Note Title 10/15/2018

WATER INFLUX:

Encroachment of external water into the HC reservoir pore volume from "Aquifer" (AQ)

(+) ⇒ Reduction of the HCPV ⇒ slows the average pressure decline during depletion

(−) ⇒ Gas reservoirs, may lead to water production leading to the "death" of producers

(+) ⇒ Oil reservoirs, displacement of oil that otherwise would not be recovered by SGD (expansion) depletion

"EOR" from mother nature

SGD 15% → 50 → 9x %

Pot Aquifer Model gives us the MAXIMUM, FASTEST encroachment of water for a finite aquifer

$$p_{R(HC)}(t) = p_{AQ}(t) = "p_R"$$

W_e = cumulative water volume encroachment from an aquifer into the HC reservoir

$$W_e^{POT} = V_{AQ}(c_f + c_w)(p_{Ri} - p_R) = W_{e,max}$$

$$G(B_g-B_{gi}) + \frac{GB_{gi}}{1-S_{wi}}\left[S_{wi}\left(\frac{B_{tw}-B_{twi}}{B_{twi}}\right) + \bar{c}_f(p_i-p)\right.$$

$$\left. + M\left(\frac{B_{tw}-B_{twi}}{B_{twi}}\right) + M\,\bar{c}_f(p_i-p)\right] \qquad \ldots\ldots \text{(A25)}$$

$$= (G_p - W_pR_{sw} - G_{inj})B_g + 5.615\left(W_p - W_{inj} - \frac{W_e}{B_w}\right)B_w$$

The p/z-cumulative plot including all terms would consider $(p/z)[1-\bar{c}_e(p_i-p)]$ versus the entire production/injection term Q

$$(p/z)[1-\bar{c}_e(p_i-p)] = (p/z)_i - \frac{(p/z)_i}{G}Q \qquad \ldots\ldots\ldots\ldots\ldots \text{(A31)}$$

with

$$Q = G_p - G_{inj} + W_pR_{sw} + \frac{5.615}{B_g}(W_pB_w - W_{inj}B_w - W_e) \qquad \ldots \text{(A32)}$$

where the intercept is given by $(p/z)_i$ and the slope equals $(p/z)_i/G$.

The water encroachment term calculated by superposition is expressed,

$$W_e = B\sum_j Q_D(\Delta t_j)_D\,\Delta p_j \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A36)}$$

where $Q_D(t_D)$ is the dimensionless cumulative influx given as a function of dimensionless time t_D and aquifer-to-reservoir radius $r_D = r_{AQ}/r_R$. Δp_j is given by $p_j - p_{j-1}$ (in the limit for small time steps), and $\Delta t_j = t - t_{j-1}$.

Radial Aquifers

The water influx equation for radial aquifers is:

$$W_e = 1.119\ \phi ch\ r_w^{\ 2} \cdot \frac{\Theta}{360} \sum_{0}^{n} \Delta p\ Q_D \qquad (11)$$

where

Θ = angle subtended by the reservoir circumference, degrees.

r_w = radius of the aquifer inner boundary, ft.

Q_D = radial efflux functions, dim.

ϕch = aquifer storage number, ft. $\cdot$ psi^{-1}.

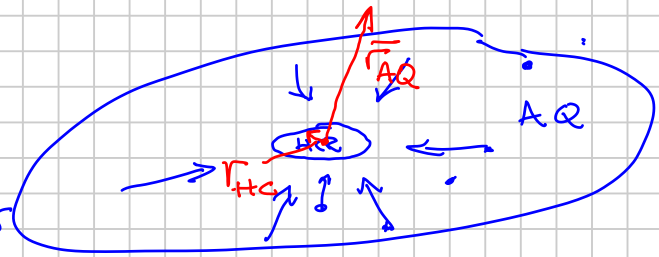
Values of Q_D for infinite and limited outer boundaries are available in equation, chart, and tabular form as a function of dimensionless wellbore time, t_{Dw}. Chart 48 in Volume 4 gives Q_D vs. t_{Dw} curves for several limited no-flow aquifers. Tabulated values can be found in Craft and Hawkin's, "Applied Petroleum Reservoir Engineering", pages 212-217.

PE/RE Books & Papers & PetroWiki

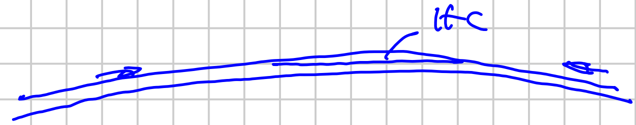
Estimation of $W_e(t)$

① GEOMETRY

- Radial Flow Geometries



- Linear Flow — " —



$$r_D = \frac{r_{AQ}}{r_{HC}} = 1.x - 10^{(+)}$$

Dimensionless Length L_D

$$x_D = \frac{x_{AQ}}{x_{HC}}$$

② $k_{AQ} \propto v_w$ in AQ

③ $p_{RHC}(t)$

Pot Aquifer : $p_{RHC}(t)$ only time dependency

$$\left.\begin{array}{l} k_{AQ} \sim \infty \\ L_D \sim \text{"small"} \end{array}\right\} \text{Instantaneous Encroachment}$$

$$W_{e,max}(t) = V_{AQ}(c_f + c_w)\left(p_{Ri} - \underline{\underline{p_{RHC}(t)}}\right)$$

How fast you empty the HC's

Water Encroachment is modeled EXACTLY as single phase fluid flow in "Well Testing" ("PTA" Pressure Transient Analysis and/or "RTA" Rate Transient Analysis)

Solves PDE using continuity Eq. (material balance) & Darcy's eq.

: Geometry (Radial, Cylindrical)
(Linear)

: Boundary Conditions:

PTA: Well Testing

$r = r_w \quad q = \text{constant} = \nu$

(a) $r = r_e \quad (dp/dr) = 0$ No Flow $(q = 0)$

(b) $p = p_e = \text{const.}$

(c) No outer boundary ("infinite" $r_e = \infty$)

$\Rightarrow \quad p(t, r) \qquad p_{wf}(t, r = r_w)$

$p_D(t_D, r_D)$ General Dimensionless Solution

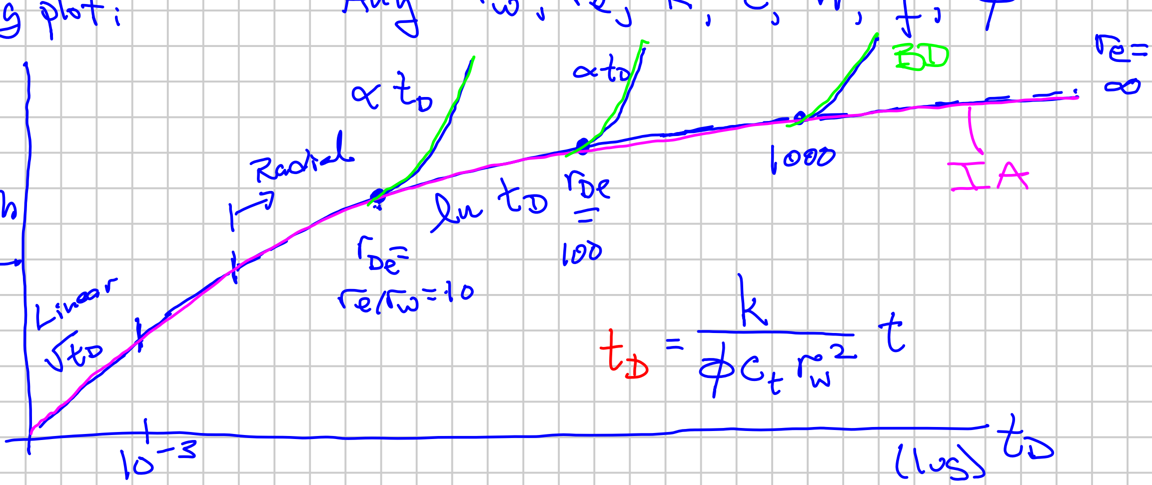
Any $r_w, r_e, k, c, h, q, \phi$

log-log plot:

$$p_D = \frac{\left[\left(p_{Ri} - p_{wf}\right)\right] kh}{q \mu B}$$

$f(t)$

const



$$t_D = \frac{k}{\phi c_t r_w^2} t$$

RTA (Rate Transient Analysis)

"Time"

B.C. $\quad r = r_w \; : \; p = p_{wf} = \text{constant}$ ("p_{RHC}")

$\Rightarrow q(t) \;|\; \underline{q_D(t_D)}$

B.C. Used for Water Influx Calculations $W_e(t)$

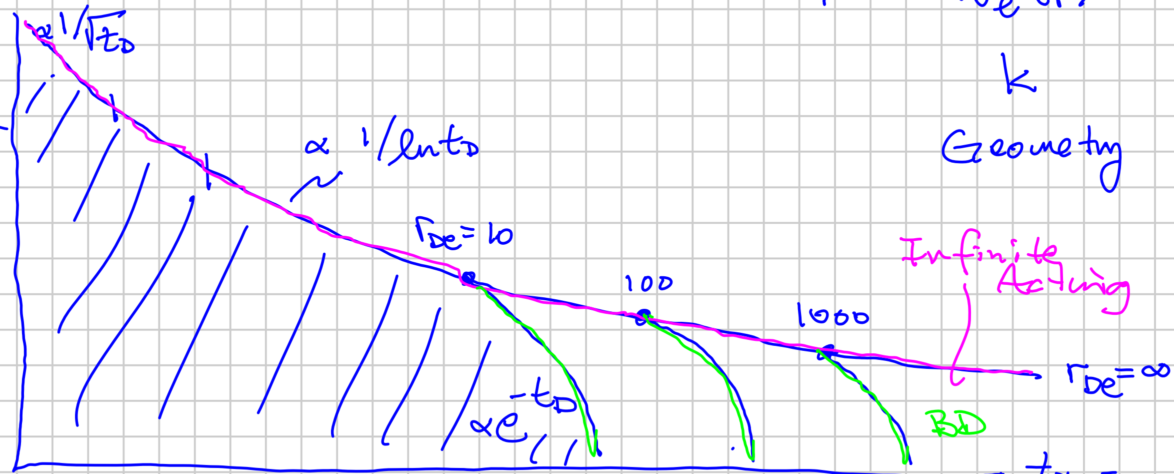
k

Geometry

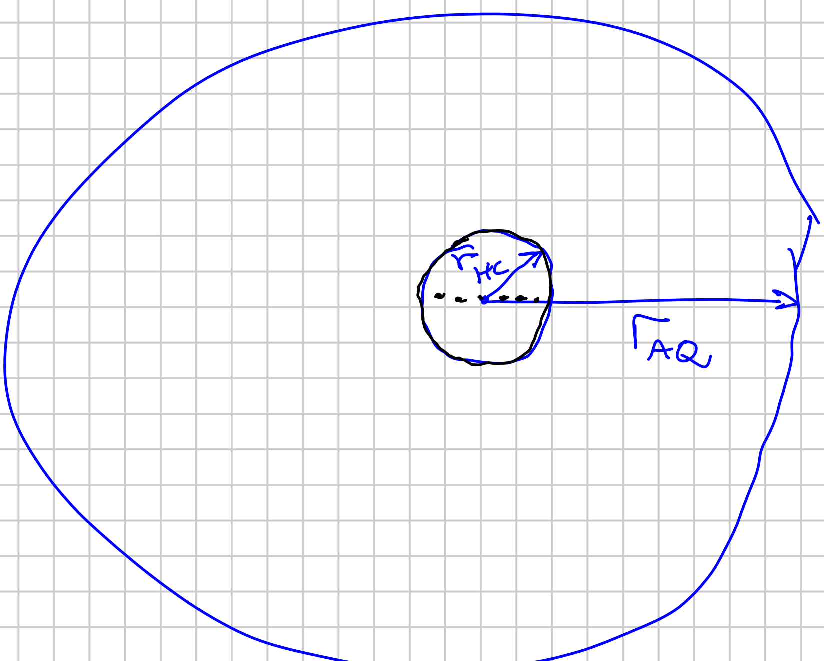
$f(t)$

$$q_D = \frac{q\mu B}{(p_{Ri} - p_{wf})\, kh}$$

const

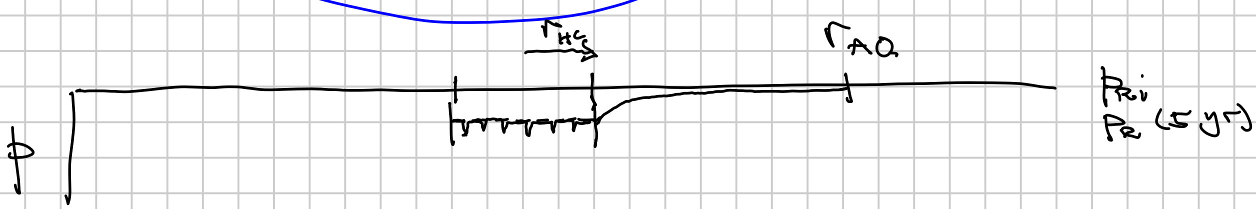


$$t_D = \frac{k}{\phi \mu c_t} t$$



Aquifer Influx

$p_{wf} = $ "p_{RHC}" (t)



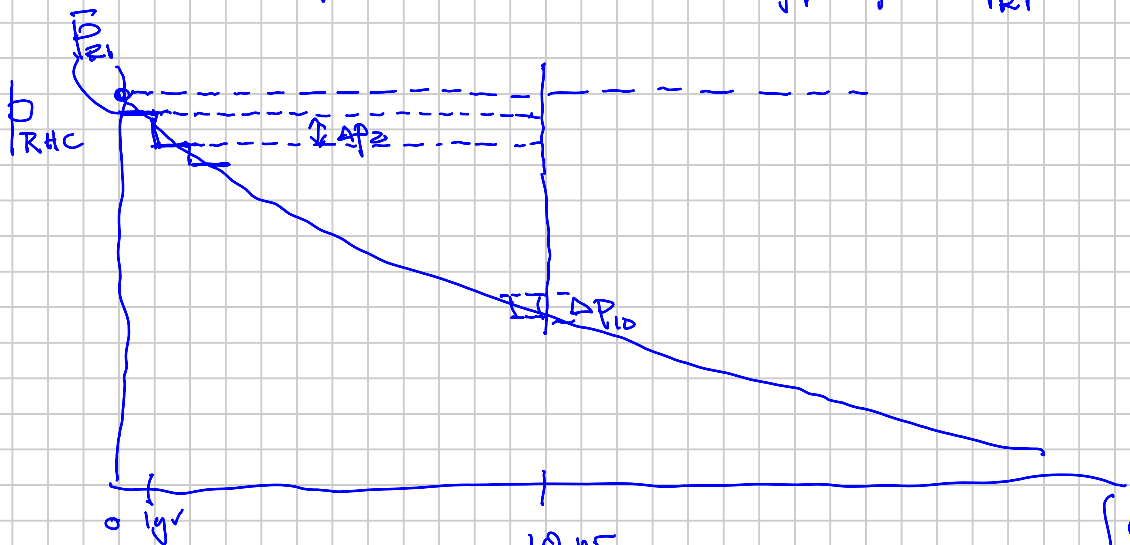
Analogy to Well Behavior $\quad r_w \quad r_e$

Aquifer $\quad r_{HC} \quad r_{AQ}$

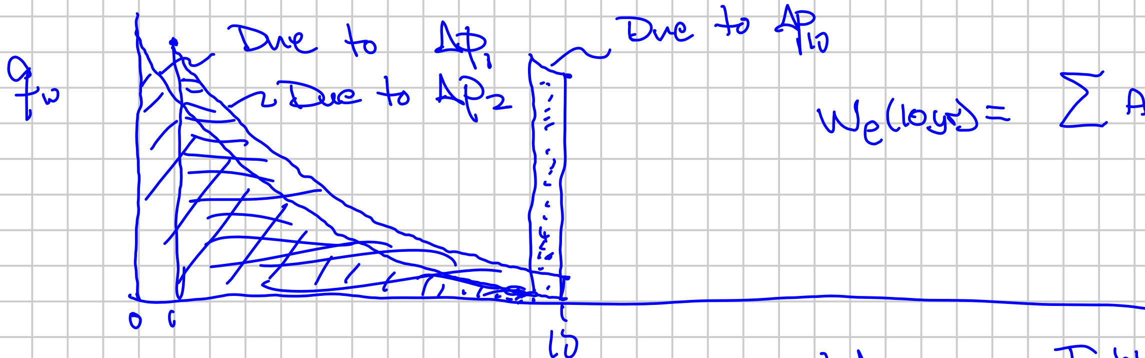
r_R

Because inner BC "p_{wf}" "p_{RHC}" (t)

use "Superposition" $\Delta p_1 = p_{Ri} - \bar{p}_{R1} \Rightarrow q_w(t)$ for 10 yrs



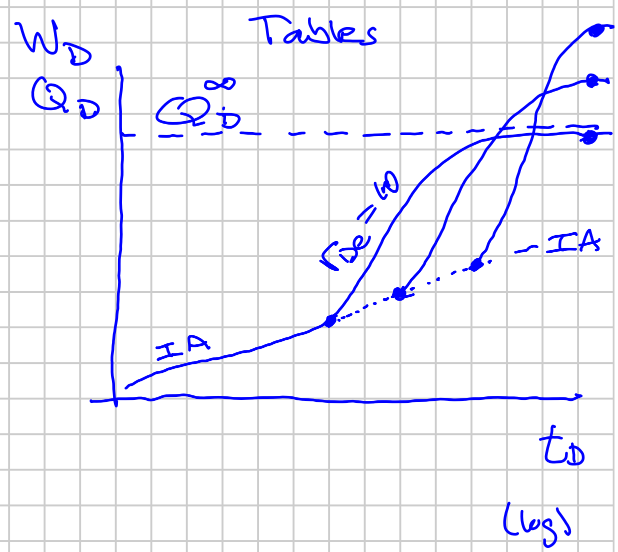
$\int q(t)$



$W_e(10\,yrs) = \sum$ Areas

Cumulative Volume =

$$W_D = Q_D = \int_0^{t_D} q_D(t_D)\, dt_D$$



$$\boxed{W_e = U \sum \Delta p_k \, W_D(\Delta t_{D_k})}$$

$$\Delta p_k = p_{k-1} - p_k$$

$Q_D^{\infty}(r_{De})$

$$\Delta t_{D_k} = t_D - t_{D_{k-1}}$$

$$\Delta t_{D_1} = 10 - 0$$

$$\Delta t_{D_2} = 10 - 1$$

$$W_e = U \sum \Delta p_{RHC,k} \, Q_D(\Delta t_{D_k})$$

$$W_{e,max} = U \sum \Delta p_{RHC,k} \, Q_D^{\infty}$$

$$= U \left(p_{RHi} - p_{RHC,k}\right) Q_D^{\infty} \qquad \text{Pot Aq.}$$

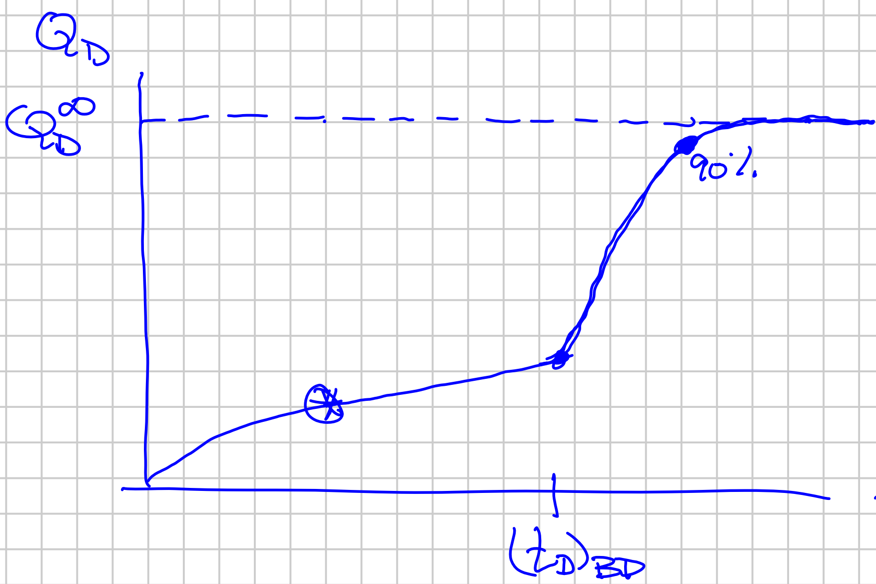
$$= (C_w + C_f) V_{AQ} \left(p_{RHCi} - p_{RHC,k}\right)$$

$$\Rightarrow U = \frac{V_{AQ}(C_w + C_f)}{Q_D^{\infty}}$$

$$\Rightarrow \boxed{W_e = \underbrace{V_{AQ}(C_w + C_f)} \sum_{k=1}^{N} \Delta p_k \underbrace{\left[\frac{Q_D(\Delta t)_k}{Q_D^{\infty}}\right]}_{}}$$

fraction of the total influx achieved in Δt_k for Δp_k

0 — 1



$$t_D = \frac{k \, (1 \text{ year})}{\phi \mu C_t r_{HC}^2}$$

units

C_t: $C_f + C_w$

$$\frac{Q_D}{Q_D^{\infty}} > 90\% \text{ for incremental "}\Delta t\text{" (e.g. 1 year)}$$

$\Rightarrow$ Pot Aq. assumption is <u>valid</u>