# Darcy's Law d'Arcy

$$v_x = \underbrace{\left(\frac{k}{\mu}\right)}_{C} \frac{dp}{dx}$$

(permeability)

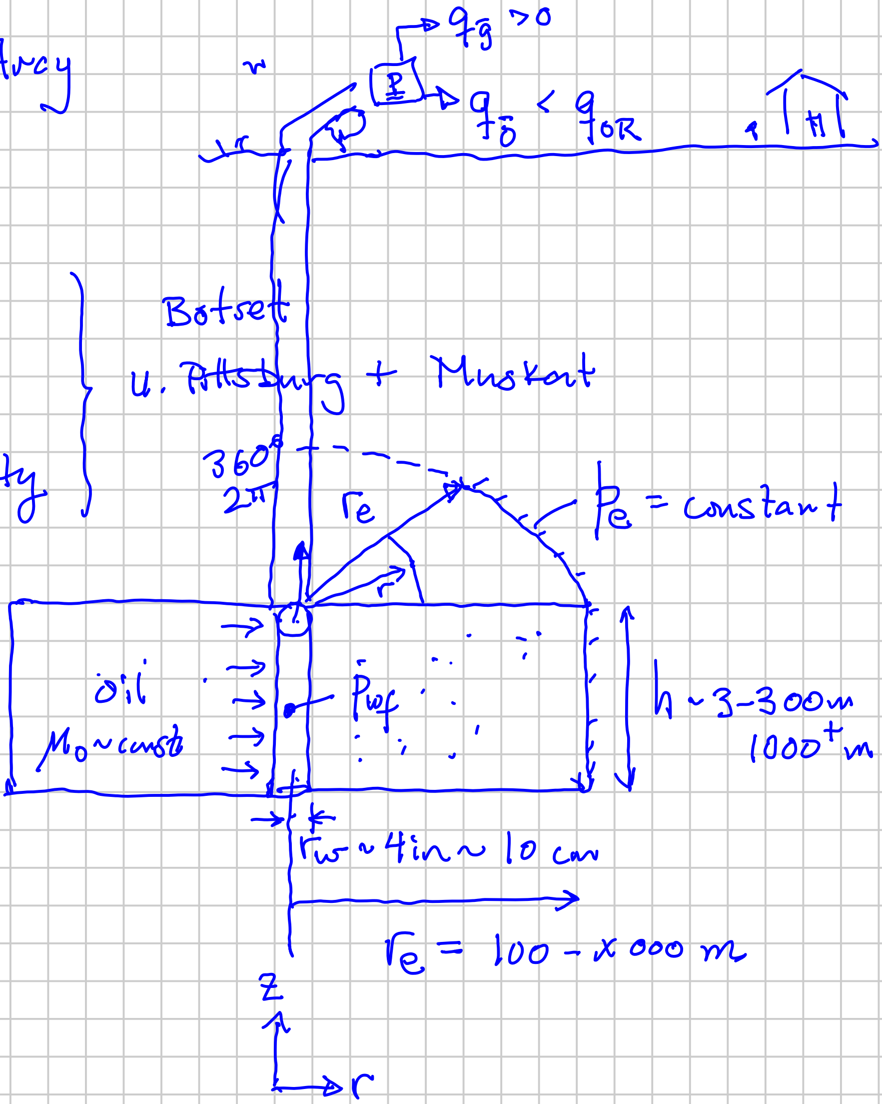
$k$ : Rock property

1930s

$\mu$ : fluid property (viscosity)

Botset

U. Pittsburg + Muskat

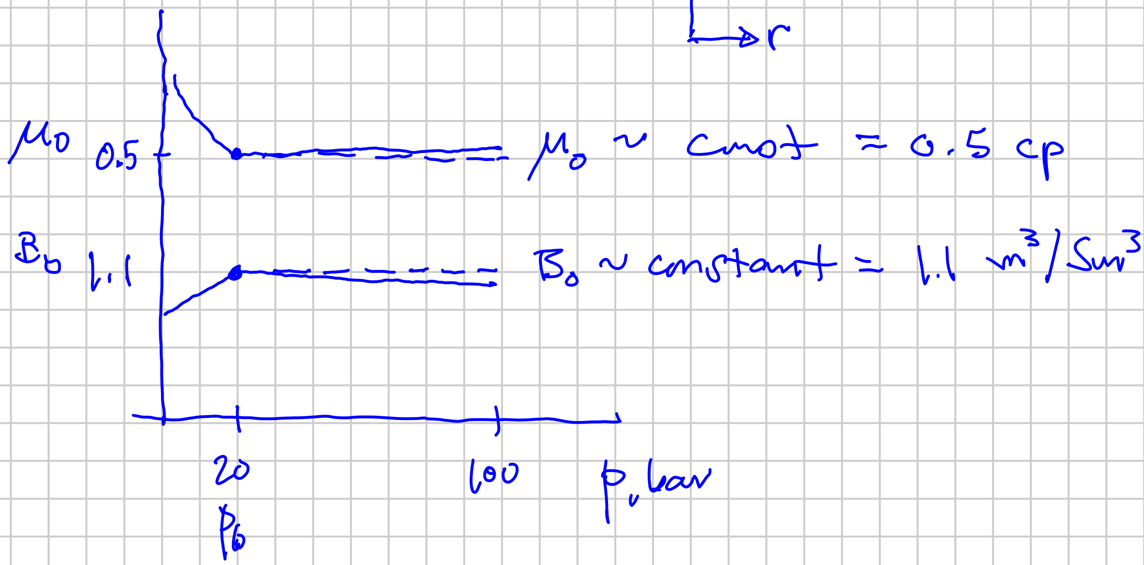



PSS : BD

$\Rightarrow q_{oR}(r, z)$ = constant

$q_m(r, z)$ = constant

$\rho_o$ = constant




By 10:30

$$\boxed{q_{\bar{o}} = f(p_{wf},\ p_e = p_i)}$$

\$40/STB

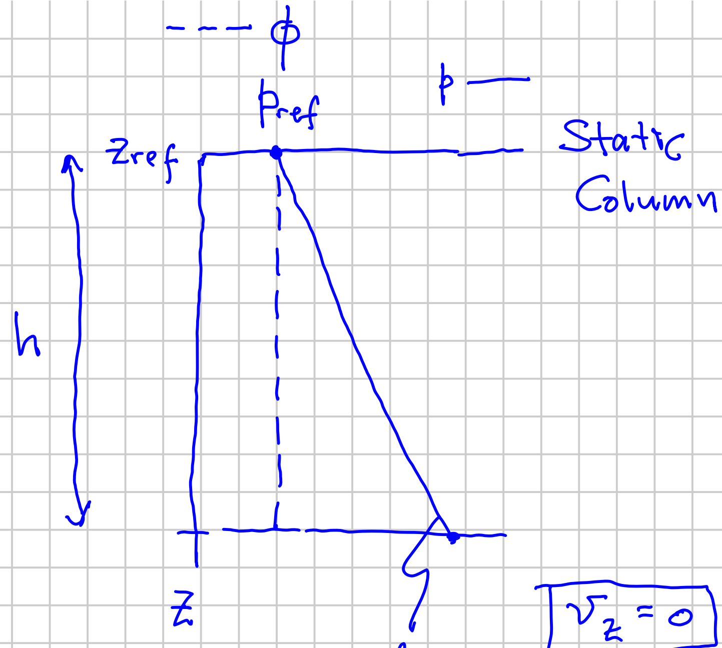
k
h
$r_e$
$r_w$
$\mu_o$
$B_o$

Vertical flow in z-direction

$$v_z = \frac{k}{\mu}\frac{d\phi}{dz}$$

$$\phi = p + \rho g z$$

$$\phi = p_{ref} + \rho g (z - z_{ref})$$




$$\boxed{v_z = 0}$$

$$\frac{dp}{dz} = \rho g$$

$$\Rightarrow \; p = p_{ref} + \rho g (z - z_{ref})$$

Derive Oil Rate Eq. $q_{\bar{o}} = q_o$ (surface oil rate)
for Radial-Cylindrical geometry

$$v_r = \frac{k}{\mu}\frac{dp}{dr}$$

$$\boxed{q_{oR}} = q_{mo} \cdot \rho_o^{-1}$$

at $p(r)$ — mass rate

$\Rightarrow$ SS : $q_{mo}(r, z) = \text{constant} = q_{mo}(r)$

$$q_{oR}(r) = \underbrace{\overset{\text{Darcy}}{v(r)}} \cdot \underbrace{A_\perp(r)}$$

$$A_\perp(r) = 2\pi r h$$

Want $q_{\bar{o}} = q_o \iff q_{oR}$

PUT:

$$\text{constant } (1.1) \sim B_o = \frac{V_{oR}(p)\frac{1}{\Delta t}}{V_{\bar{o}} \frac{1}{\Delta t}} = \frac{q_{oR}}{q_{\bar{o}}}$$

$$\boxed{q_{\bar{o}}} = q_{oR} \cdot \frac{1}{B_o} = \underbrace{\frac{k}{\mu_o} \cdot \frac{dp}{dr} \cdot 2\pi r h}_{q_{oR}} \cdot \frac{1}{B_o}$$

Assume: $\mu_o \sim$ const (0.5 cp)

$B_o \sim$ const (1.1 $m^3/Sm^3$, bbl/STB)

$$q_{\bar{o}} = \frac{2\pi \, kh}{\mu_o B_o} \; r \; \frac{dp}{dr}$$

* SS Assumption : $q_{om}$ = const at all r

$q_{oR}$ = —— " —— = $q_{om}/\rho_o$

[ $q_{\bar{o}}$ = —— " ——

$$q_{\bar{o}} \int_{r_w}^{r_e} \frac{1}{r} dr = \underbrace{\frac{2\pi}{\mu_o B_o}}_{\sim \text{ Constant}} (kh) \int_{p_{wf}}^{p_e} dp$$

$\Omega$

(include unit conversions)

$$q_{\bar{o}} \cdot \ln\frac{r_e}{r_w} = \frac{2\pi}{\mu_o B_o} kh \cdot (p_e - p_{wf})$$

$$\boxed{q_o = \underbrace{\frac{2\pi}{\mu_o B_o \cdot \ln r_e/r_w}}_{\text{Productivity Index (J)}} kh \underbrace{(p_e - p_{wf})}_{\text{Drawdown}}}$$ Pure SI

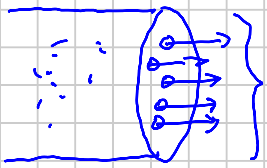
"PI" Productivity Index (J) Drawdown

(STB/D) / psi psi

$$\boxed{q_o = PI\ \Delta p}$$

Darcy Velocity

Pore Velocity (Local) : EOR only



$$v_D = \frac{q_R}{A_\perp} \ll v_p = \frac{q_R}{(A_\perp \cdot \phi \cdot (1-S_w))}$$

$A_\perp$: Bulk Flow Area

$(A_\perp \cdot \phi \cdot (1-S_w))$: Actual flow area in pores

Units:

Pure SI → $q$ [$m^3/s$] $v$ [$m/s$]

$h$ [$m$] $r$ [$m$]

$\mu$ [$Pa \cdot s$]

$B$ [$-$]

$p$ [$Pa$]

$k$ [$m^2$]

| | SI | LAB Darcy Units | Metric (Quasi-SI) | Field Units OIL | GAS |
|---|---|---|---|---|---|
| $v$ | m/s | cm/s | m/d | ft/D | |
| $q$ | $m^3/s$ | $cm^3/s$ | $m^3/d$ | STB/D | scf/D<br>Mscf/D<br>MMscf/D |
| $A$ | $m^2$ | $cm^2$ | $m^2$ | $ft^2$ | |
| $h, w, r$ | m | cm | m | ft | |
| $p$ | Pa | atm | ⟨bar⟩ = $10^5$ Pa | psi | |
| $\mu$ | Pa·s | cp | cp ≡ m Pa·s | cp | |
| $B$ | – | – | – | – | |
| $k$ | $m^2$ | D (Darcy) | (D or) $\underbrace{\text{md}}$ | md (or D) | |

Field Units Oil

$$\underbrace{q_o}_{\text{STB/D}} = \frac{\underbrace{k}_{\text{md}}\,\underbrace{h}_{\text{ft}}\,(\underbrace{p_e - p_{wf}}_{\text{psi}})}{141.2\,\underbrace{\mu_o}_{\text{cp}}\,\underbrace{B_o}_{-}\,\underbrace{\ln r_e/r_w}_{-}} \Rightarrow q_o = \frac{kh}{141.2\,\ln r_e/r_w} \cdot \underbrace{\int_{p_{wf}}^{p_e} \frac{k_{ro}}{\mu_o B_o}\,dp}$$

How to get 141.2

Have:

$$q_o\left[\frac{m^3}{s}\right] = \frac{2\pi\, k[m^2]\, h[m]\, \Delta p\,[Pa]}{\mu_o[Pa \cdot s]\, \cancel{B_o}\, \cancel{\ln r_e/r_w}}$$

Want

$$q_o\left[\frac{STB}{D}\right] = C\cdot\frac{k[md]\ h[ft]\ \Delta p\,[psi]}{\mu\,[cp]}$$

Example: $k = 1\,md$

$h = 1\,ft$

$\Delta p = 1\,psi$

$\mu = 1\,cp$

$$\frac{m^3}{s} = \frac{2\pi\left[\cdot(1\,\cancel{md})\cdot\frac{10^{-15}\,m^2}{\cancel{md}}\right]\cdot h\left[(1\,\cancel{ft})\cdot\left(\frac{m}{3.28\,\cancel{ft}}\right)\right]\cdot\Delta p\left[(1\,\cancel{psi})\cdot\left(\frac{\cancel{bar}}{14.50377\,\cancel{psi}}\right)\cdot\left(\frac{10^5\,Pa}{\cancel{bar}}\right)\right]}{\mu\left[(1\,\cancel{cp})\left(\cdot\frac{1\,\cancel{mPa\cdot s}}{1\,\cancel{cp}}\right)\cdot\left(\frac{Pa\cdot s}{10^3\,\cancel{mPa\cdot s}}\right)\right]}$$

$$= \underbrace{\left(\frac{2\pi\cdot 10^{-15}\cdot\frac{1}{3.28}\cdot\frac{1}{14.5}\cdot 10^5}{10^{-3}}\right)}\ \frac{k[md]\,h[ft]\,\Delta p\,[psi]}{\mu[cp]}$$

$$= \underbrace{\left[\left(\quad\right)\frac{k[md]\,h[ft]\,\Delta p\,[psi]}{\mu[cp]}\right]}\left(6.289\frac{STB}{\cancel{m^3}}\right)\cdot\frac{60\cdot 60\cdot 24\,\cancel{s}}{D}$$

$$q_o\left[\frac{STB}{D}\right] = \underbrace{\left(\quad\right)\left(\frac{\cancel{m^3}}{\cancel{s}}\right)}_{=\frac{1}{141.2}}\ \frac{k[md]\,h[ft]\ \Delta p\,[psi]}{\mu[cp]\,B\,\ln r_e/r_w}$$

$k[m^2]$ ? $k[md]$

$10^{-12} m^2 = 1 \mu m^2 = 1$ Darcy $= 1 D = 1000$ md

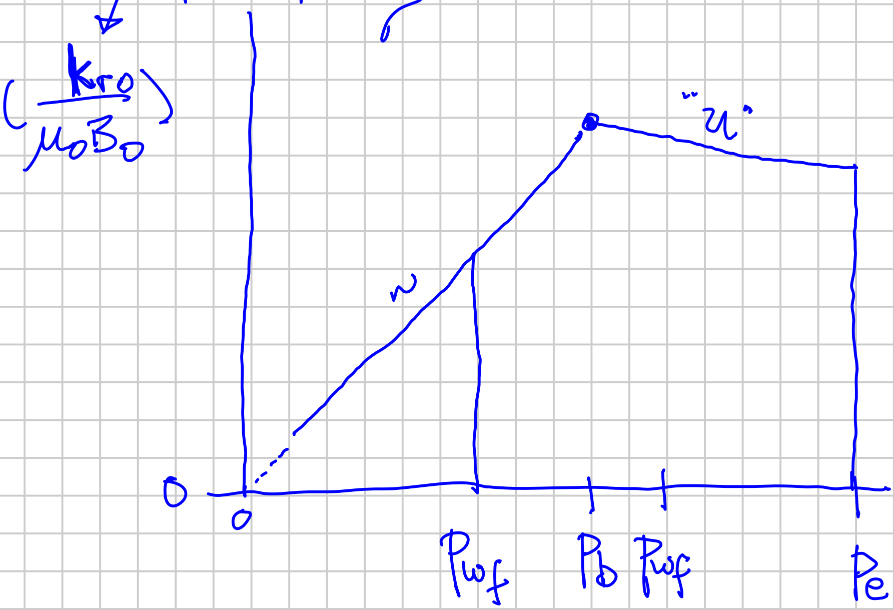
$= 1 (\mu m)^2$

$$\boxed{10^{-15} m^2 = 1 \text{ md}}$$

Real oil $\mu_o B_o$

$k_o h \frac{1}{\mu_o B_o} \int dp$ ✓ $p \geq p_b$

Oil Relative Permeability




$$\int \frac{k_o}{\mu_o B_o} dp$$

194x Muskat - Evinger
196x Vogel
197x Fetkovich

$p_{wf} < p_b$