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Multidimensional Displacement of a Viscous Phase by a Non-Viscous Phase

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ABSTRACT

A mathematical model has been formulated for simulating three-dimensional displacement of a viscous fluid by a displacing fluid of zero viscosity. The model has been incorporated into a FORTRAN IV computer program for application in low-rate, high-permeability systems. Where applicable, the zero-viscosity program reduces computer time by a factor of 5 to 10 relative to conventional two- and three-dimensional programs.

To determine the area of applicability, a gas-oil cross-section model representation of a high-dip, high-permeability reservoir was simulated with the zero-viscosity and conventional two-dimension programs for a range of flow rates up to 80 percent of the critical rate. In comparing the two solutions, the conventional one was assumed to be the correct one because its program is based upon a more physically realistic model than that of the zero-viscosity solution. The two solutions agreed at rates up to 50 percent of the critical; at 80 percent they disagreed significantly. This indicates that the zero-viscosity model, which is quite simple and inexpensive to apply, can be used with accuracy at rates up to at least 50 percent of the critical. This area of applicability is important in improving computational capability, for it is at these lower rates that the conventional programs are excessively costly. At the higher rates, where the zero-viscosity solution is not accurate, the conventional programs are easy and economical to apply.*

The zero-viscosity model accounts for capillary and gravitational forces, effects of viscosity and relative permeability for the displaced phase, and arbitrary reservoir heterogeneity. The program handles up to 1,800 blocks on an in-core basis.

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*Deitz, D. N.: "A Theoretical Approach to the Problem of Encroaching and By-Passing Edge Water", Proc., Koninkl.

INTRODUCTION

Computational difficulties caused by slow or metastable convergence in gas-oil calculations using conventional two-phase reservoir simulation programs have been correlatable with the effects of low viscosity in the gas phase. In many such problems, a very small deviation in the calculated flow potentials causes a large deviation in the calculated gas flow due to the low viscosity. Thus, the program is trying to converge on a small variation in potential, which makes the computations difficult.

A previous method of overcoming this difficulty has been to introduce in the conventional two-phase calculations an artificial resistance to gas flow; this method causes a more significant variation in the calculated flow potential. This paper describes a new method for treatment of gas-oil problems in which a zero-viscosity gas phase is used. Both methods are based on the assumption that oil mobility is the controlling factor in the displacement and that the behavior is insensitive to gas mobility over a relatively wide range. We show that the two methods give identical results, and since the correct gas mobility is bracketed by the two methods, we may conclude that either method gives valid results for low rate displacements. The chief advantage of the zero-viscosity program is lower computing costs. This report presents a mathematical description of the zero-viscosity model and compares saturation distributions calculated for several typical problems using the zero-viscosity and conventional two-phase programs.

ZERO-VISCOSITY MODEL

The zero-viscosity model simulates the immiscible displacement of a viscous fluid by a displacing fluid of zero viscosity. The method includes the effects of capillary and gravitational forces, relative permeability and viscosity in the displaced phase, and arbitrary reservoir heterogeneity. The mathematics presented here apply to incompressible

oil-gas systems with dissolved gas.

Eqs. 1 and 2 govern the flow of incompressible oil and gas through porous media:

$$\nabla \cdot \left[k \frac{k_{ro}}{\mu_o} \nabla (p_o - \rho_o g Z)\right] + \underline{i}_o = \phi \frac{\partial S_o}{\partial t}; \quad \ldots\ldots\ldots \quad (1)$$

$$\nabla \cdot \left[k \frac{k_{rg}}{\mu_g} \nabla (p_g - \rho_g g Z)\right] + \underline{i}_g = -\phi \frac{\partial S_o}{\partial t} \quad \ldots\ldots\ldots \quad (2)$$

If the gas viscosity in Eq. 2 approaches zero, then the gas flow potential must be a constant spatially. Under these conditions, as is shown in the Appendix, both flow equations may be reduced to a single equation,

$$\nabla \cdot \left[k \frac{k_{ro}}{\mu_o} \nabla R\right] - \underline{i}_o = -\phi S' \frac{\partial R}{\partial t}, \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

where

$$R \equiv P_c + (\rho_o - \rho_g) g Z \ldots\ldots \quad (4)$$

Eq. 3 thus constitutes the mathematical model to be used. In practice we have found it necessary to restrict use of the model to flow rates less than one-half of the critical rate:

$$\frac{q_o}{A} \leq \tfrac{1}{2} \frac{k(\rho_o - \rho_g) g \sin \alpha_d}{\mu_o / k_{ro}}. \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

A full derivation of all equations is included in the Appendix.

The numerical solution of Eq. 3 using the ADIP procedure has been programmed in FORTRAN IV. The program models a heterogeneous reservoir with up to 1,800 blocks and is entirely core-contained. Seven sets of relative permeability and capillary pressure tables can be read in and applied through a key word to arbitrary block positions in the reservoir. The program can be employed to simulate a one-, two-, or three-dimensional displacement. The two-dimensional calculation can represent either a cross-sectional or an areal system.

COMPARISON OF ZERO-VISCOSITY PROGRAM AND CONVENTIONAL PROGRAM RESULTS

analyze the behavior of a gas-injection project in an offshore Louisiana reservoir. Table 1 gives the data for a cross-section of this reservoir. The relative permeability data shown in Fig. 1 (Curve A) were obtained from core analysis and considered representative of the reservoir. Computer runs with a conventional two-phase program were attempted with these data but the solutions were so poor and so expensive that this attempt to simulate the reservoir had to be abandoned.

We found that reservoir performance could be predicted with either the zero-viscosity program or the conventional two-phase program in which an increased gas flow resistance was used. To achieve increased flow resistance, the gas relative permeability curve was adjusted as shown by Curve B, Fig. 1. In Fig. 2, the saturation distribution calculated with the zero-viscosity program agrees almost exactly with that of the conventional two-dimensional, two-phase program and relative permeability Curve B. The results correspond to an injection rate of 0.43 B/D/ft of width, which is 1/20th of the critical rate. The fact that these results are in such good agreement confirms that either method gives a valid prediction of reservoir performance for low flow rates. Computing costs with the zero-viscosity method, however, were much lower. To calculate 9 years of producing history with the conventional program required

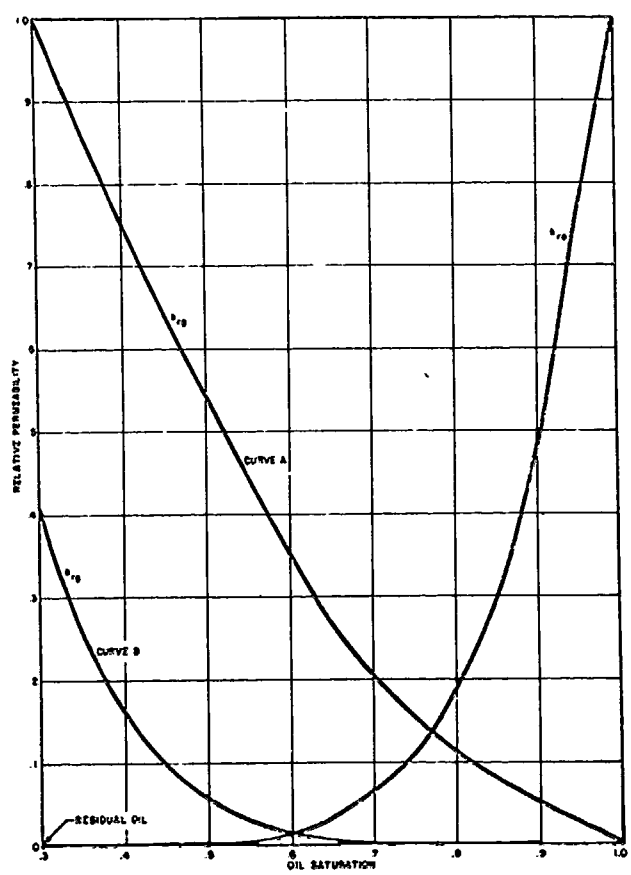

FIG. 1 — OIL AND GAS RELATIVE PERMEABILITY

about 7 hours of machine time on the IBM 7044. The same calculation with the zero-viscosity program required about 0.90 hour. Although the costs of running either method are not prohibitive, the zero-viscosity program solved the problem with an eight-fold reduction in computing expense.

Two additional runs were made at higher flow rates of 50 and 80 percent of the critical rate. Gas relative permeability Curve A, the correct field data, was used successfully in the conventional program for these higher rate runs. This was possible because the higher rates caused higher gas phase pressure gradients, so that the conventional program produced solutions that converged rapidly even with the higher gas relative permeability data. Figs. 3 and 4 compare the saturation distributions predicted by both methods for the higher rates. At 50 percent of the critical rate, the agreement between the two methods is excellent. However, at 80 percent of the critical rate, the agreement is not so good. Since the conventional method is based upon a more physically realistic model than the zero-viscosity model, the latter should not be used at rates as high as 80 percent of the critical rate.

TABLE 1 — RESERVOIR DATA FOR CROSS-SECTIONAL CALCULATION

Permeability, md =	1,000
Porosity, percent =	28
Oil density, lb/cu ft =	46.086
Gas density, lb/cu ft =	9.271
Two-dimensional grid =	32 × 8
Injection rate, B/D/ft of width =	0.43
Thickness, ft =	80
Length, ft =	1,280
Dip angle, degrees =	16

Oil Saturation	P_c (psi)	k_{ro}	Run Characteristics
0.297	2.63	0	*Field Time:* 0 to 9 years (0.335 PV injected)
0.320	2.53	0.00002	
0.360	2.39	0.00009	
0.40	2.26	0.00032	*Computer Time:* 54 minutes (350 time steps)
0.45	2.17	0.00113	(36 seconds/1,000 blocks — time step)
0.55	1.76	0.00785	
0.60	1.59	0.01706	
0.65	1.42	0.03393	
0.70	1.25	0.06285	*Iterations/time step:* Average of 28
0.75	1.08	0.10992	
0.80	.92	0.18330	
0.85	.75	0.29364	*Material Balance:* Cumulative gas balance at 9 years = 0.997
0.90	.57	0.45451	
0.926	.48	0.555	
1.	0	1.0	
1.		1.0	*Time Step:* 20 days over major portion of run; no cutbacks on Δt

CONCLUSIONS

A mathematical model based on the concept of

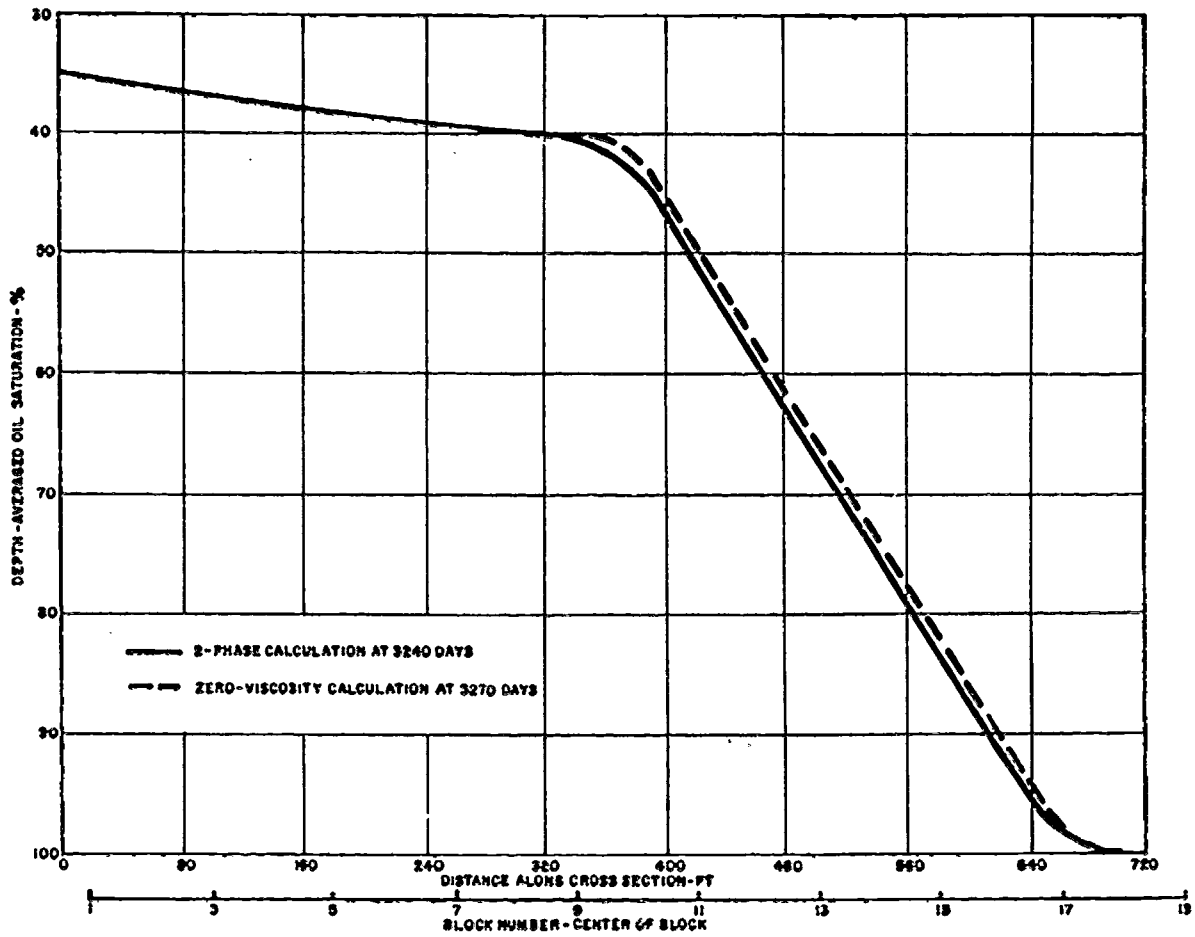

FIG. 2 — COMPARISON OF CONVENTIONAL TWO-DIMENSIONAL AND ZERO VISCOSITY PROGRAM

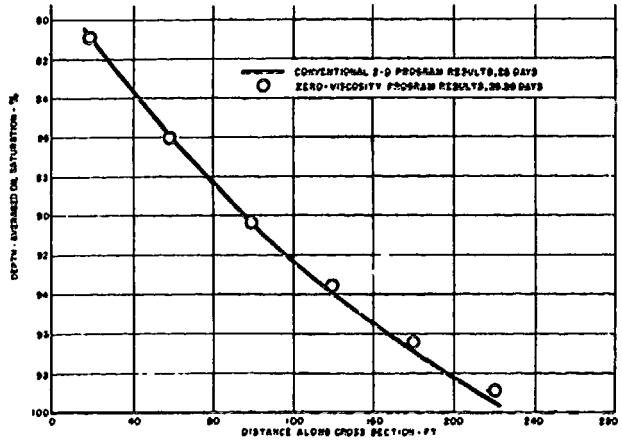

FIG. 3 — COMPARISON OF CONVENTIONAL TWO-DIMENSIONAL AND ZERO VISCOSITY PROGRAM RESULTS AT 50 PERCENT OF CRITICAL RATE.

a constant flow potential in the gas phase provides an efficient method for calculation of gas-oil displacement problems for rates of 50 percent or less of the critical. Computing time and cost are reduced significantly for reservoir situations in which use of conventional reservoir computer programs causes computational instabilities.

NOMENCLATURE

A = cross-sectional area normal to direction of flow

g = acceleration of gravity

i = specific injection rate, volume of fluid/(volume of reservoir-unit time)

k = absolute permeability

k_r = relative permeability

p = pressure

P_c = capillary pressure

q = flow rate, volume/time

R = see Eq. 4

S = saturation

$S' = dS_w/dP_c$

t = time

Z = elevation, measured positively downward

μ = viscosity

ϕ = porosity

Φ = potential, $p - \rho g Z$

ρ = fluid density

α_d = dip angle

SUBSCRIPTS

g = gas

n = nonwetting phase

o = oil

w = wetting phase

APPENDIX

DEVELOPMENT OF EQUATIONS

Consider the flow of two immiscible, incompres-

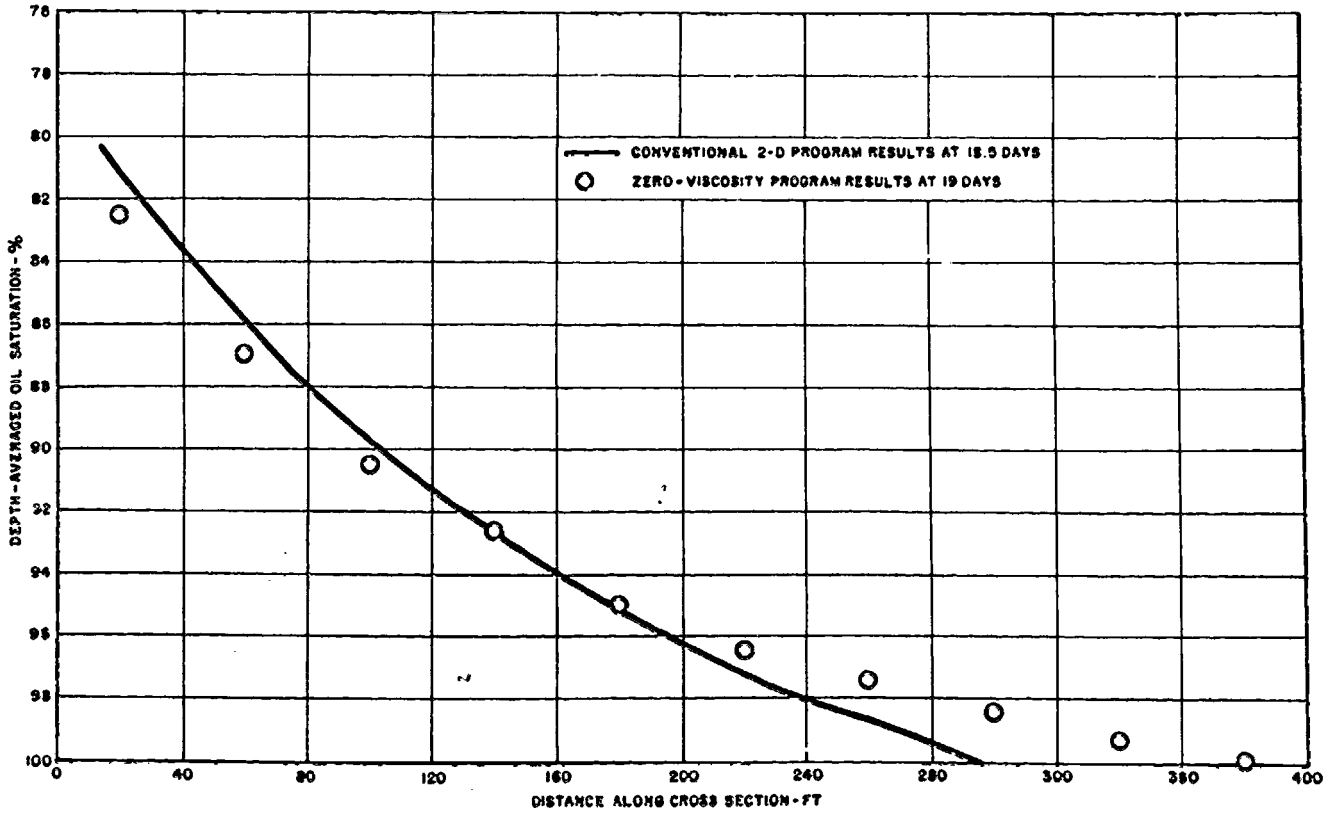

FIG. 4 — COMPARISON OF CONVENTIONAL TWO-DIMENSIONAL AND ZERO VISCOSITY PROGRAM

sible fluids — a wetting phase w, and a nonwetting phase n. The two well known differential equations describing the flow are

$$\nabla \cdot \left[\frac{k\, k_{rw}}{\mu_w} \nabla (p_w - \rho_w g Z)\right] + \underline{i}_w = \phi \frac{\partial S_w}{\partial t}, \quad \text{(A-1)}$$

and

$$\nabla \cdot \left[\frac{k\, k_{rn}}{\mu_n} \nabla (p_n - \rho_n g Z)\right] + \underline{i}_n = -\phi \frac{\partial S_w}{\partial t} \quad \text{(A-2)}$$

Assume a constant nonwetting phase viscosity, μ_n, and multiply Eq. A-2 by μ_n:

$$\nabla \cdot [k\, k_{rn} \nabla (p_n - \rho_n g Z)] + \underline{i}_n \mu_n = -\mu_n \phi \frac{\partial S_w}{\partial t} \quad \text{(A-3)}$$

No-flow, exterior reservoir boundaries are employed here so that

$$[\nabla (p_n - \rho_n g Z)] \cdot \vec{n} = 0 \quad \text{(A-4)}$$

at all points on the exterior surface bounding the reservoir, where $\vec{n}$ is the normal to that surface. As the nonwetting phase viscosity μ_n approaches zero, Eq. A-3 becomes

$$\nabla \cdot [k\, k_{rn} \nabla (p_n - \rho_n g Z)] = 0. \quad \text{(A-5)}$$

The unique solution to Eq. 5 for boundary condition described in Eq. 4 is a spatially uniform potential, so that

$$\nabla (p_n - \rho_n g Z) = 0 \quad \text{(A-6)}$$

Equivalently stated, $p_n - \rho_n g Z$ is a "constant" which may vary with time but not with the spatial coordinates x, y, z.

Now, working with the terms on the left-hand side of Eq. A-6,

$$p_n - \rho_n g Z = p_w + P_c - \rho_n g Z + \rho_w g Z - \rho_w g Z,$$

where the capillary pressure P_c is given by

$$p_n = p_w + P_c$$

If a flow potential Φ is defined as

$$\Phi = p - \rho g Z,$$

then

$$p_n - \rho_n g Z = \Phi_n = \Phi_w + P_c + (\rho_w - \rho_n) g Z.$$

Substituting this value into Eq. A-6 yields

$$\nabla [\Phi_w + P_c + (\rho_w - \rho_n) g Z] = 0,$$

and rearranging yields

$$\nabla \Phi_w = -\nabla [P_c + (\rho_w - \rho_n) g Z]. \quad \text{(A-7)}$$

Substitute Eq. A-7 into Eq. A-1:

$$-\nabla \cdot k \frac{k_{rw}}{w} \nabla [P_c + (\rho_w - \rho_n) g Z] + \underline{i}_w = \phi \frac{\partial S_w}{\partial t} \quad \text{(A-8)}$$

Now define a new variable R,

$$R \equiv P_c + (\rho_w - \rho_n) g Z, \quad \text{(A-9)}$$

and change variables in the saturation derivative,

$$\frac{\partial S_w}{\partial t} \equiv \frac{dS_w}{dP_c} \frac{\partial P_c}{\partial t} = \frac{dS_w}{dP_c} \frac{\partial R}{\partial t}. \quad \text{(A-10)}$$

Combining Eqs. A-8 through A-10,

$$\nabla \cdot k \frac{k_{rw}}{\mu_w} \nabla R - \underline{i}_w = -\phi S' \frac{\partial R}{\partial t}, \quad \text{(A-11)}$$

which is the final result.

An interesting characteristic of the solution to Eq. A-11 is that calculated gas saturation will increase in the direction of oil flow at any point in the reservoir where oil flow rate exceeds the critical rate. To show this, let x denote the direction of oil flow at some point in the reservoir and let that direction be inclined somewhat downward; e.g., $dZ/dx = 0.1$.

Starting with the definition of R,

$$R = P_c + (\rho_w - \rho_n) g Z, \quad \text{(A-9)}$$

and differentiating with respect to x,

$$\frac{dR}{dx} = \frac{dP_c}{dx} + (\rho_w - \rho_n)\, g \frac{dZ}{dx}. \quad \text{(A-12)}$$

If gas saturation is to decrease in the direction of oil flow, then

$$\frac{dS_w}{dx} \geq 0$$

or

$$\frac{dP_c}{dx} \leq 0 \quad \ldots\ldots\ldots \text{(A-13)}$$

Combining Eqs. A-12 and A-13,

$$\frac{dR}{dx} \leq (\rho_w - \rho_n) g \frac{dZ}{dx} .$$

But since $\frac{d\Phi_w}{dx} = -\frac{dR}{dx}$, then

$$-\frac{d\Phi_w}{dx} \leq (\rho_w - \rho_n) g \frac{dZ}{dx} \quad \text{(A-14)}$$

Integrating Eq. A-14 from point 1 to point 2 yields

$$\Phi_1 - \Phi_2 \leq (\rho_w \rho - \rho_n) g (Z_2 - Z_1).$$

Therefore, if calculated gas saturation is to decrease in the direction of oil flow, then

$$\Delta\Phi_w \leq (\rho_w - \rho_n) g \Delta Z \quad \text{(A-15)}$$

Eq. A-15 states that the viscous pressure drop between any two points is less than or equal to the difference in gravity head.

This restriction has a definite relation to the critical rate as defined by:

$$q_c = \frac{k(\rho_w - \rho_n) Ag \sin \alpha_d}{\mu_w/k_{rw} - \mu_n/k_{rn}} .$$

Let $\mu_n = 0$, and rearrange:

$$\frac{q_c(\mu_w/k_{rw})}{Ak} = (\rho_w - \rho_n) g \sin \alpha_d$$

But

$$\frac{q_c (\mu_w/k_{rw})}{Ak} = \frac{\Delta\Phi_w}{\Delta x} ,$$

from Darcy's law. Therefore, at the critical rate,

$$\Delta\Phi_w = (\rho_w - \rho_n) g \sin \alpha_d \Delta x,$$

$$\Delta x \sin \alpha_d = \Delta Z;$$

so

$$\Delta\Phi_w = (\rho_w - \rho_n) g \Delta Z.$$

At rates above the critical rate, the interface becomes unstable, so the critical rate concept yields

$$\Delta\Phi_w \leq (\rho_w - \rho_n) g \Delta Z,$$

which is identical with the previous result.

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