

The Use of Vertical Equilibrium in Two-Dimensional Simulation of Three-Dimensional Reservoir Performance

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ABSTRACT

This paper discusses the use of the Vertical Equilibrium (VE) concept in simulating heterogeneous reservoirs. Where VE criteria are met, this technique allows two-dimensional (2-D) simulation of three-dimensional (3-D) problems with equivalent accuracy, and with attendant substantial savings in data preparation and machine time.

The paper presents the VE concept itself and a new dimensionless group as a possible criterion for the validity of VE as applied to thick reservoirs or to reservoirs where the capillary transition zone is a small fraction of thickness. A description of the generation of the appropriate pseudo relative permeability and capillary pressure curves is presented.

In addition to the dimensionless group criterion, an actual comparison of the results of an x - z cross-section and a one-dimensional (1-D) areal run with VE illustrates the validity of the VE concept. Numerical results of such a comparison along with the attendant machine-time requirements are presented. More than an order of magnitude difference in machine-time requirements was experienced. Finally, an actual field case example shows the utility of VE as applied to a reservoir containing one or multiple gas pools residing on a common aquifer.

INTRODUCTION

Numerical simulation of reservoir performance currently encompasses a wide variety of recovery processes, reservoir types and purposes or questions to which answers are sought. A feature common to virtually all reservoir simulation studies, however,

is the choice of simulation in one, two or three dimensions. Most frequently this choice is one between an areal (x - y) study and a 3-D study. While the areal study is considerably cheaper than a 3-D simulation, the validity or accuracy of the former is often questioned in light of its apparent inability to simulate flow and fluid saturation distributions in the vertical direction. Areal studies are frequently performed with little attention to or understanding of the extent to which the x - y calculations do or can be made to account for this vertical flow and fluid distribution.

Previous papers^{1,2} describe a VE assumption or concept which leads to the definition of pseudo relative permeability and capillary pressure curves to be used in areal studies to simulate 3-D flow. A dimensionless group proposed as a criterion for the assumption's validity¹ primarily treats the case of a reservoir where the capillary transition zone is an appreciable fraction of reservoir thickness.

This paper treats the case of a reservoir where the capillary transition zone is a small fraction of reservoir thickness (e.g., < 10 percent). We propose to describe the VE concept as applied to thick reservoirs or to reservoirs where capillary transition zone is a small fraction of thickness; to describe the generation of appropriate relative permeability and capillary pressure curves for such reservoirs to represent 3-D performance by 2-D areal calculations; to propose a new dimensionless group as a criterion for the VE assumptions' validity, obtained from an analysis of countercurrent gravity segregation; and finally, to present a cross-sectional vs 1-D (VE) comparison and a 2-D areal field case study.

THE VERTICAL EQUILIBRIUM CONCEPT

Most oil and gas reservoirs extend distances areally which are at least two orders of magnitude greater than reservoir thickness. Viewed in perspective, these reservoirs appear as "blankets". For a variety of reasons, some valid and some invalid, numerical simulations of such reservoirs are performed occasionally in three dimensions as opposed to only two areal (x - y) dimensions. A 2-D

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¹References given at end of paper.

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areal calculation is strongly preferable to a 3-D simulation in that computing expense is at least an order of magnitude lower. However, uncertainty often exists in relation to the validity of the areal results or, more specifically, in relation to the ability of an areal calculation to account for effects of flow and nonuniform fluid saturation distributions in the vertical direction. The VE concept is an assumption which, in those cases where it is valid, allows simulation of 3-D performance of a reservoir with a 2-D areal calculation.

The VE concept is easily explained in reference to the heat conduction analogy to fluid flow in reservoirs. We would accept with little concern a 2-D simulation of heat conduction in a metal sheet many feet in areal expanse and 1/8 in. thick. The assumption implicit in this 2-D calculation is that of thermal equilibrium through the sheet thickness. This assumption of thermal equilibrium is in turn simply one of uniform temperature. The analogous assumption for a reservoir of large areal extent relative to thickness is also one of equilibrium throughout reservoir thickness. However, equilibrium in this case does not imply uniform pressure or uniform saturation, but rather uniform fluid phase potentials ($p - \gamma z$). Uniform potentials in turn imply capillary-gravitation equilibrium, which yields nonuniform saturation and pressure profiles through reservoir thickness. In the case of insignificant capillary forces treated here, this assumed equilibrium through reservoir thickness corresponds to a gravity segregated fluid distribution with potential of any given phase being uniform in the portion of the fluid column occupied by that phase.

This VE assumption is not equivalent to an assumption of no flow in the vertical direction; in fact, it is equivalent to the opposite assumption of infinite vertical flow rate. It is equivalent to the assumption of infinite vertical permeability. Reservoir stratification and/or injection-production well completion intervals tend to create nonequilibrium fluid distributions (water overriding oil) through the reservoir thickness. Under the assumption of infinite vertical conductivity, these vertical-direction perturbations from equilibrium instantaneously dissipate with resultant equilibrium fluid distributions. These equilibrated vertical fluid distributions are assumed to exist throughout the reservoir during the displacement process. Equilibrium is not assumed in the areal directions and the areal calculation using VE is capable of simulating gas overriding water or oil and water underrunning oil; that is, the VE assumption does not imply horizontal gas-oil and/or water-oil contacts throughout the reservoir during producing operations.

The assumption of infinite vertical conductivity is clearly incorrect in an absolute sense for any reservoir. However, the validity of the assumption as a practical matter depends upon the rapidity with which perturbations from equilibrium in the vertical-direction saturation distributions dissipate as compared with the rate of movement areally throughout the reservoir. Pore velocities of fluid

particles in the reservoir are the order of magnitude of 1 ft/D. Depending upon the reservoir thickness, vertical permeability, and levels of gravitational and capillary forces, a nonequilibrium fluid distribution may "decay" to near-equilibrium distribution after traveling a relatively short distance areally through the reservoir.

The existence of VE in a reservoir is enhanced by large vertical permeability, small thickness, high gravity and/or capillary forces, low fluid viscosities, high relative permeabilities and low rates of areal fluid movement. In any given case, the combined effect of all these properties determines the degree of validity of the VE assumption. A published criterion¹ incorporates these variables in a dimensionless group which provides rule of thumb for an *a priori* decision regarding the necessity for 3-D as opposed to 2-D areal simulation. However, that criterion is based on the assumption that the capillary transition zone is an appreciable fraction of reservoir thickness. Later in this paper we will propose a criterion based on the assumption that gravity forces alone are the driving forces toward equilibrated (segregated) vertical fluid distributions.

These criteria or dimensionless groups are simply rough rules of thumb for deciding between 2-D areal and 3-D simulation. We have found from experience that it is strongly desirable to make an actual comparison between a representative cross-section (vertical slice) calculation and a 1-D calculation using VE. Good agreement between these two calculations indicates the sufficiency of the cheaper 2-D areal simulation.

PSEUDO CAPILLARY PRESSURE

The VE concept is applicable to gas-oil, water-oil, gas-water, or three-phase systems. Field applications given below involve water-drive gas fields. We, therefore, discuss application of the VE concept to generate pseudo capillary pressure and relative permeability curves in the context of a gas-water system.

We consider a reservoir with a capillary transition zone of less than 10 percent of thickness. In such a case, the assumption of VE is one of fluid (gravity) segregation. We consider first the homogeneous, vertical slice through a flat reservoir, shown in Fig. 1, with significant thickness h and with an insignificant capillary transition zone. The initial gas-water contact is at $z = z_{ci}$. Fluid injection and production wells are scattered arbitrarily along the section. A 1-D areal (x -

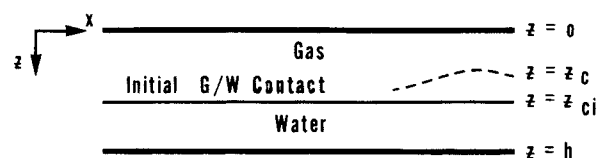


FIG. 1 — VERTICAL SLICE THROUGH A FLAT RESERVOIR.

direction) simulation of this cross-section determines depth-averaged water saturation \bar{S} as a function of position and time. Initially this depth-averaged saturation is clearly uniform and equal to

$$\bar{S}_i = (S_{wc} z_{ci} + h - z_{ci})/h, \quad (1)$$

since the reservoir is 100 percent water saturated below z_{ci} and contains only connate water above. The VE concept assumes that at any stage of the dynamic displacement the fluid distribution through reservoir thickness at any areal point x is a segregated one, consisting of connate water from the top of the sand to the moving gas-water contact z_c and residual gas or no gas below the contact. Consider a point near a water injection well as shown in Fig. 2. Let \bar{S} ($> \bar{S}_i$) denote the depth-averaged water saturation at this position at some time after initiation of water injection. By the VE assumption of segregation, we have

$$\bar{S} = (S_{wc} z_c + (1-S_{gr})(z_{ci}-z_c) + h - z_{ci})/h \quad (2a)$$

if $z_c < z_{ci}$ and

$$\bar{S} = (S_{wc} z_c + h - z_c)/h \quad (2b)$$

if $z_c > z_{ci}$.

Eqs. 2a and 2b relate the saturation \bar{S} to the gas-water contact position z_c . We now relate this contact position to the difference between water and oil pressures. We select a reference plane for the areal calculation as the top of the reservoir, in this case the horizontal plane at $z = 0$. In the areal (1-D) calculation, the pressures p_w and p_g used to calculate the Darcy flow from one grid block horizontally to its neighbors are defined as pressures at this plane $z = 0$. Now by the premise of insignificant rock capillary pressure forces, water and gas pressures are equal at the gas-water contact. Thus, we write

$$p_{wc} = p_{gc} \quad (3)$$

At any position other than $z = z_c$ we represent water and gas pressures by

$$p_w(z) = p_{wc} + \gamma_w(z - z_c) \quad (4)$$

$$p_g(z) = p_{gc} + \gamma_g(z - z_c) \quad (5)$$

At the reference plane $z = 0$, then,

$$p_g - p_w \equiv \bar{P}_c = \Delta\gamma z_c \quad (6)$$

We call this pressure difference a pseudo capillary pressure and denote it by \bar{P}_c . Note that this pseudo capillary pressure arises from gravity forces in the vertical direction and has nothing whatsoever to do with physical, rock capillary forces.

Substitution of z_c from Eq. 6 into Eqs. 2a and 2b gives a relation between depth-averaged water saturation \bar{S} and pseudo capillary pressure \bar{P}_c :

$$\bar{S} = \frac{-(1-S_{wc}-S_{gr})\bar{P}_c}{h\Delta\gamma} + 1 - \frac{S_{gr} z_{ci}}{h} \quad (7a)$$

if $z_c < z_{ci}$ and

$$\bar{S} = 1 - \frac{1-S_{wc}}{h\Delta\gamma} \bar{P}_c \quad (7b)$$

if $z_c > z_{ci}$. Fig. 3 shows a plot of Eqs. 7a and 7b representing the pseudo capillary pressure curve to be used in the areal calculation for the case $h = 300$ ft, $S_{wc} = 0.15$, $S_{gr} = 0.25$, $z_{ci} = 100$ ft, $\Delta\gamma = \gamma_w - \gamma_g = 0.38$ psi/ft.

PSEUDO RELATIVE PERMEABILITY

Fig. 2 illustrates an areal point at some time after initiation of water injection where the water-oil

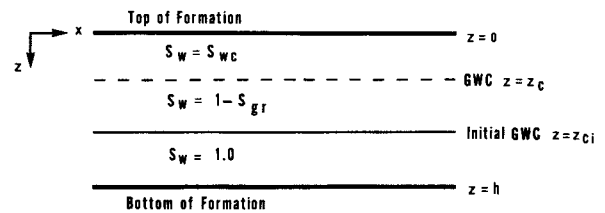


FIG. 2 — SEGREGATED FLUID DISTRIBUTION NEAR WATER INJECTION WELL.

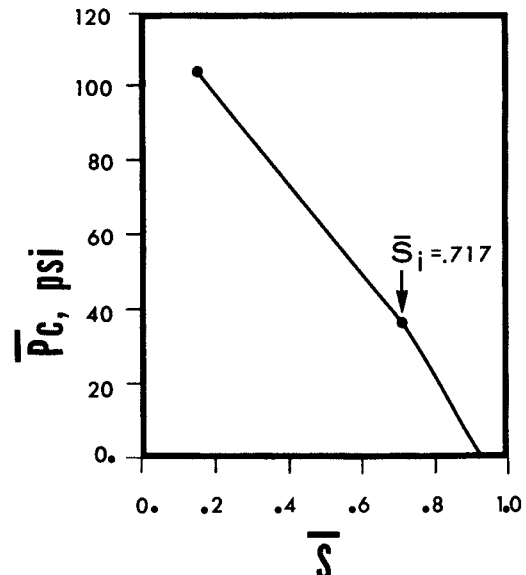


FIG. 3 — PSEUDO CAPILLARY PRESSURE CURVE.

contact has moved up. Fig. 4 shows rock or laboratory relative permeability to water for areal flow (in the x-direction) is

$$\bar{k}_{rw} = (0 \cdot z_c + k_{rwr}g(z_{ci} - z_c) + h - z_{ci})/h \quad (8a)$$

if $z_c < z_{ci}$ and

$$\bar{k}_{rw} = (0 \cdot z_c + h - z_c)/h \quad (8b)$$

if $z_c > z_{ci}$. Eliminating z_c between Eqs. 2a and 2b and 8a and 8b gives pseudo relative permeability to water as a function of depth-averaged water saturation:

$$\begin{aligned} \bar{k}_{rw} &= \frac{k_{rwr}g \bar{S}}{(1 - S_{gr} - S_{wc})} \\ &+ \frac{k_{rwr}g(z_{ci}(1 - S_{gr} - S_{wc}) - h + S_{gr}z_{ci})}{h(1 - S_{gr} - S_{wc})} \\ &+ \frac{(h - z_{ci})(1 - S_{gr} - S_{wc})}{h(1 - S_{gr} - S_{wc})} \quad \dots \quad (9a) \end{aligned}$$

if $z_c < z_{ci}$ and

$$\bar{k}_{rw} = \frac{\bar{S} - S_{wc}}{1 - S_{wc}} \quad \dots \quad (9b)$$

if $z_c > z_{ci}$. A similar deviation yields pseudo relative permeability to gas as

$$\bar{k}_{rg} = \frac{k_{rgc}w}{1 - S_{gr} - S_{wc}} [1 - S_{gr} \frac{z_{ci}}{h} - \bar{S}] \quad (10a)$$

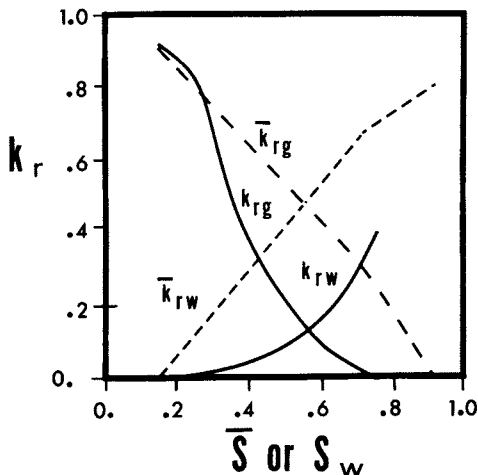


FIG. 4 — ROCK AND PSEUDO RELATIVE PERMEABILITY CURVES.

if $z_c < z_{ci}$ and

$$\bar{k}_{rg} = \frac{k_{rgc}w}{1 - S_{wc}} (1 - \bar{S}) \quad \dots \quad (10b)$$

if $z_c > z_{ci}$.

Fig. 4 shows these pseudo relative permeability curves for the case $h = 300$ ft, $z_{ci} = 100$ ft, $S_{gr} = 0.25$, $S_{wc} = 0.15$, $k_{rwr}g = 0.4$, $k_{rgc}w = 0.9$.

EFFECT OF RESERVOIR STRUCTURE

Fig. 5 illustrates the more realistic case where a vertical slice through the reservoir exhibits structure of varying dip angle and varying reservoir thickness. In this case, the areal grid block configuration is as shown by the dotted lines. We again select the top of the sand as the reference plane and assign a value of z_{ij} to each of the indicated grid points as the depth of the grid point below some arbitrary reference horizontal plane. These z_{ij} values are used to define potentials $p_{ij} - \gamma z_{ij}$ which are in turn used to calculate the Darcy flow between areal grid blocks. Each areal grid block has a unique thickness or height h_{ij} . Also, as indicated in Fig. 5, the relative position of the initial contact will, in general, vary for each grid block. The pseudo capillary pressure and relative permeability curves for a given grid block (i, j) are given by Eqs. 7a and 7b, 9a and 9b and 10a and 10b provided

z_{cij} = depth of initial contact from top of the block for block i, j

h_{ij} = reservoir thickness at areal position i, j .

It is easily seen that in this case each grid block, in general, has a different set of pseudo capillary pressure and relative permeability curves. Thus, it becomes preferable to program internally the calculation of pseudo relative permeability and capillary pressure rather than read them in as tables.

EFFECT OF RESERVOIR STRATIFICATION

We now consider a stratified reservoir as indicated in Fig. 6. If both z_{ci} and z_c lie within Layer 2 and $z_c < z_{ci}$, then

$$\begin{aligned} \bar{S} &= \{S_{wc}(\phi_1 z_1 + \phi_2(z_c - z_1)) \\ &+ (1 - S_{gr})\phi_2(z_{ci} - z_c) + \phi_2(z_2 - z_{ci}) \\ &+ \phi_3(h - z_2)\} / \Sigma \phi \Delta z \quad \dots \quad (11) \end{aligned}$$

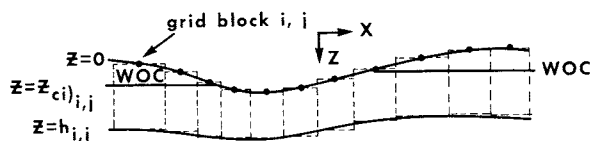


FIG. 5 — AREAL GRID BLOCK CONFIGURATION IN A RESERVOIR OF VARYING STRUCTURE.

where

$$\Sigma \phi \Delta z = \phi_1 z_1 + \phi_2 (z_2 - z_1) + \phi_3 (h - z_2).$$

Pseudo relative permeability to gas for areal flow is

$$\bar{k}_{rg} = \{k_{rgcw} k_1 z_1 + k_{rgcw} k_2 (z_c - z_1)\} / \Sigma k \Delta z, \dots (12)$$

where

$$\Sigma k \Delta z = k_1 z_1 + k_2 (z_2 - z_1) + k_3 (h - z_2).$$

Finally, the difference between water and gas pressures referred back to the reference point at $z = 0$ is given by Eq. 6. Elimination of z_c between Eqs. 11 and 6 relates \bar{P}_c to \bar{S} and elimination of z_c between Eqs. 12 and 6 relates \bar{k}_{rg} to \bar{S} .

A similar procedure allows determination of \bar{k}_{rg} or \bar{k}_{rw} , \bar{P}_c and \bar{S} for any given positions of z_{ci} and z_c . These equations can be solved (a) externally to the simulator, resulting in different pseudo relative permeability and capillary pressure curves for each areal block, or (b) internally within the simulator with reservoir stratification (which may vary with areal position) read in as input data. We have found (b) more desirable from the standpoint of minimizing storage requirements, although (a) results in lower simulator computing time.

AN EXAMPLE TEST OF THE VE ASSUMPTION

As discussed earlier, a common test one uses to determine whether a 3-D simulator is required for a particular reservoir study is to run a vertical cross-section and compare the results with a 1-D areal. If the results compare favorably, a 2-D areal model can then be used with confidence for the reservoir study. Data preparation and computer charges are then minimized.

Fig. 7 shows a reservoir system selected to

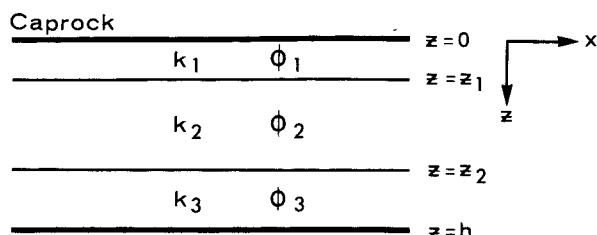


FIG. 6 — RESERVOIR STRATIFICATION.

TABLE 1 — DATA FOR 1-D AREAL AND x-z CROSS-SECTION

$N_x = 24$			
$N_z = 15$			
$\Delta x = 2,000$ ft		$4 \leq i \leq 24$	
$\Delta x_1 = 16,000$ ft			
$\Delta x_2 = 8,000$ ft			
$\Delta x_3 = 4,000$ ft			
$\Delta z = 20$ ft		$1 \leq j \leq 11$	
$\Delta z_{12} = 30$ ft			
$\Delta z_{13} = 50$ ft			
$\Delta z_{14} = 75$ ft			
$\Delta z_{15} = 125$ ft			
$\Delta y = 10,000$ ft			

Saturation Table			
S_w	P_c	k_{rw}	k_{rg}
0.1	4.5	0.0	1.0
0.55	0.0	0.5	0.5
1.0	-4.5	1.0	0.0

Fluid Data (see Table 2)	
water compressibility =	.000003 (1/psi)
rock compressibility =	.000004 (1/psi)
initial pressure =	5,050 psia
water viscosity =	0.6 cp
base water density =	62.4 lb/cu ft
base gas density =	46.6 lb/Mcf
horizontal permeability =	100 md
vertical permeability =	25 md
porosity =	0.15

Production Data	
Well	Rate (Mcf/D)
1	16,000
2	16,000
3	16,000

illustrate this comparison of 2-D cross-sectional results with 1-D areal results. Table 1 gives the reservoir rock, fluid and production data.

The reservoir system is a 500-ft thick water-bearing sand formation with two gas fields trapped in structural highs. Field B is producing and Field A is shut in.

A 24×15 grid defines the x-z cross-section. Irregular grid spacing used in both the x and z directions allows better definition of the structural heterogeneities. The 1-D areal model with VE uses

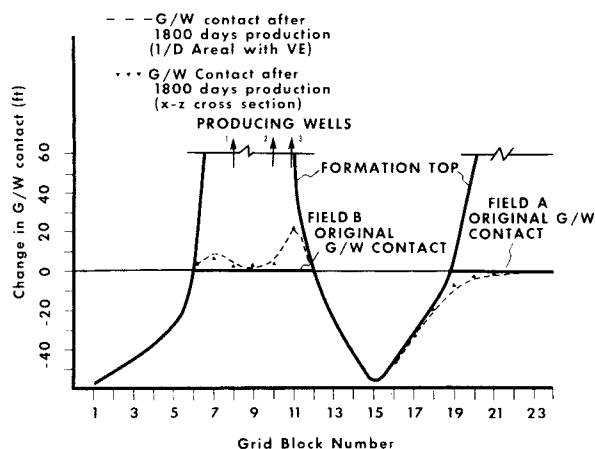


FIG. 7 — MOVEMENT OF GAS-WATER CONTACT.

a 24 × 1 grid with identical definition of the gas fields as in the *x-z* cross-section.

Both cases were run to simulate 1,800 days of production. The results show gas migrating down structure from Field A toward the gas producing area in Field B. Fig. 7 shows an amplification of the formation cross-section in the vicinity of the original gas-water contact. The contour of the gas-water contact at 1,800 days shows a lateral movement of gas some 5,000 ft down structure from the east to the west. Fig. 7 also shows the agreement between the *x-z* cross-section and the 1-D areal calculations using VE.

The computing times were 3 seconds for an 1,800-day run using the 1-D areal, and 74 seconds for the same 1,800-day run using the *x-z* model. Using these computing times, one estimates that a full 3-D study would be approximately 25 times as much machine time as that of a 2-D areal simulator using VE. This latter statement assumes all storage in-core. As core requirements for a good 3-D definition areally are usually large, a much finer areal definition of the reservoir can be attained using a 2-D simulator and still remain completely in-core.

2-D, AREAL FIELD APPLICATION

Fig. 8 shows the structure of a large aquifer with two gas fields located on structural highs. The original gas-water contacts are 6,490 and 6,375 ft subsea for Fields A and B, respectively. The reservoir and fluid properties, production data and completion intervals are given in Table 2.

A 2-D, gas-water transient simulator with the VE concept built in was used for the reservoir study. The 18,000 × 20,000-ft reservoir containing the two gas fields was defined by an 18 × 20 grid of 1,000-ft square blocks.

The objective was to study the interference effects that a producing field would have on an

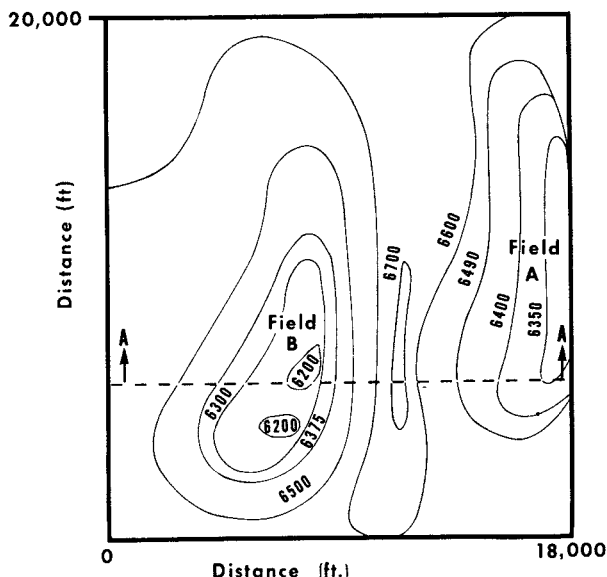


FIG. 8 — STRUCTURE OF GAS FIELD-AQUIFER SYSTEM.

adjacent field which was discovered at a later date. Since the two gas fields share a common aquifer, the production from Field B causes transients which are transmitted through the aquifer to Field A. It is the various effects of these transients which we will discuss here.

Fig. 9 shows time sequence contours of change in gas bubble thickness. The +1-ft contours at 15 years indicate that gas from Field A is migrating down structure toward Field B. The lateral extent of Field A after Field B has been producing for 15 years is some 60 percent larger than original. Because of residual gas saturation, production of Field A will result in a lower recovery than if both fields had been discovered and produced simultaneously.

After 15 years of production of Field B, a pressure gradient of 93 psi exists across Field A with the steepest part of the gradient in the area adjacent to Field B. Fig. 10 is a cross-section plot of the pressure distribution along a Line A-A (shown on Fig. 8). The 15-year pressure distribution along Line A-A shows more than a 100-psi decrease in average bubble pressure.

The implications of the gas-water contact movement down structure and the existence of a pressure gradient in Field A and the decrease in average bubble pressure during the production history of Field B are as follows: (1) Field B's production is causing gas in Field A to migrate down the west flank toward Field B, (2) Field A's gas bubble is expanding laterally as the pressure declines due to the pressure sink caused by Field

TABLE 2 — DATA FOR 2-D AREAL PROBLEM

$N_x = 18$		
$N_y = 20$		
$\Delta x = \Delta y = 1,000$ ft		
<u>Gas-Water Contacts</u>		
Pool 1 = 6,490 ft		
Pool 2 = 6,375 ft		
formation thickness = 500 ft		
permeability = 50 md		
water viscosity = 0.7 cp		
porosity = 0.15		
residual gas saturation = 0.2		
connate water = 0.1		
initial pressure = 5,050 (at G/W contact Field A)		
<u>Fluid Data</u>		
gas gravity = 0.75		
<u>Pressure</u>	<u>B_g</u>	<u>μ_g</u>
500	5.250	0.014
1,000	2.480	0.016
2,000	1.190	0.020
3,000	0.786	0.025
4,000	0.590	0.031
5,000	0.471	0.037
6,000	0.393	0.042
<u>Production Data</u>		
<u>Well</u>	<u>Rate (Mcf/D)</u>	<u>Formation Top</u>
1	2,015	6,300
2	8,150	6,300
3	5,900	6,308
4	16,100	6,200
5	5,300	6,275
6	6,850	6,275

B's production, and when Field A does come on production, its deliverability due to this pressure decline will be considerably reduced as will its ultimate gas recovery, and (3) if Field B continues to produce with Field A shut in, the gas migrating down structure will soon reach the structure saddle-point (Fig. 9) and start "leaking" into Field B.

Computer time for this 360-block, 15-year production run was 23 seconds of UNIVAC 1108 time.

A PROPOSED CRITERION FOR VE

We consider countercurrent segregation from an initially uniform saturation distribution in the closed vertical column shown in Fig. 11. Martin³ gives a detailed analysis of this problem. Neglecting fluid compressibilities, the mass conservation equations for wetting (*w*) and nonwetting (*n*) phases are

$$-\frac{\partial u_w}{\partial z} = \phi \frac{\partial S_w}{\partial t} \dots \dots \dots (13a)$$

$$-\frac{\partial u_n}{\partial z} = \phi \frac{\partial S_n}{\partial t} \dots \dots \dots (13b)$$

Addition of Eqs. 13a and 13b gives $\partial u/\partial z = 0$, where *u* is total velocity $u_w + u_n$. Since *u* is zero at the closed column ends, it is then zero everywhere.

Darcy's law gives the superficial velocities as

$$u_w = -\frac{k k_{rw}}{\mu_w} \left(\frac{\partial p_w}{\partial z} - \gamma_w \right) \dots (14a)$$

$$u_n = -\frac{k k_{rn}}{\mu_n} \left(\frac{\partial p_n}{\partial z} - \gamma_n \right) \dots (14b)$$

We neglect capillary pressure so that $p_w = p_n = p$. The gradient $\partial p/\partial z$ can be determined by adding Eqs. 14a and 14b, using the fact that $u_w + u_n = 0$. Substitution of this resulting expression for $\partial p/\partial z$ into Eq. 14a then gives

$$u_w = k \Delta \gamma \psi(S), \dots \dots \dots (15)$$

where

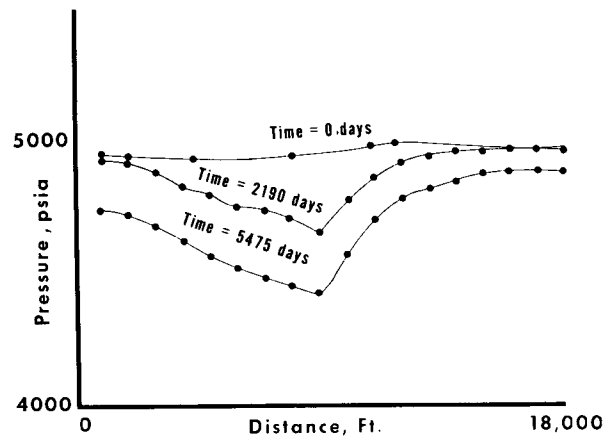


FIG. 10 — PRESSURE DISTRIBUTION THROUGH A-A (FIG. 8).

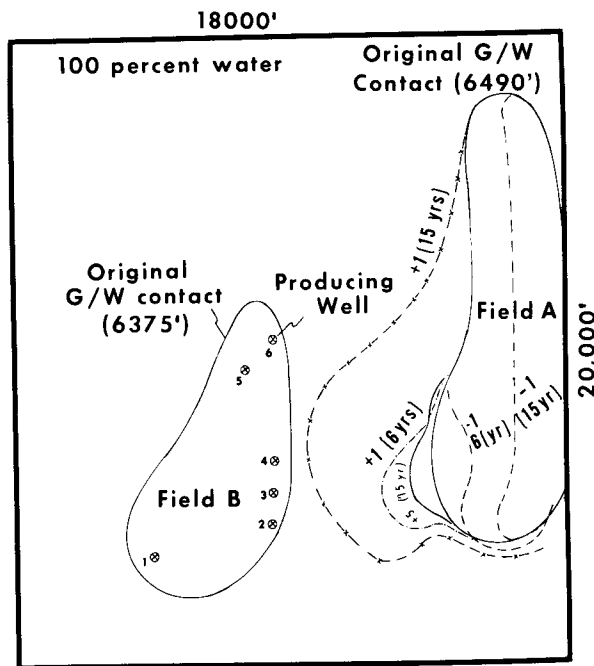


FIG. 9 — EFFECTS OF MULTIPLE GAS POOLS PRODUCING FROM A COMMON AQUIFER.

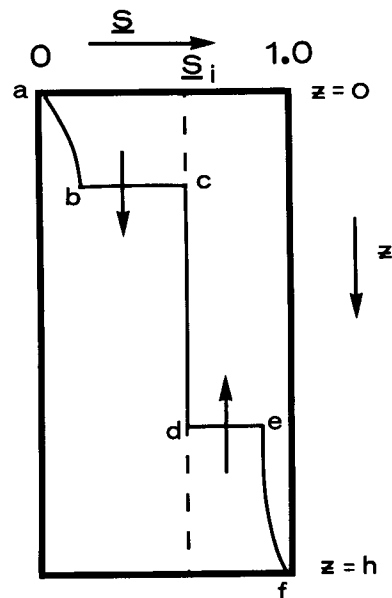


FIG. 11 — GRAVITY SEGREGATION IN A CLOSED VERTICAL COLUMN.

$$\psi(S) = 1 / \left(\frac{\mu_n}{k_{rn}} + \frac{\mu_w}{k_{rw}} \right) \dots \dots \dots (16)$$

$\Delta\gamma$ is $\gamma_w - \gamma_n$ and S is wetting-phase saturation. Substitution of u_w from Eq. 15 into Eq. 13a gives

$$- \frac{k\Delta\gamma}{\phi\Delta S} \psi' \frac{\partial S}{\partial z} = \frac{\partial S}{\partial t} \dots \dots \dots (17)$$

where

$$\Delta S = 1 - S_{nr} - S_{wr} \prime$$

$$\underline{S} = \frac{S_w - S_{wr}}{1 - S_{nr} - S_{wr}} \prime$$

$$\psi' = d\psi/d\underline{S} \prime$$

and S_{nr} , S_{wr} are residual or endpoint saturations of nonwetting and wetting phases at which their respective relative permeabilities are zero.

Since

$$d\underline{S}(z,t) = \frac{\partial \underline{S}}{\partial z} dz + \frac{\partial \underline{S}}{\partial t} dt \dots \dots (18)$$

we have

$$\left(\frac{dz}{dt} \right)_{\underline{S}} = - \frac{\partial \underline{S} / \partial t}{\partial \underline{S} / \partial z} \dots \dots \dots (19)$$

Substituting from Eq. 17 into Eq. 19 gives

$$\left(\frac{dz}{dt} \right)_{\underline{S}} = \alpha \psi'(\underline{S}) \prime \dots \dots \dots (20)$$

where α is $k\Delta\gamma/\phi\Delta S$. This equation gives the velocity of any saturation which lies on a continuous profile where $\partial \underline{S} / \partial z$ is defined.

Eq. 20 does not apply to saturation discontinuities or shocks since $\partial \underline{S} / \partial z$ is not defined at such discontinuities. A material balance about a saturation discontinuity gives shock velocity as

$$\left(\frac{dz}{dt} \right)_{\text{Shock}} = \alpha \frac{\psi^+ - \psi^-}{\underline{S}^+ - \underline{S}^-} \prime \dots \dots \dots (21)$$

where \underline{S}^+ and \underline{S}^- are saturations on the discontinuity and ψ^+ and ψ^- are the corresponding values of ψ . Eqs. 20 and 21, together with the ψ vs \underline{S} curve are sufficient to analyze the countercurrent segregation process.

Fig. 12 shows ψ vs \underline{S} for the relative permeabilities $k_{rw} = 0.3\underline{S}^3$ and $k_{rn} = (1 - \underline{S})^3$ and $\mu_w = 1$ cp, $\mu_n = 2$ cp. The dotted curve on Fig. 12 shows ψ' and \underline{S}_1 and \underline{S}_2 are defined as the two saturations at which ψ' is zero. If the initial uniform column saturation \underline{S}_i lies between \underline{S}_1 and \underline{S}_2 , then the saturation profile development is as indicated on

Fig. 11. Shocks form instantaneously at the top and bottom of the column and saturations of zero and unity remain fixed at the column ends. The upper shock saturations \underline{S}_b is determined by

$$\psi'_b = \frac{\psi_i - \psi_b}{\underline{S}_i - \underline{S}_b} \prime \dots \dots \dots (22)$$

or the point of tangency, b , on Line 1 of Fig. 12. The lower shock saturation \underline{S}_e is given by

$$\psi'_e = \frac{\psi_e - \psi_i}{\underline{S}_e - \underline{S}_i} \prime \dots \dots \dots (23)$$

or the point of tangency, e , on Line 2 of Fig. 12. The positions of the two shocks at any time are given by

$$z_b = \psi'_b \alpha t$$

$$z_e = h + \psi'_e \alpha t \prime \dots \dots \dots (24)$$

and the time at which the two shocks meet is

$$t^* = \frac{h}{\alpha(\psi'_b - \psi'_e)} \prime \dots \dots \dots (25)$$

where h is column height. This time t^* may be taken as a measure of the time necessary for segregation to occur; actually, an infinite time is required for complete segregation (unless ψ' is nonzero at $\underline{S} = 0$ and $\underline{S} = 1$).

The time t^* given by Eq. 25 might be considered as a criterion for the existence of vertical equilibrium. Quite obviously the validity of the VE assumption should be inversely proportional to the value of t^* . A dimensionless criterion can be obtained by dividing t^* by some other (reference) time. We select this reference as the time necessary for a fluid particle or column to move areally a distance equal to reservoir thickness h . If \bar{u} is the mean superficial velocity of fluids in the reservoir

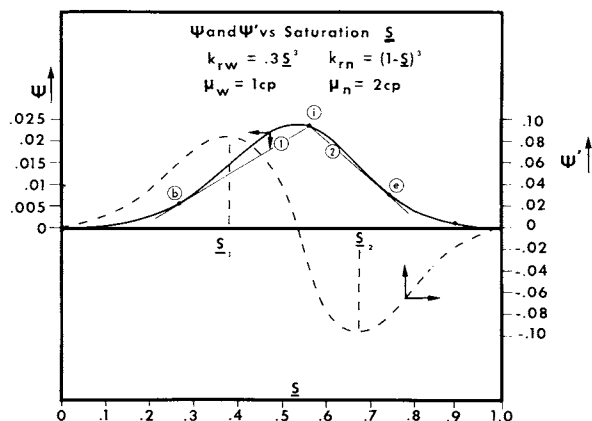


FIG. 12 — ψ VS S FOR RELATIVE PERMEABILITIES.

in the areal directions then $h\phi/\bar{u}$ is the value of this reference time. The dimensionless group $h\phi/\bar{u}t^*$, denoted by G ,

$$G = \frac{\phi\alpha(\psi'_b - \psi'_e)}{\bar{u}} = \frac{k\Delta\gamma(\psi'_b - \psi'_e)}{\bar{u}\Delta S} \quad (26)$$

is then the proposed criterion, the value of which might be directly proportional to the degree of validity of the VE assumption.

The above analysis is only trivially altered by consideration of a column inclined at an angle θ from the vertical. The term $\Delta\gamma$ is simply replaced by $\Delta\gamma \cos \theta$.

Further work is necessary to attach a meaningful "critical" value to the criterion group G . Two-dimensional cross-sectional calculations must be compared with 1-D areal calculations for a wide range of reservoir and fluid properties yielding a range of values for G . It must be determined whether the agreement between the 2-D and 1-D results (using VE) is good provided G is greater than some value. If so, that numerical value of G would provide, in any specific case an *a priori* estimate of the need for 3-D as opposed to 2-D areal simulations.

CONCLUSIONS

From our discussion of the use of the VE concept in simulating heterogeneous reservoirs, we have formed the following conclusions.

1. Use of the VE concept, when applicable, saves considerable man-time for data preparation and machine-time necessary to perform a reservoir study. When the concept is applicable, 2-D areal calculations with VE duplicate the more complex 3-D calculations.

2. A new dimensionless group, G , is derived and proposed as a possible *a priori* check on the validity of the VE assumption for small transition zone systems.

3. A comparison of a x - z cross-section with a 1-D areal with VE demonstrates the capability of solving a higher dimension reservoir problem using a lesser-dimension model with VE incorporated in the calculation.

4. An actual field case example shows the applicability of VE when studying the interference effects of multiphase gas fields located on a common aquifer.

NOMENCLATURE

- h = reservoir net pay thickness, ft
- k_r = relative permeability
- k_{rgcw} = relative permeability to gas at connate water
- k_{rwrg} = relative permeability to water at residual gas
- p = pressure, psi
- \bar{p}_c = pseudo capillary pressure, psi
- S = saturation
- \bar{S} = depth averaged saturation
- \underline{S} = saturation defined in Eq. 17
- t = time, days
- u = velocity, ft/D
- z = depth, ft
- z_{ci} = depth of initial contact, ft
- γ = fluid gradient, psi/ft
- ϕ = porosity, fraction
- ψ = defined by Eq. 16

SUBSCRIPTS

- g = gas
- i = initial
- n = nonwetting phase
- r = residual
- w = water or wetting phase

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