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Optimum Location of Trunklines in Oil and Gas Fields

KEITH H. COATS
JUNIOR MEMBER AIME

JERSEY PRODUCTION RESEARCH CO.
TULSA, OKLA.

ABSTRACT

In many oil and gas fields, significant investments are required in piping networks for collection, injection and disposal of various fluids. This paper treats the problem of optimally locating the trunkline(s) for a given system to minimize both the total length of piping connecting the individual wells or lease batteries to the trunkline and the length of the latter. Single and multiple trunklines, varying line sizes, and grouping of wells are considered. Some attention is paid to the case of a curved trunkline. A field case is treated and conclusions are that the problem of locating a single trunkline is relatively straightforward while the location of multiple trunklines requires further research.

INTRODUCTION

In many oil and gas fields, significant investments are required in piping networks for collection, injection and disposal of various fluids. A single field may concurrently require piping systems for gathering of oil, gathering of gas, injection of water or gas and disposal of salt water. Two problems arise in the consideration of any one of these networks. The first involves the sizing of the various segments of the network to minimize piping and compression costs. The second problem, treated herein, is that of optimally locating the trunkline(s) for a given piping system to minimize the total length of pipe connecting the individual wells or lease batteries to the trunkline. This minimization problem may arise in field automation if it involves replacement of individual lease batteries by a central field battery which will require trunklines and connecting pipelines to individual wells. The problem also arises in connecting a gas pipeline through a subsidiary trunkline to wells in a gas storage field.

The first analysis given below treats the case of a single trunkline and of separate connecting lines for all field units.* The analysis is then extended to consider multiple trunklines and common connecting lines for groups of field units. Varying sizes, and therefore costs, of the connecting lines are taken into account. A simplified development is given to illustrate the manner in which line sizing calculations may be integrated with the line placement problem. This study is primarily directed toward the placement of straight trunklines, although for certain field shapes or well distributions curved trunklines might be preferable. Some mathematical consideration is therefore given to the placement of a curved trunkline in a field.

The determination of optimal locations for two or more trunklines involves some difficulties which are discussed below. The need for further research in this area is emphasized.

The problem of a single trunkline is solved in relation to an actual oil field. The problem of optimally locating two trunklines in the same field is also treated and the results are compared to the single-trunkline case. All calculations reported were performed on an IBM 7072 digital computer.

The importance of right-of-way, roads and terrain in locating trunklines and connecting lines is realized. The present analysis is advanced on the premise that in planning piping networks one should start with the optimal conditions whenever possible and proceed therefrom.

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*Wells or lease batteries which are to be connected to the trunkline are referred to in this paper as "field units".

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PHYSICAL AND MATHEMATICAL CONSIDERATIONS

If an x-y coordinate system is imposed over a plan view of an oil or gas field, each field unit will be represented by a point (x_i, y_i). A single trunkline can be represented by a straight line of slope m and intercept b, given as

$$y = m\,x + b \quad . \quad . \quad . \quad . \quad (1)$$

The method of least squares determines m and b so the sum of the squares of the vertical or horizontal distances between the field units and trunkline is a minimum. Application of this method yields

$$\sum_{i=1}^{i=N} (y_i - y)^2 = \sum_{i=1}^{i=N} (y_i - mx_i - b)^2 = \text{Minimum} \quad . \quad . \quad . \quad (2)$$

If this function of m and b is to be a minimum, then the partial derivatives with respect to m and b must both equal zero. Thus

$$\sum_{i=1}^{N} x_i(y_i - mx_i - b) = 0 \quad . \quad . \quad . \quad (3)$$

and

$$\sum_{i=1}^{N} (y_i - mx_i - b) = 0 \quad . \quad . \quad . \quad (4)$$

Solving of Eqs. 3 and 4 for m and b yields

$$m = \frac{N\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{N\Sigma x_i^2 - (\Sigma x_i)^2} \quad . \quad . \quad . \quad (5)$$

$$b = \frac{\Sigma x_i^2 \Sigma y_i - \Sigma x_i \Sigma x_i y_i}{N\Sigma x_i^2 - (\Sigma x_i)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where summation from $i = 1$ to $i = N$ is implied by Σ. Eqs. 5 and 6 offer a relatively simple procedure for determination of trunkline location and are applied to an actual field below.

The use of the least-squares method is physically unjustified since the lowest piping cost requires minimization of the sum of the field unit-trunkline distances themselves, not their squares. Also, the distance from a given field unit to the trunkline should itself be minimized by making the connecting line perpendicular to the trunkline rather than vertical or horizontal. The perpendicular distance from a field unit having coordinates (x_i, y_i) to the trunkline is readily obtained as

$$l_i = \frac{1}{\sqrt{1 + m^2}} \left| y_i - mx_i - b \right| \quad . \quad . \quad . \quad . \quad (7)$$

The total length of connecting pipeline required is then

$$L = \sum_{i=1}^{i=N} l_i = \frac{1}{\sqrt{1 + m^2}} \sum_{i=1}^{i=N} \left| y_i - mx_i - b \right| \quad . \quad . \quad . \quad . \quad (8)$$

Eq. 8 applies to the case of a single, straight trunkline connected to all field units by separate lines of equal sizes (costs). The problem is now to determine m and b so that, given (x_i, y_i) for $i = 1, 2, \ldots, N$, the total length L is a minimum.

Extreme points (maxima or minima) of the function $L(m, b)$ should occur at values of m and b such that the following conditions hold:

$$\frac{\partial L}{\partial m} = \frac{\partial L}{\partial b} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Partial differentiation of Eq. 8 first with respect to m and second with respect to b yields

$$m \sum_{i=1}^{N} \left| y_i - mx_i - b \right| + (1 + m^2) \sum_{i=1}^{N} x_i \frac{y_i - mx_i - b}{\left| y_i - mx_i - b \right|} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\sum_{i=1}^{N} \frac{y_i - mx_i - b}{\left| y_i - mx_i - b \right|} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The physical meaning of Eq. 11 is: given any slope m, the optimum b is that value which results in equal numbers of field units above and below the trunkline. Simultaneous solution of Eqs. 10 and 11 for m and b will yield an extreme point of the function $L(m, b)$.

The left-hand side of Eq. 10 can be denoted by $f(m)$ and considered as a function only of m since for each assumed m value, Eq. 11 determines the "optimum" value of b. Solution of Eq. 10 by Newton's method[1] is not feasible because the second summation in Eq. 10 is a step function of m and b having zero partial derivatives at all points (m, b) except at those of discontinuity where the derivatives are not defined. The half-interval technique[2] involves increasing m by a fixed "interval" Δm repeatedly until $f(m)$ changes sign. The interval Δm is then halved repeatedly, always keeping the values of $f(m)$ of opposite sign at the end points of the interval. This half-interval search technique locates the extreme points of the function L [zeroes of the function $f(m)$] accurately and quickly.

The simplest and most informative method for determining the optimum m and b is calculation of the function L at equally spaced values of m over a range from, say, $m = -5$ to $m = +5$. At each value of m for which L is calculated, b is determined from Eq. 11. The minimum and maximum values of L as well as the optimum m and b values are then easily seen from a plot of L vs m. Such a plot also shows the sensitivity of the required pipe footage to non-optimal slopes of the trunkline.

If each field unit is to have a separate connecting line, as above, but these lines are of varying sizes, then a weight function w_i may be defined. This factor will be a function of line size and may be normalized to 1.0 at an arbitrary pipe diameter. More specifically, w_i will be directly proportional to the cost-per-foot of the pipe. We must now speak of the "cost" of the lines connecting the field units to the trunkline and must minimize the function

$$C = \sum_{i=1}^{i=N} w_i l_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where l_i is given by Eq. 7. If the trunkline length itself is included, then

$$C = \sum_{i=1}^{N} w_i l_i + w_T L_T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where the trunkline length L_T is given by

$$L_T = \frac{1}{\sqrt{1 + m^2}} \left(\operatorname*{Max}_i u_i - \operatorname*{Min}_i u_i \right) \quad . \quad . \quad . \quad (14)$$

and

$$u_i = my_i + x_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The required trunkline length is a function of slope m alone and not of the intercept b. The minimum value of C occurs at a point (m, b) where

$$\frac{\partial C}{\partial m} = \frac{\partial C}{\partial b} = 0,$$

or

$$w_T \frac{\partial L_T}{\partial m} + m \sum_{i=1}^{N} w_i \left| y_i - mx_i - b \right| + (1 + m^2) \sum_{i=1}^{N} w_i x_i \frac{y_i - mx_i - b}{\left| y_i - mx_i - b \right|} = 0 \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$\sum_{i=1}^{N} w_i \frac{y_i - mx_i - b}{\left| y_i - mx_i - b \right|} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The physical meaning of Eq. 17 is: given any slope m, the optimum intercept b is that value which results in equal "dollar value" of connecting pipe above and below the trunkline. Thus, where weight factors other than unity occur, the optimum trunkline location does not necessarily result in equal *numbers* of field units above and below the trunkline. Again, the simplest procedure for determining the optimal (m, b) is calculation of C from Eq. 13 for values of m over a range with b determined from Eq. 17 for each value of m.

The connection of several field units through a common connecting line to the trunkline(s) would simply involve altering the (x_i, y_i) data for the analysis by representing the several field units by a single point. The w factor for that point should account for the larger line size required.

When the trunkline cost is taken into account, the minimum piping cost is not necessarily given by Eq. 13 with the optimum m and b values. An assumption inherent in Eq. 13 is that a minimum cost requires that each field unit be connected perpendicularly to the trunkline. If the trunkline is appreciably more expensive (per foot) than the connecting lines, then cost reduction might be achieved by shortening the trunkline and connecting field units near the ends of the latter non-perpendicularly.

[1]References given at end of paper.

That is, the cost of the trunkline length eliminated may be greater than the cost of the additional length of the connecting lines. Fig. 1 illustrates this possible savings. The dotted lines show the type of connecting implied in Eq. 13 while the solid lines show the alteration. Let the values of u_i given by Eq. 15 be arranged in order of increasing magnitude and the (x_i, y_i) be reordered to correspond to the u_i. If the trunkline is shortened so that its ends are connected perpendicularly to the j^{th} field unit and the k^{th} unit, then the coordinates of the "lower" end of the trunkline (x_1^*, y_1^*) and of the "upper" end (x_2^*, y_2^*) are given by

$$x_1^* = \frac{1}{m}(y_1^* - b) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$y_1^* = \frac{m}{1 + m^2}\left(my_j + x_j + \frac{b}{m}\right) \quad . \quad . \quad . \quad . \quad (19)$$

$$x_2^* = \frac{1}{m}(y_2^* - b) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$x_2^* = \frac{m}{1 + m^2}\left(my_k + x_k + \frac{b}{m}\right) \quad . \quad . \quad . \quad . \quad (21)$$

The total cost of the network is then

$$C' = \sum_{i=1}^{i=j-1} w_i \sqrt{(x_i - x_1^*)^2 + (y_i - y_1^*)^2} + \sum_{i=j}^{i=k} w_i l_i + \sum_{i=k+1}^{i=N} w_i \sqrt{(x_i - x_2^*)^2 + (y_i - y_2^*)^2} + w_T \sqrt{(y_2^* - y_1^*)^2 + (x_2^* - x_1^*)^2}. \quad . \quad . \quad (22)$$

The optimum m, b, j and k are determined by calculating C' for values of m over a range of m with b determined from Eq. 17 for each m and with (j, k) determined at each such (m, b) combination so that C' is minimized.

The case of a curved trunkline given by a parabola will now be briefly considered. The trunkline is given by

$$y = a + bx + cx^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and a, b and c must be determined to minimize the total length of piping connecting N points (x_i, y_i) perpendicularly to the trunkline. The equation of the shortest line connecting the point (x_i, y_i) to the trunkline is

$$y = y_i + e(x - x_i) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where the slope e must be determined so that the line and trunkline intersect at right angles. If the point of intersection (joining) is designated as $(\bar{x}_i, \bar{y}_i)$, then the slope of the tangent to the trunkline at that point is

$$\frac{dy}{dx} = b + 2c\bar{x}_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

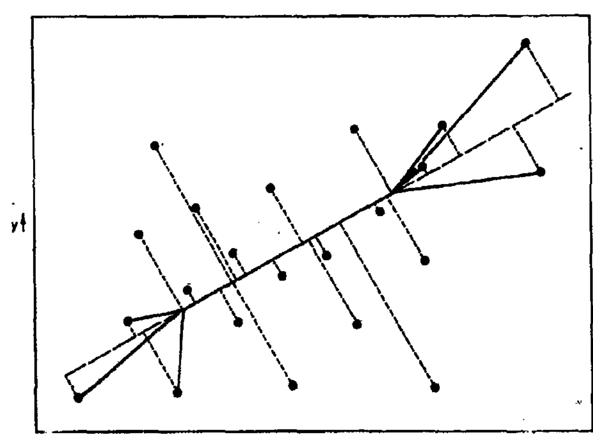

FIG. 1—NON-PERPENDICULAR CONNECTIONS OF END UNITS TO TRUNKLINE.

and if this tangent and the line drawn from (x_i, y_i) are to intersect at a right angle, then the product of their slopes must be -1.

$$2c\bar{x}_i e = -1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The point of intersection is determined by equating Eqs. 23 and 24:

$$a + b\bar{x}_i + c\bar{x}_i^2 = y_i + e(\bar{x}_i - x_i) \quad . \quad . \quad . \quad (27)$$

Eqs. 26 and 27 can be solved (e.g., by Newton's method[1]) for the two unknowns $\bar{x}_i$ and e, provided a, b, c, x_i and y_i are given. When two real solutions $(\bar{x}_i, e)$ exist, then that solution which gives the minimum value of l_i (see Eq. 28 below) must be selected. The value of $\bar{y}_i$ can then be easily found by inserting $\bar{x}_i$ for x in either Eq. 23 or Eq. 24. The perpendicular distance between the point (x_i, y_i) and the trunkline is

$$l_i = \sqrt{(\bar{x}_i - x_i)^2 + (\bar{y}_i - y_i)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The arc length along the trunkline is

$$dL_T = \sqrt{dx^2 + dy^2} = \sqrt{1 + (b + 2cx)^2}\, dx. \quad (29)$$

Thus, the required trunkline length is

$$L_T = \int_{\bar{x}_{i\,\mathrm{Min}}}^{\bar{x}_{i\,\mathrm{Max}}} \sqrt{1 + (b + 2cx)^2}\, dx \quad . \quad . \quad . \quad . \quad (30)$$

and the cost function to be minimized is obtained by inserting l_i and L_T from Eqs. 28 and 30 into Eq. 13. Since the resulting expression for C is not a simple function a, b and c, the most feasible technique of solution might be that of steepest descent.

Another possible variation of this problem arises when a gasoline plant exists and one point on the trunkline is therefore fixed. If the position of the plant is $(\bar{x}, \bar{y})$, then the equation for a straight trunkline is

$$y = \bar{y} + m(x - \bar{x}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

and the problem is simplified since only one parameter, the slope, must be determined.

The problem of line sizing, mentioned in the Introduction, can be included in part in this trunkline location problem through the weight factors w_i. To simplify the illustration of this relationship, let single-phase turbulent flow be assumed to occur in the connecting lines. The pressure drop due to friction in the connecting line for the i^{th} unit is given by

$$\Delta p_i = \frac{f\, l_i\, V_i^2\, \rho}{2\, g_c\, D_i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

D_i is pipe diameter, V_i is superficial velocity and f is friction factor. The pressure drop Δp_i is the known wellhead pressure minus the specified trunkline pressure. In terms of the production rate q_i (volume/time), Eq. 32 becomes

$$\Delta p_i = \frac{8\, f\, \rho\, l_i\, q_i^2}{g_c\, \pi^2\, D_i^5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

If turbulent flow exists, then f may be taken as roughly constant and Eq. 33 may be rearranged to give

$$D_i = a_i\, (l_i)^{1/5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Since the installed cost per foot of pipe is nearly proportional to pipe diameter, the weight factor w_i is given by

$$w_i = b_i D_i = c_i\, (l_i)^{1/5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

The dependence of w_i upon l_i can be included in the above analyses by repeating the calculation of C from Eq. 13 at each m value, updating or recalculating the w_i values and b value at each calculation until negligible change occurs. In an actual calculation along these lines, one would consider two-phase flow in the connecting lines, discrete line sizes available, minimum line size feasible, and an appropriate safety factor in the sizing equation.

Consideration of multiple trunklines complicates the problem in that each additional trunkline adds two more independent variables, a slope and an intercept, to be determined. The case of two trunklines will be outlined here since extension to three or more adds nothing new conceptually. The perpendicular distance from field unit i to trunkline $j (j = 1 \text{ or } 2)$ is

$$l_{ij} = \frac{1}{\sqrt{1 + m_j^2}} | y_i - m_j x_i - b_j | \quad . \quad . \quad . \quad (36)$$

The total cost of the piping network is

$$C = \sum_{i=1}^{i=N} w_i \, \underset{j}{\mathrm{Min}}\, l_{ij} + w_{T1} L_{T1} + w_{T2} L_{T2} \quad . \quad . \quad (37)$$

The term $\underset{j}{\mathrm{Min}}\, l_{ij}$ is the smaller of the quantities l_{i1} and l_{i2}; thus, a given field unit is to be connected to the nearest trunkline. Application of Eq. 37 will result in the division of the field units into two groups, the first containing all units lying closer to line 1, the second containing all units lying closer to line 2. The cost C may be considered as a function of the four independent variables, m_1, m_2, b_1 and b_2, which, in turn, fix the numbers of field units comprising the two groups. The minimization of $C (m_1, m_2, b_1, b_2)$ appears to be a problem subject to the method of steepest descent.[8] The method fails in this case, however, as illustrated in Fig. 2. The field units are placed to clarify the illustration. If, at some stage in the calculations of the steepest descent method, the current values of m_1, m_2, b_1, b_2 correspond to the trunkline positions shown by the solid lines, then the method will terminate with these positions even though the optimum locations are obviously not found. The total length will increase for any altered value of any of the four variables provided the other three remain fixed. That is, the total length of piping increases in all "directions" away from the point (m_1, m_2, b_1, b_2) represented by the solid lines.

The minimization method used here is as follows: (1) given a point (m_1, m_2, b_1, b_2), the field units are divided into two groups according to their proximity to the two trunklines, the trunkline lengths are determined and C is calculated from Eq. 37; (2) the group of wells closest to trunkline 1 is held fixed along with m_2 and b_2, m_1 is increased by Δm and the optimum b_1 is determined (from Eq. 17 with summation over all units in group 1), C is calculated for this new (m_1, b_1) set of values; similarly m_1 is decreased by Δm, the optimum b_1 is again determined and C is recalculated; (3) the effect of increased and decreased m_2 upon C is determined in manner identical to that described in Step 2 for m_1; (4) the new values m_1', b_1' are those at which a minimum C value occurred in Step 2, the new values m_2', b_2' are those at which a minimum C value occurred in Step 3; and (5) Steps 1-4 are repeated. This process is repeated with Δm decreased every n cycles (10-20 in this study) until Δm is less than some prescribed value.

As pointed out in the application section, this method has several flaws. First, the final trunkline locations depend upon the starting positions—thus, the search terminates in "local" optimums rather than at the true optimum point where C is minimized. This disadvantage can be partially overcome by performing calculations for a number of different starting positions. The method also ignores two cost-saving aspects which are obvious upon examination of Fig. 3. Let the trunkline positions shown represent the optimum positions as defined by a minimum value of C. The circles represent field units lying closer to line 1 while the dots are units lying closer to line 2. A savings could obviously be attained by connecting units a and b "non-optimally" to line 1 (even though they lie closer to line 2) since the relatively expensive trunkline 2 could then be shortened to point C'. Also savings might be gained by shortening the trunklines and connecting the units near their ends non-perpendicularly, as discussed in relation to the single trunkline problem.

APPLICATION TO FIELD A

Field A is an oil field, has 40 wells and is currently being considered for automation. Automation will involve laying an oil-gathering pipeline network to bring each well directly into a trunkline which will lead to a central separating facility for the field. The separate connecting lines are to be of equal diameter. Fig. 4 shows the positions of existing wells on an x-y co-ordinate system in which unit distance represents 500 ft.

The least-squares method was applied by solving Eqs. 5 and 6 for the slope and intercept. The results were $m = .54$ and $b = 2{,}541.08$ which correspond to line 1 on Fig. 4. These values of m and b were employed in Eq. 13 with all w_i equal to unity and $w_T = 3.0$. The

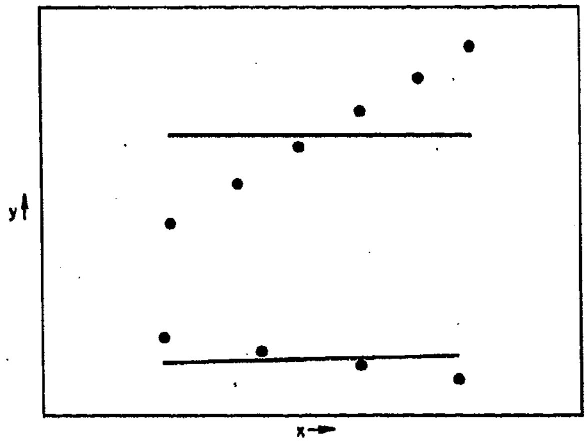

FIG. 2—POSSIBLE FINAL TRUNKLINE LOCATIONS GIVEN BY METHOD OF STEEPEST DESCENT.

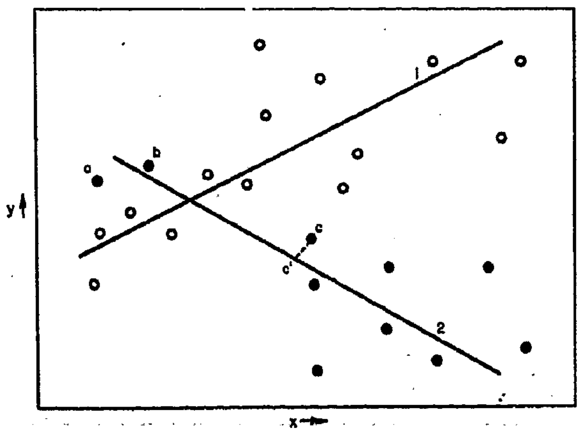

FIG. 3—OPTIMUM POSITIONS OF TWO TRUNKLINES.

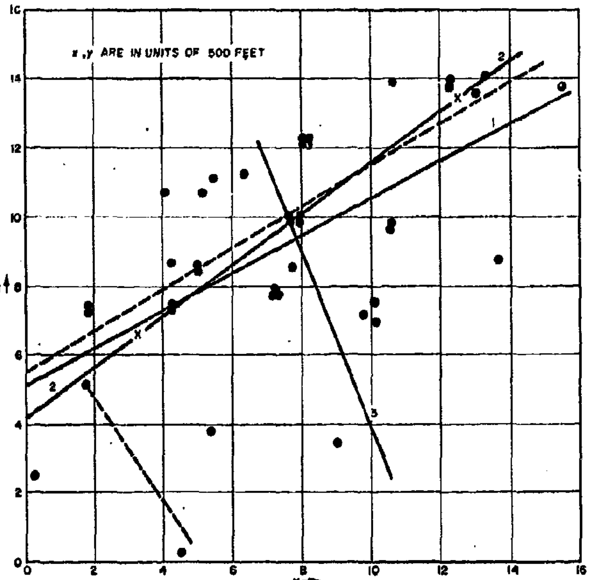

FIG. 4—WELL LOCATIONS IN FIELD "A".

resulting cost was $64,782. If the trunkline is shortened an optimum amount (four and five wells connected non-perpendicularly at the ends), then the cost falls to $57,750. If the slope of 0.54 is retained but the intercept b is changed to 2,878.5 (which results in equal numbers of wells above and below trunkline), then the cost (with all connections perpendicular) falls from $64,782 to $63,330.

The optimum location of a single trunkline was determined by calculating C from Eq. 13 for increments in m of 0.05 from $m = -5$ to $m = +5$ and for increments of 0.01 in the region of minimum C. All w_i were unity and w_T was 3.0. The optimum b for each m was determined from Eq. 17. The results are shown by the solid curve in Fig. 5. The minimum cost of $61,429 was attained at $m = 0.75$, $b = 2,075$. The maximum cost of $79,456 occurred at $m = -2.5$, $b = 14,450$. The positions of the trunklines giving minimum and maximum costs are given by lines 2 and 3, respectively, in Fig. 4. Also shown in Fig. 4 is the trunkline length as a function of slope. The trunkline length is seen to be a maximum at the slope where total cost is a minimum. As the weight factor w_T is taken larger, the optimum slope will tend toward -1.0 which minimizes the trunkline length.

The saving in cost due to shortening the trunkline was determined from Eq. 22. At each value of m the optimum b was determined from Eq. 17 and the numbers of wells connected non-perpendicularly at both ends were determined so that C' was a minimum. The results are shown by the dashed curve in Fig. 5. The minimum cost of $54,780 occurred at $m = 0.75$, $b \approx 2,075$. Thus, an additional saving of $6,650, or over 10 per cent of the $61,429 cost, is obtained by shortening the trunkline to an optimum length. The optimum numbers of wells connected non-perpendicularly at the trunkline ends ranged from 12 and seven at $m = -4$ to five and four at $m = 0.75$. The trunkline length giving the $54,780 cost is indicated by the X's in Fig. 4.

TABLE 1—RESULTS OF TWO-TRUNKLINE CALCULATIONS*

Starting Values				Final Values						
m_1	m_2	b_1	b_2	m_1	m_2	b_1	b_2	C	N_1	N_2
.5	.5	8	4	.5	.5	8	4	67,315	21	19
0	0	12	8	.42	.08	8.6	7.16	60,237	18	22
0	0	12	6	.42	.08	8.6	7.16	60,237	18	22
−.5	−.5	10	15	.42	.08	8.6	7.16	60,237	18	22
0	0	12	10	.44	.54	8.47	4.01	69,724	17	23
0	0	8	4	.5	−1.5	6.1	7.8	60,583	35	5
0	0	10	4	.6	−1.5	5.5	7.8	60,185	35	5
1	1	2	1.4	1.08	1.56	3.27	−6.75	66,825	22	18
**1	1	2	1.4	1.10	1.56	3.14	−6.72	17,846	22	18
***1	1	2	1.4	1.10	1.56	3.14	−6.72	50,511	22	18

*b_1 and b_2 are given in units of 500 ft, $w_T = 3.0$
**$w_T = 0$
***$w_T = 2.0$

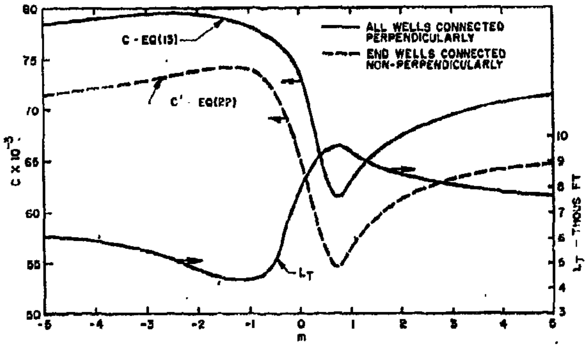

FIG. 5—PIPELINE NETWORK COSTS VS TRUNKLINE SLOPE.

The optimum location of two trunklines in Field A was attempted by use of the method outlined under Eq. 37. Results were obtained by employing an initial Δm of 0.5 and reducing Δm twice by a factor of five. A given Δm value was employed until no change of m_1 and m_2 occurred. The results are tabulated in Table 1. The final values of m_1, m_2, b_1, b_2 are seen to depend upon the starting point. Thus, there is no assurance that any of the results in Table 1 give the true minimum value of C. The minimum cost attained, $60,185, corresponds to the trunkline locations indicated by the dashed lines in Fig. 4.

In summary, the results of application of above equations to Field A are: (1) the least-squares method gives a pipeline network cost 5.5 per cent greater than that given by Eq. 13 which involves perpendicular connection of wells to the trunkline; (2) the difference in cost between the best and worst slopes for a single trunkline (intercept b being optimum in both cases) is 29 per cent of the minimum cost; (3) an additional savings of 10 per cent beyond the minimum cost given by Eq. 13 is obtained by shortening the trunkline to an optimum extent and connecting end wells non-perpendicularly; (4) with all wells connected perpendicularly in both cases, the use of two trunklines gives a slightly lower cost than one trunkline, although the true minimum cost for the former case has not necessarily been obtained; and (5) if the least-squares slope is employed but the least-squares intercept altered so that equal numbers of wells lie above and below trunkline, then a cost of $63,330, only 3.1 per cent greater than the $61,429 cost for perpendicular connections, is obtained. For this field case, at least, a nearly satisfactory location for a single trunkline is obtained from least-squares slope, optimum b, and optimum trunkline shortening—a cost of $56,250 compared to minimum cost for perpendicular connection (with line shortening) of $54,780, a 2.7 per cent difference.

CONCLUSIONS

Methods are described for locating a single trunkline to minimize piping cost for the various fluid-gathering or injection systems required in field operations. The methods require knowledge of locations of all wells or lease batteries, can treat uniform or varying sizes of connecting lines and allow a number of wells to be lumped together and connected through a common line to the trunkline. The least-squares slope can be used in conjunction with the optimal intercept and line shortening to give a satisfactory line location in the field case considered.

A partially satisfactory method is discussed and applied for optimum location of two trunklines in a field. Further research is needed to develop a method capable of optimally locating two or more trunklines, taking into account various non-optimal individual well connections which result in over-all network savings (discussed under Eq. 37). Further study is also needed to integrate line-sizing calculations into the line placement problem. Some simplification may occur in situations where the point of joining of multiple trunklines is fixed due to plant or main pipeline location.

The savings obtained by adhering as closely as possible to the optimum trunkline position may be appreciable in large fields requiring 80 to 150 miles of piping for a single system. Similarly, the savings may be appreciable when a number of smaller fields are considered.

NOMENCLATURE

b = trunkline intercept
C = cost of connecting pipe in field
l_i = perpendicular distance from field unit i to trunkline
L = total length of connecting piping in field
L_T = trunkline length
m = slope of trunkline
N = number of field units
w_i = weight factor for connecting pipe for field unit i
w_T = weight factor for trunkline
x_i, y_i = co-ordinates of field unit i

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