MULTIPOINT TESTING OF GAS WELLS

Multipoint tests consist of a series of at least three or more flows with pressures, rates and other data being recorded as a function of time. The tests are usually conducted for one of the following reasons:

1. Required by a state regulatory body for proration purposes or to obtain an allowable.

2. Required for a pipeline connection.

3. Company policy.

4. Obtain sufficient information for reservoir and production engineering studies which can consist of:
   a. Production forecasting (deliverability type or reservoir simulation).
   b. Determining number of wells and location for development of the field.
   c. Sizing tubing.
   d. Sizing gathering lines.
   e. Sizing trunklines.
   f. Designing compression requirements.
   g. Determining necessity for stimulation.
   h. Correctly evaluating damage (skin effect).
   i. Establish base performance curves for future comparison.

We will limit our present discussion on multipoint testing to Item 4.

There are two basic types of multipoint tests:

1. Flow After Flow Test (1) (No shut-in between flows)
   a. Normal sequence (Fig. 1)
   b. Reverse sequence (Fig. 2)

2. Isochronal Test (Well is shut-in between flows)
   a. True isochronal (2) (Fig. 3)
   b. Modified isochronal (3) (Fig. 4)
Flow After Flow

The flow after flow test starts from a shut-in condition after which a series of increasing flows (normal sequence) or decreasing flows (reverse sequence) is imposed upon the well. No (or very small) shut-in periods occur between each of the flows. Flow times are usually arbitrary or can be set by a regulatory body when conducted for that purpose.

If "stabilized" flows are obtained, the test may be considered to be as valid as if one were to have conducted a true isochronal test. This condition is normally obtained in high permeability reservoirs. Stabilization is defined in the IOCC Manual\(^4\): "A constant flowing wellhead pressure or static column wellhead pressure and rate of flow for a period of at least 15 minutes shall constitute stabilization ....". If a well is tubing capacity limited, a pseudo-stabilization can occur if one uses only flowing tubing pressures as the criteria. Pseudo-stabilization can also occur as a result of flowing tubing temperature increase. Therefore, bottomhole or static column pressure stabilization is preferable for this definition.

The different performance curves one could obtain on the same well from an increasing or decreasing sequence multipoint test and an isochronal test is demonstrated by the results shown\(^2\) in Fig. 5. These type of results are normally limited to tests conducted in low permeability reservoirs.

Isochronal Tests

The isochronal method of multipoint testing gas wells is the only certain way of obtaining reliable performance curves. Each flow starts from a comparable shut-in condition. The shut-in must be close enough to a fully built up condition that any pressure rise still occurring will not affect pressure during the drawdown of the subsequent flow; i.e., no prior transits exist during any flow period. Although the flow periods for an isochronal test are usually of equal duration, they need not be. However, when a performance curve is plotted, data from flow periods of the same duration are plotted to obtain the correct value of slope (n), Fig. 6. Note that rates and pressures at a specific time are plotted - NOT AVERAGE RATE.

The isochronal test is based on the principle that the drainage radius established during a flow period is a function only of dimensionless time and is independent of the flow rate; i.e., for equal flow times the same drainage radius is established for different rates of flow. It follows then that an isochronal test would yield a valid performance curve if conducted as either a constant rate or constant flowing pressure test. In fact, many low permeability gas well tests that exhibit severe rate declines on test are really constant wellbore pressure cases and should be analyzed as such. (In a paper by Winestock and Colpitts\(^5\), their rate decline data analyzed as constant pressure case gives the same permeability value as a build-up test.) A constant rate is not required for a valid isochronal test. If one is attempting to short-cut the isochronal test using superposition, then and only then could a constant rate flow condition be required - but only for the purpose of using superposition.
Modified Isochronal Test

In very low permeability reservoirs it may require days to obtain a completely built-up pressure after even relatively short periods of flow (2 to 3 hours). In an attempt to shorten testing time, the modified isochronal test was proposed. It is conducted with shut-in periods equal to the flow periods. The unstabilized shut-in pressures are used to calculate the difference in pressure relationship used with the next flow rate. This method of testing has never been adequately justified, either theoretically or by field comparisons with true isochronal tests. What little discussion published justifying this method theoretically has been based on the assumption that flowing pressure behavior with time (superposition) is a function of the log of time $P = f (\ln t)$. Practically however, most low permeability wells where the modified test would be practically applied require stimulation (hydraulic or acid fracs) to be commercial. In these cases pressures are more likely to be a function of the square root of time, $P = f \sqrt{t}$. Modified tests under these conditions can have flowing pressure behavior as functions of $\sqrt{t}$, transitional or $\ln t$ each for different flow rates. Fig. 7 is a type curve plot of drawdown data from a 3 hr isochronal flow, note the linear flow behavior for the entire period of flow. For maximum reservoir information purposes, the author does not recommend the modified isochronal test, nor any other method that depends on the application of superposition techniques to shorten test times for low permeability wells. If time is of such importance in low permeability formations, one can be further ahead by simply conducting one long duration flow period (making certain we are out of wellbore storage, $\sqrt{t}$ and transitional period prior to $\ln (t)$ behavior) and assuming a back-pressure curve slope ($n$) of one. Better still, the Two Flow Method of Carter, Miller and Riley would be preferred - i.e., two isochronal points.

Isochronal Testing

For maximum information and minimum confusion, the writer prefers and recommends the isochronal test method when multipoint tests are required - particularly on wildcat or initial development wells. Once the basic characteristics of the reservoir and fluid properties have been defined from valid isochronal tests, one should consider the possibility of reducing testing time without sacrificing information.

The number of flows and flow and shut in times can often be reduced with shut in periods even eliminated in some cases.

Without getting into the detailed mechanics of testing and taking data, a few remarks on test procedure are appropriate. Whenever possible bottom hole pressure gauges should be used. Surface pressures should be recorded with a dead weight tester and measured on both the tubing and annulus along with flowing temperatures. The frequency of taking the surface drawdown and buildup data should be sufficient for type curve analysis, i.e., early time data is critical for this analysis. Similarly, with about the same frequency, flow rate data should be recorded and reported. A constant wellbore pressure analysis or a Winestock and Colpitts analysis ($\Delta p^2/Q$ vs time) may be
required. Plotting and analysis of the test data, drawdown, buildup and backpressure curves on site during the test are rather critical to obtaining valid tests. Most important of all, the well must be cleaned up prior to conducting the test. The importance of a clean up flow is dramatically illustrated by the test results obtained on Well C, Fig. 8. As a general rule for selecting rates one should attempt to flow the well at or near the expected continuous sales rate. If sand, water coning or other problems could develop, now is the time to find out.

Following is a typical isochronal test procedure used on a wildcat or early development well:

1. Initial Flow (± 15 min)
2. Initial Shut-in (± 2 hrs)
3. Clean Up Flow at Maximum Separator Capacity (± 10 hrs)
4. Shut-in Period (± 12 hrs)
5. Flow at ± 1/4 Maximum Rate (± 6 hrs)
6. Shut-in Period (± 9 hrs)
7. Flow at ± 1/2 Maximum Rate (± 6 hrs)
8. Shut-in Period (± 9 hrs)
9. Flow at ± 3/4 Maximum Rate (± 6 hrs)
10. Shut-in Period (± 9 hrs)
11. Flow at Maximum Rate (± 6 hrs)

The above time periods are subject to change depending on an on-site analysis of the initial data. Severe wellbore storage effects or total linear flow as demonstrated in Figs. 9 and 7 respectively would cause the multipoint test to be aborted, and one would then settle for a single long duration drawdown and buildup.

Conventional Well Test Analysis (P_w < 2500 psi)

Gas well analysis can be divided into two pressure regions, low to medium pressure and high pressure wells. Much of the basic theory of testing and analysis was developed from well tests with reservoir pressure levels under 2500 psi. This resulted in the familiar back pressure curve plotting of log q vs log Δ(p^2) and pressure build-up and drawdown analysis using p^2 vs log \left(\frac{t + Δt}{Δt}\right) plot and p^2 vs log t.
With the advent of deeper drilling, gas wells have been discovered with reservoir pressures approaching 10,000 psi. In these cases, and down to about 2500 psi the conventional methods of analysis break down and the real gas potential theory approach must be resorted to. (This will be discussed later.)

**Basic Equations (Reservoir and Surface Datums)**

The familiar transient gas flow equation is usually given in standard engineering units as:

\[
\frac{1}{P_R^2 - P_{wf}^2} = \frac{1424 \overline{\mu T} \left( \frac{1}{K_h} \right)}{2\left( \ln t_0 + 0.809 \right) + S} Q + \left( \frac{2.226 \times 10^{-15} \beta KG}{hr_w \mu} \right) Q^2
\]

Where
- \( Q \) = gas flow rate, Mscfd
- \( K \) = effective permeability to gas, md
- \( h \) = net pay, ft
- \( T \) = reservoir temperature, °R
- \( P_R \) = static reservoir pressure, psia
- \( P_{wf} \) = bottom hole flowing pressure, psia
- \( G \) = gas gravity
- \( \mu \) = gas viscosity, cps (evaluated at \( P_R \))
- \( \overline{\mu} \) = gas viscosity, cps [evaluated at average pressure, \((P_R + P_{wf})/2\)]
- \( \beta \) = turbulence factor, ft\(^{-1}\)
- \( S \) = laminar flow skin effect, dimensionless
- \( r_w \) = wellbore radius, ft
- \( t_0 \) = dimensionless time

The dimensionless time equation, with time \( t \) in days is:

\[
t_0 = \frac{6.33 \times 10^{-3} K_P R t}{\phi \mu r_w^2}
\]

The basic equation used to describe reservoir drawdown when stabilized flow exists is given as:

\[
\frac{1}{(\mu T) \overline{T}} = \frac{1424 \overline{\mu T} \left( \frac{1}{K_h} \right)}{2\left( \ln \left( \frac{.472 r_e}{r_w} \right) + S \right) Q + DQ^2}
\]

For further discussion if we define:

\[
B = \frac{1424 \overline{\mu T} \left( \frac{1}{K_h} \right)}{D}
\]
we can obtain

$$\overline{P_R}^2 - P_{wf}^2 = \frac{1424 (\mu \bar{z}) T}{Kh} \left[ \ln \left( \frac{.472 \bar{r}_{e}}{r_w} \right) + S \right] Q + BQ^2 \quad ..(5)$$

Defining

$$A = \frac{1424 (\mu \bar{z}) T}{Kh} \left[ \ln \left( \frac{.472 \bar{r}_{e}}{r_w} \right) + S \right] \quad ..(6)$$

or for the transient period

$$A(t) = \frac{1424 (\mu \bar{z}) T}{Kh} \left[ \frac{1}{2} (\ln t_D + 0.809) + S \right] \quad ..(7)$$

We obtain the familiar form of the Forscheimer equation (either A or A(t) is applicable in all that follows)

$$\overline{P_R}^2 - P_{wf}^2 = AQ + BQ^2 \quad ..(8)$$

Which can also be written in the more familiar form as an approximation to the above equation as:

$$Q = C_{bh} (\overline{P_R}^2 - P_{wf}^2)^n \quad ..(9)$$

Breaking the total pressure drop into the laminar and turbulent pressure drop contributions we have

<table>
<thead>
<tr>
<th>Total Drop</th>
<th>Laminar Drop</th>
<th>Turbulent Drop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{P_R}^2 - P_{wf}^2$</td>
<td>$(\overline{P_R}^2 - P_{Lf}^2)$</td>
<td>$(P_{Lf}^2 - P_{wf}^2)$</td>
</tr>
</tbody>
</table>

The laminar contribution equation can then be written at reservoir datum as

$$Q = \frac{1}{A} (\overline{P_R}^2 - P_{Lf}^2)^{1.0} = \frac{Kh (\overline{P_R}^2 - P_{Lf}^2)^{1.0}}{1424 (\mu \bar{z}) T \left[ \ln \left( \frac{.472 \bar{r}_{e}}{r_w} \right) + S \right]} \quad ..(11)$$

for the laminar drop and

$$Q = \frac{1}{\sqrt{B}} (P_{Lf}^2 - P_{wf}^2)^{0.5} \quad ..(12)$$

for the turbulent drop.

Rearranging equation (8) we obtain

$$\frac{\overline{P_R}^2 - P_{wf}^2}{Q} = A + BQ \quad ..(13)$$
A plot of \((\bar{P_r}^2 - P_{wf}^2)/Q\) vs \(Q\) of the back pressure test data will readily yield \(A\) from the intercept and \(B\) from the slope. Plotting transient flow data the intercept would be \(A(t)\) from which we could readily calculate skin \((S)\). Fig. 10 and Table 1 illustrate the plot and calculations for Well C. A separate back pressure curve plot, \textit{that is a straight line}, can be made for both the turbulent and laminar pressure drop contribution. The laminar curve will have a slope \(n = 1.0\) and the turbulent curve a slope \(n = 0.5\). A composite or total pressure drop curve can then be readily constructed by summing the pressure drops from each curve at the same value of rate of flow. The composite curve or total pressure drop curve may then be a curved line except for the limiting slopes of \(n = 0.5\) and \(n = 1.0\). In most cases, however, the composite will yield a dominant slope \(n\) between 0.5 and 1.0 with its actual value being a function of the relative contribution of laminar and turbulent flow.

\section*{Slopes of the Back Pressure Performance Curves}

Examination of field performance curves indicates that low permeability gas wells will normally yield bottom-hole back pressure curves with slopes more nearly approaching 1.0, while high permeability gas wells yield slopes more nearly approaching 0.5. Popular belief has usually been based simply on the concept of permeability value, i.e., low permeability develops turbulent flow \((\beta\text{ is large})\), high permeability laminar flow. Also, it is often stated that the value of the exponent \(n\) is 0.5 for completely turbulent flow. In a radial flow system, there is no possible way of physically having turbulent flow throughout the drainage radius.

Returning again to the Forscheimer equation\(^8\)
\[
\frac{\bar{P_r}^2 - P_{wf}^2}{Q} = A Q + B Q^2
\]  \(..(8)\)

When \(Kh\) is large, the term \(AQ\) becomes small and we would have
\[
Q = \frac{1}{\sqrt{B}} (\bar{P_r}^2 - P_{wf}^2)^{0.5}
\]  \(..(14)\)

Similarly when \(Kh\) is small, the \(AQ\) becomes large and the \(BQ^2\) term can become negligible (not necessarily zero) when compared to the laminar pressure drop term. We could then write
\[
Q = \frac{1}{A} (\bar{P_r}^2 - P_{wf}^2)^{1.0}
\]  \(..(15)\)

It is clear then that it is not necessary for flow to be completely turbulent throughout the reservoir for the slope \((n)\) to be equal to 0.5.

The following table summarizes results obtained from isochronal tests on a few high and low permeability wells. All tests were run with sub-surface gauges in the well.
### Bottomhole Curve

<table>
<thead>
<tr>
<th>Well</th>
<th>Formation</th>
<th>Potential Flow Rate (Mscfd)</th>
<th>Slope (n)</th>
<th>Build-Up (K - md)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sandstone</td>
<td>57,000</td>
<td>0.554</td>
<td>960</td>
</tr>
<tr>
<td>2</td>
<td>Sandstone</td>
<td>170,000</td>
<td>0.532</td>
<td>1331</td>
</tr>
<tr>
<td>3</td>
<td>Sandstone</td>
<td>310,000</td>
<td>0.658</td>
<td>978</td>
</tr>
<tr>
<td>4</td>
<td>Sandstone</td>
<td>88,000</td>
<td>1.000</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>Sandstone</td>
<td>68,000</td>
<td>1.000</td>
<td>7</td>
</tr>
</tbody>
</table>

Back Pressure Curves at Surface Datum, "Wellhead Curves"

Gas cannot be sold at the bottom of the hole. It must first be produced through tubing (or casing, of course) surface equipment, a gathering line, then finally through a pipeline. Until the gas enters a trunkline, we must continue to predict the pressure drops through the entire system to obtain the ability of a well to deliver gas to a pipeline.

We can carry the discussion about back pressure curves and slopes one step further to include the tubing pressure drop effect and the resulting wellhead back pressure curve and its slope. If we divide the Forscheimer form of the bottom hole flow equation by the gas well hydrostatic head term \( e^s \), we obtain

\[
\frac{-Pr^2}{e^s} - \frac{Pwf^2}{e^s} = A \frac{Q}{e^s} + B \frac{Q^2}{e^s} \quad (16)
\]

where \( S = 0.0375 \frac{GH}{T_a^2} Z_a \) (This \( S \) should not be confused with skin)
and 
- \( e = \) natural log
- \( G = \) gas gravity
- \( H = \) vertical depth, ft
- \( T_a = \) average temperature, °R
- \( Z_a = \) gas deviation factor at average pressure
- \( P_c^2 = \frac{Pr^2}{e^s} \)
- \( P_w^2 = \frac{Pwf^2}{e^s} \)
- \( Q = \) Mscfd

\( P_c \) is the wellhead shut-in pressure and \( P_w \) is the wellhead static column flowing pressure. \( P_w \) is that pressure that would be recorded on the annulus while flowing if there were no packer in the well. Even if there is a packer in a well, it is a useful pressure to evaluate and a useful concept to use in back pressure curve performance analysis. (The simplifying assumption of a constant \( e^s \) does not introduce serious errors and we end up with some very useful equations that can be easily manipulated.)
We can now write the reservoir flow equation in terms of the more convenient wellhead pressures and now at a surface datum

\[ P_c^2 - P_w^2 = A_{wh} Q + B_{wh} Q^2 \]  \hspace{1cm} (17)

which can also be written in the more familiar form as

\[ Q = C_{wh} (P_c^2 - P_w^2)^n \]  \hspace{1cm} (18)

where now

\[ A_{wh} = \frac{1424 \ (\mu z) \ T}{Kh \ e^s} \left[ \ln \left( \frac{.472 \ r_e}{r_w} \right) + S \right] \]  \hspace{1cm} (19)

\[ B_{wh} = \frac{B}{e^s} \]  \hspace{1cm} (20)

**Tubing Friction Curves**

The basic equation relating wellhead static column pressure \( P_w \) and the wellhead flowing tubing pressure \( P_t \) as given in the IOCC Manual(4) is

\[ P_{wf}^2 = e^s P_t^2 + \left( \frac{F_r Q T_a Z_a}{31.62} \right)^2 (e^s - 1) \]  \hspace{1cm} (21)

where

\[ F_r = \frac{0.10797}{p^2.612} \]  \hspace{1cm} with \( D \) in inches.

Dividing both sides by \( e^s \), we obtain

\[ \frac{P_{wf}^2}{e^s} = P_t^2 + \left( \frac{F_r Q T_a Z_a}{31.62} \right)^2 \left( \frac{e^s - 1}{e^s} \right) \]  \hspace{1cm} (22)

with \( P_{wf}^2/e^s = P_{w}^2 \) we can rearrange and obtain

\[ Q = \frac{31.62 \ e^{s/2}}{\sqrt{(e^s - 1) F_r T_a Z_a}} (P_w^2 - P_t^2)^{0.5} \]  \hspace{1cm} (23)

The general form is simply

\[ Q = T (P_w^2 - P_t^2)^{0.5} \]  \hspace{1cm} (24)

or \( P_w^2 - P_t^2 = (Q/T)^2 \)  \hspace{1cm} (25)

that will plot as a straight line on log-log paper with a slope of 0.5. Thus \( (1/T^2) Q^2 = (P_w^2 - P_t^2) \) defines the pressure drop through the tubing string. (For convenience, in later discussions, let's define \( T_{wh} = 1/T^2 \).)
Combining the tubing pressure drop equation with the equation describing the pressure drops through the reservoir in terms of wellhead pressures, we obtain the wellhead back pressure curve equation that accounts for the total pressure drop through the system. The total pressure drop expressed at the tubing wellhead (surface datum) is then given by

\[ (P_c^2 - P_w^2) + (P_w^2 - P_t^2) = A_{wh} Q + B_{wh} Q^2 + T_{wh} Q^2 \]  \hspace{1cm} (26)

or

\[ (P_c^2 - P_t^2) = A_{wh} Q + (B_{wh} + T_{wh}) Q^2 \]  \hspace{1cm} (27)

which can also be represented as

\[ Q = C_{wh} (P_c^2 - P_t^2)^n \]  \hspace{1cm} (28)

Note that as a limiting condition, if \( T_{wh} \) is large compared to \( A_{wh} \) and/or \( B_{wh} \) (a very large bottom hole potential), the equation reduces to

\[ Q = C_{wh} (P_c^2 - P_t^2)^{0.5} \]  \hspace{1cm} (29)

Figures 11, 12, and 13 illustrate the effects of tubing friction on wellhead deliverability.

This indicates that the wellhead curve could in some instances be totally described by the pressure drop through the tubing string. A significant point is that we can use this equation to establish, for flow through any given tubing size, a maximum position for a wellhead curve, its potential, and it will have a slope of 0.500. For other diameter flow strings one need only \( \frac{D^{2.612}_{\text{New}}}{D^{2.612}_{\text{Present}}} \) \( C_{wh} \) to draw in its curve. This approach of changing flow string diameter can also be used when compositing total drops from each of the pressure drop curves, laminar, turbulent and tubing.

**Gathering Line or Pipeline Equation**

A pressure drop equation for a gathering line and a pipeline can be developed using the general flow equation as given in the Natural Gas Processors Suppliers Association, Engineering Data Book, 1966. (The fact that we can often treat a total field as a single back pressure curve can even allow us to include the pipeline pressure drop in our analysis or forecasts.)

The flow equation (assuming negligible elevation differences between inlet and outlet) can be written as:

\[ Q = \frac{5.487 D^{2.5}}{\sqrt{T_a' Z_a GL}} \log_{10} \left[ \frac{3.7 D}{K_e} \right] \left( \frac{P_{up}^2 - P_{down}^2}{ \sqrt{P_{up}^2 - P_{down}^2} \sqrt{P_{up}^2 - P_{down}^2}} \right)^{0.5} \]  \hspace{1cm} (30)
where \( Q \) = gas flow rate Mscfd @ 14.7 & 60° F  
\( D \) = inside diameter, in  
\( K_e \) = absolute roughness, in \((K_e = 0.0015 \text{ is suggested})\)  
\( T_a' \) = average flowing line temperature, °R  
\( Z_a \) = gas deviation factor at average pressure  
\( G \) = gas gravity  
\( L \) = pipe length, miles  
\( P_{up} \) = upstream pressure, psia (equal to \( P_t \) for gathering line connected to tubing)  
\( P_{dwn} \) = downstream pressure, psia  
\( Q = C_p L \left( P_{up}^2 - P_{dwn}^2 \right)^{0.5} \)

In the general form we have  
\[ Q = L \left( P_t^2 - P_{dwn}^2 \right)^{0.5} \]  
\( \ldots (30-A) \)

(Again for convenience, we will define \( L_{wh} = 1/L^2 \).) Now we can write equations at surface datum representing the total pressure drop through the system.  
\[ \Delta(P^2)_{total} = P_c^2 - P_{dwn}^2 \]

\[ = \Delta(P^2)_{darcy} + \Delta(P^2)_{skin} + \Delta(P^2)_{turb} + \Delta(P^2)_{tbg} + \Delta(P^2)_{line} \]

or  
\[ P_c^2 - P_{dwn}^2 = (P_c^2 - P_L^2) + (P_L^2 - P_w^2) + (P_w^2 - P_t^2) + (P_t^2 - P_{dwn}^2) \]

or  
\[ P_c^2 - P_{dwn}^2 = A_{wh} Q + (B_{wh} + T_{wh} + L_{wh}) Q^2 \]

recaptitulating  
\[ A_{wh} = \frac{1424}{Kh} \left( \frac{\mu z}{T} \right) \left[ \ln \left( \frac{.472 r_e}{r_w} \right) + s \right] \]  
\[ B_{wh} = B/e^s \]

\[ T_{wh} = \frac{(e^s - 1) \frac{F_r^2}{T_a'} \frac{Z_a^2}{G - 2}}{(31.62)^2} \]

\[ L_{wh} = \frac{T_a' Z_a GL}{(5.487)^2 D^5} \left[ \frac{\log_{10} \left( \frac{3.47 D}{K_e} \right)}{D^5} \right]^{2} \]

The total pressure drop equation finally can be represented as  
\[ Q = C_{total} \left( P_c^2 - P_{dwn}^2 \right)^n \]  
\( \ldots (31) \)
For deliverability or production forecasting this is the equation that should be used.

Still further, to account for the pressure drop through a well's required well site surface equipment (separator and dehydrator), the basic flow equation \( Q = C_e (p_1^2 - p_2^2)^{0.5} \) can be included into the above equations. A single data point on the equipment either measured or obtained from the manufacturer can be used to define \( C_e \).

Note that all the above pressure drop components can be graphed individually as \( \log Q \) vs \( \log \Delta(p^2) \). A total curve can then be constructed by a composition of all curves, thus relating pressure drop through the total system for a given flow rate.

Examination of the total pressure drop equations indicates that in wells with large bottom hole potentials we should expect the slope of the wellhead deliverability curve to approach 0.5, the slope of all pipe flow and turbulent flow pressure drop components. Conversely for small potential wells we would expect the slope of the wellhead curve to approach the slope of the bottom hole or Darcy flow curve, 1.0.

**Real Gas Flow** \( (p_g > 2500 \text{ psi}) \)

Al Hussainy, Ramey and Crawford\(^{(7)}\) showed that it was possible to consider gas physical property dependence on pressure by means of the real gas pseudo-pressure \( m(p) \). Although they indicated that it was important for the case of gas flow in tight high pressure formations with large drawdowns, it is equally important for high permeability formations with normal drawdowns.

The real gas pseudo-pressure \( m(p) \) was defined by them as:

\[
m(p) = 2 \int_{p_m}^{P} \frac{P}{\mu(P) Z(P)} \, dp \tag{32}
\]

where \( P = \) pressure, psia
\( \mu = \) gas viscosity, cps
\( Z = \) gas deviation factor
\( p_m = \) base pressure, psia (\( P = 0 \) is most convenient)

An \( m(p) \) could also be defined in a more familiar form as

\[
m(p) = \int_{p_m}^{P} \frac{1}{\mu_g \beta_g} \, dp \tag{33}
\]
where \( \beta_g = \frac{P_{sc}}{T_{sc}} \frac{T}{P} \) (sub sc indicates standard conditions)

then

\[
m(p) = \frac{P_{sc}}{P_{sc}} \int_{P_m}^{P} \frac{P}{\mu(P) Z(P)} \, dp
\]  

where temperature is in °R.

In all further discussions we will deal with the \( m(p) \) in terms of \( 1/\mu \beta_g \).

For simplicity of discussion, let us use the general steady state radial flow equation:

\[
q = \frac{7.088k}{\ln \left( \frac{r_e}{r_w} \right)} \int_{P_{wf}}^{P_e} \frac{1}{\mu \beta_g} \, dp
\]  

where \( q = \) surface rate of flow, bbl/day
\( K = \) effective permeability, Darcy
\( h = \) thickness, ft
\( r_w = \) wellbore radius, ft
\( r_e = \) external boundary radius, ft
\( P_{wf} = \) bottom hole flowing pressure, psia
\( P_e = \) external boundary pressure, psia

It is perfectly general and is equally applicable to either liquid or gas flow. For gas flow we can simply write

\[
q_g = \frac{7.088k}{\ln \left( \frac{r_e}{r_w} \right)} \int_{P_{wf}}^{P_e} \frac{1}{\mu \beta_g} \, dp
\]  

The integral can be expressed in terms of pseudo-pressures \( m(p) \)

\[
\int_{P_{wf}}^{P_e} \frac{1}{\mu \beta_g} \, dp = \int_{0}^{P_e} \frac{1}{\mu \beta_g} \, dp - \int_{0}^{P_{wf}} \frac{1}{\mu \beta_g} \, dp
\]  

or

\[
\int_{P_{wf}}^{P_e} \frac{1}{\mu \beta_g} \, dp = m(P_e) - m(P_{wf})
\]

The quantity \([m(P_e) - m(P_{wf})]\) is simply the area under the \( 1/\mu \beta_g \) curve from \( P_e \) to \( P_{wf} \). \( m(P_e) \) is the area under the curve from \( P_e \) to 0, and \( m(P_{wf}) \) is the area under the curve from \( P_{wf} \) to 0. The ABSOLUTE OPEN FLOW POTENTIAL can be expressed by

\[
AOFP = \frac{7.088k}{\ln \left( \frac{r_e}{r_w} \right)} m(P_e)
\]
Let us examine the basic shape of $1/\mu_g \beta_g$ with pressure. Fig. 14 is a plot of $1/\mu_g \beta_g$ for a gas reservoir with an initial shut in pressure of 5567 psia. At high pressures $1/\mu_g \beta_g$ is nearly constant, only slightly changing with pressure. Again, for simplicity of discussion, we can approximate the pressure function by two straight line sections as:

$$f(p) = \frac{1}{\mu_g \beta_g}$$

For the region where the pressure function is a constant ($1/\mu_g \beta_g$ is constant), we can evaluate the integral as

$$\int_{P_{wf}}^{P_e} f(p) \, dp = \frac{1}{\mu_g \beta_g} \int_{P_{wf}}^{P_e} dp$$

which when integrated between limits, yields

$$\int_{P_{wf}}^{P_e} f(p) \, dp = \frac{P_e - P_{wf}}{\mu_g \beta_g}$$

Then

$$q_g = \frac{7.08kh}{\ln \left( \frac{r_e}{r_w} \right)} \frac{(P_e - P_{wf})}{\mu_g \beta_g}$$

(Note that this is identical to the single phase liquid flow equation commonly used for oil wells.) A multipoint test conducted with drawdowns over the constant portion of the $1/\mu_g \beta_g$ curve should yield a straight line on a $q_g$ vs $\Delta p$ plot. This in fact is the case for a gas well isochronal test, Fig. 15, conducted in the reservoir represented by the $1/\mu_g \beta_g$ plot of Figure 14.

The same data when plotted in the conventional manner of log $q$ vs log $\Delta(p^2)$ yields a back pressure curve with a slope $n = 1.265$, Fig. 16. This is greater than the normally accepted maximum value of 1.0. (A curve with slopes greater than 1 is characteristic of a $\Delta p$ behavior plotted in the $\Delta(p^2)$ form.)
Neither a $\Delta p$ nor a $\Delta(p^2)$ extrapolation of the multipoint test can be justified to define the remainder of the back-pressure curve and to determine its absolute open flow potential. (It's only coincidental that the $\Delta(p^2)$ extrapolation of the example results in the same AOFP as the $m(p)$ plot. It should only be considered a result of trend plotting.) The only correct method of extrapolating the test results from Well D is by means of an $m(p)$ plot, Fig. 17.

Even though we cannot extrapolate the $\Delta(p)$ plot of Well D to a correct AOFP, it can and was used to validate what "appeared" initially to be an invalid isochronal test. All high pressure gas well test data should be field checked with a $\Delta p$ plot until $m(p)$ data can be developed.

Let us next examine the pressure function at pressures below $\approx 2500$ psi. Approximating $f(p)$ in this region with an equation of a straight line.

$$f(p) = aP + b$$

With the intercept $b = 0$, and $P_e < 2500$ psi

$$\int_{P_{wf}}^{P_e} f(p) \, dp = \int_{P_{wf}}^{P_e} aP \, dp = \frac{a}{2} \left( P_e^2 - P_{wf}^2 \right)$$

The slope $a$ for $b = 0$, is simply $(1/\mu_g \beta_g)/P_e$. We can then write

$$q_g = \frac{7.08kh}{\ln \left( \frac{r_e}{r_w} \right)} \left( \frac{P_e^2 - P_{wf}^2}{2P_e} \right)$$

with $\beta_g$ evaluated at $P_e$, we obtain the familiar $\Delta(p^2)$ form of equation

$$q_g = \frac{3.54kh T_{sc}}{\mu Z T P_{sc}} \ln \left( \frac{r_e}{r_w} \right)$$

Clearly then in the high pressure region where flow is behaving as a liquid, one can plot $p$ to analyze drawdown or buildup data. With all pressures over the low pressure region, one should plot $p^2$ to analyze drawdown or build-up data. If the pressure data covers both the high and low pressure regions, as one might expect in high pressure low permeability formations, one must plot $m(p)$. When in doubt, always use $m(p)$. 

- 15 -
Table 1
Well C

ISOCHRONAL TEST (BOTTOM-HOLE GAUGE)

\( \bar{P}_R = 1370 \) psia

<table>
<thead>
<tr>
<th>Flow No.</th>
<th>Q (MSCFD)</th>
<th>( P_{wf} ) (PSIG)</th>
<th>( \frac{\bar{P}<em>R^2 - P</em>{wf}^2}{psia^2} )</th>
<th>( \frac{\bar{P}<em>R^2 - P</em>{wf}^2}{psia^2/Mscfd} )</th>
<th>Flow Duration (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11,300</td>
<td>1327</td>
<td>75,096</td>
<td>6,646</td>
<td>16</td>
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<tr>
<td>2</td>
<td>6,700</td>
<td>1343</td>
<td>32,376</td>
<td>4,832</td>
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<tr>
<td>3</td>
<td>11,850</td>
<td>1325</td>
<td>80,400</td>
<td>6,785</td>
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<tr>
<td>4</td>
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<td>53,800</td>
<td>1,830</td>
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</table>

\( K_h = 306,060 \) md-ft (From Build-Up); \( K = 978 \) md
\( h = 313 \) ft
\( \bar{P} = 1346 \) psia
\( \mu = 0.014 \) cps
\( Z = 0.868 \)
\( T = 120^\circ F \)
\( t = 1 \) hr
\( Q = 18.1 \)
\( G = 0.655 \)
\( r_w = .33 \) ft
\( h_p = 70 \) ft (perforated at top)
EXAMPLE CALCULATIONS FROM ISOCHRONAL TEST
Well "C"

\[ A(t) = \frac{1424 \, (\mu Z) \, T}{K_h} \left[ \frac{1}{2} \left( \ln t_D + 0.809 \right) + S \right] = 1.00 \frac{\text{psi}^2}{\text{Mscfd}} \]

\[ S = \frac{A(t) \, K_h}{1424 \, (\mu Z) \, T} \left[ \frac{1}{2} \left( \ln t_D + 0.809 \right) \right] \]

With

\[ t_D = \frac{0.00633 \, K \, \bar{P} \, t}{\phi \mu r_w^2} = \frac{.00633 \, (978) \, (1346) \, 1}{(.181) \, (.014) \, (.33)^2 \, 24} \]

\[ t_D = 1.258 \times 10^6 \]

\[ S = \frac{1.0 \, (306,060)}{1424 \, (.014)(.868)(580)} - \left[ 7.43 \right] = +22.8 \]

PARTIAL PENETRATION SKIN (BROWN AND MARTING) \(^8\)

TOP 70 ft PERFORATED OUT OF 313 ft PAY

\[ b = \frac{70}{313} = 0.22 \]

\[ h = \frac{313}{.33} = 948 \]

\[ r_w = +19 \quad \text{Good Check} \]
EXAMPLE CALCULATIONS FROM ISOCHRONAL TEST

WELL "C"

\[
B = \frac{1424 (\mu Z) T}{Kh} \quad D = 27.322 \times 10^{-6} \quad \left( \frac{\text{psi}^2}{\text{Mscfd}} \right)
\]

\[
D = \frac{2.226 \times 10^{-15} \beta K G}{h \mu r_w}
\]

\[
B = \frac{3.17 \times 10^{-12} Z G T}{h^2 r_w} \beta
\]

\[
\beta = \frac{B h^2 r_w}{3.17 \times 10^{-12} Z G T} = \frac{27.322 \times 10^{-6} (313)^2 (.33)}{3.17 \times 10^{-12} (.868) (.655) (580)}
\]

\[
\beta = 8.45 \times 10^8 \text{ ft}^{-1}
\]

From Katz Curve, \( \beta = 3.4 \times 10^6 \text{ ft}^{-1} \)

NOTE: This difference is consistent with the author's in evaluating \( \beta \) values from field data over a large range of permeabilities. Actual \( \beta \)'s are usually 100 times larger than those obtained from Katz's Curve.\(^{(3)}\)
### CALCULATION OF \( m(p) \), WELL D

\[
m(p) = \frac{1}{\mu_g \beta_g} \sum_{P=0}^{P} \Delta P \left( \frac{1}{\mu_g \beta_g} \right)
\]

\[
\Delta P = \frac{1}{\mu_g \beta_g} \cdot \frac{P - P_{\text{avg}}}{Mscf - \text{psi}}
\]

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<tr>
<th>Pressure</th>
<th>( \Delta P )</th>
<th>( P_{\text{avg}} )</th>
<th>( Mscf )</th>
<th>( \text{psi} )</th>
<th>( \text{psi} )</th>
<th>( \text{Res Bbl-cps} )</th>
<th>( \text{psi} )</th>
<th>( \text{Res Bbl-cps} )</th>
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</table>
REFERENCES


PURPOSE OF MULTIPoint TESTING

1. Required by a state regulatory body for proration purposes, or to obtain an allowable.

2. Required to obtain a pipeline connection.

3. Company Policy.

4. Obtain sufficient information for reservoir and production engineering studies. Some of which are:
   a) Production forecasting (deliverability type or reservoir simulation).
   b) Determining number of wells for field development.
   c) Sizing tubing.
   d) Sizing gathering lines.
   e) Sizing trunklines.
   f) Compressor requirements.
   g) Determining necessity for stimulation.
   h) Proper evaluation of damage or skin effect.
   i) Establish a base performance curve for future comparison (Reconditioning Studies).
GAS WELL PRESSURE DROP EQUATION

\[ Q_{\text{TOTAL}} = C_{\text{TOTAL}} \left( P_C^2 - P_{\text{DWN}}^2 \right)^n \]

\[ Q = C_{\text{WH}} \left( P_C^2 - P_w^2 \right)^n \]

\[ Q = C_{\text{WHT}} \left( P_C^2 - P_L^2 \right)^n \]
FLOW AFTER FLOW (NORMAL SEQUENCE)

FLOW AFTER FLOW (REVERSE SEQUENCE)
NORMAL ISOCRATIONAL TEST

MODIFIED ISOCRATIONAL TEST

FIG. 3

FIG. 4
Figure 5.

Curve A: 24-hour, reverse sequence back-pressure test (slope=1.097).
Curve B: 24-hour, normal sequence back-pressure test (slope=0.701).
Curve C: 24-hour, normal sequence back-pressure test (slope=0.776).
Curve D: 24-hour, isochronal performance curve (slope=0.867).

Figure 6.

Curve A: 0.1-hour duration of flow
Curve B: 0.2-hour duration of flow
Curve C: 0.5-hour duration of flow
Curve D: 1.0-hour duration of flow
Curve E: 3.0-hour duration of flow
Curve F: 24.0-hour duration of flow
WELL "E" DRAWDOWN
3 HR. (ISCOHRNAL
PR = 468.9 Psi.; Q = 120.4 Mcfd
TYPE CURVE PLOT

Figure 7.

WELL "C" RESULTS FROM A CLEAN UP FLOW

Fig. 8.
WELL E

Δ DRAWDOWN - 16 Hrs.
Q = 82 Mcf/d
Ο BUILD UP
EXTENDED FLOW AFTER 4 HR.
ISOCHRONAL TEST
Pv 753 Pslg
TYPE CURVE PLOT
ALL STORAGE

FIG. 9

WELL C

A = (B) + 1.00 \frac{Pv^2}{Mcf/d}
B = 27,322 \times 10^{-6} \left( \frac{Pv^2}{Mcf/d} \right)

FIG. 10
Figure 13.
WELL "D" PLOT
CORE LAB PVT DATA

FIG. 14
WELL "D" (ΔP) PLOT
LIQUID FLOW BEHAVIOR
24 HR ISOCRONAL

FIG. 15

WELL "D"
CONVENTIONAL PLOT
$P_r^2 - P_w^2 = 2$ HR ISOCRONAL

FIG. 15
Figure 17.

WELL "D" mP) PLOT
BOTTOM HOLE POTENTIAL CURVE
2 HR ISOCRINAL TEST

Figure 18.