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Gas Field Development: Automatic Selection of Locations for New Producing Wells

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ABSTRACT

Recent advancements in technology have made possible studies of gas field development programs heretofore impossible. With the use of a mathematical model, a method for solving the simulator equations, and adequate computing equipment, various development plans can be tested and evaluated. In evaluating a development scheme, we are concerned not only with the number of wells drilled, but also when each is drilled and the cost of each well.

In order to obtain a plan for developing a gas field, there should be some criterion on which to base a decision on the location of additional wells. In this paper, we consider schemes resulting from the use of three distinct criteria. From the alternate schemes, a computer program determines the one which ultimately achieves the lowest cumulative present value cost.

The subject field of this study is a portion of the Crossfield reservoir located near Calgary, Alta., Canada. The flow was considered single-phase, unsteady-state, isothermal, in a closed heterogeneous reservoir with a uniform porosity. An alternating direction iterative technique was used to solve the partial differential equations in the mathematical model.

INTRODUCTION

The goal in the natural gas industry is to locate adequate amounts of deliverable reserves and to produce them efficiently. Once a gas reserve has been located and dedicated to market, there are many things that must be considered in order to produce it efficiently: The production rate on the average and maximum day is usually specified. Gas sales contracts often require that the average delivered quantity be one Mcf/D/8 MMcf of reserve. This rate depletes about 90 percent of the reserve in 20 years. The operator drills sufficient wells to ensure that he can maintain the maximum daily requirement. Well productivity is checked at intervals, and the deliverability of the field is computed to ensure that it can continue to meet the market requirement. Well reconditioning or new wells may be required at certain stages in the life. If the reservoir is producing by pressure depletion and the gas sales contract calls for a minimum pressure for delivering gas, field compressors may be installed as the field approaches abandonment pressure.

References and illustrations at end of paper.

With all these considerations, it becomes immediately evident that a plan is needed for developing the field and placing the gas on the market. Estimates of gas reserves are needed as a basis for marketing the gas. Sufficient wells must be planned to produce the gas at the rates projected for delivery of the gas.

At any stage of reservoir depletion, a correct estimation of the change in delivery capacity with time caused by depletion is essential for determining the number of wells required to meet a given contract to deliver gas. Fundamentally, the problem is the familiar one of obtaining data during the drilling, testing and early production life of a well and applying these data within a simulation model to predict long-term behavior. In planning the operation of a multiwell gas reservoir, it is desirable to predict its performance for each alternate development and producing schedule under consideration.

Mathematical models of petroleum and natural gas reservoirs have been the object of much concentrated study in recent years. These models permit the engineer to examine and evaluate the physical and economic consequences of various alternative production policies. The reduction in cost of solving such models of late has made possible their use as an almost routine management tool. This reduction is the result not only of improved computer hardware, but also of more efficient mathematical techniques.

In this paper, a mathematical model is presented for optimizing the development plan for a gas reservoir. A specific reservoir is studied considering the provisions of a specific gas contract. However, the computer program as written will simulate any gas reservoir with any sales contract. Problems such as this one have been presented in technical literature, along with general attempts at solving the problem of additional well locations. The unique feature of the optimization scheme presented here is the automatic selection by a computer program of new well locations according to certain criteria. Following evaluation of several schemes for drilling wells that will allow the gas contract to be met with the minimum drilling effort, each scheme is analyzed according to its economic benefits. Thus at the completion of the study, we have obtained a selection procedure which is the most economically feasible for the particular gas reservoir under consideration.

This paper is written as a result of research done at The U. of Texas at Austin in partial fulfillment of the requirements for the degree of MS in Petroleum Engineering.[3]

POSSIBLE CRITERIA FOR SELECTING NEW WELL LOCATIONS

Henderson, et al. showed the considerable dependence of gas storage reservoir deliverability upon locations of wells and the order in which wells are opened to flow during the withdrawal season.[14] This dependence upon well locations results from a rather complex interaction of effects due to well interference and reservoir heterogeneity. Coats later presented a method for automatically determining optimal new well locations in gas fields producing under semisteady-state conditions.[6] He indicated that the proper selection of additional well sites is difficult, if not practically impossible, to make by intuition. In the past, the use of intuition as the primary basis for decision has led to conflicting opinions. One faction contends that the heart [high permeability-thickness portion] of the field is the most favorable area for drilling additional wells, while another proponent maintains locations in the tight portions would give the best results. Arguments for each side might draw support from particular case histories without evidence that a method has been found that would be suitable for any reservoir to be studied. Therefore, in order to develop a field with the most favorable economics, a method for locating the additional wells must be defined so that the properties of the particular reservoir under consideration are taken into account. In addition, any criterion associated with a selection method must be constructed so that the rate of withdrawal from the reservoir meets the field producing schedule.

In a heterogeneous gas field, one might consider the properties of the formation as some criterion for a decision on the location of a new well. In this study, we have considered tests on the relative sizes of the permeability-thickness product in all parts of the field as well as well productivity as suitable criteria for locating new wells. In case an estimate of the variation in porosity is available, the relative sizes of the porosity-thickness product might offer a decision on the advantage of drilling in the heart or the tight portions of the field.

Irregular spacing of existing wells, the unequal producing rates of the wells, and the well interference effects might also provide some means for forming an opinion about the optimum location of the next gas well to be drilled. A two-dimensional pressure distribution may be calculated which will reflect all of these effects.

In the case where a scheme is discovered which will reduce the number of wells ultimately drilled in the field or postpone the drilling of some wells until a later time in the life of the contract, the final judgment on the value of the scheme is left to economic analysis. The use of the particular scheme might incur costs in excess of those demanded by another scheme in which the number of wells ultimately drilled is not the least. Therefore, we must find the selection method that offers the highest present value profit in order to ensure an optimal development program.

MATHEMATICAL MODEL

Eq. 1 describes the isothermal, transient gas flow in two dimensions in a region R:

$$\frac{\partial}{\partial x}\left(\alpha kh \frac{p}{z\mu}\frac{\partial p}{\partial x}\right) - \frac{\partial}{\partial y}\left(\alpha kh \frac{p}{z\mu}\frac{\partial p}{\partial y}\right) - \frac{q_a}{f} = \phi h \frac{\partial p/z}{\partial t} . \qquad \text{. . . . .[1]}$$

A complete derivation of the equation is given in the Appendix.

A problem is defined by Eq. 1 and appropriate boundary and initial conditions if the various functions of pressure are specified and means for determing $[q_a(x,y,t)]/f$ are provided. The solution to the problem is the function p[x,y,t], the pressure distribution in the field as a function of time.

To estimate the pressure trend in a gas field the above equation, along with the equations expressing the boundary and initial conditions, is solved for the case of a specified field producing rate schedule, under the condition that flowing well pressures have to remain above a fixed minimum value. These nonlinear equations are expressed in finite difference form and are solved numerically on a digital computer.

The numerical methods involve superimposing a two-dimensional grid on the reservoir, resulting in a number of rectangular elements or blocks with dimensions $\Delta x, \Delta y$. Mathematically, the procedure is to simply replace the differentials in Eq. 1 with finite difference approximations, giving a set of ordinary differential equations to solve. An alternating direction iterative technique has been applied successfully to numerical integration of the nonlinear partial differential equations describing flow in two-dimensional gas reservoirs. The procedures used and the accuracy of the solutions are discussed in the Appendix.

DESCRIPTION OF THE CROSSFIELD RESERVOIR

The mathematical model presented in the preceding section is a set of mathematical relationships which simulate the flow of natural gas in a reservoir. This model has been applied to the unsteady-state flow behavior in that portion of the Crossfield field designated as the Crossfield Wabamum A pool near Calgary, Alberta. The Crossfield reservoir is modelled within a rectangular area approximately 8 miles in width and 25 miles in length. The reservoir is considered to be heterogeneous, bounded by impermeable media, where the initial pressure throughout the reservoir at the beginning of gas production is known. We assume single-phase, isothermal flow of nonideal gas at velocities for which Darcy's law is applicable. Flow is permitted in the x and y areal directions, but is considered negligible in the vertical or z direction. For purposes of numerical solution, the reservoir is divided into square blocks, each having an area of 640 acres, forming a two-dimensional grid of approximately 200 blocks. It is assumed that each well is located in the center of a block, though not every block will contain a well. Because sufficient information was not available throughout the field, an average value for porosity was used for the entire reservoir. The permeability-thickness product is variable in the reservoir and is defined at the mid-point of each grid block. [Fig. 1 is the grid map, with kh values shown as defined in the center of each block.] The natural gas volumes are given as raw gas at a base pressure of 14.696 and a base temperature of 60F.

Relationships between the gas deviation factor and viscosity with pressure and depth were determined at the reservoir temperature of 176 F [Fig. 2]. Under the hypothetical gas contract in this study, we are required to maintain a constant field producing rate at a specified flowing bottom-hole pressure throughout a certain specified time period. At the beginning of the study, there are 10 wells producing in the field.

The data for this reservoir were obtained from the report, "Crossfield Wabamum A Pool, Reservoir Model Study I".[19]

RESULTS

Field Development Plan for Each Selection Method

The computer program in this study was written so that producing wells were added in order to keep the total field producing rate constant. A field development plan [i.e., location and time of drilling each new required well] was obtained from each of three schemes for selecting additional well locations. The three selection methods tested here involve tests on the value of the kh product in each block: selection of the block possessing [1] the lowest value of kh for the next well location, [2] the highest kh value, and [3] the highest product, $[P_{ij} - P_w]kh_{ij}$. Selecting low kh wells is generally selecting wells in sections with a relatively small amount of depletion but wells of low productivity index. Selecting wells by high kh generally means selecting wells of high productivity index but high depletion. The third method is selecting wells of high productivity.

A comparison of the development plans produced by the three methods is shown by a plot of the number of wells drilled vs time for the life of the contract [Fig. 3]. From the plot, the third scheme is immediately recognized as the optimum, as far as we can determine at this point. However, as mentioned earlier, there are economic considerations involved in every plan for developing a new gas field. Combining our information about the reservoir itself with what we can determine about the well costs and, indeed the cost of developing the field according to the particular plan we have in mind, we may discover that an alternate plan should be chosen so as to minimize our investment throughout the contract. At this point, an economic analysis of the alternate development schemes becomes the important study.

Economic Analysis of the Development Program

The value of all investments and of all incomes are discounted at a certain discount rate. The difference between the two discounted values is the present, or discounted value of the profit.

The present value of the total income is

$$I_{pv} = I\left[\frac{1 - e^{-jt_a}}{jt_a}\right]$$

where I is the total income, j is the interest rate and t_a is the time at which the field is abandoned.

The cost of an additional well drilled at time t_i is C. Then the present value cost of n wells drilled until abandonment time t_a is

$$C_{pv} = C\sum_{i=1}^{n} e^{-jt_i} .$$

Assigning an arbitrary cost per well of \$60,000, an arbitrary income of \$.15/Mcf [after deduction of costs per unit of production, which are assumed to be independent of the number of wells] at the contracted rate of 150,000 Mcf/D, and a value 0.08 to j, the present value cost over the time period is represented by

$$C_{pv} = (\$60,000) \sum_{i=1}^{n} e^{-.08t_i} .$$

The present value of the income is represented by

$$I_{pv} =$$

$$(\$.15)\ (150,000\ \text{Mcf/d})\ (365) \left[\frac{1 - e^{-.08t_a}}{.08t_a}\right]$$

In the present case the present value of the income is the same for all schemes; therefore, in order to maximize the present value of the profit, the present value of the total well costs must be minimized.

The results of these computations when applied to the three selection methods are shown in Fig. 4 and Tables 2 through 4. It is obvious from our use of a constant cost per well that the third location scheme remains the optimum.

CONCLUSION

A mathematical model of transient, two-dimensional gas flow has been described and applied to evaluate three methods for locating additional wells in a producing gas field. In this study, data describing the Crossfield resservoir was used. However, the program as written will accept any variation in porosity, thickness, or any other variable used to describe a particular gas reservoir, and consider these values properly in the computations. Therefore, a study such as this one, tailored to fit any reservoir whose development plan is questioned, would seem to be a profitable undertaking. For the particular variable kh, uniform ϕh field considered here, the third method, i.e., the scheme containing the $[P_{ij} - P_w]kh_{ij}$ criterion, was clearly the optimum.

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NOMENCLATURE

b_g = gas formation volume factor, Mcf/cu ft

$C = \dfrac{d[p/z]}{dP}$

$f = \dfrac{T_s}{1,000 p_s T}$

h = thickness, ft

k = permeability, md

p = pressure, psia

p_s = standard pressure

$P = \displaystyle\int \frac{p}{z\mu}\,dp$ [potential]

P_{ij} = block calculated potential

P_w = potential at the wellbore

q = production rate, Mcf/cu ft of reservoir/D

q_m = mass production rate per unit of volume

q_v = production rate per unit of volume

q_a = production rate per unit area

r_e = exterior radius, ft

r_w = wellbore radius, ft

t = time, days

T = reservoir temperature, °R

T_s = standard temperature

u = superficial velocity

V_p = total reservoir pore volume, cu ft

$w = \dfrac{2\pi\alpha kh}{\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2}}$

z = gas compressibility factor

α = conversion constant for field units equal to 0.00633

ρ = density, lb/cu ft

μ = viscosity, cp

ϕ = porosity, fraction

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APPENDIX

Derivation of the Diffusivity Eq. 1

The basic partial differential equation for the unsteady-state flow of natural gas has been given in technical literature. Here the equation which simulates the gas flow in a particular reservoir, with its associated rock and fluid properties, is derived.

Using the principle of conservation of volume converted to standard conditions, expressed in volume per unit of area, gives

$$-\vec{\nabla} \cdot (b_g h\vec{u}) - q_a = \phi h \frac{\partial b_g}{\partial t} \quad . . . . . . [A\text{-}1]$$

where $b_g = \dfrac{pT_s}{1{,}000\ p_s Tz}$ is the volume at standard conditions [in Mcf] of 1 cu ft of gas at reservoir conditions and q_a is the rate of withdrawal per unit area. Neglecting gravitational effects, Darcy's law, using conventional units may be expressed as

$$\vec{u} = -\frac{\alpha k}{\mu}\vec{\nabla}p, \text{ with } \alpha = 0.00633. \quad . . . [A\text{-}2]$$

Substitution of Darcy's law in Eq. A-1 gives

$$\vec{\nabla} \cdot \left(b_g \frac{\alpha kh}{\mu} \vec{\nabla}p\right) - q_a = \phi h \frac{\partial b_g}{\partial t} \quad . . . . [A\text{-}3]$$

In two dimensions, Eq. A-3 becomes

$$\frac{\partial}{\partial x}\left(b_g \frac{\alpha kh}{\mu} \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(b_g \frac{\alpha kh}{\mu} \frac{\partial p}{\partial y}\right) - q_a =$$

$$\phi h \frac{\partial b_g}{\partial t} \quad . . . . [A\text{-}4]$$

Because $b_g = \dfrac{pT_s}{1{,}000\ p_s Tz} = \dfrac{fp}{z}$, where $f = \dfrac{T_s}{1{,}000\ p_s T}$, Eq. A-4 becomes Eq. 1 of the text.

$$\frac{\partial}{\partial x}\left(\alpha kh \frac{p}{z\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\alpha kh \frac{p}{z\mu}\frac{\partial p}{\partial y}\right) - \frac{q_a}{f} = \theta h \quad \ldots\ldots [A\text{-}5]$$

Introducing $dP = p/z\mu[dp]$, and therefore $\frac{\partial P}{\partial x} = \frac{p}{z\mu}\frac{\partial p}{\partial x}$, $\frac{\partial P}{\partial y} = \frac{p}{z\mu}\frac{\partial p}{\partial y}$ and using the relationship $\frac{\partial\, p/z}{\partial t} = \frac{d\, p/z}{dP}\cdot\frac{\partial P}{\partial t}$ we obtain

$$\frac{\partial}{\partial x}\left(\alpha kh \frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial y}\left(\alpha kh \frac{\partial P}{\partial v}\right) - \frac{q_a}{f} = \phi hC\frac{\partial P}{\partial t} \quad \ldots\ldots [A\text{-}6]$$

with $C = \frac{d\, p/z}{dP}$. Note: As a matt of interest $C = c\mu$, where $c = \frac{1}{p} - \frac{1}{z}\frac{dz}{dp}$ is the g s compressibility.

Using $\frac{q_a}{f} = \frac{w[\bar{P} - P_w]}{\Delta x\Delta y}$, with $w = \frac{2\pi kh\alpha}{\ln[r_e/r_w] - 1/2}$ and substituting this in Eq. A-6 gives

$$\frac{\partial}{\partial x}\left(\alpha kh \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\alpha kh \frac{\partial p}{\partial y}\right) - \frac{w(\bar{P} - P_w)}{\Delta x\Delta y} = \phi hC\frac{\partial \bar{P}}{\partial t} \quad \ldots\ldots [A\text{-}7]$$

Note: Because crossflow between blocks is considered, no semisteady-state will be reached; the actual form of w is uncertain, but the form as presented seems to be a good approximation.[3,14]

In a block of size $\Delta x\Delta y$, Eq. A-7 becomes

$$\Delta x\Delta y\frac{\partial}{\partial x}\left(\alpha kh\frac{\partial p}{\partial x}\right) + \Delta x\Delta y\frac{\partial}{\partial y}\left(\alpha kh\frac{\partial p}{\partial y}\right) - w(\bar{P} - P_w) = \Delta x\Delta y\phi hC\frac{\partial P}{\partial t}. \quad \ldots\ldots [A\text{-}8]$$

In standard second-order difference form, Eq. A-8 becomes:

$$\Delta(T\Delta P)_{i,j} - w_{ij}\,(P_{i,j} - P_w) = C_{ij}\frac{V_{p\; i,j}}{\Delta t}\Delta_t P_{i,j} \quad \ldots\ldots [A\text{-}9]$$

with:

$$\Delta(T\Delta P)_{i,j} = \Delta_x(T\Delta_x P)_{i,j} + \Delta_y(T\Delta_y P)_{i,j}$$

where:

$$\Delta_x(T\Delta_x P)_{i,j} = T_{i+1/2,j}\,(P_{i+1,j} - P_{i,j}) - T_{i-1/2,j}(P_{i,j} - P_{i-1,j})$$

$$\Delta_y(T\Delta_y P)_{i,j} = T_{i,j+1/2}(P_{i,j+1} - P_{i,j}) - T_{i,j-1/2}(P_{i,j} - P_{i,j-1})$$

Further:

$$w_{ij} = w = \frac{2\alpha\pi(kh)_{i,j}}{\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2}}$$

$$P_{ij} = \bar{P}$$

$$C_{ij} = \left(\frac{dp/z}{dP}\right)_{i,j}$$

$$V_{p\; i,j} = \Delta x\Delta y(kh)_{i,j}$$

$$\Delta_t P_{i,j} = P_{i,j,n+1} - P_{i,j,n}$$

Note: All P values on the left side of Eq. A-9 are values at the new time level n+1. This is the backward-difference or implicit-difference approximation to the diffusivity equation.

Description of the Alternating Direction Iterative Technique

Consider Eq. A-9 as the difference equation whose solution we wish to obtain:

$$\Delta T\Delta P - w_{ij}(P - P_w) = G\Delta_t P$$

where $w_{ij} = \frac{2\pi(kh)_{ij}\alpha}{\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2}}$ and $G = C\frac{V_p}{\Delta t}$.

Note that w_{ij} is zero unless block i,j contains a well. The equation becomes, using the Douglas-Rachford Alternating Direction Iterative Method,

$$\Delta_x T\Delta_x P^* + \Delta_y T\Delta_y P^k - w_{ij}(P^* - P_w) = G(P^* - P_n) + H_k\Sigma T(P^* - P^k), \quad \ldots\ldots [A\text{-}10]$$

and

$$\Delta_x T\Delta_x P^* + \Delta_y T\Delta_y P^{k+1} - w_{ij}(P^{k+1} - P_w) = G(P^{k+1} - P_n) + H_k\Sigma T(P^{k+1} - P^k), \quad \ldots [A\text{-}11]$$

where

$$\Sigma T = T_{i+1/2,j} + T_{i-1/2,j} + T_{i,j+1/2} + T_{i,j-1/2} \text{ and}$$

H_k is the iteration parameter. $P^k_{i,j}$ is the kth iterate or approximation to $P_{i,j,n+1}$. That is, $\lim_{k\to\infty} P^k_{i,j} = P_{i,j,n+1}$. Subtract and add $\Delta_x T\Delta_x P^k$ to Eq. A-10, and let $PX = P^* - P^k$:

$$\Delta_x T\Delta_x PX - (w_{ij} + H_k\Sigma T + G)\, PX = -R_{ij} \quad . [A\text{-}12]$$

where R_{ij} is the residual [i.e., the equation itself being solved with the latest iterate

$P^k_{i,j}$ inserted for $P_{i,j,n+1}$].

$$R_{ij} = \Delta T \Delta P^k + w_{ij}\ (P_w - P^k) - G(P^k - P_n)$$

Subtract Eq. A-10 from Eq. A-11 and let PY = $P^{k+1} - P^k$:

$$\Delta_y T \Delta_y PY - (w_{ij} + H_k \Sigma T + G) PY = - (w_{ij} + H_k \Sigma T + G) PX \quad \text{[A-13]}$$

Eq. A-12 is first solved to yield PX_{ij} for all i,j. Eq. A-13 is then solved for PY_{ij}. The new iterate P^{k+1} is then obtained as

$$P^{k+1}_{i,j} = P^k_{i,j} + PY_{i,j}.$$

Solution of Eqs. A-12 and A-13 using H_k constitute one iteration. K iterations constitute one cycle. Here K was 5 and $H_1, \ldots, H_5$ were used as [0, .02, .06, .18, .54]. Iterations were continued until the sum $\sum_{i,j} R_{ij}$ of the residuals over the grid was less than .002 times the total field producing rate.

Each of Eqs. A-12 and A-13 are of one-dimensional type and their solution is given by Douglas, Peaceman and Rachford.[10]

The Computer Program

The computer program for this study may be considered in three distinct sections: [1] definition of the reservoir with its particular rock and fluid characteristics, [2] alternating direction iterative technique solution of the simulator equations and the "drilling" of new wells at optimum field location when necessary, and [3] economic analysis and output section.

Immediately following the initialization of storage areas, a description of the reservoir fluid and rock characteristics and a description of the grid map are read in. Values for z and zμ for particular values of p are obtained from a graph of the information [Fig. 2] and read in at this time. Later in the computation, values for p/z and P are needed; therefore, they are computed and stored at this time. The values for P are computed using the trapezoidal rule for integration, using the definition: $P = \frac{1}{2} \int \frac{dp^2}{z\mu}$. The transmissibilities are computed along with the pore volumes in each grid block. In the computation of the transmissibilities, harmonic averaging is used to obtain an average value of permeability between two grid blocks. A contract well rate schedule is read in with a rate value for each successive time period. The number of initial wells is read in along with each location; also, the time step data and iteration parameters for the alternating direction technique. The withdrawal terms are computed for each block where a well exists. The initial and minimum wellbore potentials are found in the computed table by linear interpolation. An initial value for C, or d[p/z]dP, is computed. The volume of gas contained in the reservoir is computed by finding the volume of pore space containing gas and the gas content of a unit space from temperature, pressure and the gas compressibility factor.

The first part of the next section of the program contains the computations involved in the alternating direction iterative technique. This method is employed in solving the partial differential equation to obtain our pressure distributions at each time step. A description of the procedure is given above. The last part of the second section contains the routine for automatic selection of well location. A test is made on the total field wellbore potential. If the value has dropped below the minimum, the program will branch to "drill" a well according to the criterion given for the decision. This procedure includes locating the block where the well is to be drilled, computing a withdrawal term of well rate term for the block, and storing the new location for future reference.

The third section of the program merely organizes the output and prints when instructed by the program to do so. Cards are punched for input to the economic analysis program and the computation is complete.

Table 1 - Initial reservoir properties.

Pressure Base-psia	14.696
Temperature Base-degrees Fahrenheit	60
Original Reservoir Pressure at 5000 ft.-psia	3600
Reservoir Temperature-degrees Fahrenheit	176
Average Gross Porosity-percent	2.2
Average Gross Thickness-ft.	58.0
Total Original Gas-In-Place-Mmcf	1,589,700
Total Productive Area-acres	128,000

Table 2 - Economic analysis of Case 1 (low kh).

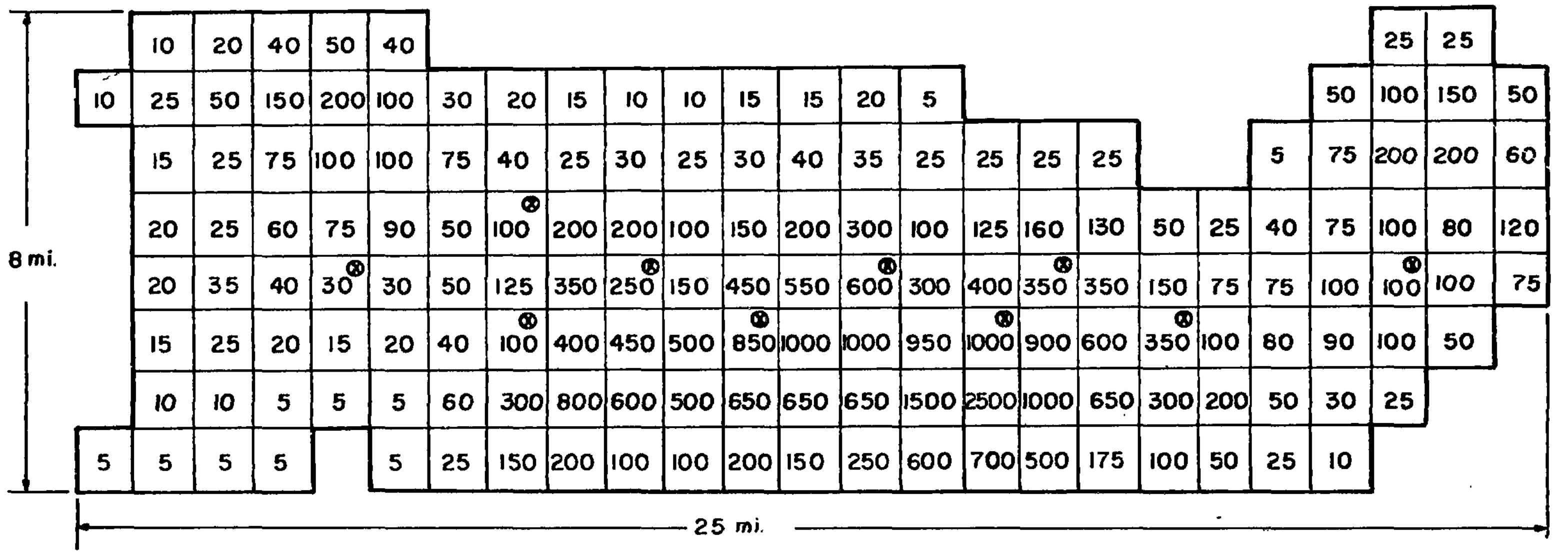
TIME(YRS.)	NEW WELLS	PV COST	PV INC	CPV COST	PV PROFIT
2.4658E-01	16	$ 5.8828E+05	$ 8.1320E+06	$ 5.8828E+05	$ 7.5437E+06
4.9315E-01	0	$ 0.	$ 8.0526E+06	$ 5.8828E+05	$ 8.0526E+06
7.3973E-01	0	$ 0.	$ 7.9742E+06	$ 5.8828E+05	$ 7.9742E+06
9.8630E-01	0	$ 0.	$ 7.8969E+06	$ 5.8828E+05	$ 7.8969E+06
1.2329E+00	0	$ 0.	$ 7.8205E+06	$ 5.8828E+05	$ 7.8205E+06
1.4795E+00	0	$ 0.	$ 7.7451E+06	$ 5.8828E+05	$ 7.7451E+06
1.7260E+00	0	$ 0.	$ 7.6707E+06	$ 5.8828E+05	$ 7.6707E+06
1.9726E+00	0	$ 0.	$ 7.5973E+06	$ 5.8828E+05	$ 7.5973E+06
2.2192E+00	0	$ 0.	$ 7.5248E+06	$ 5.8828E+05	$ 7.5248E+06
2.4658E+00	0	$ 0.	$ 7.4532E+06	$ 5.8828E+05	$ 7.4532E+06
2.7123E+00	0	$ 0.	$ 7.3826E+06	$ 5.8828E+05	$ 7.3826E+06
3.2055E+00	0	$ 0.	$ 7.2440E+06	$ 5.8828E+05	$ 7.2440E+06
3.6986E+00	16	$ 7.1411E+05	$ 7.1090E+06	$ 1.3024E+06	$ 6.3949E+06
4.1918E+00	9	$ 3.8615E+05	$ 6.9773E+06	$ 1.6885E+06	$ 6.5912E+06
4.6849E+00	6	$ 2.4747E+05	$ 6.8490E+06	$ 1.9360E+06	$ 6.6015E+06
5.1781E+00	6	$ 2.3790E+05	$ 6.7239E+06	$ 2.1739E+06	$ 6.4860E+06
5.6712E+00	6	$ 2.2870E+05	$ 6.6020E+06	$ 2.4026E+06	$ 6.3733E+06
6.1644E+00	5	$ 1.8321E+05	$ 6.4831E+06	$ 2.5858E+06	$ 6.2999E+06
6.6575E+00	5	$ 1.7612E+05	$ 6.3671E+06	$ 2.7620E+06	$ 6.1910E+06
7.1507E+00	3	$ 1.0159E+05	$ 6.2540E+06	$ 2.8635E+06	$ 6.1525E+06
7.6438E+00	4	$ 1.3021E+05	$ 6.1438E+06	$ 2.9937E+06	$ 6.0135E+06
8.1370E+00	3	$ 9.3878E+04	$ 6.0362E+06	$ 3.0876E+06	$ 5.9423E+06
8.6301E+00	4	$ 1.2033E+05	$ 5.9312E+06	$ 3.2080E+06	$ 5.8109E+06
9.1233E+00	4	$ 1.1567E+05	$ 5.8289E+06	$ 3.3236E+06	$ 5.7132E+06
9.6164E+00	3	$ 8.3399E+04	$ 5.7290E+06	$ 3.4070E+06	$ 5.6456E+06
1.0603E+01	5	$ 1.2845E+05	$ 5.5364E+06	$ 3.5355E+06	$ 5.4080E+06
1.1589E+01	7	$ 1.6619E+05	$ 5.3530E+06	$ 3.7017E+06	$ 5.1868E+06
1.2575E+01	8	$ 1.7552E+05	$ 5.1782E+06	$ 3.8772E+06	$ 5.0027E+06

Table 3 - Economic analysis of Case 2 (high kh).

TIME (YRS.)	NEW WELLS	PV COST	PV INC	CPV COST	PV PROFIT
2.4658E-01	10	$ 5.8828E+05	$ 8.1320E+06	$ 5.8828E+05	$ 7.5437E+06
4.9315E-01	0	$ 0.	$ 8.0526E+06	$ 5.8828E+05	$ 8.0526E+06
7.3973E-01	0	$ 0.	$ 7.9742E+06	$ 5.8828E+05	$ 7.9742E+06
9.8630E-01	0	$ 0.	$ 7.8969E+06	$ 5.8828E+05	$ 7.8969E+06
1.2329E+00	0	$ 0.	$ 7.8205E+06	$ 5.8828E+05	$ 7.8205E+06
1.4795E+00	0	$ 0.	$ 7.7451E+06	$ 5.8828E+05	$ 7.7451E+06
1.7260E+00	0	$ 0.	$ 7.6707E+06	$ 5.8828E+05	$ 7.6707E+06
1.9726E+00	0	$ 0.	$ 7.5973E+06	$ 5.8828E+05	$ 7.5973E+06
2.2192E+00	0	$ 0.	$ 7.5248E+06	$ 5.8828E+05	$ 7.5248E+06
2.4658E+00	0	$ 0.	$ 7.4532E+06	$ 5.8828E+05	$ 7.4532E+06
2.7123E+00	0	$ 0.	$ 7.3826E+06	$ 5.8828E+05	$ 7.3826E+06
3.2055E+00	0	$ 0.	$ 7.2440E+06	$ 5.8828E+05	$ 7.2440E+06
3.6986E+00	1	$ 4.4632E+04	$ 7.1090E+06	$ 6.3291E+05	$ 7.0643E+06
4.1918E+00	0	$ 0.	$ 6.9773E+06	$ 6.3291E+05	$ 6.9773E+06
4.6849E+00	0	$ 0.	$ 6.8490E+06	$ 6.3291E+05	$ 6.8490E+06
5.1781E+00	1	$ 3.9650E+04	$ 6.7239E+06	$ 6.7256E+05	$ 6.6843E+06
5.6712E+00	2	$ 7.6233E+04	$ 6.6020E+06	$ 7.4880E+05	$ 6.5258E+06
6.1644E+00	3	$ 1.0993E+05	$ 6.4831E+06	$ 8.5872E+05	$ 6.3731E+06
6.6575E+00	1	$ 3.5224E+04	$ 6.3671E+06	$ 8.9395E+05	$ 6.3319E+06
7.1507E+00	6	$ 2.0317E+05	$ 6.2540E+06	$ 1.0971E+06	$ 6.0509E+06
7.6438E+00	6	$ 1.9531E+05	$ 6.1438E+06	$ 1.2924E+06	$ 5.9484E+06
8.1370E+00	9	$ 2.8163E+05	$ 6.0362E+06	$ 1.5741E+06	$ 5.7546E+06
8.6301E+00	4	$ 1.2033E+05	$ 5.9312E+06	$ 1.6944E+06	$ 5.8109E+06
9.1233E+00	5	$ 1.4459E+05	$ 5.8289E+06	$ 1.8390E+06	$ 5.6843E+06
9.6164E+00	6	$ 1.6680E+05	$ 5.7290E+06	$ 2.0058E+06	$ 5.5622E+06
1.0603E+01	14	$ 3.5967E+05	$ 5.5364E+06	$ 2.3655E+06	$ 5.1768E+06
1.1589E+01	16	$ 3.7986E+05	$ 5.3530E+06	$ 2.7453E+06	$ 4.9731E+06
1.2575E+01	20	$ 4.3880E+05	$ 5.1782E+06	$ 3.1841E+06	$ 4.7394E+06

Table 4 - Economic analysis of Case 3 (high P_{ij}-P_w)kh).

TIME(YRS.)	NEW WELLS	PV COST	PV INC	CPV COST	PV PROFIT
2.4658E-01	10	$ 5.8828E+05	$ 8.1320E+06	$ 5.8828E+05	$ 7.5437E+06
4.9315E-01	0	$ 0.	$ 8.0526E+06	$ 5.8828E+05	$ 8.0526E+06
7.3973E-01	0	$ 0.	$ 7.9742E+06	$ 5.8828E+05	$ 7.9742E+06
9.8630E-01	0	$ 0.	$ 7.8969E+06	$ 5.8828E+05	$ 7.8969E+06
1.2329E+00	0	$ 0.	$ 7.8205E+06	$ 5.8828E+05	$ 7.8205E+06
1.4795E+00	0	$ 0.	$ 7.7451E+06	$ 5.8828E+05	$ 7.7451E+06
1.7260E+00	0	$ 0.	$ 7.6707E+06	$ 5.8828E+05	$ 7.6707E+06
1.9726E+00	0	$ 0.	$ 7.5973E+06	$ 5.8828E+05	$ 7.5973E+06
2.2192E+00	0	$ 0.	$ 7.5248E+06	$ 5.8828E+05	$ 7.5248E+06
2.4658E+00	0	$ 0.	$ 7.4532E+06	$ 5.8828E+05	$ 7.4532E+06
2.7123E+00	0	$ 0.	$ 7.3826E+06	$ 5.8828E+05	$ 7.3826E+06
3.2055E+00	0	$ 0.	$ 7.2440E+06	$ 5.8828E+05	$ 7.2440E+06
3.6986E+00	1	$ 4.4632E+04	$ 7.1090E+06	$ 6.3291E+05	$ 7.0643E+06
4.1918E+00	0	$ 0.	$ 6.9773E+06	$ 6.3291E+05	$ 6.9773E+06
4.6849E+00	0	$ 0.	$ 6.8490E+06	$ 6.3291E+05	$ 6.8490E+06
5.1781E+00	1	$ 3.9650E+04	$ 6.7239E+06	$ 6.7256E+05	$ 6.6843E+06
5.6712E+00	1	$ 3.8116E+04	$ 6.6020E+06	$ 7.1068E+05	$ 6.5639E+06
6.1644E+00	1	$ 3.6642E+04	$ 6.4831E+06	$ 7.4732E+05	$ 6.4464E+06
6.6575E+00	2	$ 7.0449E+04	$ 6.3671E+06	$ 8.1777E+05	$ 6.2967E+06
7.1507E+00	2	$ 6.7724E+04	$ 6.2540E+06	$ 8.8549E+05	$ 6.1863E+06
7.6438E+00	2	$ 6.5104E+04	$ 6.1438E+06	$ 9.5060E+05	$ 6.0787E+06
8.1370E+00	3	$ 9.3878E+04	$ 6.0362E+06	$ 1.0445E+06	$ 5.9423E+06
8.6301E+00	2	$ 6.0164E+04	$ 5.9312E+06	$ 1.1046E+06	$ 5.8711E+06
9.1233E+00	3	$ 8.6756E+04	$ 5.8289E+06	$ 1.1914E+06	$ 5.7421E+06
9.6164E+00	5	$ 1.3900E+05	$ 5.7290E+06	$ 1.3304E+06	$ 5.5900E+06
1.0603E+01	9	$ 2.3122E+05	$ 5.5344E+06	$ 1.5616E+06	$ 5.3052E+06
1.1589E+01	13	$ 3.0864E+05	$ 5.3530E+06	$ 1.8702E+06	$ 5.0444E+06
1.2575E+01	23	$ 5.0462E+05	$ 5.1782E+06	$ 2.3749E+06	$ 4.6736E+06



(1 BLOCK = 640 ACRES)

⊗ = EXISTING GAS WELL

Fig. 1 - Grid map showing permeability-thickness product (md ft) for each block.

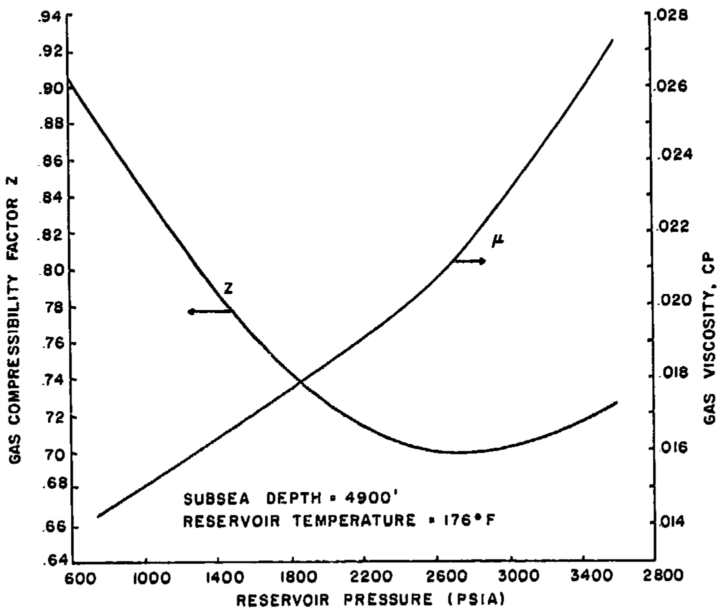


FIGURE 2 GAS COMPRESSIBILITY FACTOR (Z) AND VISCOSITY (μ) CURVES AT RESERVOIR CONDITIONS

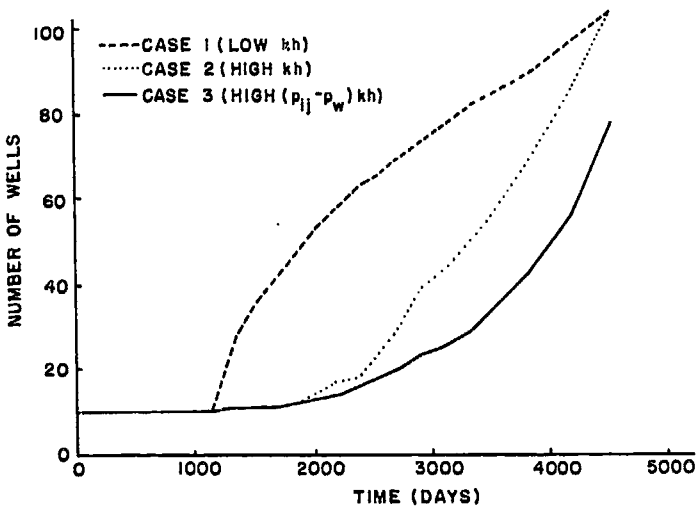


FIGURE 3 CUMULATIVE NUMBER OF WELLS REQUIRED vs TIME

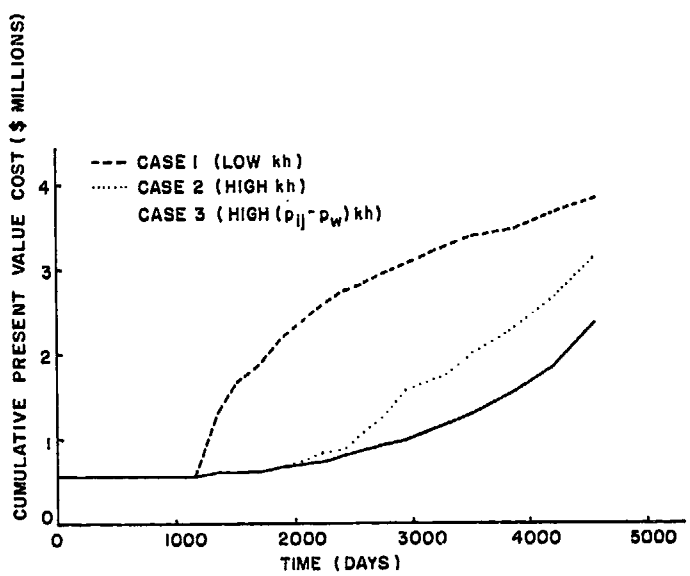


FIGURE 4 CUMULATIVE PRESENT VALUE COST vs TIME