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An Approach to Locating New Wells in Heterogeneous, Gas Producing Fields

Keith H. Coats, SPE-AIME, The U. of Texas at Austin

Introduction

The decline of average field pressure during the producing life of a gas field causes a corresponding decline in well deliverability. Additional wells are therefore drilled during the producing life to maintain the desired or contractual total field producing rate. Maximizing the economic return from the producing operation requires minimizing the number of wells drilled. Even if a fixed total number of wells are to be drilled (ultimately), the usual discounted cash flow or present-value analysis dictates that the wells be drilled as late as possible in project life.

This paper treats the problem of determining an optimum drilling schedule for remaining field development, starting from an initial (present) time corresponding to an arbitrary degree of depletion and arbitrary number and locations of existing wells. This optimum schedule consists of specified well locations to drill and the time at which each is to be drilled.

Conventionally, additional well requirements for maintenance of field productivity are determined from back-pressure curves that relate well deliverability to the difference between average field pressure and flowing wellbore pressures. The assumptions involved in this approach render it satisfactory for only a homogeneous field with regularly spaced wells having equal producing rates.

The method described here minimizes new well requirements at each successive stage of depletion by selecting optimal locations for additional wells. The method accounts for the effects on well deliverability of reservoir heterogeneity, irregular spacing, and well interference.

Following sections state the problem more fully, describe the conventional and proposed methods of solution, and present example applications. The Appendix describes in detail the mathematics involved in the proposed technique for determining optimal well locations.

The Problem

The development drilling program necessary to meet contractual producing rates from a gas field is characterized by a plot, such as Fig. 1, of cumulative wells drilled vs time or vs remaining gas reserves. Time and gas in place are related through the specified field producing rate. An obvious economic incentive exists to keep this curve as "low" as possible. Thus Curve 2 is preferable to Curve 1 even though the total number of wells drilled over project life is the same in both cases. Curve 3 is preferable to both of the other curves. A lower limit on the number of wells drilled is imposed by the requirement of a specified total field deliverability, which may vary with time. The problem is to meet this required field productivity with as few wells as possible. The degrees of freedom or decision variables in attacking this problem are the locations of new wells. For example, let the set of 100 locations $\{1, 2, 3, \ldots, 100\} \equiv \{i\}$ represent possible

A two-dimensional numerical calculation of semi-steady-state flow in a gas field can be used to determine the best possible drilling schedule for developing the remainder of the field. The technique can be used also to estimate gas field performance under alternate spacings, to examine lease-line drainage, and to determine the degree of drainage or depletion of different portions of a field.

(admissible) drilling sites. Each location, 1, 2, . . . , 100 is a certain (x, y) position in the reservoir. A development drilling program consists of an ordered subset of $\{i\}$ and the times $\{t_n\}$ at which each member of the subset is to be drilled. A constraint on this subset and corresponding times of drilling is that specified field producing rate must be met at all times. An example drilling program is the set of wells {10, 3, 91, 70, 97, 32, 41} drilled at times $t_1, t_2, \ldots, t_7$ where $t_7 > t_6 > \ldots > t_2 > t_1$. The optimal policy or drilling program is the ordered subset i_n that results in a curve (Fig. 1) of cumulative wells drilled vs time representing minimum present-value cost of all additional wells drilled. If the total cost per well drilled is c, then a well drilled at time t_n represents a present-value cost of ce^{-rt_n}, where r is the discount rate (e.g., 15 percent). The objective function to be minimized is therefore

$$c \sum_{n=1}^{N} e^{-rt_n} \quad , \quad \ldots \ldots \ldots \quad (1)$$

where N is the total number of additional wells drilled to time of abandonment t_a.

A further expanded statement of the problem of minimizing the Function 1 leads to a dynamic programming formulation[1] where the drilling of each additional well constitutes one stage. Alternative definitions of a stage (e.g., one stage equals a time increment Δt or, equivalently, a decrement ΔG of gas in place) also lead to dynamic programming problems. These formulations are omitted here because they do not lead to a practical solution process. That is, the dependence of the incremental deliverability due to adding a certain well upon the locations of all existing wells results in prohibitive arithmetic labor using dynamic programming methods. This is not to deny the existence of a feasible dynamic programming formulation, but only to say that I have not discovered it.

The problem statement utilized in this work is as follows. Given, for a producing gas reservoir, the geometry, heterogeneity (kh and ϕh distributions), present gas in place, existing well locations and the required total field producing rate schedule, and given, also, a set of admissible new well locations and a minimum allowable flowing bottom-hole pressure, determine the order of drilling any subset of these locations so as to keep the curve of Fig. 1 as low as possible when traversed from left to right. More specifically, the object is to add wells one at a time in such a way that each well added is that one of the remaining undrilled sites that contributes the most to field deliverability. Equivalently stated, the well added should be the one that results in the specified field producing rate with the lowest gas in place. This statement is the "optimality principle" used in this work. The "optimal" drilling programs discussed below are those calculated using this principle.

This principle of choosing successive well locations so as to minimize the number of wells drilled at each successive stage of depletion may or may not be equivalent to minimization of the cost Function 1. The question is which of two curves, Type 1 or Type 2 on Fig. 2, might lead to minimization of Function 1. Curve 1 on Fig. 2 corresponds to that obtained by adding each well in accordance with the principle in the above paragraph; Curve 2 violates the principle. Both curves satisfy the required producing rate schedule. However, Curve 2 is intended to represent a case of smaller present-value cost than Curve 1. It seems intuitively plausible to me that no curve such as Curve 2 exists. Example calculations have never shown a curve of Type 2 with smaller present-value cost. However, equivalence of the criterion actually applied to cost function minimization has not been established.

The problem just described could be expanded somewhat by allowing flowing bottom-hole pressures below line pressure, thus introducing compression cost considerations. The method described below would then be modified to account for the balance between additional wells and additional compression during late stages of depletion in order to meet the required field producing rate.

Conventional Approach to Determining New Well Requirements

The back-pressure curve[2,3] relates well deliverability to the difference between average field pressure and flowing wellbore pressure. Estimation of this curve for a proposed well requires knowledge of the shape, size and permeability-thickness product of the drainage area of the well. The shape and size in turn depend upon the reservoir heterogencity and locations and producing rates of other wells. Even if the back-pressure curve were reliably known, the deliverability of the well for a given flowing wellbore pressure depends upon the pressure level in the drainage area of the well. This local pressure level is dependent upon reservoir heterogeneity and upon locations and producing rates of other wells. The net result of all this is the extreme difficulty if not impossibility of determining, by the conventional, back-pressure curve approach, what the net increase in field productivity will be if a new well is added in a certain location.

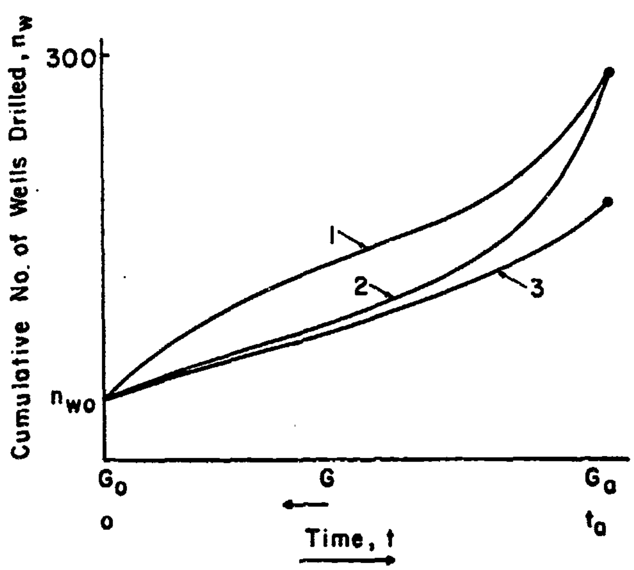

Fig. 1—Cumulative wells vs time for a producing gas field.

· The use of back-pressure curves to calculate well requirements is justified for homogeneous fields developed on regular spacing. At any specified stage of depletion, the average field pressure can be calculated from the remaining gas-in-place figure. Use of this average pressure in the back-pressure equation then gives deliverability (Mcf/D) per well, and dividing this into the total required field rate yields the number of wells required. An easily performed trial and error is generally required since a coefficient in the back-pressure equation is a function of the well spacing and the spacing is not known until the required number of wells is determined.

Proposed Method for Determining the Optimal Drilling Program

The basic element of the method described here is a calculation of the semi-steady-state, two-dimensional pressure distribution corresponding to a specified number of flowing wells and their locations and a given total required field producing rate. The calculation is a numerical solution of the partial differential equation describing semi-steady-state gas flow in a reservoir. It accounts for effects of reservoir geometry and heterogeneity (i.e., kh and ϕh distributions) and yields pressure distributions that reflect any pattern (regular or irregular) of well locations, the unequal producing rates of the wells and the well interference effects. This two-dimensional calculation is described in detail in the Appendix.

For brevity, this two-dimensional pressure distribution calculation will be referred to hereafter as the TDP calculation.

The producing rate of a well is related to the difference between block pressure (potential) and flowing wellbore pressure by Eq. 3, which is discussed later. The coefficient in that equation reflects the kh and size of the square grid block in which the well is located.

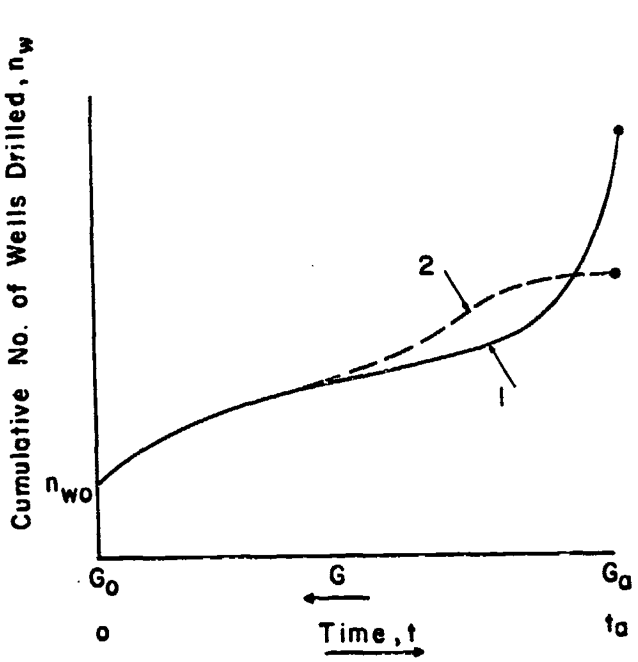

Fig. 2—Cumulative wells vs time for a producing gas field.

In addition to reservoir geometry and heterogeneity and the number of wells and their locations, certain additional variables must be specified for the TDP calculations: total field producing rates (Mcf/D) and the bottom-hole pressure against which all the wells are flowing. The pressure distribution yielded by the calculation allows determination of gas in place in the reservoir.

In short, the TDP calculation determines the gas in place (or equivalently, the average field pressure level) necessary for any proposed combination of flowing wells to meet the required total field rate. The fewer and "poorer" wells we allow on stream, the higher will be this determined G.

Let us say that a set of 100 admissible new well sites $\{i\}$ is specified. The sequence of calculations to determine the optimum drilling program is as follows:

1. For the presently existing wells in the field, perform the TDP calculation to determine G. This point (n_{wo}, G_o) is the first point on the curve of Fig. 1.

2-a. Perform the TDP calculation 100 times, each time with the n_{wo} existing wells plus 1 of the admissible set of 100.

2-b. "Drill" that well that in Step 2-a resulted in the lowest value of G.

3-a. Perform the TDP calculation 99 times, each time with the existing wells plus the well drilled in Step 2-b plus 1 of the admissible remaining 99 wells.

3-b. "Drill" that well which in Step 3-a resulted in the lowest value of G.

4. Repeat Steps 2 and 3, for the remaining 98, 97, . . . wells, each time "drilling" the best well found. Quit when the wells are all drilled or when the minimum G of the last step is less than G_a, whichever occurs first.

The result of these calculations is a sequence of points (n_w, G) on the Fig. 1 curve. More specifically, the result is an ordered set of well sites to be drilled. The ordered set is that unique set that is optimum as defined in the problem statement above. That is, at each successive stage of depletion, optimal locations are found so that as few wells are drilled at that stage as are necessary to maintain the required field producing rate. The times of drilling the wells are obtained directly from the equivalent time scale on the plot of $n_w - G$ (see Fig. 1).

Limitations and Other Capabilities of the Proposed Method

The assumption of semi-steady-state flow[4,5] in the reservoir is reasonable for most gas field producing operations that require 20 to 30 years to achieve depletion. A physical definition of semi-steady-state flow is simply the nearly uniform field pressure decline rate throughout the entire field. This nearly uniform pressure decline rate is a phenomenon that occurs after a sufficient elapse of time following a rate change at any well. This assumption would not be valid in general for gas storage reservoirs that are essentially depleted within 3 to 4 months. Henderson *et al.*[6] treat this latter case, showing that the strong transient character of the flow can be taken advantage of not only in locating wells optimally but also in staging their flow (i.e., turning them on in an optimal order during the withdrawal season to meet peak, end-of-season withdrawals).

A quantitative measure of the semi-steady-state assumption is offered by the equation

$$t^* = \frac{\mu\phi c r_e^2}{4(0.00633)k} \text{ days} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

given by Craft and Hawkins[4] and discussed by Katz and Coats.[8] If a well at the center of the reservoir is turned on at zero time, then flow will be transient for $t < t^*$, and it will reach semi-steady state for $t > t^*$. As an example,

$\mu = 0.015$ cp,

$\phi = 0.15$,

$c = 1/1500$ = approximate value of compressibility in a gas reservoir at 1,500 psia,

$r_e = 2.5$ miles, and

$k = 75$ md,

give in Eq. 2

$$t^* = \frac{0.015\ (0.15)\ (1/1500)\ (2.5 \times 5280)^2}{4\ (0.00633)\ (75)}$$
$$= 138 \text{ days.}$$

Thus, after a well is added, the field will assume a new semi-steady-state pressure distribution in about 4 months. Note that the r_e used here should really be the radius of the entire field, not just the radius of the drainage area of the well, since the assumption here is one of semi-steady-state flow throughout the entire field. From a practical point of view, the time given by Eq. 2 using field radius can be relaxed by a factor of 5 (possibly more in some cases) and the semi-steady-state flow calculations still give a good estimate of field and well deliverability (comparisons between transient and steady-state calculations have shown this).

For this same example, the assumption of semi-steady-state flow would be rather poor for a storage field since the 138 days equals or exceeds the entire withdrawal season. Indeed, as Henderson *et al.* point out, transient effects over time periods of even a few days can be extremely important in meeting the storage field peak withdrawal requirements.

The two-dimensional pressure calculation described in the Appendix has several uses apart from its role in determining an optimal, ordered set of drilling sites.

1. At any point of field depletion, the calculation can be performed with various kh and ϕh distributions in an attempt to match the observed deliverabilities of producing wells. In this sense, the calculation is an aid in reservoir description.

2. The calculation can be used to estimate field performance under alternate spacing or different well locations. The calculation of pressure distribution, gas in place, and individual well deliverabilities for any given set of wells requires only about one second (UNIVAC 1108) of computing time. Thus alternate types of well patterns can be evaluated in light of required field producing rate at any stage of depletion.

3. Since the two-dimensional calculation yields the pressure distribution throughout the field, the results give the degree of depletion or drainage of each grid block or of any areas of the field. Questions of lease-line drainage can be examined in detail, in general with reasonably large grid spacing.

4. The calculation can be used to determine the shape factor C_A for a single well producing in a drainage area of any shape whatever (i.e., not necessarily a rectangle). This shape factor, discussed by several authors,[7,8] relates the semi-steady-state well producing rate to block-average pressure minus flowing well-bore pressure.

5. This method for estimating optimal drilling programs or gas field performance under given well patterns can be applied to oil fields, provided the assumption of single-phase flow is valid for practical purposes.

Example Field Calculations

Fig. 3 shows the geometry, kh distribution and the 15 existing well locations of producing gas field A. The uniform ϕh product is 2. The kh product ranges from a high of 400 md-ft in the northwest portion to a low of 35 md-ft in the southeast portion of the field, and most of the existing wells are located in the permeable, northwest part of the field. The field is 6.7 miles long and 4.3 miles wide, discovery pressure is 2,500 psia and initial gas in place is 250.8 Bcf. The required total field producing rate is specified as 30,000 Mcf/D. The minimum bottom-hole flowing pressure, determined by pipeline pressure, is specified as 1,000 psia. Table 1 lists the gas compressibility factor z and the $z\mu$ product as functions of pressure.

On Fig. 3 are noted 57 additional well sites. The wells are numbered so that the order 1, 2, . . . , 57 represents essentially a progression from high kh to low kh. Application of the TDP calculation shows that the 15 existing wells will produce the required 30,000 Mcf/D at a gas-in-place (G) level of 190 Bcf. Simply adding the 57 wells in order (1 to 57) results in the calculated n_w vs G curve (Curve 1) of Fig. 4. Adding the wells in reverse order (57 to 1) results in Curve 2 of Fig. 4.

Curves 1 and 2 of Fig. 4 show that addition of wells in order of increasing kh is preferable to the reverse. This is primarily because the concentration of existing wells in the highly permeable, northwest

TABLE 1—COMPRESSIBILITY FACTOR z AND $z\mu$ PRODUCT VS PRESSURE

p (psia)	$z\mu$ (cp)	z
547.5	.01175	.95
843.0	.01184	.9182
1138.5	.01207	.8925
1434.1	.01243	.8727
1729.6	.01291	.8583
2025.2	.01351	.8492
2320.7	.01421	.8448
2616.3	.01503	.8452

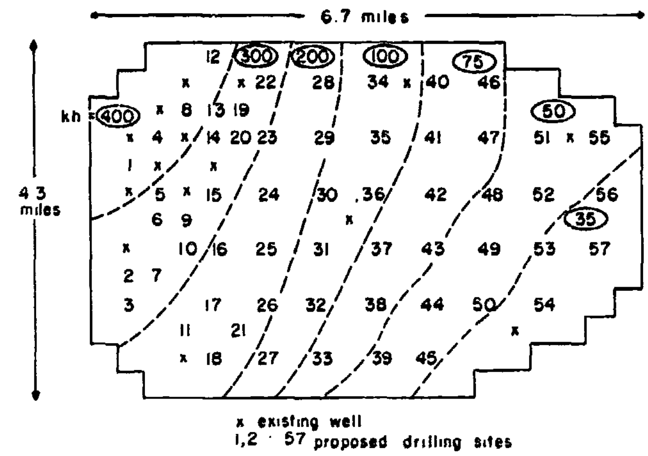

Fig. 3—Description and well sites of producing gas field A.

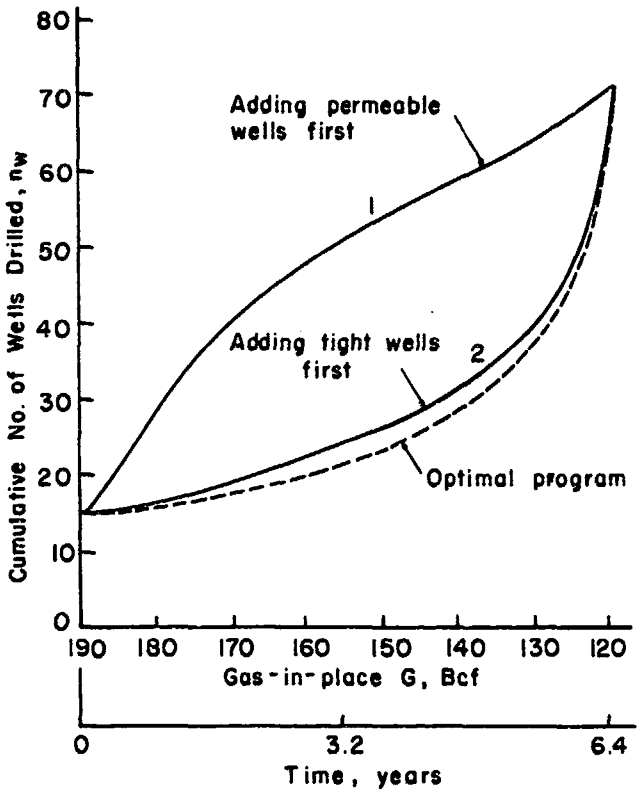

Fig. 4—Cumulative wells drilled vs time for gas field A.

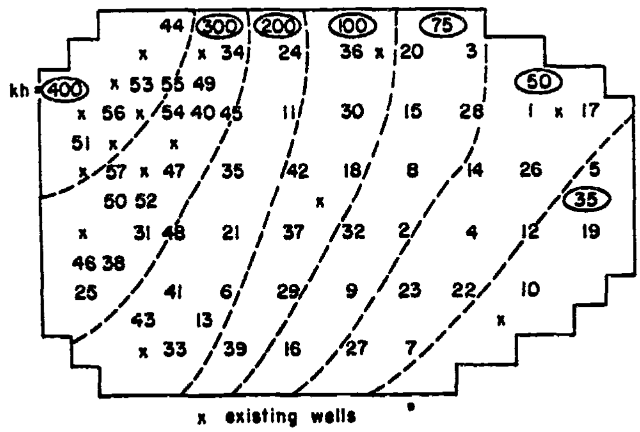

Fig. 5—Optimum order for drilling wells in Field A.

part of the field has lowered the pressure level in that area, which causes low deliverability of additional wells located there.

Use of the sequence of calculations described above yields the optimal drilling program, also shown on Fig. 4. The present-value drilling costs of Curves 1, 2, and the Optimal Curve on Fig. 4 are $1,935,000, $1,357,000 and $1,297,000, respectively, using a well cost of $50,000 and a discount rate of 15 percent. Fig. 5 shows the optimal order in which the sites should be drilled.

A question frequently asked is whether a fixed number of wells in a heterogeneous field should be concentrated in the high or in the low *kh* parts of the field. Two cases were run for the field described in Fig. 3, the first with 27 wells spread over the field but concentrated in the high permeability portion, the second with 27 wells concentrated in the low *kh* portion. In the first case, use of the TDP calculation yielded a field producing rate of 30,000 Mcf/D with an average pressure level corresponding to a gas in place of 153 Bcf. In the second case the calculation yielded the same rate with a *G* of only 141 Bcf. This indicates that if ϕh is approximately constant (and therefore the *kh* distribution is due primarily to variation in *k*, not in *h*), then well spacing should be less (i.e., higher concentration of wells) in the lower *kh* than in the higher *kh* portions of the field.

The same calculation just described was repeated with a variable of ϕh distribution. The ϕh of each region on Fig. 3 was set equal to 0.01 times the *kh* value. In this case, the field has uniform *k* and ϕ but has thickness varying markedly from northwest to southeast. The same 27 wells concentrated in the high *kh* portion produce 30,000 Mcf/D with a *G* value of 93 Bcf (this 93 should not be compared with the 153 and 141 figures above since initial gas in place is only 188.5 Bcf in this variable ϕh case). The 27 wells concentrated in the tight portion of the field produce 30,000 Mcf/D at a *G* value of 108.7 Bcf. This result is the opposite of that obtained for the homogeneous ϕh case above. Thus the answer to the question of spacing as a function of field area depends strongly upon the ϕh as well as the *kh* distributions. In any specific case this question can be answered nearly rigorously by simply determining the optimal drilling program from a set of admissible additional well sites.

Fig. 6 shows the geometry, *kh* distribution and nine existing well locations of producing gas field B. The ϕh product is also variable, equalling 0.01 times the *kh* value. The field has a permeable heart of 400 md-ft, falling off gradually to 80 md-ft to the northwest and sharply to 20 md-ft to the northeast. Contrary to the previous case, the existing wells in Field B are scattered throughout the field with no preferential concentration in any permeability region. The field is 8.2 miles long and 6.4 miles wide, discovery pressure is 2,500 psia and initial gas in place is 240.6 Bcf. The required total field producing rate is 30,000 Mcf/D against a flowing bottom-hole pressure of 1,000 psia.

Noted on Fig. 6 are 28 additional well sites. The wells are numbered from left to right and represent a progression from low to high and then again to low *kh*. All well positions in this case are on regular spacing. The TDP calculation shows that the nine existing wells would produce the 30,000 Mcf/D at an average field pressure corresponding to a gas-in-place G of 178 Bcf. Adding the well sites in order (1 to 28) results in the n_w vs G curve (Curve 1) of Fig. 7. Drilling the wells in reverse order gives a curve not much different from Curve 1.

The optimal drilling program for Field B is shown by the dashed curve on Fig. 7. Present-value drilling costs for Curve 1 and the Optimal Curve on that figure are \$819,000 and \$674,000, respectively. Fig. 8 shows the order in which the wells should be drilled. This optimal order shows a significant preference for adding wells in order of decreasing kh or ϕh; i.e., drill the most permeable or high capacity (ϕh) areas first.

The optimal drilling program for Field B was calculated again using a uniform ϕh of 2. Fig. 9 shows that in this case the optimal order of adding wells does not correspond to drilling the high kh areas first and progressing to tighter areas.

The calculations described above indicate that the optimal order of drilling sites depends upon the ϕh and kh distributions, and that significantly different "policies" should be followed depending upon whether ϕh is approximately uniform or highly variable. The Field A results show the preference for early drilling of areas that are removed from an existing area of high well concentration, in spite of the fact that such remote areas may possess low permeability.

Representation of a Producing Well in a Grid Block

The relationship

$$q_{wi,j} = \frac{2\pi kh}{S + \ln\frac{r_e}{r_w} - \frac{1}{2}} \frac{T_s}{1000\, p_s} (\Phi_{i,j} - \Phi_w) \frac{\text{Mcf}}{\text{day}} \qquad (3)$$

is employed to relate the producing of $q_{wi,j}$ of a well centered in a square grid block (i, j) to the difference in block-average potential and the flowing potential Φ_w at the wellbore. The constant ½ in this equation should be ½ if flow into the grid block equals q_w, and ¾ if flow into the block is 0. The term r_e is the block dimension Δx divided by $\sqrt{\pi}$.

Eq. 3 can be modified to account for the extra pressure drop caused by turbulent flow near the wellbore. Alternatively, an experimentally determined back-pressure curve can be employed in place of Eq. 3. In either case an iterative correction of the coefficient a_{ij} in Eq. A-9 is then required.

The steady- or semi-steady-state equation (Eq. 3) is derived and discussed by several authors.[4,5,7] Brons and Miller[7] show that an appropriate shape factor can be used in this equation to represent a well that is off center in a rectangular block. As a practical matter, for the problem considered here it is preferable to overlay the field by a regular grid and consider wells at the centers of the square blocks.

An especially pertinent point regarding Eq. 3 is its ability to represent accurately a well's performance without resorting to a fine grid to reflect the sharp pressure profiles near the well. A number of calculations have been performed showing agreement between results calculated using Eq. 3 with large grid sizes and results obtained using much finer grids.

The following example calculation demonstrates that wells can be located in adjacent blocks with little loss of accuracy. That is, grids need not be constructed so as to avoid placing wells in adjacent grid

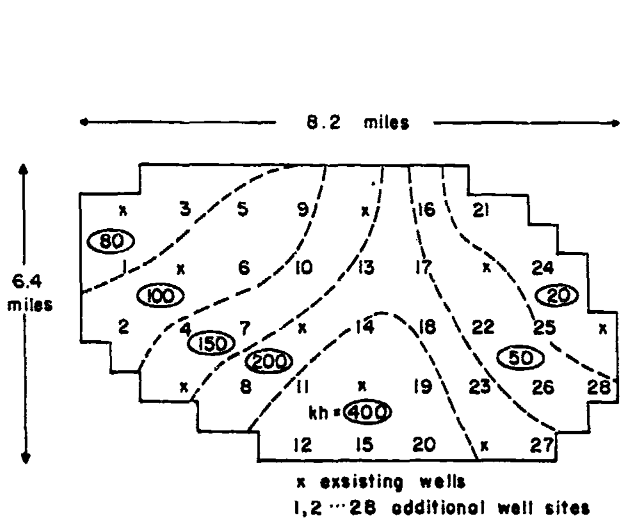

Fig. 6—Description and well sites of gas field B.

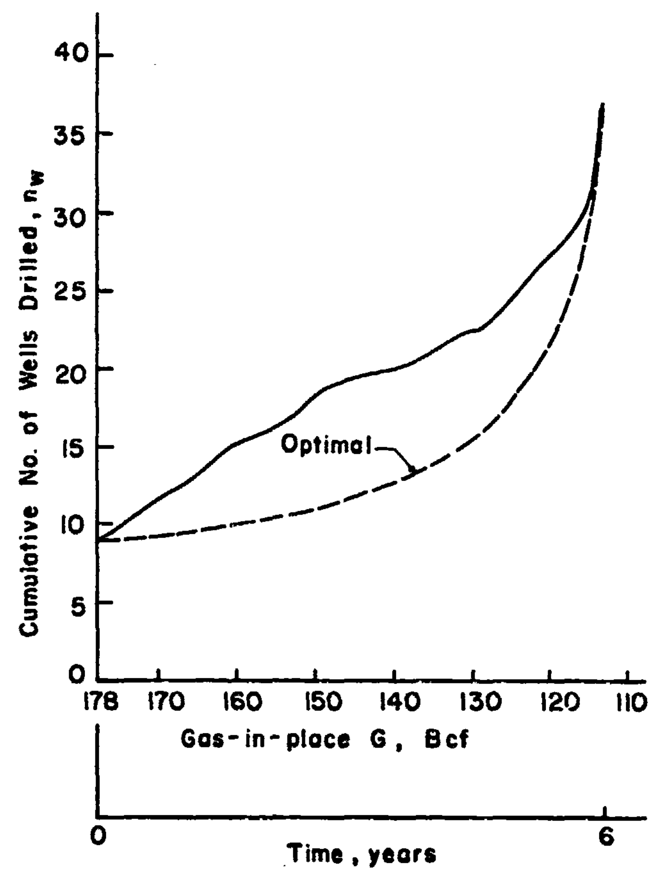

Fig. 7—Cumulative wells drilled vs time, Field B.

blocks. Fig. 10 shows a square reservoir, 18,000 ft on a side, closed on the exterior boundary, with Well No. 1 at the center and Well No. 2 to the right. The ϕh product is uniform at 2 ft and the kh product is 300 md-ft in the left-hand 12,000 ft of the reservoir and 100 md-ft in the right-hand 6,000 ft.

Two semi-steady-state, two-dimensional calculations were performed using 9 × 9 and 3 × 3 grids. Fig. 10 shows that the 9 × 9 grid results in 2 blocks between the wells while the 3 × 3 grid places the wells in adjacent blocks. In each calculation, a combined producing rate of 5,000 Mcf/D was specified and well deliverability was represented by Eq. 3. The calculated results are:

	Deliverability, Mcf/D		
	Well No. 1	Well No. 2	Gas in Place (G) (Mcf)
9 × 9 grid	3732.8	1258.1	68,869,784
3 × 3 grid	3723.1	1274.8	68,897,904

The differences are negligible for practical purposes.

Computing Accuracy and Efficiency

As previously mentioned, the primary element of the method proposed here is a two-dimensional pressure calculation over an x-y grid representing the reservoir. As discussed in the Appendix, an iterative alternating-direction technique is employed for numerical solution of the partial differential equation describing the semi-steady-state gas flow. This iterative calculation is continued until the sum of the calculated well producing rates is equal to $1 \pm \varepsilon$ times the specified total field producing rate. The computing accuracy demanded in calculations reported here corresponds to an ε of 0.002. For these calculations, iteration to a closer tolerance produced no changes in well selections and only negligible changes in individual well producing rates and the calculated levels of gas in place. Generally, the sum of individual well producing rates (calculated from the converged solution for Φ_{ij}) differed by less than 10 Mcf/D from the total specified field rate of 30,000 Mcf/D in calculations for Fields A and B. The sum of well rates often was 29999.x or some fraction of one above 30,000 Mcf/D. The absolute sum of "residuals" (see Appendix for definition) divided by the 30,000 was in general 0.5 to 1 percent.

The computing time required for the proposed method depends upon the use to which it is put and upon the number of grid blocks representing the reservoir. A single TDP calculation for a proposed set of flowing wells requires 0.28 seconds (UNIVAC 1108) for the 20 × 13 Field A grid shown on Fig. 3. Required computing time increases slightly more than proportionately to the total number of blocks in the grid. Convergence of the TDP calculation generally was reached on Iteration 1 of Cycle 2, which was the 6th iteration since 5 iterations/cycle were employed.

If the TDP calculation is used to determine the optimal drilling program from a set of N admissible well sites, then about $N^2/2$ total TDP calculations must be performed if termination is reached by running out of wells. If instead of depleting the remaining set of admissible well sites by 1 when we "drill" a well, we add another new admissible site in replacement, then the total number of TDP calculations is NM, where N is the constant number of admissible well sites remaining and M is the total number of additional wells drilled when the specified terminal (low) value of G is reached. The total computing time necessary on a UNIVAC 1108 to find an optimal drilling program is then approximately

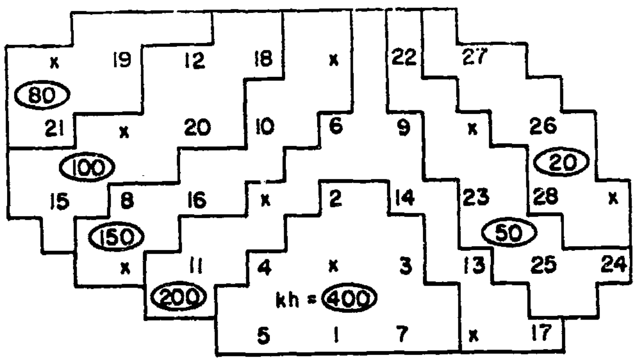

Fig. 8—Optimum order for drilling wells in Field B, variable ϕh case.

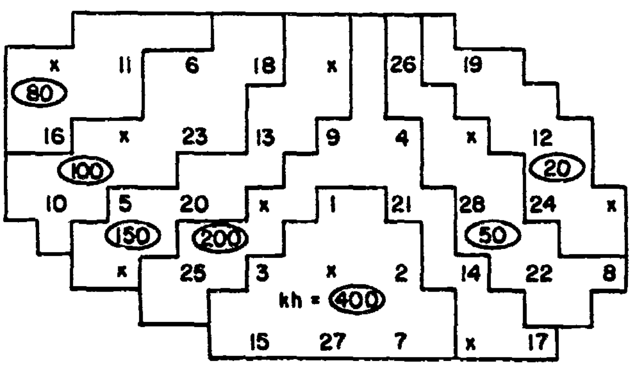

Fig. 9—Optimum order for drilling wells in Field B, uniform ϕh case.

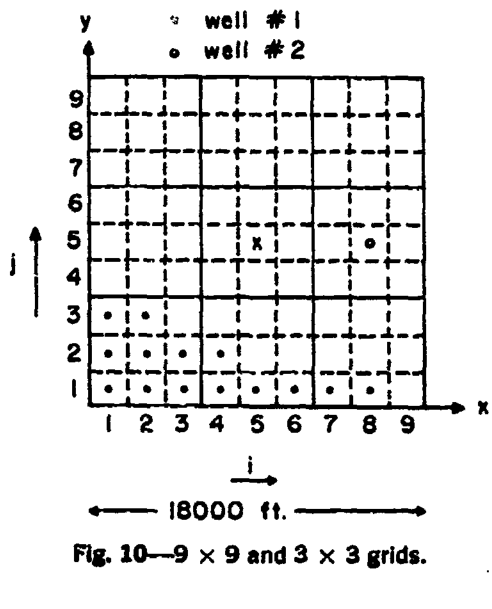

Fig. 10—9 × 9 and 3 × 3 grids.

$$0.28 \frac{N_x N_y}{260} \bullet NM \text{ seconds}$$

where $N_x N_y$ is the number of blocks in the grid.

Conclusions

A two-dimensional numerical calculation has been proposed for calculating the semi-steady-state pressure distribution and individual well deliverabilities in a gas field producing under a specified total rate schedule. The calculation accounts for reservoir heterogeneity, irregular well spacing and drainage areas, unequal well rates, and well interference effects.

The calculation may be employed to estimate field performance for any given combination of producing well locations or may be used to determine an optimal order of drilling a given set of admissible well sites.

The use of the well known semi-steady-state relation between well rate and average "block" pressure allows accurate representation of well deliverability in the context of large block numerical simulations of over-all reservoir performance. This accuracy is reduced only negligibly by placing wells in adjacent grid blocks.

The proposed method selects additional well sites in that order that ensures the drilling of as few wells as possible at each successive stage of depletion. This optimal selection depends upon well interference phenomena as well as upon the reservoir kh and ϕh distributions. The proper selection is difficult, if not practically impossible, to make by intuition.

Extensions of the proposed method can achieve some optimum balance between the cost of additional wells vs additional compression during middle and late stages of depletion.

Nomenclature

Bcf = billion standard cu ft
c = fluid compresibility, psi^{-1} or well cost, dollars
C = $d(p/z)/d\Phi$
G = gas in place, Mcf
G_a = gas in place at abandonment, Mcf
h = reservoir net pay thickness, ft
k = permeability, md
n_w = cumulative number of wells drilled
n_{wo} = number of wells drilled at the present time
q = production rate, Mcf/cu ft of reservoir/day
Q_w = well production rate, Mcf/D
Q_F = total field producing rate, Mcf/D
p = pressure, psia
p_s = standard pressure
r_e = exterior radius, ft
r_w = wellbore radius, ft
S = skin factor
t = time, days
t_a = abandonment time
T = reservoir temperature, °R
T_s = standard temperature
V = total reservoir bulk volume, cu ft
V_p = total reservoir pore volume, cu ft
V_b = block pore volume, cu ft
z = gas compressibility factor
μ = gas viscosity, cp
ϕ = porosity, fraction
Φ = potential, see Eq. 5
ρ = density, lb$_{mass}$/cu ft

References

1. Bellman, Richard: *Dynamic Programming*, Princeton U. Press, Princeton, N. J. (1957).
2. Katz, D. L., Cornell, D., Kobayashi, R., Poettmann, F. H., Vary, J. A., Elenbaas, J. R. and Weinaug, C. F.: *Handbook of Natural Gas Engineering*, McGraw-Hill Book Co., Inc., New York (1959).
3. Katz, D. L. and Coats, K. H.: *Underground Storage of Fluids*, Ulrich's Books, Ann Arbor, Mich. (1968).
4. Craft, B. C. and Hawkins, M. F., Jr.: *Applied Petroleum Reservoir Engineering*, Prentice-Hall, Inc., Englewood Cliffs, N. J. (1959).
5. Matthews, C. S. and Russell, D. G.: *Pressure Buildup and Flow Tests in Wells*, Monograph Series, Society of Petroleum Engineers, Dallas, Tex. (1967) **1**.
6. Henderson, J. H., Dempsey, J. R. and Tyler, J. C.: "Use of Numerical Models to Develop and Operate Gas Storage Reservoirs", *J. Pet. Tech.* (Nov., 1968) 1239-1246.
7. Brons, F. and Miller, W. C.: "A Simple Method for Correcting Spot Pressure Readings", *J. Pet. Tech.* (Aug., 1961) 803-805.
8. Earlougher, R. C., Jr., and Ramey, H. J., Jr.: "The Use of Interpolation to Obtain Shape Factors for Pressure Buildup Calculations", *J. Pet. Tech.* (May, 1968) 449-450.
9. Al-Hussainy, R., Ramey, H. J., Jr., and Crawford, P. B.: "The Flow of Real Gases Through Porous Media", *J. Pet. Tech.* (May, 1966) 624-636.
10. Douglas, J., Jr., and Rachford, H. H., Jr.: "On the Numerical Solution of Heat Conduction Problems in Two and Three Space Variables", *Trans.*, AMS (1956) **82**, 421.
11. Richtmyer, R. D.: *Difference Methods for Initial-Value Problems*, Interscience, Inc., New York (1957) 103.

APPENDIX

Numerical Simulation of Semi-Steady-State Gas Flow

The partial differential equation expressing conservation of mass for transient gas flow in a reservoir is*

$$\nabla \bullet \left(\frac{kh}{\mu} \rho \nabla p \right) - h\, q_m = h\phi \frac{\partial \rho}{\partial t}, \quad . \quad \text{(A-1)}$$

where q_m is gas production rate in units of pounds mass per bulk reservoir volume per day. Use of the gas law, $\rho = Mp/zRT$, and definition of the potential[9]

$$\Phi \equiv \int^{p} \frac{p}{z\mu}\, dp \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A-2)}$$

allows writing Eq. A-1 as

$$\nabla \bullet (kh \nabla \Phi) - qh \frac{1000\, T\, p_s}{T_s} = \phi h \frac{\partial (p/z)}{\partial t}$$

$$= \phi h C \frac{\partial \Phi}{\partial t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A-3)}$$

where C is $d(p/z)/d\Phi$, a single-valued function of potential Φ.

The total field producing rate Q_F Mcf/D can be equated to

*Note that k must be md × 0.00633 for units consistency here.

$$Q_F = -\frac{d}{dt}\int_V \frac{p}{z}\frac{T_s}{1000\,p_s\,T}\,\phi dV, \quad . \quad \text{(A-4)}$$

where V is total reservoir bulk volume. Interchanging the order of differentiation and integration yields

$$Q_F = -\frac{T_s}{1000\,p_s\,T}\int_V \frac{\partial(p/z)}{\partial t}\,\phi dV \quad . \quad \text{(A-5)}$$

The definition of semi-steady state employed here is "that flow regime in which the derivative $\partial(p/z)/\partial t$ is nearly independent of spatial position". This independence never truly occurs in gas flow due to the dependence of C in Eq. A-3 upon Φ. However, it is a useful approximation; and comparison between semi-steady-state calculations and transient flow calculations has shown it to be a close approximation.

Under this semi-steady-state flow regime, $\partial(p/z)/\partial t$ is nearly uniform throughout the field and, from Eq. A-5, is equal to

$$\frac{\partial(p/z)}{\partial t} = -\frac{Q_F}{V_p}\frac{1000\,P_s\,T}{T_s}, \quad . \quad . \quad . \quad . \quad \text{(A-6)}$$

where V_p is total field pore volume. Substitution of this result into Eq. A-3 gives

$$\frac{T_s}{1000\,p_s\,T}\nabla\bullet(kh\,\nabla\Phi) - q_h = -\frac{\phi h\,Q_F}{V_p}. \quad \text{(A-7)}$$

Using the standard second-order differences, for example,

$$\nabla_x kh\nabla_x\Phi \cong [(kh)_{i+½,j}(\Phi_{i+1,j}-\Phi_{i,j}) - (kh)_{i-½,j}(\Phi_{i,j}-\Phi_{i-1,j})]/\Delta x^2,$$

and multiplying Eq. A-7 throughout by $\Delta x\Delta y$, gives

$$\Delta A\Delta\Phi - Q_{ij} = -\frac{V_{bij}}{V_p}Q_F, \quad . \quad . \quad . \quad \text{(A-8)}$$

where

$$\Delta A\Delta\Phi \equiv \Delta_x A_x \Delta_x \Phi + \Delta_y A_y \Delta_y \Phi,$$

$$\Delta_x A_x \Delta_x \Phi = A_{xi+½,j}(\Phi_{i+1,j}-\Phi_{i,j}) - A_{xi-½,j}(\Phi_{i,j}-\Phi_{i-1,j}),$$

$$A_{xi+½,j} = \frac{T_s}{1000\,T\,p_s}(kh)_{i+½,j}\frac{\Delta y}{\Delta x}\bullet 0.00633,$$

k in md, h in ft,

Q_{ij} = Mcf/D production rate from block i, j, and

V_{bij} = pore volume $\phi h\,\Delta x\,\Delta y$ of block i, j.

The interblock $(kh)_{i+½,j}$ is calculated as the harmonic average $2(kh)_{ij}\,(kh)_{i+1,j}/[(kh)_{ij} + (kh)_{i+1,j}]$.

Representation of a producing well by Eq. 3 finally gives

$$\Delta A\Delta\Phi - a_{ij}(\Phi_{ij}-\Phi_w) = -V_{bij}Q_F/V_p \quad \text{(A-9)}$$

where a_{ij} is the coefficient in Eq. 3. Boundary conditions for Eq. A-9 are no-flow, so that for block-centered grids of the type shown on Fig. 10,

$$\Phi_{o,j} = \Phi_{1,j}\;\Phi_{N_x,j} = \Phi_{N_x+1,j}\quad j = 1, N_y, \text{(A-10a)}$$

$$\Phi_{i,o} = \Phi_{i,1}\;\Phi_{i,N_y} = \Phi_{i,N_y+1}\quad i = 1, N_x, \text{(A-10b)}$$

where N_x and N_y are the numbers of grid blocks in the x and y directions.

The specified quantities in Eq. A-9 are the transmissibilities A_x, A_y, the coefficients a_{ij} (note that a_{ij} is 0 in all blocks containing no well), the producing potential Φ_w, the block and field pore volumes $V_{bi,j}$, V_p, and the total required field producing rate Q_F. The unknowns are $\Phi_{i,j}$ at each grid point.

The unknown $\Phi_{i,j}$ values are "block average" values in the sense that the corresponding $(p/z)_{i,j}$ values (recall p/z is a single valued function of Φ) are the block average values which, when multiplied into the block pore volumes, give the quantity, in Mcf, of gas in place in the block. Thus, after Eq. A-9 is solved for $\Phi_{i,j}$, the corresponding values of $(p/z)_{i,j}$ are obtained, and the gas in place in the reservoir is found as

$$G\,\text{Mcf} = \sum_{i,j} V_{bi,j}\,(p/z)_{i,j}\frac{T_s}{1000\,T\,p_s} \quad . \quad \text{(A-11)}$$

This gas in place can be thought of equivalently as average pressure level, although the average pressure (obtained from the average p/z corresponding to G) is actually slightly different from the true volumetric average

$$\int_V p dV_p/V_p.$$

Solution of Eq. A-9 could be performed by fixing the gas in place (i.e., average pressure) and then determining the total field deliverability Q_F. However, it is much simpler to fix the rate Q_F; the solution $\Phi_{i,j}$ then automatically seeks a level such that, for the given well locations, the wells taken together produce exactly Q_F Mcf/D. The term Φ_w is held constant at the value corresponding to the specified minimum allowable flowing wellbore pressure. A mathematical statement of this fact is obtained by summing Eq. A-9 over all grid points (i, j). Due to the boundary conditions,[13] the

$$\sum_{i,j}\Delta A\Delta\Phi_{i,j}$$

is identically 0 (for any $\Phi_{i,j}$ whatever) and we have

$$\sum_{i,j} a_{ij}(\Phi_{i,j}-\Phi_w) = Q_F\sum_{i,j}V_{bij}/V_p = Q_F. \quad \text{(A-12)}$$

It should be noted that the solution $\Phi_{i,j}$ to Eq. A-9 gives not only the potential distribution and gas in place but also the deliverability Q_w Mcf/D for each well and the remaining gas in place in each grid block. Thus the extent of depletion or drainage of each block or certain areas of the reservoir can be printed out by the computer program solving Eq. A-9. Questions of lease-line drainage can be examined in detail, in general with reasonably large grid spacings.

Eq. A-9 is solved by use of the Douglas-Rachford iterative alternating-direction technique.[10] The x- and y-direction sweeps are

$$\Delta_x A_x \Delta_x \Phi^* + \Delta_y A_y \Delta_y \Phi^k + a_{ij} (\Phi_w - \Phi_{i,j}^*)$$
$$H_k \Sigma A (\Phi_{ij}^* - \Phi_{ij}^k) = - \frac{V_{bij}}{V_p} Q_F +$$
$$\Delta_x A_x \Delta_x \Phi^* + \Delta_y A_y \Delta_y \Phi^{k+1} + a_{ij} (\Phi_w - \Phi_{ij}^{k+1}) =$$
$$- \frac{V_{bij}}{V_p} Q_F + H_k \Sigma A (\Phi_{ij}^{k+1} - \Phi_{ij}^k) \quad . \quad \text{(A-13)}$$

The superscript k is iteration index, ΣA is the sum of the transmissibilities around the four sides of the block, and H_k is iteration parameter. Definition of $PX \equiv \Phi^* - \Phi^k$ and $PY = \Phi^{k+1} - \Phi^k$ allows writing Eqs. A-13 as

$$\Delta_x A_x \Delta_x PX - (a_{ij} + H_k \Sigma A) PX = - B_{ij}, \quad \text{(A-14a)}$$

$$\Delta_y A_y \Delta_y PY - (a_{ij} + H_k \Sigma A) PY = - (a_{ij} + H_k \Sigma A) PX \quad , \quad \text{(A-14b)}$$

where B_{ij} is the residual of the Eq. A-8 itself; i.e.,

$$B_{ij} = \Delta A \Delta \Phi^k + a_{ij} (\Phi_w - \Phi_{ij}^k) + V_{bij} Q_F / V_p.$$

Note that the residual B approaches 0 as the solution converges. The units of the residual are Mcf/D and hence any non-zero value represents erroneous injection or production. Natural closure tolerances are, then,

$$\varepsilon_1 = \sum_{i,j} B_{ij}/Q_F, \text{ and}$$
$$\varepsilon_2 = \sum_{i,j} |B_{i,j}|/Q_F .$$

The first is a material balance check, and iterations are continued until $\varepsilon_1 < 0.002$. The second tolerance is simply printed out and generally declines to about 1 percent or less at "convergence".

Eqs. A-14a and A-14b are each of "one-dimensional" type and their solution is given by Richtmyer (see page 103 of Ref. 11). After Eq. A-14b is solved for PY, the new iterate Φ^{k+1} is calculated as $\Phi^k + PY$ at each grid point. **JPT**

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