APPLICATION OF A GENERAL MATERIAL BALANCE FOR HIGH-PRESSURE GAS RESERVOIRS


SPE Members

Abstract

This paper presents the derivation of a general material balance that has particular application to high pressure gas reservoirs. The material balance is valid for both normal-pressure and over-pressured (geo-pressured) reservoirs. Its main application is to calculate original gas in place and assist in calculating remaining recoverable reserves from pressure-production data.

The form of the material balance equation is

\[(p/p_c)(1 - \xi_2(p)/p_c(p_p - p_c)) = (p/p_c)(1 - G/G_0) \]  \hspace{1cm} (1)

which includes a pressure-dependent cumulative effective compressibility term \(\xi_2(p)\) that is defined in terms of the following reservoir parameters: (1) pore compressibility, (2) water compressibility, (3) gas solubility, and (4) total water associated with the gas reservoir volume. "Associated" water includes connate water, water within interbedded shales and non-pay reservoir rock, and any limited aquifer volume \(\xi_2\) physically represents the cumulative change in hydrocarbon pore volume caused by compressibility effects and encroaching water.

High pressure gas reservoirs typically have curved \(p/z-G\) plots (concave downward). Incorrect extrapolation of early data may result in serious overestimation of original gas in place and remaining recoverable reserves. The proposed form of the gas material balance equation provides a method to linearize the \(p/z-G\) plot and thereby predict the true original gas in place. A method is suggested to determine initial gas in place by analyzing the behavior of cumulative effective compressibility backcalculated from pressure-production data. The \(\xi_2(p)\) function determined by this procedure, or estimated from logs and geological maps (when sufficient production data is not available), is then used to forecast pressure-cumulative behavior.

For most reservoirs \(\xi_2\) is fairly constant through most of depletion. The magnitude of \(\xi_2(p)\) at initial pressure usually ranges from 15 to 100(10^4) psi^1, depending mostly on the volume of water associated with the gas reservoir. As defined in this paper, all components of \(\xi_2\) represent cumulative volume changes; i.e., instantaneous water and rock compressibilities are not used directly.

References and illustrations at end of paper.

We show that the effect of pore collapse on high pressure gas reservoirs is generally positive, providing additional pressure support. There is not a clear discontinuity in the behavior of \(p/z-G\) where pore collapse occurs, and pore collapse tends to flatten or increase \(p/z-G\) at lower pressures.

\(\xi_2\) may increase significantly at lower pressures because of gas solubility effects. An example is given for a large gas reservoir with high-CO_2 content that requires an increasing \(\xi_2\) term at lower pressures to linearize the \(p/z-G\) plot; the increasing \(\xi_2\) behavior is substantiated by calculations based on gas solubility effects.

The proposed gas material balance is applicable (and should be applied) to any high pressure gas reservoir with an appreciable volume of associated water. Numerous field examples are provided showing the application of the material balance equation to high pressure gas reservoirs.

Introduction

High pressure gas reservoirs experiencing depletion drive typically have downward curving \(p/z-G\) behavior. Incorrect extrapolation of early depletion data may result in serious overestimation of original gas in place and remaining reserves. Bruns et al. work in 1965 was a result of a field study conducted on a large moderately overpressured gas reservoir in the Texas Gulf Coast area. Investments were made, and never needed, based on linear extrapolation of the early field \(p/z-G\) performance to an apparent original gas in place that was later found to be overstated by about 200 Bscf. Fig. 5 in Ref. 1 (Run 20) shows the concave downward curvature typical for the pressure response of a conventional limited external aquifer system that simulated the reservoir's response.

This type of "limited" aquifer behavior where pressure in the reservoir and aquifer are virtually equal led to the derivation in 1969 of a general material balance for high pressure gas reservoirs. The derivation includes pressure-dependent rock and water compressibility (with gas evolving from solution). All water and rock volumes associated with the reservoir and available for expansion, including a limited aquifer volume, were included in a cumulative effective compressibility term \(\xi_2(p)\). Rock and water compressibilities were defined to account for cumulative changes in pore volume to be multiplied by the cumulative pressure drop \((p-p_c)\); instantaneous compressibilities are not used at all. The final form of the material balance is similar to that published by Ramoost and Farshad, except
that Ref. 2 considered \( \xi \) as a constant. The 1969 derivation as presented in this paper defines a cumulative effective compressibility \( \xi(p) \) as a function of pressure expressed in terms of reservoir properties and volumes.

### Literature Review

Harville and Hawkins\(^3\) and Hamerlinck\(^4\) attribute the concave downward shape of \( p/z-G \) curves obtained in abnormally pressured gas reservoirs entirely to pore collapse and formation compaction. No definition of pore collapse is given in Ref. 3, but a plot of backcalculated pore volume change indicated a system compressibility change from 28(10\(^{-6}\)) psi\(^{-1}\) at initial pressure to about 6(10\(^{-6}\)) psi\(^{-1}\) at low pressures. This magnitude of pore volume change implies associated water volume. The decreasing "system" compressibility is expected for an overpressured reservoir with pressure dependent pore volume compressibility and based on this paper pore collapse is not a necessary condition for such behavior.

The Anderson \"L\" reservoir performance presented by Duggan\(^8\) shows curved \( p/z-G \) behavior which was primarily attributed to shock wave influx with no evidence of reservoir pore compression. The water influx drive mechanism was supported by the fact that several wells watered out. Wallace\(^9\), in a 1969 paper, also concluded that shale water influx is an important drive mechanism in abnormally pressured gas reservoirs. Bass\(^8\) discounts shale water influx and attributes curved \( p/z-G \) behavior to peripheral water influx from a limited aquifer and formation compaction treated with a constant pore volume compressibility \( c \). For a limited aquifer Bass defines a term \( F \) as the ratio of peripheral water pore volume to the pore volume of gas-bearing rock.

Roach\(^9\) and Ramagost and Farshad\(^6\) both utilize the term \( p/z-c \) for geopressed and abnormally pressured gas reservoirs. Both authors consider \( c \) as a constant and they consider only the Anderson \"L\" example.

Bernard\(^10\) does not accept the rock collapse theory as the cause for overpressured \( p/z-G \) behavior, concluding that water influx is the basic drive mechanism. He also uses \( p/z-c \) where \( c \) is a "catch-all" approximation for treating the effects of rock and water compressibility, a small steady-state non-aqueous, and steady state shale water influx. He further states that the term \( c \) is virtually impossible to quantify in terms of reservoir properties.

Begild and Whitehead\(^11\), Prasad and Rogers\(^12\), and Wang and Teasdale\(^13\) in all present studies of overpressured gas reservoirs based on computer models. Refs. 10 and 11 treat \( c \) as functions of pressure, including the effect of solution gas in the water. External water sources are also included in Refs. 11 and 12. The differential forms of the material balance used in these references correctly apply instantaneous compressibility in a history matching approach to determine initial gas in place. A direct plot of \( p/z-c \) versus \( G \) was not made because the \( \xi \) term had not been defined.

Ponson and Chen\(^14\) analyzed several abnormally pressured gas reservoirs, and recognized that calculated values of \( c \) > 3(10\(^{-6}\)) psi\(^{-1}\) required to linearize the material balance equation reflected the influence of water influx.

Bourgoyne\(^15\) demonstrates that reasonable values of shale permeability and compressibilities treated as a function of pressure can be used to match abnormal gas reservoir performance behavior. He points out, however, that determining \( k \) and \( c \) of the shale necessary for modelling this behavior is virtually impossible.

Ambastha\(^16\) uses Bourgoyne's general material balance equation to develop a gas material balance based on a constant initial gas compressibility \( c \). The example considered shows a lack of uniqueness in determining initial gas in place.

### General Material Balance

The general form of the gas material balance derived in Appendix A is:

\[ \frac{p}{2} (1 - \xi(p)(p,))-\xi(p) = \frac{p}{2} \left( \frac{G_s - G_a + W'R'_m + 5.615 \left[ (WB_s - W'BL - W') \right]}{B_1} \right) \]

which reduces to Eq. 1 when water terms and gas injection can be neglected. The cumulative effective compressibility term \( \xi(p) \) is pressure dependent consisting of a cumulative pore volume compressibility \( \xi(p) \), cumulative total water compressibility \( \xi_w(p) \), and total pore and water volume associated (i.e. in pressure communication) with the gas reservoir,

\[ \xi(p) = \frac{S_G \xi_w(p) + \xi_G(p) + \xi_w(p)}{1 - S_w} \]

The formation and total water compressibility terms \( \xi_G \) and \( \xi_w \) account for cumulative changes in pore volume from initial pressure to the current pressure.

The interbedded non-pay volume and limited aquifer contributions to pressure support are quantified in terms of the M ratio,

\[ M = \frac{V_{NON-NET-PAY} + V_{LIMITED-AQUIFER}}{V_{NET-PAY-RESERVOIR}} \]

An important aspect of the material balance for high-pressure gas reservoirs is that the gas in solution in the connate and associated water provide both pressure support and additional gas available for production. The level of pressure support provided by the evolved solution gas depends on the level of depletion, and it is shown that this support is significant below about 1500 psi. The solution gas available for production also depends on the level of depletion, i.e. how much of the original solution gas has evolved \( \xi \). The cumulative effective compressibility term \( \xi(p) \) functions.

The stage for accurate determination of \( \xi(p) \) functions. Cumulative Pore Volume Compressibility \( \xi \). The material balance presented in this paper uses a cumulative pore volume compressibility \( \xi \), defined as

\[ \xi(p) = \frac{1}{V_{p}} \left( \frac{V_p - V_{p}}{p_{i} - p} \right) \]
The term in brackets is the slope of the chord from the initial condition $(p_i, V_{i0})$ to any lower pressure $(p_f, V_f)$, as shown in Fig. 2. This implies that $c_i$ is a function of both pressure and the initial condition. The instantaneous pore volume compressibility $c_i$ is defined as

$$c_i(p) = \frac{1}{V_i} \frac{\partial V_i}{\partial p}$$

(6)

and is only a function of pressure. At initial pressure the two pore volume compressibilities are equal: $c_i(p) = c_0(p)$. The instantaneous compressibility function $c_i(p)$ should be used for instantaneous correlation and differential forms of the material balance, while the cumulative compressibility function $c_i(p)$ should be used with forms of the material balance that apply the cumulative pressure drop $(p; p)$, i.e. $p/z$ vs $G_c$, plots.

The pressure dependence of $c_i$ is best determined by special core analysis under appropriate reservoir conditions. Table 1 summarizes the calculation of $c_i$ as a function of pressure using laboratory data for a Gulf Coast sandstone. Fig. 3 shows how $c_0$ and $c_i$ vary as a function of pressure for this overpressured reservoir rock.

In the absence of pore collapse $c_i$ is always greater than or equal to $c_0$. The cumulative pore volume compressibility remains higher than the instantaneous compressibility because of an averaging effect that reduces the pressure dependence of $c_i$ compared with $c_0$. An important consequence of this behavior is that a rock exhibiting large pore volume change because of a high level of overpressure (and consequently with a high initial $c_i$ value dropping rapidly to a "normal" value) will initially have and maintain a high cumulative compressibility $c_i$ as shown in Fig. 3.

Pore collapse is defined as the condition when a rock's instantaneous pore volume compressibility starts to increase at decreasing reservoir pressure. Pore collapse provides greater pressure support when collapse occurs at a high pressure. However, pore collapse is not reflected by the $c_i(p)$ function and will not therefore be seen on the $p/z-G_c$ plot at the pressure when pore collapse occurs. In fact, pore collapse may not be identifiable at all on the cumulative compressibility term. For example, the Gulf Coast sandstone in Fig. 3 exhibits pore collapse at 4000 psia (about 5000 psi less than initial pressure). Despite the increase in $c_i$ from 4 to 25(10^-4) psi^-1 in the pressure range 4000 to 1000 psia, the change in $c_i$ over the same pressure range is almost insignificant. Fig. 4 shows a North Sea Chalk sample from a reservoir with initial pressure of 7000 psia exhibiting pore collapse at 6000 psia. Here the effect of pore collapse is greater, causing $c_i$ to increase from 6 to 20(10^-4) psi^-1 in the pressure range from 6000 to 2000 psia. In general, however, pore collapse in and of itself does not have a significant effect on the $p/z-G_c$ plot.

In the absence of laboratory data, pore volume compressibilities can be estimated from correlations presented by Hall and by Von Gonten and Choudhary. Hall's correlation (his Fig. 2) gives instantaneous pore volume compressibility as a function of porosity, i.e. there is no pressure dependence. The Hall correlogram simulation and adequate for normal pressured reservoirs. Von Gonten develops correlations for instantaneous pore volume compressibility $c_i$ as a function of net overburden pressure $(p_o)$, where $p_o = p - p_i$, equals the overburden gradient times depth minus reservoir pressure.

Table 2 gives example values of initial pore volume compressibility $c_i(p)$ for overpressured and normal pressured reservoir conditions. Typically there are not large differences in $c_i$ values for these two conditions.

Cumulative Total Water Compressibility $\xi_w$. The pressure support provided by water is made up of two components. First, the water expansion with decreasing pressure, and second, the release of solution gas and its expansion. The total or composite compressibility effect is expressed as

$$\xi_w(p) = \frac{1}{B_w(p)} \frac{B_w(p) - B_w(p_i)}{p - p_i}$$

(7)

in terms of the total water formation volume factor $B_w$.

$$B_w(p) = B_w(p_i) + \frac{(R_w - R_w(p_i))B_w(p_i)}{5.615}$$

(8)

Fig. 5 shows typical behavior for $B_w$ and $B_w$ as a function of pressure; the figure also shows the behavior of $\xi_w(p)$ where it is seen that little increase occurs before a pressure of about 1500 psia, and that at pressures below 1000 psia there is a significant increase in $\xi_w$ with a limiting relationship $\xi_w = 1/p$ at low pressures.

$$\xi_w(p=0) = \frac{1}{5.615} \frac{T_{w0} R_w}{P_{w0} B_w}$$

(9)

Specifically at standard conditions $(p_{w0})$, $\xi_w$ is given by

$$\xi_w(p_{w0}) = \frac{1}{5.615} \frac{T_{w0} R_w}{P_{w0} B_w}$$

(10)

To calculate $\xi_w$ values of $B_w$, $R_w$, and $B_i$ are tabulated with pressure as shown in Table 3. These properties can be obtained from correlations at pressures less than about 10,000 psia and 300°F. At more extreme conditions of pressure and temperature, and for gases with high concentrations of nonhydrocarbons CO$_2$, N$_2$, and H$_2$S, we have used the Peng-Robinson equation of state with variable translation, and using binary interaction coefficients that are dependent on both temperature and salinity.

Another approach for high pressures is simply to extrapolate $B$ linearly and $R_w$ with a flattening curvature towards a constant value. Nonhydrocarbons can be treated by evaluating $R_w$ of each component separately at its partial pressure, and summing the values for all soluble components

$$\frac{R_w}{B_w(p_i)} = \sum_j \frac{R_y(p_i)}{B_y}$$

(11)

$y_j$ is the reservoir gas mole fraction of component $j$. Typically only the components with appreciable solubility are methane, CO$_2$, and H$_2$S.

Associated Water Volume Ratio $M$. The total compressibility effect on the gas material balance depends on the magnitudes of rock and total water compressibilities and on the total pore and water volumes in pressure communication with the gas reservoir (including connate water and the pore volume within the net pay).

Associated water and pore volumes external to the net pay include non-net pay (NPP) such as interbedded shales and dirty sands, plus external water volume found in limited aquifers. Including these water volumes in reservoir simulation is referred to as using a "gross" model. In the proposed material balance equations this associated volume is expressed as a ratio relative to the pore volume of the net-pay reservoir.

$$M = M_{NPP} + M_{AQ}$$

(12)

where

$$M_{NPP} = \frac{V_{NPP} \text{(INTERBEDDED NONNET PAY)}}{V_{NPP} \text{(NET PAY)}}$$

(13)
When the aquifer is sufficiently large and requires treatment with either superposition or the Schliibius infinite aquifer model, the \( \xi \) term should still be used but it will only contain the effect of net pay and non-net pay volumes, i.e. \( M = M_{a \phi p} \).

Cumulative Effective Compressibility \( \xi \). Total cumulative effective compressibility represents all available pressure support from rock and water. The equation for \( \xi \) is

\[
\xi(p) = \frac{S_{\infty}(p) + \xi_{\epsilon}(p) + M_{e}(p) + \xi_{i}(p)}{1 - S_{\infty}}
\]

For overpressurized reservoirs exhibiting a pressure-dependent \( \xi(p) \), the family of \( \xi(p) \) curves at high pressures will tend to decrease with depletion. In the absence of pore collapse \( \xi(p) \) decreases to a constant value at lower pressure and the \( \xi(p) \) curves at lower pressure are dominated by the increasing \( \xi_{e}(p) \) function. If pore collapse occurs, but not early in depletion, the pore collapse is almost insignificant because (a) the \( \xi_{e}(p) \) function does not start increasing until low pressures because it represents a cumulative pore volume change, and (b) when the \( \xi(p) \) function finally starts to increase it will be masked by the \( \xi_{e}(p) \) function which is increasing as \( 1/p \). Fig. 7 illustrates this point for a Gulf Coast overpressured reservoir with \( p_i = 9000 \) psia, \( T = 300^\circ \mathrm{F} \), and \( \gamma_r = 0.71 \) (air = 1). Although pore collapse occurs at 4000 psia (Fig. 3), \( \xi \) does not start increasing until 2000 psia. The increase is insignificant relative to the increase in \( \xi_{e}(p) \) at lower pressures.

The next example is a North Sea chalk (Fig. 4) that shows pore collapse at a pressure only 1000 psia below initial pressure of 7000 psia. The \( \xi(p) \) function increases almost simultaneously with instantaneous \( c_r \) and the effect of \( \xi(p) \) on \( \xi \) is shown in Fig. 8. Although \( \xi(p) \) has an impact on \( \xi(p) \) at moderate and high pressures for this example, the \( \xi_{e}(p) \) function still dominates the behavior of \( \xi(p) \) at pressures less than 1500 psia.

Estimating Gas-In-Place. A method is proposed for estimating the initial (free) gas in place \( G \) based on historical pressure- cumulative data. The procedure also determines the water volume ratio \( M \) and the \( \xi_{e}(p) \) function. First, a plot of \( p_{zG} \) versus cumulative gas production \( G \) should have the characteristic concave downward shape of a high-pressure reservoir influenced by associated water and pore volume compressibility.

A range of values for \( G \) should then be assumed, with the largest value based on an extrapolation of the early depletion data and the lowest value being somewhat larger than the current \( G_r \). For an assumed value of \( G \), calculate for each measured \( p_{zG} \) and \( G \) data the \( \xi(p) \) from the rearranged material balance equation (Eq. 1),

\[
G_{\text{backcalculated}} = \left[ 1 - \left( \frac{p_{zG}}{p_{zG}} \right) \left( \frac{G}{G_r} \right)^{1/\gamma_r} \right] \frac{1}{\gamma_r} \]

At this point, a plot can be made of backcalculated \( \xi(p) \), as a function of pressure given the assumed \( G \). Using reservoir rock and water properties, a family of \( \xi(p) \) curves at various \( M \) values can be generated to match against the backcalculated \( \xi(p) \) values. The data should
honor the shape and magnitude of one \( \xi(p) \) curve, where this match yields \( G \), the \( M \) value, and a \( \xi(p) \) function that can be used to forecast future \( p/z-G \), behavior. This procedure gives a sound physical significance to the estimation of \( G \) as opposed to a pure statistical best-fit that may lead to unrealistic solutions. The Field Examples section discusses criteria for matching field data, and the expected behavior of \( \xi(p) \).

Characteristics of \( p/z-G \), Plots for High-Pressure Reservoirs

Pore volume reduction, water expansion, and solution gas evolution, expressed in terms of \( \xi(p) \) in the general material balance equation, provide pressure support for all reservoirs during depletion. The reservoir does not have to be overpressured or geopressed. The term \( \xi(p) \) determines whether the conventional \( p/z-G \) plot yields a straight line. For most low-pressure reservoirs this term is small and is often neglected because a straight-line \( p/z-G \) plot is obtained. Reservoirs undergoing depletion with initial pressure exceeding 3000 psia are automatically candidates for being treated with the complete material balance equation.

Fig. 9 presents three generated \( p/z-G \) curves for a Gulf Coast overpressured sandstone reservoir using \( M = 0 \) (i.e., \( \xi(p) = (1 - S,)/(1 - S,) \)). Curve A uses the same \( \xi(p) \) function as Curve A down to 4000 psia (where pore collapse occurs) and thereafter uses a constant instantaneous compressibility of 4(10\(^{-6}) \) psi\(^{-1}\). \( p/z-G \) plots for A and B are almost identical, showing only a slight separation at pressures less than 3500 psia. This clearly shows the limited effect of pore collapse on the \( p/z-G \) plot when collapse occurs late in depletion. Curve A assumes, unrealistically, that the initial pore volume compressibility of 13(10\(^{-6}) \) psi\(^{-1}\) remains constant throughout depletion. The difference between the two \( p/z-G \) curves A and C is a result of the actual decrease in pore volume compressibility. Including an external water volume quantified with \( M = 2 \) produces more curvature in the \( p/z-G \) plots, but the separation between curves with and without pore collapse is still very small (not shown).

Another example relates to a North Sea chalk reservoir where pore collapse occurs just below initial pressure. Fig. 10 presents generated \( p/z-G \) plots for \( M = 0 \) with pore collapse (curve A) and with no pore collapse (curve B). The effect of pore collapse is more significant than in the previous example because it occurs at a relatively high pressure.

Field Examples

Ellesbanger Gas Reservoir. This field example is for a normal pressured (0.5 psia/ft) 1600-foot thick, dry gas reservoir with initial reservoir pressure of 6675 psia at 200°F. Average porosity is about 50.0% with connate water saturation in the pay of about 35.0%. Permeability is high because of an extensive microfracture system that results in a high degree of interwell pressure communication and almost instantaneous pressure buildup to static conditions. Initial \( \text{CO}_2 \) concentration was about 28 mol-%, and a gradual increase in \( \text{CO}_2 \) concentration to 31 mol-% has been observed. The reservoir has produced about 3.1 Tscf and currently has a fieldwide bottomhole pressure of approximately 1000 psia. The \( p/z-G \) plot shows a characteristic concave downward behavior, with an initial gas in place estimate of more than 4.4 Tscf, using early data (Fig. 11). The \( p/z-G \) data at low pressures has started flattening.

The procedure outlined earlier for determining initial free gas in place \( G \) was used for this reservoir. Fig. 12 shows a plot of backcalculated \( \xi(p) \) versus pressure for a range of \( G \) from 3.0 Tscf to 3.6 Tscf. Another plot of \( \xi(p) \) was generated for several values of \( M \) using \( S, = 0.35, \xi = 6.5(10^{-6}) \) psi\(^{-1}\) from Hall\(^{6}\), and \( \xi(p) \) calculated from equation of state results. Fig. 13 shows the best-fit of data on the \( \xi(p) \) curve for \( M = 3.5 \), corresponding to an initial free gas in place \( G = 3.15 \) Tscf.

The total water volume including connate and associated waters is given by

\[
W = \frac{1}{5.615} \left( \frac{S_n + M}{1 - S_n} \right) \, (1 - S_n) \, (1 - S_w) \, (1 - S_o).
\]

which yields 8.45(10\(^6\)) STB. The initial solution gas in place \( G \) is equal to \( W \) times the initial solution gas-water ratio \( R_{GW} \).

\[
G = W \, R_{GW} \, (1 - S_o).
\]

Because of the high \( \text{CO}_2 \) concentration in this reservoir the solution gas-water ratio \( (R_{GW} = 67.5 \, acf/STB) \) is about three times larger than for hydrocarbon gas systems. This yields a solution gas in place of \( G = 0.55 \) Tscf and a total initial gas in place of \( G + G_0 = 3.70 \) Tscf. Fig. 11 shows the \( p/z-G \) forecast using the \( M \) value determined from the match to calculate the \( \xi(p) \) function from \( S_o, M, \xi, \) and \( \xi(p) \). Also shown on this figure is the plot of \( p/z-G \) vs. \( p/z \) for historical performance data, \( p/z-G \) for the forecast, where \( p/z \) is obtained from the current cumulative gas produced equals the original free gas in place.

The associated water volume given by \( M = 3.3 \) consists of non-net pay and an external limited aquifer. Log analysis indicates a net-to-gross ratio NGR = 0.5, \( \phi_w = 0.05 \), and \( \phi_{GW} = 0.03 \), yielding \( M_{GW} = 0.6 \). External water is known to exist but has not been mapped due to lack of well control. The calculated aquifer water volume ratio \( M_{GW} = 2.7 \times 0.03 \), or an equivalent \( r_{GW}/r_a = 1.9 \), seems reasonable for a limited aquifer.

Anderson ".L.". This reservoir has been studied by several authors and it is perhaps the best recognized example of a high-pressure gas reservoir with connate downward \( p/z-G \) behavior (Fig. 14). The reservoir was abandoned after producing 25 Bscf, but pressure tests of public record were discontinued after 40 Bscf had been produced.

Different analyses by other authors have indicated original free gas in place between 65 to 75 Bscf. Fig. 15 shows backcalculated \( \xi(p) \) versus pressure for values of \( G \) equal to 65, 72, and 90 Bscf. The 72 Bscf volume is chosen based on a best-fit match with the \( \xi(p) \) function calculated using \( M = 2.25, S_o = 0.35, \xi = 3.2(10^{-6}) \) psi\(^{-1}\), and a \( \xi(p) \) function from equation of state results. Although the first four data do not fall on the slightly-increasing \( \xi(p) \) curve, data at pressures below this value do follow the trend down to the last pressure data near 3000 psia.

The 90 Bscf estimate produces unrealistically low \( \xi(p) \) values, lower than would be calculated using the net reservoir pore volume and connate water compressibilities. The lowest estimate of 65 Bscf gives a shape for \( \xi(p) \) that cannot be accounted for using normal \( \xi(p) \) and \( \xi(p) \) functions.

The forecasted \( p/z-G \) performance (Fig. 14) is calculated using the match determined above. Total gas in place of 72 Bscf which includes 72 Bscf of original free gas plus 4 Bscf of solution gas.

Cajus Field. This reservoir was originally reported by Stelly and Farshad\(^{14}\) and recently analyzed by Ambath\(^{3}\). Initial pressure is 11450 psi at 13,300 ft (0.86 psia/ft). Constate water saturation is reported as 22%. Production data is reported to a pressure of 6850 psia and a cumulative of 145 Bscf.

Using the \( p/z-G \) data shown in Fig. 16, backcalculated values of \( G \) are shown in Fig. 17. The range of values for \( G \) are the same as considered by Ambath: 410 to 760 Bscf. The 360 Bscf estimate yields unrealistically low \( \xi(p) \) values, less than 2(10\(^{-6}) \) psi\(^{-1}\). Values lower than 565 Bscf produce \( \xi(p) \) functions that increase more steeply than would be expected from a \( \xi(p) \) behavior. The expected magnitude and shape of cumulative effective compressibility is exhibited by the backcalculated \( \xi(p) \) values for an assumed \( G \) of 650 Bscf. This
corresponds to \( M = 0.2 \) using \( S_\text{w} = 0.22 \), \( \xi_0 = 4 \times 10^{-6} \text{ psi}^{-1} \), and \( \xi_w(p) \) from correlations.

Gulf Coast Reservoir C. This example is taken from Bernard\(^7\) his Fig. 1) and represents a high-pressure, overpressured gas reservoir taken to a low abandonment pressure. The \( p/z-G \) plot (Fig. 18) shows significant concave downward character with an extrapolation of early data giving an initial free gas in place of 380 Bscf. Depletion from \( p_\text{i} = 11500 \text{ psia} \) to about 1200 psia produced 180 Bscf, verifying that the early-data extrapolation of 380 Bscf was incorrect.

Fig. 19 shows backcalculated \( \xi(p) \) for values of \( G \) from 160 to 240 Bscf. The only curve that produces an approximately constant \( \xi \) at high pressures is \( G = 160 \text{ Bscf} \), a value 30 Bscf less than the volume already produced. The curve for \( G = 240 \text{ Bscf} \) has a downward sloping \( \xi(p) \) that becomes negative, also an unrealistic solution. The \( G = 200 \text{ Bscf} \) curve has a downward sloping \( \xi(p) \) function that can be explained by a pressure-dependent pore volume compressibility. A highly overpressured formation can readily have a pressure-dependent \( \xi(p) \) function, i.e. one that decreases with depletion.

Reservoir data was not presented by Bernard\(^7\) for this field example, but making some assumptions about typical Gulf Coast reservoir properties we matched the \( \xi(p) \) backcalculated behavior using \( G = 185 \text{ Bscf} \), and a \( \xi(p) \) function that decreased linearly by a factor of about 2 from initial conditions to abandonment pressure. The backcalculated \( \xi(p) \) behavior at 2000 psia started increasing, indicating that pore collapse could have occurred earlier in depletion.

Duck Lake Field. Casen\(^3\) presents production performance data from the Discorbia 1 reservoir in the Duck Lake field of southern Louisiana. This high-pressure gas reservoir was waterflooded for more than 10 years after first being depleted to about 1000 psia. Fig. 20 shows the \( p/z \)-cumulative plot for data prior to the waterflooding project, showing typical concave downward curvature.

Casen reports an initial gas in place of 680 Bscf using traditional water influx analysis. Based on the high reservoir permeability \( k = 1750 \text{ md} \) we established that the reservoir performance could be analyzed with the general material balance where the external aquifer was treated as part of the \( \xi \) term. Using \( G = 680 \text{ Bscf} \), \( \xi(p) \) was backcalculated from the general material balance as shown in Fig. 21. The \( \xi(p) \) behavior is flat throughout depletion. This behavior should be compared with the dashed line representing the expected \( \xi(p) \) behavior based on Eq. 3 using a constant \( \xi_0 = 3.4 \times 10^{-9} \text{ psi}^{-1} \), \( M = 4.8 \), and an appropriate \( \xi_w(p) \) function. The dashed curve has significant increase in \( \xi_w(p) \) already at 2000 psia, and the more-or-less constant \( \xi(p) \) behavior backcalculated from production data can not be readily explained. It is expected, however, that conventional water influx analysis which assumes constant water and pore volume compressibility will yield an estimate of initial gas in place that reflects a constant \( \xi \) when backcalculated from the general material balance.

Using a smaller estimate of \( G = 625 \text{ Bscf} \) yields backcalculated \( \xi(p) \) behavior that is very similar to \( \xi(p) \) calculated from Eq. 3 using a constant \( \xi_0 = 3.4 \times 10^{-9} \text{ psi}^{-1} \), \( M = 6.5 \), and \( \xi_w(p) \) function. Cumulative production at abandonment was about 650 Bscf, indicating that about 25 Bscf of the total produced gas came from solution. Based on \( G = 625 \text{ Bscf} \), \( R_\text{w} = 20.6 \text{ scf/STB} \), \( S_\text{w} = 0.18 \), and \( M = 6.5 \) the initial solution gas in place is \( G = 65 \text{ Bscf} \).

Conclusions

1. A general form of the material balance equation for gas reservoirs has been presented. This equation has particular application to high-pressure reservoirs. The cumulative effective compressibility term \( \xi(p) \) has been defined in terms of pressure-dependent pore volume and total water cumulative compressibilities, \( \xi_0(p) \) and \( \xi_w(p) \), and the total volume of water associated with the net pay reservoir expressed as a ratio \( M \).

2. The general material balance equation applies to all high-pressure reservoirs, both normal pressured and abnormally pressured (overpressured and geopressed).

3. The effect of a limited aquifer can be included as part of the \( M \) term for most depletion-type reservoirs. Using the water volume ratio \( M \) in the cumulative effective compressibility term, together with normal values of \( \xi_0 \) and \( \xi_w(p) \) explains the "large" \( \xi \) values commonly reported for high-pressure gas reservoirs when linearizing the material balance equation. In fact, large values of \( \xi \) backcalculated from field performance data indicate that associated water influx is a dominant drive mechanism.

4. Only cumulative compressibilities \( \xi_0(p) \) can be used in the general gas material balance equation because they are applied against the cumulative pressure drop \( (p-p_i) \) in \( p/z-G \) plots. A method is given for calculating cumulative total water and pore volume compressibility \( \xi_w(p) \) and \( \xi_w(p) \).

5. A method is proposed for estimating the original free gas in place from production data. This method uses backcalculated cumulative effective compressibility \( \xi \), which is plotted versus pressure and compared with expected \( \xi(p) \) behavior calculated from rock and water properties for a range of values of the associated water volume ratio \( M \).

6. In lieu of laboratory data for pore volume compaction we recommend Hall\(^1\) correlation for normal-pressured reservoirs, and Von Conten's\(^1\) correlation for abnormally-pressed reservoirs.

7. Pore collapse in and of itself does not contribute significantly to pressure support in overpressured gas reservoirs. In fact, pore collapse has little effect unless it occurs early in depletion at a relatively high pressure. The effect of pore collapse, if present, is a positive effect and tends to flatten the \( p/z \)-G curve, not bending the curve downward as has been implied by others.

8. Gas found initially in solution in the connate and associated water is an important component of pressure support late in depletion (below 1500 psia) and may contribute additional producible volumes of gas. Typically the solution gas in place \( G_s \) represents 2 to 10 percent of the original free gas in place, the value depending primarily on total water volume \( M + S_\text{w} V/(1-S_\text{w}) \) and the initial solution gas-water ratio \( R_\text{sw} \). Gas reservoirs with high CO\(_2\) concentration (>20 mol-%) can have even higher solution gas in place, \( G_s \).

Nomenclature

\( A = \) area, \( \text{ft}^2 \) [\( \text{m}^2 \)]

\( B = \) formation volume factor, reservoir per standard volume

\( c = \) instantaneous compressibility, \( \text{psi}^{-1} \) [\( \text{kPa}^{-1} \)]

\( G = \) cumulative compressibility, \( 1/\text{psi} \) [\( 1/\text{kPa} \)]

\( G_s = \) original free gas-in-place, Bscf [\( \text{std} \text{m}^3 \)]

\( G_i = \) cumulative gas production, Bscf [\( \text{std} \text{m}^3 \)]

\( G_s = \) initial solution gas in place, Bscf [\( \text{std} \text{m}^3 \)]

\( G_s = \) early overestimate of \( G \), Bscf [\( \text{std} \text{m}^3 \)]

\( G_i = \) cumulative gas injection, Bscf [\( \text{std} \text{m}^3 \)]

\( h = \) thickness, \( \text{ft} \) [\( \text{m} \)]

\( M = \) volume ratio, dimensionless

\( NGR = \) net-to-gross ratio, dimensionless

\( p = \) reservoir pressure, psia [\( \text{kPa} \)]

\( P_\text{i} = \) initial reservoir pressure, psia [\( \text{kPa} \)]

\( p_\text{sat} = \) net overburden pressure, psia [\( \text{kPa} \)]

\( r = \) radius of reservoir, \( \text{ft} \) [\( \text{m} \)]

\( r_\text{aq} = \) radius of aquifer, \( \text{ft} \) [\( \text{m} \)]

\( R_\text{sw} = \) solution gas water ratio, SCF/STB [\( \text{std} \text{m}^3/\text{m}^3 \)]

\( S_\text{w} = \) initial water saturation, function

\( T = \) reservoir temperature, °R [K]
\[ V = \text{volume, ft}^3 \]
\[ V_{\text{p}} = \text{pore volume, cm}^3 \text{ and ft}^3 \]
\[ V_{\text{b}} = \text{bulk volume, cm}^3 \]
\[ W = \text{total water in place, bbl} \text{ [m}^3 \text{]} \]
\[ W_{\text{c}} = \text{cumulative water influx, bbl} \text{ [m}^3 \text{]} \]
\[ W_{\text{w}} = \text{cumulative water production, bbl} \text{ [m}^3 \text{]} \]
\[ z = \text{gas compressibility factor, dimensionless} \]
\[ \phi = \text{porosity, fraction} \]

Subscripts
- A = associated water
- AQ = limited aquifer
- \( f \) = pore volume ("formation")
- G = gas
- GROSS = gross interval thickness
- inj = injection
- NNP = non-net pay
- R = reservoir
- sc = standard conditions
- tw = total water
- w = water

Acknowledgements

We thank the management of Phillips Petroleum Company for permission to publish this paper. We also acknowledge Fred Kent for work done on the Ellenburger example.

References


Appendix A - Derivation of General Gas Material Balance

The derivation that follows is based on the following assumptions:

1. Any pressure change caused by production or injection into the reservoir will be felt immediately throughout the total system including:
   a. Net Pay Reservoir ("R").
   b. Non-Net Pay ("NNP") including interbedded shales and poor quality rock assumed to be 100% water saturated.
   c. Limited Aquifer ("AQ"), when present, also assumed to be water saturated.

2. The non-net pay and aquifer volumes are referred to as "associated" water volumes and both contribute to water influx during depletion.

3. Simple modifications to the material balance equations can be made to generalize for non-net pay that has an initial free gas saturation.

4. All water in the system is initially saturated with solution gas.

Fig. 22 shows a schematic of the reservoir/associated water system.
Derivation. The volumetric balance at any pressure states that the total pore volume \( (V_{pt} + V_{we}) \) equals the net reservoir pore volume occupied by gas and water \( (V_{pt} + V_{we}) \) plus the associated (non-net pay and aquifer) pore volume which also is occupied by gas and water \( (V_{pt} + V_{we}) \):

\[
(V_{pt} + V_{we}) = (V_{pt} + V_{we}) + (V_{pt} + V_{we}) \quad \text{.................................................................. (A1)}
\]

The net-reservoir pore volume \( V_{an} \) is given by the initial volume \( V_{an} \) less the change in pore volume \( \Delta V_{an} \):

\[
V_{an} = V_{an} - \Delta V_{an} \quad \text{.................................................................. (A2)}
\]

\[
V_{an} = V_{an} + V_{we} \quad \text{.................................................................. (A3)}
\]

\[
\Delta V_{an} = \frac{G_{s}}{1-S_{w}} \bar{e}_{f}(p_{t}-p) \quad \text{\( \bar{e}_{f} = (\bar{e}_{f})_{a} \) \quad \text{.................................................................. (A4)}}
\]

yielding

\[
V_{an} = \frac{G_{s}}{1-S_{w}} M - \frac{G_{s}}{1-S_{w}} \bar{e}_{f}(p_{t}-p) \quad \text{.................................................................. (A5)}
\]

Pore volume of the associated rock is given by the initial pore volume less the change in pore volume,

\[
V_{an} = V_{an} - \Delta V_{an} \quad \text{.................................................................. (A6)}
\]

\[
V_{an} = \frac{G_{s}}{1-S_{w}} M \quad \text{.................................................................. (A7)}
\]

\[
\Delta V_{an} = \frac{G_{s}}{1-S_{w}} M \bar{e}_{f}(p_{t}-p) \quad \text{\( \bar{e}_{f} = (\bar{e}_{f})_{a} \) \quad \text{.................................................................. (A8)}}
\]

yielding

\[
V_{an} = \frac{G_{s}}{1-S_{w}} M - \frac{G_{s}}{1-S_{w}} M \bar{e}_{f}(p_{t}-p) \quad \text{.................................................................. (A9)}
\]

The net reservoir gas volume is given by the sum of unproduced free gas, gas released from solution, and any injected gas,

\[
V_{pt} = (V_{pt})_{un-produced} + (V_{pt})_{solution} + (V_{pt})_{injected} \quad \text{.................................................................. (A10)}
\]

\[
(V_{pt})_{un-produced} = [G - (G_{w} - W_{B})] B_{s} \quad \text{.................................................................. (A11)}
\]

\[
(V_{pt})_{solution} = \frac{G_{s}}{1-S_{w}} (R_{sw} - R_{w}) B_{s} \quad \frac{1}{5.615} \quad \text{.................................................................. (A12)}
\]

\[
(V_{pt})_{injected} = G_{in} B_{s} \quad \text{.................................................................. (A13)}
\]

resulting in

\[
V_{pt} = [G - (G_{w} - W_{B})] B_{s} + \frac{G_{s}}{1-S_{w}} (R_{sw} - R_{w}) B_{s} + 5.615 \quad \text{.................................................................. (A14)}
\]

PVT properties \( B_{s} \) and \( R_{w} \) are evaluated at current reservoir pressure. \( G \) for a gas condensate is the wet gas volume calculated by adding separator gas to liquid condensate converted to an equivalent surface gas volume. Also, the two-phase Z-factor must be used to calculated \( B_{s} \) for gas condensate reservoirs. Strictly speaking the cumulative water production term \( W_{B} \) represents "free" water production and not the water condensed out of solution from the produced gas wellstream.

The gas volume in the associated pore volume is a function of the amount of gas that has come out of solution,

\[
V_{an} = \frac{G_{s}}{1-S_{w}} M \frac{1}{1-S_{B}} (R_{sw} - R_{w}) B_{s} \quad \frac{1}{5.615} \quad \text{.................................................................. (A15)}
\]

The water volume in the net-pay reservoir equals the unproduced initial water plus injected water plus water encroachment from an external aquifer,

\[
V_{we} = (V_{we})_{un-produced} + (V_{we})_{injected} + [(V_{we})_{solution}] \quad \text{.................................................................. (A16)}
\]

\[
(V_{we})_{un-produced} = \frac{G_{s}}{1-S_{w}} B_{s} - W_{B} \frac{1}{1-S_{B}} \quad \frac{1}{5.615} \quad \text{.................................................................. (A17)}
\]

\[
(V_{we})_{injected} = 5.615 W_{B} B_{s} \quad \text{.................................................................. (A18)}
\]

\[
(V_{we})_{solution} = 5.615 W_{B} B_{s} \quad \text{.................................................................. (A19)}
\]

yielding

\[
V_{we} = \frac{G_{s}}{1-S_{w}} B_{s} + 5.615 (W_{B} B_{s} + W_{B} W_{B}) \quad \text{.................................................................. (A20)}
\]

The aquifer encroachment term \( W_{B} \) represents any external water volume that is not already included in the "M" term. Later in the derivation we show the conditions required so that water encroachment (treated rigorously by the method of superposition) can be included as part of the M term used in the cumulative effective compressibility \( \bar{e}_{f} \).

The water volume in the associated pore volume is given by simple expansion,

\[
V_{an} = \frac{G_{s}}{1-S_{w}} M \frac{1}{1-S_{B}} B_{s} \quad \text{.................................................................. (A21)}
\]

Combining terms gives

\[
GB_{s} = \frac{G_{s}}{1-S_{w}} M - \frac{G_{s}}{1-S_{w}} M \bar{e}_{f}(p_{t}-p) - \frac{G_{s}}{1-S_{w}} M \bar{e}_{f}(p_{t}-p) \quad \text{.................................................................. (A22)}
\]

Changing signs and grouping terms yields
Defining the total water-gas formation volume factor $B_{w,g}$, and noting that $B_{w,g} = B_w$ gives

$$G(B_w - B_g) + GB_w \left[ \frac{B_w - B_m}{1 - \frac{M}{B_w}} - \frac{B_m}{B_w} \right] + \xi(p_1 - p)$$

Dividing through by $GB_w$ and expressing $B_g = (p/T) (p/T) + C$ gives the final form of the material balance

$$(p/z) \left[ 1 - \xi(p_1 - p) \right] =$$

$$(p/z), \left\{ 1 - \frac{1}{G} \left[ \frac{G - G_m + W R_w - 5.615 (W B_w - W B_m - W_g)}{B_g} \right] \right\} \right.$$

$$(p/z) \left[ 1 - \xi(p_1 - p) \right] = (p/z), - \frac{(p/z)_1}{Q}$$

with

$$Q = G_g - G_m + W R_w - 5.615 (W B_w - W B_m - W_g) \ldots (A32)$$

where the intercept is given by $(p/z)_1$ and the slope equals $(p/z)/G$. Setting $G_m = W_m = W = 0$ gives the common form of the gas material balance,

$$G(B_g - B_m) + GB_g \xi(p_1 - p)$$

Treating Limited Aquifers in $\xi$ Term. The material balance thus far has considered any associated water volume expressed in terms of the $M$ parameter. In fact $M$ may include a limited aquifer with up to 25 times the reservoir pore volume for a system permeability greater than about 100 md, and even larger aquifer volumes for higher permeabilities. The condition that determines when a limited aquifer can be treated as part of the $\xi$ term is outlined below. We start with the general material balance equation including a water encroachment term $W_a$ and a $\xi$ term that considers only non-net pay.

$$(p/z) \left[ 1 - \xi(p_1 - p) \right] = (p/z)_1 \left[ 1 - \frac{G_g}{G} \right]$$

$$(p/z) \left[ 1 - \xi(p_1 - p) \right] = (p/z)_1 \left[ \frac{G_g}{G} + 5.615 \frac{W_a}{GB_g} \right]$$

$$(p/z) \left[ 1 - \xi(p_1 - p) \right] = (p/z)_1 \left[ \frac{G_g}{G} + \frac{5.615}{GB_g} W_a \right]$$

$$(p/z) \left[ 1 - \xi(p_1 - p) \right] = (p/z)_1 \left[ \frac{G_g}{G} + \frac{5.615}{GB_g} W_a \right]$$

The water encroachment term calculated by superposition is expressed,

$$W_a = B \sum_j Q_j (\Delta t_j) \Delta p_j$$

where $Q_j (\Delta t_j)$ is the dimensionless cumulative influx given as a function of dimensionless time $\Delta t$, and aquifer-to-reservoir radius $r_a = r_a/r_k$. $\Delta p_j$ is given by $p_j - p_j^*$ (in the limit for small time steps), and $\Delta t = r_a/r_k$. Assuming that permeability is reasonably high and the ratio $r_a/r_k$ is not
too large, $Q_0$ for the smallest time step will approach the limiting value $Q_0^*$, and the summation can be closely approximated by

$$\sum Q_0 (\Delta t_j) \Delta p_j = Q_0^* (p_i - p_f) \quad \text{........................................ (A37)}$$

giving a simple expression for $W_*$ which is independent of time and only dependent on reservoir pressure,

$$W_* = B Q_0^* (p_i - p_f) ; W_0 (bbl) \quad \text{................................. (A38)}$$

$$B = \frac{2\pi \phi h (\bar{\varepsilon}_w + \bar{\varepsilon}_g)}{3.613}$$

$$Q_0^* = \frac{1}{2} \left( \frac{V_{mq}}{t_a} \right)^2 - 1 \quad \text{........................................ (A39)}$$

Expressing $W_*$ in terms of aquifer pore volume $V_{mq}$,

$$W_* = \frac{\pi \phi h (\bar{\varepsilon}_w + \bar{\varepsilon}_g) (p_i - p_f)}{V_{mq}} \quad \text{........ (A40)}$$

$$\text{The material balance equation can then be written:}$$

$$(p/z)^2 [1 - \delta_0 (p_i - p_f)] = (p/z) \left[ 1 - \frac{Q_0^*}{S} \right] + (p/z)^2 \frac{W_*}{GB_a} \quad \text{5.615} \quad \text{........................................ (A41)}$$

and simplified in a form where the $\delta_*$ term includes the aquifer contribution to pressure support,

$$\frac{W_*}{GB_a} = \frac{(p/z)^2}{(p/z) - \frac{W_*}{GB_a} \text{ (p/z)}}$$

$$\frac{W_*}{GB_a} = \frac{(p/z)^2}{W_* \text{ GB}_a \text{ (p/z)}} \quad \text{GB_a} = V_{mq} (1 - S_*W) \quad \text{........................................ (A42)}$$

$$\frac{W_*}{GB_a} = \frac{(p/z)^2}{V_{mq} (\bar{\varepsilon}_w + \bar{\varepsilon}_g) (p_i - p_f)} \quad \text{........................................ (A43)}$$

Rearranging we arrive at the general form of the material balance (without water production and gas/water injection terms):

$$(p/z)^2 [1 - \delta_0 (p_i - p_f)] = (p/z) \left[ 1 - \frac{Q_0^*}{S} \right] \quad \text{........................................ (A43)}$$

where

$$\delta_* = \frac{S \bar{\varepsilon}_w + \bar{\varepsilon}_g + \left[ \frac{V_{mq} + V_{mq}}{V_{mq} + V_{mq}} \right] (\bar{\varepsilon}_w + \bar{\varepsilon}_g)}{1 - S_*W} \quad \text{........................................ (A44)}$$

$$M = \frac{V_{mq} + V_{mq}}{V_{mq} + V_{mq}} \quad \text{........................................ (A45)}$$

$$\bar{\varepsilon}_* = \frac{S \bar{\varepsilon}_w + \bar{\varepsilon}_g + M (\bar{\varepsilon}_w + \bar{\varepsilon}_g)}{(1 - S_*W)} \quad \text{........................................ (A46)}$$
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<th>$V_p$ (cm$^3$)</th>
<th>$V_b$ (cm$^3$)</th>
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<th>$c_r$</th>
<th>$p$ (psia)</th>
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<th>$V_{pl}-V_p$ (cm$^3$)</th>
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All compressibilities in $10^{-6}$ psi$^{-1}$. 
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<th>Normal Pressure ( c_f ) (psi(^{-1}))</th>
<th>Over-Pressured ( c_f ) (psi(^{-1}))</th>
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*Normal Pressured* is 0.5 psi/ft x Depth;
*Overpressured* is 0.8 psi/ft x Depth. Depth Used is 10,000 ft.
### TABLE 3A - EXAMPLE CALCULATION OF TOTAL WATER CUMULATIVE COMPRESSIBILITY FOR THE ANDERSON "L" RESERVOIR

<table>
<thead>
<tr>
<th>Pressure psia</th>
<th>(B_w) bbl/STB</th>
<th>(R_{nw}) scf/STB</th>
<th>(Z_g)</th>
<th>(B_g) ft³/scf</th>
<th>(B_{tw}) bbl/STB</th>
<th>(c_{tw}) (10^6) psi⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>9510</td>
<td>1.0560</td>
<td>31.8</td>
<td>1.4401</td>
<td>0.00282</td>
<td>1.056</td>
<td>2.40</td>
</tr>
<tr>
<td>9000</td>
<td>1.0569</td>
<td>31.0</td>
<td>1.3923</td>
<td>0.00288</td>
<td>1.057</td>
<td>2.43</td>
</tr>
<tr>
<td>8000</td>
<td>1.0586</td>
<td>29.2</td>
<td>1.2991</td>
<td>0.00303</td>
<td>1.060</td>
<td>2.51</td>
</tr>
<tr>
<td>7000</td>
<td>1.0604</td>
<td>27.2</td>
<td>1.2072</td>
<td>0.00322</td>
<td>1.063</td>
<td>2.65</td>
</tr>
<tr>
<td>6000</td>
<td>1.0621</td>
<td>25.0</td>
<td>1.1176</td>
<td>0.00347</td>
<td>1.066</td>
<td>2.78</td>
</tr>
<tr>
<td>5000</td>
<td>1.0638</td>
<td>22.5</td>
<td>1.0325</td>
<td>0.00385</td>
<td>1.070</td>
<td>2.98</td>
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<tr>
<td>4000</td>
<td>1.0654</td>
<td>19.6</td>
<td>0.9562</td>
<td>0.00446</td>
<td>1.075</td>
<td>3.28</td>
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<tr>
<td>3000</td>
<td>1.0669</td>
<td>16.1</td>
<td>0.8977</td>
<td>0.00558</td>
<td>1.083</td>
<td>3.86</td>
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<td>2000</td>
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<td>11.8</td>
<td>0.8744</td>
<td>0.00815</td>
<td>1.097</td>
<td>5.19</td>
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<tr>
<td>1500</td>
<td>1.0686</td>
<td>9.3</td>
<td>0.8832</td>
<td>0.01098</td>
<td>1.113</td>
<td>6.69</td>
</tr>
<tr>
<td>1000</td>
<td>1.0691</td>
<td>6.5</td>
<td>0.9078</td>
<td>0.01693</td>
<td>1.145</td>
<td>9.95</td>
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<tr>
<td>750</td>
<td>1.0692</td>
<td>5.0</td>
<td>0.9258</td>
<td>0.02302</td>
<td>1.179</td>
<td>13.30</td>
</tr>
<tr>
<td>500</td>
<td>1.0693</td>
<td>3.3</td>
<td>0.9472</td>
<td>0.03533</td>
<td>1.249</td>
<td>20.24</td>
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<td>1.459</td>
<td>41.20</td>
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<tr>
<td>100</td>
<td>1.0694</td>
<td>0.5</td>
<td>0.9835</td>
<td>0.18341</td>
<td>2.092</td>
<td>104.23</td>
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<td>14.7</td>
<td>1.0694</td>
<td>0.0</td>
<td>1.000</td>
<td>1.26860</td>
<td>8.254</td>
<td>717.86</td>
</tr>
</tbody>
</table>

\(B_w\) and \(R_{nw}\) were calculated from the Peng-Robinson EOS with volume translation using binary interaction parameters that are functions of temperature and salinity (28,000 PPM for this example); the gas Z-factor was calculated from the Standing-Katz correlation.
### TABLE 3B - EXAMPLE CALCULATION OF TOTAL WATER CUMULATIVE COMPRESSIBILITY FOR THE ELLENBURGER RESERVOIR WITH INITIAL 28% CO$_2$ CONCENTRATION

<table>
<thead>
<tr>
<th>Pressure psia</th>
<th>$B_w$ bbl/STB</th>
<th>$R_w$ scf/STB</th>
<th>$Z_g$ ft$^3$/scf</th>
<th>$B_g$ bbl/STB</th>
<th>$c_{rw}$ $10^{-6}$ psi$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6675</td>
<td>1.0761</td>
<td>67.5</td>
<td>1.0464</td>
<td>0.00292</td>
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<td>6000</td>
<td>1.0765</td>
<td>64.5</td>
<td>0.9962</td>
<td>0.00310</td>
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<tr>
<td>5000</td>
<td>1.0768</td>
<td>59.5</td>
<td>0.9262</td>
<td>0.00345</td>
<td>3.12</td>
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<tr>
<td>4000</td>
<td>1.0770</td>
<td>53.5</td>
<td>0.8732</td>
<td>0.00407</td>
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</tr>
<tr>
<td>3000</td>
<td>1.0767</td>
<td>46.1</td>
<td>0.8493</td>
<td>0.00528</td>
<td>5.24</td>
</tr>
<tr>
<td>2500</td>
<td>1.0764</td>
<td>41.5</td>
<td>0.8513</td>
<td>0.00635</td>
<td>6.61</td>
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<tr>
<td>2000</td>
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<td>36.1</td>
<td>0.8638</td>
<td>0.00805</td>
<td>8.89</td>
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<tr>
<td>1750</td>
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<td>33.0</td>
<td>0.8742</td>
<td>0.00932</td>
<td>10.67</td>
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<td>1500</td>
<td>1.0749</td>
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<td>0.8872</td>
<td>0.01103</td>
<td>13.15</td>
</tr>
<tr>
<td>1250</td>
<td>1.0743</td>
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<td>0.9028</td>
<td>0.01347</td>
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<td>0.9208</td>
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<td>22.56</td>
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<tr>
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<td>0.9408</td>
<td>0.02339</td>
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<td>0.9621</td>
<td>0.03588</td>
<td>52.99</td>
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<tr>
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<td>5.8</td>
<td>0.9833</td>
<td>0.07335</td>
<td>115.75</td>
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<tr>
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<td>0.0</td>
<td>1.0000</td>
<td>1.26860</td>
<td>2126.80</td>
</tr>
</tbody>
</table>

$B_w$ and $R_w$ were calculated from the Peng-Robinson EOS with volume translation using binary interaction parameters that are functions of temperature and salinity (50,000 PPM for this example); the gas $Z$-factor was calculated from the Standing-Katz correlation.
\[
\frac{G}{G_x} = \frac{1}{1 + (2 \times 10^{-4}) (p_i^2) (\bar{c}_e)}
\]

with \( p_i \) (psia) & \( \bar{c}_e \) (1/psi)

\( p_i = 12000 \text{ PSIA} \)

\( p_i = 10000 \text{ PSIA} \)

\( p_i = 8000 \text{ PSIA} \)

\( p_i = 6000 \text{ PSIA} \)

\( p_i = 4000 \text{ PSIA} \)

\( p_i = 2000 \text{ PSIA} \)

Fig. 1 Effect of \( p_i \) and \( \bar{c}_e \) on Overestimating \( G \)
Fig. 2 Cumulative Pore Volume Compressibility as a Chord Slope
Fig. 3  Cumulative and Instantaneous $c_f$ vs $p$ for a Sandstone With Pore Collapse
Fig. 4 Cumulative and Instantaneous $c_f$ vs $p$ for a Chalk With Pore Collapse
Fig. 5 Cumulative Total Water Compressibility, $\tilde{c}_{tw}$, vs $p$
Fig. 6  Cumulative Effective Compressibility vs \( p \) at Various \( M \) Ratios
Fig. 7  Cumulative Effective Compressibility vs p for a Sandstone W/Pore Collapse
Fig. 8 Cumulative Effective Compressibility vs p for a Chalk W/Pore Collapse
Fig. 9  Effect on $p/z$ vs $G_p$ With and Without Pore Collapse, Sandstone
Fig. 10  Effect on p/z vs Gp With and Without Pore Collapse, Chalk
Fig. 11  Pressure vs Cumulative Production, Ellenburger Gas Reservoir
Fig. 12  Backcalculated $\bar{c}_e$ vs $p$ at Various OGIP, Ellenburger Gas Reservoir
Fig. 13 Matching Backcalculated $\bar{c}_e$ to Generated $\bar{c}_e$ Curves, Ellenburger Gas Reservoir
ANDERSON "L" RESERVOIR

- HISTORICAL PERFORMANCE DATA

FORECAST USING M = 2.25

Fig. 14  p/z vs Cumulative Production, Anderson "L" Reservoir
Fig. 15  Backcalculated $\bar{c}_e$ vs p at Various OGIP, Anderson "L" Reservoir
Fig. 16  p/z vs Cumulative Production, Cajun Reservoir
Fig. 17  Backcalculated $\bar{c}_e$ vs $p$ at Various OGIP, Cajun Reservoir
Fig. 18  \( p/z \) vs Cumulative Production, Gulf Coast Reservoir "C"
Fig. 19  Backcalculated $\bar{c}_e$ vs $p$ at Various OGIP, Gulf Coast Reservoir "C"
Fig. 20  Pressure vs Cumulative Production, Duck Reservoir
DUCK RESERVOIR

\( \bar{c}_e \) BACK CALCULATED ASSUMING OGIP

- 625 BSCF
- 680 BSCF

\[ M = 6.5 \]

\[ M = 4.8 \]

Fig. 21 Backcalculated \( \bar{c}_e \) vs p at Various OGIP, Duck Reservoir
<table>
<thead>
<tr>
<th>PRODUCTION / INJECTION TERMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_p$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NET RESERVOIR</th>
<th>ASSOCIATED WATER-BEARING VOLUMES</th>
</tr>
</thead>
<tbody>
<tr>
<td>FREE GAS</td>
<td>1. INTERBEDDED SHALES AND POOR QUALITY RESERVOIR ROCK (CONTAINING SOLUTION GAS)</td>
</tr>
<tr>
<td>CONNATE WATER (CONTAINING SOLUTION GAS)</td>
<td>2. LIMITED AQUIFER (CONTAINING SOLUTION GAS)</td>
</tr>
</tbody>
</table>

EXTERNAL "LARGE" AQUIFER WITH TRANSIENT ENCROACHMENT BEHAVIOR

Fig. 22 General Material Balance Schematic