probably the problem. The type of multirate test chosen will depend on the permeability of the formation: High-permeability reservoirs can be tested with a flow-after-flow sequence, whereas low-permeability reservoirs should be tested with a true isochronal sequence, having adequate shut-in periods to erase the transients of previous flows.

Many measures are available today to avoid or mitigate formation damage. Table 3.1 lists the main mechanisms of formation damage, well operations that may produce the damage, precautions to prevent it, and methods to cure it. Avoiding formation damage is of paramount importance to the completion engineer. Yet, measures to prevent formation damage are costly and often more expensive than treatments to cure the damage. Avoiding formation damage involves costs for determining specifications for nondamaging fluid, obtaining the fluid, filtering the fluid, cleaning the well, and placing the fluid. Therefore, the strategy for handling formation damage is usually a matter of economics. (Bell 1985)

Finally, a treatment to cure formation damage should be planned and executed only after a skin due to damage is verified. That is, after skin components attributable to other effects have been subtracted from the measured composite skin. The procedure for this skin analysis is covered in detail in section 3.8.

3.3 **PARTIAL PENETRATION AND LIMITED ENTRY.** Since the beginning of petroleum production, wells have been drilled and completed through only a fraction of the total formation, to avoid contact with the water zone that may underlie the oil or gas zone, or perhaps because of difficulties in controlling mud circulation in the pay zone.

A completion arrangement where only limited pay-zone interval is open to production is referred to as partial penetration or limited entry. Numerous cases of partial penetration are mentioned and discussed in the petroleum literature. One example is the completion of wells in the North Antioch field in Oklahoma (Culter and Rees 1970). The wells were completed in the Oil Creek reservoir and were perforated only 5–10 ft near the top of approximately 100 ft of a fairly uniform and clean sandstone pay zone. The partial penetration reflects the concern regarding water coning from bottom water that underlies the oil reservoir.

Another example is the high-rate (50 MMscf/D) gas wells of the North Sea Frigg field (Barril and Gay 1983). They are only partially penetrated so as to keep the production interval a safe distance above the water level. High-rate oil wells in the Statfjord field (Norway) completed in the Brent sandstone formation are not perforated close to the gas–oil contact at the top of the formation, nor close to the oil–water contact, to avoid both gas and water coning. Furthermore, the wells are perforated only across intervals where the sandstone is highly consolidated, to avoid sand production.

In many wells the problem of gas and water coning restricts the production interval to a small fraction of the oil zone located near the midpoint of the formation. In general, reasons for limited entry into the hydrocarbon-bearing formation are:

Table 3.1 Causes of Formation Damage and Precautions to Control It

Operation	Causes of formation damage	Accelerating factors	How to prevent it	How to cure the damage
1. Drilling	— mud filtrate invasion — mud solids invasion — sealing of pores and flow tunnels by the trowelling action of the bit, drill collars, and drill pipes — plugging by rock cuttings	— high-permeability formation — water-based mud — abrupt reduction in salinity — drilling with high water loss — bentonite mud — strongly overpressured drilling — high solids mud	— drilling the production zone with nondamaging fluids — use of removable bridging loss and of circulation material — use of clay-migration and clay-swelling inhibitors	— backflush — acid wash, matrix acidizing
2. Running casing and cementing	— plugging/blockage of pore space by mud or cement solids — filtrate invasion — chemical reactions with cement additives and spacers	— high-permeability formation	— use fluid-loss additives — pretreat for clay stabilizing	— deep perforations — matrix acidizing, acid wash
3. Perforating	— plugging of perforations and formation with debris — compaction of pores around perforations	— use of low performance or expendable guns — perforate overbalanced in drilling mud	— perforate underbalanced — use of clean, solid-free fluid — use premium charges and large guns — use deep-penetrating charges	— backflow — acidizing
4. Running completion string	— plugging by solids from completion fluids and diverting agents — filtrate invasion — dissolutions of rock cementing materials	— overbalanced conditions with damaging completion fluids — improper bridging materials — high-permeability formation — uncleaned wellbore and production equipment	— underbalanced operation — remove all bulk solids — clean casing and tubing before use — use nondamaging fluids and bridging materials	— acid treatment — solvent wash — same as for drilling

Table 3.1 (continued)

Operation	Causes of formation damage	Accelerating factors	How to prevent it	How to cure the damage
5. Production	— fines movement — clay migration — condensate and water blockage — deposits of salt crystals, wax, and paraffines — hydrate and emulsions forming	— high production rates — increase water/oil ratio — pressure decrease — communication with water zones — poor gravel-packing or sand-control measures	— control water/oil ratio — inject clay-migration inhibitors — inject scale inhibitors — keep clean wellbore — avoid abrupt increase of production rate	— acidizing — chemical treatments
6. Gravel packing	— invasion of filtrate from gravel-pack slurries — invasion of solids and contaminations — mixing of gravel with formation sand — plugging by diverting agents	— variation of permeability along the producing interval — nonuniform sand — clay-rich sand	— use nondamaging clean fluids — operate in clean wellbore — properly designed pre- and post-gravel-packing acidizing — proper design and placement of gravel and gravel-pack equipment	— acidizing (through gravel pack) — replace the gravel pack

7. Acidizing	— insoluble precipitates — iron precipitation in the wellbore — plugging by solids scoured from the tubing	— incompatibility between acid, acid additivies, and formation materials — damaging diverting agents — large variations in permeability	— proper injection sequence — use only nondamaging additives — proper diverting procedure — use damage-inhibition additives	— reacidize with proper additives
8. Fracturing	— plugging by formation fines or damaged by gelled frac fluids	— poorly designed frac	— clean, properly sorted and sized proppant — use of proper and sufficient breakers in the fracturing fluids and slurries	— soak with gel breaker
9. Workover	— residual cement plugging — wireline loosened iron scale or paraffin from tubing plugging — plugging by metallic particles resulting from casing repair operations — damaging workover fluids — damaging bridging materials	— operate at overpressured conditions — high-permeability formation — large variation in permeabilities — uncleaned wellbore — use of corrosion inhibitors or emulsion breakers	— underpressured workover operation — use of nondamaging fluids — operate in clean wellbore — clean the working string	— acid stimulation — chemical treatment

1. to avoid coning of water and/or gas
2. the well cannot be drilled throughout the pay zone for mechanical or safety reasons
3. to avoid producing sand or other friable formation particles
4. to test an exploratory well in selected intervals
5. to leave portions of the casing unperforated for future needs of setting mechanical assemblies such as packers, spacers, bridge plugs, and centralizers
6. erroneous log interpretation that fails to define the true total pay zone
7. plugged perforations that do not contribute to production

Most of the reasons for partial penetration and limited entry are intentional, designed to improve the overall performance of a well or reservoir (though not necessarily productivity). The last two reasons listed are not planned, and in fact the operator may not be aware of the unintended restriction to flow.

The general characteristics of a well with limited entry are shown in figure 3.5. Flow lines converge from above and below the open interval, gradually changing to radial flow away from the wellbore. Because of the deformed flow path and localized pressure gradients near the ends of the open interval, lower wellbore flowing pressure is required to produce a given rate. Muskat's original work (1932) on partial penetration serves as the fundamental solution to the steady-state problem. He illustrated the influence of partial penetration, as a plot of rate versus flowing pressure, for the case of limited entry without any other flow restriction. Figure 3.6 reproduces some of Muskat's results. Muskat (1937) also discussed the effect of rock unisotropy on a partially penetrating well. In unisotropic rock, permeability in various directions may vary considerably. Corresponding to the particular sedimentation process, permeability in the direction of the bedding plane is in most cases larger than permeability in a direction perpendicular to the bedding plane.

In relation to partial penetration, when the vertical permeability is less than the horizontal permeability, the anisotropy acts to hinder convergence to the limited completion interval and will attempt to keep the radial nature of the flow. Consequently, it tends to limit the contribution of the part of the reservoir not penetrated by the well and thus decreases the production capacity. In conclusion, anisotropy, where k_v is less than k, acts to reduce production capacity, a reduction that is magnified as the penetrating ratio becomes smaller.

Brons and Marting (1961) suggested that the effect of partial penetration and limited entry can be expressed as a skin factor. They gave the simple relation

$$s_c = (1/b - 1)[\ln(h_D) - G(b)], \tag{3.6}$$

where

b $= h_p/h$,
h_D = dimensionless pay thickness, $(k/k_v)^{0.5}(h/r_w)$,
h_p = limited interval open to flow (ft),
h = total formation thickness (ft),
k = horizontal formation permeability (md),

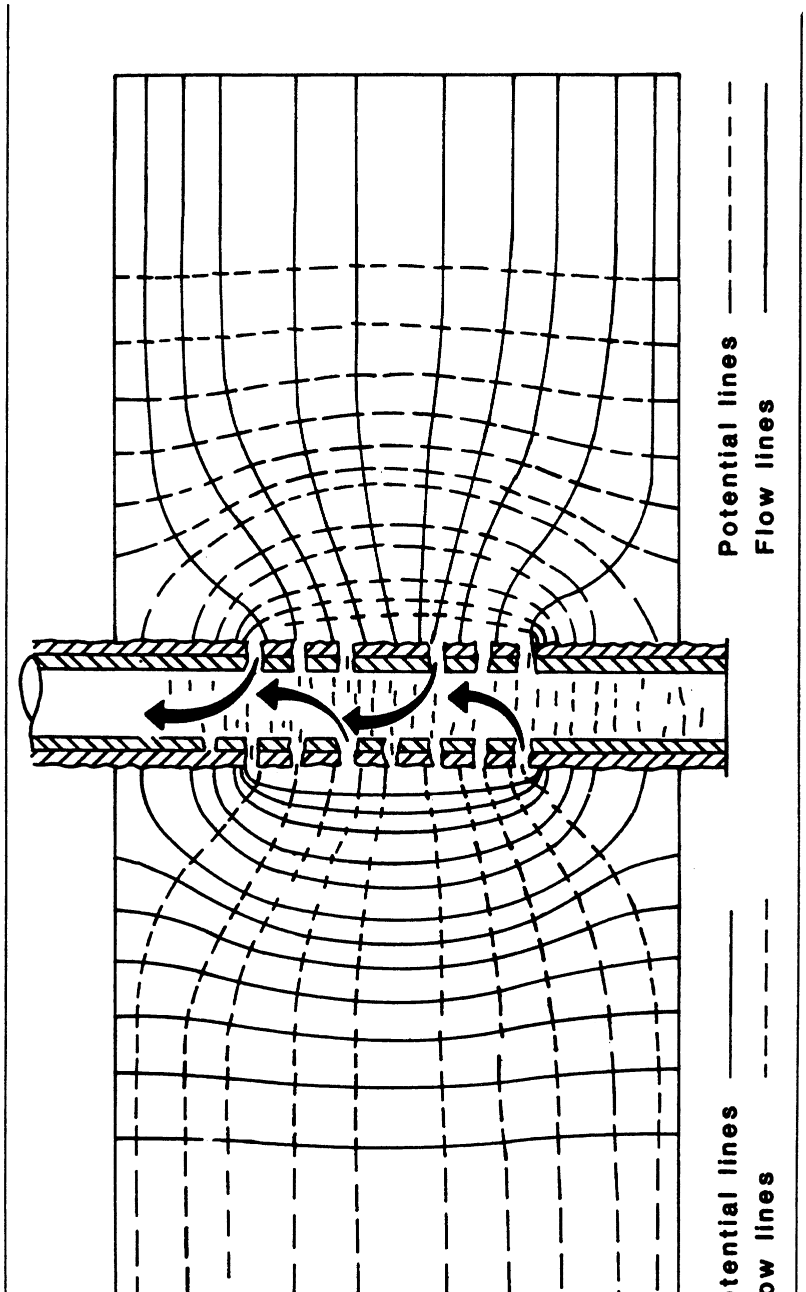


Figure 3.5 Flow behavior of a well with limited entry.

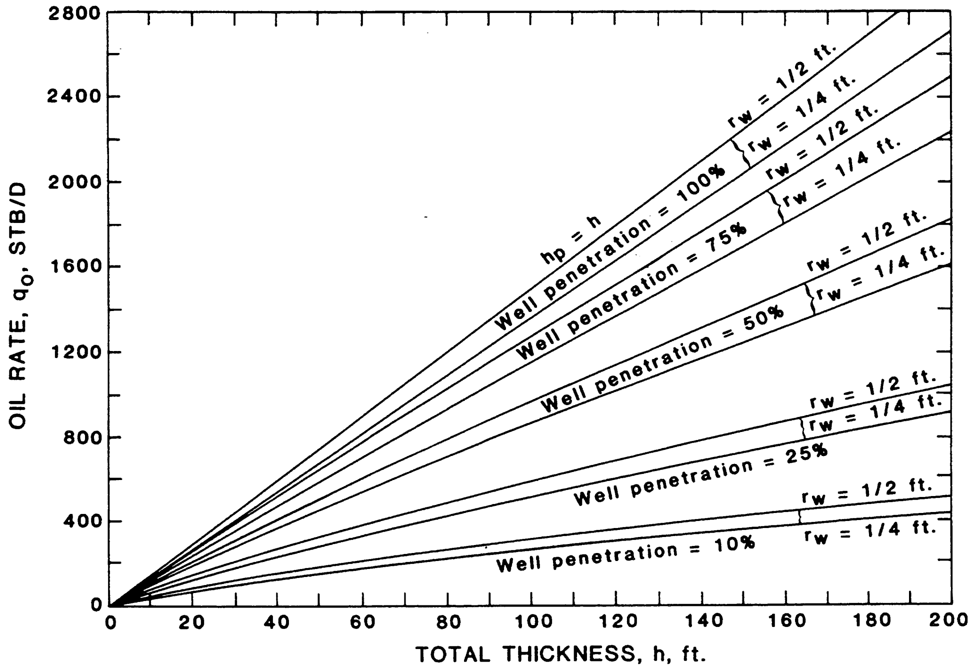


Figure 3.6 Effect of partial penetration on productivity of a well. Reprinted by permission of IHRDC Press from Muskat 1981, fig. 5.12.

k_v = vertical formation permeability (md),
$G(b)$ is a function of the fractional penetration b.

Brons and Marting solved Nisle's (1958) analytical formulation by numerical integration, to arrive at a table of values for $G(b)$. Their table is given in example 3.2, together with the analytical expression for $G(b)$ found directly by algebraic manipulation of Muskat's original solution (1932) as developed in Example 3.2. An approximate relation for $G(b)$ is

$$G(b) = 2.948 - 7.363b + 11.45b^2 - 4.675b^3. \tag{3.7}$$

EXAMPLE 3.2 DEVELOPMENT OF THE BRONS AND MARTING (1961) SKIN EQUATION, BASED ON MUSKAT'S ORIGINAL SOLUTION

Muskat (1932) studied the effect of partial penetration on well performance and proposed the following expression as a reasonable approximation to the general steady-state equation for isotropic sand:

$$q = \frac{2\pi kh\Delta p/\mu B}{\{2\ln(4h/r_w) - \ln[C(b)]\}/(2b) - \ln(4h/r_e)},$$

EXAMPLE 3.2 continued

where $b = h_p/h$ (penetrating interval divided by the total thickness), and

$$C(b) = \frac{\Gamma(0.875b)\Gamma(0.125b)}{\Gamma(1-0.875b)\Gamma(1-0.125b)}$$

where $\Gamma(x)$ is the gamma function of x. If field units are used (STB/D, md, ft, etc.), the constant 2π is replaced by $(141.2)^{-1} = 0.00708$. Note that the isotropic assumption implies that $k/k_v = 1.0$ and $h_D = h/r_w$. However, as noted by Muskat, the same solution is valid for anisotropic systems by using the transformation $(k/k_v)^{0.5}$ multiplied by h/r_w [i.e., $h_D = (k/k_v)^{0.5}h/r_w$].

Tasks

1. Show that it is possible to use Muskat's solution to derive the Brons and Marting expression $G(b)$ for the partial-penetration skin factor relation

   $$s_c = (1/b - 1)[\ln(h/r_w) - G(b)].$$

   Based on the results, comment on the statement by Brons and Marting (1961), "Since this function $[G(b)]$ cannot be expressed analytically, it has been calculated numerically."
2. How would you apply the Muskat or Brons and Marting skin factor expression if the producing interval is in the middle of the formation?

SOLUTION

1. The derivation of the function $G(b)$ in terms of Muskat's analytical solution is obtained by arranging Muskat's flow equation so it explicitly expresses the skin factor. Equating Muskat's denominator with $[\ln(r_e/r_w) + s_c]$ and solving for s_c gives

   $$\begin{aligned} s_c &= (1/b - 1)\ln(4h/r_w) - \ln[C(b)]/(2b) \\ &= (1/b - 1)\ln(h/r_w) + (1/b - 1)\ln(4) - \ln[C(b)]/(2b) \\ &= (1/b - 1)\{\ln(h/r_w) + \ln(4) - [b/(1-b)]\ln[C(b)]/(2b)\} \\ &= (1/b - 1)\{\ln(h/r_w) + \ln(4) - \ln[C(b)]/[2(1-b)]\} \\ &= (1/b - 1)[\ln(h/r_w) - G(b)], \end{aligned}$$

   where

   $$G(b) = -\ln(4) + \frac{\ln C(b)}{2(1-b)}.$$

   Table E3.2 shows values of $G(b)$ from the Muskat analytical approximation compared with the numerically calculated values by Brons and Marting. It is

EXAMPLE 3.2 continued

Table E3.2 Comparison of Analytical (Muskat 1932) and Numerical (Brons and Marting 1961) Values of the $G(b)$ Function

b	Brons–Marting $G(b)$	Muskat $G(b)$	Muskat $C(b)$
0.1	2.337	2.337	814.16
0.2	1.862	1.862	180.65
0.4	1.569	1.569	34.71
0.6	1.621	1.620	11.08
0.8	1.995	1.992	3.86

obvious that the statement of Brons and Marting regarding the possibility of analytical solution is unjustified in cases of isotropic sand. A similar procedure can be developed for anisotropic sand, where the horizontal permeability is larger than the vertical permeability. The procedure for anisotropic formation, however, is beyond the scope of this example.

2. The skin factor developed by Muskat and by Brons and Marting assumes the open interval is at the top of the formation and penetrates a fraction $b = h_p/h$ of the total thickness. Symmetry is used to apply the skin factor to a situation when the open interval is located near the midpoint of the formation (e.g., when avoiding both water and gas coning).

Basically, the formation is cut in two layers, split at the midperforations. Assuming the upper and lower sections are of equal thickness, the definition of b for each layer is $[(h_p/2)/(h/2)]$ or h_p/h, the same as for penetration from the top of the formation. The expression h/r_w [or more correctly, $(k/k_v)^{0.5}(h/r_w)$] becomes $(h/2)/r_w$ or $h/2r_w$, half the value used when penetration starts at the top of the formation. This value results in a lower skin, which is reasonable, considering the convergence in flow lines toward the wellbore. For an interval at the midperforations there is less flow convergence than for penetration from the top of the formation.

Figure 3.7 plots skin s_c versus fractional penetration b for several values of h_D. These curves are useful for three types of limited-entry configurations, shown in figure 3.8:

1. a well penetrating the top of the formation
2. a well open to flow from the midsection of the formation
3. a well with open intervals equally spaced along the entire height of the formation

For each case the value of b remains unchanged, but h_D is different for each configuration. For limited entry starting at the top of the formation, total formation thickness h is used to define h_D.

If the well is open at the midsection of the formation, $h/2$ is used to define h_D

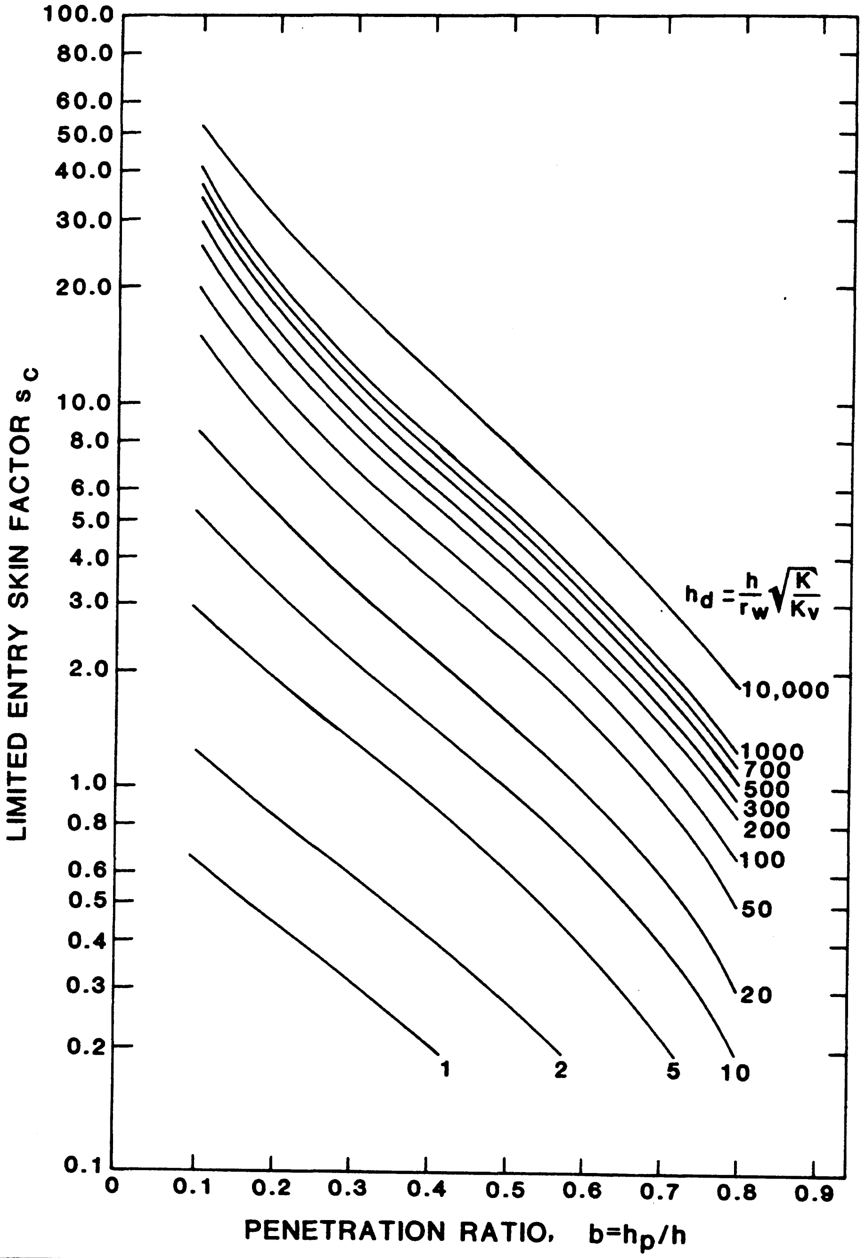
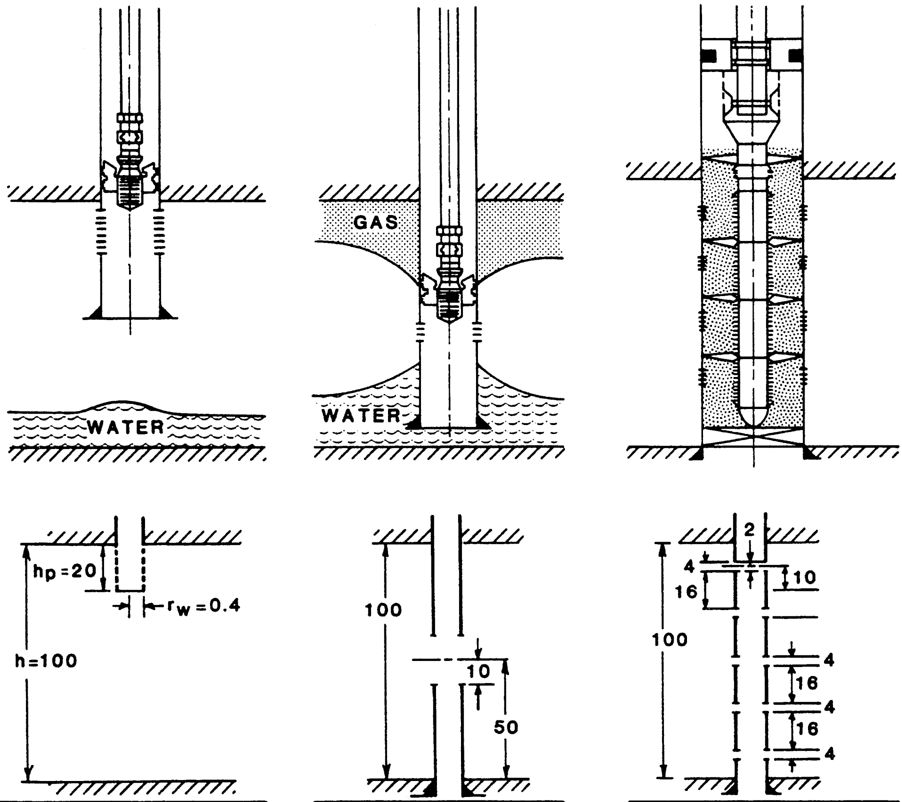


Figure 3.7 Partial-penetration skin factor. Reprinted by permission of the University of Trondheim from Standing 1980, fig. 5; after Brons and Marting 1961.



$b = \frac{h_p}{h}$	$\frac{20}{100} = 0.2$	$\frac{20}{100} = \frac{10}{50} = 0.2$	$\frac{20}{100} = \frac{2}{10} = 0.2$
h_D	$(h/r_w)\left(\frac{K}{K_v}\right)^{0.5}$	$\left(\frac{h}{2Nr_w}\right)\left(\frac{K}{K_v}\right)^{0.5}$	$\left(\frac{h}{2Nr_w}\right)\left(\frac{K}{K_v}\right)^{0.5}$
	$\frac{K}{K_v} = 1$	$N = 1$ $\quad \frac{K}{K_v} = 1$	$N = 5$ $\quad \frac{K}{K_v} = 1$
h_D	250	125	25
S_c	15	12	6.5

Figure 3.8 Partial penetration and three geometries of limited entry.

[i.e., $h_D = (k/k_v)^{0.5}(h/2r_w)$]. For N sections of open interval located symmetrically about the middle of the formation, with equal thickness and equally spaced, the correct expression for h_D is $(k/k_v)^{0.5}(h/2Nr_w)$. The definition of the dimensionless pay thickness, h_D, for the three configurations in figure 3.7 is also valid when using equation (3.6). Example 3.3 shows the use of figure 3.7 for estimating the skin due to limited entry.

EXAMPLE 3.3 ESTIMATION OF PARTIAL-PENETRATION SKIN FROM THE CHARTS OF BRONS AND MARTING (1961)

The Java No. 2 is a low-pressure, high-permeability gas well reported by Fetkovich (1975). An eight-point multirate test was run to determine the backpressure curve. Buildup analysis indicates a permeability thickness of 306,060 md-ft. Total thickness is 313 ft, giving a permeability of 978 md. The interval open to flow is only 70 ft (open from the top to avoid water coning from the underlying aquifer). Table E3.3 gives relevant rock and fluid data for the Java No. 2 well. The complete analysis of the test is given in example 3.13. However, we may indicate here that only the last four points of the eight test points were used to draw and interpret the backpressure curve, since cleanup had not been completed prior to the fifth point.

A plot of $(p_R^2 - p_{wf}^2)/q_g$ versus q_g given in Example 3.13 indicates an intercept of $A = 0.000966\,\text{psia}^2/(\text{scf/D})$, which calculates a steady-state skin factor of $s = +22.18$. Since the plot is based on one-hour isochronal test data, the skin is calculated from the intercept using the transient rather than the steady-state intercept expression.

The transient expression for A obtained by substituting equation (2.139) in equation (2.144) is

$$A(t) = \frac{T\mu_g Z}{0.703kh}\{0.5[\ln(t_D) + 0.80907] + s\},$$

Table E3.3 Reservoir Data for the Java No. 2 Well

Initial reservoir pressure p_i	1370 psia
Reservoir temperature T	120°F
Total reservoir thickness h	313 ft
Perforated thickness h_p	70 ft
Permeability (buildup) k	978
Horizontal/vertical permeability ratio (assumed) k/k_v	10
Gas gravity γ_g	0.655 (air = 1)
Wellbore radius r_w	0.33 ft
Initial gas viscosity μ_{gi}	0.0144 cp
Initial gas compressibility c_{gi}	832×10^{-6} 1/psia
Initial total compressibility c_{ti}	582×10^{-6} 1/psia
Initial gas Z-factor Z	0.837

EXAMPLE 3.3 continued

and t_D is given by

$$t_D = \frac{0.000264kt}{\phi \mu_{gi} c_{ti} r_w^{\,2}}$$

$$= \frac{0.000264(978)(1)}{(0.181)(0.0144)(0.000582)(0.33)^2}$$

$$= 1.56 \times 10^6,$$

so

$$A(t = 1\text{hr}) = 0.000966$$

$$= \frac{(120 + 460)(0.0144)(0.837)}{0.703(978)(313)} \times \{0.5[\ln(1.56 \times 10^6) + 0.80907] + s\}$$

or

$$0.000966 = 3.25 \times 10^{-5}(7.54 + s).$$

Solving for s yields the steady-state skin:

$$s = 29.72 - 7.54$$

$$= +22.18.$$

Task
Compare the back-calculated skin factor ($s = +22.18$) with a predicted partial-penetration skin factor.

SOLUTION

The fraction of total thickness open to flow is 70/313 or 0.22, and h_D is given by

$$h_D = (k/k_v)^{0.5}(h/r_w)$$

$$= (10)^{0.5}(313/0.33)$$

$$= 3000.$$

From the Brons and Marting figure (Fig. 3.7), $s_c = +21.0$, which checks with the back-calculated skin. Based on equation (3.7) for $G(b)$, where $b = 70/313 = 0.22$.

$$G(b) = 2.948 - 7.363b + 11.45b^2 - 4.675b^3$$

EXAMPLE 3.3 continued

$$= 2.948 - 7.363(0.22) + 11.45(0.22)^2 - 4.675(0.22)^3$$

$$= 1.833.$$

The Brons and Marting equation for skin gives

$$s_c = (1/b - 1)[\ln(h_D) - G(b)]$$

$$= (1/0.22 - 1)[\ln(3000) - 1.833]$$

$$= +21.9,$$

which closely matches the steady-state skin back-calculated from multirate data ($s = +22.18$). Example 3.13 gives a more detailed discussion of the entire test performed on the Java No. 2, including the first four points in the multirate test, which were dominated by a large damage skin.

Several other publications give expressions for s_c, among the more relevant being Odeh (1968), and Streltsova-Adams (1979). A discussion of approximate analytical expressions developed prior to Muskat's (1932) solution is given by Muskat (1937, 1982). Odeh (1980) gives an empirical relation for skin due to an arbitrarily located open interval h_p:

$$s_c = 1.35(1/b - 1)^{0.825} \{\ln(r_w h_D + 7) - 1.95 - [0.49 + 0.1\ln(r_w h_D)]\ln(r_{wc})\}, \tag{3.8}$$

where

$$r_{wc} = r_w \exp[0.2126(2.753 + z_m/h)], \quad 0 < z_m/h < 0.5, \tag{3.9}$$

$$r_{wc} = r_w, \quad y = 0, \tag{3.10}$$

$$z_m = y + h_p/2, \tag{3.11}$$

y = distance from the top of the formation to the top of the open interval.

If z_m/h is greater than 0.5, then use $1 - z_m/h$ instead of z_m/h in the equation for r_{wc} (eq. [3.9]). Example 3.4 shows use of Odeh's correlation for partial-penetration skin.

EXAMPLE 3.4 ODEH CORRELATION FOR ESTIMATING PARTIAL-PENETRATION SKIN

Arthur (1944) gives data for a well perforated only in the lower middle section of a 160-ft formation (fig. E3.4). An overlying gas cap and an underlying aquifer posed the problem of coning. This led to a completion interval with perforations in only

EXAMPLE 3.4 continued

62 ft of the total thickness (38.8%). Arthur claimed that as a result of the close proximity to the water–oil contact a pressure drop of only 1.3 psi would lead to water coning. Arthur also reported that up to 103 psi drawdown could be tolerated without gas coning from above.

Additional hypothetical well data, $kh = 83{,}200$ md-ft ($k = 520$ md); $\mu_o = 0.85$ cp; $B_o = 1.32$ bbl/STB; $\ln(r_e/r_w) - 0.75 = 7.73$ ($r_e = 1000$ ft); and $k/k_v = 1.0$.

Calculate (1) the oil rate resulting in the 1.3-psi pressure drop to produce a water cone; (2) the oil rate resulting in the 103-psia pressure drop to produces a gas cone. Assume in the second case that water coning is not a problem if a shale break separates the oil zone from the aquifer.

SOLUTION

We use the Odeh equation, equation (3.8), which is valid for calculating limited-entry completion skin for a single completion interval located anywhere within the formation. The distance from the top of the formation to the top of the perforations is $y = 7623 - 7535 = 88$ ft. The perforated interval is

$$h_p = 7685 - 7623$$

$$= 62\text{ ft},$$

which calculates

$$z_m = y + h_p/2$$

$$= 88 + 62/2$$

$$= 119\text{ ft}.$$

The effective pay zone is

$$h = 7695 - 7535$$

$$= 160\text{ ft},$$

which calculates

$$z_m/h = 119/160$$

$$= 0.744.$$

Since z_m/h is larger than 0.5, we use the symmetry of r_{wc}/r_w about $z_m/h = 0.5$ to substitute