_g y_1)_{n+1} = (q_o x_1 + q_g y_1)_n + q_{on} \delta x_1 + q_{gn} \delta y_1 + x_{1n+1} \delta q_o + y_{1n+1} \delta q_g \dots (36)$$

The terms  $x_{1n+1}$  and  $y_{1n+1}$  are approximated by their latest iterates,  $\delta y_1$  is given by Eq. 25,  $\delta q_o$  is  $q_{on+1} - q_{on}$  from Eq. 34b, and  $\delta q_g$  is  $q_{gn+1} - q_{gn}$  from Eq. 34c.

The energy production term  $q_H$  in Eq. 1 is

$$q_H = q_w \rho_w H_w + q_o \rho_o H_o + q_g \rho_g H_g \dots (37)$$

The molar densities are expressed at time level  $n$ . If free gas is present, enthalpies are treated explicitly in temperature

$$q_{Hn+1} = q_{Hn} + \rho_{wn} H_{wn} \delta q_w + \rho_{on} H_{on} \delta q_o + \rho_{gn} H_{gn} \delta q_g \dots (38)$$

where  $\delta q_w$ ,  $\delta q_o$ , and  $\delta q_g$  are given by Eq. 34. If no free gas is present,  $q_g$  is zero and the enthalpies are treated implicitly in temperature,

$$q_{Hn+1} = q_{Hn} + \rho_{wn} H_{wn+1} \delta q_w + \rho_{on} H_{on+1} \delta q_o + \rho_{wn} q_{wn} H_{wT} \delta T + \rho_{on} q_{on} H_{oT} \delta T \dots (39)$$

#### SOLUTION OF MODEL EQUATIONS

The model formulation uses "variable substitution" in the simultaneous-equation logic to maximize stability for a fixed amount of arithmetic. For example, in compositional mode the model treats four equations in the four unknowns — water saturation, gas saturation, mol fraction of light Component 1 in the oil phase, and pressure. Transmissibilities are treated implicitly in saturations and compositions. If free gas is present, convective heat transmissibilities are explicit in temperature. Since temperature is not an

independent variable in this case, little or no stability is lost due to these explicit transmissibilities. However, if gas saturation is zero, temperature is substituted for gas saturation in the variables, and convective heat transmissibilities are treated implicitly in lieu of implicit gas-phase transmissibilities. Replacement of gas saturation by temperature during a time step, together with the implicit treatment, allows rigorous treatment of the repressurization problem where gas disappears upon pressure increase or temperature decrease. If water disappears through vaporization, then the model handles this superheated condition through substituting temperature for the water-saturation unknown.

Combining the general forms of the left-hand-side expansions, the implicit interblock flow-term expressions and the implicit production-rate representations give

$$\sum_{j=1}^5 C_{ij} P_j = R_i + \sum_{j=1}^5 \Delta(T_{ij} \Delta P_j),$$

$$i = 1, 5, \dots \dots \dots (40)$$

as a general form of the model Eqs. 1 through 5. These five equations in five unknowns are reduced to four equations in four unknowns in three different ways for the three cases of (1)  $S_g > 0, S_w > 0$ ; (2)  $S_g = 0, S_w > 0$ ; and (3)  $S_g > 0, S_w = 0$  (the superheated case).

For the first case, the energy transmissibilities are explicit in temperature as discussed above. Thus,  $T_{i1}$  in Eq. 40 is 0 for  $i = 1, 2, \dots, 5$ . With  $C_{i1}$  in Eq. 40 as the pivot element, Gaussian elimination is performed on both sides of Eq. 40 to zero the coefficients  $C_{i1}, i = 2, 3, 4, 5$ . The resulting four equations  $i = 2$  through 5 are then four equations in the four unknowns  $p_1 = \delta S_g, p_2 = \delta S_w, p_3 = \delta x_1$ , and  $p_4 = \delta p$ .

For the second case ( $S_g = 0$ ), the coefficients  $C_{i2}$  and transmissibilities  $T_{i2}, i = 1$  through 5, are zero and we simply transpose columns, Column  $\{C_{i1}\}$  into position  $\{C_{i2}\}$  and  $\{T_{i1}\}$  into  $\{T_{i2}\}$ . As discussed above, if  $S_g = 0$ , then energy transmissibilities are implicit in temperature so that  $T_{i1}$  is nonzero. Eq. 5 for this case does not apply so that the result of this column shift is a set of four equations in the four unknowns,  $p_1 = \delta T, p_2 = \delta S_w, p_3 = \delta x_1$ , and  $p_4 = \delta p$ .

For the third case ( $S_g > 0, S_w = 0$ ), the coefficients  $C_{i3}$  and transmissibilities  $T_{i3}$  are zero and the columns  $\{C_{i1}\}$  and  $\{T_{i1}\}$  are transposed to Column 3. Eq. 5 is not necessary as Eqs. 1 through 4 are four equations in the four unknowns  $p_1 = \delta S_g, p_2 = \delta T, p_3 = \delta x$ , and  $p_4 = \delta p$ .

In the matrix form the set of four equations, obtained from Eq. 40 as just described, can be written as

$$CP = R + \Delta(T\Delta P), \dots \dots (41)$$

where  $C$  and  $T$  are  $4 \times 4$  matrices and  $P$  and  $R$  are  $4 \times 1$  column vectors. This equation is solved by the reduced band-width direct solution technique.<sup>6</sup> Direct solution is used in preference to iterative techniques (e.g., ADIP, SIP, LSOR) because we have found the latter unable to cope with the physically real negative transmissibilities that occur when colder fluid moves into a hotter grid block containing steam.

The matrices  $C$  and  $T$  and column vector  $R$  involve unknown terms evaluated at time level  $n+1$ . Thus, the solution of Eq. 41 proceeds in iterations as follows. Before the first iteration, all  $n+1$  level coefficients are evaluated as their values at the end of the previous time step. With  $C, T$ , and  $R$  evaluated accordingly, Eq. 41 is solved by the reduced band-width direct solution technique to give first iterate approximations to four of the five unknowns —  $T_{n+1}, S_{gn+1}, S_{wn+1}, x_{1n+1}$ , and  $p_{n+1}$ . If free gas is present, the first iterate approximation to  $T_{n+1}$  is obtained from Eq. 5. These first iterates are then used in re-evaluation of  $C, T$ , and  $R$  and Eq. 41 is solved for the second iterate approximations. These iterations are continued until the maximum pressure change over the last iteration is less than a specified tolerance. In practice, a tolerance of 0.1 psi is used; dead-oil problems typically require two or three iterations and compositional problems typically require three or four iterations per time step.

In many cases, solution gas and distillation effects are absent or negligible. In this case,  $x_1 = y_1 = 0, x_2 = 1$ , Eq. 3 does not apply, and we have four equations in the four unknowns  $\delta T, \delta S_g, \delta S_w$ , and  $\delta p$ . These four equations are reduced to three equations for the three saturation configurations described above and the unknowns for the three cases are (1)  $\delta S_g, \delta S_w, \delta p$ ; (2)  $\delta T, \delta S_w, \delta p$ ; and (3)  $\delta S_g, \delta T, \delta p$ . The model has the user-specified option of solving three simultaneous equations for the dead-oil case or four simultaneous equations for the compositional case.

## WORK COMPARISON

MacDonald and Coats<sup>7</sup> introduced a concept of improved stability for the IMPES (implicit pressure, explicit saturation) technique<sup>8,9</sup> by following the pressure equation solution with a saturation equation solution using implicit transmissibilities. Spillette *et al.*<sup>10</sup> extended this concept from two- to three-phase flow and called the resulting model a "sequential" model. A previously described steamflood model<sup>1</sup> solves a pressure equation using explicit transmissibilities and then solves two saturation equations simultaneously using implicit transmissibilities. That model is referred to hereafter as the sequential steamflood model to distinguish it from the implicit steamflood model described here.

The implicit model requires more arithmetic per time step than the sequential model. Computing time for solution of Eq. 41 is roughly proportional

to the cube of the number of equations solved simultaneously. Thus, the sequential model incurs time of "1" for the pressure equation and "8" for the two simultaneous saturation equations. The implicit model incurs time of "27" for the three simultaneous equation solutions in the dead-oil mode. Thus, the implicit model requires about three times more computing time per time step than the sequential model for "large" problems, where most of the time step's arithmetic is spent in the solution of Eq. 41 as opposed to coefficient generation. For the compositional case, the implicit model solves four equations simultaneously and the ratio is about seven times more computer time per time step for the implicit model.

Most steamflood model applications involve fewer than 200 grid blocks for a symmetrical element of a pattern. In practice on such problems, the implicit model has required less than twice the sequential model's computing time per time step in the dead-oil case and less than four times for the compositional case. For cross-sectional and single-well  $r$ - $z$  problems having 50 or so grid blocks, the two models require about the same time for the dead-oil case.

In many cases, greater stability of the implicit model has resulted in time steps sufficiently larger so that total computing time for the problem was less than that using the sequential model. No generalization can be made in this respect. The implicit model will not be more efficient than the sequential model on problems where the latter performs quite stably. Quantitative comparisons of the models for two reservoir problems are given below.

Comparison of the model efficiencies in the compositional case is only marginally meaningful, since the formulation of the sequential model is incapable of handling certain compositional problems. The sequential calculation results in interblock flow rates of water, oil, and gas calculated from the saturation equations that differ from the rates calculated in the pressure equation. While the pressure equation's solution preserves material balances, the saturation equations' solution does not. If gas composition varies markedly in two adjacent blocks and the altered gas flow rate from the saturation equations' solution differs greatly from that of the pressure equation's solution, then material balances on a component basis will deteriorate. Thus, the sequential model performs well in general for dead-oil problems where the gas phase is 100% steam but performs poorly for compositional problems having high gas-phase flow rates, potentially unstable characteristics, and high gas-phase composition gradients.

The computing time of both the implicit and sequential steamflood models can increase markedly with increased number of grid blocks. This is a point in favor of preliminary or grid sensitivity work designed to minimize the number of grid blocks used. Both models use direct, reduced band-width solution of the sets of simultaneous equations. The computing time of this direct solution is proportional

to the cube of the band width that is the product of the two smaller of ( $NX$ ,  $NY$ , and  $NZ$ ). The computing time is proportional to  $NX \cdot NY^3 \cdot NZ^3$ , assuming  $NY$  and  $NZ$  are the smaller two. The time per grid-block time step is proportional to  $NY^2 NZ^2$ . For example, consider a case where  $NX = 8$ ,  $NY = 5$ , and  $NZ = 5$  for a total of 200 blocks. If we increase to  $10 \times 8 \times 5$  for 400 blocks, the computing time per grid-block time step will increase by a factor of  $8^2 \cdot 5^2 / 5^2 \cdot 5^2$  or  $64/25$ , a factor of nearly 3. Since there are twice as many grid blocks, the total computing time per time step will increase by a factor of  $128/25$ , or 5. For most steamflood problems, the time step size is controlled by saturation change per time step. Therefore, with twice as many blocks, the blocks are half as large and the saturation change for the same size time step may increase by a factor of 2. Thus, twice as many time steps may be required and total computing time for the run may increase for the 400-block description by an over-all factor of roughly 10.

If iterative techniques could be used in place of direct solution, we could avoid the cube of  $NY$  and  $NZ$  in the above example. That is, the work per time step using an iterative method would be proportional to  $aNX \cdot NY \cdot NZ$ , where  $a$  would increase somewhat with increased total grid blocks. We have found iterative methods unusable due to the occurrence of negative compressibilities or transmissibilities in the equations caused by cold fluids occasionally moving into blocks having steam saturations. However, even if iterative methods are devised to handle this problem, computer time for a run may still increase roughly by a factor of 4 if the number of grid blocks is doubled, since the time step may be half as large.

The above comparison of efficiencies of the implicit and sequential steamflood models is necessarily weak in that their relative total computing time requirements depend entirely on the problem difficulty. We have dropped use and maintenance of the sequential model using the following reasoning. The implicit model is considerably more reliable than the sequential model. In a multiple user environment, we feel reliability is extremely important, subject only to an excessive penalty in computing expense. Even in our most adverse comparisons, we have not seen more than 30% increase in total computing expense for the implicit model compared with the sequential one. In most comparisons, the expense has been comparable and in some dead-oil cases the implicit model has reduced expense by as much as 60%. We feel the reliability of the implicit model, especially in compositional cases, has reduced our man-time requirements significantly in several recent reservoir studies.

#### MODEL COMPARISON FOR TWO RESERVOIR PROBLEMS

In a recent reservoir steamflood study, we matched a rapid fingering of injected steam through a water layer underlying the oil and then performed

three-dimensional runs to estimate project economics under various sets of operating conditions. The first phase of this simulation study used a wedge-shaped cross-section to approximate the well configuration. The purpose of the cross-sectional work was approximate matching of the observed vertical conformance of the injected steam. This horizontal cross-section proved to be one of the more unstable steamflood problems we have encountered. The section was represented by 10 grid blocks in the  $x$  (radial) direction and four blocks in the  $z$  direction. The oil was dead oil with no solution gas or distillable component.

We injected 70% quality steam (at 250 psia) at a rate of 100 B/D cold water equivalent (CWE) into Layers 3 and 4 at  $x = 0$  with a total well injectivity index (PI) of 500 B/D/psi. Specified production rate was 100 RB/D of water plus oil from Layers 3 and 4 at  $x = 280$  ft. Initial water saturation was 1.0 in the bottom two layers and 0.35 in the top two layers. Initial oil pressure was 117 psia, 10 ft

below top of sand. Layer thicknesses from the top were 20, 20, 15, and 15 ft. The radial or  $x$ -direction grid block lengths were 50.9, 50.9, 50.9, 50.9, 50.9, 50.9, 50, 100, 200, and 500 ft.

Table 1 gives data for this problem. Relative permeability was assumed independent of temperature.

Horizontal permeability and porosity were specified as uniform values of 3,500 md and 0.37, respectively, with  $k_V/k_H$  ratios of 0.05 between Layers 1 and 2 and of 0.1 among Layers 2, 3, and 4. The wedge angle corresponds to a cross-section width ( $y$  direction) of 0 at  $x = 0$  and a width of 152 ft at  $x = 280$  ft.

Table 2 compares time step sizes on this problem for the sequential and implicit steamflood models. The implicit model required 108 time steps compared with 262 steps for the sequential model for the first 200 days of field time. Average time steps for the implicit and sequential models were 1.85 and 0.76 days, respectively.

The time step increases with time as the steam front expands radially outward from the small grid blocks at the well. A common feature of steamflood simulation problems is the high steam flow rates that in turn lead to high grid-block throughput rates requiring implicit model formulation for stability. In this problem, for example, the injection rate of 100 B/D (CWE) of 70% quality steam translates into 70 B/D (CWE) of saturated steam injection. The combined pore volume of the two injection blocks is 244 res bbl. If we were to think in terms of 100 RB/D of fluid injection, a 1-day time step gives a throughput rate (flow through the grid block in one time step divided by block pore volume) of only 0.41. However, the steam front forms at the well in this problem at about 120 psia, where the steam occupies 3.73 cu ft/lb. The 70-B/D (CWE) steam injection rate is thus 16,293 RB/D and a 1-day time step corresponds to a throughput rate of 16,293/244, or 67.

The implicit model required 36.5 seconds of CDC 6600 CPU time for the 200-day run or 0.0084 seconds per grid-block time step. The sequential model required 85.8 seconds or 0.0082 seconds per block step. The implicit model required 57.5% less total computing time than the sequential model.

Both models were run using automatic time step control. The implicit model used a control of 7.5% maximum saturation change per time step while the sequential model used 5%. The sequential model required 390, 377, and 262 time steps for saturation change controls of 2, 3, and 5%, respectively. The

TABLE 1 — DATA FOR WEDGE-SHAPED CROSS-SECTION PROBLEM

$$c_w = 3.5 \times 10^{-6} \text{ 1/psi}$$

$$c_o = 10 \times 10^{-6} \text{ 1/psi}$$

$$c_r = 3 \times 10^{-6} \text{ 1/psi}$$

$$C_{TO} = 0.0004 \text{ vol/vol-}^\circ\text{F}$$

$$C_{p2} = 0.5 \text{ Btu/lb-}^\circ\text{F}$$

$$(\rho C_p)_R = 35 \text{ Btu/cu ft rock-}^\circ\text{F}$$

$$T_i = 75^\circ\text{F}$$

Oil formation volume factor = 1.0 RB/STB  
 Stock-tank oil density = 58.4 lb/cu ft  
 Reservoir thermal conductivity = 29 Btu/D-ft- $^\circ$ F  
 $K_{OB} = 29 \text{ Btu/ft-D-}^\circ\text{F}$   
 $C_{POB} = 35 \text{ Btu/bulk cu ft-}^\circ\text{F}$   
 $\rho_{cwo} = 0$

Relative Permeability and Capillary Pressure Data

Water-Oil Data			Gas-Oil Data			
$S_w$	$k_{rw}$	$k_{row}$	$S_{wc} + S_o$	$k_{rg}$	$k_{rog}$	$P_{cgo}$
0.35000	0.00000	1.00000	0.50000	0.80000	0.00000	4.00
0.40000	0.01000	0.77000	0.55000	0.30000	0.03000	3.20
0.45000	0.01500	0.60000	0.60000	0.20000	0.08000	2.40
0.50000	0.02500	0.48000	0.65000	0.12000	0.15000	1.60
0.55000	0.03700	0.34000	0.70000	0.07000	0.25000	0.80
0.60000	0.05500	0.23000	0.75000	0.04000	0.36000	0.00
0.65000	0.08000	0.16000	0.80000	0.02000	0.46000	-0.80
0.70000	0.11500	0.10500	0.85000	0.01000	0.58000	-1.60
0.75000	0.16000	0.06500	0.90000	0.00500	0.70000	-2.40
0.80000	0.21500	0.03500	0.95000	0.00000	0.86000	-3.20
0.85000	0.30000	0.00000	1.00000	0.00000	1.00000	-4.00
1.00000	1.00000	0.00000				

Viscosity Data (cp)

Temperature °F	Oil Viscosity	Water Viscosity	Steam Viscosity
60	2,470	1.130	0.0100
75	1,200	0.935	0.0103
80	1,018	0.875	0.0108
100	525	0.685	0.0111
120	270	0.560	0.0116
150	100	0.430	0.0124
200	32	0.308	0.0135
250	10	0.230	0.0146
300	6.3	0.182	0.0157
400	2.5	0.145	0.0177
500	1.0	0.120	0.0200

TABLE 2 — COMPARISON OF TIME-STEP SIZE

Time (days)	Cumulative Number of Time Steps		Average Time Step (days)	
	Sequential Model	Implicit Model	Sequential Model	Implicit Model
0 to 10	50	34	0.20	0.29
10 to 50	164	67	0.35	1.21
50 to 100	221	87	0.75	2.50
100 to 200	262	108	2.44	4.76
Over-all average time step			0.76	1.85



TABLE 3 — SEQUENTIAL MODEL  
(Calculated Pressure, Temperature, and Saturations at 200 Days)

Oil Pressure at Elevation H (psi)										
	1	2	3	4	5	6	7	8	9	10
1	142.3	142.8	141.8	137.6	132.3	127.6	124.1	120.7	118.2	116.9
2	143.0	142.2	143.6	143.0	139.0	133.9	130.3	127.6	126.2	125.0
3	156.2	145.9	142.7	135.1	134.5	134.2	134.0	133.6	133.1	132.5
4	158.3	148.9	143.6	141.1	140.9	140.6	140.4	140.1	139.6	139.0
Temperature (°F)										
	1	2	3	4	5	6	7	8	9	10
1	231.5	143.3	95.5	79.3	76.1	75.2	75.0	75.0	75.0	75.0
2	353.9	353.5	354.1	127.1	90.7	78.0	75.1	75.0	75.0	75.0
3	360.4	355.3	353.7	338.3	197.8	108.2	76.7	75.0	75.0	75.0
4	361.4	356.8	354.2	303.8	114.9	82.4	75.3	75.0	75.0	75.0
Oil Saturation										
	1	2	3	4	5	6	7	8	9	10
1	0.5640	0.6294	0.6464	0.6483	0.6491	0.6496	0.6499	0.6500	0.6500	0.6500
2	0.2189	0.2611	0.4267	0.6428	0.6477	0.6489	0.6495	0.6498	0.6499	0.6500
3	0.1855	0.2001	0.2900	0.1826	0.0223	0.0111	0.0061	0.0030	0.0014	0.0010
4	0.0044	0.0937	0.0696	0.0011	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
Water Saturation										
	1	2	3	4	5	6	7	8	9	10
1	0.4360	0.3706	0.3536	0.3517	0.3509	0.3504	0.3501	0.3500	0.3500	0.3500
2	0.3935	0.3985	0.4509	0.3572	0.3523	0.3511	0.3505	0.3502	0.3501	0.3500
3	0.3793	0.3885	0.4121	0.8174	0.9777	0.9889	0.9939	0.9970	0.9986	0.9990
4	0.4561	0.4697	0.5915	0.9989	0.9990	0.9990	0.9990	0.9990	0.9990	0.9990
Gas Saturation										
	1	2	3	4	5	6	7	8	9	10
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.3876	0.3404	0.1224	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.4352	0.4115	0.2978	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.5394	0.4366	0.3389	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE 4 — IMPLICIT MODEL  
(Calculated Pressure, Temperature, and Saturations at 200 Days)

Oil Pressure at Elevation H (psi)										
	1	2	3	4	5	6	7	8	9	10
1	142.4	142.8	141.7	137.5	132.2	127.5	124.0	120.6	118.1	116.9
2	144.0	142.8	142.8	142.2	138.3	133.3	129.9	127.4	126.0	125.0
3	160.5	146.3	142.1	134.8	134.0	133.7	133.5	133.3	133.0	132.5
4	155.2	148.0	143.4	140.7	140.5	140.2	140.0	139.8	139.4	139.0
Temperature (°F)										
	1	2	3	4	5	6	7	8	9	10
1	251.8	148.8	97.6	79.8	76.2	75.2	75.0	75.0	75.0	75.0
2	354.4	353.8	353.8	128.6	91.0	78.3	75.2	75.0	75.0	75.0
3	362.5	355.5	353.4	342.3	190.4	106.6	76.9	75.0	75.0	75.0
4	359.9	356.3	354.1	290.4	122.4	83.9	75.4	75.0	75.0	75.0
Oil Saturation										
	1	2	3	4	5	6	7	8	9	10
1	0.5412	0.6260	0.6460	0.6481	0.6489	0.6495	0.6498	0.6500	0.6500	0.6500
2	0.2162	0.2549	0.3975	0.6423	0.6475	0.6488	0.6495	0.6499	0.6499	0.6500
3	0.1865	0.1997	0.2931	0.1922	0.0238	0.0118	0.0065	0.0031	0.0014	0.0010
4	0.0019	0.1048	0.0901	0.0011	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
Water Saturation										
	1	2	3	4	5	6	7	8	9	10
1	0.4588	0.3740	0.3540	0.3519	0.3511	0.3505	0.3502	0.3500	0.3500	0.3500
2	0.3897	0.3959	0.4339	0.3577	0.3525	0.3512	0.3505	0.3501	0.3501	0.3500
3	0.3803	0.3880	0.4048	0.8078	0.9762	0.9882	0.9935	0.9969	0.9986	0.9990
4	0.4571	0.4692	0.5919	0.9989	0.9990	0.9990	0.9990	0.9990	0.9990	0.9990
Gas Saturation										
	1	2	3	4	5	6	7	8	9	10
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.3941	0.3492	0.1685	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.4332	0.4124	0.3021	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.5410	0.4260	0.3180	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE 5 — DATA FOR  
EXAMPLE COMPOSITIONAL PROBLEM

$c_w = 3.1 \times 10^{-6}$  1/psi  
 $c_o = 5 \times 10^{-6}$  1/psi  
 $c_r = 2 \times 10^{-6}$  1/psi  
 $C_{To} = 0.00041$  1/°F  
 $C_{p1} = 0.5$  Btu/lb-°F  
 $C_{p2} = 0.5$  Btu/lb-°F  
 $(\rho C_p)_R = 35$  Btu/cu ft-°F  
 $T_i = 90^\circ\text{F}$   
 $MW_1 = 120$   
 $MW_2 = 300$   
 Stock-tank oil density, at  $x_1 = 0$ , = 55 lb/cu ft  
 Stock-tank oil density, at  $x_1 = 0.3$ , = 52 lb/cu ft  
 $K_1$  at stock-tank conditions = 0  
 $S_{wc} = 0.2$   
 $S_{gr} = 0.25$   
 $k = 4,000$  md  
 $\phi = 0.38$   
 $\rho_{cwo} = 0$   
 Hydrocarbon gas z-factor = 1.0  
 Hydrocarbon gas viscosity = 0.01 cp  
 $y_1 = K_1 x_1$  with  $K_1$  dependent on temperature only as follows

Temperature (°F)	$K_1$	$x_2$	Oil-Phase Density (mol/RB)	Relative Oil-Phase Viscosity
80	0.00065	0.1667	1.5316	0.25
180	0.0095	0.5	1.0701	0.35
280	0.07	0.667	0.9303	0.5
380	0.3	0.75	0.8731	1.0
480	0.5	0.8571	0.8091	2.5
		1.0	0.7372	6.0

  

Temperature (°F)	Oil-Phase Viscosity (cp)	Water Viscosity (cp)	Steam Viscosity (cp)
75.0	5,780.0000	0.9200	0.0095
100.0	1,380.0000	0.6810	0.0102
150.0	187.0000	0.4350	0.0115
200.0	47.0000	0.3050	0.0127
250.0	17.4000	0.2350	0.0138
300.0	8.5000	0.1870	0.0149
350.0	5.2000	0.1560	0.0158
500.0	2.5000	0.1180	0.0174

Relative Permeability and Capillary Pressure Data

$S_w$	$k_{rw}$	$k_{row}$	$S_{wc} + S_o$	$k_{rg}$	$k_{rog}$	$\rho_{cgo}$
0.20000	0.00000	1.00000	0.29000	0.17000	0.00000	1.5000
0.25000	0.01020	0.76900	0.39500	0.11200	0.02940	1.0563
0.29400	0.01680	0.72410	0.43300	0.10220	0.04610	0.8958
0.35700	0.02750	0.62060	0.51500	0.08560	0.08830	0.5493
0.41400	0.04240	0.50400	0.56900	0.07610	0.11720	0.3211
0.49000	0.06650	0.37140	0.61400	0.06540	0.14330	0.1310
0.55700	0.09700	0.30290	0.66300	0.05000	0.17640	-0.0761
0.63000	0.11480	0.15550	0.71900	0.03720	0.21700	-0.3127
0.67300	0.12590	0.09560	0.75000	0.02850	0.22550	-0.4437
0.71900	0.13810	0.05760	0.80500	0.01950	0.29190	-0.6761
0.78900	0.16360	0.00000	0.85000	0.01210	0.33730	-0.8662
1.00000	0.25000	0.00000	0.89900	0.00260	0.51690	-1.0732
			1.00000	0.00000	1.00000	-1.5000

model diverged when 7.5% saturation control was used. The implicit model required 131 steps using 5% saturation control and 108 steps using 7.5% control.

Tables 3 and 4 show that the calculated pressures, temperatures, and saturations at 200 days for the two models are nearly identical. The difference in pressures in injection grid blocks  $i = 1$  and  $k = 3, 4$  is due to different mobility allocation schemes used in the two models. The sequential model allocates total well injection rate among layers on the basis of pressure and total mobility. The implicit model uses pressure and a mobility equal to gas-phase mobility at residual liquid saturation.

Table 5 gives data for a three-dimensional simulation of one-eighth of a 2.5-acre five-spot. The original reservoir oil is about 2,200-cp viscosity and 30 mol % distillable Component 1, and 70% nonvolatile Component 2. The PVT data giving oil phase moles per reservoir barrel and relative oil viscosity factor apply at original reservoir temperature. Reservoir and overburden thermal conductivity is 38.4 Btu/ft-D-°F. Overburden specific heat is 35 Btu/°F-bulk cu ft.

The one-eighth five-spot is represented by a  $5 \times 3 \times 2$  (x-y-z) grid system with  $\Delta x = \Delta y = 38.9$  ft and  $\Delta z = 31.5$  ft. An amount of 37.5 B/D (CWE) of 70% quality steam (at 200 psia) is injected into a well at  $i = 1, j = 1$ , and  $k = 2$ . The production well at  $i = 5$  and  $j = 1$  is completed in both layers and produces at deliverability with a PI of 300 RB/D/psi against a bottom-hole flowing pressure of 60 psia. Initial reservoir pressure is 75 psia at 32.5 ft below top of sand and initial water and oil saturations are uniformly 30 and 70%, respectively.

Initial oil and water in place are 31,084 and 17,334 STB, respectively. After 730 days, total steam injected is 27,375 bbl CWE and cumulative oil and water produced are about 9,500 and 30,000 STB, respectively. Production rate at 730 days is about 20 STB/D oil with a water-oil ratio of 1.8 STB/STB. Oil-steam ratio for the first 730 days is 0.35 STB/bbl (CWE).

This problem was run to 730 days using the sequential and implicit models with automatic time-step control of maximum saturation change per time step. This latter saturation change will be referred to simply as "time step control" and will be expressed in percent. A maximum temperature-change control of 30°F was used with the saturation-change control, but the latter was the primary control on time step size.

Table 6 summaries number of steps and average time step for the 730-day run for the two models.

TABLE 6 — MODEL COMPARISON FOR EXAMPLE COMPOSITIONAL PROBLEM

Time-Step Control, Saturation Change (%)	Number of Time Steps		Average Time Step (days)		Material Balance Error In Sequential Model	
	Sequential Model	Implicit Model	Sequential Model	Implicit Model	Component 1	Component 2
5.0	144	114	5.1	6.4	13.5	1.91
7.5	121	87	6.0	8.4	18.6	2.63
10.0	111	70	6.6	10.4	24.6	3.45

At a time step control of 10%, the implicit model required 70 steps for an average time step of 10 days and required 35.9 CDC 6600 CPU seconds or 0.020 seconds per active grid-block time step. In comparison, the sequential model required 45.5 CPU seconds or 0.016 seconds per active block step. The implicit model required 21% less total computing time.

The material balance errors of the sequential model are significant. Material balance errors are defined as follows. The correct mass of a component (e.g., H<sub>2</sub>O, hydrocarbon Component 1) is initial mass in place plus cumulative injection minus cumulative production. The actual mass in place may differ significantly from this correct mass if the model formulation does not preserve material balances and/or if iteration is not continued sufficiently toward convergence. We define material balance error as the absolute value of (actual mass in place minus correct mass in place) divided by cumulative production, and express this error as a percentage.

At 730 days, the implicit-model energy and H<sub>2</sub>O material balance errors were less than 0.001% and hydrocarbon Component 1 and 2 material balance errors were 0.55 and 0.09%, respectively. For the sequential model, the water and energy balance errors were 0.23 and 0.25%, respectively. Component 1 material balance error ranged from 13 to 24% and Component 2 error from 1.9 to 3.5%, as shown in Table 6.

The sequential-model results exhibit more sensitivity to time step size for this problem than do those of the implicit model. We cannot determine whether this is due to time truncation error, to marginal stability, or to the significant material balance errors in the sequential model. Tables 7 through 9 show calculated pressures, saturations, etc., at 730 days for the sequential model with 5% time-step control and for the implicit model with 5 and 10% time-step control. For brevity, we dispense with a detailed comparison of these results. In summary, the sequential- and implicit-model results using 5% time-step control agree rather well. The sequential-model results at 5 and 7.5% time-step controls differed significantly more than the implicit-model results at 5 and 10% time-step controls.

#### AN ALTERNATE STEAMFLOOD-MODEL FORMULATION

We will discuss briefly another possible model formulation for steamflooding. For three-phase flow, the sequential method described by Spillette *et al.*<sup>10</sup> solves a pressure equation and then solves two saturation equations simultaneously. Our experience in developing this implicit steamflood model, and experience with a black-oil model derived from it, indicates advantages of an implicit gas scheme where two equations in pressure and gas saturation are solved simultaneously, followed by a sequential water saturation equation. Notation

TABLE 7 — SEQUENTIAL MODEL  
(Calculated Results at Time = 730 Days)  
Time Step Control = 5%

Oil Pressure at Elevation H (psi)					
K = 1					
	1	2	3	4	5
3		68.7	68.1	68.0	
2	73.3	70.7	68.1	65.9	64.8
1	74.9	71.6	68.1	65.1	60.9
K = 2					
	1	2	3	4	5
3		76.3	76.7	76.1	
2	77.8	76.1	75.1	74.9	74.3
1	80.0	76.4	74.7	74.0	72.7
Temperature (°F)					
K = 1					
	1	2	3	4	5
3		300.5	267.2	142.0	
2	304.8	302.3	299.8	295.2	180.7
1	306.2	303.3	300.2	295.3	287.5
K = 2					
	1	2	3	4	5
3		204.4	140.7	113.2	
2	308.7	306.2	242.8	175.3	141.2
1	310.7	307.6	294.7	242.8	214.0
Oil Saturation					
K = 1					
	1	2	3	4	5
3		0.3164	0.7914	0.7997	
2	0.1606	0.2115	0.3579	0.4978	0.7958
1	0.1417	0.1784	0.2466	0.4167	0.7803
K = 2					
	1	2	3	4	5
3		0.6977	0.7467	0.7593	
2	0.3995	0.5021	0.5518	0.6843	0.7533
1	0.2187	0.4517	0.5071	0.5189	0.3888
Water Saturation					
K = 1					
	1	2	3	4	5
3		0.2014	0.2086	0.2003	
2	0.2003	0.2004	0.2010	0.2028	0.2042
1	0.2002	0.2003	0.2005	0.2013	0.2197
K = 2					
	1	2	3	4	5
3		0.3023	0.2533	0.2407	
2	0.2269	0.3089	0.4482	0.3157	0.2467
1	0.2181	0.2304	0.4929	0.4811	0.6312
Gas Saturation					
K = 1					
	1	2	3	4	5
3		0.4821	0.0000	0.0000	
2	0.6391	0.5882	0.4412	0.2993	0.0000
1	0.6581	0.6213	0.5529	0.3820	0.0000
K = 2					
	1	2	3	4	5
3		0.0000	0.0000	0.0000	
2	0.3736	0.1890	0.0000	0.0000	0.0000
1	0.5632	0.3179	0.0000	0.0000	0.0000
Mol Fraction of Component 1 in Oil					
K = 1					
	1	2	3	4	5
3		0.0345	0.3887	0.3074	
2	0.0000	0.0239	0.0550	0.4284	0.3816
1	0.0000	0.0000	0.0001	0.2930	0.3906
K = 2					
	1	2	3	4	5
3		0.3392	0.3011	0.3002	
2	0.0000	0.1248	0.3437	0.3082	0.3013
1	0.0000	0.0000	0.3160	0.3218	0.3331
Mol Fraction of Component 1 in Gas					
K = 1					
	1	2	3	4	5
3		0.0040	0.0243	0.0019	
2	0.0000	0.0029	0.0064	0.0447	0.0038
1	0.0000	0.0000	0.0000	0.0308	0.0341
K = 2					
	1	2	3	4	5
3		0.0082	0.0018	0.0011	
2	0.0000	0.0162	0.0163	0.0028	0.0018
1	0.0000	0.0000	0.0328	0.0153	0.0100

**TABLE 8 — IMPLICIT MODEL**  
(Calculated Results at Time = 730 Days)  
Time Step Control = 5%

Oil Pressure at Elevation H (psi)					
K = 1					
	1	2	3	4	5
3		69.6	68.9	68.8	
2	74.2	71.5	68.9	67.0	65.7
1	76.7	72.5	69.0	66.3	61.2
K = 2					
	1	2	3	4	5
3		77.1	77.3	76.8	
2	78.6	76.9	75.8	75.5	74.9
1	80.8	77.3	75.4	74.6	72.9
Temperature (°F)					
K = 1					
	1	2	3	4	5
3		301.2	289.8	144.9	
2	305.6	303.1	300.5	296.3	172.4
1	306.9	304.1	301.0	296.8	270.0
K = 2					
	1	2	3	4	5
3		205.8	145.0	115.7	
2	309.5	306.9	243.0	176.0	141.9
1	311.4	308.3	295.4	241.3	210.8
Oil Saturation					
K = 1					
	1	2	3	4	5
3		0.3309	0.7922	0.7997	
2	0.1687	0.2244	0.3703	0.5498	0.7968
1	0.1439	0.1863	0.2702	0.4626	0.7845
K = 2					
	1	2	3	4	5
3		0.7004	0.7438	0.7600	
2	0.4089	0.5145	0.5445	0.6672	0.7528
1	0.2241	0.4615	0.5240	0.5444	0.3988
Water Saturation					
K = 1					
	1	2	3	4	5
3		0.2015	0.2078	0.2003	
2	0.2003	0.2004	0.2010	0.2028	0.2032
1	0.2003	0.2003	0.2036	0.2013	0.2155
K = 2					
	1	2	3	4	5
3		0.2996	0.2562	0.2400	
2	0.2264	0.3034	0.4555	0.3328	0.2472
1	0.2184	0.2299	0.4760	0.4556	0.6012
Gas Saturation					
K = 1					
	1	2	3	4	5
3		0.4675	0.0000	0.0000	
2	0.6310	0.5752	0.4286	0.2477	0.0000
1	0.6558	0.6134	0.5292	0.3361	0.0000
K = 2					
	1	2	3	4	5
3		0.0000	0.0000	0.0000	
2	0.3847	0.1820	0.0000	0.0000	0.0000
1	0.5575	0.3086	0.0000	0.0000	0.0000
Mol Fraction of Component 1 in Oil					
K = 1					
	1	2	3	4	5
3		0.0455	0.3926	0.3089	
2	0.0000	0.0216	0.0689	0.4249	0.3635
1	0.0000	0.0000	0.0005	0.2999	0.3727
K = 2					
	1	2	3	4	5
3		0.3340	0.3010	0.3002	
2	0.0000	0.1204	0.3273	0.3065	0.3011
1	0.0000	0.0001	0.2946	0.3080	0.3194
Mol Fraction of Component 1 in Gas					
K = 1					
	1	2	3	4	5
3		0.0054	0.0251	0.0020	
2	0.0000	0.0027	0.0081	0.0457	0.0032
1	0.0000	0.0000	0.0001	0.0293	0.0238
K = 2					
	1	2	3	4	5
3		0.0084	0.0019	0.0011	
2	0.0000	0.0159	0.0156	0.0028	0.0018
1	0.0000	0.0000	0.0311	0.0143	0.0090

**TABLE 9 — IMPLICIT MODEL**  
(Calculated Results at Time = 730 Days)  
Time Step Control = 10%

Oil Pressure at Elevation H (psi)					
K = 1					
	1	2	3	4	5
3		69.3	68.7	68.6	
2	73.9	71.2	68.7	66.9	65.8
1	75.4	72.2	68.7	66.0	61.2
K = 2					
	1	2	3	4	5
3		77.0	77.3	76.7	
2	78.5	76.8	75.8	75.5	74.9
1	80.6	77.1	75.4	74.5	72.9
Temperature (°F)					
K = 1					
	1	2	3	4	5
3		300.9	267.8	143.8	
2	305.4	302.8	300.2	296.1	170.9
1	306.7	303.9	300.8	296.5	271.6
K = 2					
	1	2	3	4	5
3		204.6	144.9	115.7	
2	309.3	306.8	242.5	174.6	141.7
1	311.2	308.2	295.6	242.2	212.0
Gas Saturation					
K = 1					
	1	2	3	4	5
3		0.4725	0.0000	0.0000	
2	0.6306	0.5765	0.4324	0.2372	0.0000
1	0.6549	0.6133	0.5305	0.3423	0.0000
K = 2					
	1	2	3	4	5
3		0.0000	0.0000	0.0000	
2	0.3635	0.1791	0.0000	0.0000	0.0000
1	0.5565	0.3070	0.0000	0.0000	0.0000
Mol Fraction of Component 1 in Oil					
K = 1					
	1	2	3	4	5
3		0.0468	0.3912	0.3079	
2	0.0000	0.0217	0.0752	0.4252	0.3590
1	0.0000	0.0001	0.0008	0.2697	0.3792
K = 2					
	1	2	3	4	5
3		0.3328	0.3012	0.3002	
2	0.0000	0.1217	0.3265	0.3063	0.3010
1	0.0000	0.0001	0.2933	0.3060	0.3175
Oil Saturation					
K = 1					
	1	2	3	4	5
3		0.3258	0.7917	0.7997	
2	0.1690	0.2230	0.3664	0.5997	0.7872
1	0.1448	0.1864	0.2689	0.4561	0.7846
K = 2					
	1	2	3	4	5
3		0.7067	0.7456	0.7593	
2	0.4094	0.5099	0.5496	0.6758	0.7544
1	0.2249	0.4616	0.5303	0.5449	0.4042
Water Saturation					
K = 1					
	1	2	3	4	5
3		0.2017	0.2083	0.2003	
2	0.2003	0.2005	0.2012	0.2031	0.2028
1	0.2003	0.2003	0.2006	0.2015	0.2154
K = 2					
	1	2	3	4	5
3		0.2933	0.2544	0.2407	
2	0.2271	0.3110	0.4504	0.3244	0.2456
1	0.2185	0.2314	0.4697	0.1551	0.5958
Mol Fraction of Component 1 in Gas					
K = 1					
	1	2	3	4	5
3		0.0057	0.0245	0.0019	
2	0.0000	0.0027	0.0088	0.0458	0.0031
1	0.0000	0.0000	0.0001	0.0291	0.0242
K = 2					
	1	2	3	4	5
3		0.0081	0.0019	0.0011	
2	0.0000	0.0160	0.0155	0.0028	0.0018
1	0.0000	0.0000	0.0311	0.0144	0.0092

becomes burdensome here, but for brevity we will refer to Spillette *et al.*'s sequential method as the  $p/S_w - S_g$  scheme and the altered method discussed here as the  $p - S_g/S_w$  scheme. The two pressure-gas saturation equations include implicit treatment of gas-oil capillary pressure, gas-phase transmissibilities, and the  $S_g$  dependence of oil-phase transmissibilities. Water-phase transmissibilities, water-oil capillary pressure, and  $S_w$  dependence of oil-phase transmissibilities are explicit in the pressure-gas saturation equations and implicit in the sequential water saturation equation. The advantages of the  $p - S_g/S_w$  scheme over the  $p/S_w - S_g$  scheme in steamflood simulation are threefold:

1. An instability arises in the  $p/S_w - S_g$  scheme at an interface across which steam is flowing from a block having steam to a block where  $S_g = 0$  due to condensation. This steam flow rate is often large in the vertical direction as it percolates upward from a bottom-completed steam injection well. We have found occasionally severe instability in the earlier model using the  $p/S_w - S_g$  scheme evidenced by oscillating gas saturation in the lower block from which steam is flowing. The instability follows from the inability of the sequential  $S_w - S_g$  equations to alter the explicit steam flow rate calculated in the pressure equation. Any alteration of the steam flow rate results in the upper undersaturated ( $S_g = 0$ ) block having a gas (steam) saturation—either positive if the flow rate alteration is positive or negative if the alteration is negative. This positive or negative  $S_g$  in a cold (undersaturated) block results in pressure surges and oscillations over a sequence of time steps and often causes divergence and termination of the run. In the  $p - S_g/S_w$  scheme, the gas flow rate with implicit transmissibility is solved simultaneously with pressure and this instability does not arise.

2. The primary or most severe instabilities in steamflooding, and in black-oil problems, are associated with gas-oil flow interactions and water-oil flow instabilities are generally secondary. Solution for gas saturation with implicit gas-phase transmissibilities simultaneously with pressure, therefore, is more stable than divorcing the implicit gas-phase treatment from pressure. In fact, in the black-oil case many problems require implicit treatment of the gas phase only and the water saturation can be calculated explicitly—i.e., without solution of the sequential, implicit-transmissibility water saturation equation.

3. In the  $p - S_g/S_w$  scheme, compositional steamflood problems do not experience the material balance deterioration of the  $p/S_w - S_g$  scheme described above in connection with the earlier steamflood model. The reason for this is that in the  $p - S_g/S_w$  scheme, there is no altered gas-phase flow rate that differs from that corresponding to the pressure solution.

The computing time requirements of the two schemes are comparable since they both involve solution of a set of two simultaneous equations and of a single equation.

We chose not to use this  $p - S_g/S_w$  scheme for our steamflood model for the following reason. Unlike most black-oil problems, a strong instability arises in steamflood simulation due to a steam-water flow interaction. That is, at various points in time and space, steam- and water-phase mobilities considerably exceed oil-phase mobility. An extreme example of this is a region where oil is reduced to residual oil saturation and its mobility is zero. In the  $p - S_g/S_w$  scheme, the sequential water saturation controls water saturation instability or oscillations only in respect to the relative water- and oil-phase flow rates, and if oil rate is zero, there is no control at all. In this respect, the  $p/S_w - S_g$  scheme is superior since the two saturation equations treat all three-phase flow interactions through three-phase fractional flow considerations.

For compositional steamflood problems, we would have to extend the  $p - S_g/S_w$  scheme to a  $p - S_g - x_1/S_w$  scheme since transmissibilities explicit in  $x_1$  and  $y_1$  would leave an instability requiring time steps nearly as small as those required due to transmissibilities explicit in gas saturation.

To summarize, in our steamflood model development, we found the  $p - S_g/S_w$  or  $p - S_g - x_1/S_w$  scheme to have significant advantages over the  $p/S_w - S_g$  scheme; however, one disadvantage cited immediately above led to abandonment. In development of a black-oil model from our steamflood model, we found the  $p - S_g/S_w$  scheme satisfactory and superior in stability to the  $p/S_w - S_g$  scheme.

The above discussion compares the  $p - S_g/S_w$  scheme with the  $p/S_w - S_g$  scheme that was used in the earlier steamflood model. In comparison with the highly implicit steamflood model described in this paper, the  $p - S_g/S_w$  scheme offers an additional advantage of significantly less computing time requirement per grid-block time step. In the dead-oil case, the  $p - S_g/S_w$  scheme requires time of  $2^3$  or 8 for the pressure-gas saturation equations and 1 for the water saturation equation. This compares with the previously mentioned  $3^3$  or 27 for the implicit model. In the compositional case, the  $p - S_g - x_1/S_w$  scheme requires time of  $27 + 1$ , while the implicit model requires time of 64. In spite of this and other attractive features of the  $p - S_g/S_w$  scheme, we chose not to use it due to occasionally strong steam-water flow interactions in steamflooding.

## SUMMARY

An implicit, three-dimensional steamflood model has been developed. The model is fully implicit except for explicit treatment of certain PVT variables and approximation of new time level relative permeabilities. With these exceptions, the model is fully implicit in water saturation, composition, pressure, and either gas saturation or temperature depending on whether free gas is present or absent, respectively, in the grid block. The model solves three simultaneous equations for

the dead-oil case and four equations for the compositional case treating solution gas or inert gas or distillation effects.

This implicit model is compared in capability, stability, and efficiency with a previously published sequential steamflood model that solves first a pressure equation, then two simultaneous saturation equations. For some compositional problems involving high gas-phase flow rates with sharp concentration gradients, the sequential model formulation fails to preserve good material balances on light hydrocarbon components. The implicit model eliminates this problem. The implicit model requires more arithmetic per grid-block time step than does the sequential model. To compete in efficiency with the sequential model, the implicit model therefore must take larger time steps. We have found in practice that this is the case for most problems and the implicit model requires comparable or less total computer time. For "easy" problems where the stability of the sequential model is adequate, the latter requires less computer time; for "difficult" problems where the stability of the sequential model is marginal, the latter can require two or more times as much computer time.

The material balance problem in the sequential model and above observations regarding efficiencies of the two models are illustrated through model results for two field-scale steamflood problems. The second of these problems and other unreported results indicate that sensitivity of calculated results to time step size is less for the implicit than for the sequential model.

An alternate sequential steamflood-model formulation is described and its advantages over the published sequential model are cited. While this alternate sequential formulation lacks the reliability of the implicit formulation in steamflood simulation, we have found it highly stable and satisfactory in isothermal black-oil saturation.

#### NOMENCLATURE

$A$  = cross-sectional area normal to flow  
 $c$  = compressibility, 1/psi  
 $c_r$  = rock formation compressibility, 1/psi  
 $c_p$  = specific heat, Btu/lb-°F  
 $C_{POB}$  = overburden heat capacity  
 $(\rho C_p)_R$  = reservoir formation specific heat, Btu/cu ft rock-°F  
 $C_{T_o}$  = oil thermal expansion coefficient, vol/vol-°F  
 $H$  = enthalpy, Btu/mol  
 $H_{oT} = \partial H_o / \partial T$   
 $H_{wT} = \partial H_w / \partial T$   
 $k$  = absolute permeability  
 $k_H$  = horizontal permeability  
 $k_r$  = relative permeability  
 $k_{rg}$  = relative permeability to gas  
 $k'_{rg} = dk_{rg} / dS_g$   
 $k_{rw}$  = relative permeability to water

$k'_{rw} = dk_{rw} / dS_w$   
 $k_{rog}$  = relative permeability to oil in gas-oil two-phase system with irreducible water present  
 $k_{row}$  = relative permeability to oil in water-oil two-phase system  
 $k_{rosG} = \partial k_{ro} / \partial S_g$   
 $k_{rosW} = \partial k_{ro} / \partial S_w$   
 $k_v$  = vertical permeability  
 $K_{OB}$  = overburden thermal conductivity  
 $K_R$  = thermal conductivity of reservoir formation, Btu/°F-ft-D  
 $K_1$  = equilibrium  $K$ -value for hydrocarbon Component 1  
 $K_{1p} = \partial K_1 / \partial p$   
 $K_{1T} = \partial K_1 / \partial T$   
 $\ell$  = distance between adjacent grid-block centers  
 $MW_i$  = molecular weight of hydrocarbon component  $i$   
 $NX, NY, NZ$  = number of grid blocks in  $x, y, z$  directions  
 $p$  = oil pressure, psia  
 $p_i$  = initial reservoir pressure  
 $p_g$  = gas-phase pressure  
 $p_w$  = water-phase pressure  
 $p_{sat}$  = steam saturation pressure  
 $p_{cgo} = \text{gas-oil capillary pressure, } p_g - p$   
 $p'_{cgo} = dp_{cgo} / dS_g$   
 $p_{cwo} = \text{water-oil capillary pressure, } p - p_w$   
 $q_H$  = enthalpy production rate, Btu/D  
 $q_L$  = heat loss rate, Btu/D  
 $q_w, q_o, q_g$  = phase production rates, RB/D  
 $S$  = fluid saturation, fraction  
 $S_{gr}$  = residual gas saturation  
 $S_{wc}$  = connate water saturation  
 $t$  = time, days  
 $\Delta t$  = time step,  $t_{n+1} - t_n$   
 $T$  = temperature, °F  
 $T_i$  = initial reservoir temperature  
 $U$  = internal energy, Btu/mol  
 $V$  = grid-block volume,  $\Delta x \cdot \Delta y \cdot \Delta z / 5.6146$ , res/bbl  
 $x, y, z$  = Cartesian coordinates, ft  
 $\Delta x, \Delta y, \Delta z$  = grid-block dimensions  
 $x_i$  = mol fraction of hydrocarbon component  $i$  in the oil phase  
 $y_i$  = mol fraction of hydrocarbon component  $i$  in the gas phase  
 $y_s$  = mol fraction of steam in the gas phase  
 $Z$  = depth, measured vertically downward, ft  
 $\gamma$  = specific weight, psi/ft  
 $\delta$  = time difference operator, e.g.,  $\delta T = T_{n+1} - T_n$

- $\rho$  = molar density, mol/RB
- $\rho_g T = \partial \rho_g / \partial T$
- $\rho_o T = \partial \rho_o / \partial T$
- $\rho_{ox1} = \partial \rho_o / \partial x_1$
- $\mu$  = viscosity, cp
- $\phi$  = porosity, fraction
- $\phi_i$  = porosity, at  $p_i$
- $\lambda$  = mobility,  $k_r / \mu$
- $\lambda_{ws} = \partial \lambda_w / \partial S_w$
- $T$  = transmissibility
- $T_c$  = heat conduction transmissibility
- $T_H = T_w H_w + T_o H_o + T_g H_g$

**SUBSCRIPTS**

- $g$  = gas
- $i$  = hydrocarbon component number ( $x_i, y_i$ , or initial ( $p_i, T_i$ ))
- $\ell$  = latest iterate (superscript)
- $n$  = time level
- $o$  = oil
- $s$  = steam
- $w$  = water
- $OB$  = overburden

**DIFFERENCE OPERATORS**

$$\delta X \equiv X_{n+1} - X_n, \text{ where } X \text{ is any quantity}$$

$$\Delta(T_w \Delta p_w) = \Delta_x(T_w \Delta_x p_w) + \Delta_y(T_w \Delta_y p_w) + \Delta_z(T_w \Delta_z p_w)$$

$$\Delta_x(T_w \Delta_x p_w) = T_{wi+\frac{1}{2},j,k}(p_{wi+1,j,k} - p_{wi,j,k}) - T_{wi-\frac{1}{2},j,k}(p_{wi,j,k} - p_{wi-1,j,k})$$

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**APPENDIX**

Oil-phase molar density is a nonlinear function of pressure, temperature, and composition  $x_1$ , as given by Eq. 7. To obtain  $\delta \rho_o$ , we write

$$\rho_{on+1} = \rho_o^\ell + \rho_{oT}(T_{n+1} - T^\ell) + \rho_{ox1}(x_{1n+1} - x_1^\ell) + \rho_{op}(p_{n+1} - p^\ell) \dots \dots \dots (A-1)$$

where  $\rho_{oT}$ ,  $\rho_{ox1}$ , and  $\rho_{op}$  are partial derivatives of  $\rho_o$  with respect to  $T$ ,  $x_1$ , and  $p$ , respectively, evaluated at latest iterate ( $\ell$ ) values. Subtracting  $\rho_{on}$  from both sides of Eq. A-1 then gives

$$\delta \rho_o = \rho_{oT} \delta T + \rho_{ox1} \delta x_1 + \rho_{op} \delta p + \rho_o^\ell - \rho_{on} - \rho_{oT}(T^\ell - T_n) - \rho_{ox1}(x_1^\ell - x_{1n}) - \rho_{op}(p^\ell - p_n),$$

which is Eq. 13. Eq. 16 for  $\delta \rho_g$  is obtained in a similar manner.

The term  $\delta y_1$  is obtained by writing

$$\delta y_1 = x_{1n} \delta K_1 + K_{1n+1} \delta x_1 \dots \dots \dots (A-2)$$

$$\delta K_1 = K_{1T} \delta T + K_{1p} \delta p + K_1^\ell - K_{1n} - K_{1T}(T^\ell - T_n) - K_{1p}(p^\ell - p_n) \dots (A-3)$$

where  $K_{1T}$  and  $K_{1p}$  are partial derivatives of  $K_1$  with respect to  $T$  and  $p$ , respectively, evaluated at latest iterate values. Substituting  $\delta K_1$  from Eq. A-3 into Eq. A-2 gives  $\delta y_1$  as given by Eq. 15.

Eq. 14 follows the equation for porosity

$$\phi = \phi_i (1 + c_r (p - p_i)) \dots (A-4)$$

where  $\phi_i$  is porosity at initial pressure  $p_i$ .

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