2344

A New Technique for Determining Reservoir Description from Field Performance Data

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ABSTRACT

Reservoir description data largely determine the validity of simulated reservoir performance. This paper presents a method that employs the least squares and linear programming techniques to determine a reservoir description from given performance data. The method handles multiphase as well as single-phase flow problems.

The description parameters determined by the method may be any physical properties that influence calculated field performance. We believe the technique offers considerably greater efficiency than previously reported techniques.

Example applications presented include cases of single-phase gas flow, single-phase oil flow and two-phase gas-water flow. In these particular applications the method gave accurate results with a large range of uncertainty in the reservoir parameters, and with a small number of simulation runs.

INTRODUCTION

The purpose of reservoir simulation is estimation of future reservoir performance under alternative well configurations or operating conditions. This estimation is increasingly being performed using rather complex, numerical reservoir models. Reservoir description data constitute the bulk of the required input data for these models, and the accuracy of these data largely determine the validity of the calculated results. Thus an obvious problem is the determination of an accurate reservoir description.

We treat the problem of determining a reservoir description that, when used as input data to a reservoir simulator, results in close agreement between calculated and observed field performance. Field history or performance data are presumed available for some period of time designated the "match period". The available field history may reflect single- or multiphase, multidimensional flow, and the performance data to be matched may be any mix of observed pressures, producing rates, gas-oil and/or water-oil producing ratios. The observed field performance may correspond to a period of depletion and/or injection, or to an interference test.

Our method for determining a viable reservoir description requires a number of runs using a reservoir simulator, each run using a reservoir description that is random within limits specified by the engineer. We then use a second, small program, that utilizes least squares and linear programming techniques, to process the data output from those runs to determine a reservoir description.

To illustrate and test this new method, we constructed three example reservoirs experiencing single-phase gas, single-phase oil and two-phase (gas-water) flow, respectively, in two spatial dimensions. Simulator runs were made using a given set of reservoir description parameters. The results of these runs were then treated as "data" and the description parameters considered unknown. The automatic history matching method described in this paper was applied to back out description parameter values from the performance "data". The agreement between these values and the true parameter values is given below.

Reed *et al.*[9] present an actual field case where the manual approach to matching production history proved prohibitive in both man and machine time. Our least squares, linear programming technique was then used to achieve a satisfactory and economical match of the reservoir performance data.

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[1]References given at end of paper.

THE PROBLEM

Field history or performance data are available over the match period $0 < t < t_m$. The set $d_1, d_2, \ldots, d_I$ or simply (d_i) represents the performance data. For example, d_1 may be observed pressure at Well 12 at a time of 90 days, d_4 may be producing gas-oil ratio of Well 6 at a time of 270 days, d_8 may be the total WOR of several wells in a certain lease or area of the reservoir at some time, or an average of same over some period of time. Several d_i values may represent observed pressures at a single well at different times.

The reservoir simulator selected as a model for the field accounts for reservoir heterogeneity and geometry and the multidimensional, single- or multiphase flow regime active in the field. A list of parameters $x_1, x_2, \ldots, x_J$, or simply (x_j), is selected, which constitutes part of the reservoir description data required by the model. Typically the reservoir is divided into zones or areas and, for example, x_1 may be the permeability of Zone 1, x_3 the porosity of Zone 4, etc. Variables representing aquifer strength or degree of communication across a suspected fault may be included in (x_j).

Restrictions on the performance data (d_i) and the description parameters (x_j) are that $I > J$ and that each parameter value x_j should have some effect on the calculated value of at least one of the d_i.

A single simulator run using certain values for (x_j) yields calculated values of the performance data, designated here by (d_i^{calc}). The observed performance data are designated by (d_i^{obs}). The error set (ϵ_i) is then defined as $(d_i^{\text{obs}} - d_i^{\text{calc}})$. In terms of this notation, the reservoir description or history match problem is that of determining the set of description parameters (x_j^*) that minimizes some norm of the errors, say $\sum_{i=1}^{I} |\epsilon_i|$.

A past and current approach to this history matching problem is the simple trial-and-error procedure of visually examing the output of each simulator run and adjusting one or more of the description parameters x_j by intuition or experience prior to performing the next run. This process can become frustrating and expensive in man-time and machine time, even in cases involving as few as 5 to 10 description parameters.

An automatic computing procedure for determining a reservoir description from field pressure data was described by Jacquard and Jain[1] and Jahns.[2] Their regression technique is an adaptation of the method of steepest descent and was proposed for determining reservoir description from pressure response to interference tests in single-phase flow cases. Kruger[3] and Nelson[4] also treat this reservoir description problem.

DESCRIPTION OF THE PROPOSED METHOD

We divide the reservoir into zones and select a set of parameters $x_1, x_2, \ldots, x_J$ that constitutes the reservoir description. An upper and lower limit for each x_j is imposed, i.e., $x_{j\ell} \le x_j \le x_{ju}$. The percent variation in the description variable x_j is $100x(x_{ju} - x_{j\ell})/x_{j\ell}$.

A total of N simulator runs are performed, each with a different random set of the variables (x_j), each run resulting in calculated values of the (d_i) and the errors (ϵ_i). These N runs are performed in a single "job"—i.e., computer submittal. At the end of each simulator run, the computer punches onto cards the values of the parameter set used, (x_j^r), and the corresponding calculated error set (ϵ_i^r), $r = 1, 2, \ldots, N$. For run r, the description parameters (x_j^r) are random in that each x_j^r is selected by a uniform, random number generator as

$$x_j^r = x_{j\ell}^r + R\,(x_{ju}^r - x_{j\ell}^r) \quad \ldots\ldots \quad (1)$$

where R is a normalized (between 0 and 1) random number.

The errors (ϵ_i) are obviously single-valued functions of the parameters (x_j). The simplest possible form of this functional dependence is the linear form

$$\epsilon_i = \sum_{j=0}^{J} a_{ij}\, x_j \qquad i = 1, 2, \ldots, I \quad (2)$$

where $x_0 \equiv 1$. This linear dependence is an approximation, imposed onto surfaces $\epsilon_i(x_1, x_2, \ldots, x_J)$ which are in fact curvilinear. This approximation will be justified in proportion to the extent of success of the following analysis (which is based upon it) in backing out "correct" reservoir descriptions from given performance data.

The coefficients a_{ij} in Eq. 2 are determined by least squares. Defining the deviation

$$D_i^r \equiv \epsilon_i^r - \sum_{j=0}^{J} a_{ij}\, x_j^r$$

and applying the least squares technique (i.e., $\sum_{r=1}^{N} (D_i^r)^2 = \min$), we find

$$\sum_{j=0}^{J} \left[\sum_{r=1}^{N} x_n^r\, x_j^r \right] a_{ij} = \sum_{r=1}^{N} \epsilon_i^r\, x_n^r \qquad \begin{matrix} n = 0, J \\ i = 1, I \end{matrix} \quad \ldots\ldots \quad (3)$$

For any i, Eq. 3 are $J+1$ simultaneous linear equations in the $J+1$ unknowns $a_{i0}, a_{i1}, \ldots, a_{iJ}$.

The solution to Eq. 3 is

$$\underline{a}_i = B^{-1}\,\underline{c}_i \quad \ldots\ldots\ldots\ldots \quad (4)$$

where B is the matrix of coefficients (b_{nj}), B^{-1} is the inverse of B, c_i is the column vector $(c_{i0}, c_{i1}, \ldots, c_{iJ})^T$, a_i is the column vector $(a_{i0}, a_{i1}, \ldots, a_{iJ})^T$ and

$$b_{nj} \equiv \sum_{r=1}^{N} x_n^r\, x_j^r \qquad c_{in} \equiv \sum_{r=1}^{N} \epsilon_i^r\, x_n^r \,. \quad \ldots\ldots\ldots\ldots \quad (5)$$

Eq. 5 shows that the elements b_{nj} of the matrix B are independent of i and hence the inverse B^{-1} can be calculated once from the (x_j^r) and stored. Then application of Eq. 4 with $i = 1, 2, \ldots, I$ gives the entire set of coefficients (a_{ij}).

We are now left with the inverse problem of determining a set of variables (the reservoir description) (x_j) that result in some minimal levels of the errors (ϵ_i). Eq. 2 gives the relationship $\epsilon_i = \sum_{j=0}^{J} a_{ij}\, x_j$ where (a_{ij}) are now known. We might proceed again using least squares to determine the x_j so as to minimize $\sum_{i=1}^{I} \epsilon_i^2$. This is simple enough but leaves the strong impression that physically absurd (e.g., negative permeability) values of x_j might result. That is, Eq. 2 is approximate due to linearity forced upon the nonlinear actual relationship between ϵ_i and (x_j). This degree of approximation alone would probably result in "illegal" x_j values as determined from least squares.

The inverse problem is actually, then, one of determining a set (x_j^*) that minimizes the sum $\sum_{i=1}^{I} w_i\,|\epsilon_i|$ subject to constraints on the (x_j). The terms w_i are weight factors, used as 1.0 in the applications discussed here. This is the classical linear programming problem.[5,6]

$$\sum_{j=1}^{J} a_{ij}\, x_j + x_{J+i} - x_{J+I+i} = -\,a_{i0}$$

$$i = 1, 2, \ldots, I \quad \ldots\ldots\ldots \quad (6a)$$

$$\left.\begin{aligned} x_j &\geq x_{j\ell} \\ x_j &\leq x_{ju} \end{aligned}\right\} \; j = 1, 2, \ldots, J \quad \ldots\ldots \quad (6b), (6c)$$

$$\sum_{i=1}^{I} w_i\,(x_{J+i} + x_{J+I+i}) = \min. \quad \ldots\ldots \quad (6d)$$

Translation of x_j by $x_{j\ell}$, i.e., $(x_j)^{new} = x_j - x_{j\ell}$, $j = 1, 2, \ldots, J$, allows elimination of Eq. 6b, and a final linear programming form is

$$\sum_{j=1}^{J} a_{ij}\, x_j + x_{J+i} - x_{J+I+i} =$$

$$-\,x_{2J+i} = -\,a_{i0} - \sum_{j=1}^{J} a_{ij}\, x_{j\ell} \quad \ldots \quad (7a)$$

$$x_i + x_{2J+I+i} = x_{iu} - x_{i\ell} \qquad i = 1, J$$

$$\ldots\ldots \quad (7b)$$

$$\sum_{i=1}^{I} w_i\,(x_{J+i} + x_{J+I+i}) = \min.$$

$$\ldots\ldots \quad (7c)$$

In these equations the $x_1, x_2, \ldots, x_J$ on the left-hand side are actually the corresponding physical x_j's less $x_{j\ell}$. Solution by the simplex method of Eq. 7 is straightforward and yields that set of (x_j) values that minimizes $\sum_{i=1}^{I} w_i\,|\epsilon_i|$ subject to the minimum and maximum constraint values on the x_j.

This LSLP method, combining the Least Squares and Linear Programming techniques, automatically determines a reservoir description from given performance data.

SINGLE-PHASE GAS FLOW

The example reservoir shown in Fig. 1 represents a single-phase gas field with a closed exterior boundary. The reservoir thickness and porosity are uniform at 25 ft and 15 percent, respectively. The initial reservoir pressure, p_o, is 2,000 psi. The gas well symbols on Fig. 1 denote the locations of eight producing wells that are flowing at a constant rate of 2,500 Mcf/D. The reservoir is square, 20 $\times$ 10^3 ft on a side. An ADI iterative numerical model[7] using an 8 $\times$ 8 computing grid that simulates single-phase gas flow is the simulator for this example. Fig. 1 also shows the true reservoir zonation and permeability factors.

The match and prediction periods for this example are 360 and 1,830 days, respectively.

The six reservoir description parameters for this

example are the permeability factors ($K_{zone} = k_{zone}/k_{base}$) of each of the six zones shown in Fig. 1 (k_{base} = 5 md). The performance data (d_i) consist of the eight well block pressures, p_i, at 360 days calculated using the true zonation and the true zone permeability factors shown in Fig. 1. The differences of p_i from this run and those from the random simulator runs result in the error sets (ϵ_i).

For this example, four cases were investigated. The first case is a 100 to 150 percent variation in the zone factors. Thirty simulator runs ($N = 30$), each using a different random set of zonal permeability factors prescribed within the specified range, yield the punched set (ϵ_i). These were then input to the LSLP program in a set of $N = 30$ and three sets of $N = 10$. The second case treats the sensitivity to the parameter range variations. Ranges of 250 and 1,000 percent were studied. The third case examines the effect of an erroneous reservoir zonation. We present a comparison of two predictions using the true and erroneous reservoir zonations. Prediction runs for a period of 5 times the match period are presented. The fourth case treats the problem of backing out a zone description parameter for a zone in which no well or match point exists.

The results of Case 1 are presented in Table 1. It is interesting to note that the backed out values of zone permeability factors for $N = 30$ and those for the three runs of $N = 10$ are identical for practical purposes.

In the second case we treat the sensitivity to the reservoir parameter range variation. We used the same reservoir zonation and made two sets of 30 runs each using 250 and 1,000 percent range variation, respectively. Table 2 tabulates these results. Again in these tests the backed out parameters are essentially identical for practical purposes (4.02 average percent deviation as compared to 7.6 percent). These results indicate that in some cases the engineer may specify wide ranges on the reservoir description parameters if the performance data are reliable.

In reservoir simulation problems, the reservoir zonation for a heterogeneous case is often difficult to choose. We studied the effect of a false zonation. For this third case the zonation shown in Fig. 1 was used to calculate the performance data (d_i). The simulator was then run 30 times using the false zonation shown in Fig. 2. We imposed a 250-percent parameter variation on all zone permeabilities. The

TABLE 1 — VARIED SAMPLE SIZE
(Range Variation of 100 to 150 Percent)

Description Parameters	True Value	Backed Out Parameters			
		$N = 30$	$N = 10$	$N = 10$	$N = 10$
K_1	0.2	0.207	0.219	0.209	0.220
K_2	1.8	1.813	1.789	1.793	1.810
K_3	1.0	0.992	0.996	0.996	0.987
K_4	0.5	0.496	0.504	0.491	0.490
K_5	0.3	0.293	0.278	0.301	0.283
K_6	2.0	2.000	2.000	1.999	2.000
Avg. Percent Deviation		1.5	2.3	1.2	3.2

TABLE 2 — VARIED PARAMETER RANGES
(All With Error Set Size $N = 30$)

Description Parameters	True Value	Range Variations Backed Out Parameters	
		250 Percent	1,000 Percent
K_1	0.2	0.178	0.184
K_2	1.8	1.787	1.690
K_3	1.0	1.050	0.925
K_4	0.5	0.501	0.454
K_5	0.3	0.321	0.267
K_6	2.0	2.004	1.926
Average Percent Deviation		4.02	7.59

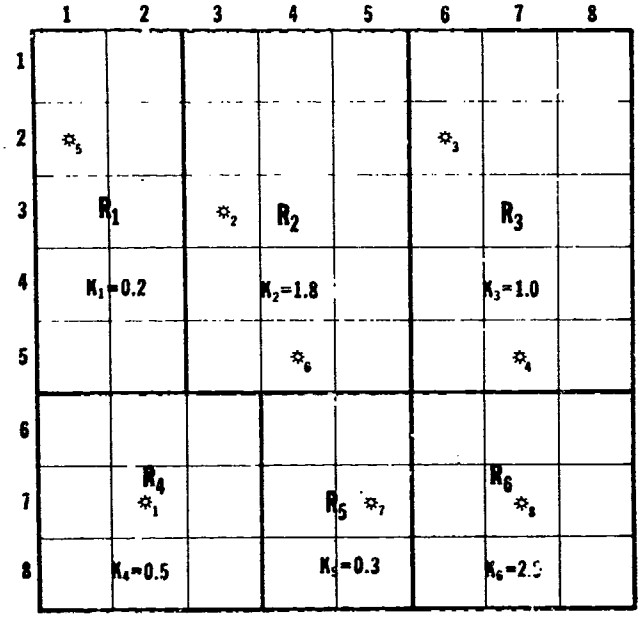

FIG. 1 — RESERVOIR ZONATION I.

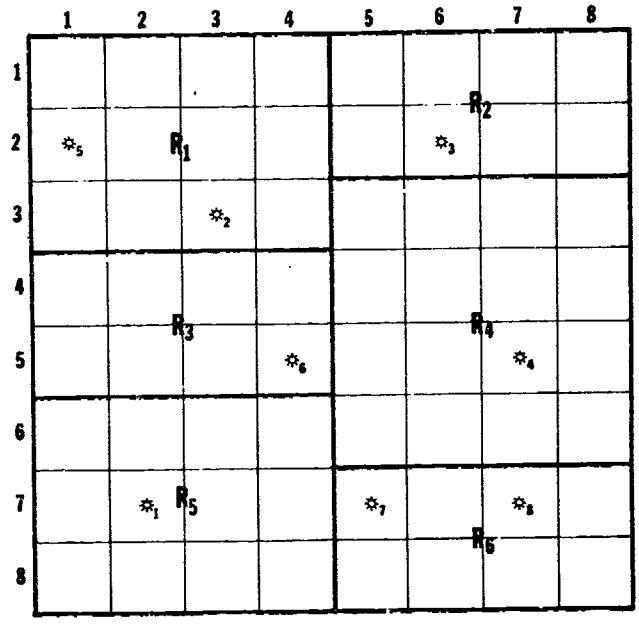

FIG. 2 — RESERVOIR ZONATION II.

reservoir description parameters were then backed out using the LSLP program. Then we made prediction runs using these values from the false zonation and those from the true zonation. The prediction period is five times the match period. Figs. 3 and 4 are plots of the well block pressure as a function of time (Wells 3 and 5). In the worst case (Well 5) at the end of 1,830 days, the false zonation differs from the actual zonation by 4.7 percent. This again is entirely within engineering accuracy for such an extended prediction period.

In order to test the hypothesis that a homogeneous reservoir description might yield satisfactory results, we made a homogeneous prediction run. This simulation run covers the same prediction period and uses a weighted average permeability. Figs. 3 and 4 also show these results. For Well 5, the percent deviation of well block pressure at 1,830 days in this case is 52.1 percent. Thus, for this system the simplifying assumption of a homogeneous reservoir yields predictions considerably in error.

Finally, we treated the problem of backing out a reservoir parameter for a zone with no well or match point. The well located in Zone 5 was removed from the simulator and 30 runs were made. Using the LSLP program, we backed out reservoir parameters for the six zones. Table 3 shows these results. The backed out parameters are essentially identical when compared to the run with a well located in each zone. We conclude that in some cases we may be able to determine good parameter values for zones in which no match points or wells exist.

TABLE 3 — NO MATCH POINT IN ZONE 5
(Range Variation of 100 to 150 Percent)

Description Parameters	True Value	Backed Out Parameters: Well in Each Zone	Backed Out Parameters: No Well in Zone 5
K_1	0.2	0.207	0.207
K_2	1.8	1.813	1.821
K_3	1.0	0.992	0.994
K_4	0.5	0.496	0.496
K_5	0.3	0.293	0.287
K_6	2.0	2.000	2.000
Average Percent Deviation		1.5	1.7

SINGLE-PHASE OIL FLOW

The example reservoir shown in Fig. 5 represents an undersaturated oil field with a closed exterior boundary. The reservoir thickness is a uniform 120 feet, and over-all reservoir dimensions are 9,000 × 6,000 feet. Each of the three producing wells flowed at a constant rate of 535 res. bbl/day; locations of eight observation wells are noted on Fig. 5. A noniterative ADI, numerical model using a 15 × 10 grid that simulates single-phase flow was the simulator for this example.

The 12 reservoir description parameters for this example are the permeability and porosity of each of the six zones shown on Fig. 5. The performance data utilized in the matching technique consist of

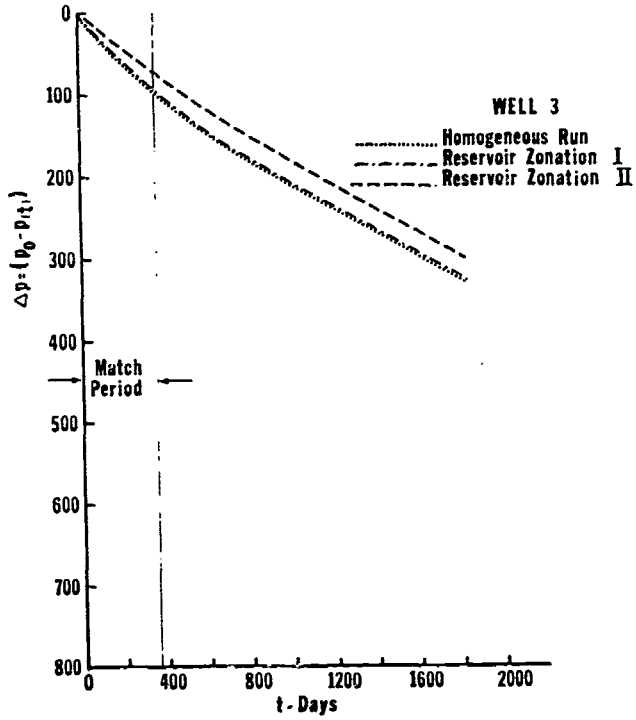

FIG. 3 — PREDICTION RUNS.

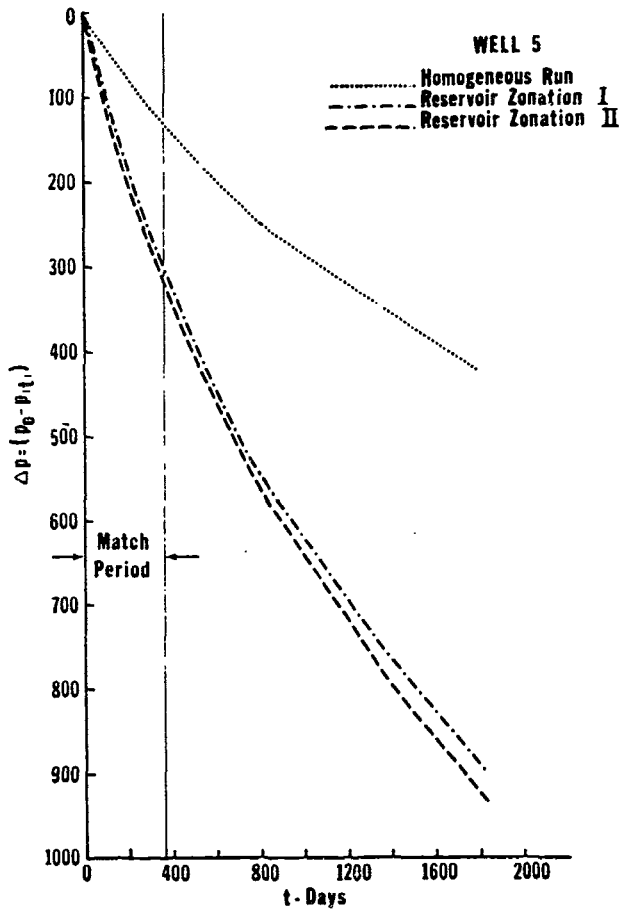

FIG. 4 — PREDICTION RUNS.

the 16 pressures corresponding to the 50-day and 300-day pressure responses at each of the eight observation wells.

For this example reservoir we used the LSLP program to back out the reservoir description from (1) 30 error sets (ϵ_i) (i.e., 30 simulator runs) calculated using a 100-percent variation on the permeabilities and a 50-percent variation on the porosities; (2) 20 error sets calculated using the same parameter variations; and (3) 30 error sets using a 200-percent variation on the permeabilities and 50 percent on the porosities. In each case a random number generator determined the true reservoir description within the prescribed range, and one simulator run determined the pressure responses (performance data) at each of the eight observation wells.

Table 4 lists the correct reservoir description and the description backed out using 30 runs and a 100-percent variation on the permeabilities. Table 5 gives calculated pressure responses vs time at observation Wells 3 and 6 throughout the 300-day match period and on through the 860-day prediction period. The first column is correct (data or observed) pressure response; the second is the response calculated using the backed out description; the third and fourth columns are calculated responses using a homogeneous reservoir of 20 md and 100 md, respectively. Calculated pressures (using the backed out description) for all eight observation wells were within 1 psi of the observed (correct) pressure responses.

Table 4 also gives the reservoir description determined in the 100-percent variation case by using only 20 of the 30 runs. The average error of 7.6 percent in the description parameters is significantly larger than the 2.06-percent average error in the former case where 30 runs were used. However, Table 6 shows that observation well pressures calculated using the erroneous description are nevertheless within 2 psi of the correct pressures (i.e., pressures calculated using the true reservoir description) throughout the prediction period of 860 days.

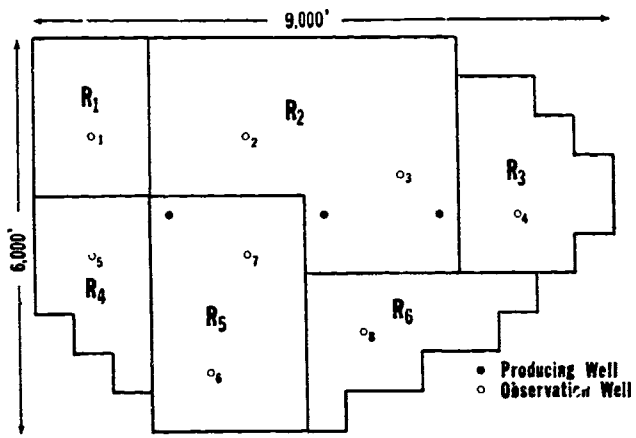

FIG. 5 — RESERVOIR DESCRIPTION, SINGLE-PHASE OIL FLOW.

TABLE 4 — RESERVOIR DESCRIPTION PARAMETERS FOR SINGLE-PHASE OIL FLOW — 100 PERCENT VARIATION CASE

Description Parameter	True Value (md)	Backed Out Using 30 Runs	Backed Out Using 20 Runs
k_1	73	69.84	82.78
ϕ_1	0.1148	0.1122	0.1270
k_2	174	169.4	194.3
ϕ_2	0.2104	0.2123	0.2171
k_3	16	15.91	16.47
ϕ_3	0.1145	0.1161	0.1171
k_4	18.6	18.55	18.36
ϕ_4	0.1045	0.1051	0.1004
k_5	33.6	35.87	36.04
ϕ_5	0.123	0.1238	0.12
k_6	291	300	215.6
ϕ_6	0.1864	0.1841	0.1746
Average Error (Percent)		2.06	7.6

TABLE 5 — PREDICTED AND CORRECT PRESSURE RESPONSES — CASE OF 100-PERCENT PARAMETER VARIATION, 30 RUNS

Time Days	Correct Pressure (psi)	Calculated Using Backed Out Description (psi)	20 md (psi)	100 md (psi)
		Well 3 (Zone 2)		
30	10.3	10.2	14.0	11.5
60	17.7	17.6	28.5	19.3
100	27.1	27.0	42.7	29.1
200	49.8	49.6	70.9	53.0
300	72.2	72.0	96.5	76.4
500	117.0	116.6	145.7	123.1
620	143.8	143.3	174.5	151.2
740	170.7	170.1	203.2	179.2
860	197.5	196.8	231.7	207.2
		Well 6 (Zone 5)		
30	1.6	1.7	0.3	3.2
60	6.4	6.6	2.3	9.3
100	14.3	14.7	7.4	18.3
200	36.1	36.6	25.5	41.2
300	58.4	58.8	46.6	64.5
500	103.2	103.5	91.3	111.1
620	130.0	130.2	118.6	139.1
740	156.9	157.0	146.1	167.1
860	183.7	183.7	173.8	195.2

TABLE 6 — PREDICTED AND CORRECT PRESSURE RESPONSES — CASE OF 100-PERCENT PARAMETER VARIATION, 20 RUNS

Time Days	Well 4 (Zone 3) Correct Pressure (psi)	Well 4 (Zone 3) Calculated Using Backed Out Description (psi)	Well 8 (Zone 6) Correct Pressure (psi)	Well 8 (Zone 6) Calculated Using Backed Out Description (psi)
30	3.6	3.5	6.5	6.4
60	8.7	8.5	13.9	13.7
100	16.2	16.0	23.2	23.0
200	37.0	36.6	46.0	45.4
300	59.0	58.2	68.3	67.6
500	103.4	102.4	113.0	112.0
620	130.2	129.0	140.0	138.6
740	157.0	155.6	166.7	165.2
860	184.0	182.2	193.5	191.8

Table 7 shows the correct reservoir description, the description backed out using 30 runs and a 200-percent variation on the permeabilities, and the percent error in the latter. The backed out description parameters differ from the true values by as much as 28.6 percent; the average error is 11.9 percent. However, the pressures calculated using this description agree within a few tenths of a psi with the correct pressures throughout the 860-day period, as shown in Table 8. This equivalence of two significantly different descriptions indicates a high degree of correlation between certain of the 12 description parameters, a point discussed below.

TWO-PHASE (GAS-WATER) CASE

We constructed the reservoir shown in Fig. 6 to test the method on a two-phase flow system. The reservoir dips in the x and y directions away from the upper left-hand corner. It contains a water leg and is open to an aquifer on portions of the lower sides. The aquifer is represented in the simulator by a "pot" aquifer that delivers a fixed number of barrels of water into the peripheral blocks per pound of pressure drop. The reservoir is divided into six permeability zones with Region 1 being designated as a partially sealing fault. Reservoir dimensions are 8,000 × 3,200 ft, with a uniform thickness of 30 ft and a uniform porosity of 10 percent. Initial gas pressure at the Gas-Water Case is 1,200 psia. There are four gas production wells and one observation well. Gas production rates correspond to a 20-year depletion. An iterative two-phase, compressible ADI model[8] performed the simulations using an 8 × 8 areal grid system.

The seven reservoir description parameters, (x_j), were defined as multipliers of the base permeability value of 5 md in each of the six regions of the reservoir (including the fault zone) and the base value of the pot aquifer strength of 350,000 B/D/psi in each of the affected blocks. The performance data set, (d_i), was defined as gas-phase pressure at the five wells measured at times of 1,170 and 1,970 days. The flow rates of the gas production wells were increased by a factor of 1.1 at 1,170 days.

The example case used a 100-percent variation of the reservoir description parameters about the true values. Thirty simulation runs were performed to generate the set of errors, (ϵ_i), as a function of the randomly generated reservoir description parameter set (x_j). These data were then processed by the LSLP program to yield the backed out parameters. Table 9 shows the results for three sample sizes, $N = 10$, 20 and 30.

Simulation runs of 3,770 days were made using the reservoir description parameter multipliers from the true case, the 30 sample–100 percent variation case and a homogeneous case. Comparative results of pressure drop from initial pressure for these runs are shown in Table 10.

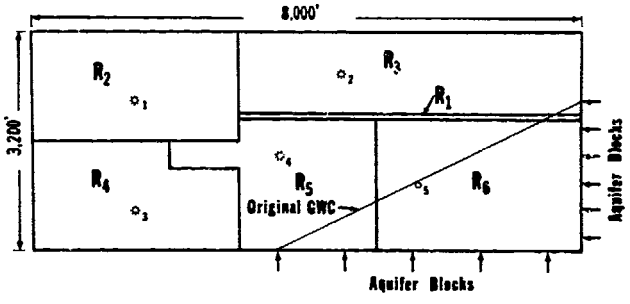

FIG. 6 — RESERVOIR SYSTEM, TWO-PHASE CASE.

TABLE 7 — RESERVOIR DESCRIPTION PARAMETERS FOR SINGLE-PHASE OIL FLOW — CASE OF 200-PERCENT PARAMETER VARIATION, 30 RUNS

Description Parameter	True Value (md)	Backed Out Value	Percent Error
k_1	78	55.67	− 28.6
ϕ_1	0.1348	0.1073	− 20.4
k_2	174	168.5	− 3.2
ϕ_2	0.1996	0.2027	+ 1.55
k_3	26.2	30	+ 14.5
ϕ_3	0.1055	0.1179	+ 11.8
k_4	58.4	54.57	− 6.5
ϕ_4	0.1030	0.1044	+ 1.36
k_5	81.6	95.89	+ 17.5
ϕ_5	0.1614	0.1756	+ 8.8
k_6	164.4	134.4	− 18.0
ϕ_6	0.2074	0.1854	− 10.6
		Average (Percent)	11.9

TABLE 8 — PREDICTED AND CORRECT PRESSURES — CASE OF 200-PERCENT PARAMETER VARIATION, 30 RUNS

	Well 1 (Zone 1)		Well 5 (Zone 4)	
Time Days	Correct Pressure (psi)	Calculated Using Backed Out Description (psi)	Correct Pressure (psi)	Calculated Using Backed Out Description (psi)
30	4.2	4.2	5.7	5.6
60	10.2	10.1	11.7	11.5
100	18.6	18.6	19.9	19.8
200	40.0	39.9	41.1	41.0
300	61.3	61.2	62.5	62.3
500	104.1	103.9	105.2	105.0
620	129.7	129.5	130.8	130.6
740	155.3	155.1	156.4	156.2
860	180.9	180.7	182.1	181.8

TABLE 9 — VARIED SAMPLE SIZE, TWO-PHASE CASE — 100-PERCENT VARIATION

Description Parameter	True Value	Backed Out Parameters		
		$N=30$	$N=20$	$N=10$
Aquifer	1.0	0.7552	0.8266	0.8178
k_1 (fault)	0.05	0.0499	0.0507	0.0484
k_2	1.3	1.269	1.261	1.299
k_3	0.8	0.7944	0.7903	0.8033
k_4	2.0	1.964	1.960	1.997
k_5	1.0	0.9926	0.9934	0.9975
k_6	1.8	1.746	1.760	1.729
Average percent deviation excluding aquifer		1.47	1.75	1.34

DISCUSSION

The reservoir description parameters (x_j) may be functional groups as well as individual properties. For example, x_1 might be e^{-k_1} or $1/k_1$ or simply k_1, where k_1 is permeability of Zone 1. The parameters (x_j) may also be composites of individual reservoir properties, such as $x_1 = k_1/\phi_1$. Ideally we select that functional group that contributes most to linearity of the $\epsilon_i(x_1, x_2, \ldots, x_j)$ surfaces. In the examples above we tried e^{-k}, $1/k$, $1/k^2$ and k. However, the Ei solution to the diffusivity equation gives, for sufficiently large time, pressure response as $q\mu/kh$ times a function of time. This suggests the use of $1/k$ and in all cases the best results followed from use of $1/k$. In the Single-Phase Oil-Flow Case we used porosity parameters as ϕ and $1/\phi$, with the former giving much better results.

Our limited experience to date in applying this method to actual field problems shows that the backed out description (x_j) usually contains several parameters at their upper or lower limits. In these cases a second "pass" or application of the method may be performed with shifted limits on those parameters. Repeated passes may be made with the method until all parameters lie within their specified ranges. One field case showed that the match of field performance continued to improve as certain parameters increased or decreased well beyond reasonable values. We prefer to use one pass with parameter limits representing a reasonable range and accept resulting extremal parameter values.

The N simulator runs and the subsequent linear programming calculation constitute the computing requirements of the proposed LSLP method. A number of runs N equal to or even less than twice the number of reservoir parameters gave acceptable results in the example problems treated above. Note that, if the surfaces ϵ_i were truly linear then, we would require exactly $J+1$ simulator runs to determine the surface $\epsilon_i = \sum_{j=0}^{J} a_{ij} x_j$, where J is the number of parameters. The computing time for these N simulator runs generally appreciably exceeds the time required by the linear programming calculation. For example, the set of $N=30$ runs in the Single-Phase Gas-Flow case required 300 seconds of Burroughs 5500 time and the LSLP calculation used less than 10 seconds. Each of the 30 runs of the Single-Phase Oil-Flow type required 3 seconds of UNIVAC 1108 time for a total of 90 seconds. The least squares-linear programming calculation required 13 seconds.

Jahns[2] reported that his regression technique required $2J$ simulator runs for each iteration; generally, 5 iterations were required or performed. This is a total of $10J$ simulator runs. In addition, the method involved several steps. A reported application involved nine parameters ($J=9$) and required 200 simulator runs, which corresponds to about $N = 22J$. A liberal estimate of required simulator runs for the LSLP method is $6J$, assuming $2J$ runs per pass and 3 passes. In most cases, considerably fewer runs should be required.

A simple definition of "correlation" is the ability of an error in one description variable (say, ϕ_4) to compensate for an error in another (say, k_4). Thus equally good matches of field performance data may be achieved by any values (within some limits) of k_4 and ϕ_4 provided only that (say) their ratio k_4/ϕ_4 have some unique value. We are currently incorporating in our method the calculation of correlation coefficients, as discussed by Jahns[2] in order to automatically detect this correlation. A partially satisfactory remedy for correlation is the fixing of one of the parameters (say, ϕ_4) and determination of the other.

The Single-Phase Oil-Flow case offers a significant degree of correlation between the permeability and porosity parameters. A semisteady-state flow regime existed during a significant portion of the 300-day match period. In this flow regime the ratio k/ϕ for each region plays a predominant role in the pressure distribution, especially for cases where the permeability level is sufficiently high that the pressure distribution in the field is relatively flat. Tables 7 and 8 indicate the presence of this correlation. The two descriptions in Table 7 are significantly different, but generally obey the rule that a higher permeability in a region is accompanied by a higher porosity. Conversely, a lower permeability is accompanied by a lower porosity. As shown in Table 8, the two different reservoir descriptions give equally good matches of reservoir performance.

The Single-Phase Gas-Flow cases offered little or no correlation. In this case we found unique descriptions backed using large ranges of uncertainty and few runs.

CONCLUSIONS

1. A new method has been presented for

TABLE 10 — TWO-PHASE CASE — PERFORMANCE COMPARISON — PRESSURE DROP AT WELLS

Well Number	True Case (psi)	100 Percent $N=30$ (psi)	Homogeneous (psi)
	t = 1,970 Days		
1	393.4	394.4	379.4
2	376.4	377.0	335.9
3	390.9	391.8	390.6
4	348.8	349.0	338.9
5	86.9	89.2	99.6
	t = 3,770 Days		
1	828.8	825.5	796.9
2	773.1	773.8	762.4
3	830.6	832.4	831.3
4	815.4	815.0	786.8
5	86.9	87.7	104.7

(pressure drop) $i, t = 1{,}200 - p_{i,t}$. $i = 1, 2, \ldots$ Number of wells, t = time

Homogeneous (x_j) = 1.2 (pot aquifer), 0.1 (fault), 1.4 (all other regions)

determining a reservoir description from field performance data. We feel that the method is more efficient than the previously described ones. This LSLP method selects a set of reservoir description parameters (x_j) which match given reservoir performance data (d_i). The advantages of the method are: (a) generality in the sense that the description parameters (x_j) may be any physical properties which affect calculated reservoir response (d_i), (b) generality in that the method handles multiphase as well as single-phase, multi-dimensional flow problems, and (c) efficiency relative to previously reported techniques.

2. An example single-phase reservoir, with six zone permeabilities as the description parameters, showed that the LSLP method determined an accurate reservoir description when: (a) as few as 10 simulator runs were used, (b) a 1,000-percent range of uncertainty was imposed on the reservoir description parameters, and (c) when no reservoir performance data were available in one of the zones. Further, for a false reservoir zonation, the method determined a reservoir description which gave excellent results for a prediction period five times the match period.

3. In an example with a significant degree of correlation between six permeabilities and six porosity parameters, the method backed out different, equally viable reservoir descriptions depending upon the parameter variation range and the number of simulator runs used.

4. A third example indicated the competence of the LSLP method for handling two-phase flow problems.

NOMENCLATURE

(d_i) = set of performance data

(ϵ_i) = set of errors, defined as $(d_i^{\text{observed}} - d_i^{\text{calculated}})$

K_i = reservoir permeability factor for Zone i

k_{base} = base reservoir permeability, md

k_{zone} = zone permeability, md

k_i = reservoir permeability for Zone i, md

ϕ_i = reservoir porosity for zone

t_m = time of match period, days

(x_j) = set of reservoir description parameters

$x_{j\ell}$ = lower limit on reservoir description parameter x_j

x_{ju} = upper limit on reservoir description parameter x_j

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REFERENCES

1. Jacquard, P. and Jain, C.: "Permeability Distribution From Field Pressure Data", *Soc. Pet. Eng. J.* (Dec., 1965) 281-294.
2. Jahns, Hans O.: "A Rapid Method for Obtaining a Two-Dimensional Reservoir Description From Well Response Data", *Soc. Pet. Eng. J.* (Dec., 1966) 315-327.
3. Kruger, W. D.: "Determining Areal Permeability Distribution by Calculations", *J. Pet. Tech.* (June, 1961) 691-696.
4. Nelson, R. W.: "In-Place Determination of Permeability Distribution for Heterogeneous Porous Media Through Analysis of Energy Dissipation", *Soc. Pet. Eng. J.* (March, 1968) 33-42.
5. Garvin, W. W.: *Introduction to Linear Programming*, McGraw-Hill Book Co., Inc., New York (1960).
6. Coats, K. H., Rapoport, L. A., McCord, J. R. and Drews, W. P.: "Determination of Aquifer Influence Functions From Field Data", *J. Pet. Tech.* (Dec., 1964) 1417-1424.
7. Henderson, J. H., Dempsey, J. R. and Tyler, J. C.: "Use of Numerical Reservoir Models in the Development and Operation of Gas Storage Reservoirs", *J. Pet. Tech.* (Nov., 1968) 1239-1246.
8. Knapp, R. M., Henderson, J. H., Dempsey, J. R. and Coats, K. H.: "Calculation of Gas Recovery Upon Ultimate Depletion of Aquifer Storage", *J. Pet. Tech.* (Oct., 1968) 1129-1132.
9. Reed, L. N., Henderson, J. H., Dempsey, J. R. and Coats, K. H.: "A Computerized Simulation of a Heterogeneous Aquifer Storage Formation", paper SPE 2236 presented at 43rd Annual SPE Fall Meeting, Houston, Tex. (Sept. 29-Oct. 2, 1968).

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