TPG 4150 RESERVOIR RECOVERY (METHODS)

Note Title 8/20/2018

CURTIS HAYS WHITSON

RESERVOIR RECOVERY METHODS
① ② ③

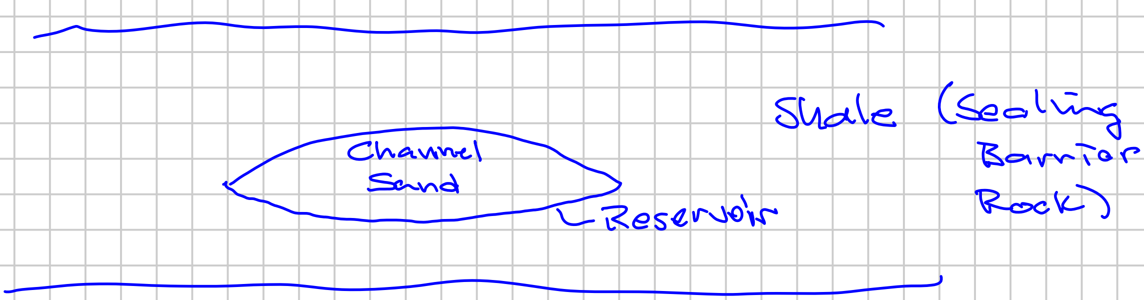
① RESERVOIR - What is it?

A porous and permeable rock containing fluid (e.g. hydrocarbons, HC) resources that can be accessed (and usually produced) by wells drilled from, connected and controlled at the surface by equipment and facilities such as pumps, compressors, separators, pipelines, and storage tanks.

(a) Geologic (and Petrophysical)
$\phi, k, P_c \ldots$

↓

- Stratigraphic Container



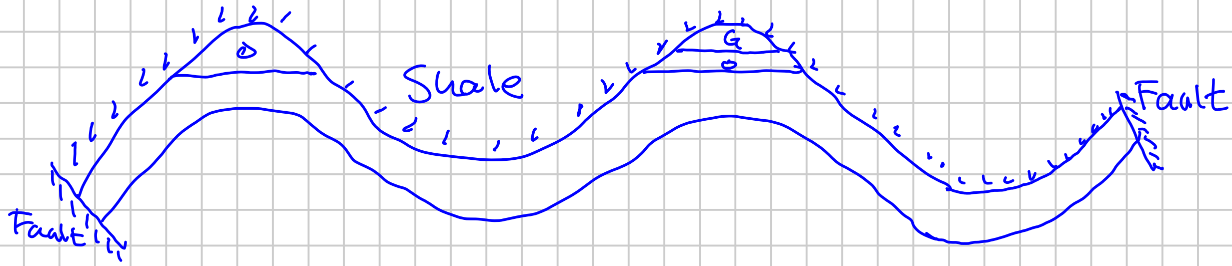
Side View

" HCs (and associated waters e.g. Aquifer AQ) are sealed in all directions by ultra-low ($k \to 0$) permeability rock/effective barriers. Sealing barrier rocks are usually "neighbor" strata to the HC-bearing reservoir, most often "shale". The sealing rock, if organic rich, may be the source of HCs." CHW

– Structural Container

Laid down ~ horizontally or flat at an angle

Transformed via folding, salt dome intrusion, faulting ...



(possible) Hydrostatic Communication with surface

"HCs trapped by overlying impermeable barriers (e.g. shale layers or faults), often with significant deformation of the ~ flat strata laid down originally – e.g. by uplift (salt domes), folding, etc.

NEXT LECTURE:

- Barriers (No Flow)
  - Types
  - Importance (Flow Communication | Well Placement | Gravity)
- Mapping
  - Structure
  - Isopach
  - ϕ, k, S_w, "Net" (N/G R), P_c
  - Initial Pressures & Temperatures (& Fluids)

http://www.ipt.ntnu.no/~curtis/courses/Reservoir-Recovery/2018-TPG4150/

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RESERVOIR ① RECOVERY ② METHODS ③

HCPV $\xrightarrow[\text{"PVT"}]{\text{TPG4145}}$ Surface Products

PV

V_b

- Surface Oil / Stock Tank Oil $\}$ $\bar{o}$

$q_{\bar{o}}$

- Surface Gas $\bar{g}$

Energy:

6 Mscf ~ 1 STB

2.5 \$/Mscf   \$70/STB

\$15/BOE

- Volumetric Material Balance
- Darcy Flow Eq.

- Barriers (No Flow)
  - Types
  - Importance (Flow Communication | Well Placement | Gravity)

- Mapping
  - Structure
  - Isopach
  - ϕ, k, S_w, "Net" (N&R), P_c
  - Initial Pressures & Temperatures (& Fluids)

Fault Block
Layer

RESERVOIR FLOW UNITS (RFU)

Reservoir volumes that are not connected or do not have flow between themselves, caused by two types of flow barriers:

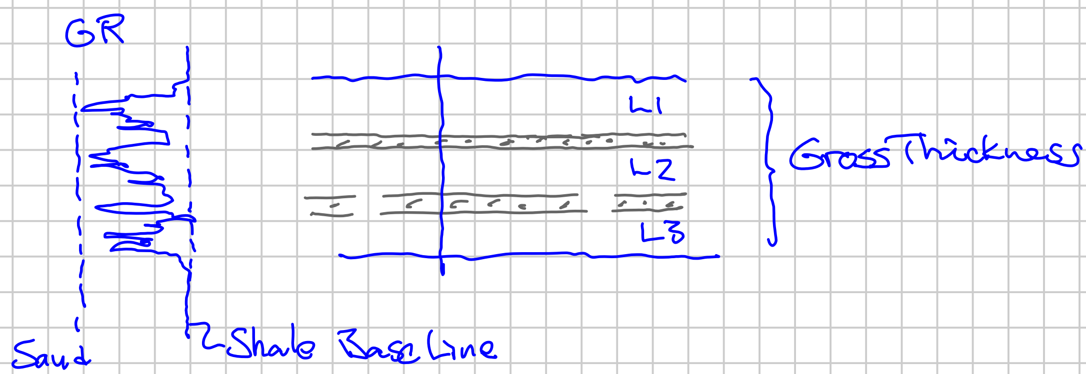
(1) Fault : (a) sealing fault $k_F = 0$
(b) Leaky fault $k_F > 0$

Always consider both cases

(2) Very-low k rock, normally shale

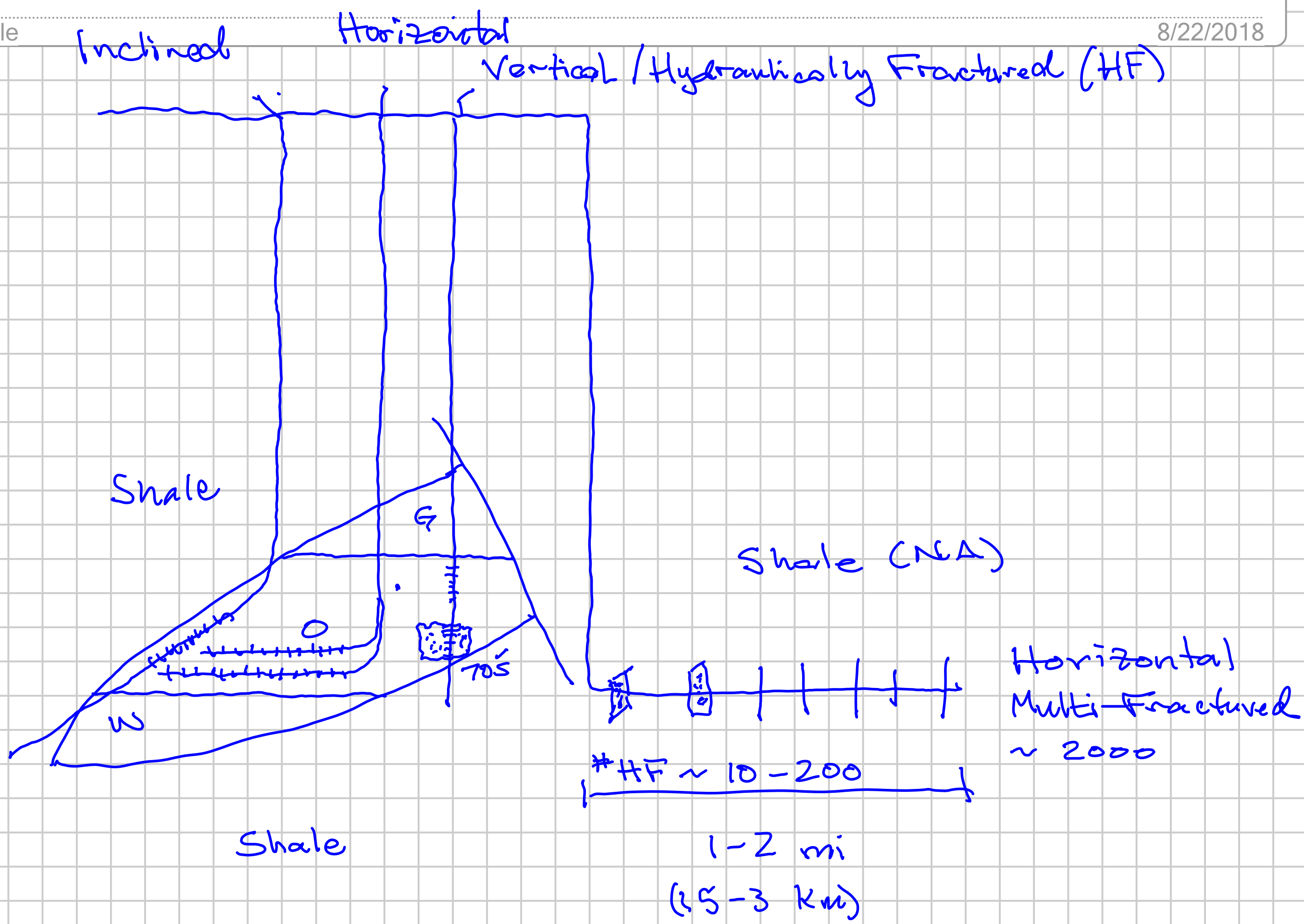
important in defining "non-net pay"

Define "very low"



net thickness (rock) ⇒ yields recovery of HCs

Well Types



RECOVERY

- Recovery of Surface Products

STO SO $\bar{o}$

SG $\bar{g}$

Categories of Recovery Processes

SO (RO)

EOR ② / IOR

EOR : Enhanced Oil Recovery

- Inject a Fluid into the Reservoir ①

W+G
- Water (saline)
- Gas (HC | CO_2 | N_2 : Flue Gas)
- Chemicals added to the above

Natural "Depletion"
- No Injection

IOR: Improved Oil Recovery
- EOR
- Well Type & Well Completion
(Depletion and/or EOR)

Impact of <u>Barriers to Vertical Flow</u> on

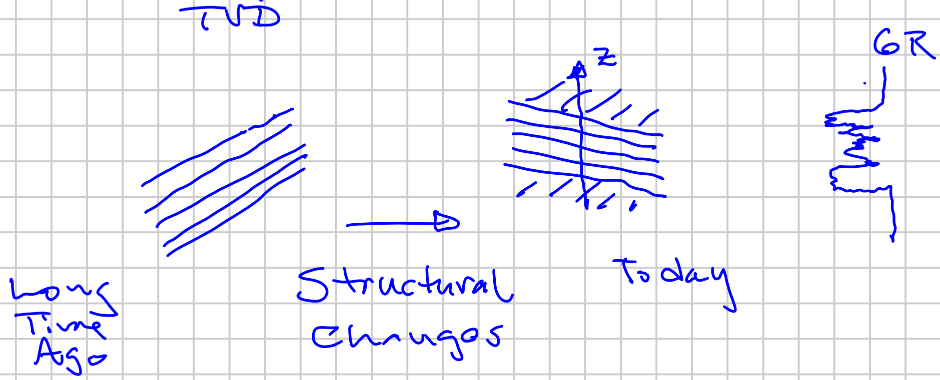
① Depletion
② EOR
} & IOR (Well Type)

Reservoir rock is made up of sequential, stacked "history".

k_z $k_v(z)$ ← TVdepth, TVD

k_{xy} : controlling flow R → WB

Wellbore



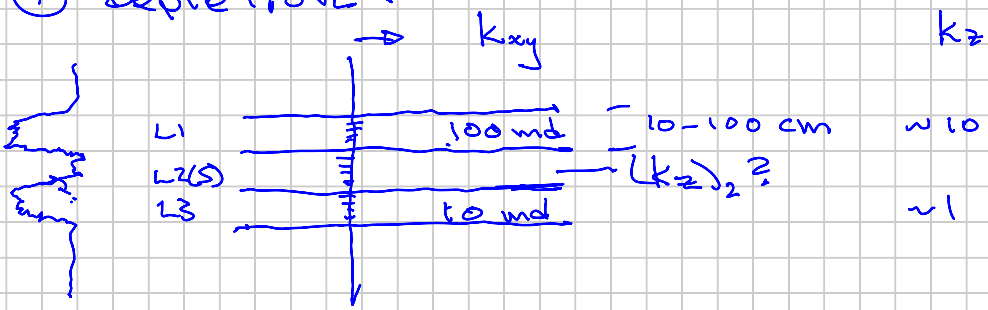
k_v : Defining "Effective" RFUs

- Depletion

- EOR (Injecting Fluid G&/or W)

$$\frac{k_z}{k_{xy}} \sim 1 \to \underbrace{0.1 \to < 0.01}$$

① Depletion :

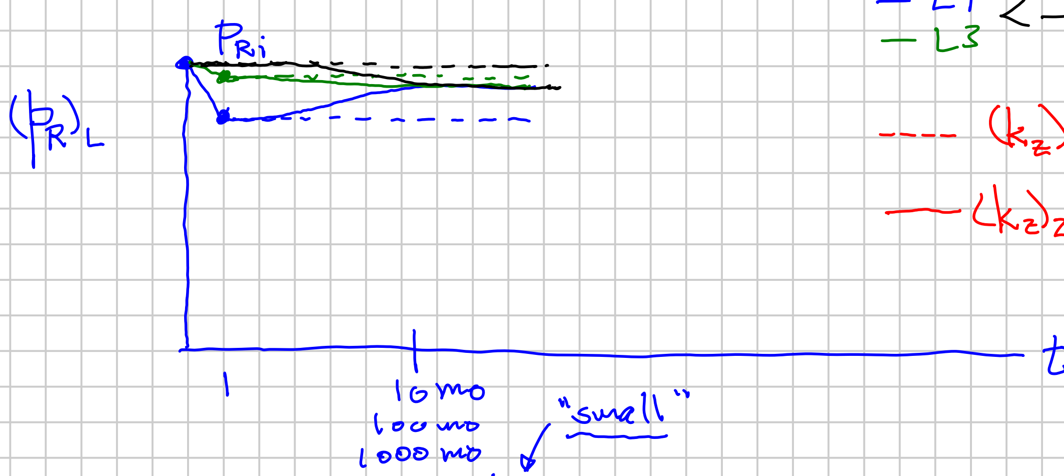


$(k_z)_2 \sim 10^{-x}$ md

Typical Shale
10 ~ 10,000 nd
$10^{-5} - 10^{-2}$ md

Time 0 :

$q_L \propto (k_{xy})_L$



— L1 < — L2
— L3

---- $(k_z)_2 \cong 0$

— $(k_z)_2$ "sufficient"

"small"

$$q_z = \frac{k_z}{\mu L} \Delta p \cdot A_\perp$$

thickness of shale

huge

$k_z \sim 10^{-3} - 10^{-4}$ md

② EOR

$p_R \sim$ constant

Driving force of IW / IG $\qquad \underbrace{\Delta \rho_{ow} \cdot g}_{\Delta \phi} \quad \underbrace{\Delta \rho_{og} \cdot g}_{\Delta \phi}$

$$q_z = \frac{k_z A_\perp}{\mu L} \Delta \phi_r$$

"small"

$\cancel{q_z}$

@ g^W ~ 0 small even for $(k_z)_{LZ}$ "large" mds

more "RFU_{EOR}" than "RFU_D"

Note Title 8/27/2018

"RESERVOIR"

① Geologic & Petrophysical

* Barriers

— To Vertical Flow

• k_v low $\Rightarrow 0$ "shaly" (& other types)

— To Areal Flow

• Faults ($T_F = 0$ → 0 > 0)

• Stratigraphic (pinchouts)

Defines RFUs
- Depl.
- EOR

* Spatial Variation of Rock (& Fluid) Properties

Mapping
Well
bore logs
Petrel
Geostatisticians

$\underbrace{k_z \quad k_x \quad k_y}_{f(x,y,z)}$ Anisotropy in all direction

- $\phi(x,y,z)$
- $S_w(x,y,z)$

$S_{HC}(x,y,z) = 1 - S_{wc}$ ← connate ("initial")

Core

$k_h(\phi)$

$k_v(\phi)$

└ Henning Omre, NTNU

② Fluid "Systems" & "Properties"

TPG4145

Fluid Systems

- Reservoir "Gas" (RG) $RF_g \propto (p_{R\,abandonment})$
  (Dry & Wet)
- Reservoir "Oil" (RO) $p_{Ri} \geq p_b$ } $p_b(x_{Ri}(Z, X_{ij}, T_R))$
- Aquifer (AQ)
  - Transition Zone ($Z < Z_{FWL}$)

    S_w from 100% to

    $k_{rw} \sim 0$

    $p_w = p_o$

    Free Water Level "(OWC / GWC)"

    Shallowest Z where $S_w = 100\%$

    "OWC, GWC"
  - Below FWC "Paleo" Zone

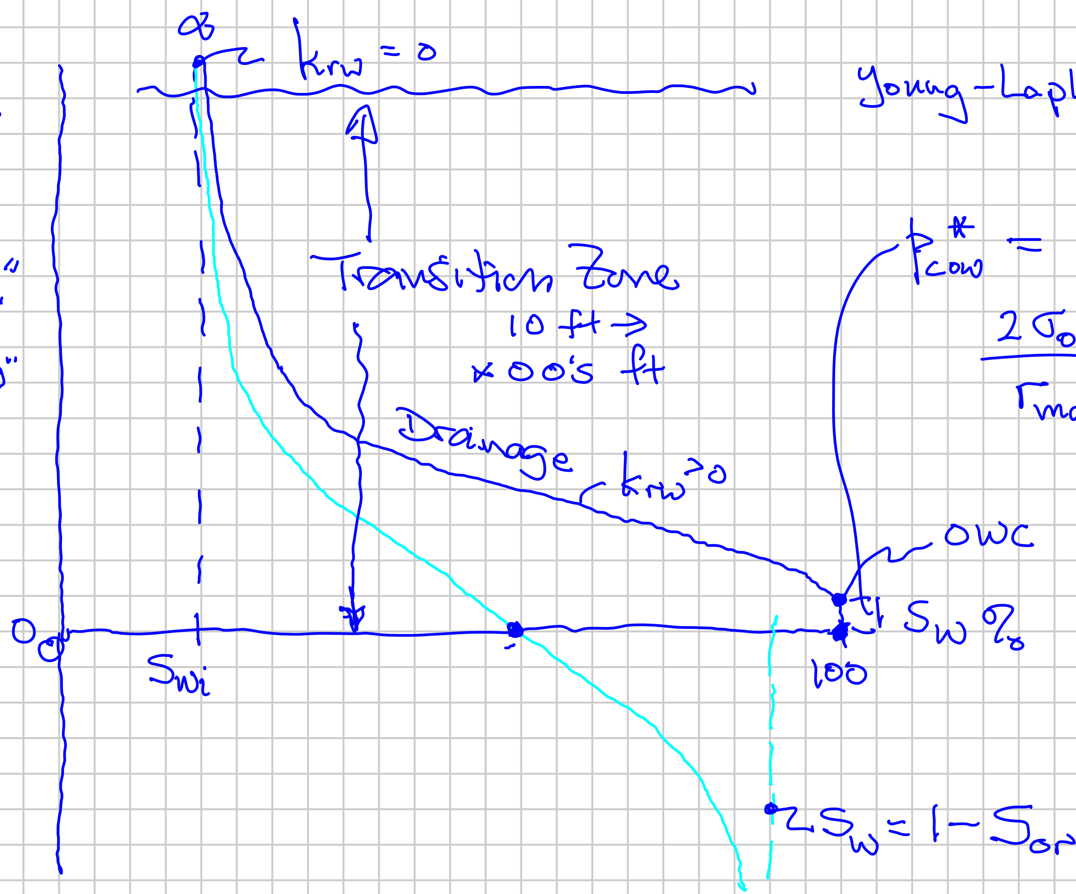
    $k_{ro} \sim$ low $\rightarrow 0$

$P_{cow} = p_o - p_w$

or

"k"

Depth = $\dfrac{P_{cow}}{(\rho_w - \rho_o)g}$ "PS"



Young-Laplace

$P^*_{cow} = \dfrac{2\sigma_{ow}}{r_{max}}$

Transition Zone : $[k_{ro} > 0 \text{ \& } k_{rw} > 0]$ $k_{ro} = 0$

$k_{ro} \simeq 0$ to $k_{rw} \simeq 0$

– RG ("Gas Cap") & RO ("Oil Rim" Zone)

- Saturated Fluid System

  Gas-Oil Contact transition $p_{R,GOC} = p_{d,GOC} = p_{b,GOC}$

  Most "common" situation assumption

  RG in Gas Cap Uniform $y_{Ri} \sim$ constant (X, Y, Z)

  RO in Oil Zone Uniform $x_{Ri} \sim$ —— " ——

  – May have a smaller gas-oil transition zone

$$\text{Depth} = \frac{P_{cgo}}{(\rho_o - \rho_g) g}$$

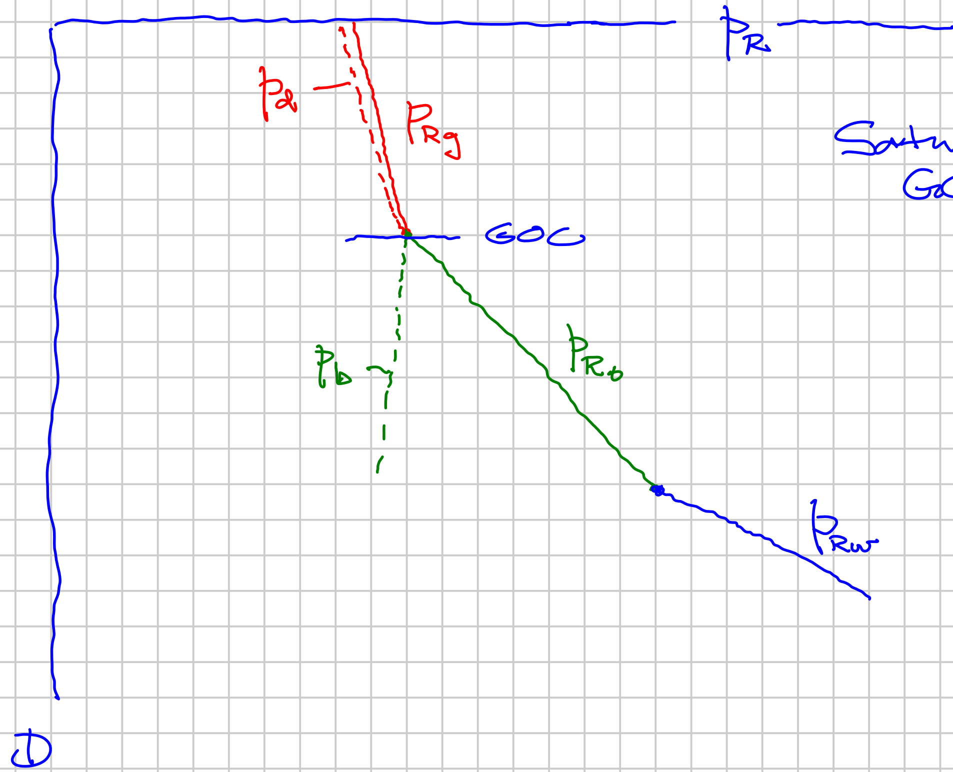
because usually $(\rho_o - \rho_g) > (\rho_w - \rho_o)$

$$P_{cgo}(S_o) < P_{cow}(S_o)$$

Lab $P_{c\,air-w}$ $\Rightarrow (P_{cgo})_R = P_{ca-w} \cdot \frac{(\sigma_{go})_R}{(\sigma_{a-w})_L}$

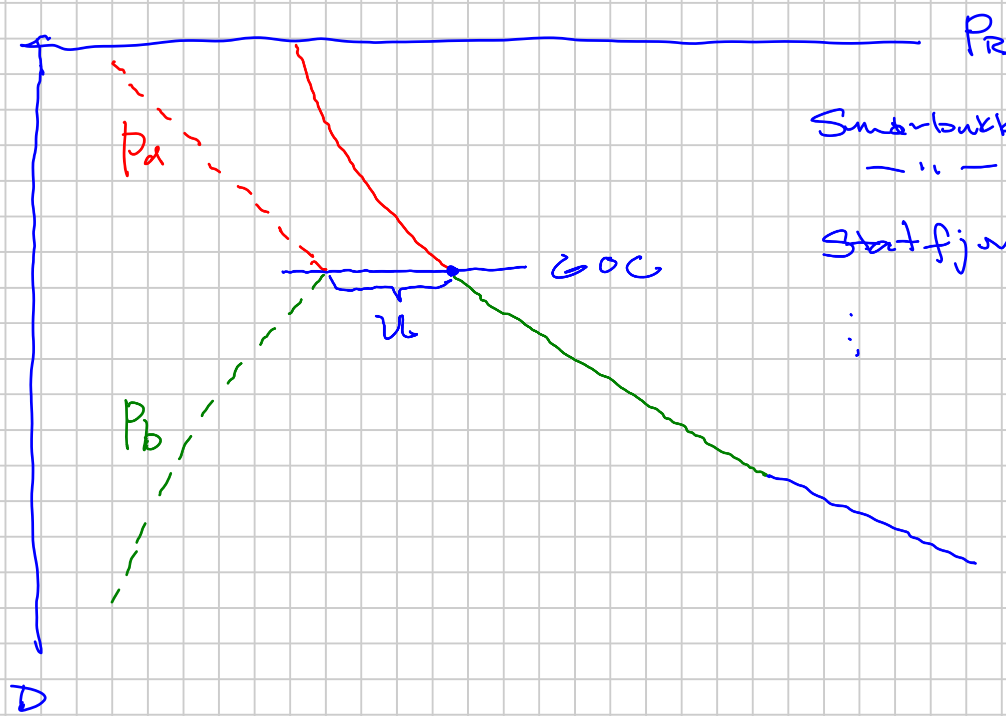
$(\sigma_{ow})_R \sim 40 - 20$ mN/m $\quad (P_{cow})_R = P_{ca-w} \cdot \frac{(\sigma_{ow})_R}{(\sigma_{a-w})_L}$

$(\sigma_{go})_R \sim 40 \rightarrow <1$ $\quad$ 70 mN/m



Saturated GOC

Undersaturated Gas-Oil Contact



Snorrebukk

— " — South

Statfjord

⋮

Strong Compositional Variation with Depth $z_{Ri}(Z)$ substantial

Final Eq. is reached by molecules moving around to minimize total energy.

no force field

Chemical Energy μ_i

Molecular Diffusion $q_i = D_i \left(\frac{d\mu_i}{dZ}\right)$

Eq. reached $q_i \Rightarrow 0$

In force field a = const.

$$\Phi_i = \mu_i + M_i \, a \, \Delta Z$$

$$q_i = D_i \left(\frac{d\Phi_i}{dZ}\right) \Rightarrow q_i = 0$$

Muskat 1930s : $\mu_i \simeq$ simplified liquid model

Sage Lacey 1937 : $\mu_i \sim$ more rigorous model

⋮

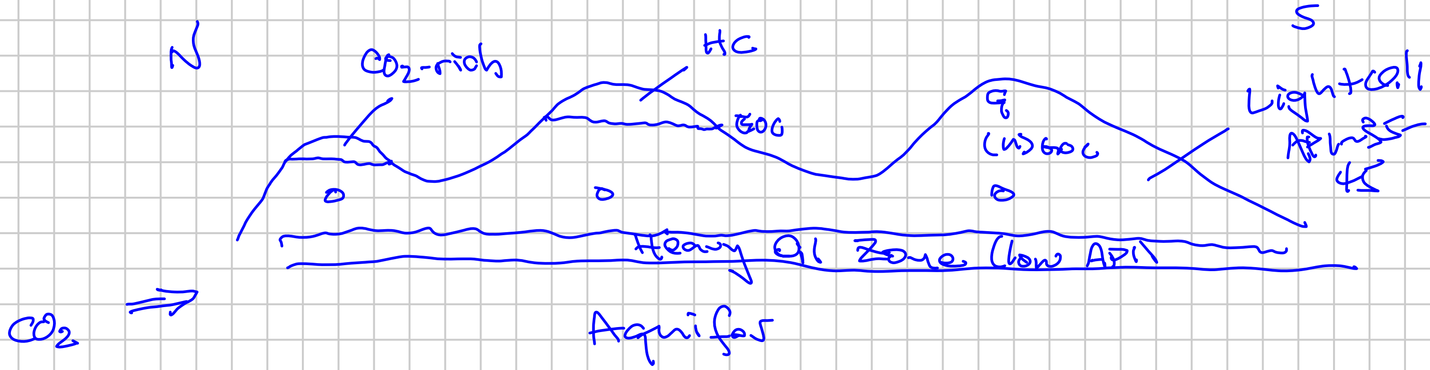
1980 : Tom Schulte (Shell)

$z_{Ri}(Z \,;\, \mu_i^{\text{Cubic EOS}})$

- Brent Field
- Birba Oman

- Non-Equilibrium Fluid Systems

Italy Val d'Agri


N
CO_2-rich
HC
GOC
S
Light oil
API 35-45
(n) GOC
Heavy Oil Zone (low API)
Aquifer
CO_2

CONVENTIONAL vs UNCONVENTIONAL RESERVOIRS

CONVENTIONAL

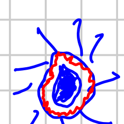
RESERVOIR TYPE (Name)	ROCK	FLUID
1. GAS (Dry | Wet | "Retrograde" Gas Condensate)	(1) $k > 0.1-1$ md (0.01); "C"; (2) $1-\phi$	$\mu < 100$ cp
2. OIL		
3. GAS + OIL		

"High" Mobility $= \frac{kh}{\mu}$ λh

UNCONVENTIONALS

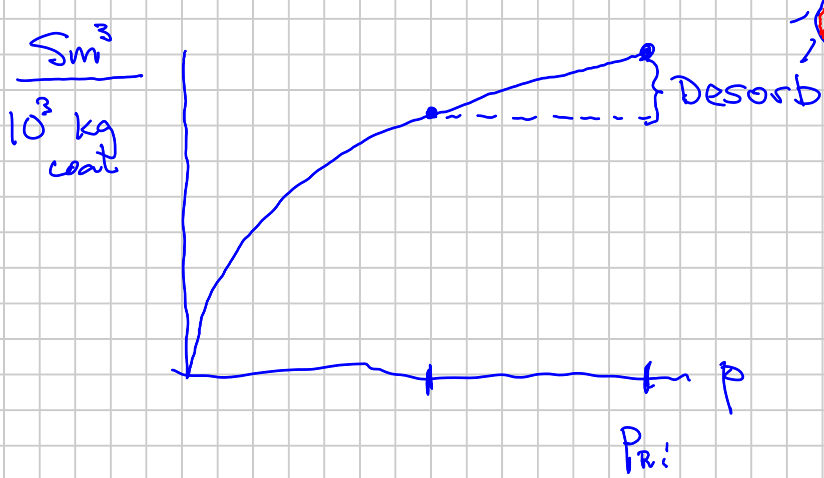
1. Heavy oil (SAGD) — p_{Ri} low; $1-\phi$, $k > 1000$ md; "U"; $\mu_o \gtrsim 1000$ cp $\downarrow 10$
2. Tight / Ultra-Tight (Shale) — p_{Ri}: high; "U" $k < 0.001 \rightarrow 10^{-5}$ md; $\mu_o < 1$ cp, Shale Gas - Sorbed Gas
3. Naturally Fractured — $2-\phi$ "U"; $k_m \sim 0.01-10$ md; $\frac{\phi_m}{\phi_t} = 90-99\%$; $\mu \lesssim 10$ cp

4. Coal Bed Methane (CBM)



Adsorbed Methane

"U"



Note Title 8/29/2018

RECOVERY FUNDAMENTALS

$\underbrace{\text{Depletion} \mid \text{EOR}}_{\text{IOR}}$

STB Sm^3

$\frac{N}{HCPV}$

RB m^3

DEPLETION

- Initial Reservoir Pressure p_{Ri}
- PVT: Initial Solution OGR (GOR) — N/HCPV r_{si} R_{si}
- Initial "Oil FVF" ($\bar{B}_{oi}$ RO or r_{si}/B_{gdi} RG) — G/HCPV STB/RB HCPV
- Total Water Volume (AQ, NGR, S_{wc})
- PVT: Oil c_o | Oil Shrinkage; Gas c_g (B_g) — μ
- LNX (# RFUs) Contrast in RFU "D" = $\left[\left(\frac{kh}{8+s}\right) / HCPV\right]_{RFU}$ (Δp)
- Rock Compressibility ("high" p_{Ri}) / Solution Gas Drive
- Gas-Oil Relative Permeabilities (RO SGD)

EOR

① – Volumetric Sweep ("Conformance")
"Macroscopic" Sweep

- Vertical (layering) $k_h(Z)$, gravity ($k_v(Z)$)
  - $k_h(Z)$ → Orders of magnitude variation $k_h(Z)$

- Areal
  - Well placement ("pattern")
  - Well type
  - (k_x k_y anisotropy)

② Microscopic Sweep
"Pore-Level" — " —

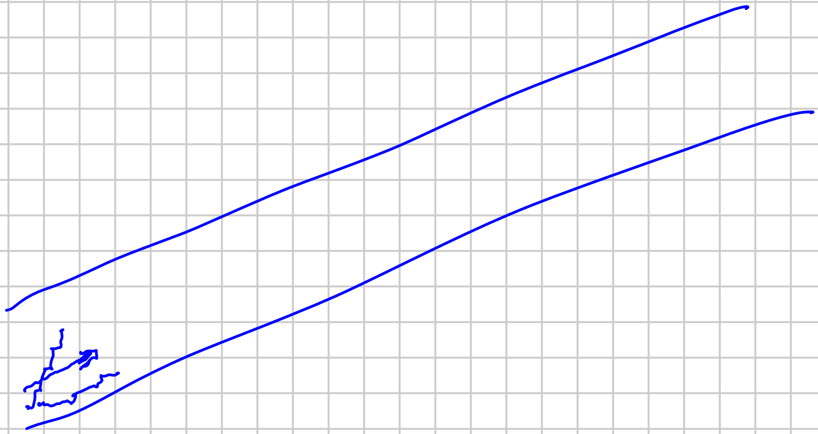
In the volumetrically swept volume
Residual Oil + S_{or} where $k_{ro} \to 0$

Buckley-Leverett 1942 80% $S_{oi} = 1 - S_{wc}$
↓
at BT Inj $\boxed{50\%}$ S_{of} > $S_{orw} \cong 25\%$
↓
Keep Injecting Many (∞) PVs S_{orw}.

- Oil-Water wettability
- Pore size distribution (k_{rw} & k_{ro})

• Gravity v_z strong

$S_{oi} \rightarrow S_{or}$



(a) BL: Immiscible, Equilibrium $\quad S_{oi} \rightarrow S_{of} \rightarrow S_{or}$

O-W $\quad$ O-G

(b) Non-Equilibrium Gas-Oil Displacement (Complex PVT)

Pore-Level

$RF_o = 100\%$

- Develop "Miscibility" $\quad S_{org} \rightarrow 0 \quad$ 100% RF microscopic
  - • Vaporizing Gas Drive (VGD)
    - Gas Inj in a Gas Condensate R
    - Gas Inj in an Oil R, (10%)
      - • High p_{inj}, Light Oils, Lean C_1-rich inj gas
  - • Condensing/Vaporizing Gas Drive (90%)

    Aaron Zick 1986

- Immiscible (Swelling | Vaporizing)

  S_{org} from $(S_{org})_{Eq.Gas\ BL} \rightarrow$ 20 10 0%

  25%

  Many PVs of gas passing over the oil

RESERVOIR RECOVERY METHODS

Note Title 9/3/2018

(A) DEVELOPMENT METHODS

① Smaller and often Onshore Projects

(G&G) Drill – Discover (HCs) – Produce – Monitor – Modeling (HM) ⇒ Forecasting (& Today's optimization) q's p's

Theory
Eqs.
Num. methods
Data
Tune

History Matching

② Larger and Usually offshore Projects

G–G – Drill – Discover (HCs) – (Might Short Production Test) – Drill & Delineation –

(a) Modeling (???) – Design – PDO
Planned Development Operation

Sensitivity Analysis
Statistics
Monte Carlo

(b) – Development – Produce – Monitor – Modeling (HM) ⇒ Forecasting (& Today's optimization)

(B) Modeling Methods $p_R(Q_p)$ N_p N

① (Volumetric) Material Balance : $p_R = f(p_{Ri}, Q_p, Q_i ; Q_{inj})$

Product Volumes

$\boxed{k \gtrsim 0.01 - 0.1}$

g o G_p IGIP G

$[\text{IOIP IGIP WIP}]^{\text{INITIAL}}$ = Produced − Injected + Remaining

Sm^3 stb Mscf

(a)

e.g. Gas M.B. "Pot Aquifer"

$$\frac{p_R}{Z_{gR}}\left[1 - c_e(p_{Ri} - p_R)\right] = \left(\frac{p_{Ri}}{Z_{gi}}\right)\left(1 - \frac{G_p}{G}\right)$$

G_p = Cum. Gas (surface Gas Product) Produced

G = IGIP (— " —)

$p_R(Q_p)$

(b) Each and All RFUs

Aside : Reservoir Simulation of Depletion

$N_{cells} \simeq N_{RFU}$

if ands and buts (k sufficiently high)

$p_R \sim p(X, Y, Z)$

during $\Delta t = 1-6$ mo.

② Approximate Flow Equations

- Mainly Darcy (In reservoir w/reservoir rates / surface product rates)
- Depletion

• Surface Rate Eq.

$$q_g = \frac{kh\,(p_{pR} - p_{pwf})}{T_R \cdot \left[\underbrace{\ln\frac{r_e}{r_w}}_{7\text{-}10} + s + Dq\right]} = \underbrace{\frac{dG_p}{dt}}$$

$$p_p = 2\int_{p_{ref}}^{p} \frac{p'}{\mu Z}\,dp'$$

Single Phase Oil

$$\frac{dQ_o}{dt} = q_o = \frac{kh\,(\overset{\text{M.B.}}{p_R} - p_{wf})}{\mu_o B_o\left[\ln\frac{r_e}{r_w} + s\right]} = \frac{dN_p}{dt}$$

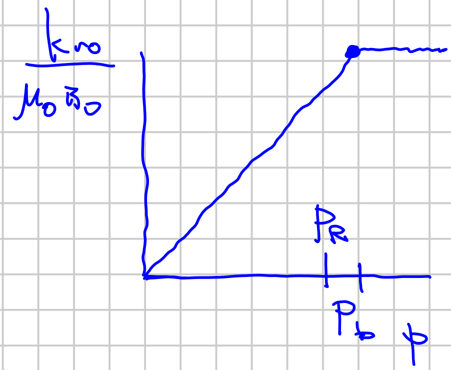
STB/D

Sm³/D

$$-6 < s < +100$$

S&D $\quad p_R < p_b$

$$q_o = \frac{k_{ro}\,kh\,(p_R^2 - p_{wf}^2)}{\mu_{ob}B_{ob}(2p_b)\left[\ln\frac{r_e}{r_w} + s\right]}$$

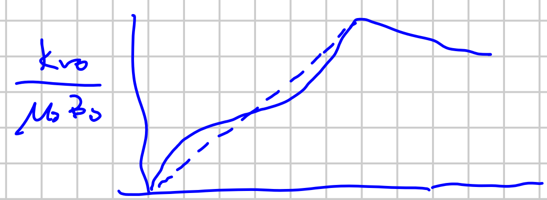


Fetkovich

Alt. ⇒ Vogel (100s of Simulations)

Muskat (1930s-40s)

Meyers



MB : $p_R(Q_p)$

Rate : $q = \frac{dQ_p}{dt} = \text{constant} = PI\,(p_R - p_{wf})$

$\boxed{q(t) \Rightarrow \text{Forecast (Today's Optimization)}}$

Arp's Decline Curve Eq. : $\underbrace{MB} + \underbrace{\text{"IPR"}} \Rightarrow q(t)$

$$q = \frac{q_i}{\left[1 + bDt\right]^{1/b}}$$

$$q_i = PI\,(P_R - P_{wf}) = \text{constant}$$

$0 < b < 0.5$ — Single phase, low c oil; Low Pressure Gas; SGD

Decline Constant

$$D \equiv \frac{q_i}{\underbrace{\underbrace{(Q_i \cdot RF_{ult})}_{\text{"EUR"}}}_{\text{"Rate of Depletion"}}} \cdot \frac{1}{1-b}$$

"Rate of Depletion"

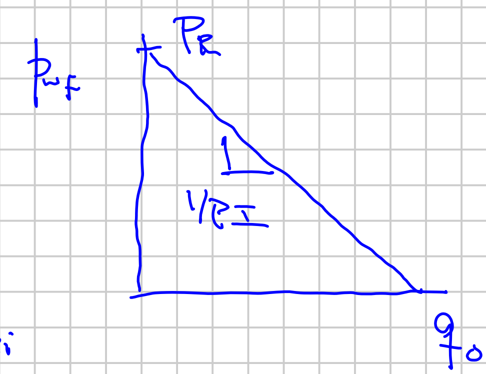
Contrast $\Rightarrow$ "Differential Depletion"

RFU_1 vs RFU_2

Homework Fun:

$$q_o = PI\,(\underbrace{P_R - P_{wf}}_{\text{const}})$$

$P_R(t)$



$$P_R = P_{Ri}\left(1 - \frac{N_p}{N_{pult}}\right)$$

$\Rightarrow$ Arp's Decline Eq.?

"EUR" = N_{pult}/N

EOR : Displacement of In-situ Fluid by Injected Fluid

- Two or more wells ($I \rightarrow Ps$)
- $v_D = \frac{k}{\mu} \cdot \frac{dp}{dx}$ Darcy Velocity

  - Injected (w)
  - Produced (o)

$$v_{wD} = \overbrace{\left(\frac{k_w}{\mu_w}\right)}^{\lambda_w} \frac{dp_w}{dx} = k\left(\frac{k_{rw}}{\mu_w}\right)\frac{dp_w}{dx}$$

$$v_{oD} = \underbrace{\left(\frac{k_o}{\mu_o}\right)}_{\lambda_o} \frac{dp_o}{dx} = k\left(\frac{k_{ro}}{\mu_o}\right)\frac{dp_o}{dx}$$

@ Reservoir Conditions

$p_w \simeq p_o$

Mobility $\lambda \equiv \frac{k}{\mu}$ md/cp

Behind Front

$$M \equiv \frac{(k_{rw}/\mu_w)}{(k_{ro}/\mu_o)} = \underbrace{\left(\frac{k_{rw}}{k_{ro}}\right)}_{\text{Rock}}\underbrace{\left(\frac{\mu_o}{\mu_w}\right)}_{\text{Fluid}}$$

Mobility Ratio

Ahead of Front

Rel. Perm. Ratio

Viscosity Ratio

Fractional Flow (Ratio)

$$f_w(S_w) \equiv \frac{v_{wD}}{v_{wD} + v_{oD}}$$

$$= \frac{1}{1 + \frac{k_{ro}}{k_{rw}}(S_w)\frac{\mu_w}{\mu_o}}$$

$k = 200$ md

$k = 100$ md

$h_1 = h_2$

Breakthrough occurs at the Same Time in both layers. $\phi_{L1} = 0.2$ $\phi_{L2} = 0.10$

Transport $v \neq v_D$

ϕ

Note Title 9/4/2018

Quiz

1 page only (front/back), Name written

① Write the Pot Aquifer Material Balance Eq.

$$\frac{p_R}{Z}\underbrace{\left[1 - c_e(p_{Ri}-p_R)\right]} = \left(1-\frac{G_p}{G}\right)\left(\frac{p_{Ri}}{Z_i}\right)$$

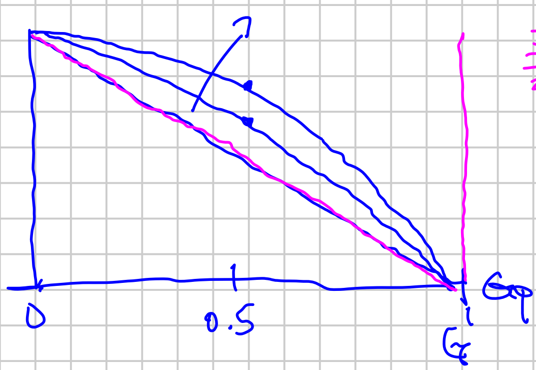
② Define the parameters needed to calculate c_e (?) M (?) c_e: M, c_w, c_f | M: V_w

③ What are the basic assumptions (list 2-3) of the Pot Aquifer model? $[p_w \simeq p_g]$

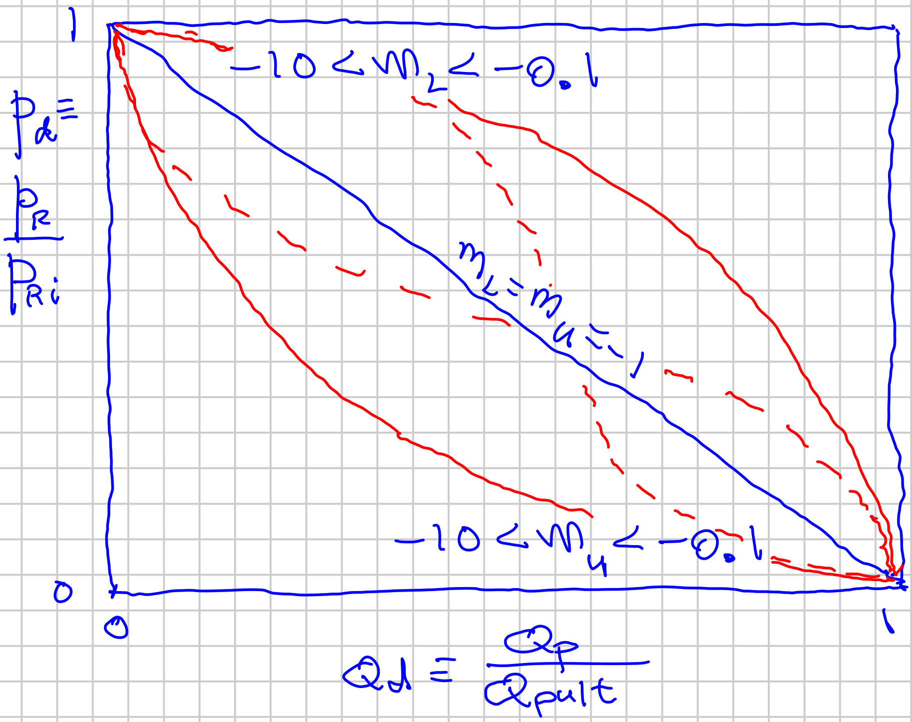
④ Draw p/Z vs Gp for straight-line material balance & for MB w/ Pot Aq.
(a) small water volume (b) large water volume

⑤ When is the $c_{tw}(p)$ important? lower p's

④ $\frac{p}{Z}$



$\frac{p}{Z}\left[1 - c_e(p_{Ri}-p_R)\right]$



Find a general relation $p_d(Q_d)$ with following constraints:

$p_d(0) = 1$, $p_d(1) = 0$

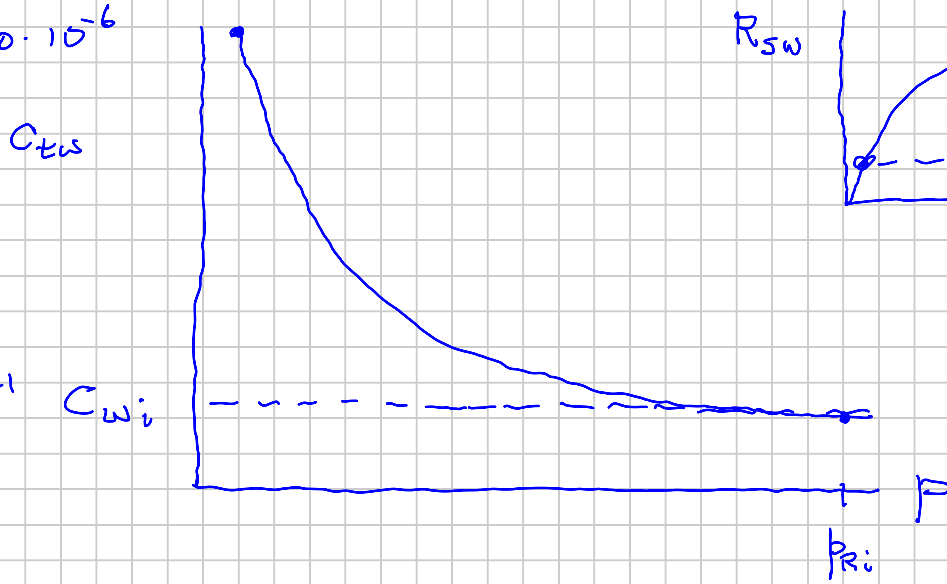
$$\left.\frac{dp_d}{dQ_d}\right|_{Q_d=0} = m_L$$

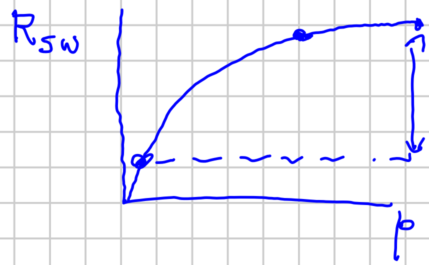
$$\left.\frac{dp_d}{dQ_d}\right|_{Q_d=1} = m_u$$

Note Title 9/5/2018

$C_{tw}(p)$: Biggest impact at lower pressures

$\times 0 = 10-50\cdot 10^{-6}$





$$c = -\frac{1}{V_t}\left(\frac{\partial V_t}{\partial p}\right)_{T,m}$$

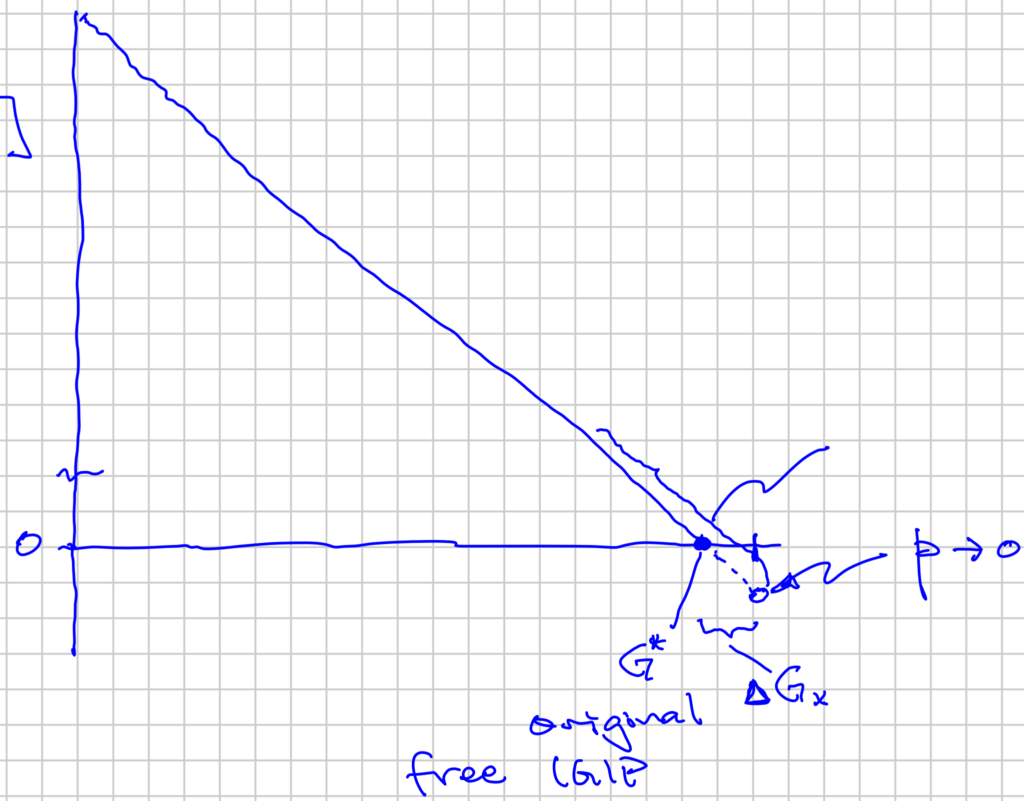
$p < p_{Ri}$ split into g+w

$$V_t = \underbrace{V_w}_{\sim\,\text{const}} + V_g(p)$$

$$V_g(p) = \underbrace{(R_{swi} - R_{sw}(p))\, B_g(p)}$$

$$\boxed{(R_{si} - R_s(p))\, B_g(p)}$$ Oil M.B.

$\frac{p}{z}\left[1 - c_e(p)(p_i - p)\right]$

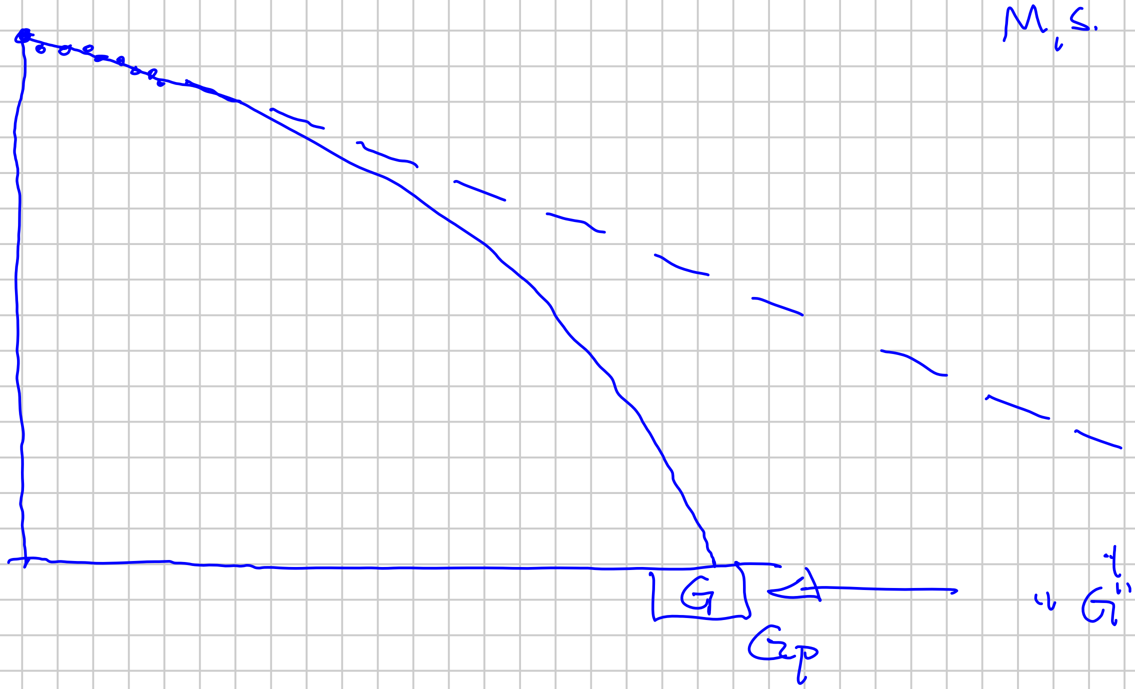


$$\text{True } G = G^* + \Delta G_x$$

$$= R_{swi} \cdot IWIP$$

M.S.

$\frac{p}{z}$



"EOR"

v_D $\qquad$ v_p

$v_D = 1 \text{ m/d}$

$t_f \neq \frac{L_f}{v_D}$

$t_f = \frac{L_f}{v_p}$

Prove:

$$v_p = \frac{v_D}{\phi \underbrace{\Delta S}_{\text{of the injected phase}}}$$

$S_{wc} = S_{wi} = 0.2$

S_w (Behind Front) In Swept Volume $= 1 - S_{orw} = 0.75$

0.25

$\Delta S_w = 0.55$

Modeling Methods

(1) Mat. Bal.

(2) Approx. Flow Eqs

- Pseudosteady State "Rate" Eqs

Oil: IPR (Inflow Performance Relationship)

Gas: BPE ("Reservoir" Backpressure Eq.)

$\left.\begin{matrix} q_{\bar{g}} \\ q_{\bar{o}} \\ q_{\bar{w}} \end{matrix}\right\} (\underline{\underline{p_R}} - \underline{\underline{p_{wf}}})$

Steady State : doesn't change with time

Pseudo SS : (a) Something does change with time

— But very slowly

(b) Now (at some time) the system behaves SS

$p_R(t)$ slowly

③ Numerical Reservoir Simulation (>1980-90s)

<1980 ⇒60s   BOAST (2) COMP1 (2) (4) "INTERCOMP" (Coats et al)
THERM   : NOLEN

>1980   ECL100   ECL300   SENSOR   VIP   NEXUS   INTERSECT ...
-MORE

Review: See paper (optional) SPE Chapter 17
by Coats et al. (2009): Optional reading.

- Finite Difference
  - Conventional   IMPES | Implicit | Adaptive Implicit
  - Streamline (key: EOR Volumetric Sweep)
    "Conformance"

- Grid Refinement
  - Each RFU!
  - Numerical dispersion (fake mixing)
    ⇒ Very problem/process dependent
    ⇒ Requires attention, always
  - Wells: PSS Rate Eqs. = $N_w \cdot N_c$
    ⇒ Needs fully implicit treatment
    - Connects Wells to Grid Cells
  - May include Reservoir-to-Surface connection
    - VL Tables (Vertical Lift)
    - VL Eqs (VIP)
  -

$p_{wf} \to p_t$
$\Delta p(q_g, q_o, q_w, p_{wf})$

DEPLETION RECOVERY

① Volumetric Material Balance

Theoretical $p_R \to 0$

$$(\bar{p}_R)_{RFu} = f\left(Q_p ; \; p_{Ri} , \; Q_{pu}\right)$$

$p_R(Q_p)$ ~ Est. Ultimate Recovery ("EUR") @ $p_R \to$ Abandonmnt

② PSS Rate Eq.

$\bar{q}$ ō

$$q = f\left(p_R , \; p_{wf} ; \; \underbrace{\frac{PI}{J}, \; C \,\&\, n ; \; A, B}_{\text{constants}}\right)$$

$$q(p_{wf}, p_R) = \frac{dQ_p}{dt}$$

③ Couple MB & Rate Eq.

[Time dependence of $q(t)$ and $Q_p(t)$ for a given, specified $p_{wf}(t)$]

- Analytically for special MB / IPR Eqs
- Iterative (Explicit) ~ Solution Excel

$$\left\{ q_o = J(p_R - p_{wf}) \qquad p_R = p_{Ri}\left(1 - \frac{N_p}{N_{pu}}\right) \right\}$$

④ Compare with Empirical (Arp's) DCA Eq.

Decline Curve Analysis

for $p_{wf} = \text{const.}$

$$\underbrace{q \simeq \frac{q_i}{\left[1 + bDt\right]^{1/b}}} = \underbrace{q_i e^{-Dt}}$$

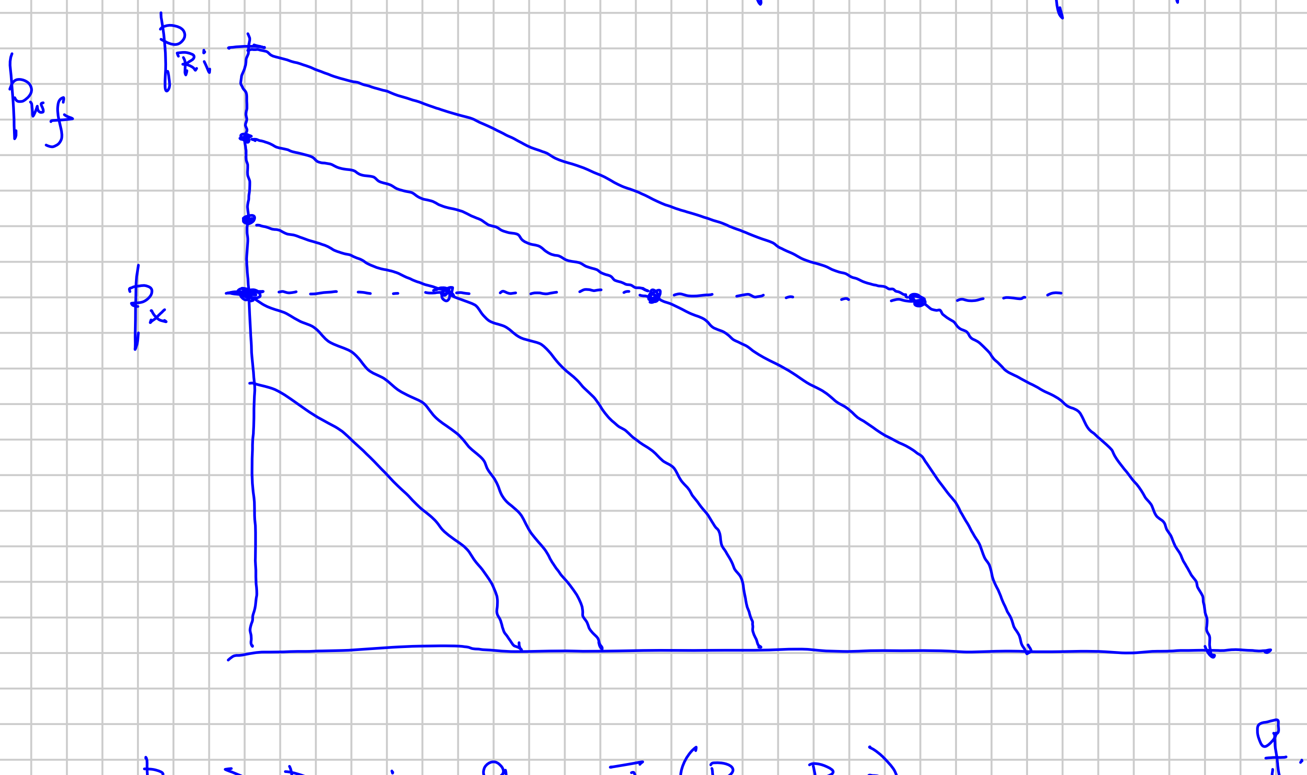
$\boxed{b = 0}$

$$\frac{dN_p}{dt} = q_o$$

Low $p_R < 200$ bar — High p, $p_R > 200$ bar

Generalized Rate Eq. Gas & Oil

Undersat. $p_R > p_b$ — Sat. (SGD) $p_R < p_b$



$p_{wf} > p_x$: $q = \underbrace{J}(p_R - p_{wf})$

$p_{wf} < p_x$: $q = J'(p_R^2 - p_{wf}^2)$

OILS : q_o $\quad p_x = \underbrace{p_b}$

GASES : q_g $\quad p_x \approx 200 \pm$ bar

Note Title 9/10/2018

Development of Gas Material Balances

② Pot Aquifer Gas MB

① Straight-Line Gas MB

ASSUMPTIONS SLMB:

* (a) $HCPV_g$ (V_{gR}) = constant during depletion

(b) Const. T_R

(c) No injected gas

(d) Produced "Surface Gas" is "surface wet gas volume"

$$G_p = \underbrace{G_{pw}}_{\text{Actual produced surface gas @ STC}} = \underbrace{G_{pd} + \underbrace{N_p \left(\frac{\rho_{\bar{o}}}{M_{\bar{o}}}\right)}_{n_{p\bar{o}}} \underbrace{\left(\frac{RT_{sc}}{p_{sc}}\right)}_{23.68\ Sm^3/kg\text{-mole}}}_{GEQ}$$

(e) Real Gas Law applies

$$pV_g = nRTZ$$

(f) $$\text{"}G_{pw}\text{"} = n\left(\frac{RT_{sc}}{p_{sc}}\right)$$

Surface Gas Volumes Used in Any/All "p/Z" gas material balances are $G_{pw} \rightarrow n$

SLMB: $pV = nRTZ \Rightarrow V_{gR} = \dfrac{n_{gR} R T_R Z}{p_R}$

$V_{gRi} = V_{gR}$ at any $p_R < p_{Ri}$

$HCPV_{gi}$

$$\frac{n_{gRi}\cancel{R}\cancel{T_R} Z_i}{p_{Ri}} = \frac{n_{gR}\cancel{R}\cancel{T_R} Z_R}{p_R}$$

Molar Material Balance

$$n_{gR} = n_{gRi} - n_p$$

$$\frac{n_{gRi} Z_i}{p_{Ri}} = \frac{(n_{gRi} - n_p) Z_R}{p_R}$$

$$\frac{p_R}{Z_R} = \left(\frac{n_{gRi}}{n_{gRi}} - \frac{n_p}{n_{gRi}}\right)\frac{p_{Ri}}{Z_i}$$

$$\frac{p_R}{Z_R} = \left(1 - \frac{n_p}{n_{gRi}}\right)\frac{p_{Ri}}{Z_i}$$

$$\frac{n_p\left(\cancel{\frac{RT_{sc}}{p_{sc}}}\right)}{n_{gRi}\left(\cancel{\frac{RT_{sc}}{p_{sc}}}\right)} = \frac{G_{pw}}{G_w} = \frac{G_p}{G}$$

$$\boxed{\frac{p_R}{Z_R} = \frac{p_{Ri}}{Z_i}\left(1 - \frac{G_p}{G}\right)} \quad \text{SLMB}$$

② Pot Aquifer Gas M.B.

(b)–(f) same assumptions

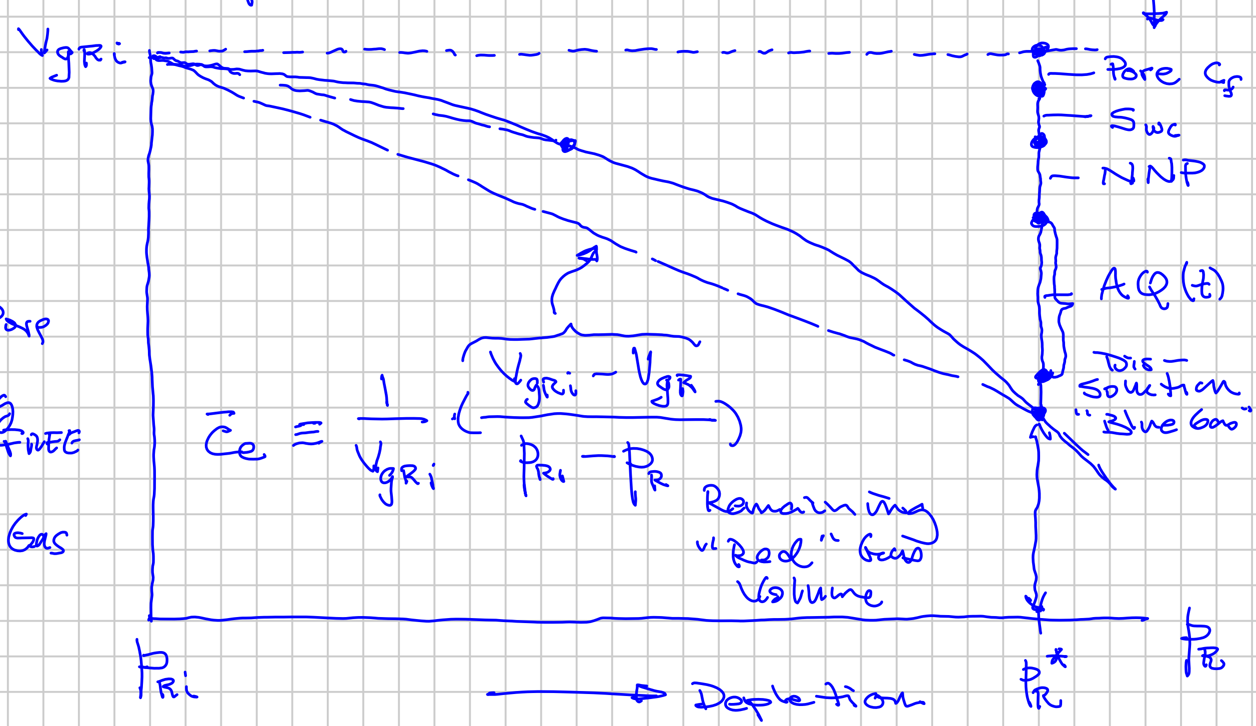
(a) $V_{gR} \neq$ constant

$HCPV_g\,(p_R)$

$$C \equiv -\frac{1}{V}\frac{dV}{dp}$$

$p = p_R$

V_{gR} = Reservoir Pore Volume containing INITIAL FREE GAS − Produced Gas



Pot Aq. $\quad C_f(p_R) \quad C_{tw}(p_R)$

99%
GMB+AQ $\quad C_f$ = const $\quad C_w$ = const $\quad R_{sw} = 0$
Blue Gas

Development of Pot Aquifer Gas M.B. Eq.

$$V_{gRi} = \left[\frac{n_{gRi}\, R\, T_R\, Z_i}{p_{Ri}} \right]$$

After Depletion to p_R, $V_{gR} < V_{gRi}$

$$\frac{V_{gRi}}{V_{gRi}} V_{gR} = \frac{n_{gR}\, R\, T_R\, Z_R}{p_R} \quad ; \quad n_{gR} = n_{gRi} - n_p$$

$$V_{gRi} = \left[\frac{(n_{gRi} - n_p)\, R\, T_R\, Z_R}{p_R} \cdot \left(\frac{V_{gRi}}{V_{gR}}\right) \right]$$

$$\frac{n_{gRi}\, \cancel{R}\, \cancel{T_R}\, Z_i}{p_{Ri}} = \frac{(n_{gRi} - n_p)\, \cancel{R}\, \cancel{T_R}\, Z_R}{p_R} \cdot \left(\frac{V_{gRi}}{V_{gR}}\right)$$

$$\frac{p_R}{Z_R}\left(\frac{V_{gR}}{V_{gRi}}\right) = \frac{p_{Ri}}{Z_i}\left(\frac{n_{gRi} - n_p}{n_{gRi}}\right)$$

$$\frac{p_R}{Z_R}\underbrace{\left(\frac{V_{gR}}{V_{gRi}}\right)} = \frac{p_{Ri}}{Z_i}\left(1 - \frac{G_p}{G}\right)$$

$$\frac{\text{Current Reservoir Volume Containing "Red" Gas}}{\text{Initial Reservoir Volume Containing "Red" Gas}} \qquad \frac{\text{" ——— "}}{\text{" - - - - - "}} \quad \text{above}$$

$$\bar{c}_e \equiv \frac{1}{V_{gRi}} \left(\frac{V_{gRi} - V_{gR}}{p_{Ri} - p_R}\right) \Rightarrow \underbrace{\left(\frac{V_{gR}}{V_{gRi}}\right)} = \left[1 - \bar{c}_e(p_R)\,(p_{Ri} - p_R) \right]$$

$$\boxed{\frac{p_R}{Z_R}\left[1 - \bar{c}_e(p_R)\,(p_{Ri} - p_R)\right] = \frac{p_{Ri}}{Z_i}\left(1 - \frac{G_p}{G}\right)}$$

$$\bar{c}_e(p_R) = \frac{\bar{c}_f(p_R) + \bar{c}_{tw}(p_R)\,S_{wc} + \overset{\text{NNP + AQ}}{M}\left(\bar{c}_f(p) + \bar{c}_{tw}\right)}{1 - S_{wc}}$$

$(1 - c_e \Delta p) < 1$ Shrinkage of Red Gas Reservoir Volume

Brief Comment on Limitation of "Pot Aquifer"

– AQ volume "encroachment"

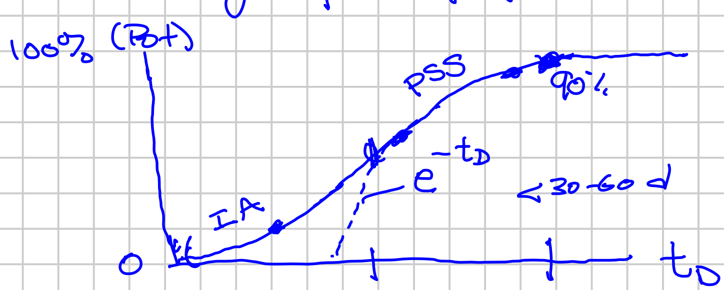
There's a delay between p_{Rg} drops and p_{RAQ} drops the same amount

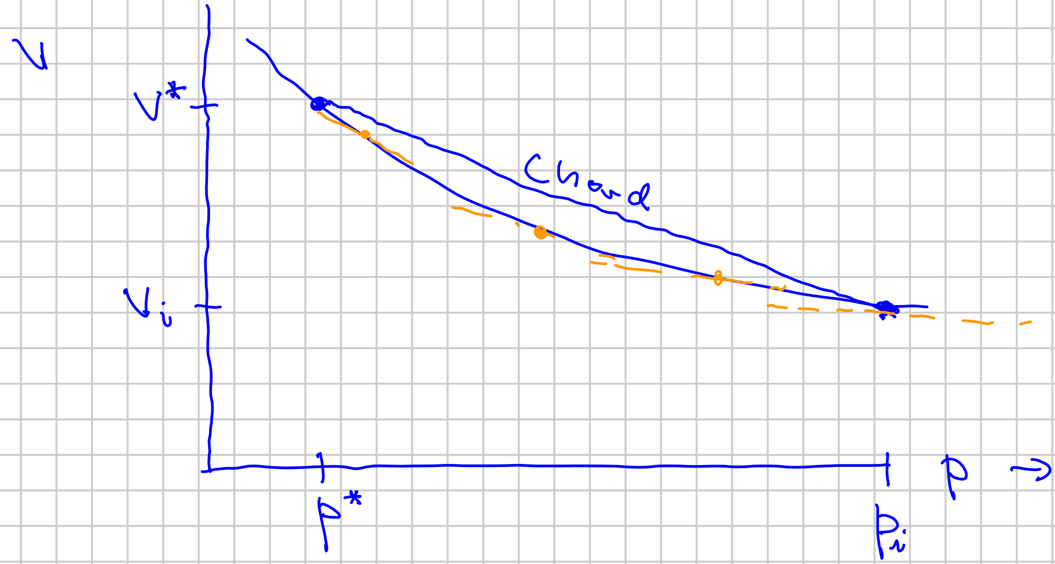
How much delay?

How much less shrinkage of $\Delta V_{gR,AQ}(t)$

Aquifer Models:

1. Hurst van Everdingen (~∞ Aq. Influx ⇒ Pot Aq.)
   - Solves the Aq. diffusivity eq. (Superposition)
2. Schilthuis (IA)
3. ?
4. Fetkovich (PSS)
5. Carter-Tracy





"Cumulative" Compressibility

$$\bar{c}(p) \equiv \frac{1}{V_i} \underbrace{\frac{V(p) - V_i}{p - p_i}}_{\text{Chord Slope from } (p_i, V_i)}$$

$\bar{c}(p)$ finds directly V^* $(p = p^*)$
from initial (p_i, V_i)

Note Title 9/11/2018

Rate Equation (PSS : $p_R(t)$) of a Surface Product

$$q_p = \underbrace{\frac{\alpha \; kh}{\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s}}_{\text{"}J\text{"}} \cdot \int_{p_{wf}}^{p_R} \underbrace{\lambda_p(p')\,dp'}$$

Product
" $\bar{o}$ "
" $\bar{g}$ "

$$\lambda_p = \frac{k_{rp}}{\mu_p B_p}$$

Phase Mobility

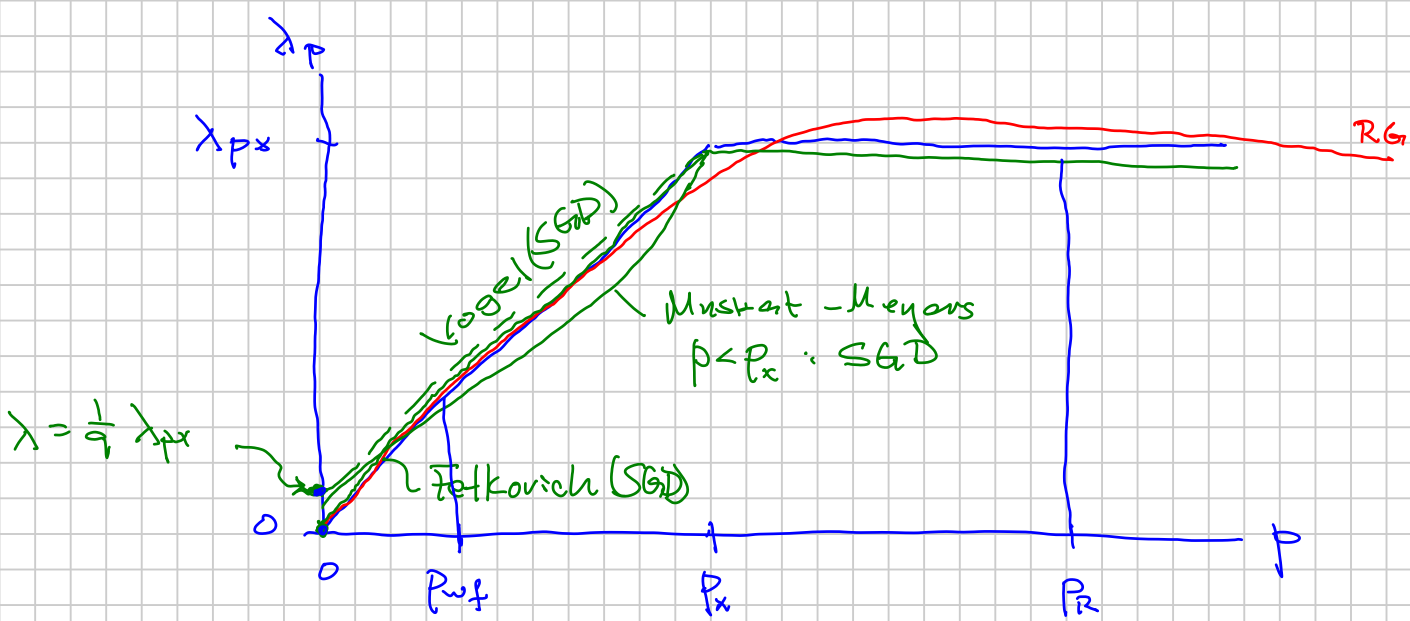
reservoir phase

Conversion from R phase to surface product

Gas $p_x \sim 200$-bar

Oils $p_x = p_b$

$$s = s' + D q_p$$



$\underline{p_x < p_{wf} < p_R}$:

$$q_p = J_p\,(p_R - p_{wf})\,\lambda_{px} \quad \text{straight-line IPR}$$

$p_{wf} < p_R < p_x$:

$$q_p = \underbrace{\frac{J_p}{2 p_x}}_{J'} (p_R^2 - p_{wf}^2)\, \lambda_{px}$$

$p_{wf} < p_x < p_R$

$$q_p = J \lambda_{px} (p_R - p_x) + \frac{J \lambda_{px}}{2 p_x} (p_x^2 - p_{wf}^2)$$

Need: J: Rock, Flow Geometry, Skin

λ_{px}: Mobility

$\underbrace{p_R \quad p_{wf}}$ p_x

$\Delta p = p_R - p_{wf}$ "Drawdown"

Dimensionless Quantities:

$$p_d \equiv \frac{p_{wf}}{p_R} \qquad q_d \equiv \frac{q_p}{[q_p(p_{wf}=0)]} = \frac{q_p}{q_{p\,max}}$$

Theoretical Absolute Open Flow Potential

$p_{wf} \to 0$ AOFP

$$\lambda_p (p_{wf}=0) = \frac{1}{q} \lambda_{px}$$

Vogel (SGD):

$$q_d = \frac{q_o}{q_{o\,max}} \cong 1 - 0.2\, p_d - 0.8\, p_d^2$$

$$1 - V p_d - (1-V)\, p_d^2 \qquad 0 < V < 1$$

Fetkovich

$$q_d \cong 1 - p_d^2$$

$s = s' + D\,q_{tp}$ $\Rightarrow$ Whitson 1984 ?

MATERIAL BALANCE APPLICATION(S)

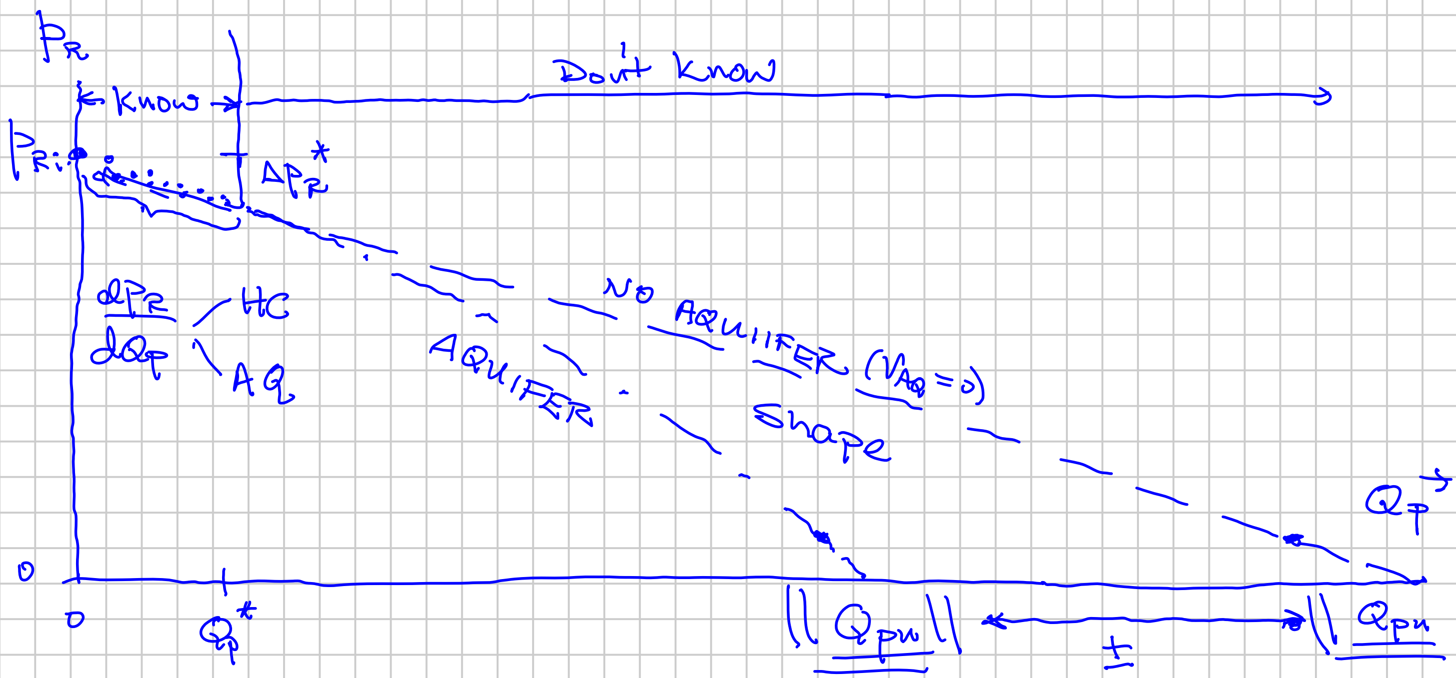
Note Title 9/12/2018

To give the relationship $\boxed{p_R(Q_p)}$ || $p_R(Q_p, Q_{inj})$ when this relationship is not known from field measurements.

$$p_R(Q_p) \rightarrow q(Q_p) \rightarrow q(t) \rightarrow Q_p(t)$$

IPR
Rate Eq.

$$\frac{dQ_p}{dt} = q$$



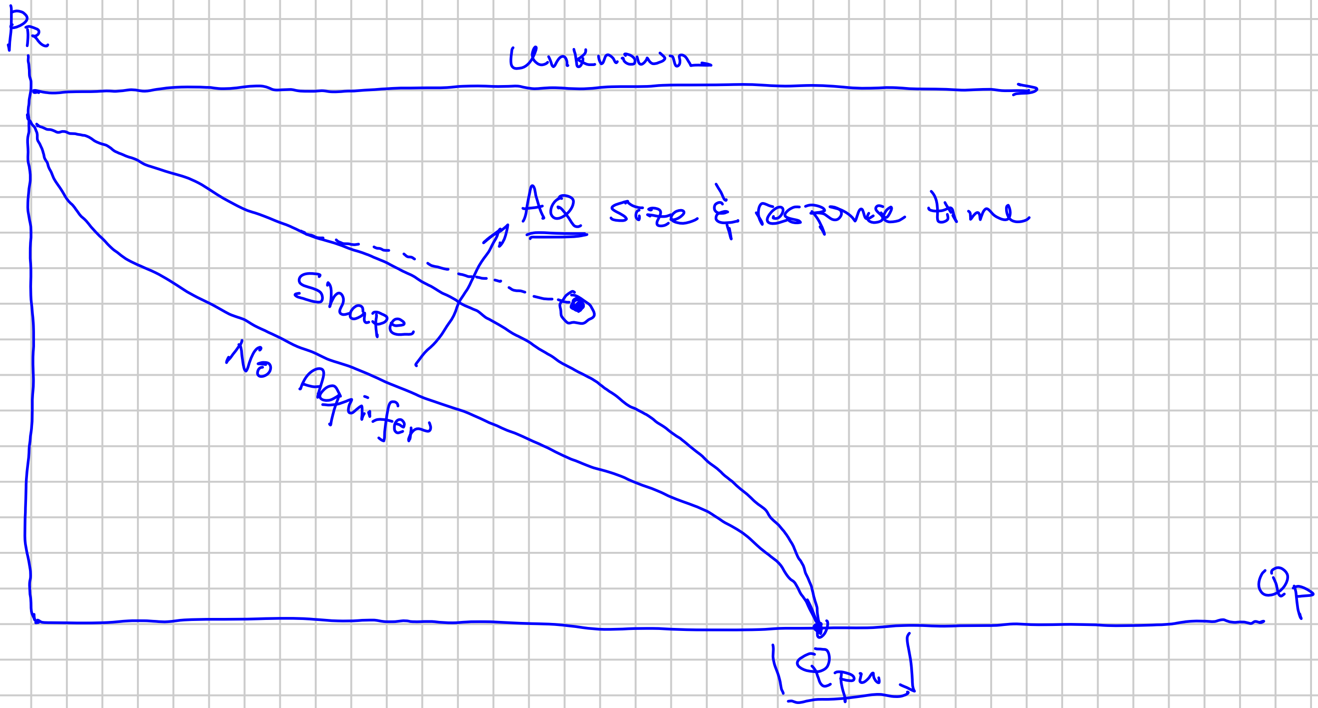
"Recovery" V_{HC}

Q_{pu} = Theoretical Estimated Ultimate Reserve Q_{pu}

$p_R \rightarrow 0$ EUR

For a given $\frac{dp_R}{dQ_p}$, and given production Q_p, there are two components contributing to dp_R/dQ_p

$$\overline{V_{HC}\, c_{HC}\, dp} + \overline{V_{AQ}\, c_{AQ}\, dp} \Rightarrow \frac{dp_R}{dQ_p} \text{ measured} \quad \Big\| \quad c_{HC} > c_{AQ}$$



Engineer the shape & Q_{pu} $\qquad$ $P_R(Q_P)$

Parameters (w/ Uncertainty) Affect Shape & Q_{pu}

$$Q_{pu} = \boxed{Q_i} \cdot \overset{10-100\%}{RF_u}$$

OOIP N $\qquad$ OGIP G $\qquad$ $P_R \to 0$

$$Q_i = \underbrace{A \cdot \bar{h}}_{G \& G} \cdot \underbrace{\bar{\phi} \cdot \overbrace{(1-\bar{S}_w)}^{S_{HC}}}_{\text{Petrophysical}} \Big/ \underbrace{B_i}_{PVT}$$

(50→100%) RG

10%–50% RO

$$\underset{\sim\sim}{RF_u} = f\left(PVT,\ RP,\ Depl/EOR,\ \overset{\#\ Wells}{Well\ Placement},\ AL\right)$$

((AQ: Size / Response Time))

└ (1) Oil Reservoirs

(2) Gas Reservoirs

SGD

20–35%

RP

Note Title 9/24/2018

Depletion Project

- Assumptions

(1) M_{AQ} ($M_{Cod} \sim M_{Frissg}$)

(2) $q_g = \alpha$

$$J = \alpha \frac{kh}{\ln\frac{r_e}{r_w} - \frac{3}{4} + S} \cdot \lambda_x$$

Units & Flow Geometry

$$\lambda_x = \left(\frac{k_{rg}}{\mu_g B_g}\right)_{P_x}$$

SOLUTION GAS DRIVE M.B. (SGD)

- Oil $p_R \le p_b$ or $p_{Ri} \to p_b \to p_{Ra}$ (Ultimate Abandonment)

- Effect of solution gas being liberated from the RO on Oil Recovery

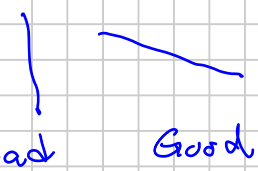
Good / Bad

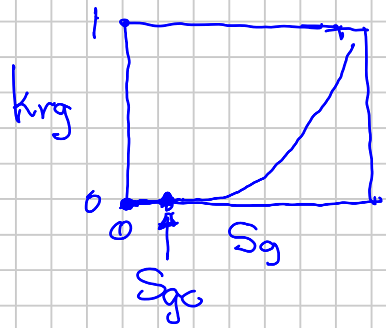
If k_{rg}, $\lambda_g \sim 0$
Internal Gas Drive
- Driving Oil $\lambda_o > 0$

$S_o^{\,n \sim 2-4} \Downarrow$

$\underbrace{k_{ro}/(\mu_o B_o)}_{\lambda_{\bar{o}}}$

$\uparrow \downarrow$ $\mu_o B_o \uparrow$





$\downarrow$ — $p_R(N_p)$

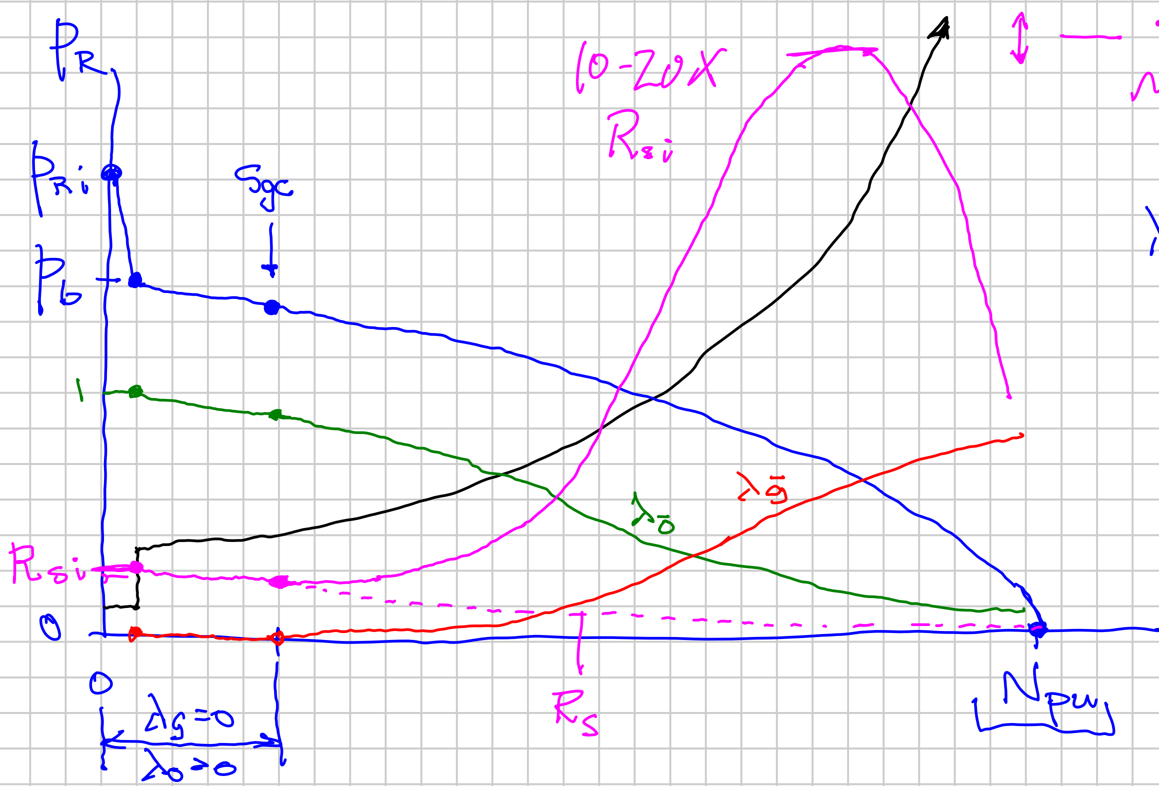
$\downarrow$ — $\lambda_{\bar{o}}(N_p)$

$\uparrow$ — $\lambda_{\bar{g}}(N_p)$

$\uparrow$ — $c_t(N_p)$

$\updownarrow$ — $R_p(N_p) = q_{\bar{g}}/q_{\bar{o}}$

$$\lambda_{\bar{p}} = \frac{k_{rp}}{\mu_p\, B_p}$$



$$R_p = \frac{q_{t\bar{g}}}{q_{t\bar{o}}}$$

$$q_{t\bar{g}} = \underset{\substack{\uparrow \\ \text{Free} \\ \text{Gas Phase}}}{q_{tgR}} / B_g + q_{t\bar{o}} R_s = C \lambda_{\bar{g}} = C \cdot \frac{k_{rg}}{\mu_g B_g}$$

$$q_{t\bar{o}} = q_{toR} / B_o + \cancel{q_{t\bar{o}g}}^{\,0} = C \lambda_{\bar{o}} = C \frac{k_{ro}}{\mu_o B_o}$$

$$R_p = \frac{q_{t\bar{g}}}{q_{t\bar{o}}} = \frac{q_{t\bar{g}g} + q_{t\bar{g}o}}{q_{t\bar{o}o} + \cancel{q_{t\bar{o}g}}_{\,0}}$$

$$= R_s + \frac{k_{rg} / \mu_g B_g}{k_{ro} / \mu_o B_o}$$

$$= R_s(p_R) + \left(\frac{\mu_o B_o}{\mu_g B_g}\right)_{p_R} \cdot \underbrace{\frac{k_{rg}}{k_{ro}}(S_g)}_{\text{Relative Permeability Ratio}}$$

$$\simeq R_s + \frac{\lambda_{\bar{g}}}{\lambda_{\bar{o}}}$$

GAS-OIL REL. PERM. RATIO – 1st Order Impact on $p_R(N_p)$

$\downarrow$

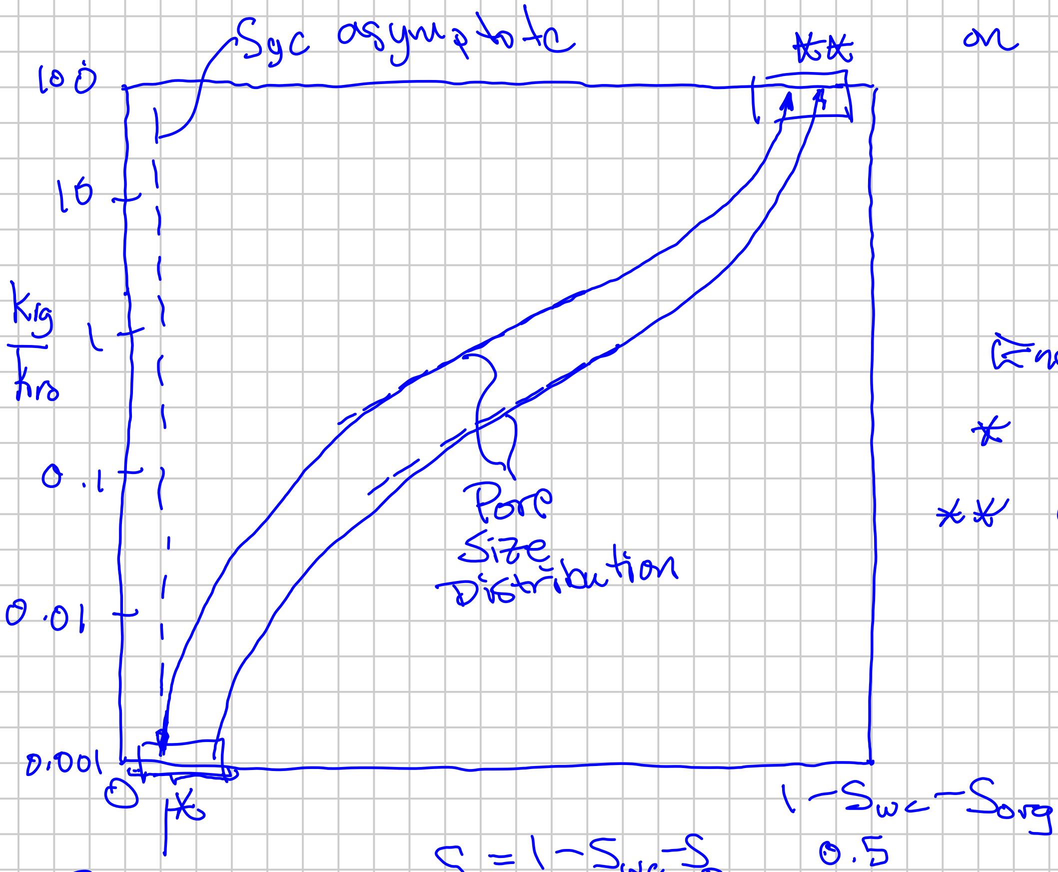
N_{pu}

10% 25-30%

End-Point Sat.

* S_{gc}

** S_{org}



$S_{gc} =$ 10–30%

(5) (25)

Only Oil Mobility

Only Gas Mobility

Estimation of S_o from knowledge of p_R & N_p $p_R(N_p)$

Assume HCPV = constant

$$V_{oi} = HCPV_i = HCPV = \overset{\text{IOIP}}{N} \cdot B_{oi}$$

$$HCPV = N B_{oi} \quad \frac{STB}{Sm^3}$$

$$V_o\,(@\bar{p}_R) = \underbrace{(N - N_p)}_{\text{STB Remaining in Reservoir}} \cdot B_o\,(@\bar{p}_R)$$

$$S_o = \frac{V_o}{HCPV} = \frac{(N-N_p)\,B_o}{N\,B_{oi}}$$

$$\boxed{S_o = \Big(1 - \underbrace{\frac{N_p}{N}}_{RF_o}\Big)\frac{B_o}{B_{oi}}} \longrightarrow \frac{k_{rg}}{k_{ro}}(S_o)$$

Saturation Relationship used in conventional S&D M.B

- HCPV = constant
- $\left.\begin{array}{l} q_{\bar{o}} \\ N_p \end{array}\right\}$ only from Res. Oil Production $q_{\bar{o}g} \sim 0$

SOLUTION GAS DRIVE MAT. BAL.

Note Title 9/26/2018

Ch. 7 Borthne MBO Mat. Bal.

$V_b = 1$ bbl

$V_p = V_b \phi = \phi$

$V_{HC} = V_p(1-S_w) = V_{oR} + V_{gR}$

(Modified)
Black-Oil PVT Formulation: $\rho_{\bar{g}g} = \rho_{\bar{g}o} = \text{const.}$ $\rho_{\bar{o}o} = \rho_{\bar{o}g} = \text{const.}$!

ECL100

Conserve Surface Product Volumes
$(\bar{g}, \bar{o})$

G.B.: Mass conservation $\Rightarrow$ $\underbrace{\rho_{\bar{o}o} \neq \rho_{\bar{o}g} \quad \rho_{\bar{g}g} \neq \rho_{\bar{g}o}}_{f(p)}$

Vol. 30: $\dfrac{\gamma_{\bar{o}o}}{\gamma_{\bar{o}g}} = 1 \qquad \dfrac{\gamma_{\bar{g}g}}{\gamma_{\bar{g}o}} = 1$

"$\bar{o}$" $\qquad \phi\left[\underbrace{\frac{S_o}{B_o}}_{V_{\bar{o}o}} + \underbrace{\underbrace{\frac{\overbrace{S_g}^{(1-S_w-S_o)}}{B_{gd}}}_{V_{\bar{g}g}} \cdot r_s}_{V_{\bar{o}g}}\right] = V_{\bar{o}}$ @ t, p_R

$r_s = \dfrac{V_{\bar{o}g}}{V_{\bar{g}g}}$

$= (N - N_p)$

"$\bar{g}$" $\qquad \phi\left[\underbrace{\frac{S_o}{B_o} R_s}_{V_{\bar{g}o}} + \underbrace{\frac{\overbrace{S_g}^{(1-S_w-S_o)}}{B_{gd}}}_{V_{\bar{g}g}}\right] = V_{\bar{g}}$ @ t, p_R

$= (G - G_p)$

p_{Ri} S_{oi}, S_{gi} $\Rightarrow$ $V_{\bar{o}} = N = \underbrace{N_o}_{6\cdot 10^9 \text{ STB}} + \underbrace{N_g}_{0}$

p_{Ri} S_{oi}, S_{gi} $\qquad V_g = G = G_o + G_g$

$7 \cdot 10^{12}$ scf

$$S_g = (1-S_w) - S_o$$

$[\, r_s \equiv R_v \,]$

SPE $\quad$ Walsh/ Eloo

$$\frac{\phi \left[\frac{S_o}{B_o} + \frac{(1-S_w)-S_o}{B_{gd}} \cdot r_s \right]}{\phi \frac{(1-S_w)}{B_{oi}}} = \frac{N - N_p}{N}$$

For $S_{oi} = 1 - S_w$ $(S_{gi} = 0)$: "Initial Res. Oil"

$$\frac{1}{(1-S_w)} \frac{B_{oi}}{B_o} S_o + \frac{r_s}{B_{gd}} B_{oi} - \frac{S_o}{(1-S_w)} \frac{r_s}{B_{gd}} B_{oi} = 1 - \frac{N_p}{N}$$

$$S_o = (1-S_w) \left[\frac{\left(1 - \frac{N_p}{N}\right) - \frac{r_s}{B_{gd}} B_{oi}}{\frac{B_{oi}}{B_o} - \frac{r_s}{B_{gd}} B_{oi}} \right] \frac{B_o B_{gd}}{B_o B_{gd}}$$

$$\underbrace{r_s = 0}$$

$$S_o = (1-S_w) \left(1 - \underbrace{\frac{N_p}{N}}_{RF_o} \right) \frac{B_o}{B_{oi}}$$

Monday

Walsh: Eq. 2
$$S_o = (1-S_w)\left[\frac{\left(1-\frac{N_p}{N}\right) B_o B_{gd} - r_s B_o B_{oi}}{B_{oi}\left(B_{gd} - r_s B_o\right)}\right]$$

Need to deal with the surface gas also:

Using the approximate reservoir rate equations for $\bar{g}$ and $\bar{o}$:

$$q_{gR} = C \cdot \int_{P_{wf}}^{P_R} \frac{k_{rg}}{\mu_g}\, dp$$

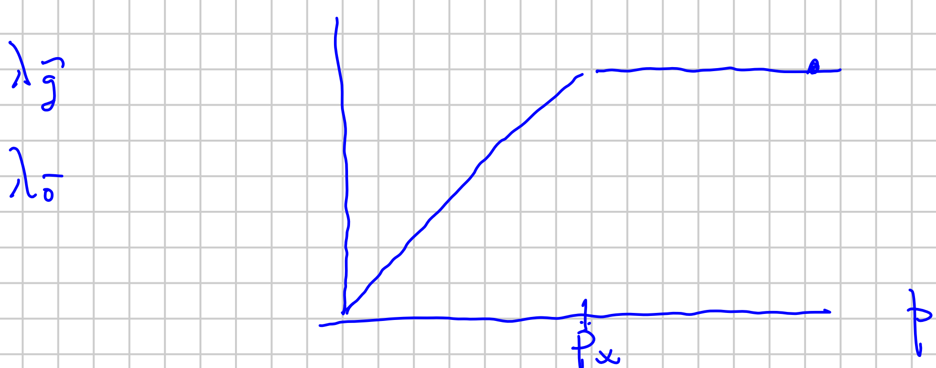
$$q_{\bar{g}g} = q_{gR} / B_{gd}$$

$$q_{\bar{g}g} = C \cdot \int_{P_{wf}}^{P_R} \underbrace{\frac{k_{rg}}{\mu_g B_{gd}}}_{\lambda_{\bar{g}}}\, dp \simeq \overline{\left(\frac{k_{rg}}{\mu_g B_{gd}}\right)}_{P_R} \cdot (P_R - P_{wf})$$

Surface Oil Product

$$q_{\bar{o}o} = C \cdot \int_{P_{wf}}^{P_R} \underbrace{\frac{k_{ro}}{\mu_o B_o}}_{\lambda_{\bar{o}}}\, dp \simeq \overline{\left(\frac{k_{ro}}{\mu_o B_o}\right)}_{P_R} (P_R - P_{wf})$$

All SGD M.B. eqs. (Tarner, Standing, Walsh, Boe the)



Conventional BOPVT

$p_{wf} > p_x \Rightarrow$ OK Approx.

$$R_p = R_s + \frac{k_{rg}}{k_{ro}} \frac{\mu_o B_o}{\mu_g B_g}$$

Modified BO : $r_s > 0$

$$R_p = \frac{q_{\bar{g}}}{q_{\bar{o}}} = \frac{q_{\bar{g}g} + q_{\bar{g}o}}{q_{\bar{o}o} + q_{\bar{o}g}}$$

$$R_s = \frac{q_{\bar{g}o}}{q_{\bar{o}o}} \qquad r_s = \frac{q_{\bar{o}g}}{q_{\bar{g}g}}$$

$$R_p = \frac{q_{\bar{g}g} + R_s\, q_{\bar{o}o}}{q_{\bar{o}o} + q_{\bar{g}g}\, r_s}$$

$$q_{\bar{g}g} \simeq C \left(\frac{k_{rg}}{\mu_g B_{gd}}\right)_{p_R} (p_R - p_{wf})$$

$$q_{\bar{o}o} \simeq C \left(\frac{k_{ro}}{\mu_o B_o}\right)_{p_R} (p_R - p_{wf})$$

$$R_p = \frac{\left[\frac{k_{rg}}{k_{ro}} \frac{\mu_o B_o}{\mu_g B_{gd}} + R_s\right]}{\left[1 + \frac{k_{rg}}{k_{ro}} \frac{\mu_o B_o}{\mu_g B_{gd}} \cdot r_s\right]} = \frac{[E_g]}{[E_o]}$$

< >

< Derive $\frac{k_{rg}}{k_{ro}} = f\left(R_p \mid PVT(p)\right)$ >

Note Title 10/1/2018

Field Units Oil Rate Eq. (Single Phase Oil: PSS)

$$q_o = \frac{kh\,(P_R - P_{wf})}{141.2\,\mu_o B_o \left[\ln \frac{r_e}{r_w} - \frac{3}{4} + s\right]}$$

$s = 0$ MBS notes

$P_b < P_w < P_R$

q_o [STB/D]
k [md]
h [ft]
p [psia]
μ_o [cp]
B_o [RB/STB]
r [ft]

$$\ln \frac{r_e}{r_w} - \frac{3}{4} \simeq \ln \frac{0.472 r_e}{r_w}$$

$$= \ln \frac{r_e}{r_w} + \underbrace{\ln 0.472}_{-3/4}$$

$1 - \left(\frac{P_{wf}}{P_R}\right)^2$ Fetkovich Eq.

$$q_o = \frac{\bar{k}_o h \bar{p}\left[1 - 0.2(p_{wf}/\bar{p}) - 0.8(p_{wf}/\bar{p})^2\right]}{254.2\,\bar{\mu}_o \bar{B}_o (\ln 0.47\, r_e/r_w)} \qquad (19)$$

(141.2×2)

$254.2 = 141.2 \times \frac{9}{5}$

Field Units ✓

$\bar{p} = P_R$

$k_o = k \cdot k_{ro}$

Vogel:

$P_R \leq P_b$

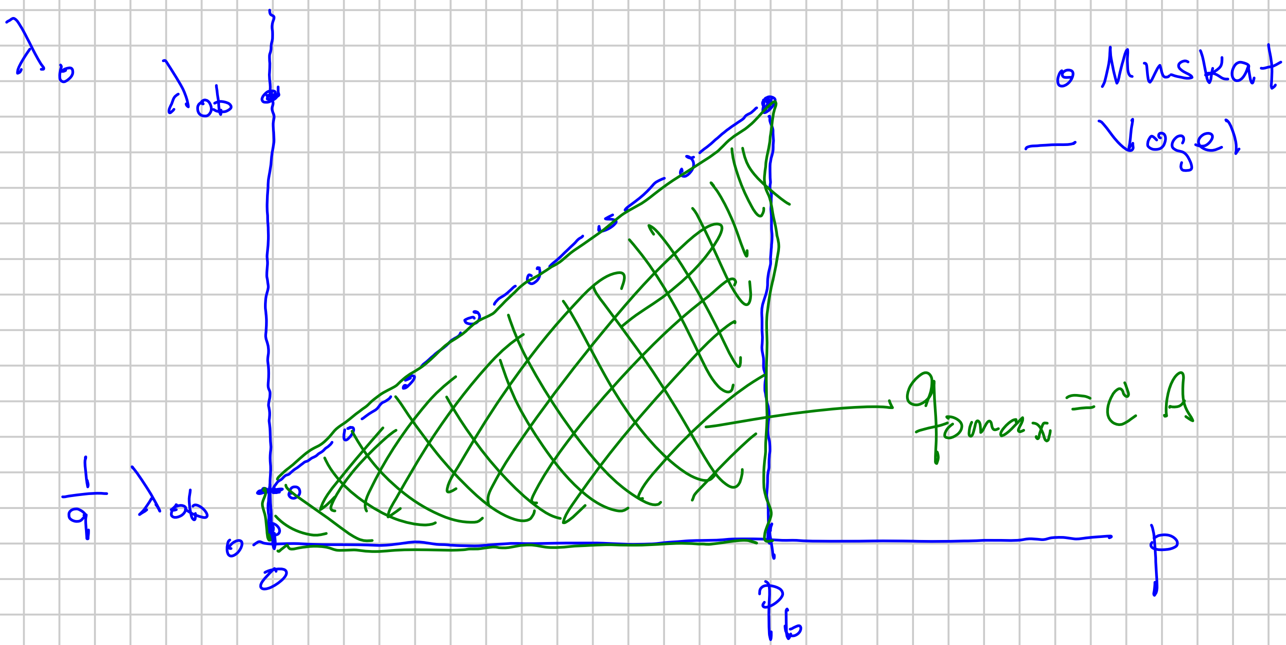
$\bar{k}_o$, $\bar{\mu}_o$, $\bar{B}_o$ @ $\bar{p} = p_R$

"Vogel" SGD saturated oil rate eq.

$$\frac{q_o}{\boxed{q_{omax}}} \cong 1 - 0.2 \frac{p_{wf}}{p_R} - 0.8 \left(\frac{p_{wf}}{p_R}\right)^2$$

$$= 1 - V \frac{p_{wf}}{p_R} - (1-V) \left(\frac{p_{wf}}{p_R}\right)^2$$

Fetkovich (197x) : $V = 0$



$k_{ro}(p)$!

Evinger - Muskat (1942) :

$$q_o = \underbrace{\frac{kh}{141.2\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s\right]}}_{C} \cdot \int_{p_{wf}}^{p_R} \underbrace{\frac{k_{ro}}{\mu_o B_o}}_{\lambda_o} dp$$

Foundation of Two-Phase Gas-Oil Flow

⇒ "Evinger-Muskat" methods (1942)

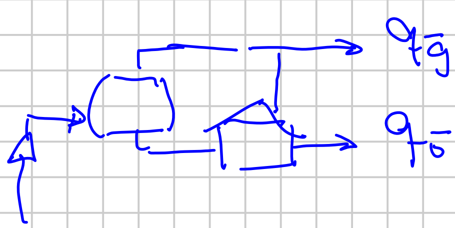
· SGD (Saturated Oil R)

⇒ Fevang - Whitson

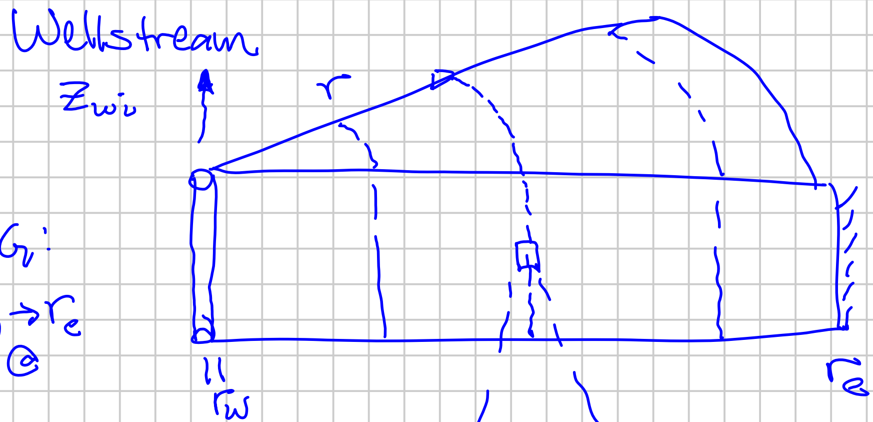
Gas Condensate R

E-M:

$p_R \leq p_b$



$$R_p = \frac{q_{\bar g}}{q_{\bar o}}$$



Steady State Flow Region

SGG: $r_w \to r_e$ @

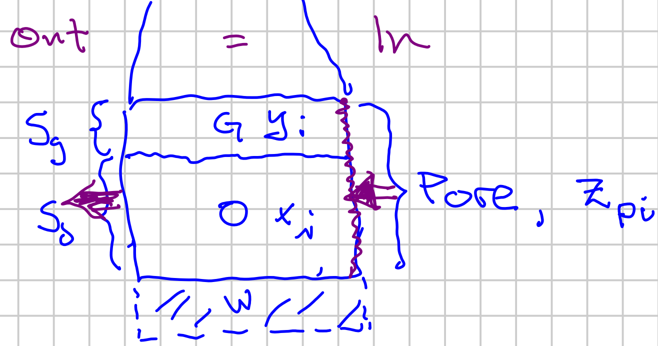
E-M

Assumption for SGD

$\underbrace{r_w < r < r_e}$

$z_{wi} = z_{fi} =$ constant

$R_p = R_{pf}$



$$\frac{S_o \frac{\rho_o}{M_o} x_i + S_g \frac{\rho_g}{M_g} y_i}{S_o \frac{\rho_o}{M_o} + S_g \frac{\rho_g}{M_g}} = z_{pi} \neq z_{fi} = \frac{\left(\frac{k_{ro}}{\mu_o}\frac{\rho_o}{M_o}\right) x_i + \left(\frac{k_{rg}}{\mu_g}\frac{\rho_g}{M_g}\right) y_i}{\left(\frac{k_{ro}}{\mu_o}\frac{\rho_o}{M_o}\right) + \left(\frac{k_{rg}}{\mu_g}\frac{\rho_g}{M_g}\right)}$$

Pore Composition

Flowing Composition

When SS Flow Region is Valid

$$\underset{@\ r=r_w}{R_p} = \underbrace{R_s + \frac{k_{rg}}{k_{ro}}\,\frac{\mu_o B_o}{\mu_g B_g}}_{\substack{\text{any and all}\\ p_{wf} < p \le p_R}} = R_{pf}$$

$r > r_w \rightarrow r_e$

SGD M.B.

At given time, with R_p known $\underbrace{(p_{wf},\ p_R)}$

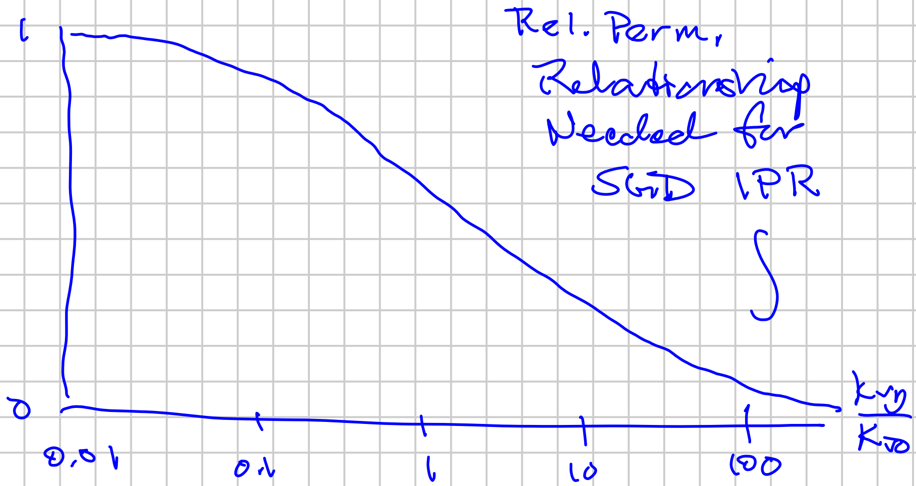
$$\frac{k_{rg}}{k_{ro}}(p) = \underbrace{\left(R_p - R_s(p)\right)\left(\frac{\mu_g B_g}{\mu_o B_o}\right)(p)}_{f(p)}$$

$p_{wf} < p < p_R$

$$\underbrace{\int \frac{k_{ro}(p)}{\mu_o(p) B_o(p)}}$$

Rel. Perm. (G-O) : S_{wc}

$\underbrace{\cancel{k_{ro}} \quad \cancel{k_g} \quad k_{rg} \quad k_{ro}}_{\text{Measure "SCAL"}}$ $\quad \boxed{k_{rg}/k_{ro}} \rightarrow k_{ro}\left(k_{rg}/k_{ro}\,(p)\right)$



Rel. Perm. Relationship Needed for SGD IPR $\int$

Before 1994

Gas Condensates:

$$q_g = \frac{0.703\; kh}{T_R\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s\right]} \cdot \underbrace{\int_{p_{wf}}^{p_R} \left(\frac{k_{rg}}{\mu_g B_{gd}} + \frac{k_{ro}}{\mu_o B_o} R_s\right) dp}_{<1994\ ?}$$

SS Assumption:

$r_w < r < r^*$ $\quad r^*$ $\quad r_d$ $\quad r_e$

Region 1 SS $\quad$ 2 $\quad$ Single Phase Gas 3

$\boxed{z_{wi}} \longleftarrow z_{fi} \longrightarrow \boxed{p^*_2} = p_d(z_{wi})$ $\qquad p = p_d$

$$\underbrace{\int_{p_{wf}}^{p^* = p_{dw}} \frac{k_{rg}(p)}{\mu_g B_g} + \frac{k_{ro}(p)}{\mu_o B_o} R_s}_{EM} + \underbrace{\int_{p^*}^{p_d} \frac{k_{rg}}{\mu_g B_g}}_{} + \underbrace{\int_{p_d}^{p_R} \frac{1}{\mu_g B_g}}_{}$$

Note Title 10/2/2018

$k_{ro} = 0.5 - m\, S_g$

Sat. Exponent to oil > 1

$$\boxed{k_{ro} \simeq k_{roi} \cdot S_{on}^{\,n_o}}$$

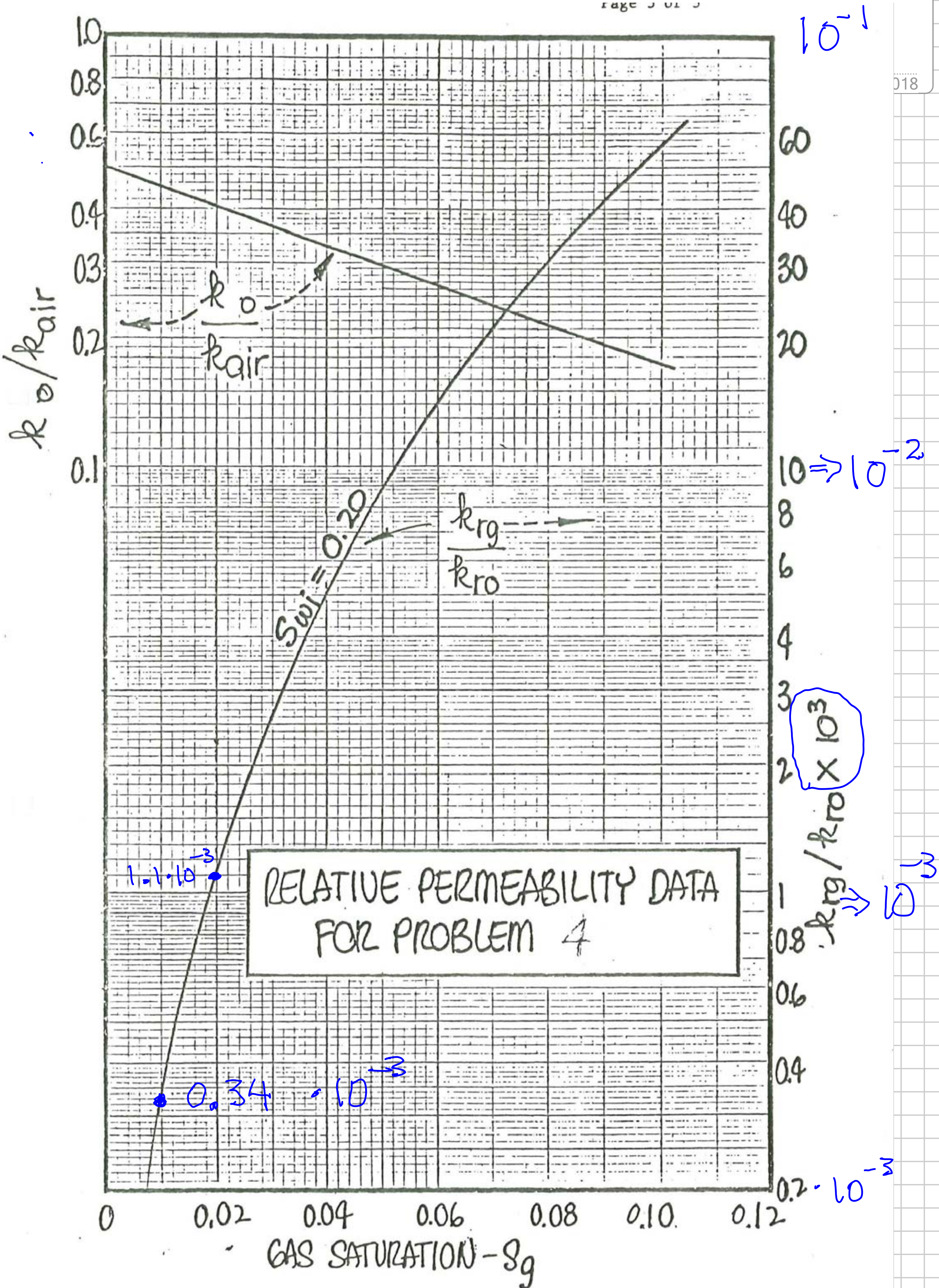
0.5

k_{ro} @ S_{wi} @ $S_o = 1 - S_{wi}$

$$S_{on} \equiv \frac{S_o - S_{org}}{1 - S_{wi} - S_{org}}$$

Initially $S_o = 1 - S_{wi} \Rightarrow S_{on} = 1$

$S_o \rightarrow S_{org} \Rightarrow S_{on} = 0$


RELATIVE PERMEABILITY DATA FOR PROBLEM 4
k_o/k_{air}
$k_{rg}/k_{ro} \times 10^3$
$S_{wi} = 0.20$
GAS SATURATION – S_g
1.0
0.8
0.6
0.4
0.3
0.2
0.1
60
40
30
20
10
8
6
4
3
2
1
0.8
0.6
0.4
0.2
0
0.02
0.04
0.06
0.08
0.10
0.12
10^{-1}
$10 \Rightarrow 10^{-2}$
$\Rightarrow 10^{-3}$
$0.2 \cdot 10^{-3}$
$1.1 \cdot 10^{-3}$
$0.34 \cdot 10^{-3}$

SGD MB : $p_R(Q_p)$: $\boxed{\text{Single RFU}}$

$$q_{RFU} = \frac{dQ_p}{dt} = \underbrace{\sum_{w=1}^{N_w} q_w}_{\text{Single Well}} \simeq \underbrace{N_w \bar{q}_w}$$

$$q_o = J\,(p_R^2 - p_{wf}^2)$$

Fetkovich SGD

$$J_w = \frac{\bar{k}_{ro}(kh)\cancel{p_R}\,\cancel{\left[1-\left(\frac{p_{wf}}{p_R}\right)^2\right]}}{2(141.2)\mu_o B_o\left[\underbrace{\ln\frac{r_e}{r_w} - \frac{3}{4}}_{\sim 8} + s\right]}$$

All wells in a RFU $\sim p_R$ same

$\Rightarrow \simeq (\overline{k_{ro}/\mu_o B_o}), p_R$

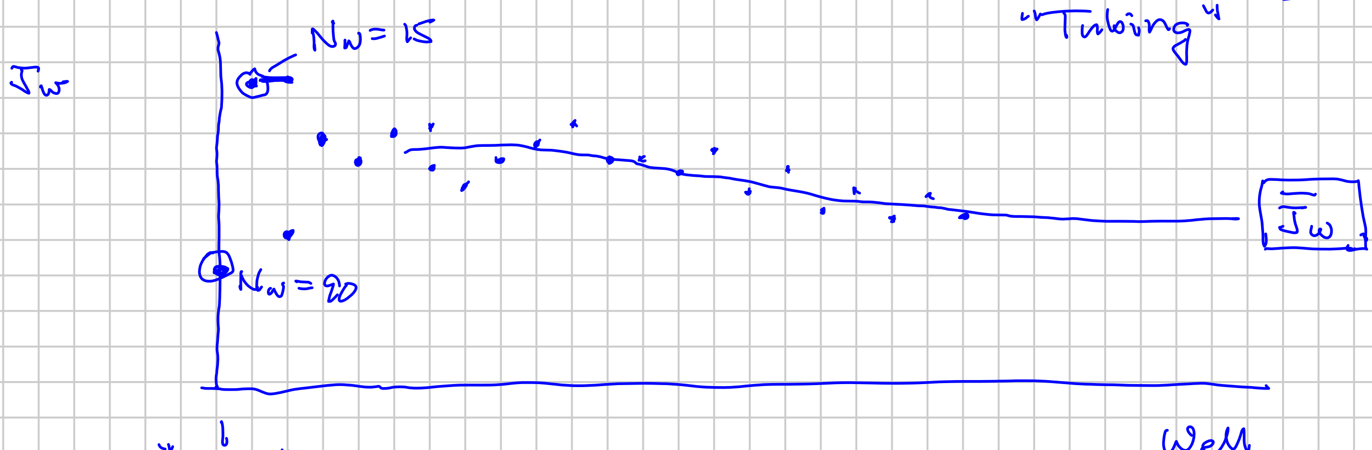
Varies for each well in RFU ?

- $(kh)_w$
- S_w
- (p_{wf}) ~ similar

Artificial Lift

Production Pipe

"Tubing"



"PDO" Development & Operation

PUD : Plan for Utvikling og Drift

(Production)

Rate-Time Forecasting $\quad q_F(t) \Rightarrow$ Revenue (t)

vs

Cost & Expenditures

Prof. Milan Stanko : Field Development Course

$$q_F(t) = \sum_{w=1}^{N_w} q_w\Big(\underbrace{J_w}_{\substack{\Delta p_R \\ \text{"R"}}} ;\ \overbrace{d_t,\ L_w,\ TVD,\ \ldots}^{\substack{\Delta p_T \\ \text{"T"}}} ;\ p_{\text{surface constraint}}\Big)$$

M.B. $\begin{cases} p_R(t) \\ Q_p(t) \end{cases}$

$(kh)_w,\ s_w$

Gas

oil : Artificial Lift

p_{sep}

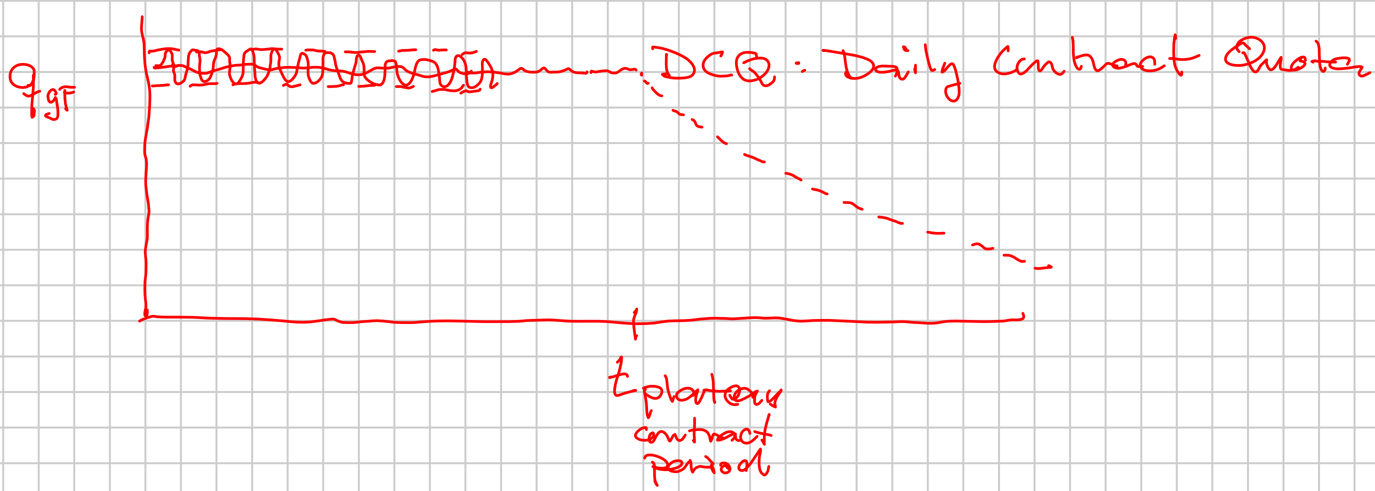
$p_{pipeline}$

p_{atm}

Frigg : $\quad \Delta p_R \quad \Delta p_t$

$\quad\quad <5\% \quad >95\%$

* Gas Field Development : Offshore (Large Onshore)



Oil Field Development :

* (a) - Huge (Giant) Fields - $q_F \sim$ const $\quad t_p$

(b) - Other : Produce what you can $\quad p_{wf,min}$

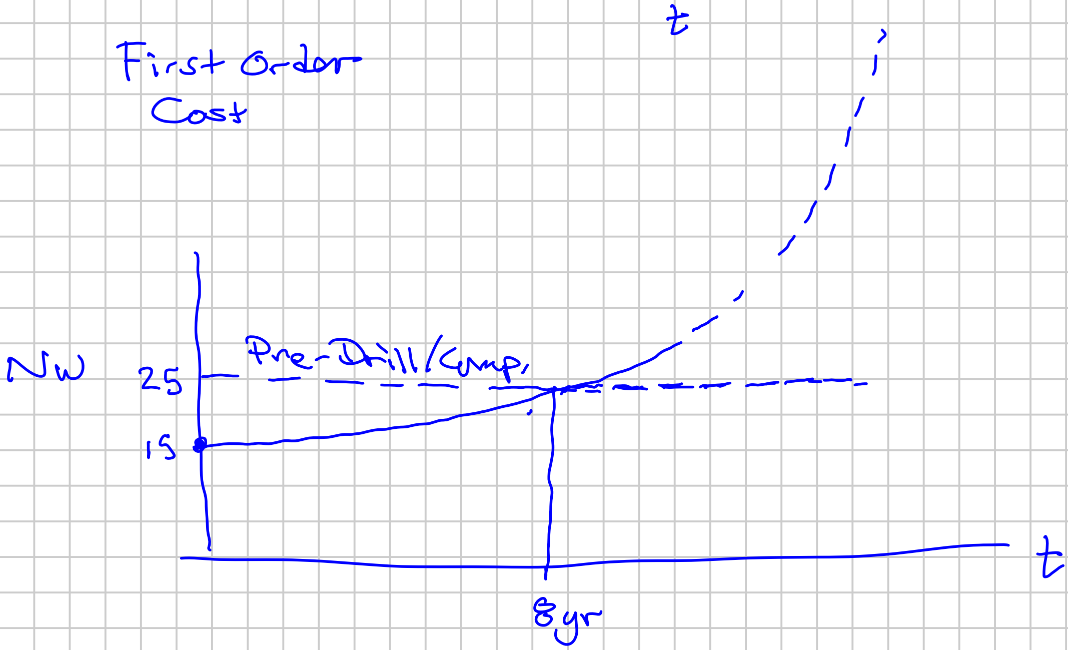
$q_F(t)$: Given

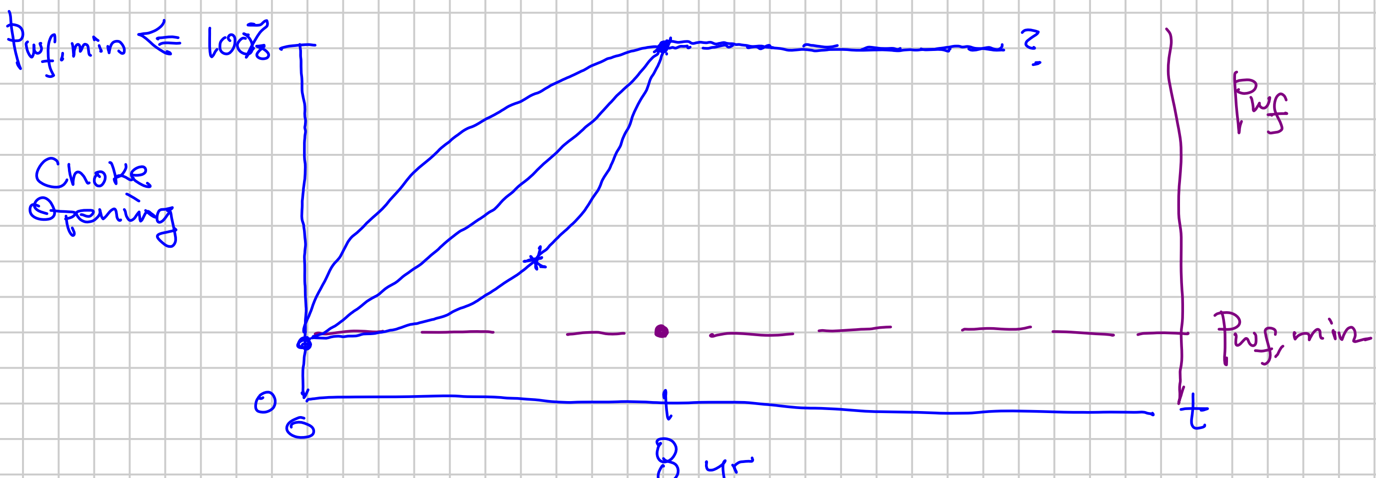
(MB + IPR) ?

Define $\left\{ \begin{array}{l} \underline{p_{wf,min}} \\ \underline{\bar{J}_w} \end{array} \right\}$ at any time $\bar{p}_R$ (M.B.)

$q_{w,max}(\bar{p}_R)$

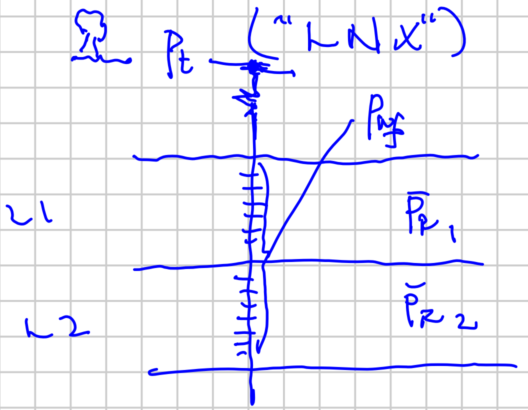
$$\text{min} \quad \underbrace{N_w(t)}_{\text{First Order Cost}} = \frac{q_F \; (=DCQ)}{q_{w,max}(\underset{\uparrow \, t}{\bar{p}_R})}$$



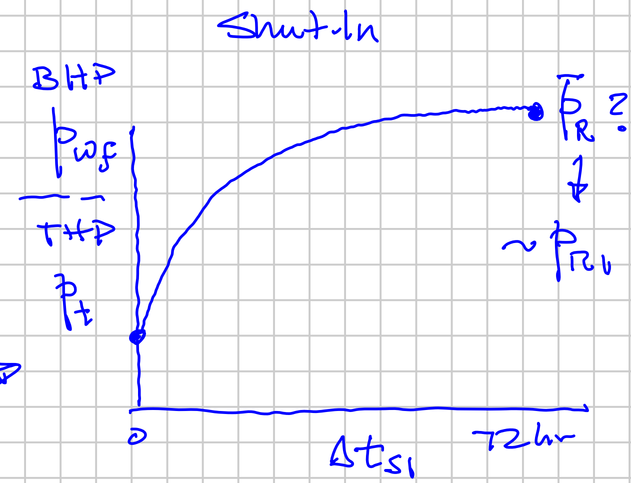


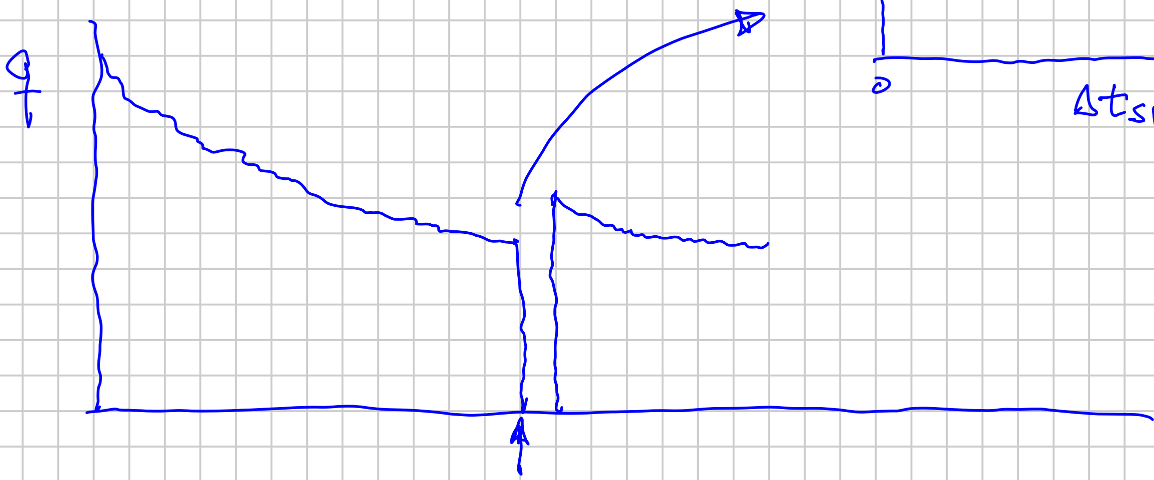
LAYERED NO-CROSSFLOW RESERVOIRS

("LNX")



$k_v = 0$

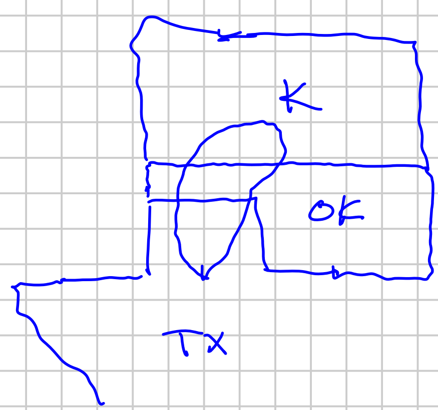




Hugoton | W. Texas

Three Geologic Zones

H K W



99%

LNX ⇒ Differential Depletion

$$\underbrace{\frac{k_1}{V_1}}\ > \ \underbrace{\frac{k_2}{V_2}} \qquad \bar{p}_{R1} \ne \bar{p}_{R2}$$

"VR" *

$$\frac{\{J \mid C_R\}}{\{IFIP \mid HCPV\}} \quad \left[\frac{\{kh,\, s\}_{N_w}}{HCPV}\right]_{RFU} \sim \text{Rate of Drainage Factor}$$

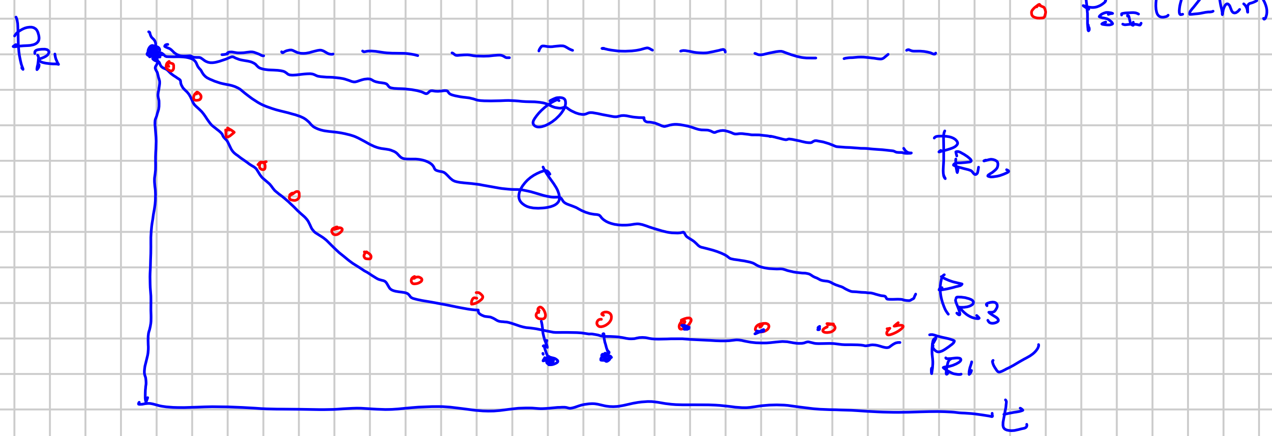
"Voidage Ratio"

RFU	VR
1	210
2	35
3	100

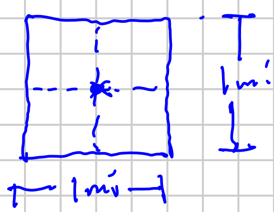
$\Rightarrow$ Rank High-to-Low VR

RFU	VR	To Equalize VR, N_w
1	210	5
3	100	10
2	35	30

* VR ≃ Decline Constant "D" in Arps's Eq.



640 acres / section



Section

1 mi ≈ 5280 f ~ 1600 m

43560 ft / acre

10 ft² ~ 1 m²

4360 m² / acre

Drilling Spacing Unit.

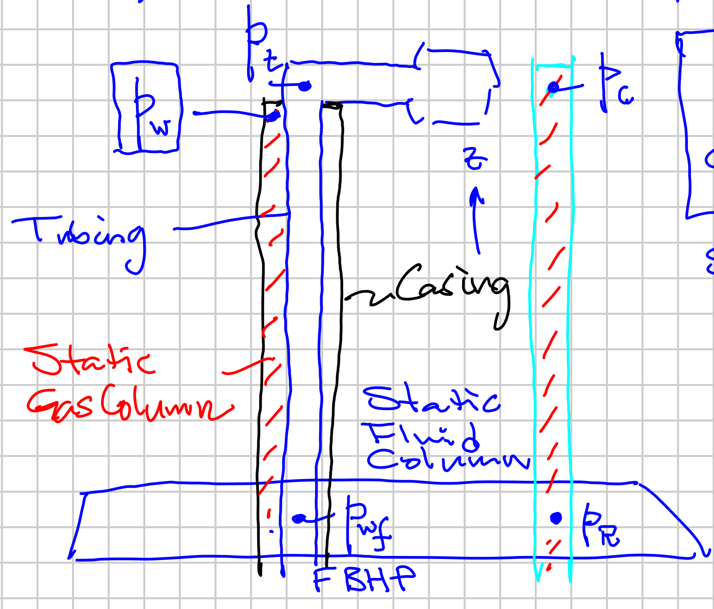
1 gas well / section

1000 m² / mål

4 mål = 1 acre

$p_{wf} - p_t =$
(1) gravity
(2) friction

$p_{wf} - p_w =$
(1) gravity



$$\boxed{\text{Darcy L.P. Gas} \quad q_g = \text{"}C_R\text{"}\left(p_R^2 - p_{wf}^2\right)}$$

Surface Datum using gas gradient correction BH→Surf

$$\underbrace{\rho_g(p,T)}: \quad \underbrace{\frac{dp}{dz} = \rho_g g}$$

$$\Downarrow$$

$$\frac{p_{wf}}{p_w} = \text{const}$$

The standard backpressure equation for a well producing from a single layer reservoir is given by Fetkovich (1975). $\approx \frac{p_R}{p_c}$

$$\boxed{q_g = C_R(p_c^2 - p_w^2)} \tag{1}$$

p_R p_{wf}

The backpressure constant, C_R, is defined as:

$$C_R = \frac{4.18khe^S}{T_R\left(\ln\frac{r_e}{r_w} - \frac{3}{4} + s\right)\mu_g z} \tag{2}$$

with q_g in std m3/d, p in bar, k in md, h in m, T_R in K, and μ_g in cp. The gravity term, S, is defined as:

"Static"

$$\boxed{S = \frac{0.0684\gamma_g D}{\bar{T}\bar{z}}} \tag{3}$$

656 TVD

This S must not be confused with the skin factor, s.

The surface datum pressures, p_c and p_w, are converted to bottomhole pressures through the gravity term. The different pressure datums are shown in **Fig. 1**.

$$p_R = e^{S/2}p_c\,; \quad p_{wf} = e^{S/2}p_w \tag{4}$$ *

$$\frac{p_R}{p_c} \approx \frac{p_{wf}}{p_w} \approx e^{S/2}$$

p_c "p_R"

p_w "p_{wf}"

$$q_g = \underline{C_R}(p_c^2 - p_w^2)$$

Same Eq. for 1-layer or n-layer system

LNX

$$\bar{C}_R \quad \Big| \quad \bar{p}_c$$

$$\bar{C}_R = \sum_{l=1}^{N_l} C_{Rl}$$

$$\underbrace{\bar{p}_c^{\,2}(t)}_{\text{Mesure}} = \frac{\sum_{l=1}^{N_l} C_{Rl} \cdot \bar{p}_{cl}^{\,2}(t)}{\bar{C}_R}$$

$$p_R^2(t) = \frac{\sum_{l=1}^{N_l} C_{Rl} \cdot p_{Rl}^2}{C_R}$$

Note Title 10/9/2018

LNX DEPLETION CHARACTERISTICS

① Differential Depletion of RFUs

$$(\bar{P}_R)_{RFU1}(t) \neq (\bar{P}_R)_{RFU2\ldots}(t)$$

$$D_{RFU1} \neq D_{RFU2\ldots}$$

Requires Voidage Ratio (VR)

"D"

Arp's Decline Eq.

$$q = \frac{q_i}{[1 + bDt]^b}$$

$$\frac{(kh)_1}{HCPV_1} = \frac{(100\ md)(\cancel{100}\ ft)}{\cancel{100}\ ft}$$

$$D \equiv \frac{q_i}{Q_{pui}(1-b)}$$

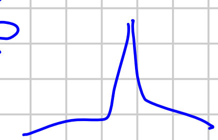
$$\frac{(kh)_2}{HCPV_2} = \frac{(100\ md)(10\ ft)}{10\ ft}$$

@ start decline

$$q_i = \frac{kh(\bar{P}_R - P_{wf})}{\left[\ln\frac{r_e}{r_w} + s\right]}$$

$$\left(\frac{k_1}{k_2}\right) \Rightarrow \frac{VR_1}{VR_2} = \frac{D_1}{D_2}$$

$$Q_{pu} = \underbrace{\frac{HCPV}{B_i}}_{IFIP} \cdot RF_u$$



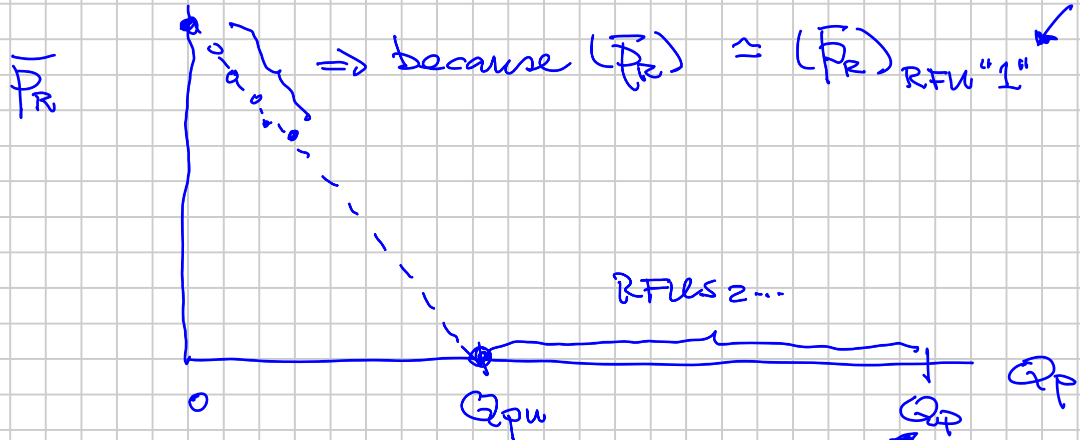
Oils 5–35%

Gas 60–98%

SGD (15–25)%

(2) Difficult to "see" Differential Depletion from conventional data used in material balance and rate (PTA, RTA)

t $\quad \bar{p}_{SI}$ / $\bar{p}_R$ $\quad (Q_p)_{Field}$

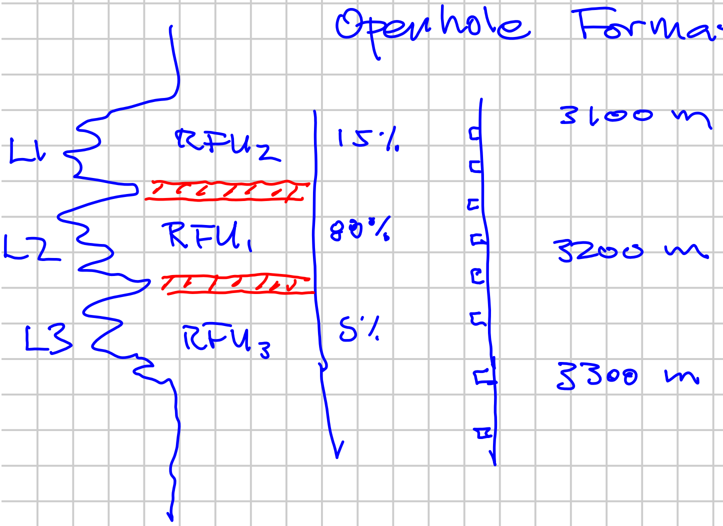


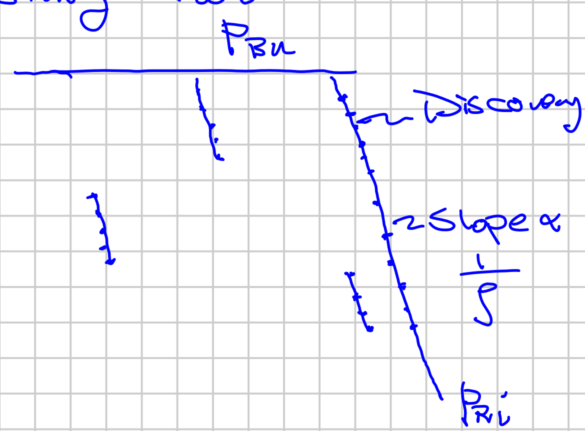
$\Rightarrow$ because $(\bar{p}_R) \simeq (\bar{p}_R)_{RFU\text{"}1\text{"}}$ $\leftarrow$ Highest Decline Constant D

Q_{pu}
$\Downarrow$
IFIP $\ll$ (IFIP) Logs / Geo
$\approx$
$(IFIP)_{RFU1}$

③ How to Verify Differential Depletion?

- "Late" Infill Wells with "MDT" (SLB)
Openhole Formation Testing Tool





④ What happens during a well shut-in?

Metered Surface $q = 0$

$q_1 \neq 0$ $\qquad p_{R1}$

$q_2 \neq 0$ $\qquad p_{wf}(\Delta t_{s1}) < p_{R2}$

$q_3 \neq 0$ $\qquad p_{R3}$

$\underbrace{\qquad\qquad\qquad\qquad}_{\text{Wellbore Storage}}$

Thereafter (hours)

$$\underbrace{\bar{p}_{R3} > \bar{p}_{R1}}_{\text{Producing into Wellbore L1 \& L3}} > \underbrace{p_{wf} > \bar{p}_{R2}}_{\text{Injection into L2 from Wellbore}}$$

$\Sigma (q)_{RFU}$ = Surface $q = 0$

"Backflow"

"Wellbore Crossflow"

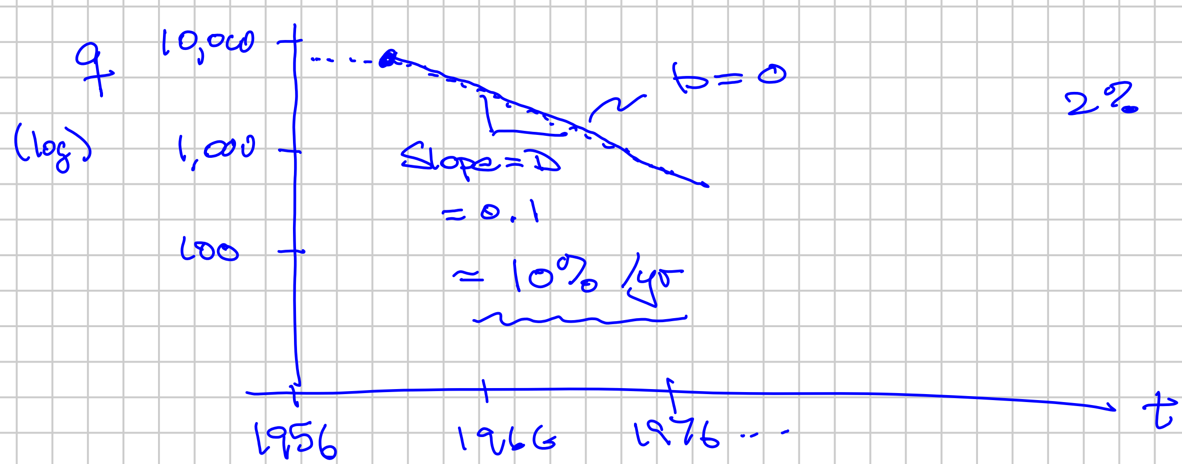
Shut-In

$$J_{L1}(\bar{p}_{R1} - p_{wf}) + J_{L3}(\bar{p}_{R3} - p_{wf}) = -J_{L2}(\bar{p}_{R2} - p_{wf})$$

$$= J_{L2}(p_{wf} - \bar{p}_{R2})$$

$p_{wf}^{SI}(\Delta t_{s1})$ satisfies this rate balance equation

(5) Longer Well & Field Lives in LNX systems because Lower-"D" RFUs deplete slowly



Note Title "GENERAL" 10/10/2018

LNX M.B. + IPR PRODUCTION FORECAST STRATEGY

① Multiple RFUs

② All wells produce from all RFUs

③ Average well | Each RFU is "uniform" | Same p_{wf} all wells

④ Gas Reservoir or Oil Reservoir all RFUs
$\bar{g}$ $\bar{o}$

⑤ Discretize in time Δt (1 month | 1 year)

⑥ Specify Target Rate of Wells
(a) Lower FBHP p_{wf} constraint } As reservoir simulators work

(b) Specify $p_{wf}(t)$: Simpler

M.B. M.B.

$R_p(p_R)$ $r_p(p_R)$

SGD: $p_R(N_p)$ GC: $p_R(G_p)$

RFU-1 : MB $p_R(Q_p)$ & IPR : $q_w(p_R, p_{wf})$

t	Target q_w	Min p_{wf}	Q_{pt}	* p_R	M.B. Q_p	IPR q_w	$[N_w] \cdot q_w$ q_t	$q_t \Delta t$ ΔQ_p	$\Sigma \Delta Q_p$ Q_p	*
$t_0=0$										
t_1										
t_2										
$\vdots$										

$\boxed{\Delta t}$

=

*new columns

M.B. (SGD) : $Q_{pz} = G_p$

$R_p(p_R)$ $\Delta Q_{p(z)} = \Delta Q_p \bar{R}_p$ $Q_{p(z)} = \Sigma \Delta Q_{p(z)}$

M.B (G.C) : $Q_{pz} = N_p$

$r_p(p_R)$ $\Delta Q_{p(z)} = \Delta Q_p \cdot \bar{r}_p$ $Q_{p(z)} = \Sigma \Delta Q_{p(z)}$

Simpler to specify p_{wf}

$\Rightarrow$ accept q_w, q_t

$N_{ts} \cdot N_{RFU}$: Variables (Unknown) : $p_R(t)$

$N_{ts} \cdot N_{RFU} + N_{ts}$

$N_{ts}(N_{RFU}+1)$
if q_w targets
are given

Objective (Target) : Minimize

Q_p^{MB} : Q_p^{calc}

$$\sum_{i=1}^{N_{RFU}} \sum_{j=1}^{N_t} \left(\frac{Q_p^{calc} - Q_p^{MB}}{Q_{p\,ref}} \right)^2$$

(a) Q_p^{MB}

*(b) Q_{pu}

Note Title 10/15/2018

WATER INFLUX:

Encroachment of external water into the HC reservoir pore volume from "Aquifer" (AQ)

(+) ⇒ Reduction of the HCPV ⇒ slows the average pressure decline during depletion

(−) ⇒ Gas reservoirs, may lead to water production leading to the "death" of producers

(+) ⇒ Oil reservoirs, displacement of oil that otherwise would not be recovered by SGD (expansion) depletion

"EOR" from mother nature

SGD 15% → 50 → 9x %

Pot Aquifer Model gives is the MAXIMUM, FASTEST encroachment of water for a finite aquifer

$$p_{R(HC)}(t) = p_{AQ}(t) = "p_R"$$

W_e = cumulative water volume encroachment from an aquifer into the HC reservoir

$$W_e^{POT} = V_{AQ}(c_f + c_w)(p_{Ri} - p_R) = W_{e,max}$$

$$G(B_g - B_{gi}) + \frac{GB_{gi}}{1-S_{wi}}\left[S_{wi}\left(\frac{B_{tw}-B_{twi}}{B_{twi}}\right) + \bar{c}_f(p_i - p)\right.$$

$$\left. + M\left(\frac{B_{tw}-B_{twi}}{B_{twi}}\right) + M\,\bar{c}_f(p_i - p)\right] \quad \ldots\ldots \quad (A25)$$

$$= (G_p - W_p R_{sw} - G_{inj})B_g + 5.615\left(W_p - W_{inj} - \frac{W_e}{B_w}\right)B_w$$

The p/z-cumulative plot including all terms would consider $(p/z)[1-\bar{c}_e(p_i-p)]$ versus the entire production/injection term Q

$$(p/z)[1-\bar{c}_e(p_i-p)] = (p/z)_i - \frac{(p/z)_i}{G}Q \quad \ldots\ldots\ldots\ldots\ldots \quad (A31)$$

with

$$Q = G_p - G_{inj} + W_p R_{sw} + \frac{5.615}{B_g}(W_p B_w - W_{inj} B_w - W_e) \quad \ldots \quad (A32)$$

where the intercept is given by $(p/z)_i$ and the slope equals $(p/z)_i/G$.

The water encroachment term calculated by superposition is expressed,

$$W_e = B\sum_j Q_D(\Delta t_j)_D \Delta p_j \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (A36)$$

where $Q_D(t_D)$ is the dimensionless cumulative influx given as a function of dimensionless time t_D and aquifer-to-reservoir radius $r_D = r_{AQ}/r_R$. Δp_j is given by $p_j - p_{j-1}$ (in the limit for small time steps), and $\Delta t_j = t - t_{j-1}$.

Radial Aquifers

The water influx equation for radial aquifers is:

$$W_e = 1.119 \, \phi ch \, r_w^{\,2} \cdot \frac{\Theta}{360} \sum_{0}^{n} \Delta p \, Q_D \qquad (11)$$

where

Θ = angle subtended by the reservoir circumference, degrees.

r_w = radius of the aquifer inner boundary, ft.

Q_D = radial efflux functions, dim.

ϕch = aquifer storage number, ft. $\cdot$ psi^{-1}.

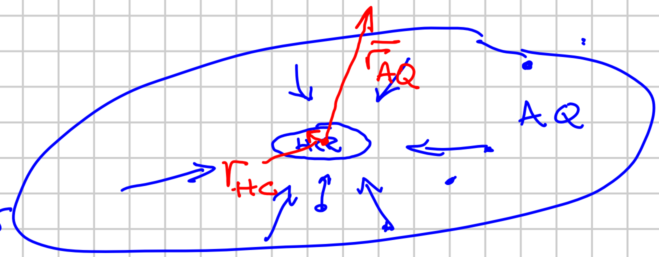
Values of Q_D for infinite and limited outer boundaries are available in equation, chart, and tabular form as a function of dimensionless wellbore time, t_{Dw}. Chart 48 in Volume 4 gives Q_D vs. t_{Dw} curves for several limited no-flow aquifers. Tabulated values can be found in Craft and Hawkin's, "Applied Petroleum Reservoir Engineering", pages 212-217.

PE/RE Books & Papers & PetroWiki

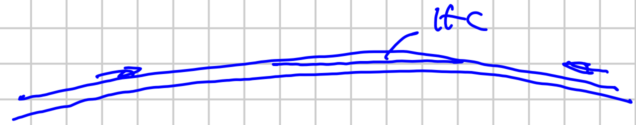
Estimation of $W_e(t)$

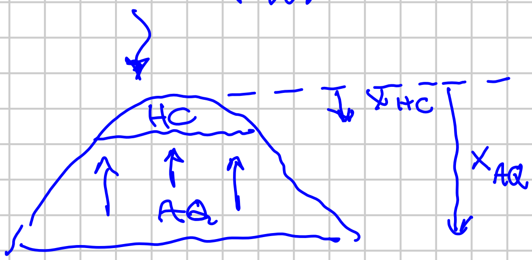
① GEOMETRY

- Radial Flow Geometries



- Linear Flow — " —





$$r_D = \frac{r_{AQ}}{r_{HC}} = 1.x - 10^{(+)}$$

Dimensionless Length L_D

$$x_D = \frac{x_{AQ}}{x_{HC}}$$

② $k_{AQ} \propto v_w$ in AQ

③ $p_{RHC}(t)$

Pot Aquifer: $p_{RHC}(t)$ only time dependency

$k_{AQ} \sim \infty$

$L_D \sim$ "small" } Instantaneous Encroachment

$$W_{e,max}(t) = V_{AQ}(c_f + c_w)\left(p_{Ri} - \underline{\underline{p_{RHC}(t)}}\right)$$

How fast you empty the HC's

Water Encroachment is modeled EXACTLY as single phase fluid flow in "Well Testing" ("PTA" Pressure Transient Analysis and/or "RTA" Rate Transient Analysis)

Solves PDE using continuity Eq. (material balance) & Darcy's eq.

: Geometry (Radial, Cylindrical)
(Linear)

: Boundary Conditions :

PTA: Well Testing

$r = r_w \quad q = \text{constant} = \nu$

(a) $r = r_e \quad (dp/dr) = 0$ No Flow $(q=0)$

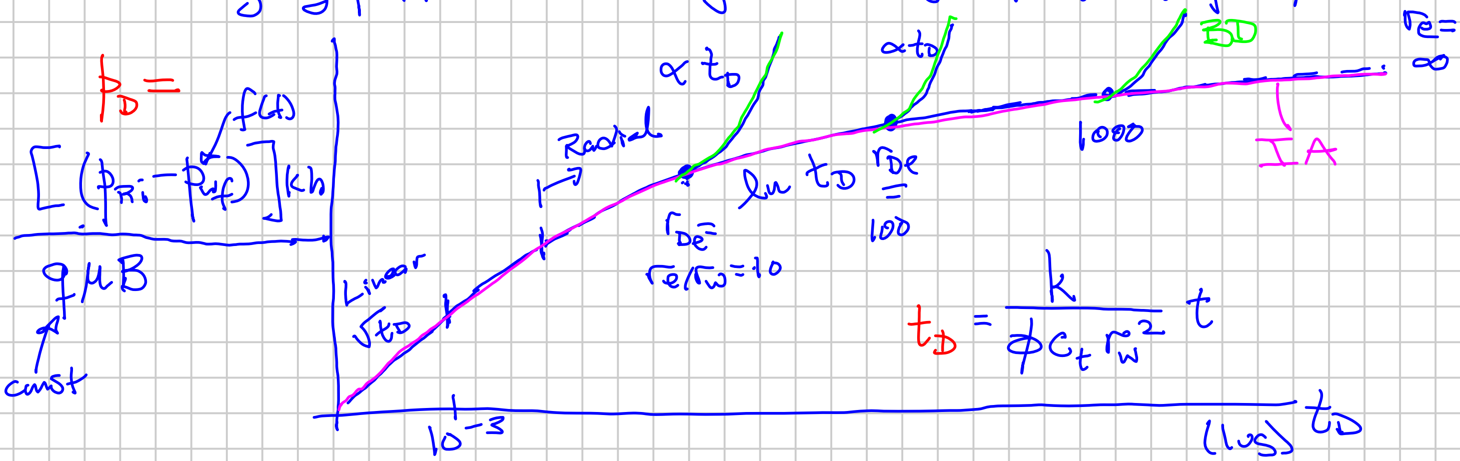
(b) $p = p_e = \text{const.}$

(c) No outer boundary ("infinite" $r_e = \infty$)

$\Rightarrow \quad p(t, r) \qquad p_{wf}(t, r = r_w)$

$p_D(t_D, r_D)$ General Dimensionless Solution

log-log plot: Any $r_w, r_e, k, c, h, q, \phi$



$$t_D = \frac{k}{\phi c_t r_w^2}\, t$$

RTA (Rate Transient Analysis)
"Time"

B.C. $r = r_w$: $p = p_{wf} = \text{constant}$ ("p_{RHC}")

$\Rightarrow q(t) \quad | \quad \underbrace{q_D(t_D)}$

B.C. used for Water Influx Calculations $W_e(t)$

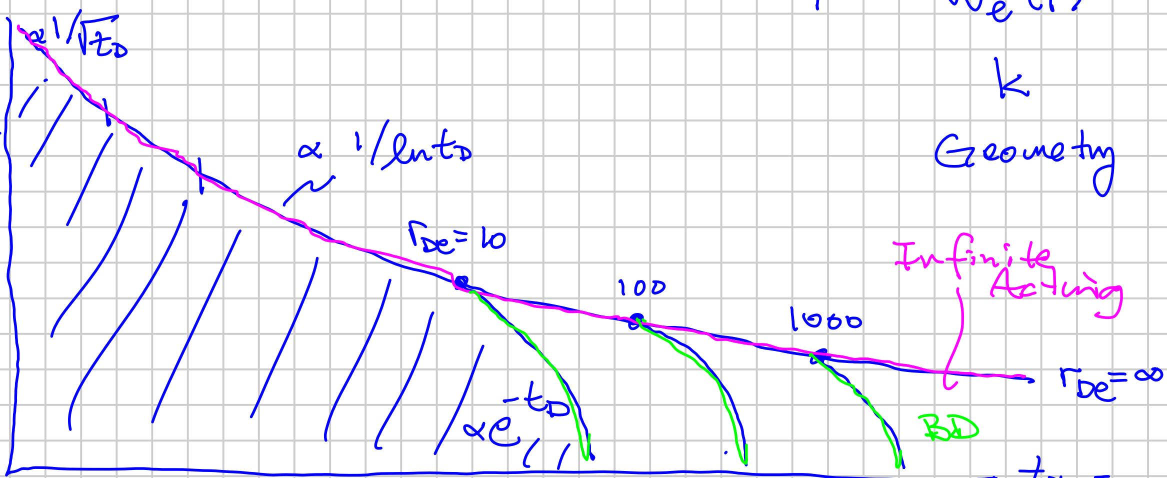
k

Geometry

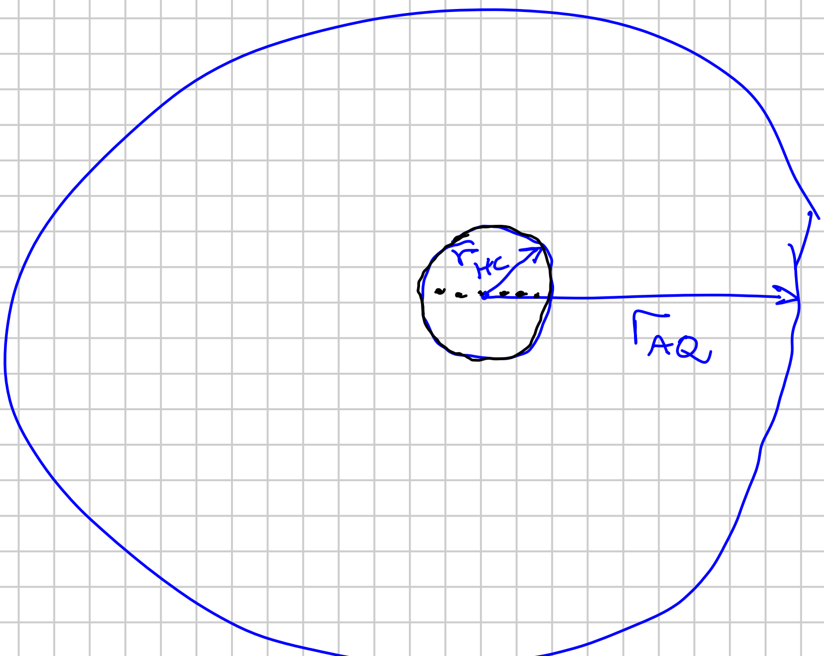
$f(t)$

$$q_D = \frac{q \mu B}{(p_{Ri} - p_{wf})\, kh}$$

ϕ const

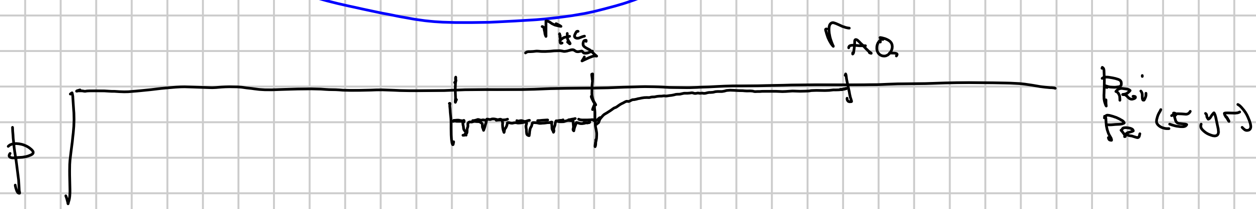


$$t_D = \frac{k}{\phi \mu c_t} t$$



Aquifer Influx

$p_{wf} = $ "p_{RHC}" (t)



Analogy to Well Behavior $\quad r_w \quad r_e$

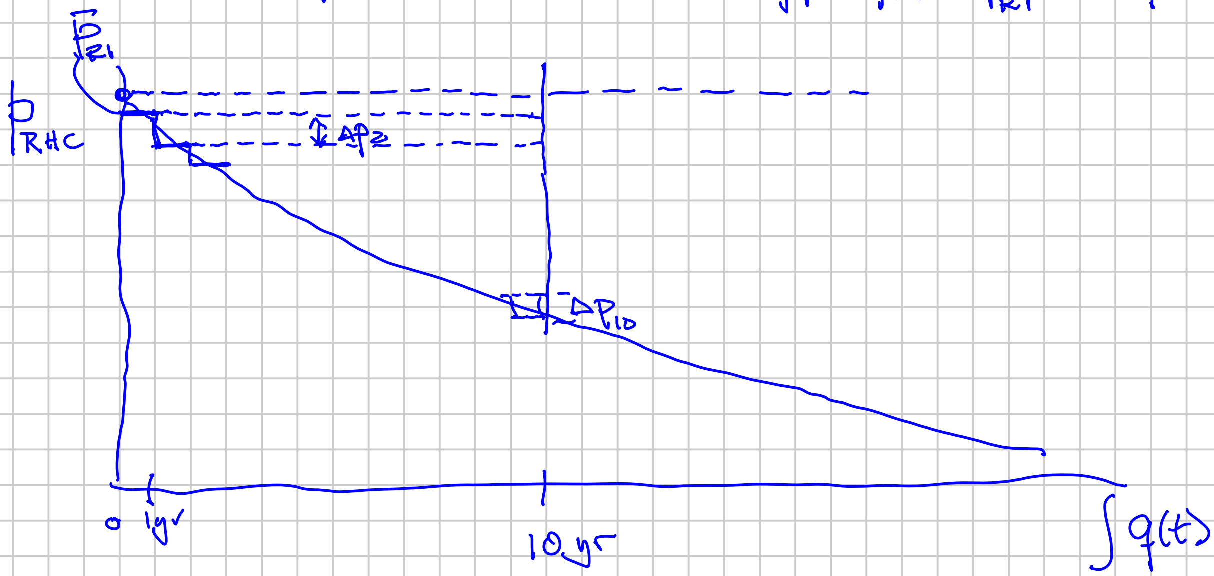
Aquifer $\quad r_{HC} \quad r_{AQ}$

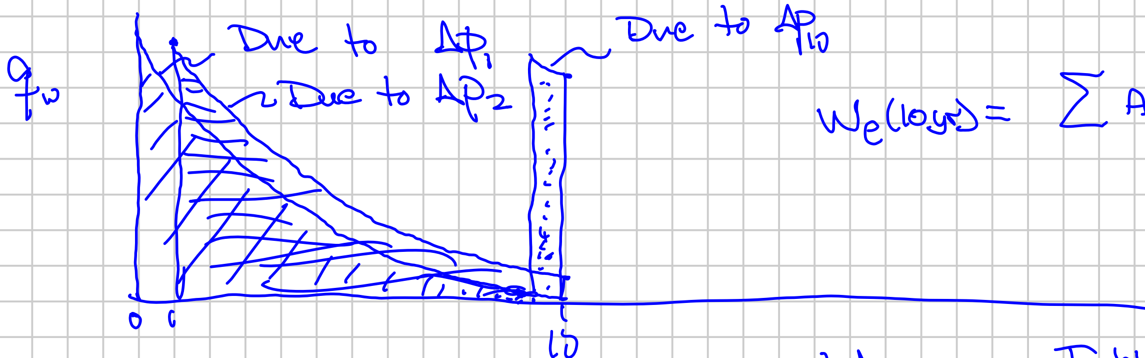
r_R

Because inner BC "p_{wf}" "p_{RHC}" (t)

use "Superposition"

$\Delta p_1 = p_{Ri} - \bar{p}_{R1} \Rightarrow q_w(t)$ for 10 yrs

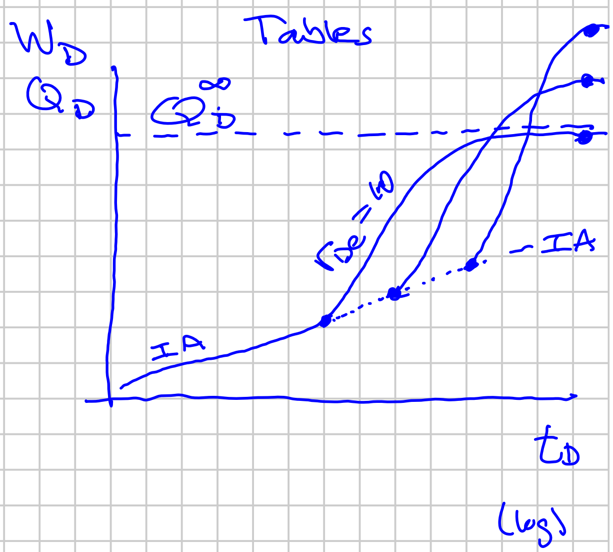




$W_e(10\,yrs) = \sum$ Areas

Cumulative Volume =

$$W_D = Q_D = \int_0^{t_D} q_D(t_D)\, dt_D$$



$$\boxed{W_e = U \sum \Delta p_k\, W_D(\Delta t_{Dk})}$$

$$\Delta p_k = p_{k-1} - p_k$$

$Q_D^\infty(r_{De})$

$$\Delta t_{Dk} = t_D - t_{D_{k-1}}$$

$$\Delta t_{D1} = 10 - 0$$

$$\Delta t_{D2} = 10 - 1$$

$$W_e = U \sum \Delta p_{RHC,k}\, Q_D(\Delta t_{D_k})$$

$$W_{e,max} = U \sum \Delta p_{RHC,k}\, Q_D^{\infty}$$

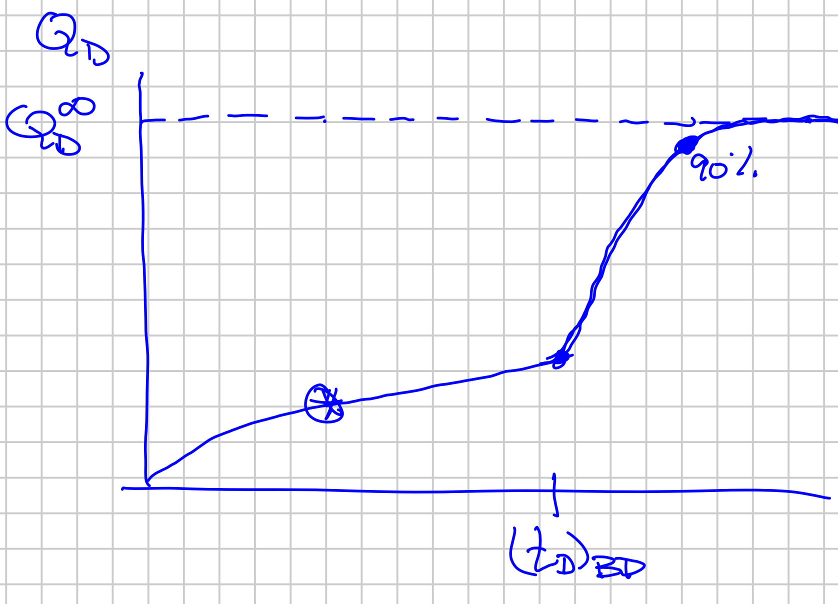
$$= U\,(p_{RHi} - p_{RHC,k})\, Q_D^{\infty} \qquad \text{Pot Aq.}$$

$$= (c_w + c_f)\, V_{AQ}\, (p_{RHCi} - p_{RHC,k})$$

$$\Rightarrow U = \frac{V_{AQ}(c_w + c_f)}{Q_D^{\infty}}$$

$$\Rightarrow \boxed{W_e = \underbrace{V_{AQ}(c_w + c_f)} \sum_{k=1}^{N} \Delta p_k \underbrace{\left[\frac{Q_D(\Delta t)_k}{Q_D^{\infty}}\right]}_{\substack{\text{fraction of the total influx}\\ \text{achieved in } \Delta t_k \text{ for } \Delta p_k}}}$$

$0 - 1$



$$t_D = \frac{\overset{\text{units}}{\bigcirc}\, k\ (1\ \text{year})}{\phi\, \mu\, \underset{c_f + c_w}{c_t}\, r_{HC}^2}$$

$$\frac{Q_D}{Q_D^{\infty}} > 90\% \quad \text{for incremental } \text{“}\Delta t\text{”} \ (\text{e.g. } 1\ \text{year})$$

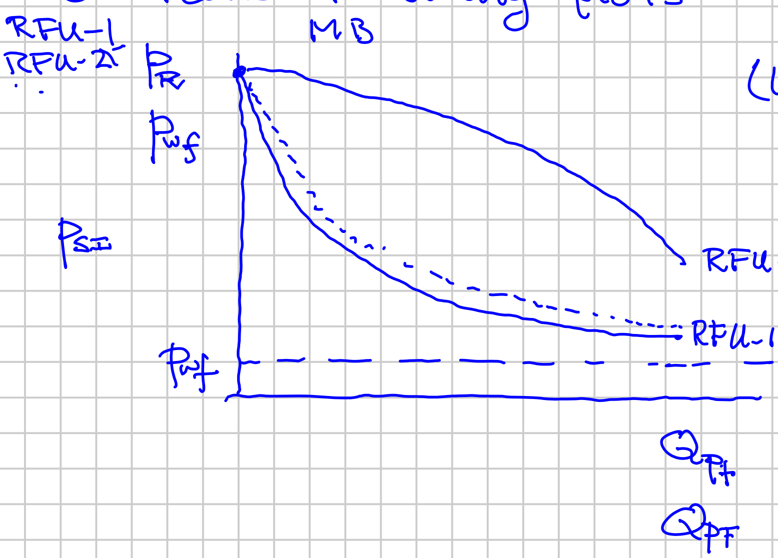
$\Rightarrow$ Pot Aq. assumption is <u>valid</u>

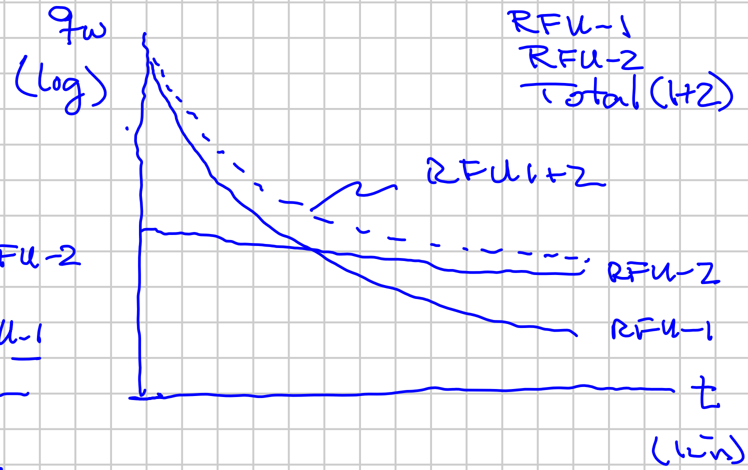
Note Title 10/16/2018

Excel

Q & A on MB-IPR Solution for q(t) Forecasting and understanding of LNX reservoir behavior

① Create following plots





SI : P_{SI} =

SI $\quad q_w = 0 \qquad q_1 = -q_2 \quad @ \; \underline{P_{wf}} \qquad P_{R1} < \underline{P_{wf}} < P_{R2}$

$$J_1 (P_{wf} - P_{R1}) = J_2 (P_{R2} - P_{wf})$$

$$\Rightarrow \quad P_{wf} = \frac{J_1 P_{R1} + J_2 P_{R2}}{J_1 + J_2}$$

Note Title 10/17/2018

* New Required Reading paper:
  1973 - Fetkovich - DCA (JPT - 1980) {supplement to lectures}

* IPR Eq. for "Wellbore Backflows" in LNX systems during a shut-in period $q_t = 0 = \underset{<0}{q_1} + \underset{>0}{q_2}$

(1)
$$q = J(p_R - p_x) + J_x(p_x^2 - p_{wf}^2) \quad | \quad p_{wf} < p_x < p_R$$

(2)
$$q = J(p_R - p_{wf}) \quad | \quad p_{wf} > p_x$$

(3)
$$q = J_x(p_R^2 - p_{wf}^2) \quad | \quad p_R < p_x$$

How to handle $p_R < p_{wf}$?

$$q < 0$$

$$J_x = \frac{J}{2p_x}$$

DECLINE CURVE ANALYSIS (DCA)

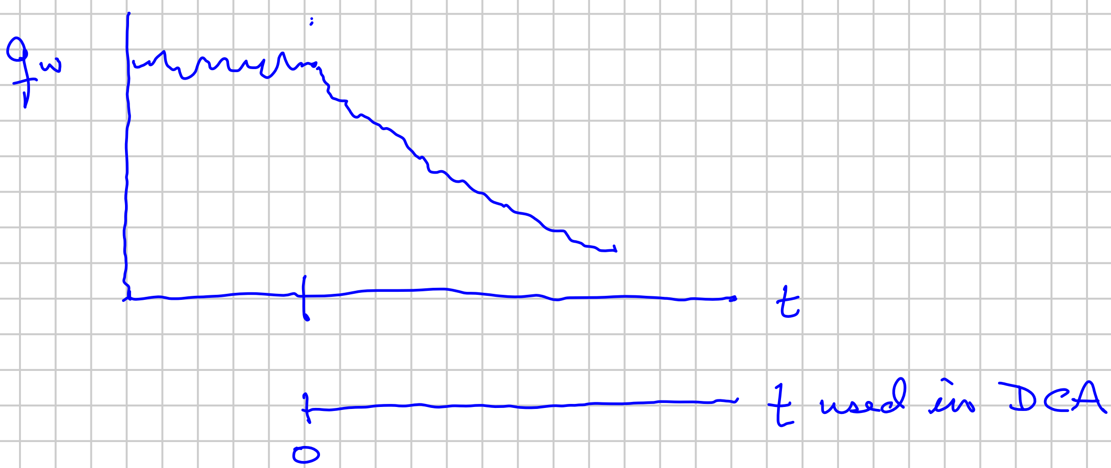
(RTA)

① Arp's Eq

$$q = \frac{q_i}{[1 + bDt]^{1/b}} \qquad 0 < b < 1$$

$$q = q_i \cdot e^{-Dt} \qquad b = 0$$

Applies after a well goes on decline



② Fetkovich (1973)

(a) Arps 3 parameters q_i D b
are expressed in physical terms

(i) $q_i = q$ for PSS (BD) flow with a constant FBHP at the start of decline ($\bar{p}_R \le \bar{p}_{Ri}$)

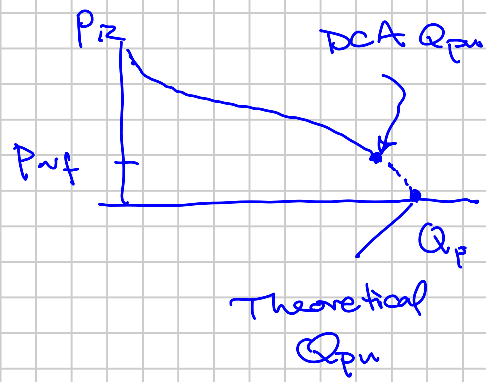
e.g. $q = J(\bar{p}_R - p_x) + J_x(p_x^2 - p_{wf}^2)$

$k \quad h \quad r_e \quad r_w \quad s \quad \mu \quad B \quad p_x$

(ii) D = "decline constant"

$$D \equiv \frac{q_i}{Q_{pu}^{DCA}} \cdot \frac{1}{1-b}$$

$$Q_{pu} = \int_0^{\infty} q \, dt$$

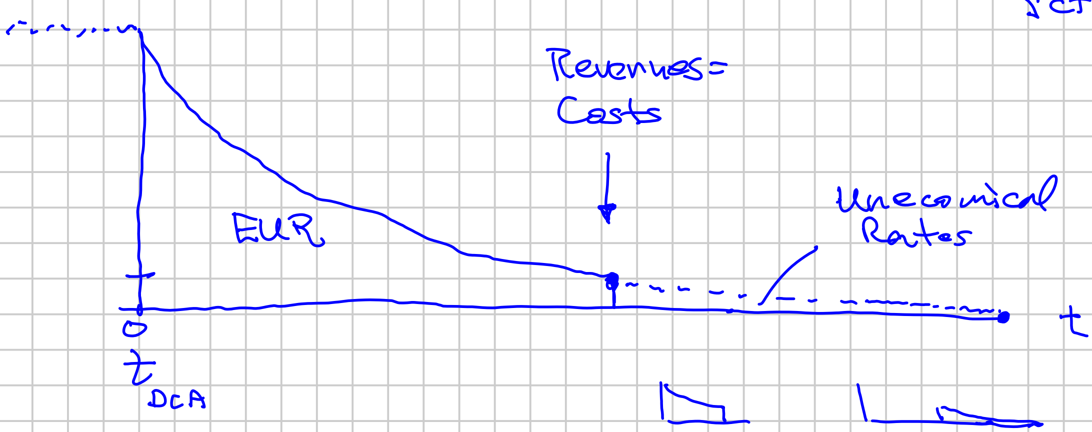


$$p_R^{\infty} = p_{wf}$$

$$q^{\infty} = 0$$

EUR = Est. Ultimate Recovery

Sm^3
STB
Scf



$$Q_{pu}^{DCA} = EUR + \text{Non-Econ}$$

$$EUR \leq Q_{pu}^{DCA} \leq Q_{pu}$$

$p_R \to p_{wf,min}$ $\qquad$ $p_R \to 0$

(iii) b : reflects the shapes of IPR & MB

$q(p_{wf})$ $\quad$ $\bar{p}_R(Q_p)$

Single-phase vs Multiphase Flow

$p_R > p_b$ $\qquad$ SGP $p_R < p_b$

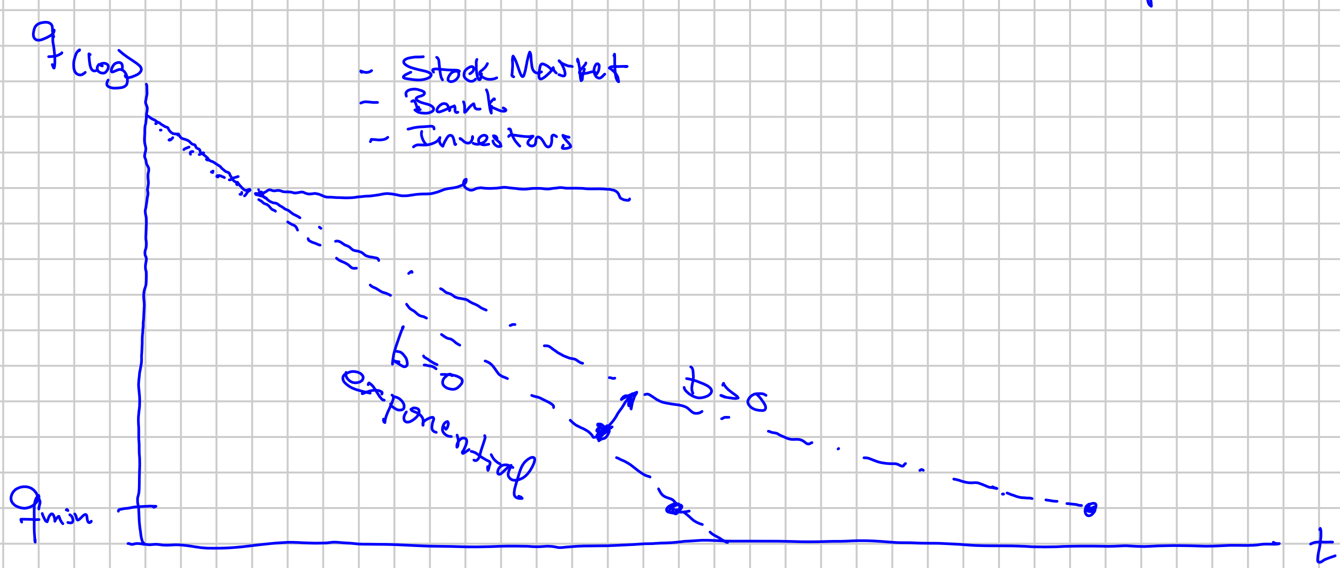
$p_{wf} > p_x$

Reservoir Flow and PVT behavior

1973 Single RFU : $0 < b < 0.5$

1990 Multiple RFUs (LNX) : $0.x \le b < 1$

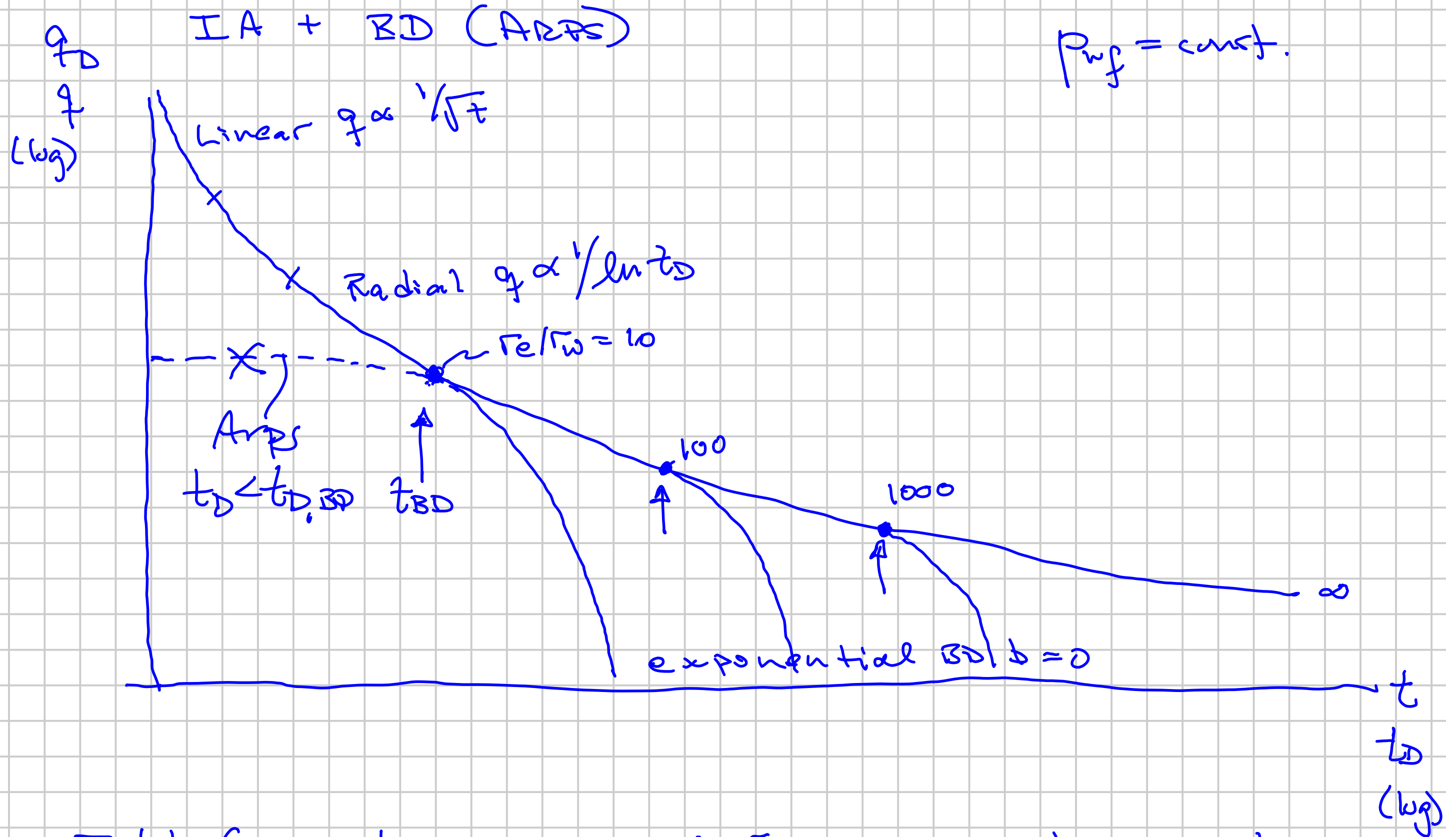
RFUs have ~ same D $\Rightarrow$ q_w "b" ~ q_{RFW} "b"



Fetkovich DCA

(b) Coupled the general flow theory in porous media (i.e. not just PSS / BD flow) which is both "Infinite Acting" (IA) & BB behavior.

IA + BD (ARPS) $p_{wf} = $ const.



Tight Gas $t_{BD} \sim 0.5$ yr $\rightarrow 5$ yr $p_{wf} \sim$ const

$\uparrow$

$\underline{\underline{k}}$ r_e

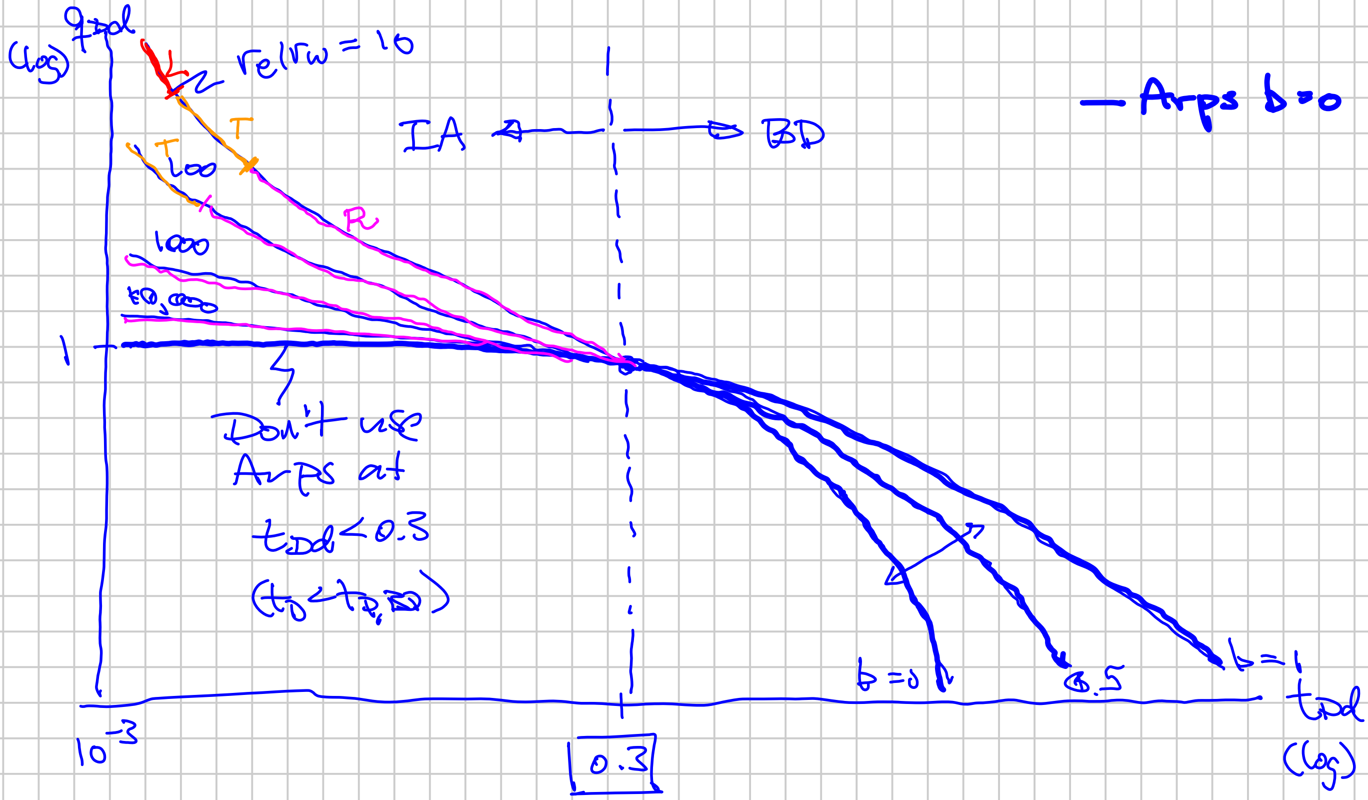
New Dimensionless variables

$$t_{Dd} \simeq \frac{t_D}{t_{D,BD}(r_e/r_w)} \quad (0.3)^{\pi/16}$$

$$q_{Dd} \simeq \frac{q_D(t_D)}{q_D(t_{D,BD})}$$

Collapses all $q_D(t_D > t_{D,BD})$ to a single exponential curve

$$q_{Dd} = e^{-t_{Dd}}$$



Generalized Fetkovich Decline Type Curve (IA & BD)

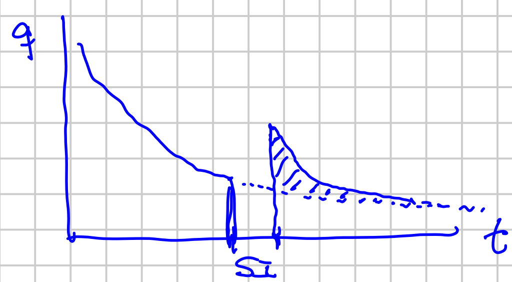
$$\left.\begin{aligned} \frac{q}{q_i} &= q_{Dd} = \frac{1}{[1+b\,t_{Dd}]^{1/b}} \\ \frac{q}{q_i} &\equiv q_{Dd} = e^{-t_{Dd}} \end{aligned}\right\} \quad t_{Dd} \equiv Dt$$

(c) $p_{wf}(t)$

(i) Rigorous Superposition - analogous to water influx

- Any $p_{wf}(t)$ variation

  e.g. $q(t > \text{shut-in})$



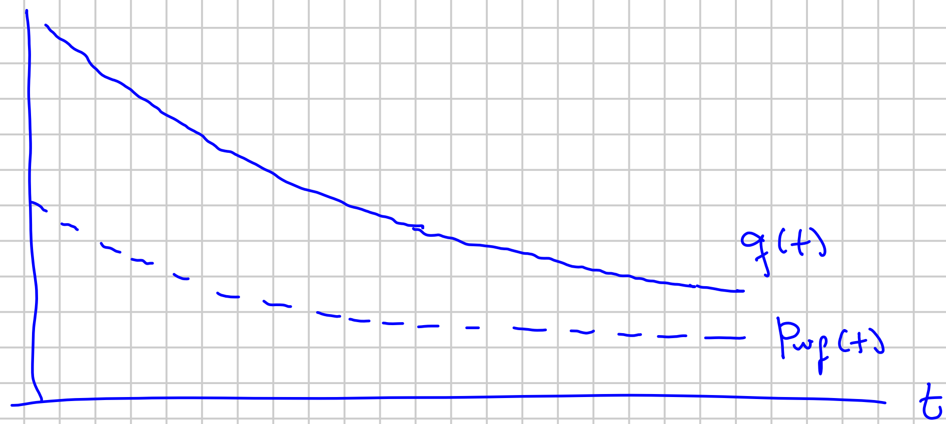
(ii) Smoothly Varying $p_{wf}(t)$:

"Rate Normalization" (Winestock & Colpitts)

$\boxed{\text{IA}}$: $q_D(t_D) \simeq p_D^{-1}(t_D)$

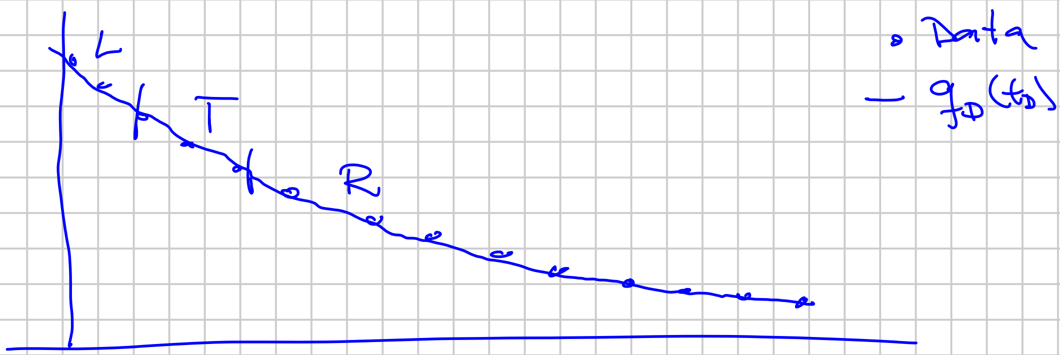
p_{wf} = const q = const

$$p_{wf}(t) \Rightarrow \frac{q(t)}{p_{Ri} - p_{wf}(t)} \simeq \frac{q(t)}{p_{Ri} - \underset{\text{const.}}{p_{wf}}} : \frac{q_D(t_D)}{T}$$



$$\left[\frac{q(t)}{p_i - p_{wf}(t)}\right]$$

$q_D(t_D)$



Rate Normalization
BD: DOES NOT WORK

Arp's

$$q_D(t_D > t_{D,BD}) \propto e^{-t_D}$$ $b = 0$

$$p_D^{-1}(t_D > t_{D,BD}) \propto \frac{1}{1 + t_D}$$ $b = 1$

③ Past Fetkovich

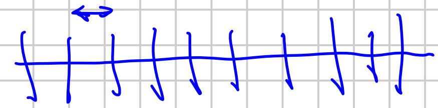
- nothing much new > 1990s LNX

~ 2005-ish "Unconventional 'Shales'" NA

$k \sim 10 - 1000$ md

$10^{-5} - 10^{-3}$ md

$L_{ff} \sim 500$ ft $\rightarrow 20$ ft



$t_{BD} \sim \underbrace{2 \text{ yr} \rightarrow 20 \text{ yr}}$

(k, L_{ff})

$10^{-5} - 10^{-3}$ md

$t < t_{BD}$

"L_{FA}" $\rightarrow$ BD

$$q \propto \frac{1}{\sqrt{t}}$$

$$q = \frac{q_i}{[1 + bDt]^{1/b}}$$

$b = 2$

$1 \ll bDt$

$$\boxed{q \sim \frac{1}{\sqrt{t}}}$$

Use Arps $b = 2$

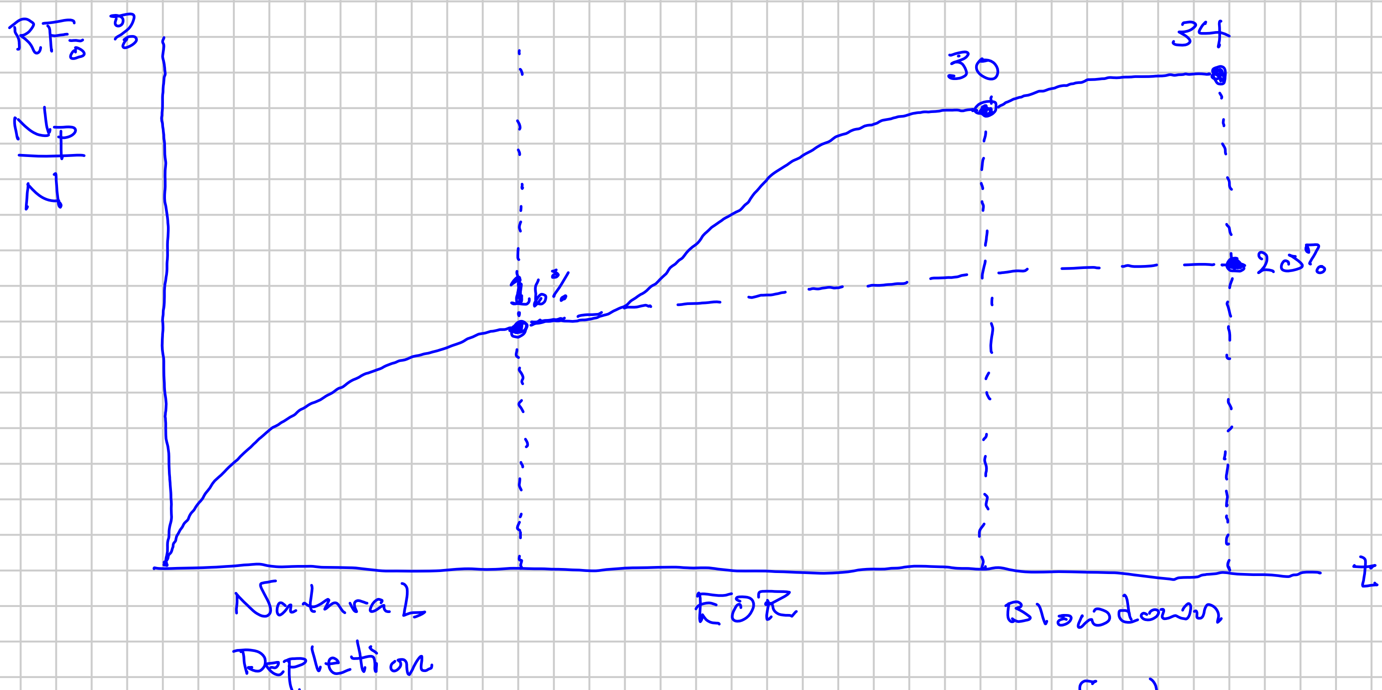
Note Title 10/22/2018

ENHANCED OIL RECOVERY (EOR)

"Enhanced"

- Recovery beyond what is expected by natural depletion: c_f, c_w, Aquifer SGD PVT (CVD) $\sqrt{s}$

- Inject $\underbrace{\text{water, gas}}_{\text{WAG}}$, (and/or chemicals)



$N_{WD} + \Delta N_{WEOR} = N_{WT}$

What is:

- EOR recovery?

- Depletion recovery?

What are:

- Depletion CAPEX & OPEX
- EOR CAPEX & OPEX

INCREMENTAL STO RECOVERY (%)

Initial STO In Place

$$N = \sum_{n=1}^{N_B} \Delta V_{pin} \left(\frac{S_{oi}}{B_{oi}} + \frac{S_{gi}\, r_{si}}{B_{gdi}} \right)_n$$

STO (RO)

"B": Block (e.g. RFU, simulator grid cells)

$$\Delta V_p = (\Delta x \cdot \Delta y \cdot \Delta z\ \phi)$$

Remaining STO in Reservoir at time after start of production

$$N_R = \sum_{n=1}^{N_B} \Delta V_{pn} \left(\frac{S_o}{B_o} + \frac{S_g\, r_s}{B_{gd}} \right)_n$$

$$N_p = N - N_R$$

$$= f\Big(\underbrace{S_o,\ S_g,\ \underset{(B_o,\, B_{gd},\, r_s)}{PVT}}\ ,\ c_f\Big)$$

- $k_r(S)$: 2ϕ, $3\text{-}\phi$, $f(PVT)$
- $PVT(p)$: Depletion
- Injection (EOR)
  - Displacement $S(x, y, z)$
  - • Gravity
  - PVT $\underbrace{B_o \quad B_{gd} \quad r_s \quad S_o}_{V \mid C \mid M}$

Vaporization Condensation Miscibility

- Imbibition (Wettability)
- Diffusion
- Capillarity

EOR METHODS

All Methods – (Common Denominator)

Many WF projects

(a) Pore-Level: Drive "S_{oi}" → minimum → S_{or} → 0

$\overbrace{S_{orw}}$ $\underbrace{S_{org}}$

$k_{ro} \to 0$ $\quad \sigma \to 0$

$$\left(\frac{S_{oi}}{B_{oi}} + \frac{S_{gi} r_{si}}{B_{gi}}\right) \to \text{minimum}$$

- Buckley-Leverett (BL) flow theory $\underbrace{S_{oi} \to S_{or}}$

Water & Gas

~~Displacing~~ Displacing Oil

$$N_c = \frac{v}{\sigma} = \frac{(dp/dx)}{(P_c)}$$

$$\frac{P_{cgo}}{P_{cwo}} \propto \frac{\sigma_{go}}{\sigma_{wo}}$$

↑

IFT

(b) CONFORMANCE – Volumetric Sweep

Composite $\begin{cases} \text{– Vertical (Layering)} & z \\ \text{– Areal } (\underbrace{\text{I-P well placement}}_{\text{"pattern"}}) & x, y \end{cases}$

* HCPV that has "seen" the injectant

– Usually the pore-level recovery $\left(\sim 1 - \frac{(S_o)_{\text{at seeing}}}{S_{oi}}\right)$

Microscopic Displacement Efficiency MDE

50% → 100%

EOR we use instead of time

we use "pore volumes injected" PV_{inj} PV_i

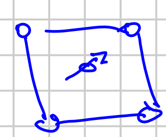
"PVI"

- "Small scale (core lab test)

Well defined, fixed, "pattern" "Volume"

$$PVI = \frac{Q_{inj}}{\text{"PV"}}$$

Core   Pattern



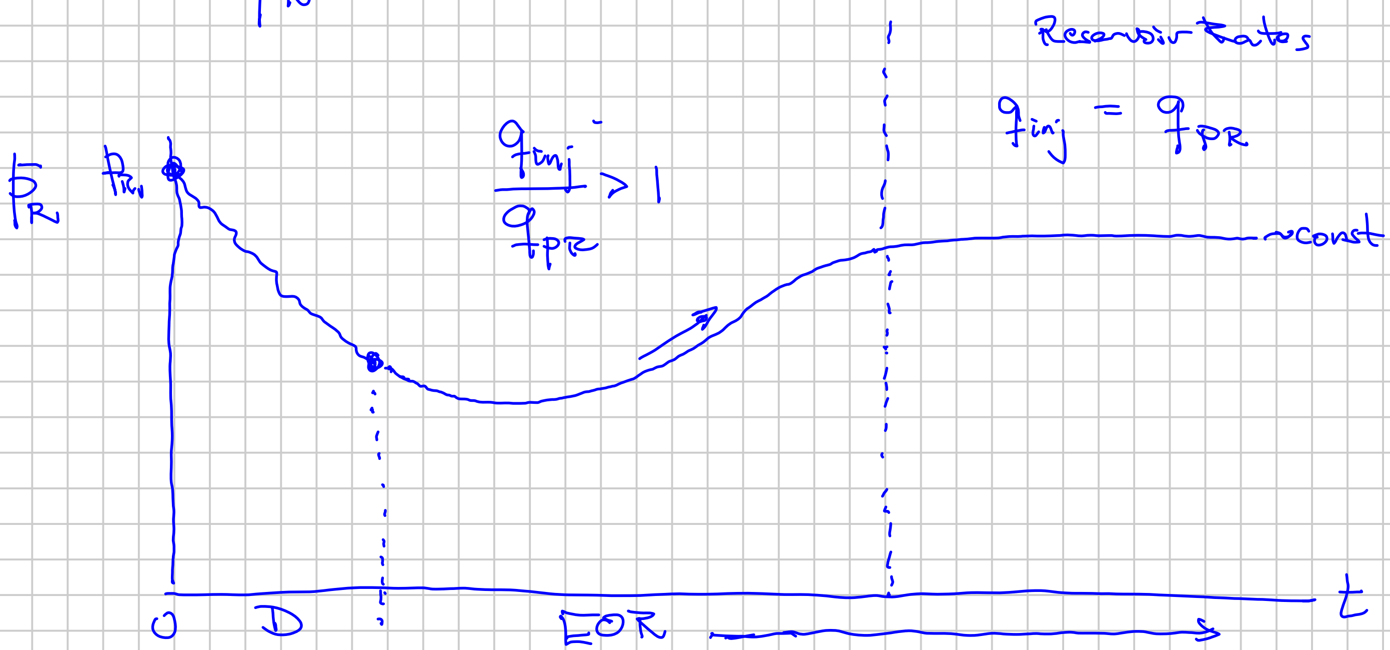
- Dynamic "PV" (t) – real fields/wells

nobody seems to talk about this...

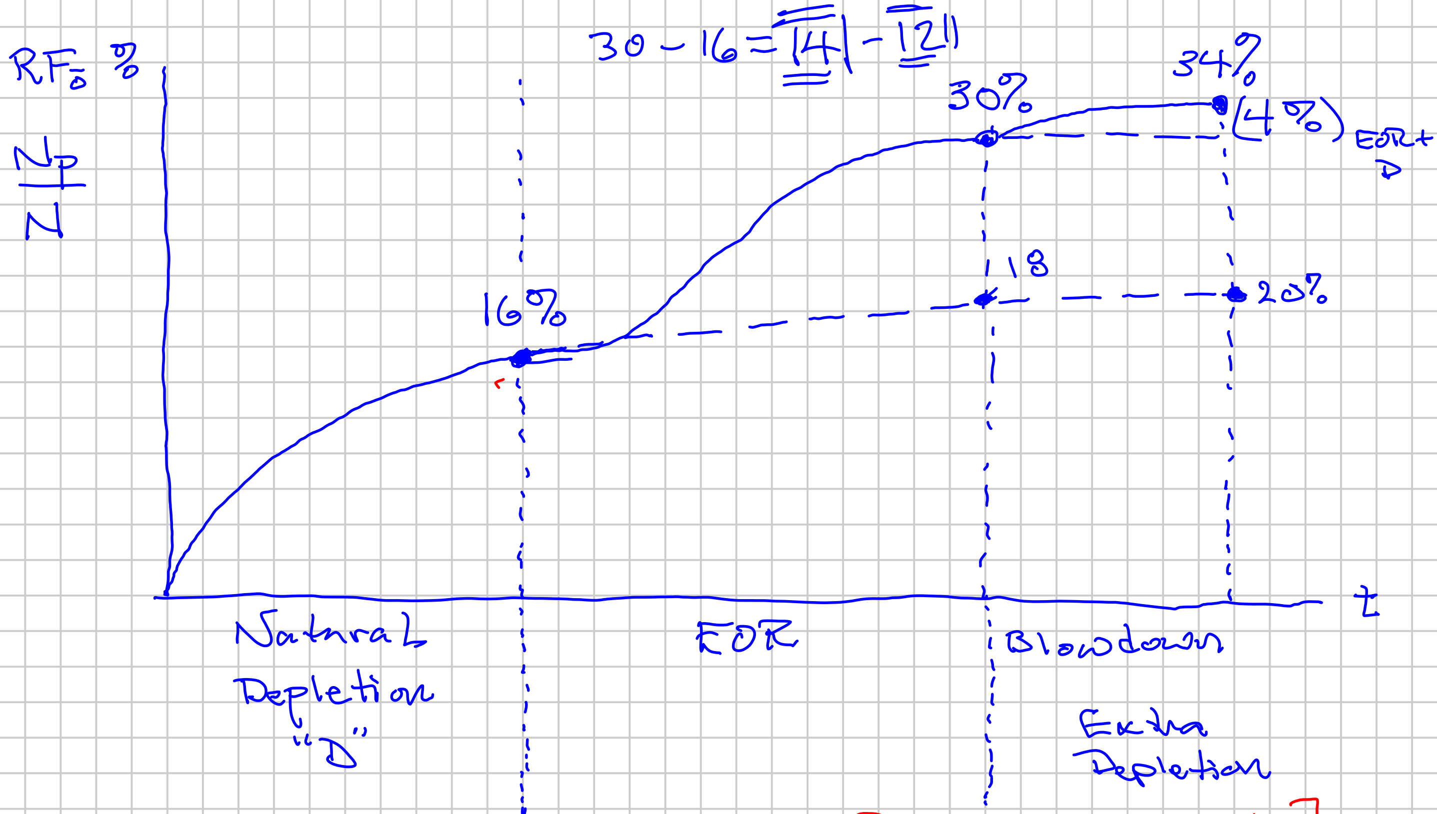
* VOIDAGE REPLACEMENT

$$q_{inj}(\bar{P}_R) \cong \underbrace{q_{PR} = q_g B_g + q_o B_o + q_w B_w}_{@\ \bar{P}_R}$$

$\Rightarrow \quad \bar{P}_R(t) \sim$ constant



Note Title 10/23/2018



CW: D: 16 + 2 + 3 = 21
EOR: 34 - 21 = 13%

(EOR, Depletion)

(20, 15) | (30, 20) | (34, 16) | (30, 16) | (34, 15) |

(30, 18) | (25, 20) | (30, 20) | (35, 15) | (38, 15) |

(34, 20) | (14, 20) | (14, 20) | (32, 17) | (18, 31) |

(35, 20) | (21, 16) | (19, 18) | (20, 16) | (14, 20)

max EOR = 34 - 16 = 18%

MOBILITY ISSUES:

Effective — Relative — Depletion

$$\lambda_p = \frac{k_p}{\mu_p} \quad \Big| \quad \frac{k_{rp}}{\mu_p}$$

$$\frac{md}{cp} \qquad \frac{1}{cp}$$

$$k_{rp} = \frac{k_p}{k} \leftarrow \text{Absolute}$$

$$\lambda_g = \frac{(k_{rg})}{\mu_g (B_g}$$

$$\lambda_o = \frac{(k_{ro})}{\mu_o (B_o}$$

EOR: Two Phases

- Displacing Phase (Injecting)
- Displaced Phase (Usually oil)

$$\boxed{\lambda : f(\Delta p)}, \quad \Delta p = P_{inj} - P_{wf}$$

λ_o (Displaced Phase)

$\underbrace{\lambda_w \quad \lambda_g}_{\lambda_D}$ (Displacing Phase

Mobility Ratio: M

Impacts directly ΔS_o (BL theory)

E_m E_p

Conformance

E_v

E_A

$$M \underset{\neq}{=} \frac{\lambda_{\text{Displacing}} \; \text{"w"}}{\lambda_{\text{Displaced}} \; \text{"o"}} = \frac{\frac{k_{rw}}{\mu_w}}{\frac{k_{ro}}{\mu_o}} = \frac{\frac{k_w}{\mu_w}}{\frac{k_o}{\mu_o}} = \frac{\underbrace{k_{rw}}_{R} \underbrace{\mu_o}_{F}}{\underbrace{k_{ro}}_{R} \underbrace{\mu_w}_{F}}$$

$$\frac{1}{cp} \qquad \frac{md}{cp}$$

* After (?) 1950

Muskat 1950 Mobility Ratio $\bar{m} \equiv \dfrac{\lambda_{Displaced}}{\lambda_{Displacing}}$

Note Title 10/24/2018

* Standing's Notes on Mobility

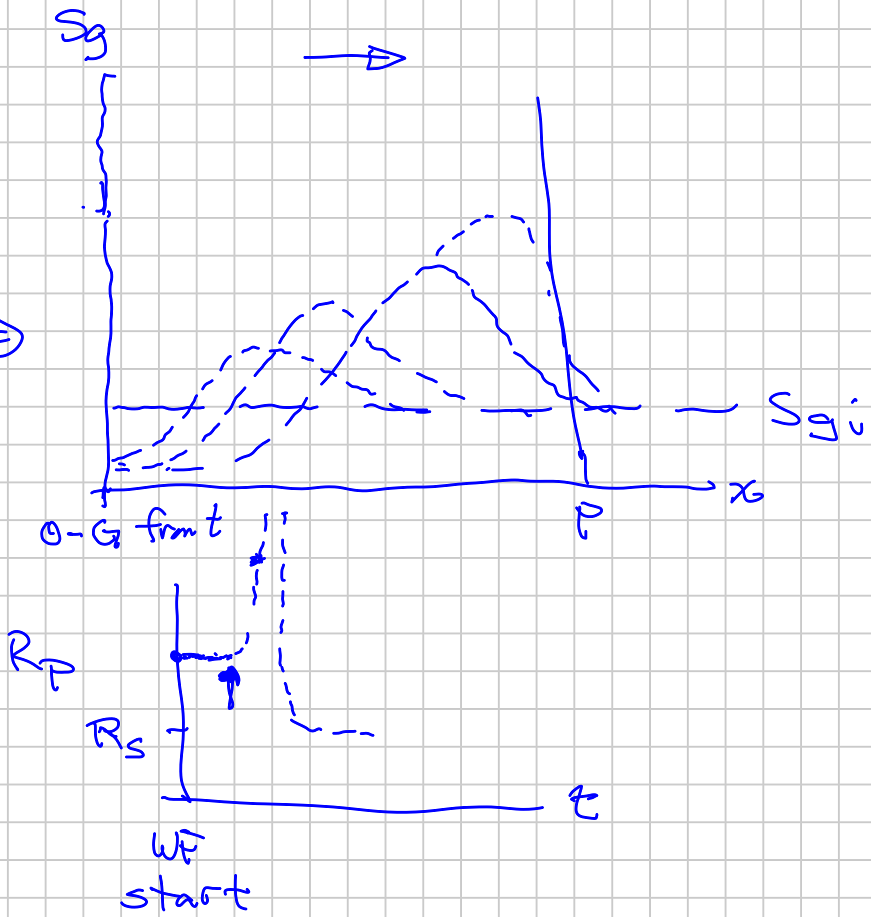
N.S. Oils $\mu_o < 2\ cp \rightarrow 0.3-0.2$

$\mu_w \sim 0.5\ cp$

(10-15 cp)

$S_o = 1 - S_g - S_{wi}$

$O \Rightarrow$



BUCKLEY-LEVERETT THEORY

* Comments on Buckley-Leverett theory and Standing notes (all books...) discussion & presentation of BL theory.

- When it may not "work"
- How to check / verify
- Why it might not work

$$\frac{q_w}{q} = f_w = \frac{1 + \frac{k_o}{q\mu_o}\left[\frac{\partial P_c}{\partial x} - \Delta\rho g \sin\alpha\right]}{1 + \frac{k_o}{k_w}\cdot\frac{\mu_w}{\mu_o}}$$

AS $k_o = k \cdot k_{ro}$ AND IGNORING $\frac{\partial P_c}{\partial x}$ BECAUSE SMALL

$$f_w = \frac{1 - a k_{ro}}{1 + \frac{k_{ro}}{k_{rw}}\cdot\frac{\mu_w}{\mu_o}} \quad \text{WHERE} \quad = \frac{1 - a k_{ro}}{1 + M(S)}$$

$$a = \frac{0.488\, k\, \Delta\rho \sin\alpha}{q\,\mu_o} \quad \text{IF} \quad \begin{array}{l} \Delta\rho = \text{gm/cc} \\ k = \text{DARCY} \\ q = \text{BBL/DAY/FT}^2 \\ \mu_o = \text{cp} \end{array}$$

$$\text{"}q\text{"} = v_D = q_R / A_\perp \quad ; \quad A_\perp(x) = \text{constant}$$

f_w = fractional flow of displacing phase/fluid

$$= \frac{q_w}{q_w + q_o} = \frac{(v_D)_w}{(v_D)_w + (v_D)_o}$$

Simplify for Horizontal flow ($\alpha = 0$)

$$f_w = \frac{1}{1 + \frac{k_{ro}}{k_{rw}} \frac{\mu_w}{\mu_o}} = \frac{1}{1 + \frac{k_{rw}}{k_{ro}}(S_w)\left(\frac{\mu_w}{\mu_o}\right)} = f_w(S_w)$$

$$S_{wi} \le S_w \le 1 - S_{orw}$$

Inclined Flow: $$f_w = \frac{1 - a\,k_{ro}(S_w)}{1 + \frac{k_{rw}}{k_{ro}} \cdot \frac{\mu_w}{\mu_o}}$$

R+F

$$\underbrace{(a\,k_{ro})}_{\text{Gravity Ratio (Velocity) Term}} = \underbrace{\left(\frac{k_o}{\mu_o}\right)}_{\lambda_o} \cdot (\Delta\rho\, g \sin\alpha) \cdot \frac{1}{q_t}$$

F F Controllable

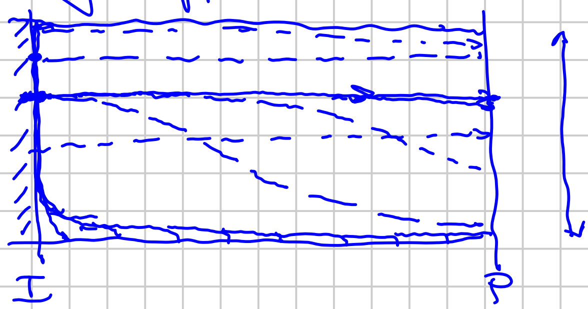
$$\Delta\rho = \rho_w - \rho_o$$

$\rho_o = 800\ kg/m^3$

$\rho_w = 1000\ kg/m^3$

Theory only applies to 1D flow

H:



1'' Core

Layer 10 m

$$\begin{Bmatrix} q \\ v \end{Bmatrix} > \underbrace{\begin{Bmatrix} q \\ v \end{Bmatrix}^*}_{?} \Rightarrow \text{get 1D flow}$$

I:

Assuming ID RL Theory Applies ... read & study notes ...

Notes on Mobility Ratios Used in Fluid Displacement Calculations.

All calculations that involve one fluid displacing a second fluid involve the ratio of the mobilities of the two fluids.

The mobility of a fluid is defined (by me(MBS)) as the ratio of effective permeability to viscosity.

Alternative:
$\lambda_p \equiv \frac{k_{rp}}{\mu_p}$
$\left[\frac{1}{cp}\right]$

$$\lambda_o = k_o/\mu_o \quad \text{md/centipoise} \quad \text{md/cp} \qquad (1)$$

$$\lambda_g = k_g/\mu_g \quad \text{"} \qquad (2)$$

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Note that effective permeability in the above equations depend on saturation, saturation history (drainage or imbibition process) as well as the character of the porous rock.

In systems in which one fluid is displacing a second, it is common practice to call the fluid that is increasing in saturation the "displacing fluid" or "displacing phase". The fluid that is decreasing in saturation is called the "displaced fluid" or "displaced phase". The mobility ratio is the ratio of the mobility of the displacing fluid to that of the displaced fluid.

* <1950: Muskat
$M = \frac{\lambda_{displaced}}{\lambda_{displacing}}$

$$M = \lambda_{displacing} / \lambda_{displaced} \qquad (4)$$

For example, when water is displacing oil, as when aquifer water moves into an oil reservoir, the mobility ratio of the operation is written

$$M_{wo} = \lambda_w/\lambda_o = \frac{k_w}{k_o} \cdot \frac{\mu_o}{\mu_w} \qquad (5)$$

changes a lot $\left\{ \frac{k_{rw}}{k_{ro}}(S's, \ldots) \right.$ R+F (F) ↳ ~ constant

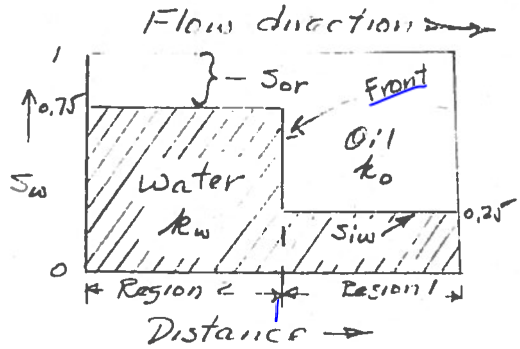


Figure 1.

Figure 1 illustrates partial displacement of oil by water. Only water is flowing in Region 2 behind the water/oil front, while only oil is flowing in Region 1 ahead of the front. In specifying the mobility ratio of this system, the effective water permeability in Region 2 is evaluated at the residual oil saturation, Sor. Sorw The effective oil permeability in Region 1 is evaluated at the irreducible water saturation Siw. The mobility ratio is,

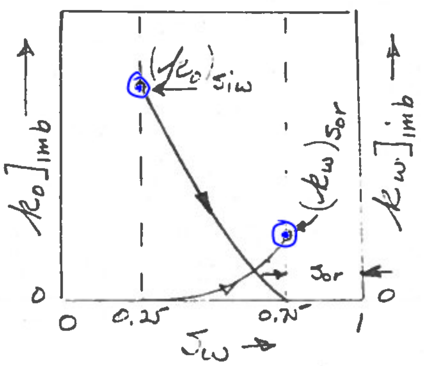


Figure 2

$$M = \frac{(k_w)_{Sor}}{(k_o)_{Siw}} \cdot \frac{\mu_o}{\mu_w} = \frac{k_{rw}(S_{orw})}{k_{ro}(S_{iw})} \qquad (6)$$

The situation of constant fluid saturation behind the front with some residual displaced phase is often refered to as a "leaky piston" displacement.

$S_{oi} \to S_{orw}$ (not $S_{oi} \to 0$)

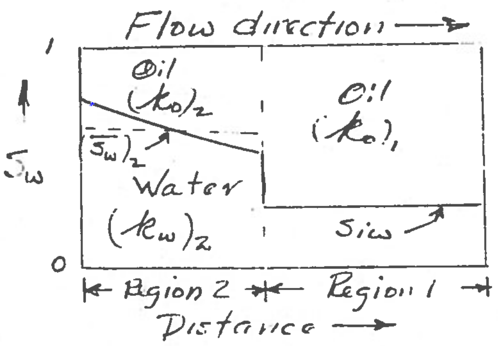


Figure 3

Figure 3 illustrates what is termed a "Buckley-Leverett" type displacement. With this type of displacement both oil and water are flowing in Region 2 (as the front advances). Only oil is flowing in Region 1. In specifying the mobility ratio of the system, the mobility of Region 2 ~~is~~ can be evaluated at the average saturation condition by the relationship:

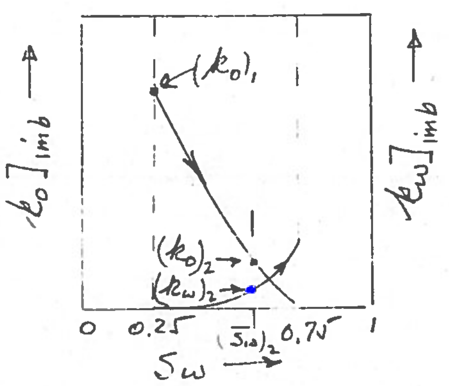


Figure 4.

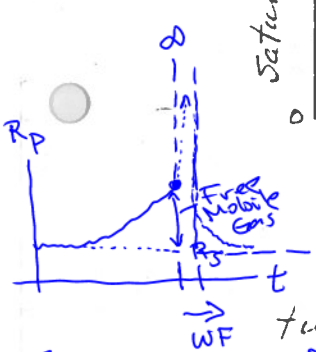
@ $\bar{S}_{w2}$

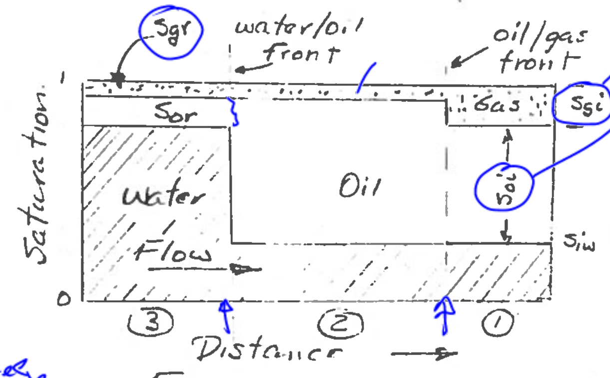
$$\lambda_2 = \left(\frac{(k_w)_2}{\mu_w} + \frac{(k_o)_2}{\mu_o} \right) \qquad (7)$$

The mobility ratio becomes.

$$M_{wo} = \frac{\lambda_2}{\lambda_1} = \left[\frac{(k_w)_2}{\mu_w} + \frac{(k_o)_2}{\mu_o} \right] \bigg/ \frac{(k_o)_1}{\mu_o} \qquad (8)$$

The above effective permeabilities are illustrated in Figure 4.





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e.g. SGD M.B.

Figure 5 illustrates two "leaky piston" displacement fronts often used in calculating a process of water displacing oil and gas. In Region 1 only gas phase is flowing. In Region 2 only oil phase is flowing. And in Region 3 only water is flowing. The two mobility ratios would be written as.

S_{HCrw} = const 35%

S_{orw} $(S_{gi} | S_{gr})$?

$$M_{og} = \frac{\lambda_2}{\lambda_1} = \frac{(k_o)_{S_{iw}, S_{gr}}}{(k_g)_{S_{gi}}} \cdot \frac{\mu_g}{\mu_o} \qquad (9)$$

and.

$$M_{wo} = \frac{\lambda_3}{\lambda_2} = \frac{(k_w)_{S_{or}+S_{gr}}}{(k_o)_{S_{iw}, S_{gr}}} \cdot \frac{\mu_o}{\mu_w} \qquad (10)$$

Mobility ratios for most reservoir displacements lie between about 0.1 and 10. Mobility ratios less than 1 are generally considered as "favorable" in that the displacement front has fairly regular (smooth) features. This is illustrated by the front appearance at three successive times in Figure 6. Mobility ratios greater than 1 give more ragged displacement fronts as illustrated by Figure 7. The term "fingering" is often used to designate the erratic front behavior of high mobility ratio displacements.

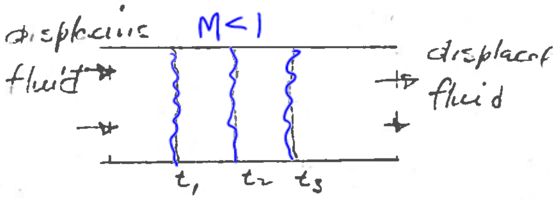
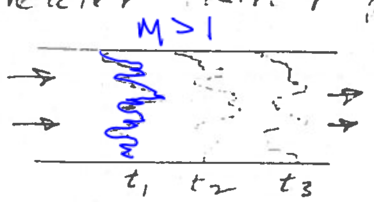


Figure 6



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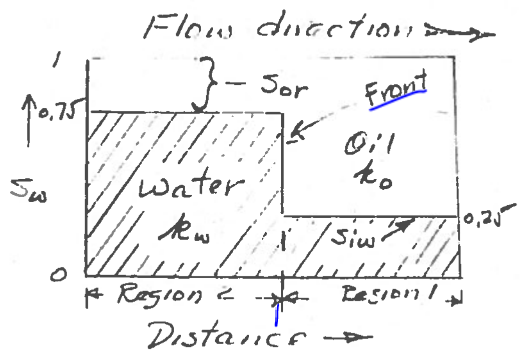

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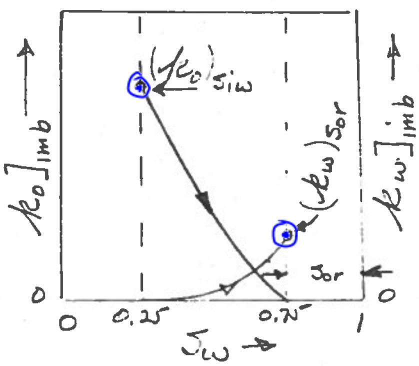

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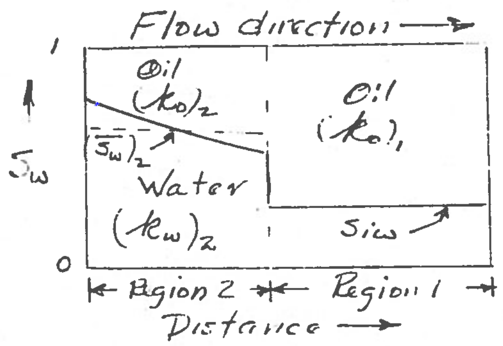

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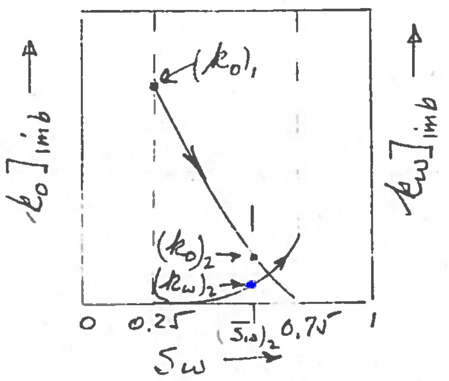

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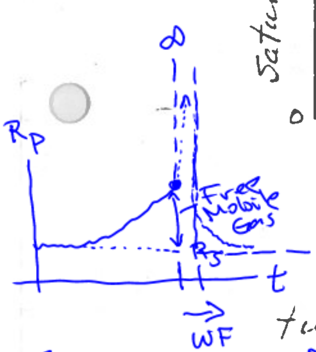

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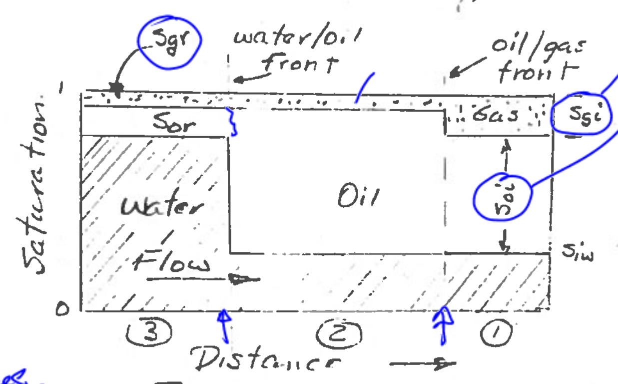

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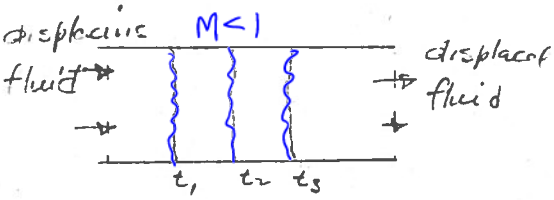


Figure 6

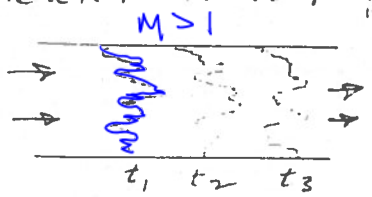


Figure 7

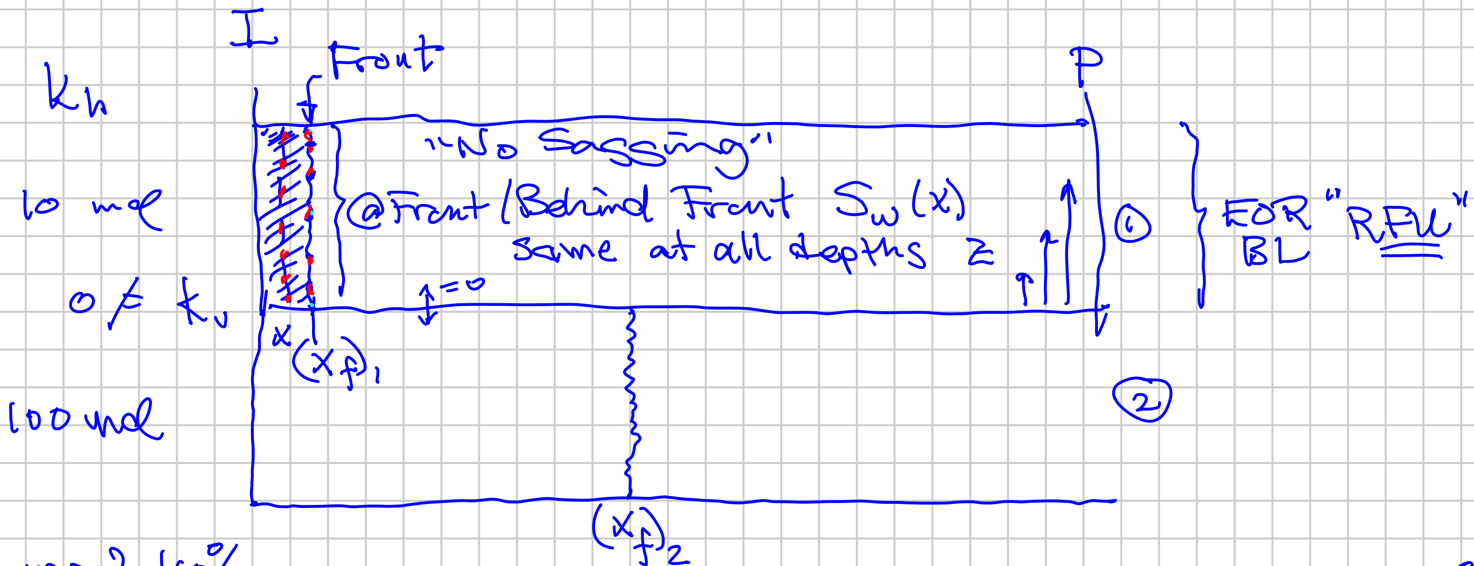
Note Title 10/29/2018

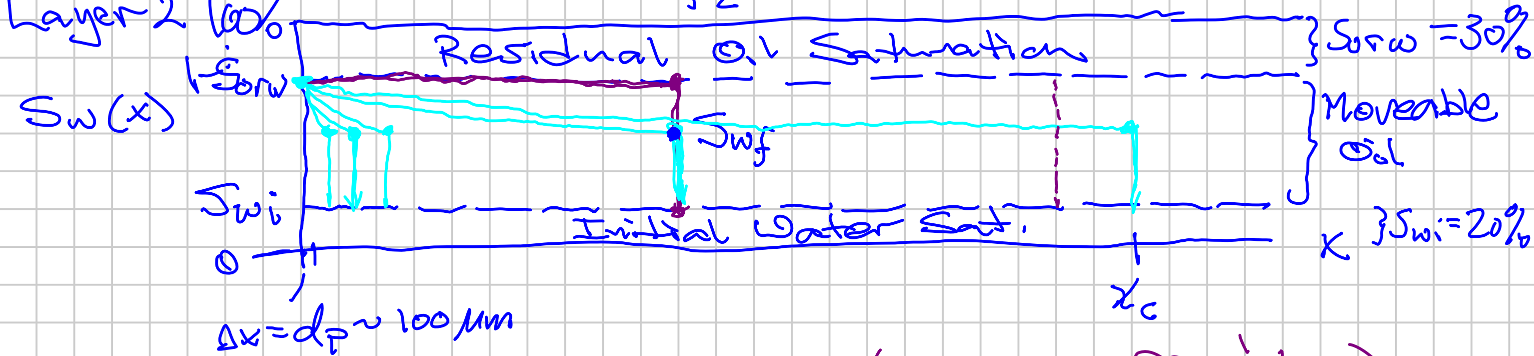
Water-Oil (Gas-Oil) Immiscible Displacement

- Buckley-Leverett Theory (1942)
  - Muskat Comments (1950)
  - Standing notes (197x) "Fractional Flow"
  - Mathematical verification (?) using 'Method of Characteristics'
  - One-dimensional flow theory ($H \mid I \mid V$) $0 < \alpha \leq 90^\circ$

$$q, k, \phi, h, k_r, \Delta\rho \ldots \} \to 1DV$$

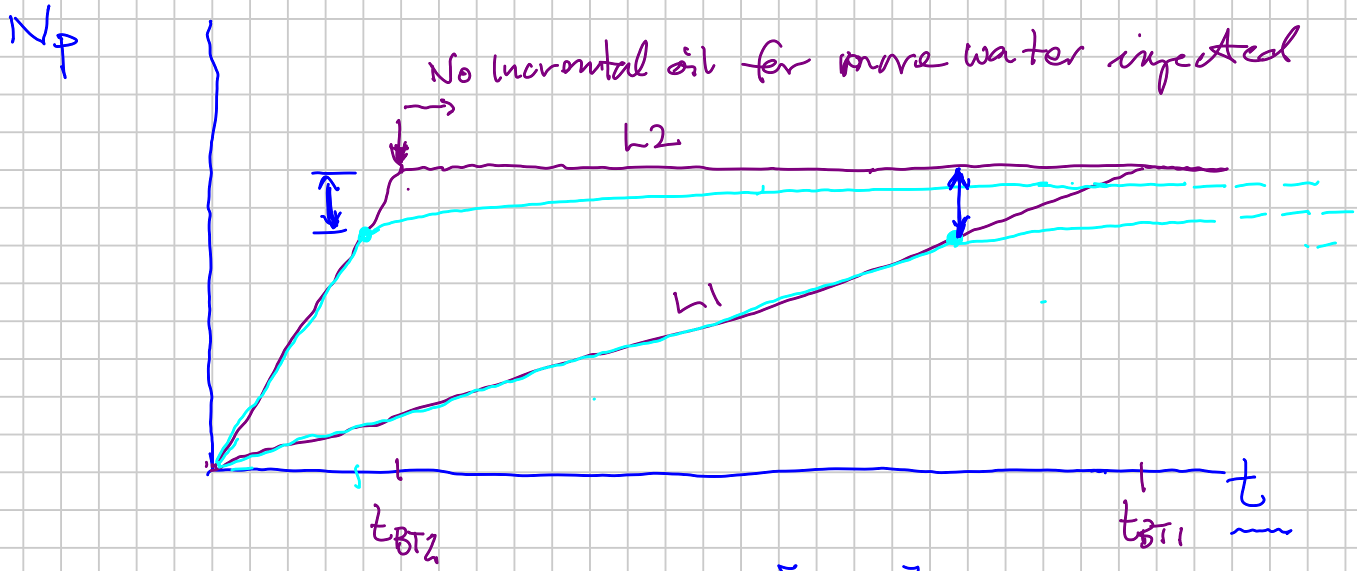
- Alternative : "Leaky Piston" (Muskat)





— Leaky Piston (Optimistic, RF_o & t_{BT})

— BL : $S_w \to 1 - S_{orw}$ requires "infinite" amount of water passing the pore

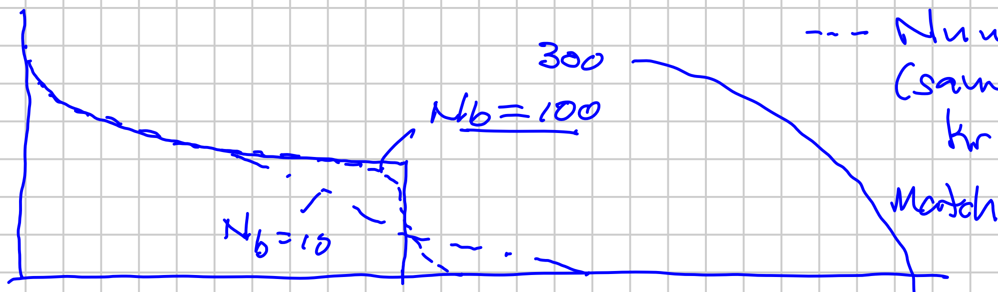


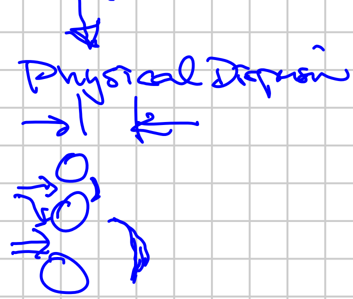
$[q_i(t)]_\ell$ for BL vs LP

Assuming the same water rate injected into L1 & L2 prior to BT

— Analytical BL

--- Numerical (same BL k_r, μ ...)





Injectant Utilization Efficiency

$$\frac{\$80/bbl \quad \Delta q_{to}}{\$10/bbl \quad \Delta q_{iw}} = \underbrace{\text{constant (max)}}_{\text{before BT}} \sim 1 \longrightarrow 0.x \longrightarrow 0$$

before BT — i.o RFW

0.x — BL — Oil Stripping After BT

0 — Leaky Piston

Standing Fraction Flow Notes

1. Fractional Flow Eqs.

2. Frontal Advance Eq. (BL) $S_w(x, t)$
   $\downarrow$
   PVI
   "dimensionless time"

3. Average Saturation ($\bar{S}_w \Rightarrow \bar{S}_o$)
   Behind Front

4. Injection Volume (PVI | V_{iD}) - Recovery $\frac{N_p}{N}$
   Relationship

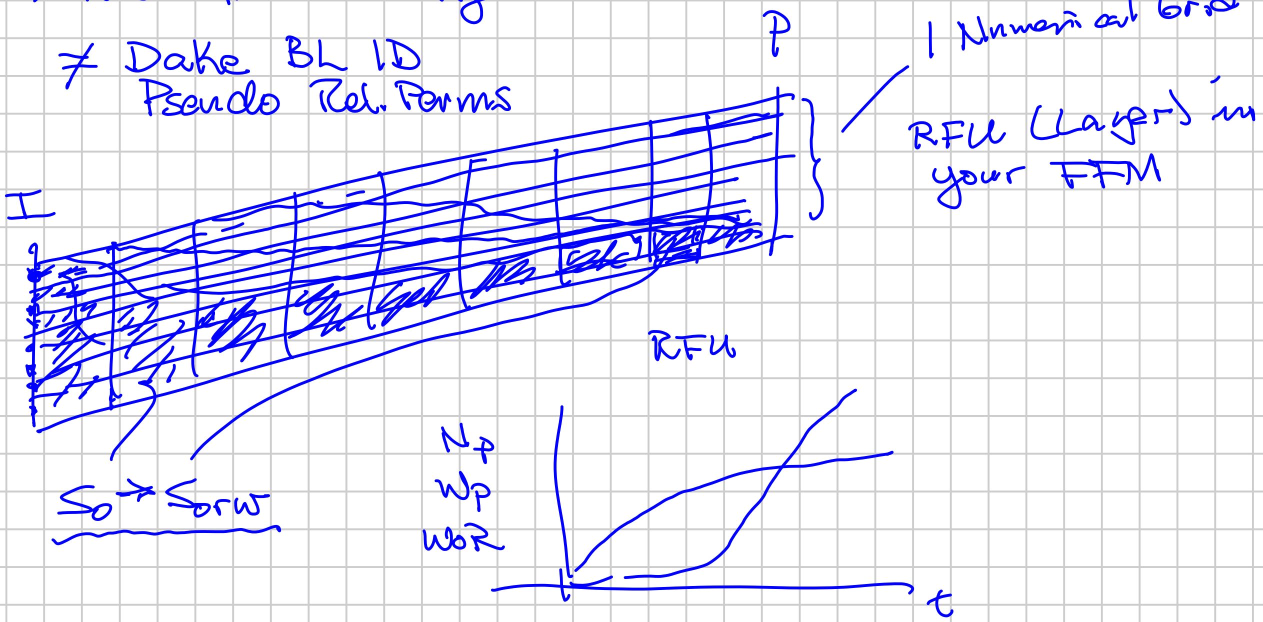
Previous Unresolved Discussion Topics Related to Applicability of BL Theory

① Residual Oil Saturation to Water Displacement in the Presence of Initial Gas Saturation

② When 1D BL theory "breaks down" because of gravitational segregation – and its impact on swept volume recovery, E_D

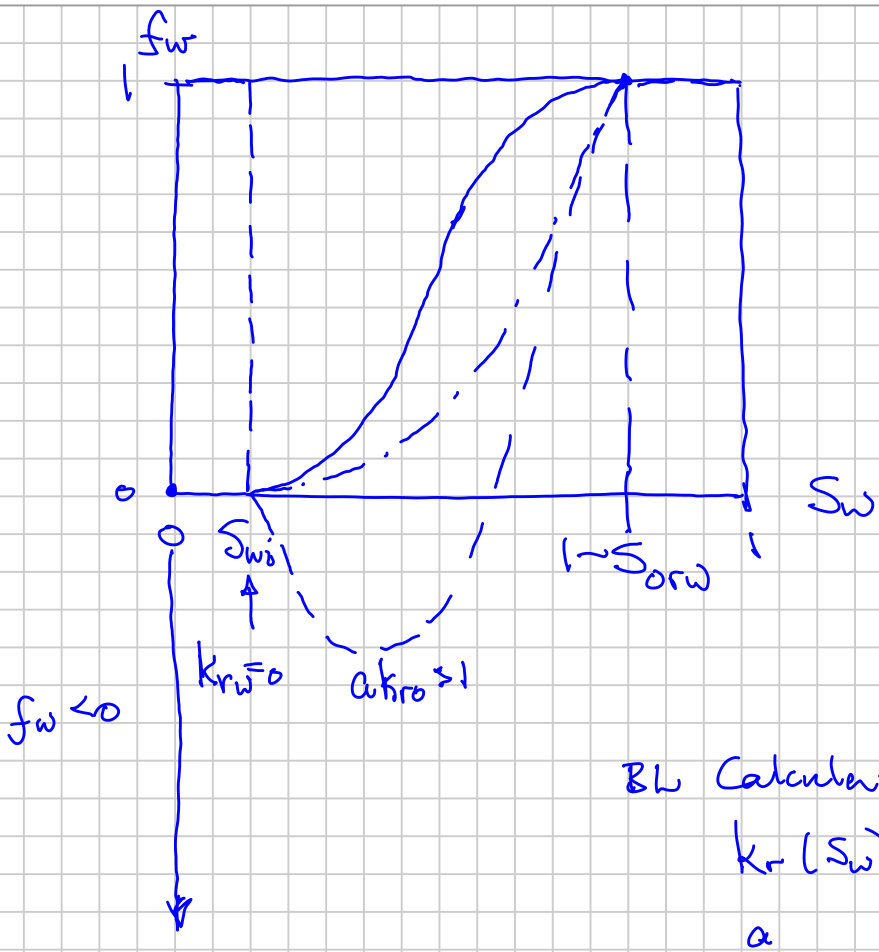
⇒ Reservoir Modeling

≠ Dake BL 1D Pseudo Rel. Perms



HAPPY HALLOWEEN 

Note Title 10/31/2018



— H

-- a > 0

In this example
--- (a>0) ⇒
Leaky Piston
No stripping

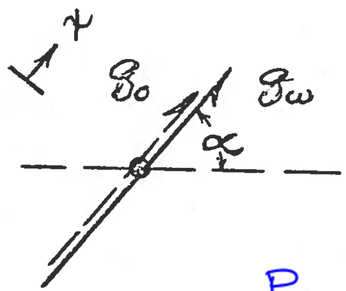
BL Calculations:

$$\left.\begin{array}{l} k_r(S_w) \\ a \\ \mu_w/\mu_o \end{array}\right\} f_w(S_w)$$

$S_w(x,t)$: Frontal Advance Eq.

Rate $q = \frac{q_R}{A_\perp} = v_D$ in Standing notes

FRACTIONAL FLOW EQUATION, I



$$q_o = -\frac{k_o}{\mu_o}\left[\frac{\partial p_o}{\partial x} + \rho_o g \sin\alpha\right]$$

$$q_w = -\frac{k_w}{\mu_w}\left[\frac{\partial p_w}{dx} + \rho_w g \sin\alpha\right]$$

Darcy 1D
H | I | V
$0 \le \alpha \le 90$

P_c

TOTAL FLOW, $q_t = q_o + q_w$

CAPILLARY PRESSURE, $P_c = P_o - P_w$

$P_o \neq P_w$

$$\frac{\partial P_c}{\partial x} = \frac{\partial P_o}{\partial x} - \frac{\partial P_w}{\partial x}$$

$$\frac{\partial P_c}{\partial x} = -\frac{(q - q_w)\mu_o}{k_o} - \rho_o g \sin\alpha + \frac{q_w \mu_w}{k_w} + \rho_w g \sin\alpha$$

LET $\Delta\rho_{wo} = \rho_w - \rho_o$

$$q_w\left[\frac{\mu_o}{k_o} + \frac{\mu_w}{k_w}\right] = \frac{q\mu_o}{k_o} + \frac{\partial P_c}{\partial x} - \Delta\rho g \sin\alpha$$

11.

FRACTIONAL FLOW EQUATION, II

DIVIDING BY $\frac{q\mu_o}{k_o}$;

$$\frac{q_w}{q}\left[1+\frac{k_o}{\mu_o}\cdot\frac{\mu_w}{k_w}\right]=1+\frac{k_o}{q\mu_o}\left[\frac{\partial P_c}{\partial x}-\Delta\rho g\sin\alpha\right]$$

$$\frac{v_{wD}}{v_{wD}+v_{oD}}$$

$$\boxed{\frac{q_w}{q}=f_w}=\frac{1+\frac{k_o}{q\mu_o}\left[\frac{\partial P_c}{\partial x}-\Delta\rho g\sin\alpha\right]}{1+\frac{k_o}{k_w}\cdot\frac{\mu_w}{\mu_o}}$$

AS $\boxed{k_o = k\cdot k_{ro}}$ AND IGNORING $\frac{\partial P_c}{\partial x}$ ~~BECAUSE~~ SMALL

$$\boxed{f_w=\frac{1-ak_{ro}}{1+\frac{k_{ro}}{k_{rw}}\cdot\frac{\mu_w}{\mu_o}}}$$

WHERE $k_r(S_w)$ | $\frac{\mu_w}{\mu_o}$ = constant

$\Delta\rho$ = constant

$$a=\frac{0.488\,k\,\Delta\rho\sin\alpha}{q\mu_o}$$

IF $\Delta\rho$ = gm/cc
k = DARCY
q = BBL/DAY/FT^2
μ_o = cp

μ_o = constant
k = constant

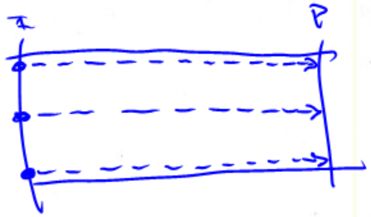
$\alpha\left(\frac{1}{q}\right)$

$a(q)$

H: $\alpha = 0 \Rightarrow a = 0$

q = constant (normal assumption)

$q > q^*$



$\alpha = 0$

FRACTIONAL FLOW EQUATION, III

$$a = \frac{\boxed{7.84(10^{-6})}\, k\, \Delta\rho \sin\alpha}{q\, \mu_o}$$

IF $\Delta\rho = \dfrac{\text{lbs}}{\text{ft}^3}$, k = md., q = BBL/DAY/FT^2, μ_o = cp. — SPE Field Units

No rate dependence

BL ff Eq.

$f_w(S_w, \underbrace{q > q^*}_{BL\checkmark})$

WHEN $\alpha = 0$ (H):

$$\boxed{f_w = \frac{1}{1 + \frac{k_{ro}}{k_{rw}} \cdot \frac{\mu_w}{\mu_o}}}$$

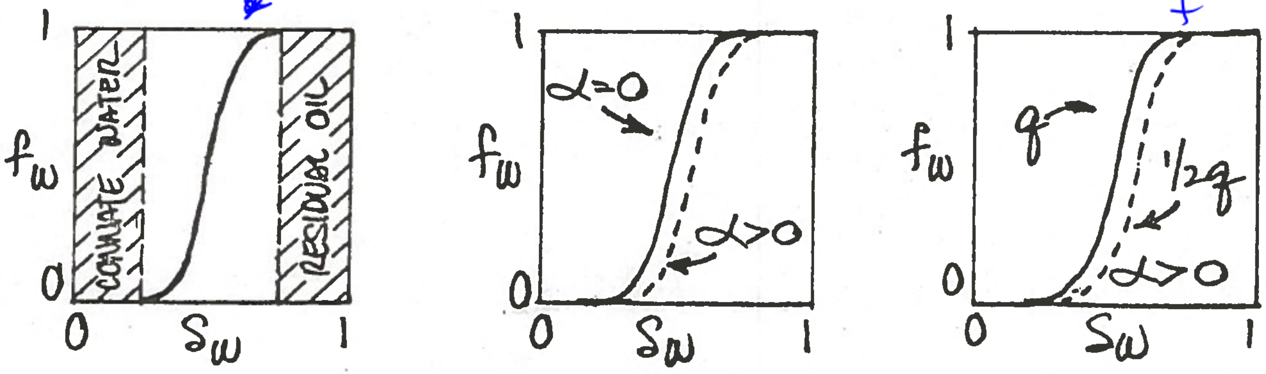
"SPE Metric"

$\Delta\rho$ — kg/m^3

k — md

q — m^3/d/m^2 = m/d

μ — cp ≡ mPa·s

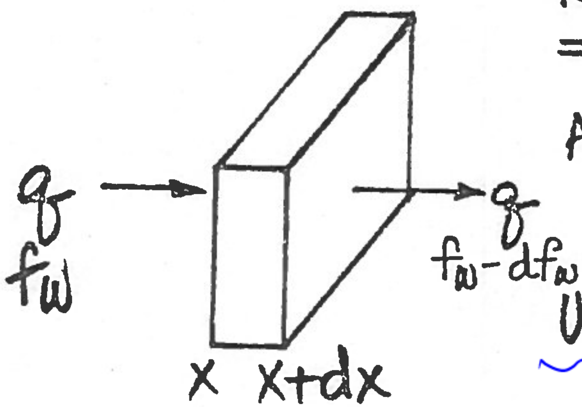


H: $\alpha = 0$

$\alpha > 0$ — q — $\frac{1}{2} q$

13.

FRONTAL ADVANCE EQUATION, I



RATE OF WATER ACCUMULATION = WATER IN MINUS WATER OUT

$$\text{ACCUM. RATE} = q f_w - q(f_w - df_w)$$
$$= q\ df_w$$

$$\text{UNIT PORE VOL.} = \frac{A \cdot dx \cdot \phi}{5.615}$$

RATE OF WATER SATURATION CHANGE, $\frac{dS_w}{dt}$, IS RATE OF WATER ACCUMULATION DIVIDED BY PORE VOLUME.

$$\frac{dS_w}{dt} = \frac{q\ df_w}{A\ dx\ \phi} \cdot 5.615$$

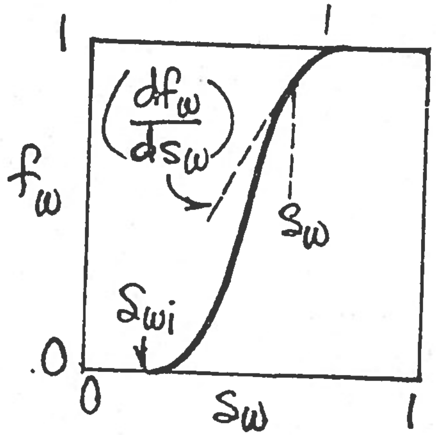
$$dx = \frac{5.615\ q}{A\ \phi}\left(\frac{df_w}{dS_w}\right) dt$$

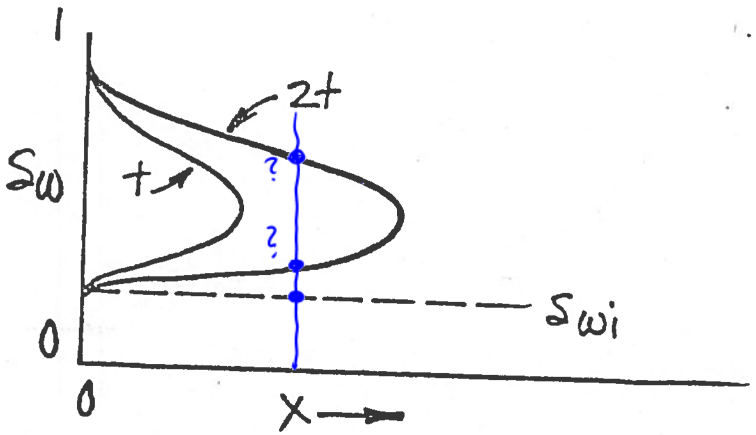
$$\int_{x_o}^{x_{S_w}} dx = 5.615 \cdot \frac{q}{A\phi}\left(\frac{df_w}{dS_w}\right)\int_{t=0}^{t} dt$$

FRONTAL ADVANCE EQUATION, II

$$(x)_{S_w} - X_o = \frac{5.615}{A\phi}\, q t \underbrace{\left(\frac{df_w}{dS_w}\right)}$$

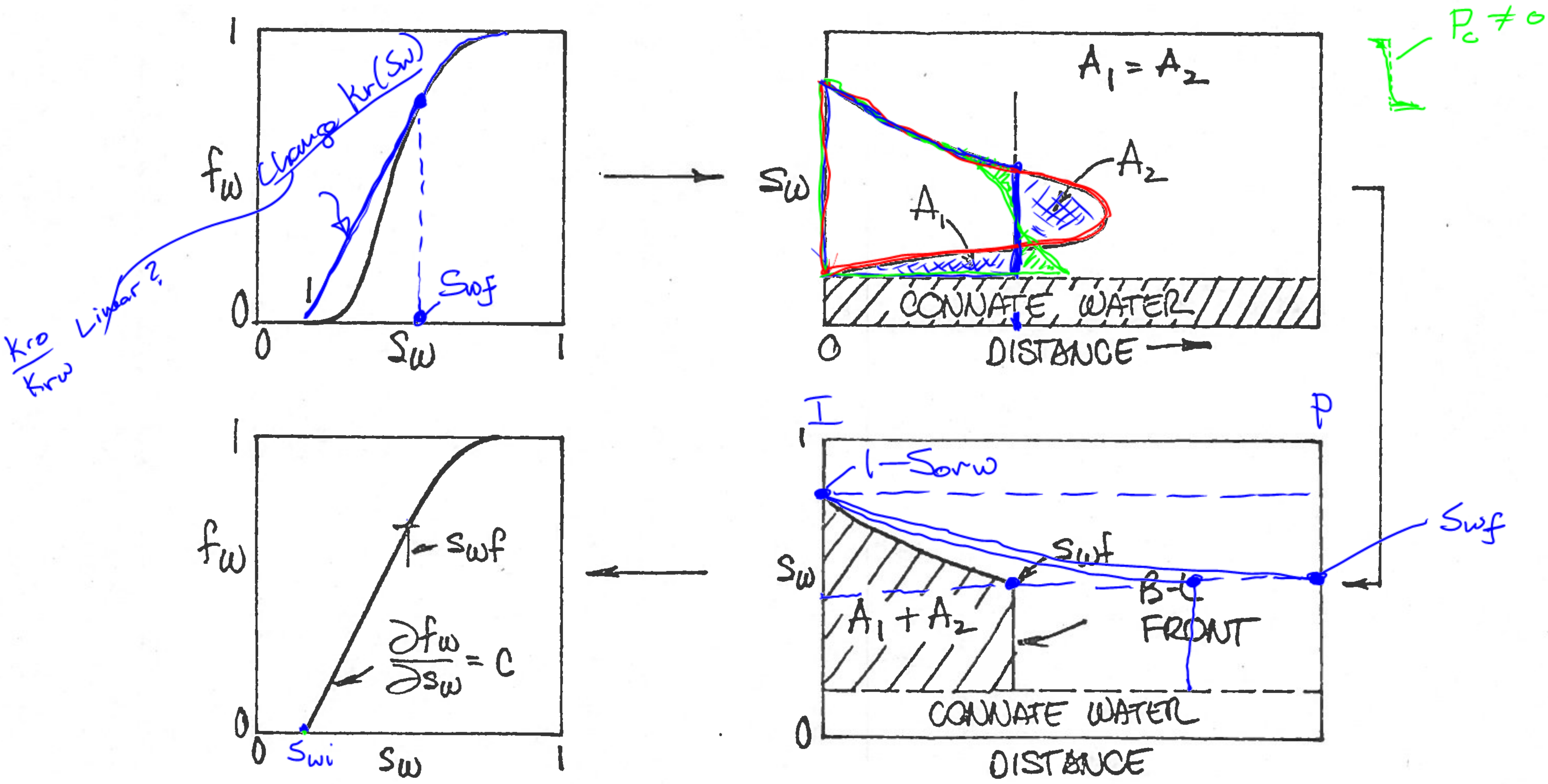
THIS IS THE BUCKLEY-LEVERETT FRONTAL ADVANCE EQUATION. IT GIVES THE DISTANCE, $[(x)_{S_w} - X_o]$, THAT A GIVEN SATURATION, S_w, MOVES IN TIME t.





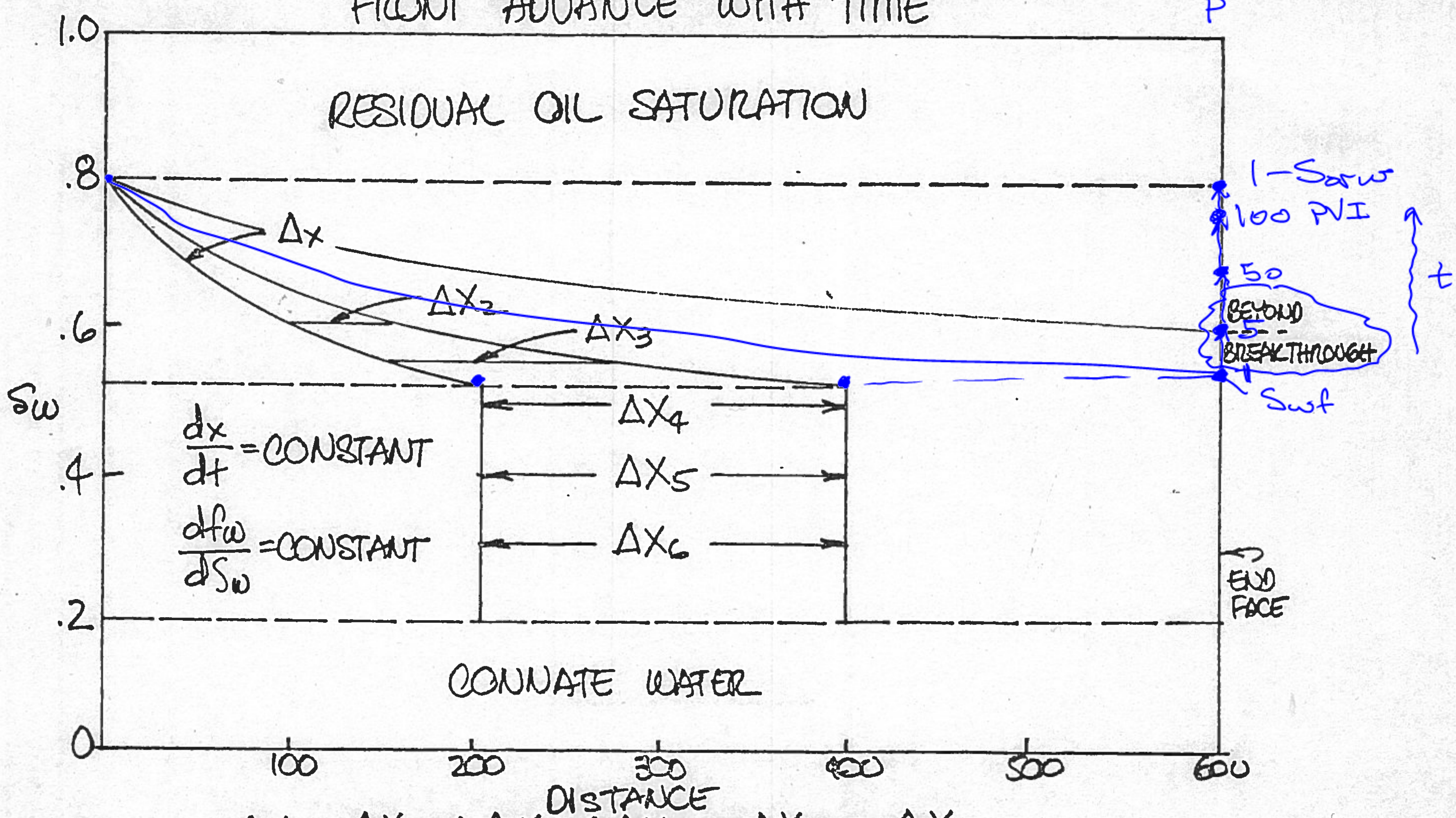
FRONTAL ADVANCE EQUATION, III

MODIFICATION OF B-L RELATIONSHIP TO AVOID TRIPLE SATURATION VALUES.



FRONTAL ADVANCE EQUATION, IV

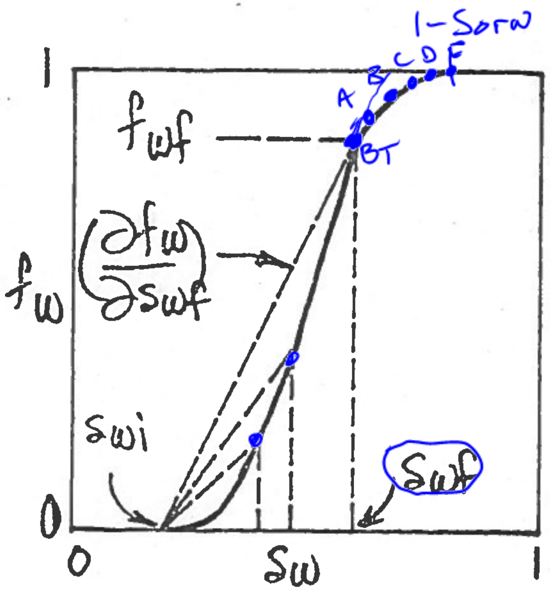
FRONT ADVANCE WITH TIME



$\Delta X_1 < \Delta X_2 < \Delta X_3 < \Delta X_4 = \Delta X_5 = \Delta X_6$

FRONTAL ADVANCE EQUATION VI

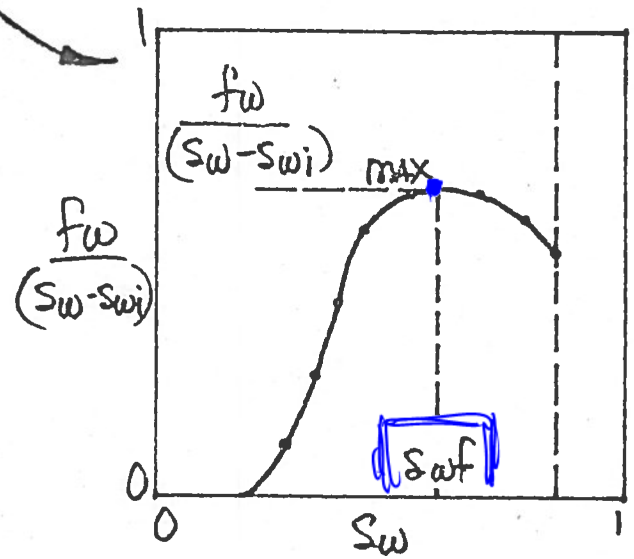
DETERMINATION OF SATURATION AND FRACTIONAL FLOW AT THE FRONT.



ALSO CAN BE OBTAINED GRAPHICALLY

CALCULATE $\frac{f_w}{(S_w - S_{wi})}$ AND PLOT AGAINST S_w.

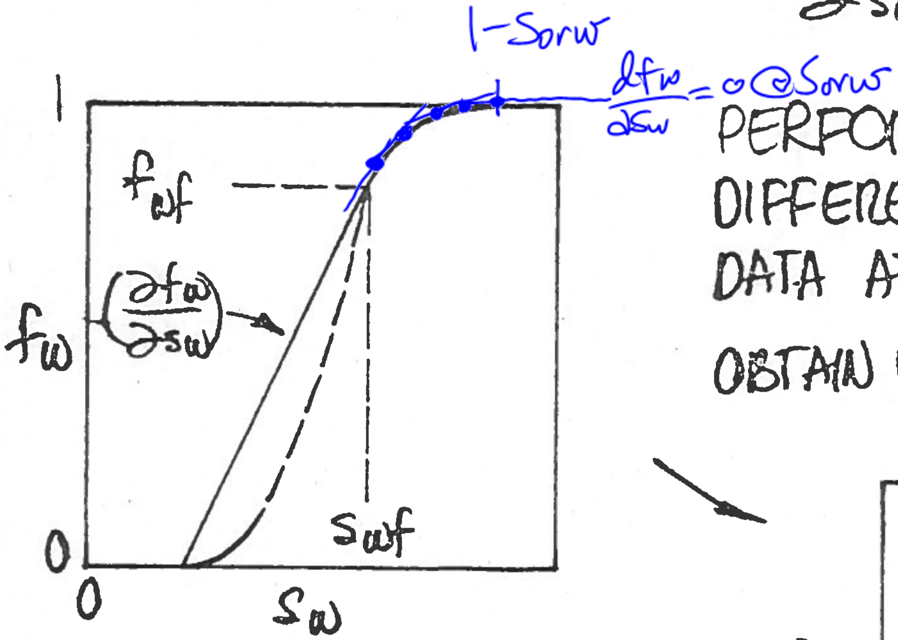
FRONT CONDITIONS ARE WHERE THIS HAS THE MAXIMUM VALUE.



Define inflection Point ⇒ S_{wf}

FRONTAL ADVANCE EQUATION VI

DETERMINATION OF $\frac{\partial f_w}{\partial S_w}$



PERFORM GRAPHICAL DIFFERENTIATION OF $f_w - S_w$ DATA AT $S_w > S_{wf}$ · to OBTAIN VALUES OF $\frac{\partial f_w}{\partial S_w}$ VS S_w

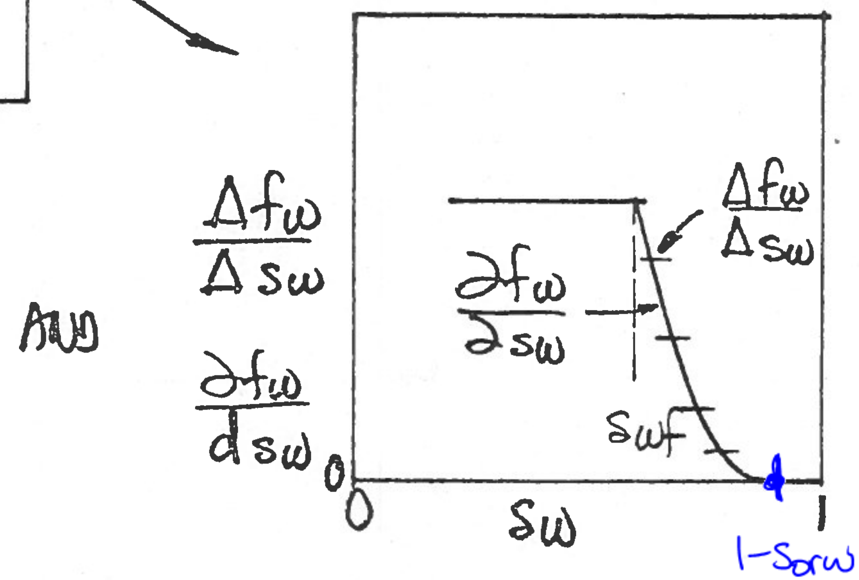
$(S_w)_p = f(PVI)$

or

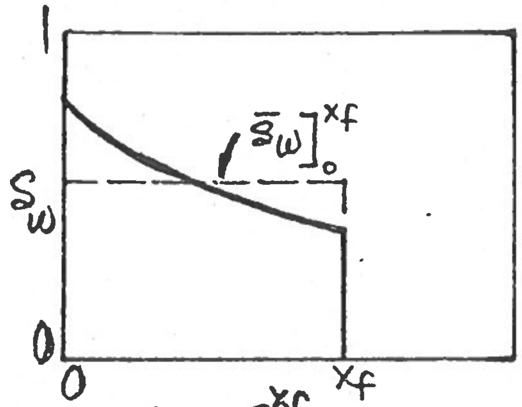
$\boxed{(\bar{S}_w)_{I-P} = f(PVI)}$

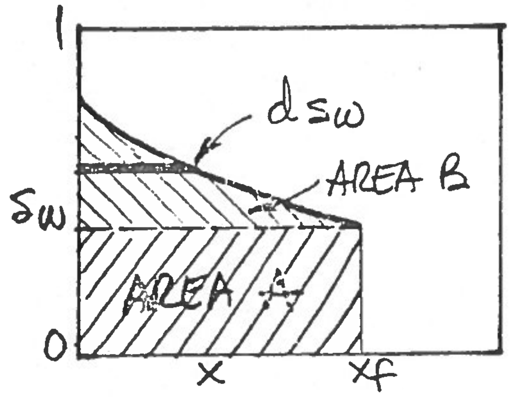
$\Downarrow$

$RF_o\,(PVI)_{I-P}$



AVERAGE SATURATION BEHIND FRONT - I





$$\bar{S}_w\Big]_0^{x_f} = \frac{\int_0^{x_f} S_w\,dx}{x_f} = \frac{\text{AREA A} + \text{AREA B}}{x_f}$$

FROM BUCKLEY-LEVERETT

$$x_f = \frac{5.615}{A\phi}\left(\frac{\partial f_w}{\partial S_w}\right)_f \int_0^t q\,dt$$

$$= \frac{5.615\;Q}{A\phi}\left(\frac{\partial f_w}{\partial S_w}\right)_f$$

$$\text{AREA A} = S_{wf}\cdot x_f$$

$$\text{AREA B} = \int_{S_{wf}}^{1} x\;dS_w$$

$$\text{AS} \quad x = \frac{5.615Q}{A\phi}\left(\frac{\partial f_w}{\partial S_w}\right)_x$$

$$\text{AREA B} = \frac{5.615\;Q}{A\phi}\int_{S_{wf}}^{1}\frac{df_w}{dS_w}\cdot dS_w$$

AVERAGE SATURATION BEHIND FRONT, II

$$\text{AREA } B = \frac{5.615\,Q}{A\phi} \int_{f_w @ S_{wf}}^{f_w @ S_w=1} df_w$$

$$= \frac{5.615\,Q}{A\phi} \left[(f_w @ S_w=1) - (f_w @ S_{wf}) \right]$$

BUT: $(f_w @ S_w=1) = 1$; $(f_w @ S_{wf}) = f_{wf}$

THEREFORE

$$\text{AREA } B = \frac{5.615\,Q}{A\phi} \left[1 - f_{wf} \right]$$

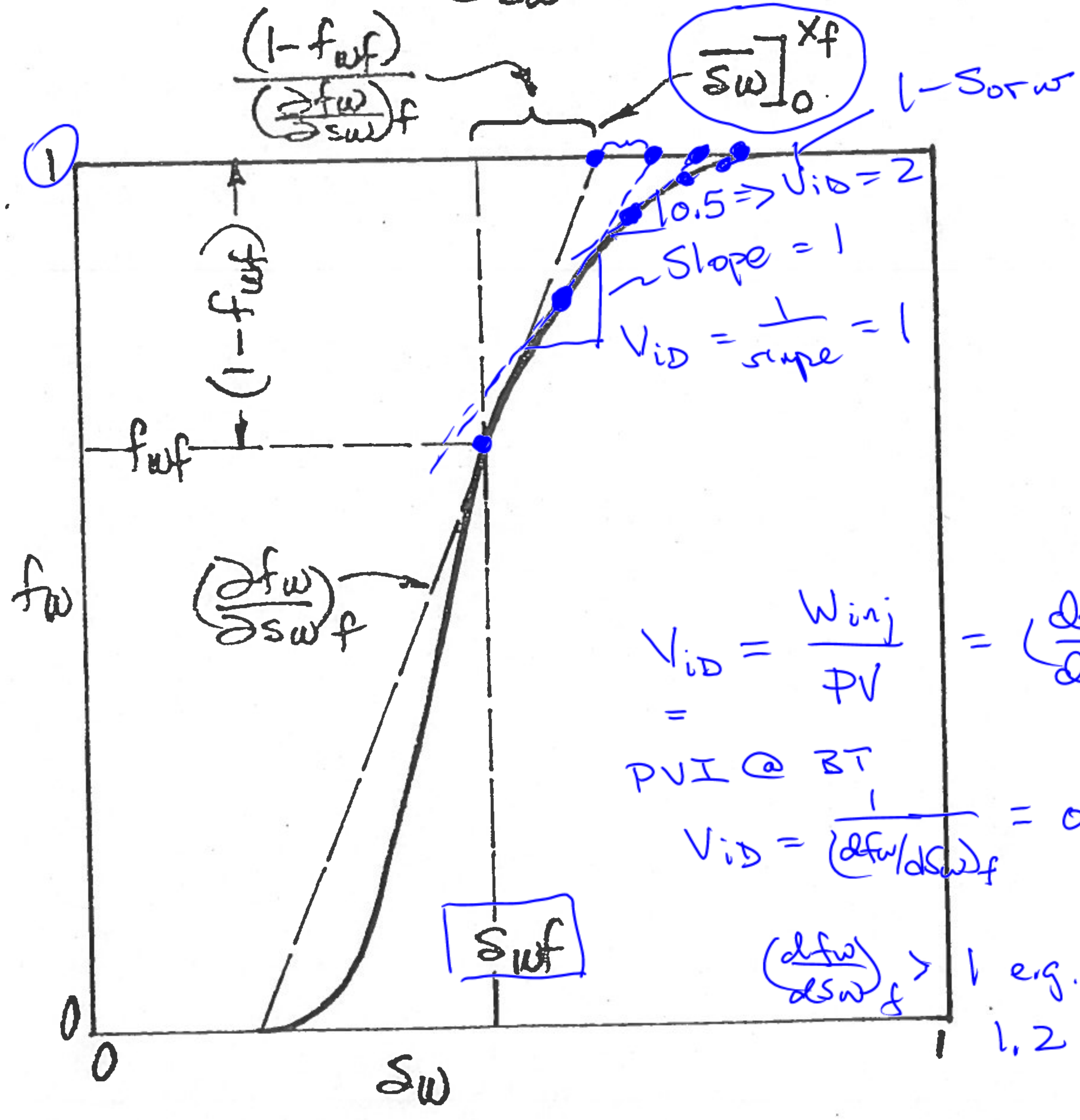
$$\text{AS } \left. \bar{S}_w \right]_0^{x_f} = \frac{\text{AREA } A + \text{AREA } B}{x_f}$$

$$\left. \bar{S}_w \right]_0^{x_f} = \frac{\frac{5.615\,Q}{A\phi} \left[S_{wf} \left(\frac{\partial f_w}{\partial S_w} \right)_f + (1 - f_{wf}) \right]}{\frac{5.615\,Q}{A\phi} \left(\frac{\partial f_w}{\partial S_w} \right)_f}$$

$$\boxed{\left. \bar{S}_w \right]_0^{x_f} = S_{wf} + \frac{(1 - f_{wf})}{\left(\frac{\partial f_w}{\partial S_w} \right)_f}}$$

AVERAGE SATURATION BEHIND FRONT III

$$\bar{S}_w\Big]_0^{x_f} = S_{wf} + \frac{(1 - f_{wf})}{\left(\frac{\partial f_w}{\partial S_w}\right)_f}$$



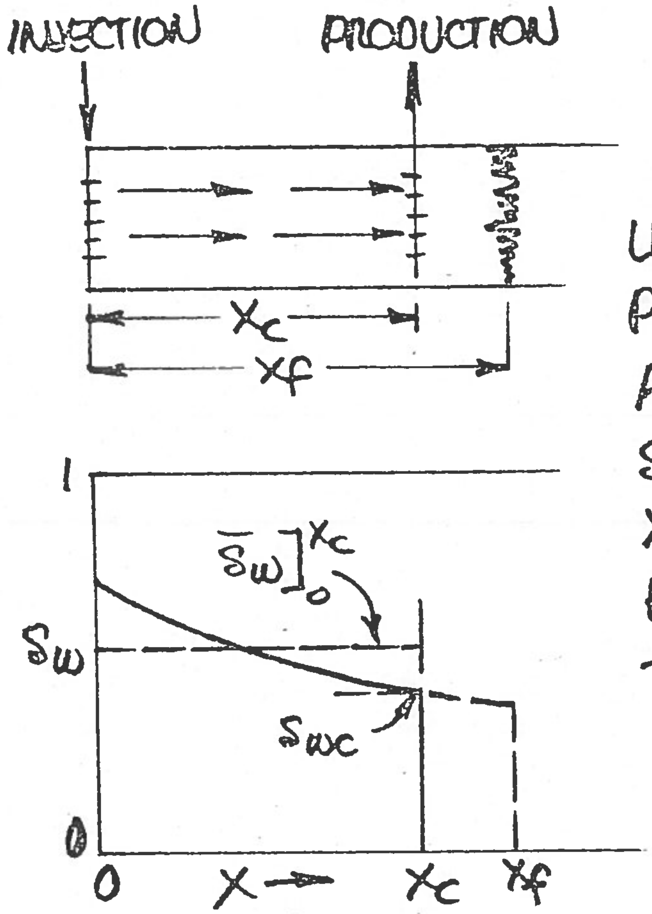
$V_{iD} = \frac{W_{inj}}{PV} = \left(\frac{df_w}{dS_w}\right)^{-1}$

= PVI @ BT

$V_{iD} = \frac{1}{(df_w/dS_w)_f} = 0.8$

$\left(\frac{df_w}{dS_w}\right)_f > 1$ e.g. 1.2

AVERAGE SATURATION BEHIND FRONT - IV

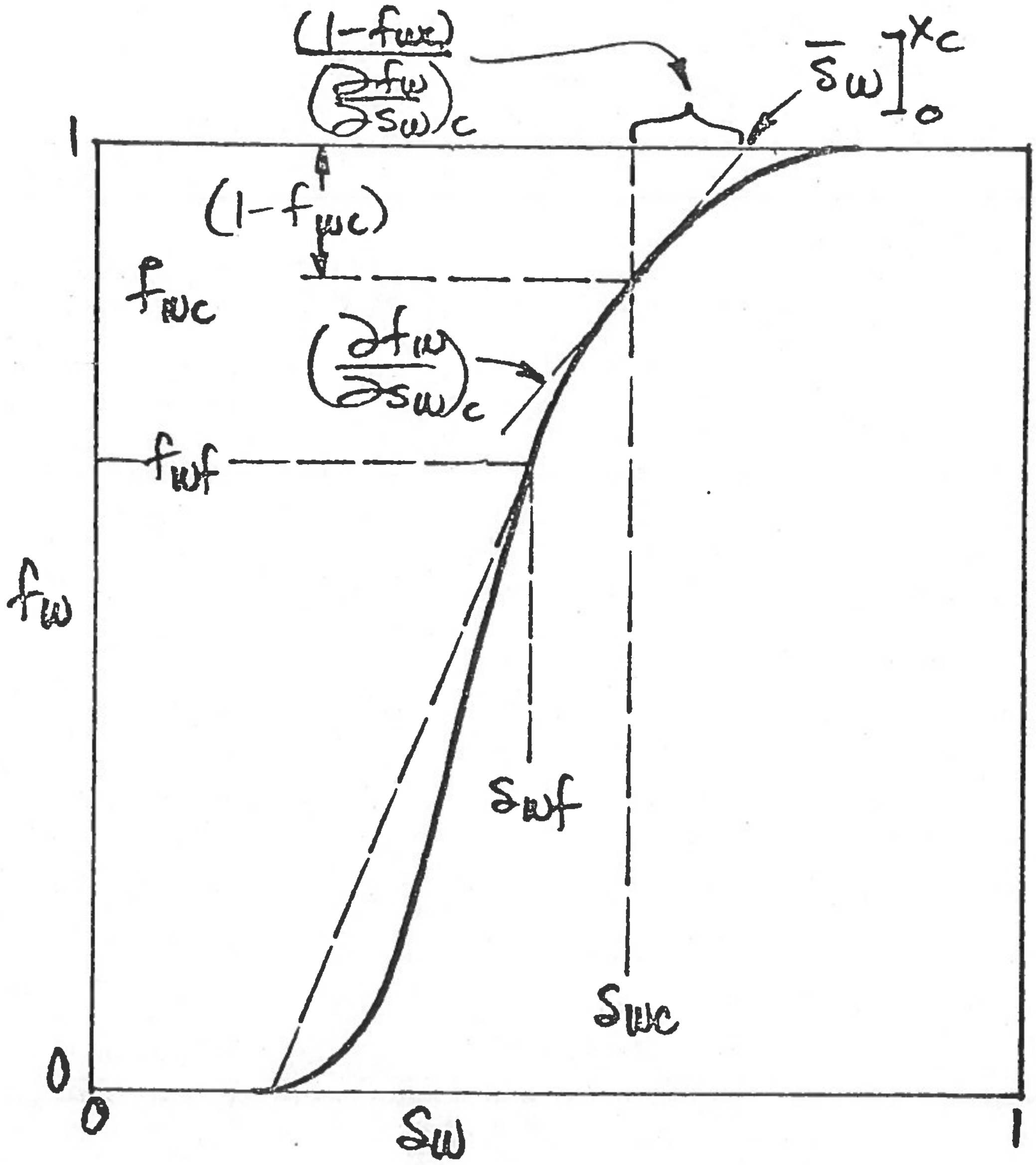


WHEN THE FRONT LINE HAS PASSED POSITION x_c, THE AVERAGE WATER SATURATION BEHIND x_c CAN BE DEVELOPED BY A SIMILAR APPROACH. THE FINAL EQUATION IS

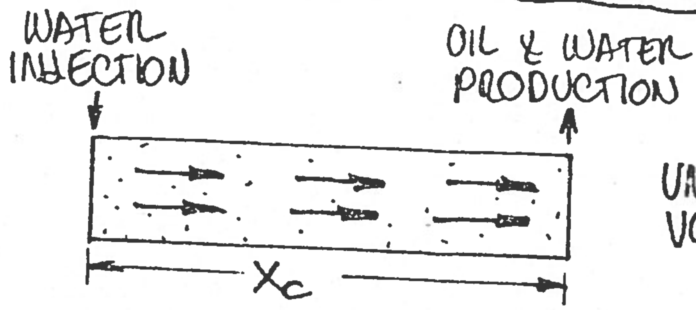
$$\bar{S}_w\Big]_0^{x_c} = S_{wc} + \frac{(1 - f_{wc})}{\left(\frac{\partial f_w}{\partial S_w}\right)_c}$$

AVERAGE SATURATION BEHIND FRONT - II

$$\bar{S}_w\Big]_0^{x_c} = S_{wc} + \frac{(1-f_{wc})}{\left(\frac{\partial f_w}{\partial S_w}\right)_c}$$



INJECTION VOLUME VS. RECOVERY RELATIONS, I



UNIT PORE VOLUME, $V_p = \dfrac{A \phi x_c}{5.615}$

DIMENSIONLESS PORE VOLUMES INJECTED

$$V_{iD} = \frac{W_i}{V_p} = \frac{W_i}{\frac{A\phi x_c}{5.615}} = \frac{5.615\, W_i}{A\phi x_c}$$

FROM BUCKLEY - LEVERETT

$$x_c = \frac{5.615\, W_i}{A\phi}\left(\frac{\partial f_w}{\partial s_w}\right)_c$$

OR

$$\frac{5.615\, W_i}{A\phi x_c} = 1\Big/\left(\frac{\partial f_w}{\partial s_w}\right)_c$$

THEREFORE THE PORE VOLUMES INJECTED TO REACH A GIVEN WATER SATURATION AT THE OUTFLOW FACE, s_{wc} IS,

$$\boxed{V_{iD} = 1\Big/\left(\frac{\partial f_w}{\partial s_w}\right)_c}$$

$\Rightarrow (\bar{S}_w)$ I-P @ "c"

INJECTION VOLUME VS. RECOVERY RELATIONS II

RESERVOIR OIL DISPLACED $= V_p (\Delta \bar{S}_w)_o^{x_c}$

$$= \frac{A \phi x_c}{5.615}\left[(\bar{S}_w)_o^{x_c} - S_{wi}\right]$$

STOCK TANK OIL PRODUCED $= \dfrac{\text{RES. OIL DISP.}}{B_o}$

$$N_p = \frac{A \phi x_c}{5.615\, B_o}\left[(\bar{S}_w)_o^{x_c} - S_{wi}\right]$$

FLOWING WATER-OIL RATIO AT OUTFLOW FACE;

$$WOR = \frac{f_{wc}}{1-f_{wc}}$$

SURFACE PRODUCING WATER-OIL RATIO,

$$F_{wo} = \frac{f_{wc} \cdot B_o}{(1-f_{wc})\, B_w}$$

SURFACE WATER CUT $= \dfrac{\dfrac{f_{wc}}{B_w}}{\dfrac{f_{wc}}{B_w} + \dfrac{(1-f_{wc})}{B_o}} = \dfrac{1}{1+F_{wo}}$

$$\text{TIME, } t = \frac{W_i}{i_w}$$

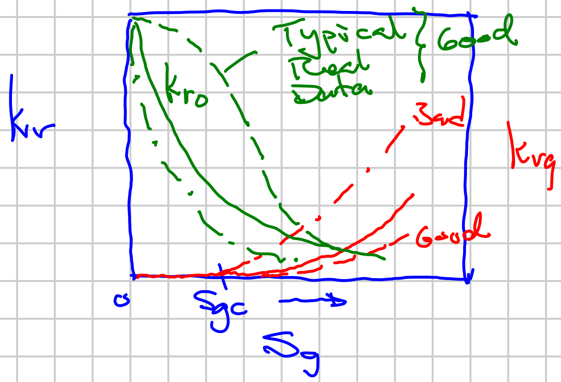
$$= \frac{V_{iD} \cdot A \phi x_c}{5.615\, i_w} \text{ (DAYS)}$$

WHERE i_w IS BBL PER DAY

LNX Modeling of EOR Processes
└ Layered No-Crossflow
$k_z \approx 0$ $v_z \sim 0$

RP Bad RP Good

Depletion SGD Oil Reservoirs : 5 – (10 – 25%)



Carey : $k_{ro} \propto (1 - S_g)^{n_o}$ $n_o \sim 2 - 5$

– – – k_{ro} : Chierici Geranld

EOR : Inject ⇒ $\underbrace{\text{Success | Failure}}$

KPI | Key Layer Permeability Variation

high-k, low-V layers

Distribution k (z)

horizontal x-y

depth (geological layer to layer)

1944 : Laws k distribution

1948 : ① Trans. AIME

Standing, Lindblad, Parsons : Gas Cycling Gas Condensate

"Charts" ② API : Dykstra & Parsons : Water Flooding

Leaky Piston Displ.

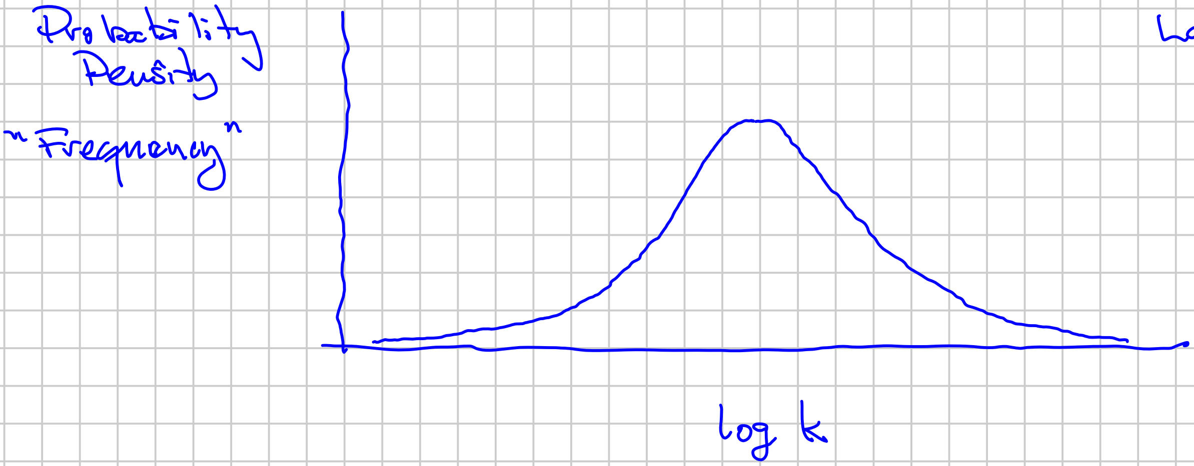
Analytical ③ Muskat : / k = exponential
: Laws log-normal

Gas Cycling Gas Condensate

June 1948 (Trans AIME 1949)

$k(z)$ using a log-normal permeability distribution

Laws 1944



Property
eg. Grain Size / Porosity

ϕ : normal dist. $\quad \bar{\Phi} = \frac{1}{N} \sum \phi_i$

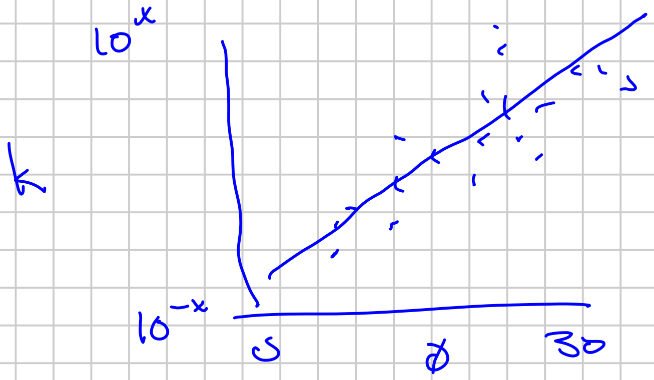
k : log-normal dist $\quad \log \bar{k} = \frac{1}{N} \sum \log k_i$

$\Rightarrow$ Geometric Average

$$\bar{k}_G = \left(\prod k_i \right)^{1/N}$$

$\Rightarrow$ Approximation

$$\log k \sim a + b\phi$$



1949 : • Muskat Trans AIME Gas Cycling

• Stiles WF EOR

Any k(z) distribution ⇒ Calculational Procedure N_{layers}

k, ϕ

Leaky Piston Displacement

1950 Muskat WF Analytical Solutions

k(z) linear

exponential

log-normal

Leaky Piston assumption

(1967) BL in each layer

Snyder & Ramey : log-normal dist

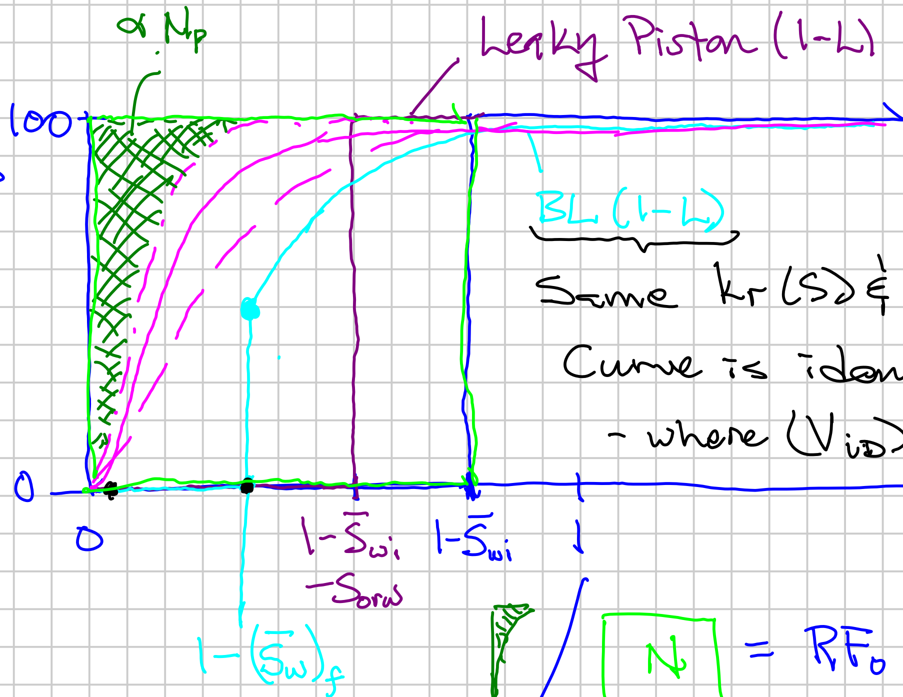
LNX EOR Publications History

⇒ Production Performance of LNX Systems

"F_w" = $\left(\frac{q_w}{q_w + q_o}\right)_P$

$\frac{STB}{STB}$

~ f_w related



Same $k_r(S)$ & μ_o/μ_w in each L →

Curve is identical in each L

– where $(V_{iD})_L\,(t) = q_{iL}\cdot t$

$\propto k_{iL}\, t$

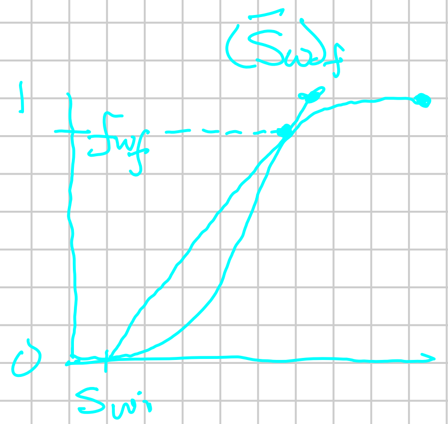
PVI

V_{iD}

$\boxed{N} = RF_o$

$$f_w = \left(\frac{q_{wR}}{q_{wR} + q_{oR}}\right)_P \qquad F_w = \frac{q_{oR}/B_o}{q_{wR}/B_w + q_{oR}/B_o}$$

B_w & B_o @ $\bar{P}_R$ in WF Unit



LNX

$$q_{wR}(t) = \sum_{L=1}^{N_L} f_w\left(V_{iD_L}(t)\right)\cdot q_{tL}$$

$f_w(V_{iD})$ same for all Layers

⇐ V_{iD} different for all Layers ∝ k_L

q_{tL} may vary somewhat over time

$$V_{iDL} \cong V_{iDt} \cdot \frac{k_L}{\bar{k}}$$

$$\bar{k} = \frac{\sum k_L}{N_L}$$

LNX : ① Earlier BT

② Lower $RF_o = N_p(t) / N$

at a given time

} Larger the $k(z)$ Variation "V"

Laws (DP / Muskat / Standing ...)

$$\text{"}V\text{"} = \frac{|\bar{k}_G - k_{SD}|}{\bar{k}_G}$$

0 – 1

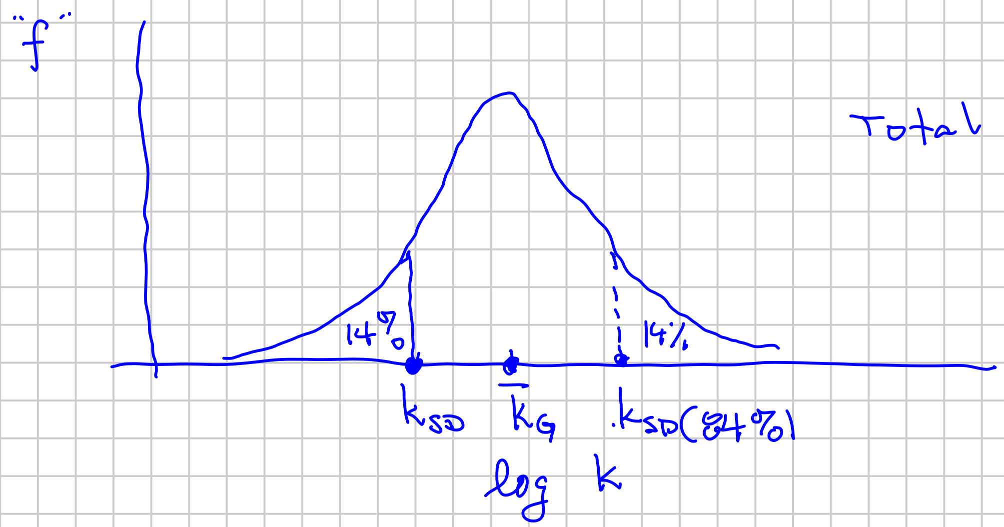
no max

(0.3 – 0.7)

real data

SD : standard deviation

Total Area = 1



Note Title 11/13/2018

Well Control and Management Considerations in EOR projects

① Well Controls: Practical & Reservoir Simulation

(a) Target Rate (volumetric)

- Surface Product Rates — ④ $\overbrace{q_g \quad q_s \quad q_w \quad q_L}$

- Reservoir Phase Rates — $q_L = q_o + q_w$

$q_{gR} \quad q_{oR} \quad q_{wR} \quad q_{LR} = q_{oR} + q_{wR}$ — ⑤

(EOR) $q_{tR} = q_{gR} + q_{oR} + q_{wR}$

$(q_{\bar{g}})$ $$q_g = \underbrace{q_{gR} / B_{gd}(p)}_{q_{\bar{g}(gR)}} + \underbrace{(q_{oR} / B_o(p)) R_s(p)}_{q_{\bar{g}(oR)}}$$

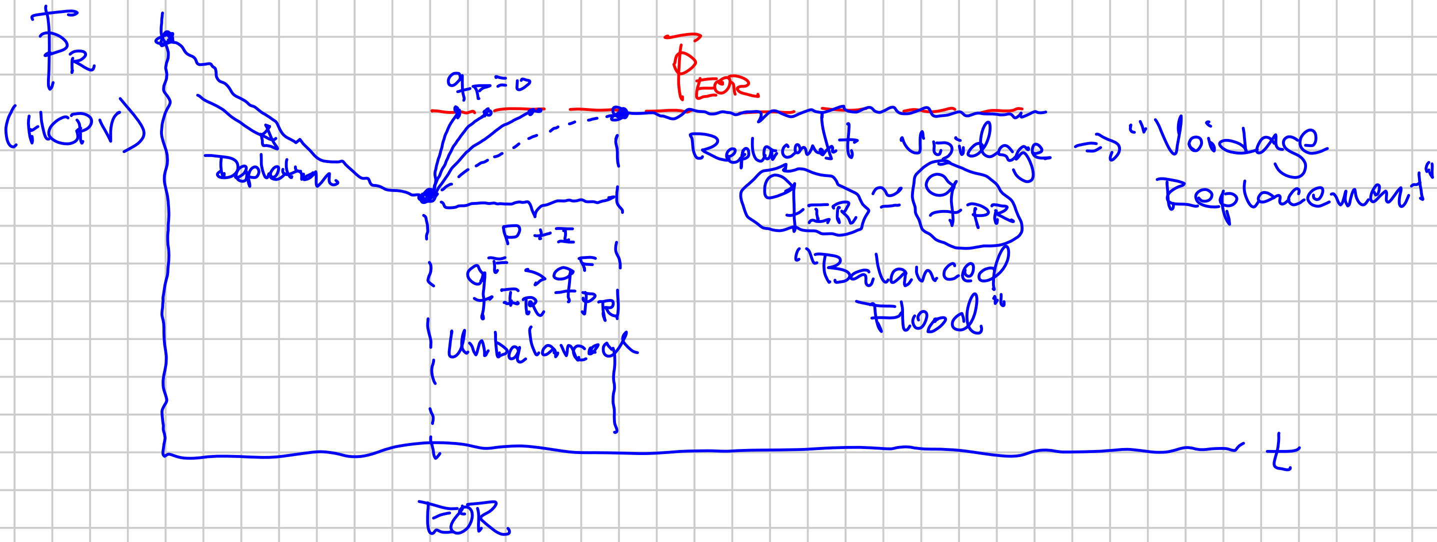
$(q_{\bar{o}})$ $$q_o = \underbrace{q_{oR} / B_o}_{q_{\bar{o}(oR)}} + \underbrace{(q_{gR} / B_{gd}) r_s}_{q_{\bar{o}(gR)}}$$

(b) Wellbore BH Pressure Constraints (alt. Wellhead Tubing Pressure Constraints)

Producer Well: $P_{wf,min}$ (= 1atm default)

Injector Well: $P_{inj,max}$

EOR Projects: ~ Target Average Reservoir Pressure $\bar{p}_{EOR}$



CSD Petroleum Industry Course

Displacement Problem 1

(This problem pertains to the displacement of oil up-structure by an influxing aquifer. The problem is to be solved using the Buckley-Leverett method).

The sketches below illustrate a section of reservoir in which water is advancing up-structure as a result of pressure reduction in the oil band section. To simplify this problem two assumptions will be used : (1) the initial water in the oil zone amounts to 32% saturation and is constant with height. To say it differently, we will neglect any effects of the initial transition zone saturation. (2) water breakthrough into the 1st line well occurs when the front reaches the elevation of the well. In other words, we will neglect effects of "cusping" of water into the well.

1980

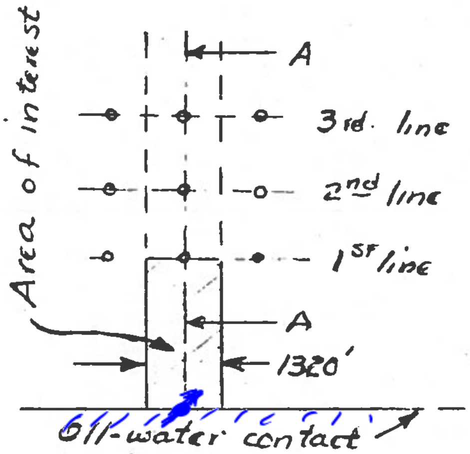


Figure 1
Plan View of "Window"

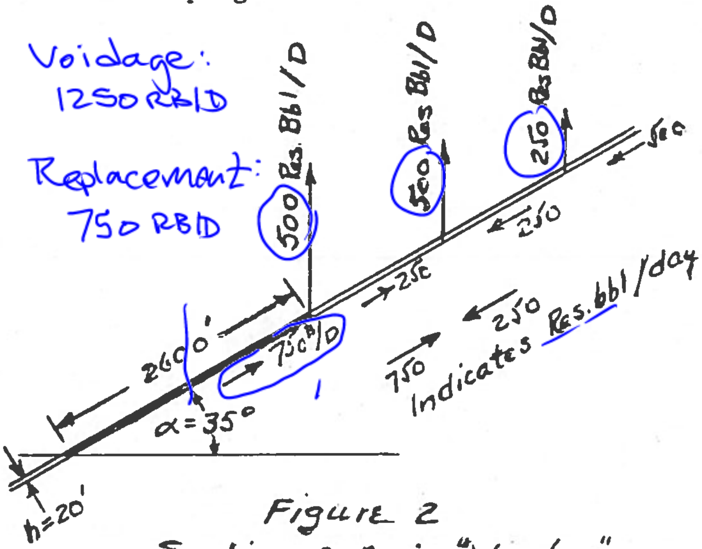


Figure 2
Section A-A in "Window"

"Window" Data

Window length = 2600'
Window width = 1320'
Formation thickness = 20'
Formation dip = +35°
Porosity = 0.235
Permeability (abs) = 100 md
Up-dip flow rate at oil-water contact = 750 Res.bbl/day

Fluid Saturations

S_{iw} = 0.30; S_{wi} = 0.32
S_{oi} = 0.68 S_{gi} = 0.00

Fluid Properties

B_o = 1.27 B_w = 1.02
μ_o = 1.16 cp μ_w = 0.38 cp
ρ_o = 45 lb/ft^3 ρ_w = 65 lb/ft^3

Relative Permeability Data

S_w	k_{ro}:imb	$k_{rw}/k_{ro}]$ imb	
0.30	0.725	0.0000	(S_{iw})
0.32	0.615	0.0195	
0.35	0.470	0.072	
0.40	0.315	0.280	
0.45	0.210	0.790	
0.50	0.133	2.000	
0.55	0.077	4.750	
0.60	0.036	11.85	
0.65	0.012	33.50	
0.67	0.007	55.53	

Displacement Problem 1 Cont.

The oil that is displaced up-structure by the influxing aquifer water will presumably be captured by the oil wells. When the water-oil front reaches the elevation of any well there will be an instantaneous jump in water-oil ratio (this is because we are considering only one layer - the jump would be more gradual if many layers were considered). We are, of course, interested in the amount of oil that can be recovered from the invaded volume (cross-hatched area in Fig 1) as we continue to produce the wells. The items to be calculated are:

(1) How many barrels of stocktank oil will be displaced from the invaded volume (and recovered) when the front first breaks through into the first line well ? What fraction of initial oil in this volume does this amount to ?

(2) What will be the surface producing water-oil ratio immediately after breakthrough ? Assuming that the well continues to produce at the same <u>total</u> fluid rate, what will the stocktank oil rate be ?

(3) When the first line well's cut reaches 95 %,what will be the amount of oil recovered from the invaded volume ? How long (years) will it take to reach this cut ? (Consider that the the aquifer influx rate into the window remains constant at 750 reservoir barrels per day.)

CSD Petroleum Industry Course

Solution Displacement Problem 1

A. Calculation of fractional flow curve

$$f_w = \frac{1 - a\,k_{ro}}{1 + \frac{k_{ro}}{k_{rw}} \cdot \frac{\mu_w}{\mu_o}} \qquad a = \frac{7.84(10^{-6})\,k_{abs}\,(\rho_w - \rho_o)\,\sin\alpha}{q/A\;\mu_o}$$

$$\alpha = 35^\circ$$

$$a = \frac{7.84(10^{-6}) \cdot 100\;(65 - 45)\;0.5736}{750/_{20 \cdot 1320} \cdot 1.16} = 0.2729$$

$S_{iw} = 0.30$

$S_{wi} = 0.32$

$$\mu_w/\mu_o = \frac{0.38}{1.16} = 0.3276$$

$$\therefore\; f_w = \frac{1 - 0.2729\,k_{ro}}{1 + \frac{0.3276}{k_{rw}/k_{ro}}}$$

S_w	k_{ro}	k_{rw}/k_{ro}	f_w	$\frac{f_w - f_{wi}}{S_w - S_{wi}}$	$\frac{\Delta f_w}{\Delta S_w}$
0.30 ← S_{iw}	0.725	0.0000	0	–	
0.32 ← S_{wi}	0.615	0.0195	0.0467 ← f_{wi}	—	
0.35	0.470	0.072	0.1571	3.680	
0.40	0.315	0.280	0.4212	4.651	
0.45	0.210	0.790	0.6664	4.767	4.904
0.50	0.133	2.000	0.8281	4.341	3.234
0.55	0.077	4.750	0.9158	3.779	1.754
0.60	0.036	11.85	0.9635	3.274	0.954
0.65	0.012	33.50	0.9871	2.850	0.472
0.67	0.007	55.53	0.9922	2.701	0.255

From plot of $\frac{f_w - f_{wi}}{S_w - S_{wi}}$ vs S_w the maximum occurs at $S_w = 0.430$. Maximum value is 4.80, which is slope of tangent line.

1. At break through at the first line well, the average saturation behind the front is

$$\bar{S}_w\Big]_{BT} = S_{wf} + \frac{1 - f_{wf}}{(f_w - f_{wi})/(S_w - S_{wi})_{max}} = 0.430 + \frac{(1 - 0.575)}{4.80} = 0.5185$$

STB recovered @ BT

$$N_p = \frac{V_p[\bar{S}_w)_{BT} - \bar{S}_{wi})}{B_o} = \frac{(2600 \cdot 1320 \cdot 20) \cdot 0.235}{1.27 \cdot 5.615}[0.5185 - 0.320]$$

$$N_p = \underline{449.0\ (10^3)}\ STB.$$ ← ans

$$\text{Rec. Fraction} = \frac{(0.5185 - 0.320)}{(1 - 0.320)} = \underline{0.292}$$ ← ans

2. What will be surface water-oil ratio at B.T.

$$\text{Subsurface } W/O = \frac{f_w}{f_o} = \frac{0.5185}{(1-0.5185)} = 1.077 = WOR$$

$$\text{Surface water-oil ratio} = 1.077 \cdot \frac{B_o}{B_w} = 1.077 \cdot \frac{1.27}{1.02} = \underline{1.34}$$ ← ans

$$q_o = \frac{q_t \cdot f_o}{B_o} = \frac{500(1-0.5185)}{1.27} = \underline{189.6\ STB/D}$$ ← ans

3. Cut @ first line well = 95%

$$Cut = 0.95 = \frac{q_w}{q_w + q_o} \quad \therefore F_{wo} = \frac{q_w}{q_o} = \frac{95}{5} = 19$$

$$WOR\ (\text{subsurface}) = F_{wo} \cdot \frac{B_w}{B_o} = 19 \cdot \frac{1.02}{1.27} = 15.26 = \frac{q_w B_w}{q_o B_o}$$

$$\therefore f_w = \frac{15.26}{15.26+1} = \frac{q_w B_w}{q_w B_w + q_o B_o} = 0.9385;\ S_{wc} = 0.569$$

$$\frac{\partial f_w}{\partial S_w} = 0.95$$

$$\therefore \bar{S}_w]_o^c = S_{wc} + \frac{(1-f_{wc})}{\partial f_w/dS_w} = 0.569 + \frac{(1-0.9385)}{0.95} = 0.6337$$

$$\therefore \text{Recovery @ 95\% cut} = \frac{(2600 \cdot 1320 \cdot 20)\,0.235}{5.615 \cdot 1.27}(0.6337 - 0.320)$$

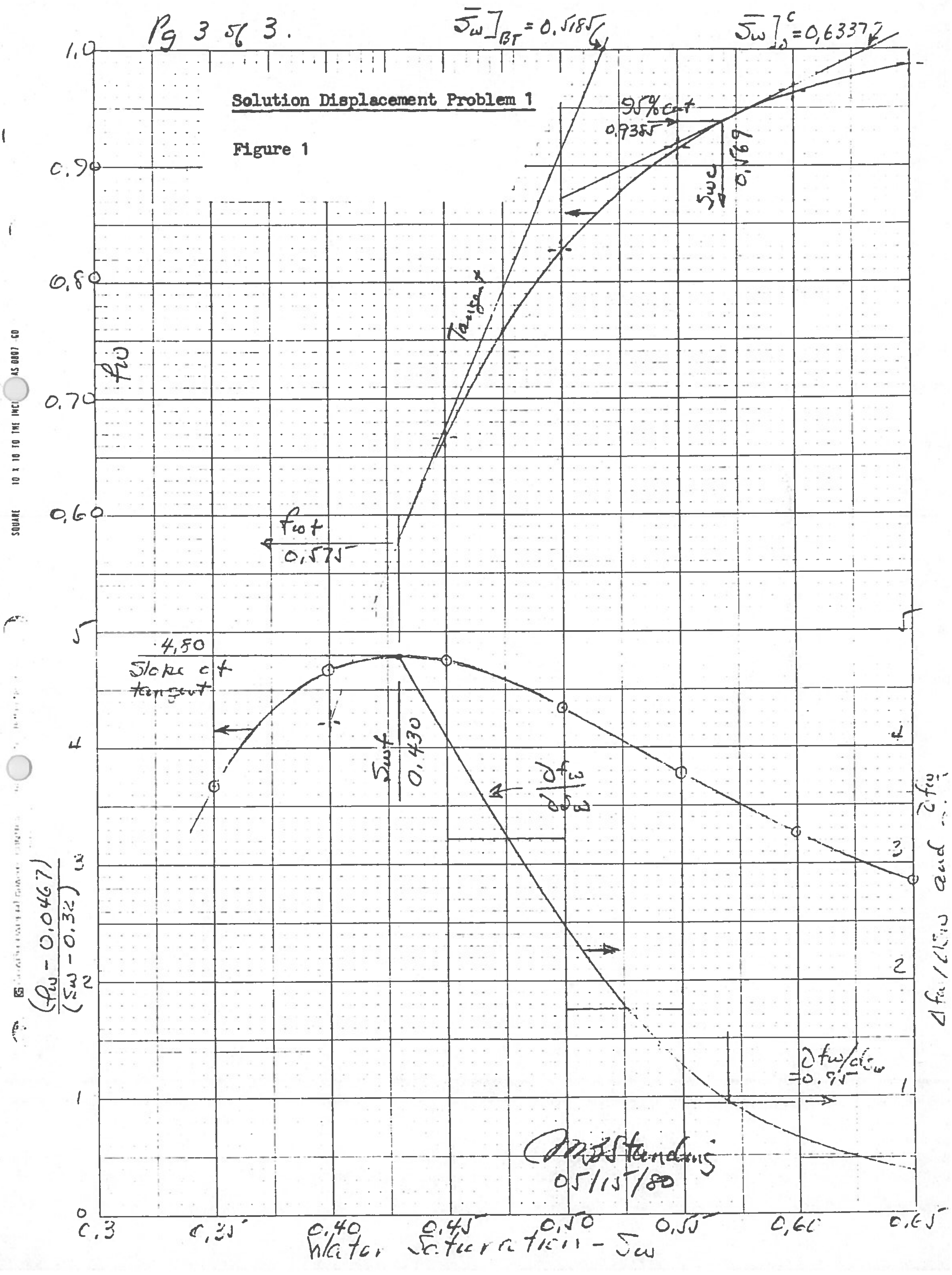
$$= \underline{709.588\ (10^3)\ STB}$$ ← ans

$$\text{Pore Vol inj} = V_{iD} = 1/\partial f_w/dS_w = \frac{1}{0.95} = 1.053$$

$\therefore$ Time to reach 95% cut

$$t = \frac{W_i}{q_i} = \frac{(2600 \cdot 1320 \cdot 20) \cdot 0.235 \cdot 1.053}{5.615 \cdot 750 \cdot 365} = \underline{11.05\ \text{years}}$$ ← ans

JM 05/15/93



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Solution Displacement Problem 1
Figure 1
$\bar{S}_w]_{BT} = 0.5185$
$\bar{S}_w]_{D}^{c} = 0.6337$
95% cut
0.9385
Swc
0.569
Tangent
fwf
0.575
4.80
Slope of tangent
Swf
0.430
$\partial f_w / \partial S_w$
$\partial f_w/\partial S_w$ = 0.95
fw
$\frac{(f_w - 0.0467)}{(S_w - 0.32)}$
df_w/dS_w and ∂f_w
Water Saturation - Sw
0.3
0.35
0.40
0.45
0.50
0.55
0.60
0.65
1.0
0.90
0.80
0.70
0.60
0
1
2
3
4
5
05/15/80

Note Title 11/20/2018

GAS INJECTION EOR
(Pore-Level Recovery)

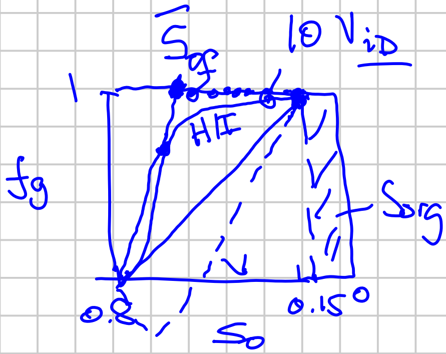
① Purely Immiscible

② Vaporization-Dominated

③ Swelling-Viscosity Reduction

④ Multi-Contact Miscible

(5) Unconventional
- Naturally Fractured
- Shale "Ultra-Tight"
- Steam



① Purely Immiscible
* No component exchange between reservoir phases
- 1D BL displacement ($k_r, \mu \Rightarrow \lambda$) $S_{oi} \to S_{org}$

slow
80% 55% → 15%
@1PVI

- 2D BL ($\underbrace{\text{Vertical Flow, Gravity}}_{\text{high } k}$)

$\Rightarrow$ Leaky Piston

$S_{oi} \to S_{org}$
Fast
@ 1PVI

② Vaporization-Dominated

- Light Oil (>35 °API) / Condensate (>45-50°) $C_{7-15(20)}$ Light Heavies

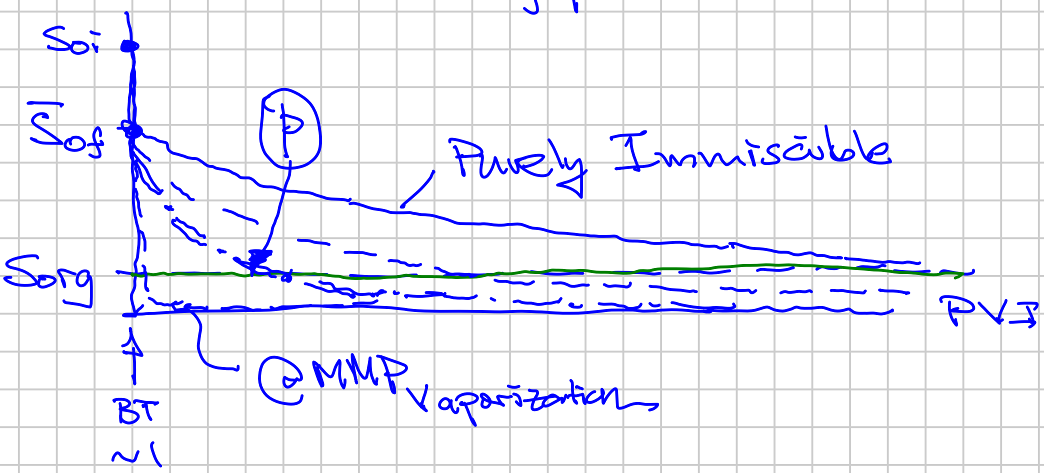
- High Pressure

- Net component transfer from RO phase to RG(IG) phase ↑ $K_i = \frac{y_i}{x_i}$

higher K_i RG ↑ i RO

$S_{oi} \Rightarrow$

- BL $\Rightarrow$ $\bar{S}_{of}$ $\Rightarrow$ 0 (low value)
$= f(p)$



③ Swelling - Viscosity Reduction

- Heavier Oil (22-32° API) $\mu_o \sim 5-50 \Rightarrow 1-10$

- Lower P_R < 3500 psia $V_{oi} \rightarrow V_{os} \sim 1.2 \rightarrow 2^+$

- Undersaturated Oils

- Injection Gas : HCs "C_1" C_2-C_4 also

CO_2

- Can be / often used together with water injection (WAG)

and alternating

Note Title 11/21/2018

MISCIBLE GAS INJECTION

(Pore Level Recovery) $S_{oi} \to 0$ where swept

- First Contact Miscible (FCM)

- Multi-contact / Developed Miscibility

Location of Front

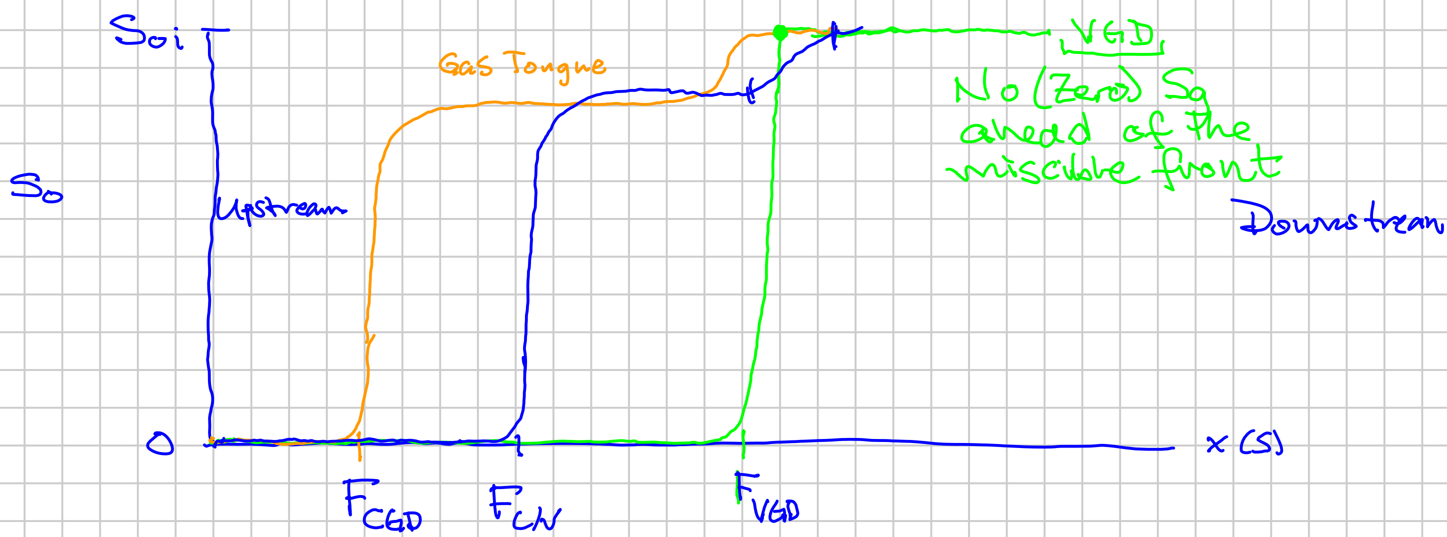
Downstream — • VGD (Vaporizing Gas Drive)

Upstream — • CGD (Condensing —"—) "non-existant"

Midstream — • C/V GD (Condensing/Vaporizing) 90% Zick 1986/87

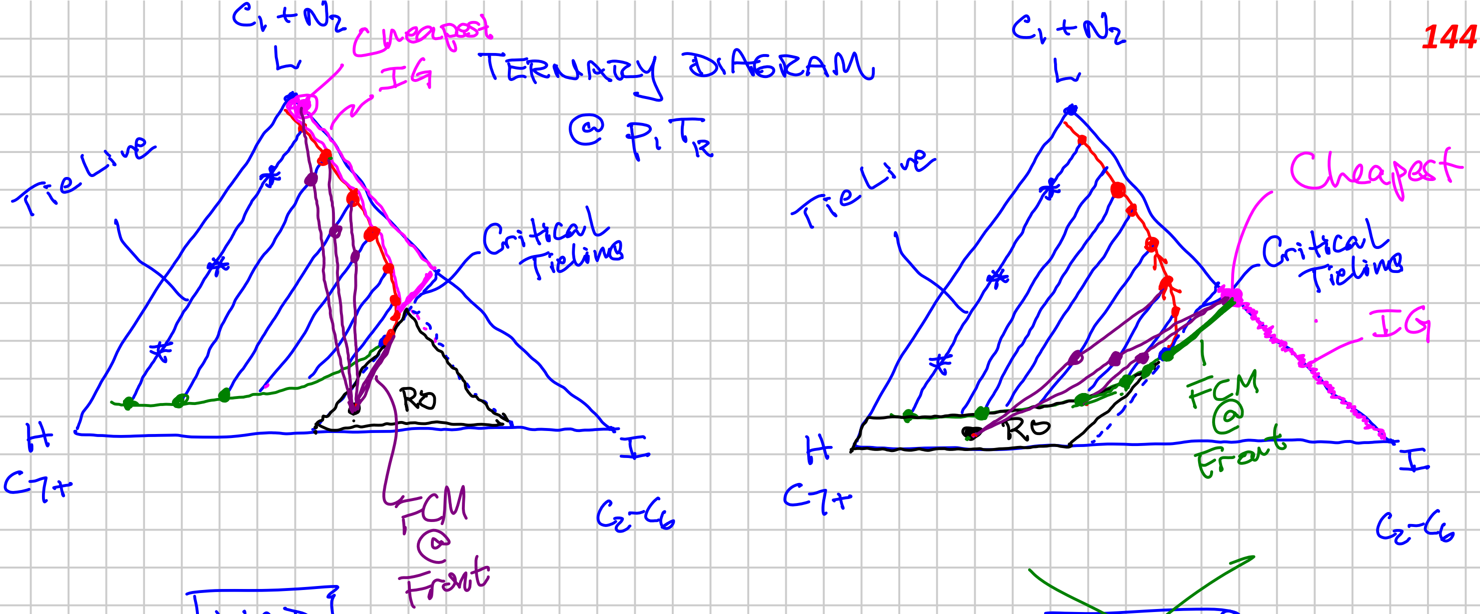
K-values! $K_i(p, z_i)$ *

Altered: Frontal Gas, Frontal Oil; Frontal Gas & Frontal Oil



* Development of an altered "frontal" gas and/or "frontal" oil ⇒ frontal phases become FCM

TERNARY DIAGRAM @ $P_i T_R$

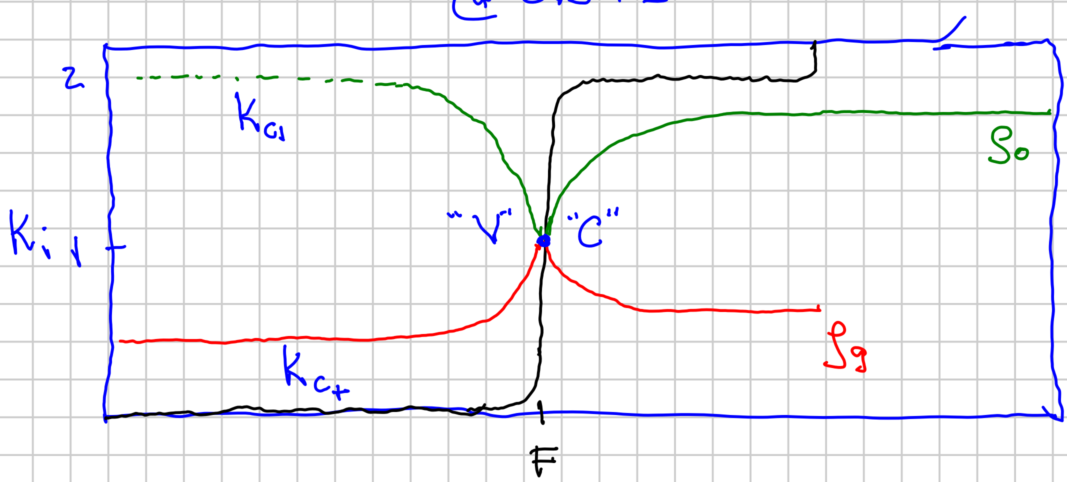


VGD

~~CGD~~

C/V Gas Drive "Signatures"

@ 0.5 PVI



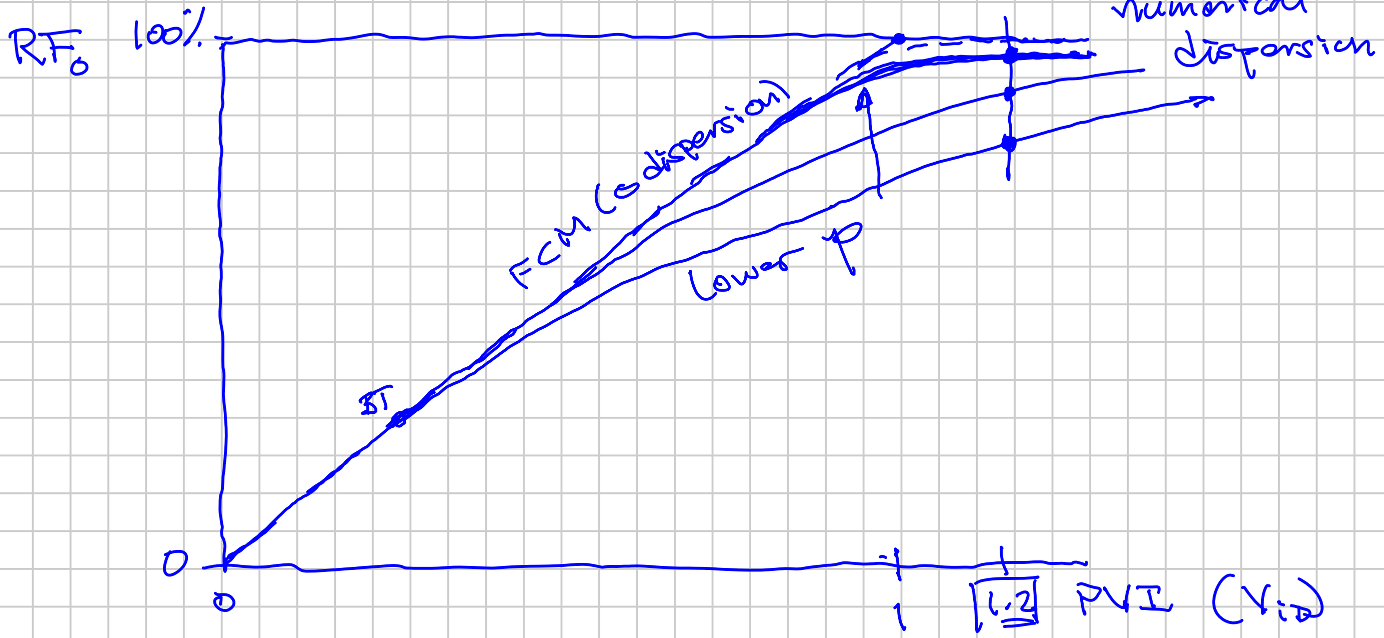
Hour-Glass Signature S_g/S_o K_{C_1}/K_{C_+}

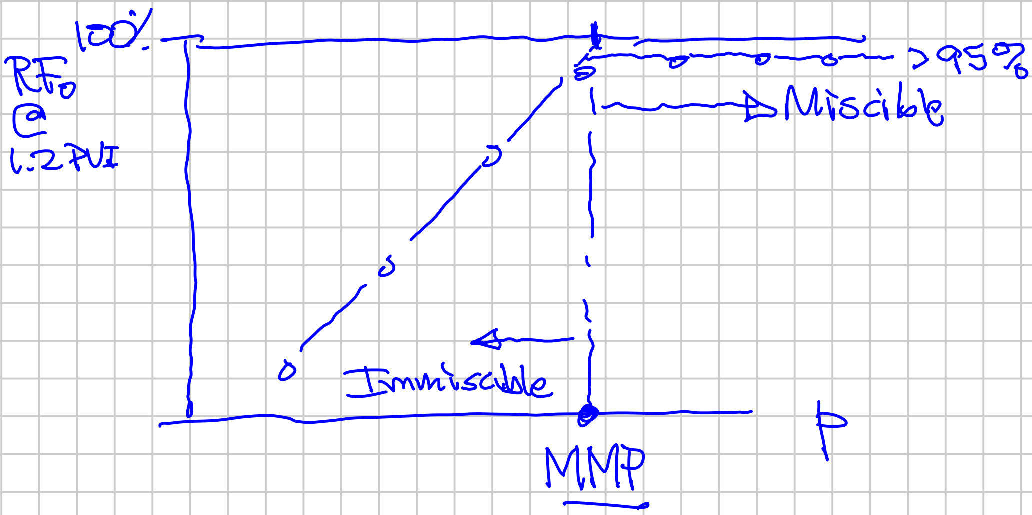
min. 4-components ⇒ C/V Miscibility

Method of Characteristics

IG + RO ⇒ Develops Miscibility, or not

- Experimentally (Slimtube : 1D packed tube)
  ⇒ min physical disp.
- Computationally (1D horizontal : many many cells ⇒ min numerical dispersion)





EOS-based 1D Simulations ⇒ MMP

- EOS model needs to be "good"
  - complex phase behavior
  - PVT lab measurements
  - near critical points

- IMPES formulation
  CFL ~ 1

- Sufficient grid cells to eliminate / minimize numerical dispersion

not real mixing of frontal gas and frontal oil

@ $p < MMP$