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Numerical Simulation of Coning Using Implicit Production Terms

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ABSTRACT

This paper describes the use of a multiphase, multidimensional mathematical model to predict two- and three-phase coning behavior. Severe computational instability in the form of saturation oscillations in grid blocks near the wellbore is commonly encountered in the mathematical simulation of coning. This instability is due to the explicit (dated at the beginning of a time step and held constant for that time step) handling of saturation-dependent transmissibilities and production terms in the finite-difference solution of the flow equations. An analysis of stability with respect to explicit handling of saturation-dependent transmissibilities is presented in this paper. This analysis shows why explicit transmissibilities can result in a severe time-step restriction for coning simulation.

The use of implicit production terms in the difference equations to reduce instabilities is discussed and examples are given. These examples show that the implicit handling of production terms alone can result in a fivefold increase in permissible time step for a coning simulation with virtually no increase in computing time per time step. A laboratory water-coning experiment was simulated and excellent agreement was obtained between computed and observed results. A three-phase coning example for a gravity-segregation reservoir is also presented.

INTRODUCTION

Simulation of coning behavior is normally done by numerically solving the flow equations expressed in cylindrical (r, z, θ) coordinates with symmetry in the θ direction. The finite-difference technique of numerical solution of differential equations requires that the portion of the reservoir being simulated be divided into grid blocks as shown in Fig. 1. Since coning is a well phenomenon and not a gross reservoir phenomenon, the grid blocks must necessarily be relatively small in the vicinity of the wellbore because both pressures and saturations vary rapidly in this region.

Severe computational instability is commonly encountered in the simulation of coning due to the relatively small grid-block sizes and high flow velocities in the vicinity of the wellbore. During a time step that would be considered normal for most reservoir simulation problems, a block near the wellbore is required to pass a volume of fluid many times its pore volume. Computational instability results when saturation-dependent quantities in the finite-difference solution of the flow equations are set at the beginning of a time step and held constant

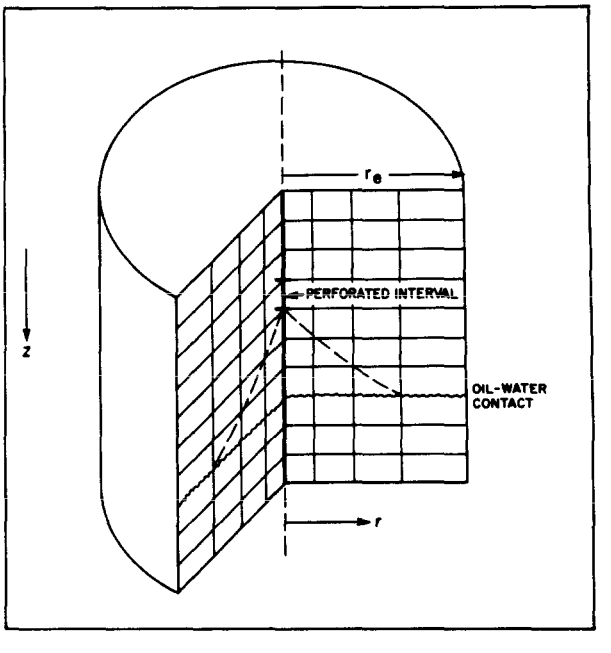

FIG. 1 — REPRESENTATION OF RESERVOIR FOR OILWELL-CONING SIMULATION.

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[1]References given at end of paper.

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for that time step. As a result, it is frequently found that very small time steps must be taken in order to obtain a stable (non-oscillatory) solution. For certain problems, particularly gas coning, time steps for a stable solution may be so small as to render simulation economically unfeasible.

Stability analysis (see Appendix) and the work of Blair and Weinaug[1] indicate that the basic problem in coning simulation is the explicit dating at the beginning of a time step of saturation-dependent quantities. Blair and Weinaug showed that a completely implicit-difference analog for the simulator equations results in a significant increase in the permissible time step for a stable solution to coning problems. A completely implicit-difference analog, however, has two disadvantages. First, the mathematical formulation and programming become extremely complex, and second, although larger time steps can be taken, the computing time per time step is significantly increased.

It is therefore of practical interest to determine a method of eliminating the instability encountered in coning simulation without increasing the complexity of the mathematical analysis or the computing time per time step. This paper is concerned mainly with handling the saturation-dependent quantities in the production (sink) terms of the difference equations. The ultimate solution to the difficult problem of coning simulation is a method whereby the changes in saturation during a time step as related to both production terms and the transmissibilities for flow between blocks are accounted for with a minimum amount of additional analysis and programming, and hence a minimum amount of additional computing time per time step.

DESCRIPTION OF THE MATHEMATICAL MODEL

The mathematical model used for the coning studies was a two-dimensional, three-phase incompressible program similar to that described by Coats.[3] The differential equations describing three-phase incompressible flow are the continuity equation and Darcy's law for each phase. In cylindrical coordinates with radial symmetry these equations are:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk_h\frac{k_{rw}}{\mu_w}\frac{\partial \Phi_w}{\partial r}\right)+\frac{\partial}{\partial z}\left(k_v\frac{k_{rw}}{\mu_w}\frac{\partial \Phi_w}{\partial z}\right)-q_{vw}=\phi\frac{\partial S_w}{\partial t} \quad \ldots \quad (1a)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk_h\frac{k_{ro}}{\mu_o}\frac{\partial \Phi_o}{\partial r}\right)+\frac{\partial}{\partial z}\left(k_v\frac{k_{ro}}{\mu_o}\frac{\partial \Phi_o}{\partial z}\right)-q_{vo}=\phi\frac{\partial S_o}{\partial t} \quad \ldots \quad (1b)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk_h\frac{k_{rg}}{\mu_g}\frac{\partial \Phi_g}{\partial r}\right)+\frac{\partial}{\partial z}\left(k_v\frac{k_{rg}}{\mu_g}\frac{\partial \Phi_g}{\partial z}\right)-q_{vg}=\phi\frac{\partial S_g}{\partial t} \quad \ldots \quad (1c)$$

Additional relationships required for the solution of Eqs. 1 are:

$$P_{cwo}(S_w)=p_o-p_w=\Phi_o-\Phi_w+(\gamma_w-\gamma_o)Z \quad \ldots \quad (2a)$$

$$P_{cgo}(S_g)=p_g-p_o=\Phi_g-\Phi_o+(\gamma_o-\gamma_g)Z \quad \ldots \quad (2b)$$

and finally:

$$S_w+S_o+S_g=1 \quad \ldots \quad (3)$$

Eqs. 1 are expressed in finite-difference form and solved simultaneously using the iterative alternating direction technique of Douglas and Rachford.[2] Details of how the finite-difference equations are handled for simultaneous numerical solution of Eqs. 1 are given in Ref. 3. As is outlined in this reference, the mathematical model for three-phase, three-dimensional incompressible flow in a porous medium may be written as:

$$\Delta T_w \Delta \Phi_w - q_w = \frac{V_p}{\Delta t}\Delta_t S_w \quad \ldots \quad (4a)$$

$$\Delta T_o \Delta \Phi_o - q_o = \frac{V_p}{\Delta t}\Delta_t S_o \quad \ldots \quad (4b)$$

$$\Delta T_g \Delta \Phi_g - q_g = \frac{V_p}{\Delta t}\Delta_t S_g \quad \ldots \quad (4c)$$

where, for the m^{th} phase:

$$\Delta T_m \Delta \Phi_m \equiv \Delta_r T_{rm} \Delta_r \Phi_m + \Delta_z T_{zm} \Delta_z \Phi_m$$

$$(T_{rm})_{i+\frac{1}{2},k}=\frac{2\pi\left(\Delta z k_h\frac{k_{rm}}{\mu_m}\right)_{i+\frac{1}{2},k}}{5.6146\ \ln\frac{r_{i+1}}{r_i}}$$

and

$$(T_{zm})_{i,k+\frac{1}{2}} = \frac{2\pi(r_{i+\frac{1}{2}}^2 - r_{i-\frac{1}{2}}^2)}{5.6146} \left(k_v \frac{k_{rm}}{\mu_m}\right)_{i,k+\frac{1}{2}} \Big/ (\Delta z_{k+1} + \Delta z_k)$$

The subscripts i and k denote spatial position in the r and z directions, respectively. r_i is the radius of the center (or representative point) of Block i in the radial direction, and $r_{i+\frac{1}{2}}$ and $r_{i-\frac{1}{2}}$ are the radii of the boundaries of Block i in the radial direction. The radius $r_{i+\frac{1}{2}}$ should be the log-mean radius between r_i and r_{i+1}.

In the simultaneous method of solution,[3] the finite-saturation changes $\Delta_t S_m$ are expressed in terms of potentials through use of the capillary pressure Eqs. 2a and 2b.

$$\Delta_t S_w = S_w' (\Delta_t \Phi_o - \Delta_t \Phi_w) \quad (5a)$$

$$\Delta_t S_o = S_w' \Delta_t \Phi_w - (S_w' - S_g') \Delta_t \Phi_o - S_g' \Delta_t \Phi_g \quad (5b)$$

$$\Delta_t S_g = S_g' (\Delta_t \Phi_g - \Delta_t \Phi_o) \quad (5c)$$

IMPLICIT HANDLING OF THE PRODUCTION TERMS IN THE SOLUTION OF THE FINITE DIFFERENCE EQUATIONS FOR INCOMPRESSIBLE FLOW

As was pointed out by Blair and Weinaug, evaluation of saturation-dependent quantities at old time levels (explicitly) results in an unstable difference equation in regions of high flow rate.[1] The Appendix discusses an analysis that shows how the explicit handling of saturation-dependent transmissibilities affects stability.

Consider the difference equation for the water phase:

$$\Delta T_w \Delta \Phi_w - q_w = \frac{V_p}{\Delta t} \Delta_t S_w \quad (4a)$$

In this equation, both the transmissibilities T_{rw} and T_{zw} and the production term q_w are functions of mobility and, hence, functions of saturation. Normally in an incompressible model, the total production rate q for each grid block is specified, and then this production is split among the phases according to their mobilities. That is, for a given producing block, q_m, the production rate of the m^{th} phase, is:

$$q_m = q \frac{\lambda_m}{\sum_{i=1}^{3} \lambda_i} = q \frac{\lambda_m}{\lambda} \quad (6)$$

Therefore, q_w in Eq. 4a is normally calculated as:

$$q_w = q \frac{\frac{k_{rw}}{\mu_w}}{\frac{k_{rw}}{\mu_w} + \frac{k_{ro}}{\mu_o} + \frac{k_{rg}}{\mu_g}}, \quad (7)$$

where mobilities are evaluated at the end of the previous time step for the next time step.

Handling the production term implicitly means taking into account the fact that *during* a step, mobilities are changing.

THREE-PHASE IMPLICIT PRODUCTION

In the implicit handling of production terms, q_w for the time step from n to $n+1$ is defined as:

$$q_w = q_{wn} + M_w' \Delta_t S_w, \quad (8)$$

where the n subscript denotes the old (previous) time level, and M_w' is defined as:

$$M_w' = \frac{\theta q \lambda_w'}{\lambda_{wn} + \lambda_{on} + \lambda_{gn}} = \frac{\theta q \lambda_w'}{\lambda_n}, \quad (9)$$

where

$$\lambda_w' = \frac{\lambda_{wn+1} - \lambda_{wn}}{S_{wn+1} - S_{wn}},$$

and

$$0 \leq \theta \leq 1.$$

That is, λ_w' is the chord slope of the water relative-permeability curve between S_{wn+1} and S_n divided by μ_w. This chord slope can generally be estimated from the relative permeability tables at the beginning of a time step and retained without updating for that time step. Another way of expressing Eq. 8 is

$$q_w = q_{wn} + \theta \frac{dq_w}{dS_w} \Delta_t S_w \quad (10)$$

If $\theta = 0$, then q_w is simply q_w based on the mobilities at time n. If $\theta = 1$, q_w is q_w at time $n + 1$. If $0 < \theta < 1$, q_w is some intermediate value during the time step.

In the definition of M_w', it was assumed that water mobility is a single-valued function of S_w. It was also assumed that the total mobility $(\lambda_w + \lambda_o + \lambda_g)$ for a producing block remains constant during the time step. Although this is not necessarily true, it is, nevertheless, a good approximation and is considerably simpler to program. As is shown later, treating the production terms implicitly for two-phase flow can easily account for the change in total mobility over the time step.

For the gas production terms, q_g during time step from n to $n+1$ can be similarly defined as:

$$q_g = q_{gn} + M'_g \Delta_t S_g \quad \ldots\ldots\ldots\ldots (11)$$

where

$$M'_g = \frac{\theta\, q\, \lambda'_g}{\lambda},$$

and

$$\lambda'_g = \frac{\lambda_{gn+1} - \lambda_{gn}}{S_{gn+1} - S_{gn}}.$$

For the oil phase, since it is assumed that total mobility is constant during the time step, we can say that for the time step from n to $n+1$:

$$\begin{aligned}\lambda_o &= \lambda - \lambda_w - \lambda_g \\ &= \lambda - (\lambda_{wn} + \theta\lambda'_w \Delta_t S_w) - (\lambda_{gn} \\ &\quad + \theta\lambda'_g \Delta_t S_g) \\ &= \lambda_{on} - \theta(\lambda'_w \Delta_t S_w + \lambda'_g \Delta_t S_g) \quad \ldots\ldots (12)\end{aligned}$$

Therefore, q_o for the time step from n to $n+1$ becomes:

$$q_o = q \frac{\lambda_{on}}{\lambda} - \frac{\theta q}{\lambda} (\lambda'_w \Delta_t S_w + \lambda'_g \Delta_t S_g) \quad \ldots\ldots\ldots\ldots (13)$$

As a result of redefining the production terms to take into account the changes in mobility during the time step, Eqs. 4 can be rewritten as

$$\Delta T_w \Delta\Phi_w - q_{wn} = \left(\frac{V_p}{\Delta t} + M'_w\right)\Delta_t S_w \quad \ldots (14a)$$

$$\Delta T_o \Delta\Phi_o - q_{on} = -\left(\frac{V_p}{\Delta t} + M'_w\right)\Delta_t S_w - \left(\frac{V_p}{\Delta t} + M'_g\right)\Delta_t S_g \quad \ldots (14b)$$

$$\Delta T_g \Delta\Phi_g - q_{gn} = \left(\frac{V_p}{\Delta t} + M'_g\right)\Delta_t S_g \quad \ldots (14c)$$

since

$$\Delta_t S_o = \Delta_t(1 - S_w - S_g) = -\Delta_t S_w - \Delta_t S_g \quad \ldots\ldots\ldots\ldots (15)$$

Therefore, when the production terms are treated implicitly, the system of equations to be solved is the set of Eqs. 14. The solution of these equations requires only minor modifications to a program that treats production explicitly. Also, the implicit production routine does not result in any appreciable increase in computing times since the only additional computations are the evaluation of M'_w and M'_g at the beginning of each time step.

THE IMPLICIT PRODUCTION ROUTINE IN TWO-PHASE FLOW

Consider two-phase oil-water flow. The instantaneous production term for a given grid block is defined as

$$q_w = q \frac{\lambda_w}{\lambda_w + \lambda_o} \quad \ldots\ldots\ldots\ldots (16)$$

Therefore, q_w for the step from n to $n+1$ may be written as

$$q_w = q_{wn} + M'_w \Delta_t S_w, \quad \ldots\ldots\ldots\ldots (17)$$

where

$$M'_w = \theta q \left(\frac{\lambda_w}{\lambda_t}\right)'$$

and

$$\left(\frac{\lambda_w}{\lambda_t}\right)' = \frac{\left(\dfrac{\frac{k_{rw}}{\mu_w}}{\frac{k_{ro}}{\mu_o} + \frac{k_{rw}}{\mu_w}}\right)_{n+1} - \left(\dfrac{\frac{k_{rw}}{\mu_w}}{\frac{k_{ro}}{\mu_o} + \frac{k_{rw}}{\mu_w}}\right)_{n}}{S_{wn+1} - S_{wn}}$$

Since $q_o + q_w = q$, q_o during the time step from n to $n+1$ is defined as $q_o = q - q_w$.

Thus in treating the production terms implicitly for two-phase flow, the total mobility need not be considered constant during the time step.

Although this paper discusses the use of implicit production terms as applied to coning problems, the technique can be applied to any type of reservoir simulation. In particular, the implicit handling of production terms should provide increased stability wherever there is converging two- or three-phase flow into a producing grid block.

EXAMPLES OF CONING CALCULATIONS USING THE IMPLICIT PRODUCTION ROUTINE

A WATER-CONING EXAMPLE

Blair and Weinaug used a water-oil coning example to show how the use of totally implicit difference equations results in increased stability.[1] The basic data for their example are given in Tables 3 through 5 of their paper. Table 1 summarizes these data. Fig. 2 is a reproduction from their paper showing the reservoir configuration, capillary pressure and relative permeability data and saturation and water cut vs time after breakthrough for the lower producing block. In a totally implicit-difference scheme, mobilities in both the production terms and transmissibilities are evaluated at the new time level. Blair and Weinaug reported that their problem was run on a 0.2-day time step with throughputs (total production from a block during a time step divided by the block pore volume) as high as 60.

Blair and Weinaug's data were used to compare results obtained using the explicit and implicit production routines.

Three runs were made in which all data were identical to those used by Blair and Weinaug except the producing interval was lowered to 50 ft above the initial water-oil contact so that breakthrough would come sooner. Fig. 3 shows water saturation in the lower producing block vs time for each run. For the first two runs, the production terms were treated explicitly. At a time step of 0.05 days the solution was stable and shows a smooth rise in water saturation with time. When the time step was increased from 0.05 days, the solution became unstable and the water saturation in the producing blocks began to oscillate. This oscillation is shown in Fig. 3 for a time step of 0.10 days.

In the third run, the implicit production routine was invoked and time steps of 0.25 days taken. Fig. 3 shows that the solution was stable for this run and was the same as that obtained for the run using explicit production terms and a time step of 0.05 days. Therefore, the implicit production routine for this problem results in a fivefold increase in the

TABLE 1 — BASIC DATA FOR BLAIR-WEINAUG WATER-CONING EXAMPLE

Number of grid blocks in r-direction = 10
Number of grid blocks in z-direction = 20

Radii of block boundaries in r-direction (ft)

2.45, 6.25, 12.88, 26.53, 54.59, 112.24, 231.69, 447.56, 984.82, 1,300

Elevation of block boundaries in z-direction (ft)

0, 7.5, 17.5, 22.5, 30, 40, 50, 62.5, 80, 100, 120, 140, 160, 180, 202.5, 227.5, 252.5, 277.5, 327.5, 352.5, 365

Fluid Properties	Rock Properties
ρ_w = 1 gm/cc	k_h = 1 darcy $0 \le r \le 1{,}300$, $o \le z \le 160$
ρ_o = 0.826 gm/cc	= 5 darcies $o \le r \le 1{,}300$, $160 < z \le 365$
μ_w = 0.31 cp	k_v = 1 darcy $0 \le r \le 1{,}300$, $0 \le z \le 365$
μ_o = 0.34 cp	ϕ = 0.207

Producing Rates

3,752 B/D from block (1,2)
1,248 B/D from block (1,3)

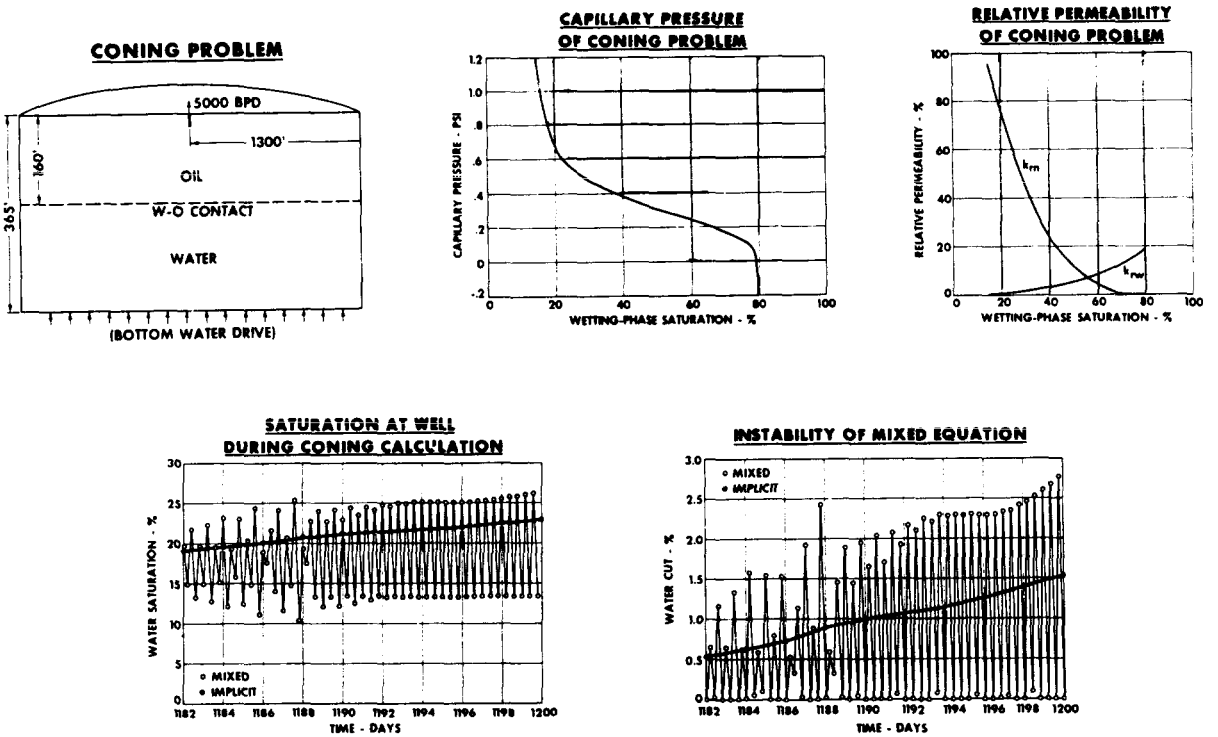

FIG. 2 — DATA AND RESULTS FOR BLAIR-WEINAUG WATER-CONING EXAMPLE.

maximum time step that can be taken.

When this run was carried out beyond 10 days, the maximum time step that could be taken using the implicit production routine decreased, and at 30 days, the maximum time step was about 0.13 days, or half the time step that could be taken from 0 to 10 days. Time steps of 0.25 and 0.13 days represent throughputs of about 33 and 17, respectively, for the lower producing block. This compares to a throughput of 60 reported by Blair and Weinaug for the completely implicit model. The increased time steps (or throughputs) that could be taken with the completely implicit program represents the additional advantage to be gained by handling transmissibilities as well as production terms implicitly.

Blair and Weinaug reported that the computing time per time step for their model was two to three times that required for a "normal" (explicit transmissibility) model, whereas the implicit handling of production terms alone requires no additional computing time per time step. It is obvious from the above that the most desirable coning program is one which would achieve the benefits of handling transmissibilities implicitly without the accompanying disadvantage of increased computing time per time step.

A GAS-CONING EXAMPLE

A gas-coning run was made for a 9 x 15 grid configuration with a total height of 150 ft and an exterior radius (r_e) of 375 ft. The vertical blocks were all 10 ft in thickness. The well was produced out of Block 8 in the vertical direction at a total producing rate of 100 RB/D. The initial gas-oil contact was 15 ft below the top, or 55 ft above the perforated interval. Fluid properties, rock properties and block centers in the radial direction are given in Table 2. The relative permeability data used are shown on Fig. 4.

Fig. 5 shows GOR vs time for two runs in which the production terms were treated explicitly and a third run in which the production terms were treated implicitly. For the explicit runs it can be seen that for a 1-day time step the solution is stable; however, at a time step of 2.5 days, the GOR oscillates wildly. For the implicit run, a 2.5-day time step is stable. The stable time steps of 1 day for the explicit and 2.5 days for the implicit were approximately the maximum time steps that could be taken for these runs. Therefore, for this gas-coning problem, the implicit production routine results in a 2½-fold increase in maximum time step and a corresponding decrease in computing time.

Throughputs for this gas-coning example were on the order of one compared to throughputs of greater than 17 for the Blair-Weinaug water-coning example. The smaller throughputs that were taken for the gas-coning example are due to the relatively high mobility of the gas phase.

TABLE 2 — BASIC DATA FOR GAS-CONING EXAMPLE IMPLICIT PRODUCTION ROUTINE

Number of grid blocks in r-direction = 9
Number of grid blocks in z-direction = 15

Radii of grid block centers in r-direction (ft)

10, 30, 60, 100, 150, 200, 250, 300, 350

Elevation of grid block centers in z-direction (ft)

0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140

Fluid Properties	Rock Properties
ρ_g = 10.9 lb/cu ft	k_h = 10 md
ρ_o = 40.6 lb/cu ft	k_v = 10 md
μ_g = 0.02 cp	ϕ = 0.10
μ_o = 0.29 cp	

Producing Rate

100 RB/D from block (1,8)

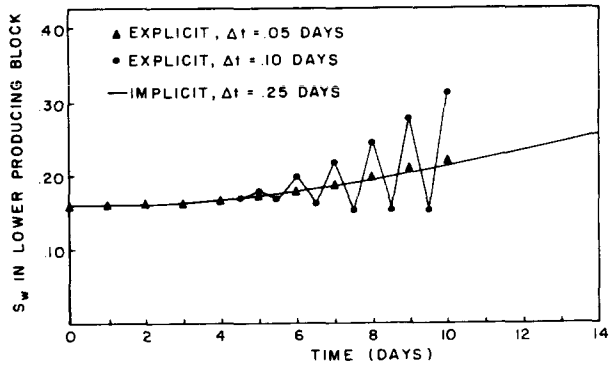

FIG. 3 — TWO-PHASE WATER-CONING, EXPLICIT VS IMPLICIT PRODUCTION ROUTINE, BLAIR-WEINAUG DATA.

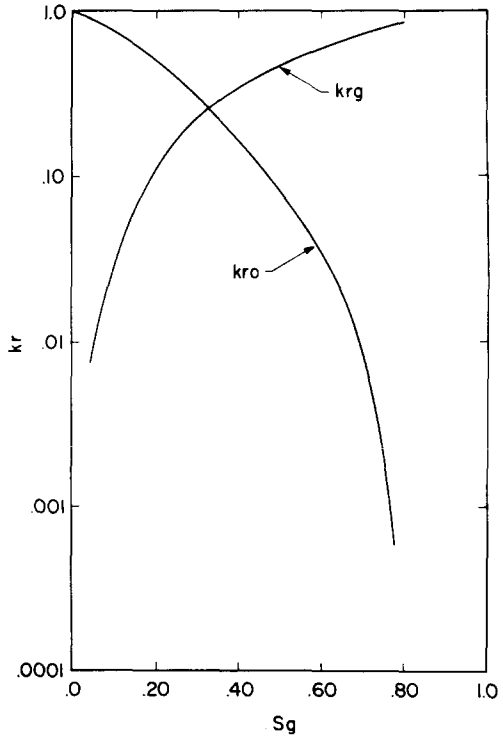

FIG. 4 — GAS-OIL RELATIVE PERMEABILITY DATA IMPLICIT PRODUCTION ROUTINE EXAMPLE.

SIMULATION OF A LABORATORY WATER-CONING EXPERIMENT

The incompressible coning model was used to simulate a laboratory water-coning experiment done by Soengkowo.[4] His work consisted of scaled model studies of water coning to investigate the production performance of reservoirs producing under bottom-water drive. The experimental apparatus consisted of a pie-shaped model with a consolidated sand oil zone and an unconsolidated sand aquifer. The model was 16 in. high and 19.3 in. in radius and the radial angle was 10°.

The fluid analogs used in the experiments were water-base solutions with glycerol added to control viscosity. The oil analog fluid was a brine solution, and potassium iodide was added to the water analog fluid to control its density. Table 3 gives all pertinent data for the particular run that was chosen for mathematical simulation.

A 9 × 5 grid system was used in the mathematical simulation model. The 3-in. oil zone was simulated using four grid blocks in the vertical direction ¾ in. in thickness, and the aquifer was simulated as a single block 13 in. in thickness. The radii of the nine block centers in the radial direction were 0.5, 1, 2, 4.25, 6.25, 8.25, 11, 14.25 and 17.25 in.

Since the fluids were miscible and viscosities were similar, straight-line relative permeability curves and a linear capillary-pressure curve from 0.045 to 0.0 psi were used in the mathematical model. The presence of a nonzero capillary pressure vs saturation relationship resulted in a small initial transition zone between the oil and water (7.25 percent water saturation in the lower block of the oil zone).

TABLE 3 — DATA FOR LABORATORY WATER-CONING EXPERIMENT

μ_w	0.9359 cp
μ_o	0.9109 cp
ρ_w	1.1755 gm/cc
ρ_o	1.0050 gm/cc
Aquifer permeability	26,200 md
Oil-zone permeability	12,100 md
Aquifer porosity	0.305
Oil-zone porosity	0.305
Aquifer thickness	13 in.
Oil-zone thickness	3 in.
Outer radius of oil zone	19.3 in.
Oil production rate	0.44 cc/min

Production was taken from the top block of the oil zone (1,1) and an equivalent amount of water was injected into the aquifer. Injection into the aquifer blocks was in proportion to their pore volumes.

Fig. 6 is a plot of oil cut vs unit volumes of fluid produced (a unit volume is defined as πh^3, where h is the height of the oil zone) for the experimental run. Excellent agreement is observed between the computed and experimental results. The earlier breakthrough that is observed in the computed results is probably due to the water-oil transition zone that was present in the simulation model but not in the laboratory model.

The laboratory experiment lasted approximately 1.2 days. The computer simulation required 8 minutes of UNIVAC 1108 time using time steps of 0.001 days.

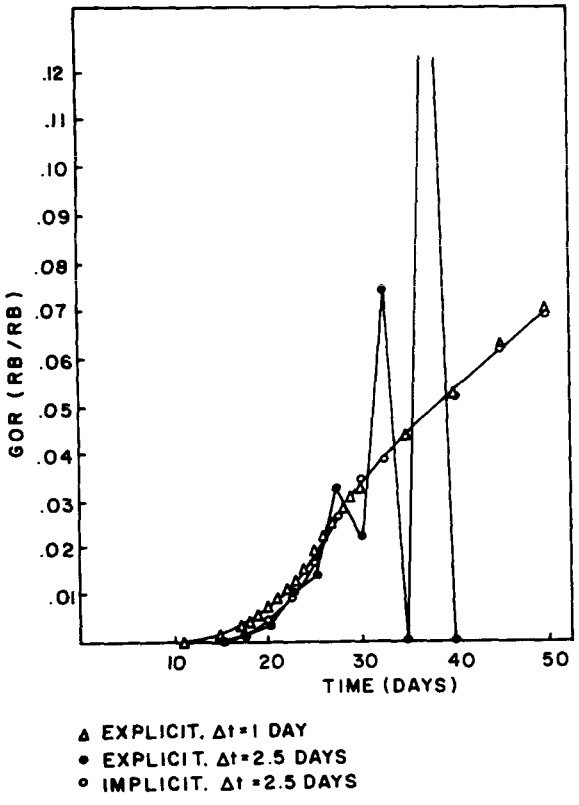

FIG. 5 — TWO-PHASE GAS-CONING, EXPLICIT VS IMPLICIT PRODUCTION ROUTINE.

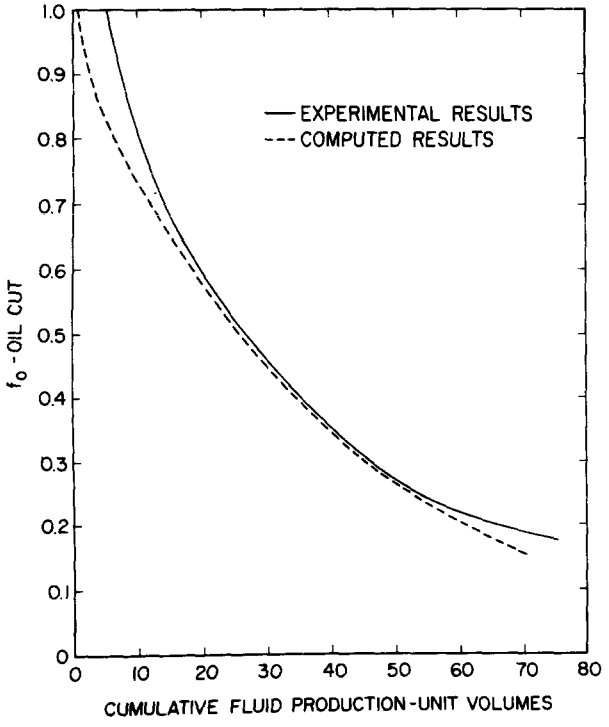

FIG. 6 — SIMULATION OF A LABORATORY CONING EXPERIMENT.

A THREE-PHASE CONING EXAMPLE

In a complete gravity-segregation reservoir where a gas cap and aquifer are present, the oil zone is a continually shrinking "sandwich" between the gas and water. Ultimate recovery from such a reservoir will be controlled by the degree of gas and/or water coning. Optimum depletion of a gravity drainage reservoir would be realized if the location and the height of perforated intervals and production rates were controlled so that ultimate economic recovery is maximized. The following section describes the results of a three-phase coning study made using the incompressible coning model with the implicit production terms.

The basic data for this example are given in Table 4. This example simulates a case where gas is being injected into the gas cap and either water is being injected into the aquifer or the aquifer is active. Three-phase relative permeability was handled using two-phase water-oil and gas-oil relative permeability data as described by Coats[3] and Peery and Herron.[5] The two-phase oil-water and gas-oil relative permeability data that were used in this example are shown in Figs. 7 and 8.

The initial gas-oil contact was in Block 4 in the vertical direction about 35 ft from the top. The initial water-oil contact was at the midpoint of Block 12 in the vertical direction or 25 ft above the bottom. Linear capillary-pressure curves from 2 to 0 psi for gas-oil and from 3 to 0 psi for water-oil were used. These gave an initial gas-oil transition zone of 10 ft and an initial oil-water transition zone of 15 ft. Production was from a single 10-ft production block (Block 9) 25 ft above the water-oil contact

TABLE 4 — BASIC DATA FOR THREE-PHASE CONING EXAMPLE

Number of grid blocks in r-direction = 9
Number of grid blocks in z-direction = 14
Thickness of blocks in vertical direction = 10 ft

Radii of grid block centers in r-direction (ft)

17.5, 35, 70, 125, 200, 400, 600, 800, 1,000

Log-mean radii of block boundaries in r-direction (ft)

0.25, 25.5, 50.5, 94.9, 159.6, 288.5, 493.3, 695.2, 896.3, 1,097

Fluid Properties	Rock Properties
ρ_w = 69 lb/cu ft	k_h = 597 md
ρ_o = 40.6 lb/cu ft	k_v = 116 md
ρ_g = 10.9 lb/cu ft	ϕ = 0.116
μ_w = 0.48 cp	
μ_o = 0.29 cp	
μ_g = 0.02 cp	
B_w = 1.03 RB/STB	
B_o = 1.40 RB/STB	
B_g = 1.00 RB/Mcf	
R_s = 800 scf/STB	

Producing Rate

1,400 RB/D from block (1,9)

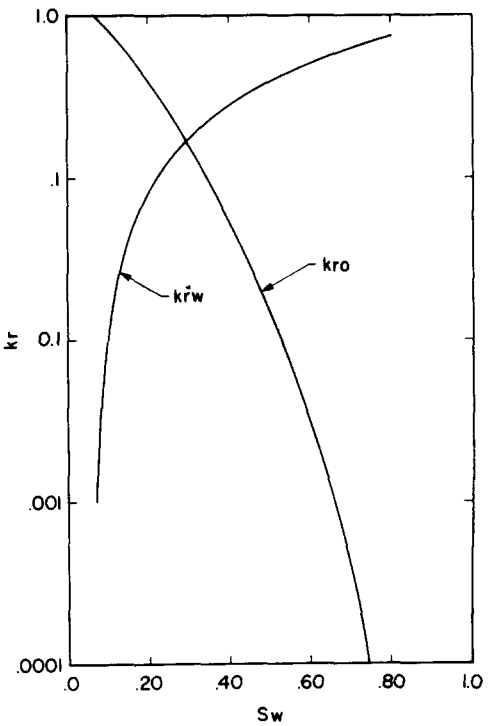

FIG. 7 —OIL-WATER RELATIVE PERMEABILITY DATA THREE-PHASE CONING EXAMPLE.

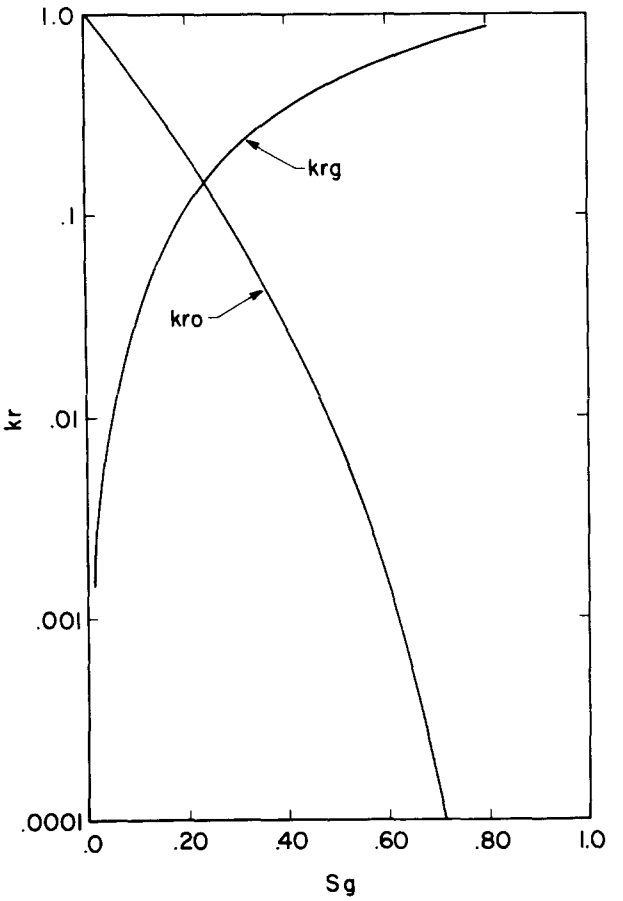

FIG. 8 — GAS-OIL RELATIVE PERMEABILITY DATA THREE-PHASE CONING EXAMPLE.

and 45 ft below the gas-oil contact. The well was produced at 1,000 STB/D or 1,400 RB/D. Injection of 700 RB/D gas into the top row of blocks and 700 RB/D water into the bottom row made the gas cap and aquifer move down and up, respectively, and maintained pressure. All injection was in proportion to block pore volumes. Fig. 9 shows schematically the portion of reservoir that was simulated.

Fig. 10 shows the gas-oil ratio and water-oil ratio vs time for this problem. The water cone broke through immediately because of the proximity of the perforations to the water-oil contact. The gas cone broke through at 420 days. After 2,200 days (6.05 years) the gas-oil ratio had increased from solution GOR (800 SCF/STB) to 2,200 SCF/STB, and the WOR had increased to 1.1 (corresponding to a water cut of 52 percent). At the end of 2,200 days the oil production had declined from an initial 1,000 STB/D to 356 STB/D.

Table 5 summarized time steps, throughputs and computing times for this example.

It should be noted that the throughputs that could be taken for this three-phase coning example were of the same order of magnitude as those that could be taken for the gas-coning example and were an order of magnitude smaller than those that could be taken for the water-coning example.

TABLE 5—COMPUTING TIMES FOR THREE-PHASE CONING

Time Period	Δt (days)	Throughput	Computing Time (min:sec)*
0– 420	1	3.40	8:36
420– 860	0.3	1.02	18:30
860–1,800	0.2	0.68	36:56
1,800–2,200	0.25	0.85	12:64
		Total computing time	77:06

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CONCLUSIONS

The use of a multiphase, multidimensional mathematical model to predict two- and three-phase coning behavior has indicated the following conclusions.

1. Instabilities in the numerical simulation of oilwell coning result from the explicit (dated at the beginning of a time step and held constant for that time step) handling of saturation-dependent quantities in the difference equations.

2. The use of implicit production terms alone in the difference equations for coning simulation can result in a fivefold increase in the permissible time step for a stable solution with no increase in computing time per time step.

3. Based on the examples presented in this paper, throughputs (volume of fluid produced from a block during a time step divided by the block pore volume) that can be taken for a water-coning simulation are an order of magnitude greater than those that can be taken for a gas or three-phase coning simulation.

NOMENCLATURE

$(T_r)_{i+\frac{1}{2}}$ = transmissibility for flow in the radial direction between blocks $i+1$ and i

$(T_z)_{k+\frac{1}{2}}$ = transmissibility for flow in the vertical direction between blocks $k+1$ and k

B_w = water-formation volume factor, RB/STB

B_o = oil-formation volume factor, RB/STB

B_g = gas-formation volume factor, RB/Mcf

f = fractional flow

$f' = df/ds$

Z = elevation measured positive vertically downward, ft

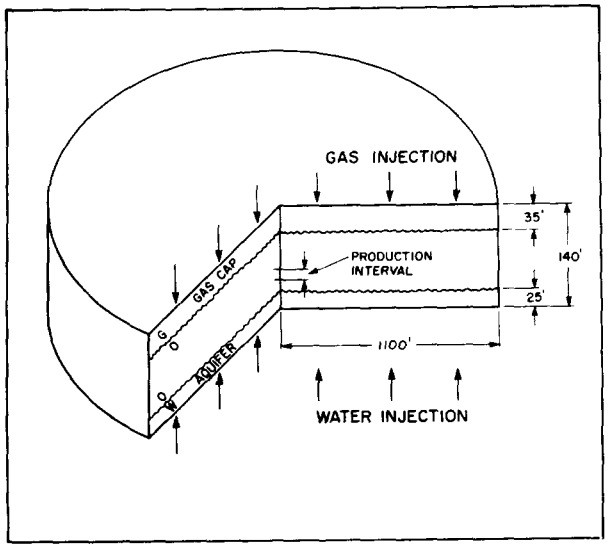

FIG. 9 — THREE-PHASE CONING EXAMPLE.

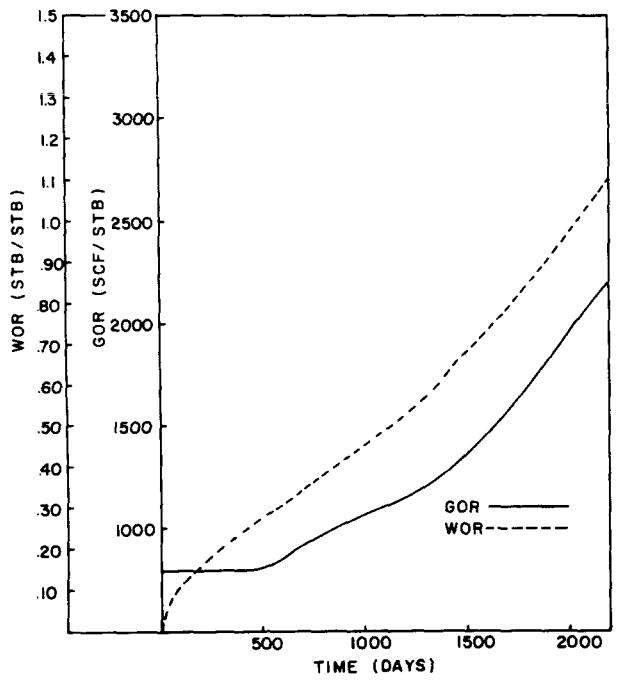

FIG. 10 — THREE-PHASE CONING, GAS AND WATER INJECTION, GOR AND WOR VS TIME.

k = absolute permeability (md × 0.00633)
k_h = horizontal permeability (md × 0.00633)
k_v = vertical permeability (md × 0.00633)
k_r = relative permeability
p = pressure, psia
P_{cwo} = water-oil capillary pressure, psia
P_{cgo} = gas-oil capillary pressure, psia
q = sink term, reservoir barrels of produced fluid per day for an entire grid block
q_v = sink term, volume of produced fluid/volume of reservoir/unit time
S = saturation
$S'_w = dS_w/dP_{cwo}$
$S'_g = dS/dP_{cgo}$
t = time, days
Δt = time increment, days
Δ_t = difference operator with respect to time
u = Darcy superficial velocity
V_p = pore volume of grid block, bbl
ϕ = porosity (fraction)
Φ = potential $(p - \gamma Z)$
γ = specific weight $\rho g/144 g_c$ (psi/ft)
ρ = density (lb/cu ft)
μ = viscosity (cp)
λ = mobility (kk_r/μ)
$\Delta T \Delta \Phi = \Delta_r T_r \Delta_r \Phi + \Delta_z T_z \Delta_z \Phi$

SUBSCRIPTS

w, o and
g = water, oil and gas, respectively

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APPENDIX

ANALYSIS OF STABILITY WITH RESPECT TO THE EXPLICIT HANDLING OF SATURATION-DEPENDENT TRANSMISSIBILITIES

The equations describing three-phase incompressible flow are:

$$-\nabla\cdot\vec{u}_w - q_{vw} = \phi \frac{\partial S_w}{\partial t}, \quad \text{(A-1a)}$$

$$-\nabla\cdot\vec{u}_o - q_{vo} = \phi \frac{\partial S_o}{\partial t}, \quad \text{(A-1b)}$$

$$-\nabla\cdot\vec{u}_g - q_{vg} = \phi \frac{\partial S_g}{\partial t}, \quad \text{(A-1c)}$$

where $\vec{u}_m$ for phase m is the Darcy superficial velocity vector:

$$\vec{u}_m = k \frac{k_{rm}}{\mu_m} \nabla\Phi_m \quad \text{(A-2)}$$

Adding Eqs. A-1 gives:

$$\nabla\cdot\vec{u} = -q_v, \quad \text{(A-3)}$$

where $\vec{u}$ is the total superficial velocity $\vec{u}_w + \vec{u}_o + \vec{u}_g$ and q_v is the total sink term $q_{vw} + q_{vo} + q_{vg}$.

If we consider three-dimensional flow in Cartesian coordinates, the fractional flow of water is defined by:

$$f_{wx} = \frac{u_{wx}}{u_x} \qquad f_{wy} = \frac{u_{wy}}{u_y} \qquad f_{wz} = \frac{u_{wz}}{u_z} \quad \text{(A-4)}$$

Thus $\vec{u}_m = f_m \vec{u}$ for each phase m. Substituting this into Eq. A-1a, using the fact that

$$\nabla\cdot f_m\vec{u} = f_m \nabla\cdot\vec{u} + \vec{u}\cdot\nabla f_m$$

and using Eq. A-3, gives

$$-\vec{u}\cdot\nabla f_w + f_w q_v - q_{vw} = \phi \frac{\partial S_w}{\partial t}.$$

Setting $q_{vw} = f_w q_v$ and using the assumption that f_w is a single-valued function of S_w, we obtain:

$$-f'_w \vec{u}\cdot\nabla S_w = \phi \frac{\partial S_w}{\partial t}, \quad \text{(A-5)}$$

where

$$f'_w = \frac{df_w}{dS_w}.$$

With no loss in generality we consider a point where u_x, u_y and u_z are all positive. In accordance with upstream weighting of explicit transmissibilities, we express Eq. A-5 in the following difference form.

$$-\frac{f'_w \Delta t}{\phi}\left[u_x \frac{S_i - S_{i-1}}{\Delta x} + u_y \frac{S_j - S_{j-1}}{\Delta y} + u_z \frac{S_k - S_{k-1}}{\Delta z}\right]_n = S_{n+1} - S_n, \quad \text{(A-6)}$$

where $S = S_w$ and centered values of i, j, k and n are omitted from the subscripts. Defining

$$U_x = \frac{f'_w u_x \Delta t}{\phi \Delta x}$$

$$U_y = \frac{f'_w u_y \Delta t}{\phi \Delta y}$$

$$U_z = \frac{f'_w u_z \Delta t}{\phi \Delta z},$$

Eq. A-6 becomes

$$-U_x(S_i - S_{i-1})_n - U_y(S_j - S_{j-1})_n - U_z(S_k - S_{k-1})_n = S_{n+1} - S_n \quad \text{(A-7)}$$

Using Von Neumann stability analysis, it can be shown that the condition for stability of Eq. A-7 is that *each* of U_x, U_y and U_z and their sum must be less than 1. This analysis and result for the water equation also applies to the oil and gas equations. This gives a general condition for stability due to the explicit dating of saturation-dependent transmissibilities as:

$$\Delta t \leq \min_{m=w,o,g}\left[\frac{\phi}{f'_m}\frac{1}{\frac{u_x}{\Delta x} + \frac{u_y}{\Delta y} + \frac{u_z}{\Delta z}}\right] \quad \text{(A-8)}$$

where u_x, u_y and u_z are the total superficial velocities in the x, y and z directions, respectively.

It can be seen from Eq. A-8 that the time-step restriction due to explicit handling of saturation-dependent transmissibilities is dependent upon superficial velocity and the grid-block sizes, such that the permissible time step for stability decreases as u gets larger and the grid-block sizes get smaller. Thus, it is to be expected that this time-step restriction is more severe in the case of coning studies due to the high flow rates and small block sizes near the wellbore.

For one-dimensional flow, Eq. A-8 may be written as:

$$\Delta t \leq \min_{m=w,o,g}\left[\frac{\phi A \Delta x}{f'_m Au}\right], \quad \text{(A-9)}$$

where A is the cross-sectional area.

$\phi A \Delta x$ is the block pore volume and Au is the producing rate. Thus it is seen that the time-step restriction is related to the throughput. However, due to the presence of the f'_m term in this equation, throughput alone cannot be used quantitatively to determine the maximum time step. In multidimensional flow, throughput becomes even less significant as a guide in the determination of maximum time step.

It should be noted that the above stability analysis of necessity assumes constant coefficients in the equations (a linear system) and, as a result, cannot be rigorously applied to nonlinear systems. However, this analysis does give valuable insight into the nature of the problem with explicit transmissibilities as partially related to throughput and, in fact, is borne out in practice.

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