Note Title 3/31/2017

Reservoir Gas Rate Equation

$$q_{\bar{g}} = \frac{\boxed{\pi}\, kh\, (\bar{p}_{pR} - p_{pwf})}{\boxed{(p_{sc}/T_{sc})}\, T_R \left[\underbrace{\ln\frac{r_e}{r_w} - \frac{3}{4}}\ + s + D q_{\bar{g}}\right]}$$

"G"

Rate-independent skin

$D \leftarrow \beta$

PSS $\quad \{ \text{IA (IA} - \text{PSS)} \quad p_D(t_D) \ \& \ p_{pRi} \}$

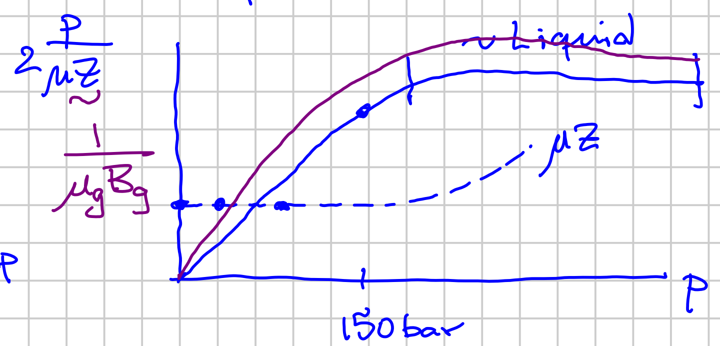
$\bar{p}_{pR}(t)$

pseudo-radial flow $\quad p_D = \frac{1}{2}\left[\ln t_D + 0.80907\right]$

Low Pressure $\lesssim$ 150 bar

$$p_p = 2\int \frac{p}{\mu Z}\, dp = 2\, \frac{1}{(\mu Z)^*_i} \int_0^p p\, dp$$

$$= \cancel{2}\, \frac{1}{(\mu Z)^*}\, \frac{1}{\cancel{2}}\, (p^2)$$



$$\int = \frac{p}{\mu Z}(\bar{p}_R - p_{wf})$$

Liquid-Like

"G"

$$q_{\bar{g}} = \frac{\boxed{\pi}\, kh\, (\bar{p}_R^2 - p_{wf}^2)}{(\mu_g Z_g)^*\, \boxed{(p_{sc}/T_{sc})}\, T_R \underbrace{\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s + D q_{\bar{g}}\right]}_{\text{dimensionless}}}$$

What is G?

Units of Interest

	Pure SI		Hybrid SI	Field
q_g	m^3/s	$\Rightarrow$	Sm^3/d	scf/D $(Mscf/D)$
h	m		m	ft
p	Pa	$\Rightarrow$	bar $\left(\begin{matrix} kPa \\ MPa \end{matrix}\right)$	$psia$
T	K		K	$°R$
μ	$Pa \cdot s$	$\Rightarrow$	cp	cp
r	m		m	in ft
D	$(m^3/s)^{-1}$		$(Sm^3/d)^{-1}$	$(scf/D)^{-1}$
k	m^2	$\Rightarrow$	md	md $\left(\begin{matrix} D \\ nd \end{matrix}\right)$

Conversion Factors

$$\underbrace{24\frac{hr}{d}}\cdot\underbrace{60\frac{min}{hr}}\cdot\underbrace{60\frac{s}{min}} = 86\,400\ \frac{s}{d}$$

$$\left[\frac{10^5\ Pa}{bar}\right]$$

$$1\ cp = 1\ mPa\cdot s = 10^{-3}\ Pa\cdot s \qquad \left[\frac{10^{-3}\ Pa\cdot s}{cp}\right]$$

SPE: $\underbrace{1\ D \equiv 10^{-12}\ m^2}$ wrong by $\dfrac{14.696}{14.50377}\ \dfrac{atm}{bar}$

$1\ md = 10^{-15}\ m^2$

1.0135

cgs units:

$$\left.\begin{aligned} v &= 1\ cm/s \\ L &= 1\ cm \\ A &= 1\ cm^2 \\ \Delta p &= 1\ atm \\ \mu &= 1\ cp \end{aligned}\right\} \quad k = 1\ D$$

$$\frac{1.8\ °R}{K} = \frac{9}{5}\ \frac{°R}{K}$$

$$q_g\left[\underline{m^3/s}\right] = \frac{\pi\, k[m^2]\cdot h[m]\cdot \Delta p_p\left[\frac{Pa^2}{Pa\cdot s}\right]}{\underline{(p_{sc}[Pa]/T_{sc}[K])}\, T_R[K]\,\left\{\ln\frac{r_e}{r_w}\right\}}$$

Want

$$q_g\left[\frac{Sm^3}{d}\right] = \frac{C\; k[md]\, h[m]\; \Delta p_p\left[\frac{bar^2}{cp}\right]}{T_R[K]\,\left\{\ln r_e/r_w \cdots\right\}}$$

How to get C?

e.g. $\left\{k[md]\cdot 10^{-15}\frac{m^2}{md}\right\} = k[m^2]$

$$\left\{\Delta p_p\left[\frac{bar^2}{cp}\right]\left(\frac{cp}{10^{-3}\,Pa\cdot s}\right)\left(\frac{10^5\,Pa}{bar}\right)^2\right\} = \Delta p_p\left[\frac{Pa^2}{Pa\cdot s}\right]$$

$$C' = \left(\frac{\pi}{P_{sc}(Pa)/T_{sc}(K)}\right)\cdot \overbrace{\left(10^{-15}\frac{m^2}{md}\right)\left(\frac{10^{10}}{10^{-3}}\right)}^{10^{-2}}$$

$P_{sc} = \underline{1.01325\; 10^5\; Pa} = 1.01325\text{ bar}$

$T_{sc} = 273.15 + 15.56\,°C = \underline{288.71\text{ K}}$

$$C' = \frac{3.14\cdot(288.71)}{(1.01325\cdot 10^5)}\left(10^{-2}\right) = 8.947\cdot 10^{-5}$$

$$q_g\left(\frac{m^3}{s}\right) = \frac{C'\; k[md]\, h[m]\; \Delta p_p\left[\frac{bar^2}{cp}\right]}{T_R[K]\left[\ln\frac{r_e}{r_w} + \cdots\right]}$$

$$q_g\left(\frac{m^3}{d}\right) = q_g\left(\frac{m^3}{s}\right)\cdot 86\,400\;\frac{s}{d}$$

$$C = (8.947 \cdot 10^{-5})(86\,400)$$

$$= 7.73$$

$$\Delta p_p = \bar{p}_{pR} - p_{pwf}$$

$$\boxed{q_{\bar{g}}\left(\frac{Sm^3}{d}\right) = \frac{7.73\; k[md]\; h[m]\; \Delta p_p\left[\frac{bar^2}{cp}\right]}{T_R[K]\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s + D q_{\bar{g}}\right]}}$$ ✓

Field units: $\Delta p_p \left[\frac{psi^2}{cp}\right]$ $T_R\,[°R]$ $h\,[ft]$

$q_{\bar{g}}\,[scf/D]$

$$C_F = \left\{ 7.73 \left(\frac{m}{3.28\ ft}\right)\left(\frac{bar}{14.50377\ psi}\right)^2 \left(\frac{1.8\,R}{K}\right)\right\}\left(\frac{35.31\ ft^3}{m^3}\right)$$

$$\underbrace{ft \to m \qquad psi^2 \to bar^2 \qquad \frac{1}{T_R[R]} \to \frac{1}{T_R[K]}}_{Sm^3/d}$$

$$q_{\bar{g}}\left(\frac{scf}{D}\right) = \frac{C_F\; k[md]\; h[ft]\; \Delta p_p\left[\frac{psi^2}{cp}\right]}{T_R[°R]\left[\ln r_e/r_w \cdots\right]}$$

$$C_F = 0.712$$

Well Performance

$\boxed{0.703}$

$D \to m^2$ exactly

Fetkovich paper: (uses p^2 instead p_p => add $\left(\frac{1}{\mu z}\right)$)

$$q_{\bar{g}} = \frac{kh\,(\bar{p}_{pR} - p_{pwf})}{1424\,T_R\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s + Dq_{\bar{g}}\right]}$$

$\left[\frac{Mscf}{D}\right]$

$$1424 = \frac{10^3\ scf/Mscf}{0.703}$$

β : Forchheimer constant

$$= \frac{10^3}{C_F}$$

$$p_p = \int \frac{1}{\mu_g \beta_g}\,dp$$

$$\beta_g = \left[\frac{p_{sc}}{T_{sc}} \cdot T_R\right] \frac{p}{z_g}$$

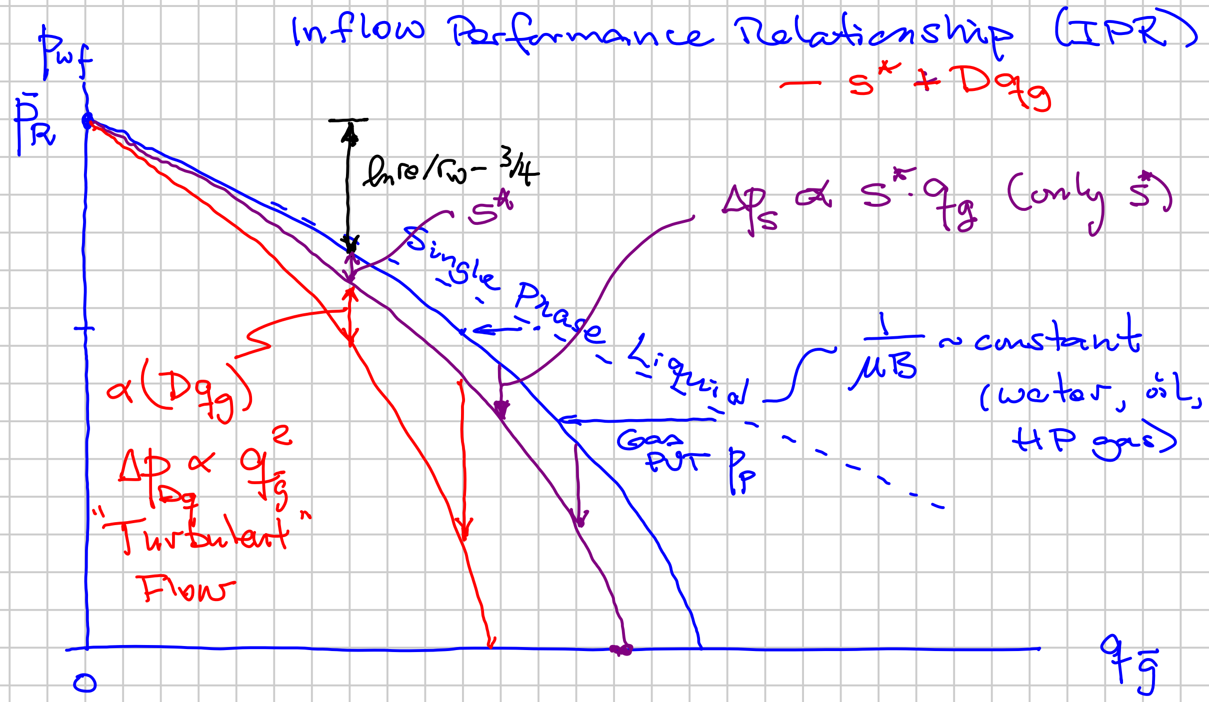
Range of Magnitude of all variables (parameters)

	Field		Orders of Mag.
$q_{\bar{g}}$		10 Mscf/D – 120 MMscf/D	4
h		(5) 10 ft – ~~1~~000 ft	2
k	Pre-Shale	0.001 md – 10,000 md	7
	Post-Shale	10 nd – 10,000 md	
		$10 \cdot 10^{-6}$ md	11
p_R		100 psi – 10,000 psi	2
$\ln r_e/r_w$		5 – 10	—
s		−6 to +100	2
T_R		540°R – 800°R	—
μ_g		0.015 – 0.05	—

10 × \$2 × 365 ~ \$5000/yr ~ Abandonment Value in USA

120 000 Mscf/D × 2 × 365 \$ $87 \cdot 10^6$ /yr

Rewrite Reservoir Gas Rate Eq as a Quadratic



$$q_g = \frac{7.73\ kh\ (p_{pR} - p_{pwf})}{T_R\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s^* + Dq_g\right]}$$

$$\underbrace{p_{pR} - p_{pwf}}_{\Delta p_{pR}} = \left(\frac{T_R\, q_g}{7.73\ kh}\right)\left\{\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s^*\right] + Dq_g\right\}$$

$$= A\, q_g + B\, q_g^2$$

$$B\, q_g^2 + A\, q_g - (\Delta p_p) = 0$$

$$ax^2 + bx + c$$

$$\boxed{q_g = \frac{-A + \left[A^2 + 4B\,\Delta p_p\right]^{0.5}}{2B}}$$

"LAMINAR" (Darcy) Pressure Drop $\quad A = \left(\frac{T_R}{7.73\ kh}\right)\left[\underbrace{\ln\frac{r_e}{r_w} - \frac{3}{4}}_{8} + s^*\right]$

$s^* = -6$

$= +10$

"TURBULENT" $\quad B = \left(\frac{T_R}{7.73\ kh}\right) \cdot D$

Forchheimer β

p_p = Low Pressure Assumption $\Delta p_p = \frac{1}{(\mu Z)^*}(p_R^2 - p_{wf}^2)$

$$A' = \frac{T_R (\mu Z)^*}{7.73\ kh}\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s^*\right]$$

$$B' = \frac{T_R (\mu Z)^*}{7.73\ kh} \cdot D$$

"Δp_p" $\boxed{\Delta p^2} = p_R^2 - p_{wf}^2$

Into the quadratic eq. term