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## Consistent Black-Oil PVT Table Modification

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### Abstract

This paper presents a physically consistent approach to modify black-oil PVT tables for (1) eliminating, in some cases, so-called negative compressibilities, (2) extrapolating saturated and undersaturated properties to conditions beyond the limits of original PVT tables, and (3) guaranteeing physical consistency of gas and oil properties as a critical condition is approached.

For a number of reasons a black-oil PVT table may contain inconsistencies that result in non-physical behavior in reservoir simulators, sometimes leading to slow run times and, sometimes, premature run termination. Physical inconsistencies can even arise when the black-oil PVT table is created by a physically-consistent EOS model. Black-oil behavior can also be inconsistent as a result of the method used to create the tables – e.g. correlations, conversion of laboratory data, or using EOS models where gas and oil phases are not in thermodynamic equilibrium.

Three main methods are developed in the paper. First, an analysis of negative compressibilities is given and an approach to eliminate the problem is proposed. Negative compressibilities can arise because derivatives are held constant from one pressure table point to the next, inconsistent with the pressure-dependent evaluation of properties themselves.

The second contribution of our paper is a method to extrapolate an existing black-oil table to higher saturation pressures than found in the original table. Extrapolation is possible to *any* higher pressure – including a critical (“convergence”) pressure where phase properties become identical. The proposed method uses a piecewise-linear log-log relationship between black-oil (surface gas and surface oil) K-values and pressure.

Our final contribution is a consistent method to calculate saturated and undersaturated black-oil PVT properties at interior and extrapolated pressures to those in the original table. A cubic EOS and the LBC viscosity correlation methods

are used to provide physical consistency, even approaching a critical condition where gas and oil properties must become equal.

### Introduction

Sometimes black-oil reservoir simulators take unexpectedly-large CPU time and experience numerical instability due to “problem” PVT data – e.g. physically-inconsistent or ill-behaved input, or “fill-in” data (in this paper, the black-oil PVT data are divided in two groups – input and fill-in data. All PVT data in the input table are termed as *input data* and all calculated (interpolated and extrapolated) PVT data are termed as *fill-in data*. Thus the fill-in data includes both interpolated and extrapolated data). The problem may be more pronounced for near critical fluid systems, but “bad” black-oil PVT data can be found for even the simplest fluid system. In this study, different types of consistency checks of the black-oil PVT data are described.

An existing black-oil PVT table may need to be extrapolated to higher saturation pressures. A method to extrapolate an existing table is described. The extrapolation of the fluid (which is defined by  $R_s$  or  $r_s$ ) is based on K-values of the surface oil and surface gas. In the paper, the fluids are divided in two groups – input fluids and extrapolated fluids. The fluid is termed “extrapolated” fluid if the solution  $R_s$  (or  $r_s$ ) is higher than the maximum solution  $R_s$  (or  $r_s$ ) in the input table.

An EOS based method is used to calculate saturated and undersaturated fill-in data. The fill-in data are calculated both for input and extrapolated fluids. The LBC correlation is used to calculate oil and gas viscosities.

### Consistency Checks

The conditions of physical and “numerical” consistency are defined for black-oil PVT tables. In general, physical consistency guarantees numerical consistency – i.e. reservoir simulation model stability with respect to phase and volumetric calculations. However, for near-critical conditions we have found that model stability is sometimes jeopardized by PVT properties with large pressure derivatives – e.g.  $dR_s/dp$  and  $dr_s/dp$ . The reason for model instability may be the fill-in method, or it may be due to inadequate numerical methods.

Our approach for checking consistency is rigorous numerical checks. The numerical checks include three types –

#### 1. Basic Physical Consistency

- $\rho_g < \rho_o$
- $R_s < 1/r_s$
- $B_o < B_{gd}/r_s$
- $B_{gd} < B_o/R_s$
- $\mu_g < \mu_o$

### 2. Critical-Point Consistency (*Gas PVT = Oil PVT*)

- $\rho_g = \rho_o$
- $R_s = 1/r_s$
- $B_o = B_{gd}/r_s$
- $B_{gd} = B_o/R_s$
- $\mu_g = \mu_o$

### 3. Compressibilities at Saturated Conditions

- $\frac{(B_{gd} - r_s \cdot B_o)}{(1 - r_s \cdot R_s)} \cdot \frac{dR_s}{dp} > \frac{dB_o}{dp}$
- $\frac{(B_o - R_s \cdot B_{gd})}{(1 - r_s \cdot R_s)} \cdot \frac{dr_s}{dp} > \frac{dB_{gd}}{dp}$

### Oil and Gas Compressibilities

The input black-oil PVT table should be checked for oil and gas compressibilities at different pressures.

The saturated oil and gas compressibilities at any pressure are calculated by following relation:

$$c_o = \frac{1}{B_o} \cdot \left( -\frac{dB_o}{dp} + \frac{(B_{gd} - r_s \cdot B_o)}{(1 - r_s \cdot R_s)} \cdot \frac{dR_s}{dp} \right) \dots\dots\dots(1)$$

$$c_g = \frac{1}{B_{gd}} \cdot \left( -\frac{dB_{gd}}{dp} + \frac{(B_o - R_s \cdot B_{gd})}{(1 - r_s \cdot R_s)} \cdot \frac{dr_s}{dp} \right) \dots\dots\dots(2)$$

As an example of the oil and gas compressibility checks, a black-oil PVT table is generated using fluid from a North Sea field (fluid A). The fluid is a near critical fluid with saturation pressure of 400 bara at reservoir temperature of 163 °C. The black-oil PVT properties of the fluid are shown in Fig. 1 through Fig. 4.

For fluid A, the saturated oil compressibility was calculated at each input saturation pressure nodes and is shown in Fig. 5.

The calculated saturated oil compressibility is negative at the highest saturation pressure. This may cause severe problem in the reservoir simulator. One way to remove the negative oil compressibility in the PVT table is to include more pressure (saturated) points near the pressure where negative compressibility was found. In this table, the negative oil compressibility can be removed by including one extra saturation pressure point (the method used to calculate the

PVT properties at the extra point is described later) as shown in Fig. 6.

The oil compressibility is also calculated at pressures between the input saturation pressure nodes (symbol triangle) as shown in Fig. 7. As shown in Fig. 7 the saturated oil compressibility with pressure is discontinuous. The discontinuity in the saturated oil compressibility occurs at pressures slightly higher than the input pressure nodes. The reason for the discontinuity is the change in the derivatives  $dR_s/dp$  and  $dB_o/dp$  at the input pressure nodes. The changes in the derivatives cause discontinuities in the compressibilities even if the black-oil PVT properties are continuous. The magnitude of the discontinuity depends on the difference in derivatives. This discontinuity in compressibility will have large impact on the simulator performance and stability. One way to reduce the discontinuity is to include more saturated pressure points in the input table.

For fluid A, a modified black-oil PVT table was generated with saturated oil properties at every 5 bar. The saturated oil compressibility in the modified black-oil PVT table is shown in Fig. 8 (blue symbol) where saturated oil compressibility is quite smooth.

The saturated gas compressibility is positive at all pressures for this fluid system as shown in Fig. 9. We have not found negative saturated gas compressibility in any fluid system during this work.

### Extrapolation of Solution GOR ( $R_s$ and $r_s$ ) What's the Problem?

An existing black-oil PVT table may need to be extrapolated for a number of reasons:

1. Saturated conditions may be encountered at pressures higher than the highest saturation pressure in a table – e.g. during pressure buildup where two-phase gas-oil flow exists prior to shutin; gas percolation; or gas injection.
2. Saturated conditions may exist initially at pressures higher than the highest saturation pressure in a table – e.g. in a reservoir with compositional grading; or a saturated gas-oil system where only a downstructure low-bubblepoint sample is available and was used to generate the black-oil PVT table.
3. Some simulators require that any gas-oil contact be “saturated”, even if the actual GOC is undersaturated with a critical mixture at the contact.
4. Some simulators require saturated PVT properties to extrapolate *undersaturated* PVT properties – e.g. gas FVF in an undersaturated gas condensate reservoir. Although this method can lead to non-physical extrapolations, it is still used and one needs to provide extrapolated saturated properties for the model to run at all.
5. Some simulators extrapolate saturated PVT properties using sub-optimal methods. These extrapolated values may never be used in a simulation, but they are still made during initialization and internal testing for “negative compressibilities”. With warnings about negative compressibilities during initialization, many engineers become unsure whether their simulation results have been

affected by this “problem” – unknowing that the simulator never experienced negative compressibilities during the run.

**“Optimal” Extrapolation Procedures**

Our approach to extrapolate starts with the use of two-component equilibrium K-value behavior. The K-values of a black-oil PVT model define the partitioning of the two components *surface gas* and *surface oil* between the two reservoir phases *gas* and *oil*. K-values are defined by the solution gas-oil ratio  $R_s$  and solution oil-gas ratio  $r_s$ .

To ensure consistency out to a critical point (or “convergence pressure”), the two-component K-values must be extrapolated to unity ( $K_{\bar{o}} = K_{\bar{g}} = 1$ ) at some pressure. This pressure should be equal to or somewhat greater than the highest reservoir pressure experiencing two-phases during reservoir modeling. The pressure dependence of K-values out to the convergence pressure can be specified, or defaulted to a log K – log p linear extrapolation.

Given the K-value extrapolation in pressure,  $R_s(p)$  and  $r_s(p)$  extrapolations are found directly.

**Steps in Extrapolation Procedure**

With this method, extrapolated fluids are generated. The steps in generation of extrapolated fluids are described below.

The equilibrium K-values of the surface oil and the surface gas at the maximum saturation pressure in the input table are calculated by the following relation (Appendix A):

$$K_{\bar{o}s} = \frac{R_s + C_o}{1/r_s + C_o} \dots\dots\dots(3)$$

$$K_{\bar{g}s} = \frac{1}{R_s r_s} \frac{R_s + C_o}{1/r_s + C_o} \dots\dots\dots(4)$$

where  $K_{\bar{o}s}$  is the equilibrium K-value of the surface oil and  $K_{\bar{g}s}$  is the equilibrium K-value of the surface gas at the maximum saturation pressure.

Convergence pressure is required to extrapolate solution GOR between maximum saturation pressure and convergence pressure. The convergence pressure ( $p_k$ ) can be defined/selected by the engineer or calculated by correlation. We recommend using saturated  $B_o$  versus  $R_s$  relation to find the convergence pressure since the relation is almost straight line. The recommended convergence pressure calculation procedure is given in Appendix B.

In the case of near-critical fluid with undersaturated GOC, the convergence pressure is equal to the maximum saturation pressure. If the reservoir simulator needs saturated properties upto reservoir pressure at the undersaturated GOC then the user defined convergence pressure should be higher or equal to reservoir pressure at the undersaturated GOC.

At any pressure between maximum saturation pressure and convergence pressure, the equilibrium constant for surface oil is calculated by linear interpolation of  $\log(K_{\bar{o}})$  versus  $\log(p)$  and similarly for surface gas.

$$K_{\bar{o}} = K_{\bar{o}s} \left( \frac{p}{p_s} \right)^{m\bar{o}} \dots\dots\dots(5)$$

$$m\bar{o} = \frac{\log(K_{\bar{o}s})}{\log(p_s / p_k)} \dots\dots\dots(6)$$

$$K_{\bar{g}} = K_{\bar{g}s} \left( \frac{p}{p_s} \right)^{m\bar{g}} \dots\dots\dots(7)$$

$$m\bar{g} = \frac{\log(K_{\bar{g}s})}{\log(p_s / p_k)} \dots\dots\dots(8)$$

$p_s$  saturation pressure

$p_k$  convergence pressure

$K_{\bar{o}}$  is equilibrium constant of surface oil and  $K_{\bar{g}}$  is equilibrium constant of surface gas at desired pressure p.

Then solution  $R_s$  and  $r_s$  at pressure p are calculated using the following relations:

$$R_s = C_o \frac{1 - K_{\bar{o}}}{K_{\bar{g}} - 1} \dots\dots\dots(9)$$

$$r_s = \frac{K_{\bar{o}} (K_{\bar{g}} - 1)}{C_o K_{\bar{g}} (1 - K_{\bar{o}})} \dots\dots\dots(10)$$

The solution GOR of fluid A was extrapolated using above method. The input (closed circle) and extrapolated (open circle) solution GOR of oil is shown in Fig. 10 and Fig. 11. In this case, the calculated convergence pressure was about 420 bara.

The extrapolation was also done after changing convergence pressure to 600 bara (user defined) and is shown in Fig. 12. In this case, the oil solution GOR versus FVF trend line in the extrapolated region is different than in the input table trend line (Fig. 13). This is because of a higher saturation pressure at the same solution GOR in the higher convergence pressure case. As shown in the figure, the extrapolated solution GOR versus pressure trend depends on the convergence pressure. But the solution GOR at the convergence pressure is independent of the convergence pressure. The solution GOR of the gas is also extrapolated and is shown in Fig. 14.

In the above method of extrapolation of solution GOR (K-value extrapolation), the extrapolated solution GOR versus pressure is almost a straight line. The  $dR_s/dp$  (and  $dr_s/dp$ ) slope in the extrapolated region depends on the convergence pressure i.e. the  $dR_s/dp$  decreases with increasing convergence pressure. The  $dR_s/dp$  slope at the highest saturation pressure in the input table is fixed. If the selected convergence pressure is high then there can be large difference in  $dR_s/dp$  slope at the highest saturation pressure (input table range versus

extrapolated range) which may cause convergence problems in the reservoir simulator.

The recommended procedure for convergence pressure ensures that the  $dR_s/dp$  slope changes smoothly from highest saturation pressure to extrapolated pressure range. But it might not always be possible if the selected convergence pressure is very high. In those cases, an alternative method (DGOR) for extrapolation of solution GOR can be used. In this method, the solution GOR with pressure is extrapolated in such a way that the  $dR_s/dp$  (and similarly  $dr_s/dp$ ) slope changes gradually (increased or decreased). The extrapolated solution GOR using both methods (K-value extrapolation and DGOR extrapolation) is shown in Fig. 15. The solution GOR at the convergence pressure is same in both methods but the solution GOR versus pressure trend is different. In the DGOR method, the solution GOR increases smoothly from the maximum input solution GOR to extrapolated GOR. It should be noted that physically  $dR_s/dp$  (and similarly  $dr_s/dp$ ) almost always increases with pressure. The main reason for reducing  $dR_s/dp$  is to “create” an artificially high convergence pressure due to reservoir simulator limitations or to reduce the CPU-time.

### Fill-in of Volumetric Properties ( $B_o$ and $B_{gd}$ )

In this study, optimal methods for calculating saturated and undersaturated PVT properties have been developed.

We use *physical* pressure-variation of black-oil PVT properties in this work to define “optimal” methods of fill-in of the saturated and undersaturated data. Using the recommended method, one can “fill in” a black-oil PVT table to minimize the impact of sub-optimal schemes used by different simulators.

Another issue regarding fill-in is that different simulators use different fill-in schemes. If simulations have been conducted with one simulator, and now the engineer wants to convert the model to another simulator, it is advantageous to minimize the difference in simulation results caused by differences in fill-in methods when the black oil PVT properties are calculated. The recommended method will allow filling-in data before running the simulators, thereby minimizing differences in simulation results due to differences in PVT treatment.

### Fill-in Procedure

First, we propose the fill-in of saturated and undersaturated volumetric properties –  $B_o$  and  $B_{gd}$ . This fill-in must also honor constraints on consistent reservoir phase densities ( $\rho_o$  and  $\rho_g$ ).

### Steps in EOS Calculations

All input FVFs are converted to molar volumes (via the density) and fit to the Peng-Robinson (PR79) cubic equation of state model assuming two surface components (surface oil and gas).

The conversion of oil FVF to molar volumes:

For reservoir oil at pressure  $p$ , solution gas-oil ratio  $R_s$ , and oil FVF  $B_o$

Reservoir oil density

$$\rho_o = \frac{\rho_{osc} + \rho_{gsc} R_s}{B_o} \dots\dots\dots (11)$$

Mole fraction of surface oil in reservoir oil<sup>1</sup> (Appendix A)

$$x_o = \frac{C_o}{C_o + R_s} \dots\dots\dots (12)$$

Mole fraction of surface gas in reservoir oil

$$x_g = \frac{R_s}{C_o + R_s} \dots\dots\dots (13)$$

Where  $C_o$  is calculated by the relation

$$C_o = \frac{\rho_o}{M_o} \frac{RT_{sc}}{p_{sc}} \dots\dots\dots (14)$$

Molecular weight of the surface oil ( $M_o$ ):

Different correlations can be used to calculate the molecular weight of the surface oil based on the input surface oil density (calculated molecular weight of surface oil from different correlation<sup>2-5</sup> is shown in Fig. 16). The Standing correlation is used in this study to calculate molecular weight of the surface oil.

$$M_o = 240 - 2.22 \gamma_{API} \dots\dots\dots (15)$$

Molecular weight of the surface gas

$$M_g = \gamma_{gsc} \cdot M_{air} \dots\dots\dots (16)$$

Molecular weight of the reservoir oil

$$M_o = M_o \cdot x_o + M_g \cdot x_g \dots\dots\dots (17)$$

Molar volume of the reservoir oil

$$v_o = M_o / \rho_o \dots\dots\dots (18)$$

Reservoir gas at pressure  $p$ , solution oil-gas ratio  $r_s$ , and gas FVF  $B_{gd}$ .

Reservoir gas densities

$$\rho_g = \frac{\rho_{gsc} + \rho_{osc} r_s}{B_{gd}} \dots\dots\dots (19)$$

Mole fraction of surface oil in reservoir gas

$$y_o = \frac{C_o}{1/r_s + C_o} \dots\dots\dots (20)$$

Mole fraction of surface gas in reservoir gas

$$y_g = \frac{1/r_s}{1/r_s + C_o} \dots\dots\dots (21)$$

Molecular weight of the reservoir gas

$$M_g = M_o \cdot y_o + M_g \cdot y_g \dots\dots\dots (22)$$

Where molecular weight of the surface oil is calculated by eq. 15 and molecular weight of the surface gas by eq. 16.

Molar volume of the reservoir gas

$$v_g = M_g / \rho_g \dots\dots\dots(23)$$

Calculation of the EOS parameters for PR79.

Peng-Robinson equation-of-state<sup>6-7</sup>:

$$p = \frac{RT}{v-b} - \frac{a}{v \cdot (v+b) + b \cdot (v-b)} \dots\dots\dots(24)$$

Unknowns or regression parameters:

$$a_{\bar{o}} \quad b_{\bar{o}} \quad s_{\bar{o}} \quad a_{\bar{g}} \quad b_{\bar{g}} \quad s_{\bar{g}} \dots\dots\dots(25)$$

In the black-oil model, the PVT system consists of two reservoir phases (oil and gas) and two surface components (surface oil and surface gas). EOS constants for the two components, surface oil ( $a_{\bar{o}} \quad b_{\bar{o}} \quad s_{\bar{o}}$ ) and surface gas ( $a_{\bar{g}} \quad b_{\bar{g}} \quad s_{\bar{g}}$ ), are obtained after regression to match the input oil and gas FVF.

Molar volume and EOS constants for oil (and similarly for gas):

$$v = v_o + (sb)_o = v_o + x_{\bar{o}} s_{\bar{o}} b_{\bar{o}} + x_{\bar{g}} s_{\bar{g}} b_{\bar{g}} \dots\dots\dots(26)$$

$$a = a_{\bar{o}} x_{\bar{o}}^2 + a_{\bar{g}} x_{\bar{g}}^2 + 2(a_{\bar{o}} a_{\bar{g}})^{0.5} x_{\bar{o}} x_{\bar{g}} \dots\dots\dots(27)$$

$$b = b_{\bar{o}} x_{\bar{o}} + b_{\bar{g}} x_{\bar{g}} \dots\dots\dots(28)$$

Starting values of EOS regression parameters:

For surface oil:

$$a_{\bar{o}} = \Omega_a \frac{R^2 T_c^2}{p_c} \alpha \dots\dots\dots(29)$$

$$b_{\bar{o}} = \Omega_b \frac{RT_c}{p_c} \dots\dots\dots(30)$$

$$s_{\bar{o}} = 0 \dots\dots\dots(31)$$

Surface oil critical properties from Matthews correlation<sup>8</sup>

$$T_c = 608 + 364 \log(M_{\bar{o}} - 71.2) + (2450 \log M_{\bar{o}} - 3800) \log \gamma_o \dots\dots\dots(32)$$

$$p_c = 1188 - 431 \log(M_{\bar{o}} - 61.1) + [2319 - 852 \log(M_{\bar{o}} - 53.7)](\gamma_o - 0.8) \dots\dots\dots(33)$$

$$\alpha = [1 + m(1 - \sqrt{T_r})]^2 \dots\dots\dots(34)$$

If  $\omega \leq 0.49$  then

$$m = 0.37464 + 1.54226\omega - 0.26992\omega^2 \dots\dots\dots(35)$$

If  $\omega > 0.49$  then

$$m = 0.3796 + 1.485\omega - 0.1644\omega^2 + 0.01667\omega^3 \dots\dots\dots(36)$$

$$\omega = 0.000003 M_{\bar{o}}^2 + 0.004 M_{\bar{o}} - 0.039 \dots\dots\dots(37)$$

For surface gas:

$$a_{\bar{g}} = \Omega_a \frac{R^2 T_c^2}{p_c} \alpha \dots\dots\dots(38)$$

$$b_{\bar{g}} = \Omega_b \frac{RT_c}{p_c} \dots\dots\dots(39)$$

$$s_{\bar{g}} = 0 \dots\dots\dots(40)$$

Surface gas critical properties from Standing<sup>9</sup> correlation:

If  $y_g < 0.75$

$$T_c = 168 + 325 \gamma_g - 12.5 \gamma_g^2 \dots\dots\dots(41)$$

$$p_c = 667 + 15.0 \gamma_g - 37.5 \gamma_g^2 \dots\dots\dots(42)$$

If  $y_g \geq 0.75$

$$T_c = 187 + 330 \gamma_g - 71.5 \gamma_g^2 \dots\dots\dots(43)$$

$$p_c = 706 + 51.7 \gamma_g - 11.1 \gamma_g^2 \dots\dots\dots(44)$$

$$\alpha = [1 + m(1 - \sqrt{T_r})]^2 \dots\dots\dots(45)$$

If  $\omega \leq 0.49$  then

$$m = 0.37464 + 1.54226\omega - 0.26992\omega^2 \dots\dots\dots(46)$$

If  $\omega > 0.49$  then

$$m = 0.3796 + 1.485\omega - 0.1644\omega^2 + 0.01667\omega^3 \dots\dots\dots(47)$$

$$\omega = 0.1637 \gamma_g - 0.0792 \dots\dots\dots(48)$$

Best fit EOS constants a, b, and s for each component

The objective function is:

$$p - \left( \frac{RT}{v-b} - \frac{a}{v \cdot (v+b) + b \cdot (v-b)} \right) = 0 \dots\dots\dots(49)$$

Regress on  $a_{\bar{o}} \quad b_{\bar{o}} \quad s_{\bar{o}} \quad a_{\bar{g}} \quad b_{\bar{g}} \quad s_{\bar{g}}$  to satisfy above equation. All input densities are used in the regression.

The regressed parameters  $a_{\bar{o}}$   $b_{\bar{o}}$   $a_{\bar{g}}$   $b_{\bar{g}}$  are used in all EOS calculations of volumetric properties.

$(sb)_o$  ( $=x_{\bar{o}} s_{\bar{o},ps \max} b_{\bar{o}} + x_{\bar{g}} s_{\bar{g},ps \max} b_{\bar{g}}$ ) and  $(sb)_g$  are re-calculated for each input table node to match exactly the input FVF. These  $(sb)_o$  and  $(sb)_g$  values are used in FVF fill-in of the input fluids.

Calculate  $s_{\bar{o},ps \max}$  and  $s_{\bar{g},ps \max}$  from the highest saturated oil FVF and highest saturated gas FVF using EOS (two unknowns and two equations).  $s_{\bar{o},ps \max}$  and  $s_{\bar{g},ps \max}$  are used in FVF fill-in of the extrapolated fluids.

### Fill-in of Oil and Gas FVF

For an oil with given composition ( $R_s$ ) at any pressure  $p$ :

(a) Calculate EOS constants “a” and “b” from surface oil and surface gas compositions

$$x_{\bar{o}} = \frac{C_o}{C_o + R_s} \dots\dots\dots(50)$$

$$x_{\bar{g}} = \frac{R_s}{C_o + R_s} \dots\dots\dots(51)$$

$$a = a_{\bar{o}} x_{\bar{o}}^2 + a_{\bar{g}} x_{\bar{g}}^2 + 2(a_{\bar{o}} a_{\bar{g}})^{0.5} x_{\bar{o}} x_{\bar{g}} \dots\dots\dots(52)$$

$$b = b_{\bar{o}} x_{\bar{o}} + b_{\bar{g}} x_{\bar{g}} \dots\dots\dots(53)$$

(b) Calculate molar volume from the EOS model: Solve the cubic equation of state to find Z.

$$Z^3 - (1-B)Z^2 + (A-3B^2-2B)Z - (AB-B^2-B^3) = 0 \dots\dots\dots(54)$$

$$A = a \frac{p}{(RT)^2} \dots\dots\dots(55)$$

$$B = b \frac{p}{RT} \dots\dots\dots(56)$$

(b) Calculate EOS molar volume

$$v = \frac{ZRT}{p} \dots\dots\dots(57)$$

(b) Calculate actual/modified molar volume from EOS calculated molar volume

$$v_o = v - (sb)_o \dots\dots\dots(58)$$

The value of  $C_o$  is obtained as given below:

- Extrapolated fluids: If the solution  $R_s$  is higher than the maximum solution  $R_s$  in the input table then  $(sb)_o$  is calculated using  $s_{\bar{o},ps \max}$  and  $s_{\bar{g},ps \max}$ .

$$(sb)_o = x_{\bar{o}} s_{\bar{o},ps \max} b_{\bar{o}} + x_{\bar{g}} s_{\bar{g},ps \max} b_{\bar{g}}$$

- Input fluids: if the solution  $R_s$  is equal or lower than the maximum solution  $R_s$  in the input table and pressure is lower than the maximum pressure in the table then  $(sb)_o$  is calculated by linear interpolation of input node  $(sb)_o$  values.
- Input fluids: if the solution  $R_s$  is equal or lower than the maximum solution  $R_s$  in the input table and pressure is higher than the maximum pressure in the table then  $(sb)_o$  is calculated by linear extrapolation of input node  $(sb)_o$  values.

(c) Calculate molecular weight of reservoir oil using equation 17.

(d) Calculate density of oil

$$\rho_o = \frac{M_o}{v_o} \dots\dots\dots(59)$$

(e) Calculate FVF of oil

$$B_o = \frac{\rho_{osc} + \rho_{gsc} R_s}{\rho_o} \dots\dots\dots(60)$$

The above procedure is also used for fill-in of gas FVF. The filled-in oil and gas FVFs are shown in Fig. 17 and Fig. 18.

### Fill-in of Oil and Gas Viscosities

Finally, viscosity fill-in is made for saturated and undersaturated conditions. The close and fundamental relation between viscosity and density are used to ensure physically meaningful viscosity extrapolations, while guaranteeing critical-point consistency.

### LBC Viscosity Correlation<sup>10</sup>

$$\left[ (\mu - \mu^o) \xi_T + 10^{-4} \right]^{1/4} = A_0 + A_1 \rho_{pr} + A_2 \rho_{pr}^2 + A_3 \rho_{pr}^3 + A_4 \rho_{pr}^4 \dots\dots(61)$$

The default values of different LBC coefficients are:

$A_0=0.10230$ ,  $A_1=0.023364$ ,  $A_2=0.058533$ ,  $A_3=-0.040758$ ,  $A_4=0.0093324$ .

Regression parameters:  $\rho_{c\bar{o}}$ ,  $\rho_{c\bar{g}}$ ,  $\xi_{T\bar{o}}$ ,  $\xi_{T\bar{g}}$ ,  $\mu_{\bar{o}}^o$ ,  $\mu_{\bar{g}}^o$

### Steps in Viscosity Calculation:

The pseudo-reduced density is calculated as:

$$\rho_{pr} = \frac{\rho}{\rho_{pc}} \dots\dots\dots(62)$$

$$\rho_{pc} = \rho_{c\bar{o}} x_{\bar{o}} + \rho_{c\bar{g}} x_{\bar{g}} \dots\dots\dots(63)$$

$$\xi_T = \xi_{T\bar{o}} x_{\bar{o}} + \xi_{T\bar{g}} x_{\bar{g}} \dots\dots\dots(64)$$

$$\mu^o = \mu_{\bar{o}}^o x_{\bar{o}} + \mu_{\bar{g}}^o x_{\bar{g}} \dots\dots\dots(65)$$

**Starting values of LBC regression parameters:**

Lohrenz et al. correlation<sup>10</sup>

$$V_{c\bar{o}} = 21.573 + 0.015122M_{\bar{o}} - 27.656\gamma_{\bar{o}} + 0.070615M_{\bar{o}}\gamma_{\bar{o}} \dots\dots\dots(66)$$

$$\rho_{c\bar{o}} = \frac{M_{\bar{o}}}{V_{c\bar{o}}} \dots\dots\dots(67)$$

$$V_{c\bar{g}} = \frac{Z_c R T_{pc}}{P_{pc}} \dots\dots\dots(68)$$

$$\rho_{c\bar{g}} = \frac{M_{\bar{g}}}{V_{c\bar{g}}} \dots\dots\dots(69)$$

For Surface oil component:

$$\xi_{T\bar{o}} = 5.35(T_{c\bar{o}} M_{\bar{o}}^3 / P_{c\bar{o}}^4)^{1/6} \dots\dots\dots(70)$$

For  $T_r \leq 1.5$

$$\mu_{\bar{o}}^o = \frac{34 \cdot 10^5 T_{r\bar{o}}^{0.94}}{\xi_{T\bar{o}}} \dots\dots\dots(71)$$

For  $T_r > 1.5$

$$\mu_{\bar{o}}^o = \frac{17.78 \cdot 10^5 (4.58 T_{r\bar{o}} - 1.67)^{5/8}}{\xi_{T\bar{o}}} \dots\dots\dots(72)$$

For Surface gas component:

$$\xi_{T\bar{g}} = 5.35(T_{c\bar{g}} M_{\bar{g}}^3 / P_{c\bar{g}}^4)^{1/6} \dots\dots\dots(73)$$

For  $T_r \leq 1.5$

$$\mu_{\bar{g}}^o = \frac{34 \cdot 10^5 T_{r\bar{g}}^{0.94}}{\xi_{T\bar{g}}} \dots\dots\dots(74)$$

For  $T_r > 1.5$

$$\mu_{\bar{g}}^o = \frac{17.78 \cdot 10^5 (4.58 T_{r\bar{g}} - 1.67)^{5/8}}{\xi_{T\bar{g}}} \dots\dots\dots(75)$$

**Best fit of LBC regression parameters:**

All input oil and gas viscosities are used in regression to find the best fit regression parameters  $\rho_{c\bar{o}}, \rho_{c\bar{g}}, \xi_{T\bar{o}}, \xi_{T\bar{g}}, \mu_{\bar{o}}^o, \mu_{\bar{g}}^o$ .

Regressed parameters  $\xi_{T\bar{o}}, \xi_{T\bar{g}}, \mu_{\bar{o}}^o, \mu_{\bar{g}}^o$  are used in the LBC correlation for all the viscosity calculations.

$\rho_{pc}$  is re-calculated from the LBC correlation for each input node to exactly match the input viscosities. These  $\rho_{pc}$  values are used in viscosity fill-in of the input fluids.

$\rho_{c\bar{o}, ps \max}$  and  $\rho_{c\bar{g}, ps \max}$  are calculated for the oil and gas with the highest saturation pressure from the LBC correlation (two unknowns and two equations).

$\rho_{c\bar{o}, ps \max}$  and  $\rho_{c\bar{g}, ps \max}$  are used in calculation of the viscosities of the extrapolated fluids.

**Fill-in of Oil and Gas Viscosity**

The oil and gas viscosities are calculated in the same way as the volumetric properties. The oil and gas viscosities at any pressure are calculated using the LBC correlation. Regressed parameters  $\xi_{T\bar{o}}, \xi_{T\bar{g}}, \mu_{\bar{o}}^o, \mu_{\bar{g}}^o$  are used in all viscosity calculations.

**Oil viscosity at pressure p and solution gas-oil ratio  $R_s$ :**

The composition of the surface oil and surface gas are calculated from solution  $R_s$ , using eqs. 12 and 13.  $\xi_T, \mu^o$  are found by eqs. 64 and 65 respectively. The  $\rho_{pc}$  at the required pressure is obtained from node  $\rho_{pc}$  for input fluids and from  $\rho_{c\bar{o}, ps \max}$  and  $\rho_{c\bar{g}, ps \max}$  for extrapolated fluids as in the volumetric calculation. Finally oil viscosity at the given pressure is calculated using LBC correlation (Eq. 61).

The gas viscosity at any pressure is calculated in the same way as oil viscosity. The filled-in oil and gas viscosities are shown in Fig. 19 and Fig. 20.

**Example of Recommended Procedures**

*In this example, CPU time was compared for the cases with and without negative saturated oil compressibility in the black-oil PVT table (fluid A).*

A synthetic reservoir simulation model was taken from SPE paper 63087<sup>11</sup>. The synthetic simulation model has 50x10x10 grid cells. The model contains fluid A throughout the reservoir i.e. no compositional variation. The saturation pressure of the fluid is 400 bara and the initial solution  $R_s$  is 512 Sm<sup>3</sup>/Sm<sup>3</sup>. The initial reservoir pressure is 493 bara. The reservoir is produced through one well on maximum withdrawal constraint (about 10% hydrocarbon pore volume per year) with a minimum well bottom hole pressure of 100 bara. The reservoir was simulated under depletion drive and the CPU time was monitored.

As mentioned earlier, negative saturated oil compressibility was found at the highest saturation pressure in the black-oil PVT table of fluid A. When the reservoir was simulated using a commercial reservoir simulator, it was not possible to run the model. The simulator had convergence problem after 230 days of simulation when the reservoir pressure reached 400 bara (saturation pressure of the fluid) as shown in Fig. 21. The pressure in the simulator did not

converge but started to oscillate when the reservoir pressure was close to 400 bara.

The black-oil PVT table was modified to remove the negative oil compressibility by adding more saturated points near the maximum saturation pressure using the recommended fill-in method. For the modified black-oil PVT table case, the simulated performance is shown in Fig. 22. The simulator runs smoothly after removing the negative saturated oil compressibility in the black-oil PVT table. (The production performance was simulated for 10 years with a CPU time of 90 seconds).

### Conclusions

1. A new method has been developed to extrapolate an existing black-oil table to higher saturation pressures than found in the original table. Extrapolation is possible to any higher pressure – including a critical (“convergence”) pressure where phase properties become identical.
2. A consistent method has been developed to calculate saturated and undersaturated black-oil PVT properties at interior and extrapolated pressures to those in the original table. A cubic EOS and the LBC viscosity correlation methods are used to provide physical consistency, even approaching a critical condition where gas and oil properties must become equal.
3. An analysis of negative compressibilities is given and an approach to eliminate the problem is proposed.

### Nomenclature

$a$	= EOS parameter $a$
$b$	= EOS parameter $b$
$B_{gd}$	= dry-gas FVF, $m^3/Sm^3$
$B_o$	= oil FVF, $m^3/Sm^3$
$c_g$	= gas isothermal compressibility, 1/bar
$c_o$	= oil isothermal compressibility, 1/bar
$C_o$	= conversion from STO to equivalent surface gas
GOR	= gas-oil ratio, $Sm^3/Sm^3$
$K$	= equilibrium K-value
$M$	= molecular weight
OGR	= oil-gas ratio, $Sm^3/Sm^3$
$p$	= pressure, bara
$p_b$	= bubblepoint pressure, bara
$p_c$	= critical pressure, bara
$p_d$	= dewpoint pressure, bara
$p_k$	= convergence pressure, bara
$p_R$	= reservoir pressure, bara
$p_s$	= saturation pressure, bara
$p_{sc}$	= standard pressure, bara
$R$	= universal gas constant
$R_s$	= solution gas-oil ratio, $Sm^3/Sm^3$
$r_s$	= solution oil-gas ratio, $Sm^3/Sm^3$
$s_{\bar{g}}$	= EOS parameter volume shift for surface gas
$s_{\bar{o}}$	= EOS parameter volume shift for surface oil
$(sb)_g$	= EOS parameter volume shift for reservoir gas
$(sb)_o$	= EOS parameter volume shift for reservoir oil
$T$	= reservoir temperature
$T_c$	= critical temperature

$T_r$	= reduced temperature, $T/T_c$
$T_{sc}$	= standard temperature
$v$	= molar volume (EOS calculated)
$x_o$	= mole fraction of surface oil from reservoir oil
$x_g$	= mole fraction of surface gas from reservoir oil
$y_o$	= mole fraction of surface oil from reservoir gas
$y_g$	= mole fraction of surface gas from reservoir gas
$\rho$	= mass density, $kg/m^3$
$\rho_c$	= critical density, $kg/m^3$
$\rho_r$	= reduced density
$\mu$	= viscosity, cp
$\xi_T$	= LBC gas viscosity correlation parameter, $cp^{-1}$
$\gamma_g$	= specific gravity of surface gas
$\gamma_o$	= specific gravity of surface oil
$\gamma_{API}$	= API density of surface oil
Subscript	
$\bar{o}$	= surface oil
$\bar{g}$	= surface gas
$o$	= reservoir oil
$g$	= reservoir gas

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## Appendix

### Appendix A - Equilibrium K-values for Surface Oil and Surface Gas.

For reservoir oil, assuming one Sm<sup>3</sup> of oil and R<sub>s</sub> Sm<sup>3</sup> of gas at surface conditions.

$$n_{\bar{o}o} = \frac{\rho_{\bar{o}o}}{M_{\bar{o}o}}, n_{\bar{g}o} = R_s \frac{P_{sc}}{RT_{sc}}$$

$$n_{\bar{o}o} + n_{\bar{g}o} = \frac{P_{sc}}{RT_{sc}} (C_{oo} + R_s)$$

$$C_{oo} = \frac{\rho_{\bar{o}o}}{M_{\bar{o}o}} \frac{RT_{sc}}{P_{sc}}$$

For reservoir gas, assuming one Sm<sup>3</sup> of gas and r<sub>s</sub> Sm<sup>3</sup> of oil at the surface condition

$$n_{\bar{g}g} = \frac{P_{sc}}{RT_{sc}}, n_{\bar{o}g} = r_s \frac{\rho_{\bar{o}g}}{M_{\bar{o}g}}$$

$$n_{\bar{o}g} + n_{\bar{g}g} = \frac{P_{sc}}{RT_{sc}} (1 + r_s C_{og})$$

$$C_{og} = \frac{\rho_{\bar{o}g}}{M_{\bar{o}g}} \frac{RT_{sc}}{P_{sc}}$$

Assuming  $\rho_{\bar{o}o} = \rho_{\bar{o}g} = \rho_{osc}$  and  $M_{\bar{o}o} = M_{\bar{o}g} = M_{osc}$  so

$$C_{oo} = C_{og} = C_o$$

$$x_{\bar{o}} = \frac{n_{\bar{o}o}}{n_{\bar{o}o} + n_{\bar{g}o}} = \frac{C_o}{R_s + C_o}$$

$$x_{\bar{g}} = \frac{n_{\bar{g}o}}{n_{\bar{o}o} + n_{\bar{g}o}} = \frac{R_s}{R_s + C_o}$$

$$y_{\bar{o}} = \frac{n_{\bar{o}g}}{n_{\bar{o}g} + n_{\bar{g}g}} = \frac{r_s C_o}{1 + r_s C_o} = \frac{C_o}{1/r_s + C_o}$$

$$y_{\bar{g}} = \frac{n_{\bar{g}g}}{n_{\bar{o}g} + n_{\bar{g}g}} = \frac{1}{1 + r_s C_o} = \frac{1/r_s}{1/r_s + C_o}$$

$$K_{\bar{o}} = \frac{y_{\bar{o}}}{x_{\bar{o}}} = r_s \frac{R_s + C_o}{1 + r_s C_o} = \frac{R_s + C_o}{1/r_s + C_o}$$

$$K_{\bar{g}} = \frac{y_{\bar{g}}}{x_{\bar{g}}} = \frac{1}{R_s} \frac{R_s + C_o}{1 + r_s C_o} = \frac{1}{R_s r_s} \frac{R_s + C_o}{1/r_s + C_o}$$

### Appendix B – Convergence Pressure Calculation

The solution GOR at convergence pressure (p<sub>k</sub>) is calculated using following relation:

$$R_{s,pk} = -C_o \frac{\ln K_{\bar{o}s}}{\ln K_{\bar{g}s}}$$

$K_{\bar{o}s}$  and  $K_{\bar{g}s}$  are calculated from equations 3 and 4. The above relation is applicable between the maximum saturation pressure in the input table and convergence pressure with log-log relationship between K-values and pressure.

The saturated B<sub>o,pk</sub> at R<sub>s,pk</sub> is calculated by linear extrapolation of input table B<sub>o</sub> versus R<sub>s</sub> correlation.

$$\rho_o = \frac{\rho_{osc} + \rho_{gsc} R_{s,pk}}{B_{o,pk}}$$

$$x_{\bar{o}} = \frac{C_o}{C_o + R_{s,pk}}$$

$$x_{\bar{g}} = \frac{R_{s,pk}}{C_o + R_{s,pk}}$$

$$M_{\bar{g}} = \gamma_{gsc} \cdot M_{air}$$

$$M_o = M_{\bar{o}} \cdot x_{\bar{o}} + M_{\bar{g}} \cdot x_{\bar{g}}$$

$$v_o = M_o / \rho_o$$

$$a = a_{\bar{o}} x_{\bar{o}}^2 + a_{\bar{g}} x_{\bar{g}}^2 + 2(a_{\bar{o}} a_{\bar{g}})^{0.5} x_{\bar{o}} x_{\bar{g}}$$

$$b = b_{\bar{o}} x_{\bar{o}} + b_{\bar{g}} x_{\bar{g}}$$

$$v = v_o + (x_{\bar{o}} s_{\bar{o},ps \max} b_{\bar{o}} + x_{\bar{g}} s_{\bar{g},ps \max} b_{\bar{g}})$$

$$p_k = \frac{RT}{v-b} - \frac{a}{v \cdot (v+b) + b \cdot (v-b)}$$

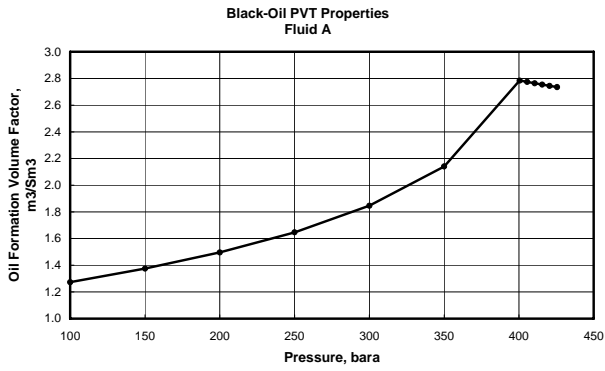


Fig. 1 — Oil formation volume factor for fluid A.

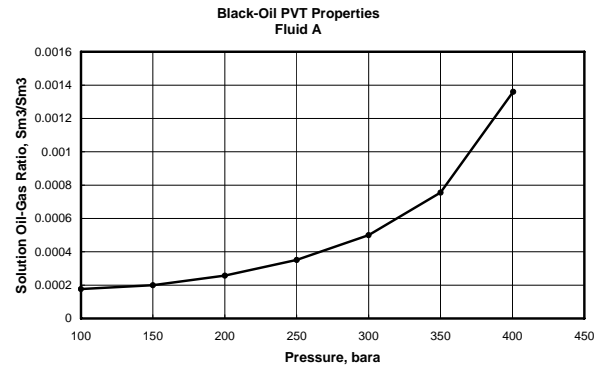


Fig. 4 — Solution oil-gas ratio for fluid A.

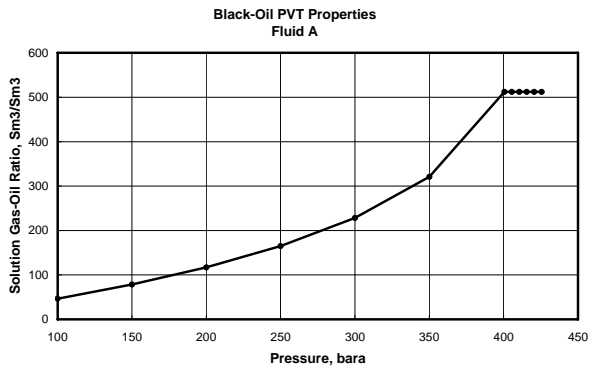


Fig. 2 — Solution gas-oil ratio for fluid A.

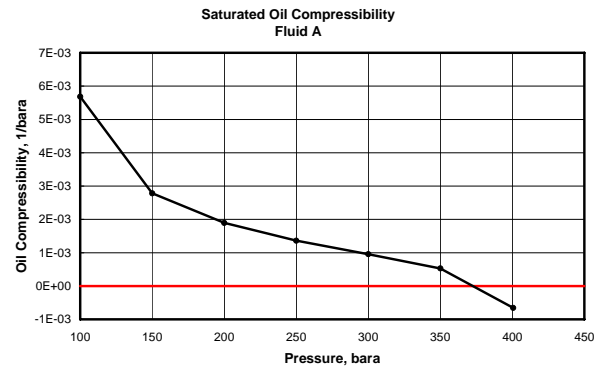


Fig. 5 — Oil compressibility at input pressure nodes in PVT table (fluid A).

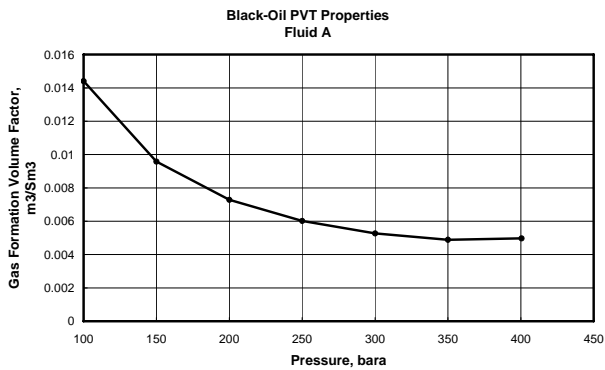


Fig. 3 — Gas formation volume factor for fluid A.

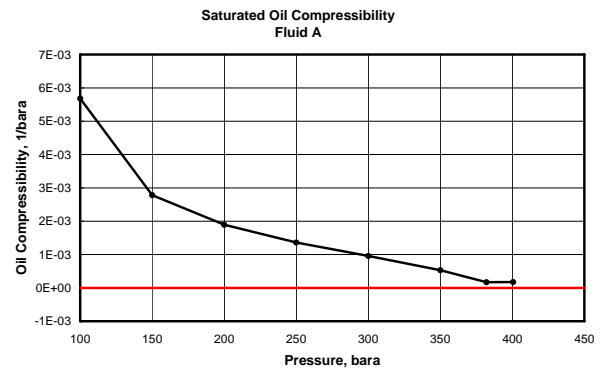


Fig. 6 — Oil compressibility at input pressure nodes in PVT table and at one extra pressure added to remove negative compressibility (fluid A).

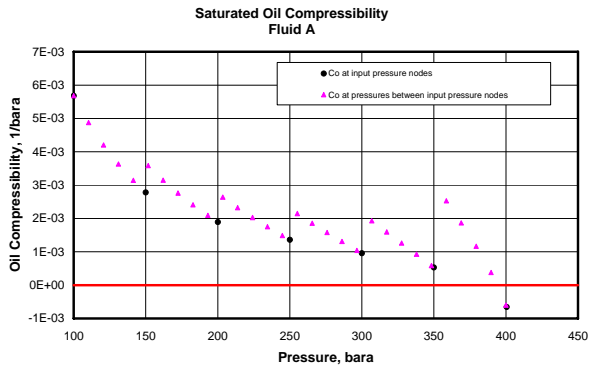


Fig. 7 — Oil compressibility at (a) input pressure nodes and (b) pressures between the input pressure nodes in PVT table (fluid A).

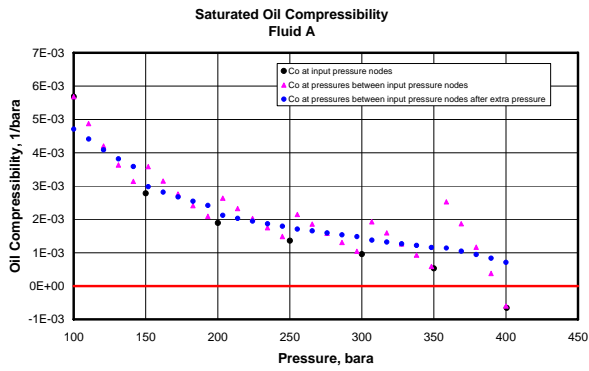


Fig. 8 — Oil compressibility at (a) input pressure nodes (b) pressures points between the input pressure nodes in PVT table and (c) after fill-in table every 5 bara pressure interval (fluid A).

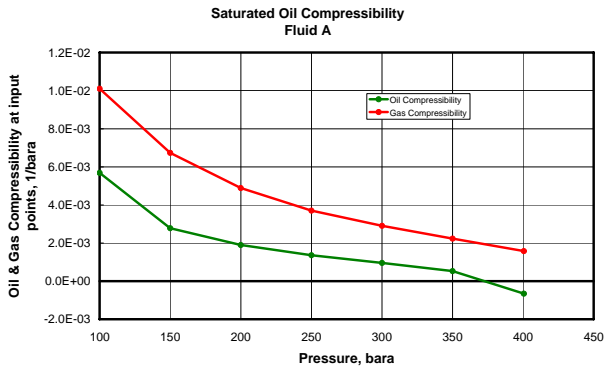


Fig. 9 — Oil and gas compressibilities at input pressure nodes (fluid A).

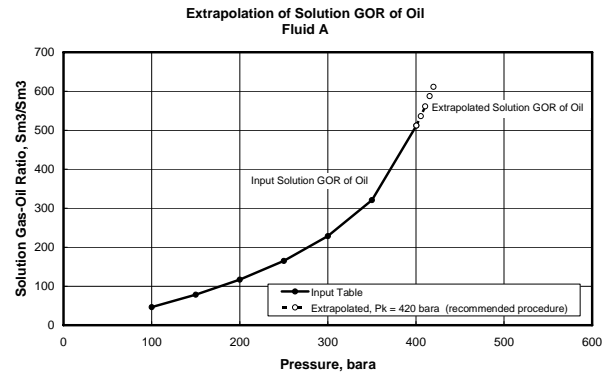


Fig. 10 — Extrapolation of solution GOR of oil at the convergence pressure using recommended procedure.

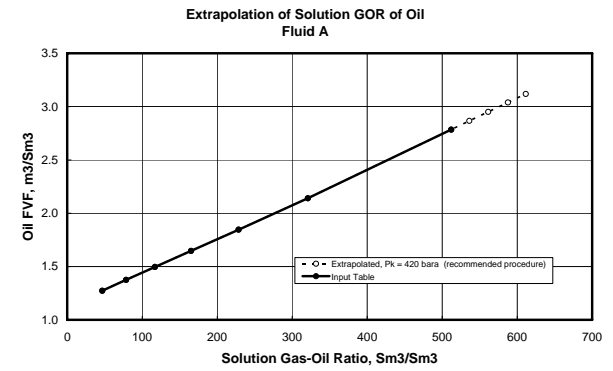


Fig. 11 — Extrapolated solution GOR of oil versus solution FVF of oil. The solution GOR of oil was extrapolated using recommended procedure.

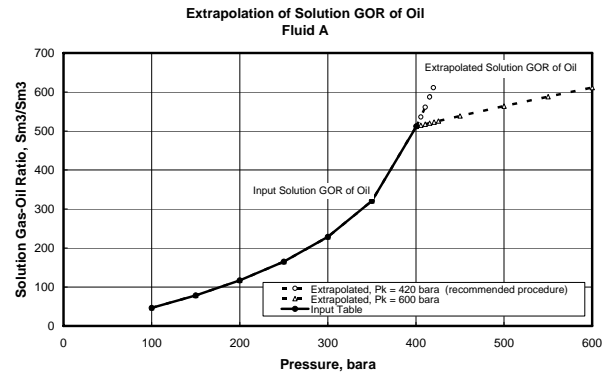


Fig. 12 — Extrapolation of solution GOR oil oil with different convergence pressures (a) 420 bara, recommended procedure and (b) 600 bara.

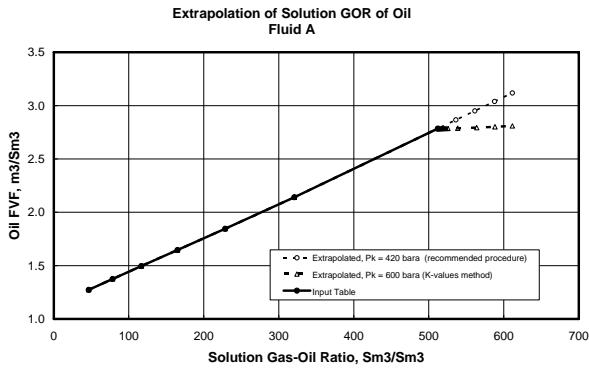


Fig. 13 — Extrapolated solution GOR versus FVF of oil with different convergence pressures (a) 420 bara, recommended procedure and (b) 600 bara.

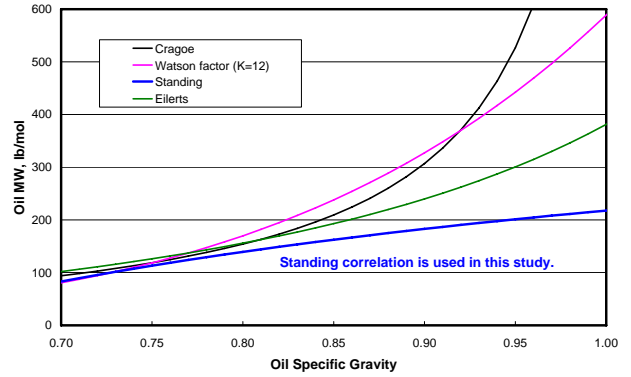


Fig. 16 — Molecular weight of the surface oil based on surface oil density from different correlations. The Standing correlation is used in this study.

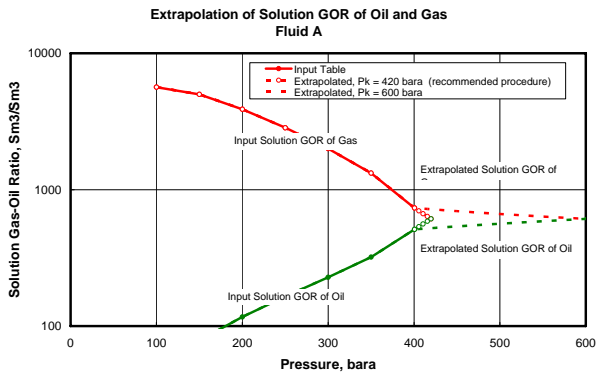


Fig. 14 — Extrapolation of solution GOR of oil and gas with different convergence pressures (a) 420 bara, recommended procedure and (b) 600 bara.

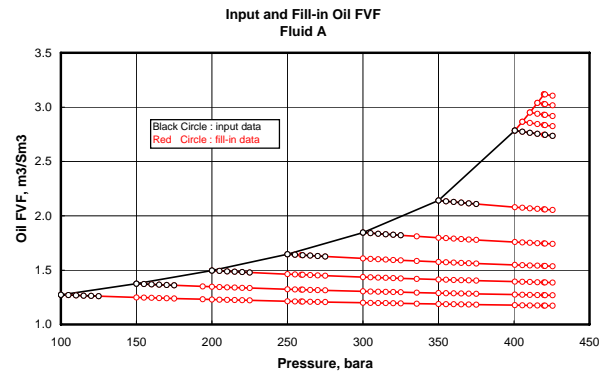


Fig. 17 — Input and fill-in saturated and undersaturated oil FVF. The fill-in data are calculated with EOS model. The convergence pressure is about 420 bara.

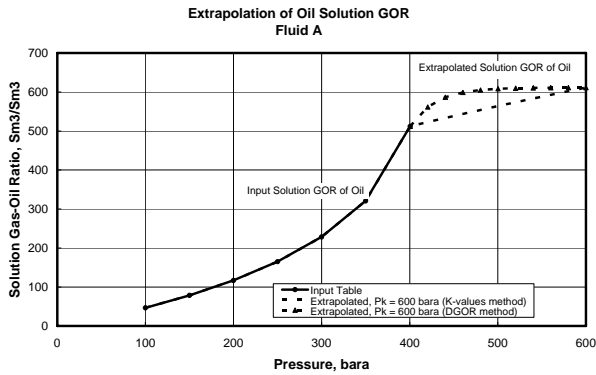


Fig. 15 — Extrapolated solution GOR versus pressure using (a) K-value extrapolation and (b) DGOR extrapolation methods. The convergence pressure in both methods is 600 bara.

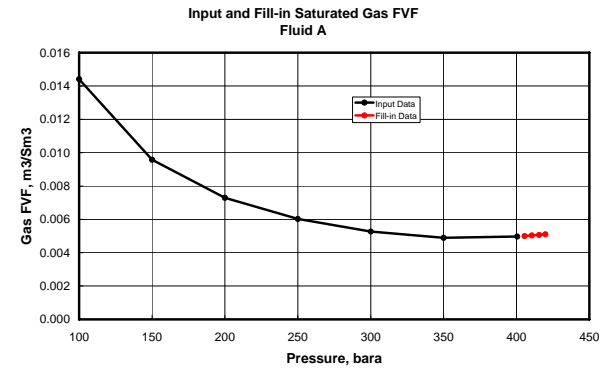


Fig. 18 — Input and fill-in saturated gas FVF. The fill-in data are calculated with EOS model. The convergence pressure is about 420 bara.

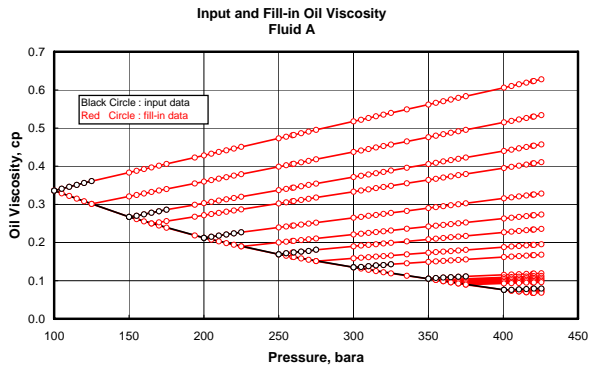


Fig. 19 — Input and fill-in saturated and undersaturated oil viscosity. The fill-in data are calculated with LBC correlation. The convergence pressure is about 420 bara.

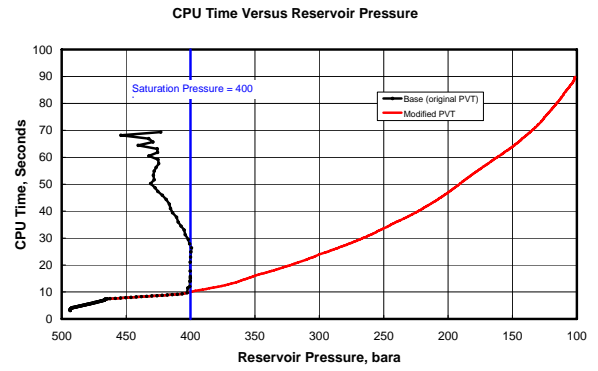


Fig. 22 — Reservoir simulation result : CPU time for the simulation case using (a) base and (b) modified black oil PVT table for fluid A.

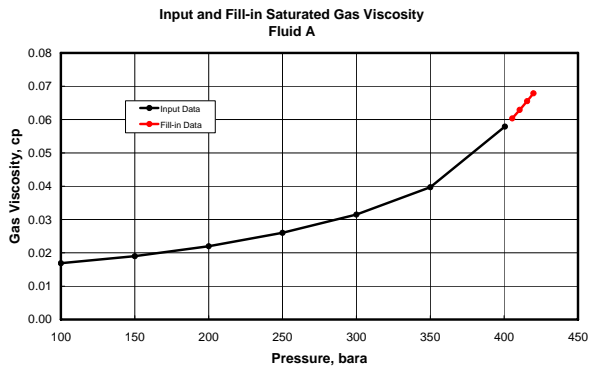


Fig. 20 — Input and fill-in saturated gas viscosity. The fill-in data are calculated with LBC correlation. The convergence pressure is about 420 bara.

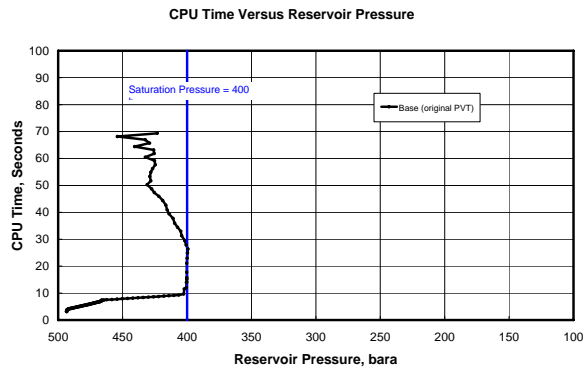


Fig. 21 — Reservoir simulation result : CPU time for the simulation case using base black-oil PVT table for fluid A.