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Backpressure Equation for Layered Gas Reservoirs

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Abstract

This paper presents a backpressure equation (BPE) for wells producing from layered gas reservoirs with or without
communication. The proposed BPE handles backflow between the layers through the wellbore for non-communicating layered
systems, and accurately describes performance of wells experiencing differential depletion.

The proposed multi-layer BPE has the same form as the familiar backpressure equation for single-layer gas reservoirs, where
the correct averages are defined for reservoir pressure and backpressure constants.

The BPE is validated against numerical simulation models, as well as field data which include decades of historical production
performance and annual shut-in pressures. All numerical models and field data used to validate the BPE are publicly available.
This paper gives guidelines on welltest design to quantify reservoir parameters in layered systems, based on systematic studies
with numerical simulation models.

Background

Layered reservoirs without communication, also referred to as layered no-crossflow reservoirs, consist of separate layers
without communication within the reservoir; layers only communicate through the wellbore.

One of the first attempts to study the transient performance of layered reservoirs was Lefkovits et al. (1961). They show
individual layer gas rates as a function of each layer *kh* product, but do not consider production performance solutions for
boundary-dominated (pseudosteady state, PSS) conditions.

Fetkovich et al. (1990) studied and identified all key performance characteristics of layered no-crossflow systems producing
under boundary-dominated conditions. One of their many important observations is Curve 6 in their Fig. 12, showing that the
backpressure relation for a differentially depleting system is, in fact, a straight line with exponent n~1. We show, in this paper,
that this is an expected and general observation for any layered system, and that the layered no-crossflow backpressure
equation is the same as for a single-layer system with equal total *kh*, but using the layer PI-averaged shut-in pressure.

El-Banbi and Wattenbarger (1996) developed a model to match production data from a layered no-crossflow system during
boundary-dominated conditions, using individual-layer coupling of material balance and PSS rate equations. This model is
used to estimate individual layer properties, for the assumption of constant bottomhole flowing pressure. Another attempt to
estimate layer properties and gas in place for layered no-crossflow reservoirs was Kuppe et al. (2000). This work allows
changes in bottomhole flowing pressure, but does not handle extended shut-ins resulting in backflow through the wellbore.

This paper will primarily consider layered no-crossflow reservoirs, but some results are shown to be applicable to reservoirs
with partially- or fully-communicating layers. The backpressure equation presented is valid for all layered reservoirs, but the
coupled material balance approach is only valid for non-communicating layer systems.

Standard Backpressure Equation

The standard backpressure equation for a well producing from a single layer reservoir is given by Fetkovich (1975).

$$q_g = C_R(p_c^2 - p_w^2) \tag{1}$$

The backpressure constant, C_R, is defined as:

$$C_R = \frac{4.18 k h e^S}{T_R \left(\ln \dfrac{r_e}{r_w} - \dfrac{3}{4} + s\right) \mu_g z} \tag{2}$$

with q_g in std m3/d, p in bar, k in md, h in m, T_R in K, and μ_g in cp. The gravity term, S, is defined as:

$$S = \frac{0.0684 \gamma_g D}{\bar{T} \bar{z}} \tag{3}$$

This S must not be confused with the skin factor, s.

The surface datum pressures, p_c and p_w, are converted to bottomhole pressures through the gravity term. The different pressure datums are shown in **Fig. 1**.

$$p_R = e^{S/2} p_c ; \quad p_{wf} = e^{S/2} p_w \tag{4}$$

A more generalized rate equation using pseudopressure can be used instead of the pressure-squared (p^2) approximation. In that case all equations developed in our work, should use $C_R = 4.18 kh/[T_R(\ln(r_e/r_w) - 0.75 + s)]$, and ($p_c^2 - p_w^2$) should be replaced by bottomhole pressures expressed as pseudopressures ($p_{pR} - p_{pwf}$), where $p_p = 2 \int_0^p p/\mu z \, dp$.

Multi-Layer Backpressure Equation

For a well producing from a layered no-crossflow reservoir, we have:

$$q_g = \sum_{l=1}^{N} q_{gl} = \sum_{l=1}^{N} C_{Rl}\left(p_{cl}^2 - p_w^2\right) \tag{5}$$

where p_w is common for all layers, assuming no pressure drop in the wellbore throughout the perforated interval. Eq. 5 can be rewritten as:

$$\frac{q_g}{\sum_{l=1}^{N} C_{Rl}} = \frac{\sum_{l=1}^{N}\left(C_{Rl} p_{cl}^2\right)}{\sum_{l=1}^{N} C_{Rl}} - p_w^2 \tag{6}$$

where we define the total productivity index (PI) as the sum of layer PI's:

$$C_R = \sum_{l=1}^{N} C_{Rl} \tag{7}$$

and,

$$\bar{p}_c^2 = \frac{\sum_{l=1}^{N}\left(C_{Rl}p_{cl}^2\right)}{C_R} \tag{8}$$

The average wellhead shut-in pressure, $\bar{p}_c$, represents the PI-averaged reservoir pressure of all the layers at surface datum. Shut-in pressure $\bar{p}_c$ can be shown to represent the pressure recorded at the wellhead, as Eq. 5 is satisfied for $q_g = 0$ when $p_w = \bar{p}_c$.

Now, Eq. 6 can be written in the familiar backpressure form:

$$q_g = C_R(\bar{p}_c^2 - p_w^2) \tag{9}$$

Numerical Model

A 2D numerical radial single well simulation model was constructed to validate the multi-layer backpressure equation. The simulation model is based on a model presented by Fetkovich et al. (1990). The reservoir properties are given in **Table 1**.

Both reservoir layers have a drainage radius of 908 m. The wellbore radius is 0.091 m. The model consists of 50 cells in the radial direction, and one horizontal cell-layer per reservoir layer. The reservoir gas has a specific gravity of 0.7.

The numerical model was produced for 10 years against a constant bottomhole pressure of 2.95 bara, and then shut in for one year. It was subsequently produced for three years, and shut in for another year. This was continued until the simulation time reached 20 years. The production profile of the model is shown in **Fig. 2**.

In addition to the BPE, a material balance is used to calculate the depletion performance of the numerically simulated well.

$$\left(\frac{p_c}{z}\right)_l = \left(\frac{p_{ci}}{z_i}\right)_l \left(1 - \frac{G_p}{G}\right)_l \tag{10}$$

TABLE 1 – NUMERICAL MODEL PARAMETERS			
	Layer 1	Layer 2	Unit
k	100	1	md
h	0.61	6.7	m
	2	22	ft
φ	0.15	0.15	–
p_i	29.5	29.5	bara
	428	428	psia
s	0	0	–
S_{wi}	0.514	0.514	–
C_R	17.79	1.96	std m^3/d/bar^2
	2.99E-3	3.29E-4	scf/d/psi^2

The production profile, together with the reservoir parameters given in Table 1, was used as input to the backpressure equation presented in this paper. Bottomhole pressures, individual layer pressures, and layer gas rates were calculated, and compared with the output from the numerical simulator. The bottomhole pressures are presented in **Fig. 3**, layer pressures are presented in **Fig. 4**, and individual layer gas rates are presented in **Fig. 5**.

As is seen from the comparison of the BPE calculations and the output from the numerical simulator, the BPE gives an accurate description of the layered reservoir.

Field Data

Production data from the Buf #3 well (API #3513900778), producing from the Guymon-Hugoton field in Oklahoma, USA was used to test the BPE against actual field data. This reservoir consists of three non-communicating productive layers. The well was completed in 1946. Production data from 1967 – 2009 was publicly available. Only the cumulative production in the beginning of 1967 was available to estimate the gas production rates prior to 1967. For simplicity, a constant gas rate was assumed between 1946 and 1967 (amounting to the known cumulative production in 1967). Annual wellhead shut-in pressure

data was available for the period 1967 – 2000. This pressure data was used to validate the BPE.

The wellhead shut-in pressures were collected during annual 72-hour (96-hour since 1975) shut-ins. These shut-in periods were incorporated in the gas rate table used as input to the BPE, and the calculated tubinghead pressure at the end of the shut-ins was compared with the reported test pressures.

Reservoir parameters for the field are taken from Fetkovich et al. (1994), and are presented in **Table 2**.

TABLE 2 – RESERVOIR PARAMETERS FOR THE GUYMON-HUGOTON FIELD				
	Layer 1	Layer 2	Layer 3	Unit
Name	Herrington	Krider	Winfield	–
k	0.1	9.0	3.3	md
h	17.7	15.2	12.2	m
	58	50	40	ft
φ	0.06	0.08	0.08	–
p_i	33.8	33.8	33.8	bara
	490	490	490	psia
s	-5.44	-5.44	-5.44	–
S_{wi}	0.76	0.60	0.67	–
S	0.157	0.157	0.157	–
T_R	26.7	26.7	26.7	°C
	80	80	80	°F
C_R	2.02	139.96	42.14	std m^3/d/bar^2
	3.39E-4	2.35E-2	7.07E-3	scf/d/psi^2
C_t	1.15E4	1.15E4	1.15E4	std m^3/d/bar
	2.80E4	2.80E4	2.80E4	scf/d/psi

The specific gravity of the reservoir gas is 0.73, the depth to the top of the reservoir is 853.5 m, and the tubing outer diameter is 2-3/8 inches.

The available pressure data is all measured at the wellhead. It is therefore necessary to calculate tubinghead pressures, p_t, from the bottomhole pressures calculated by the BPE.

$$q_g = C_t(p_w^2 - p_t^2)^{0.5} \tag{11}$$

where the tubing constant, C_t, is:

$$C_t = \frac{13.0\exp(S/2)}{\sqrt{e^S - 1}F_r\bar{T}\bar{z}} \tag{12}$$

Eq. 11 is used to convert the bottomhole pressures, p_w, from the BPE to tubinghead pressures, p_t. A friction factor, F_r, of 0.00612 was assumed. These tubinghead pressures are compared with the recorded pressure tests performed on the well.

The only unknowns remaining in the model are the drainage radii, r_{el}, for the individual layers. These drainage radii are used as regression variables to fit the tubinghead pressures calculated by the BPE to the test pressures recorded at the well.

$$F_{SSQ} = \sum_{t=0}^{T}\left(\frac{p_t^{BPE} - p_t^{obs}}{p_{ref}}\right)^2 \tag{13}$$

The accuracy of the BPE is evaluated from the sum of squares (SSQ), F_{SSQ}, and is calculated from Eq. 13. The drainage radii are fit to minimize the SSQ. These layer radii, r_{el}, represent layer gas in place, G_l, and realistically over such a long period of time their value could change because of infill drilling.

In addition to the tubinghead shut-in pressures, a bottomhole pressure test was conducted for the individual reservoir layers in the beginning of 1989. This gives an extra independent verification of the BPE, but is not included in the fitting of the drainage radii.

The production profile for the well is presented in **Fig. 6**. As seen from this figure, the gas rate is set constant prior to the start of the publicly available production data.

The resulting best-fit values for the drainage radii are 832 m, 1119 m, and 1041 m, for layers $1 - 3$ respectively (908 m represents spacing of 640 acres). The tubinghead pressures for the best-fit model are presented in **Fig. 7**, and the layer pressures are presented in **Fig. 8**. The individual layer gas rates are shown in **Fig. 9**. The BPE accurately predicts the performance of the well, and the differential depletion is represented correctly by the model, as seen from Fig. 8.

Fig. 7 shows some of the measured tubinghead pressures between 11500 and 14500 days (May 1978 – August 1986) are under predicted by the BPE. A possible cause could be longer shut-ins of the well than the required 96 hours. The BPE is also under predicting the two measured pressures between 7180 and 7550 days (June 1966 – June 1967), just at the start of publicly available rate data. Because the rate profile is not known prior to this, the under prediction could be caused by a long shut in of the well.

Numerical Model Based on Field Example

It is evident that the BPE fits the measured pressure data from the Buf #3 well reasonably. To illustrate how well the BPE would fit a gas well where all pressure data and individual layer rates were available; a numerical simulation model was built. The model is constructed based on the best-fit parameters from the field case. The model consists of 100 cells in the radial direction, and one horizontal cell-layer per reservoir layer.

The wells' negative skin, *s*, is implemented in the model according to Hawkins (1956).

$$s = \left(\frac{k}{k_a} - 1\right) ln \frac{r_a}{r_w}$$
(14)

The extent of the stimulated region, r_a, was selected to be 21.8 m (corresponding to the outer boundary of radial cell 58 in the model). This gives permeabilities in the stimulated region of 17, 1537, and 564 md for the three layers respectively.

A comparison of the tubinghead pressures from the numerical model and the BPE is shown in **Fig. 10**. The reservoir layer pressures are presented in **Fig. 11**, and the individual layer gas rates are shown in **Fig. 12**. As can be seen from these figures, the predictions of the BPE are at all times within symbol thickness of the numerical model results.

Backpressure Analysis for Monitoring Well Performance

Backpressure analysis may be used as a tool to detect deterioration in well performance for wells producing from layered gas reservoirs. **Fig. 13** shows the backpressure plot for a well with different shut-in durations. Each set of shut-ins corresponding to a specific shut-in duration fall on a straight line on a log-log plot. Thrasher (1995), as well as Golan and Whitson (2003), illustrates this behavior for a single-layer system. As seen from Fig. 13, this also applies for a multi-layer system. The model used to generate the data in Fig. 13 is a two layer model with layer thicknesses of 6.1 and 61 m, layer permeabilities of 0.1 and 0.01 md, and a stimulated region with permeability of 100 md extending 22.9 m into the reservoir. The initial reservoir pressure is138 bara. All other properties are equal to the two layer model previously described.

When using layered backpressure analysis to monitor a well's performance, it is important to be consistent from test to test. Each shut-in period should be of ~ equal length, and the shut-in pressure, $\bar{p}_c$, should be recorded at the end of the shut-in. The gas rate, q_g, and associated flowing pressure, p_t, to be used in backpressure analysis together with $\bar{p}_c$ should be recorded *following* the shut-in. We recommend a post shut-in flow period equal to the shut-in time, e.g. if the shut-in lasted 48 hours, the well should flow for 48 hours prior to recording the gas rate and flowing pressure.

The BPE presented in this paper is a reservoir-only equation. Fetkovich 1975 shows that the reservoir BPE (Eq. 9) and the tubing equation (Eq. 11) can be combined to yield the wellhead BPE:

$$B_{wh}q_g^2 + A_{wh}q_g = \bar{p}_c^2 - p_t^2$$
(15)

or,

$$q_g \approx C_{wh}(\bar{p}_c^2 - p_t^2)^{n_{wh}}$$
(16)

In our work using the Darcy equation for reservoir flow $B_{wh} = 1/C_t^2$, and $A_{wh} = 1/C_R$.

Any deviation from the established wellhead backpressure curve signifies a change in either reservoir or tubing performance. If the deviating point lies above the established line, the performance of the well has deteriorated. If the point lies below the line, the performance has improved. **Fig. 14** shows a well where a large positive skin (+20) was introduced after 10 years. A clear shift in the wellhead backpressure curve is seen after this point. When a deviation from the backpressure curve is detected, actions should be considered to restore the well's productivity.

Test conditions during the flow period often differ from the normal flowing conditions of the well. This may affect backpressure analysis, because rate contributions from the different layers can vary greatly with small changes to the flowing pressure. **Fig. 15** shows the wellhead backpressure curve for a well that was flowed with a tubinghead pressure of 3.5 bara during tests, and 1 bara otherwise. As seen from the figure, all the tests still fall on a straight line, and the backpressure analysis is valid.

The numerical model was altered to allow varying degrees of communication between the two layers throughout the reservoir. This was achieved by increasing the z-direction transmissibility multiplier (TZ) from 0 to 0.01 to 1 between the two layers. The reservoir is still experiencing differential depletion for the TZ = 0.01 case, and the layer reservoir pressures are 122 and 131 bara after 20 years (layer pressures are 105 and 133 bara for the corresponding no-crossflow model after 20 years). As seen from **Fig. 16**, backpressure analysis is still applicable.

Model Limitations

When layer permeabilities are low enough, the well performance may be dominated by transient effects, and the BPE looses accuracy, mainly because the steady-state assumption is violated.

The layer permeabilities of the two-layer numerical model based on Fetkovich et al. (1990) were reduced by $1 - 2$ orders of magnitude to test when the BPE no longer is able to reproduce the performance of the simulator. Both layer permeabilities were scaled by the same multiplier.

Fig. 17 shows the bottomhole pressure behavior for the BPE applied to a model with permeabilities one order of magnitude lower than the original example (10 and 0.1 md for the high- and low-permeability layers, respectively). As can be seen from the figure, the predicted bottomhole pressure is mismatched because of transients, but the model still replicates the general pressure behavior fairly well. When lowering the permeabilities another order of magnitude (1 and 0.01 md), the BPE is no longer able to calculate the bottomhole pressure with any certainty, as shown in **Fig. 18**. This is due to the BPEs inability to reproduce the transient behavior of the numerical model gas rates, as seen in **Fig. 19**.

Discussion

Layered backpressure analysis as proposed in this paper should be valid:

- For reservoirs with permeability greater than ~ 0.01 md.
- When using surface pressures, p_c and p_t, as long as the well hydraulics are accurately described by the gas tubing equation (e.g. Eq. 11).
- When the reservoir pressure squared assumption is applicable. Higher pressure reservoirs require the use of the pseudopressure rate equation.
- Wells not significantly affected by rate dependent skin. We were not successful in developing a layered (Forchheimer) quadratic rate equation using average rate constants (A_R and B_R) and $\bar{p}_c^2$, though we suspect an extension of our work using the quadratic rate equation, even if approximate, should exist and deserves further study.

Conclusions

1. The presented backpressure equation (BPE) for layered gas reservoirs accurately predicts pressure and rate data from field examples and numerical simulation models. The form of the BPE is identical with the single layer equation.

2. Backpressure analysis with the layered (wellhead) BPE can be used as a monitoring tool to detect deterioration in tubing and/or reservoir performance. Any deviation from the established wellhead backpressure curve indicates a change in the wells performance.
3. The layered BPE can be used to forecast depletion performance for layered no-crossflow gas reservoirs when coupled with layer material balances.

Nomenclature

A	=	Quadratic rate equation constant
B	=	Quadratic rate equation constant
C	=	Backpressure constant (std m^3/d/bar^2)
D	=	Depth (m)
F_r	=	Friction factor
F_{SSQ}	=	Sum of squares
G	=	Gas in place (std m^3)
G_p	=	Cumulative gas produced (std m^3)
h	=	Layer thickness (m)
k	=	Permeability (md)
n	=	Backpressure exponent
p	=	Pressure (bara)
q_g	=	Gas flow rate (std m^3/d)
r_e	=	Drainage radius (m)
r_w	=	Wellbore radius (m)
s	=	Skin factor
S	=	Gravity term
S_{wi}	=	Irreducible water saturation
T	=	Temperature (K)
z	=	Z-factor

Greek Symbols

μ	=	Viscosity (cp)
φ	=	Porosity
γ	=	Specific gravity

Subscripts

a	=	Altered region (stimulated)
c	=	Reservoir property at surface datum
g	=	Gas
i	=	Initial
l	=	Layer number
R	=	Reservoir
ref	=	Reference value used for normalization
t	=	Tubing
w	=	Bottomhole property at surface datum
wf	=	Bottomhole
wh	=	Wellhead

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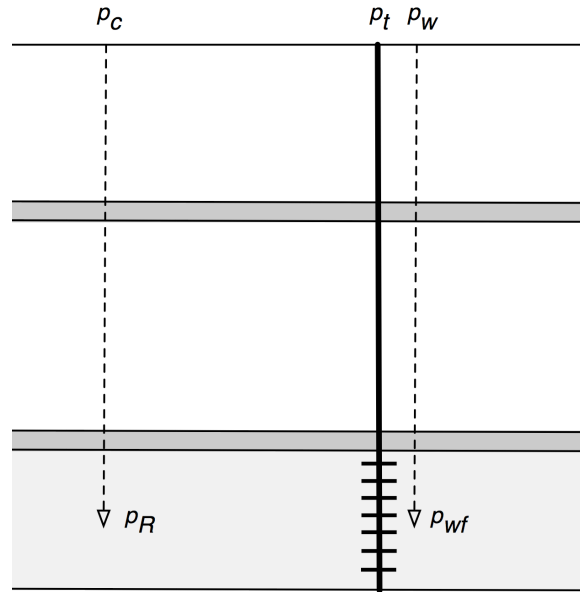
Fig. 1 – Pressure datums for the backpressure equation. The gravity term, *S*, is used to convert from surface to bottomhole pressures for static gas columns.
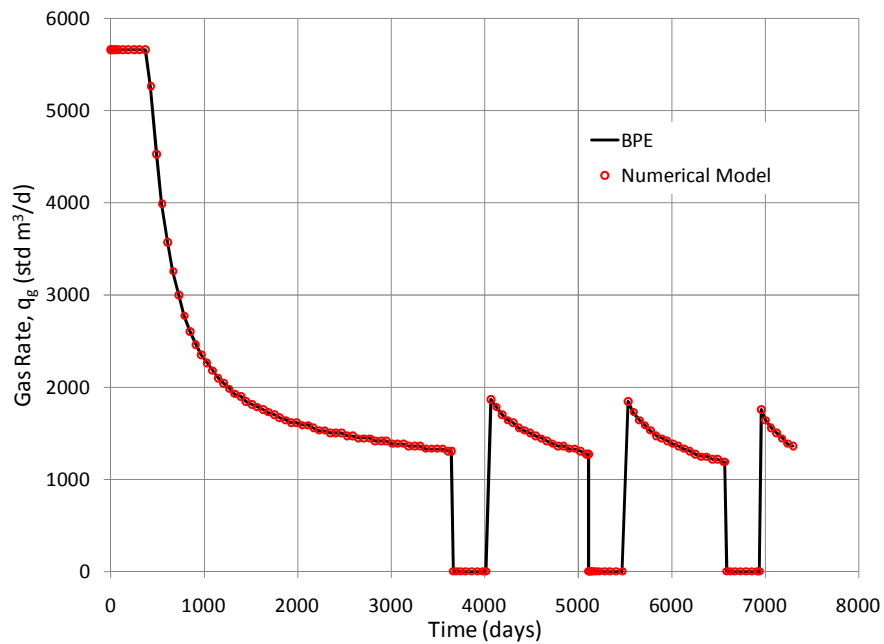
Fig 2. – Production profile for the numerical two-layer model based on Fetkovich et al. (1990).
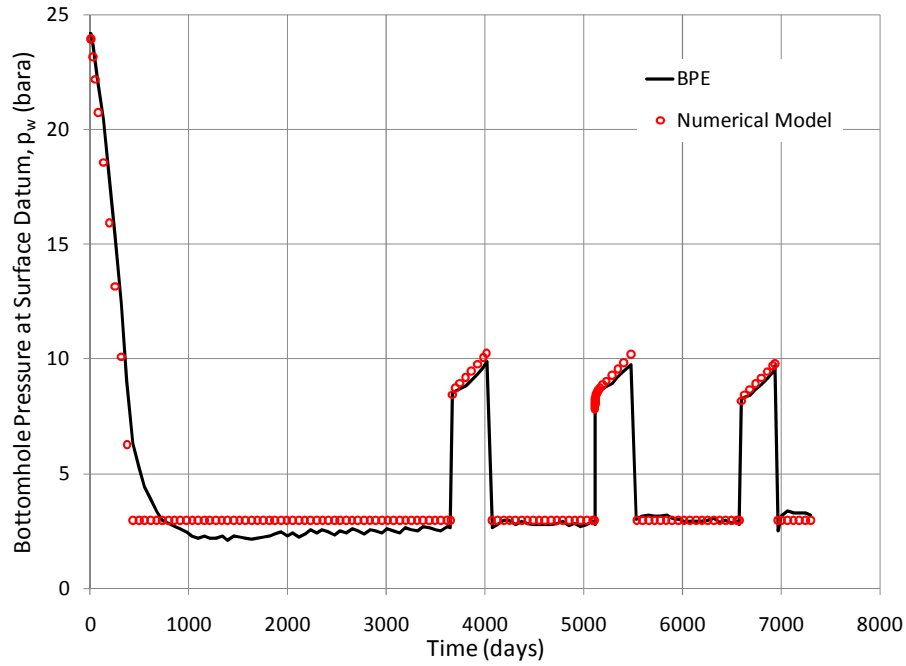
Fig. 3 – Bottomhole pressures predicted by the BPE vs. the numerical two-layer model based on Fetkovich et al. (1990).
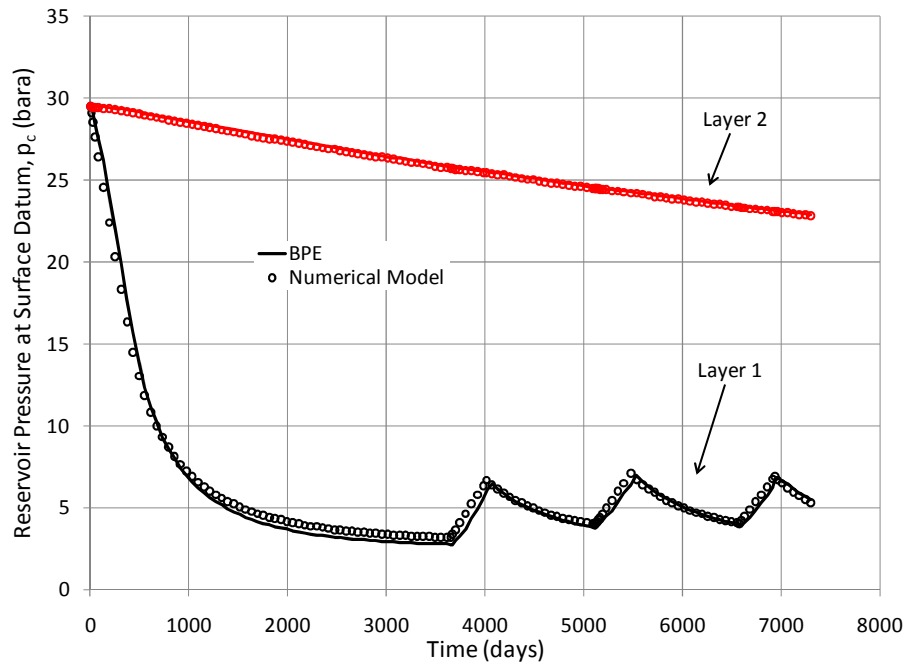
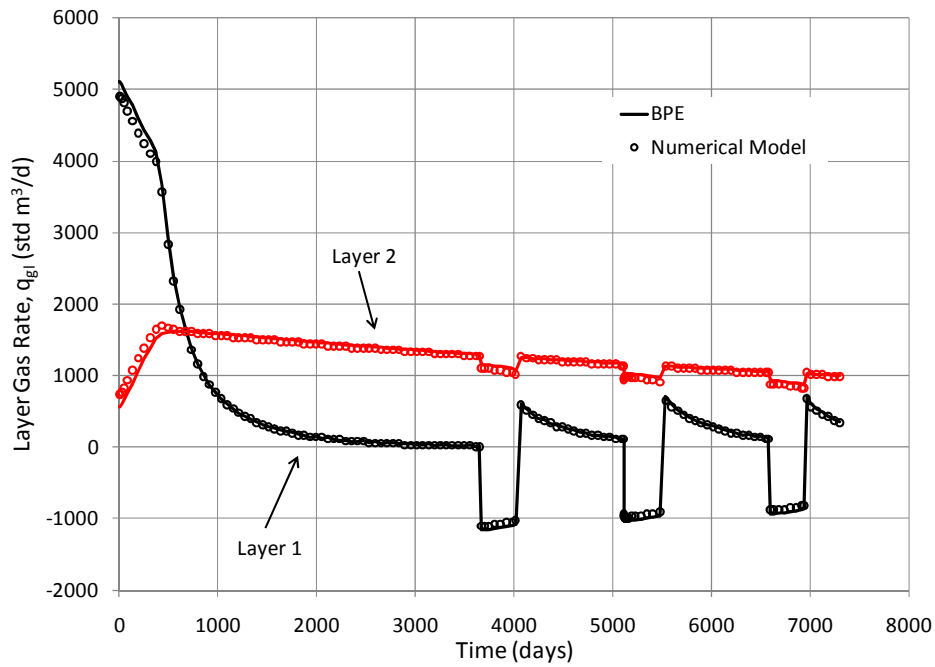**Fig 4. – Layer pressures predicted by the BPE vs. layer pressures from the numerical two-layer model based on Fetkovich et al. (1990).**

Fig. 5 – Layer gas rates from the BPE vs. layer rates from the numerical two-layer model based on Fetkovich et al. (1990).
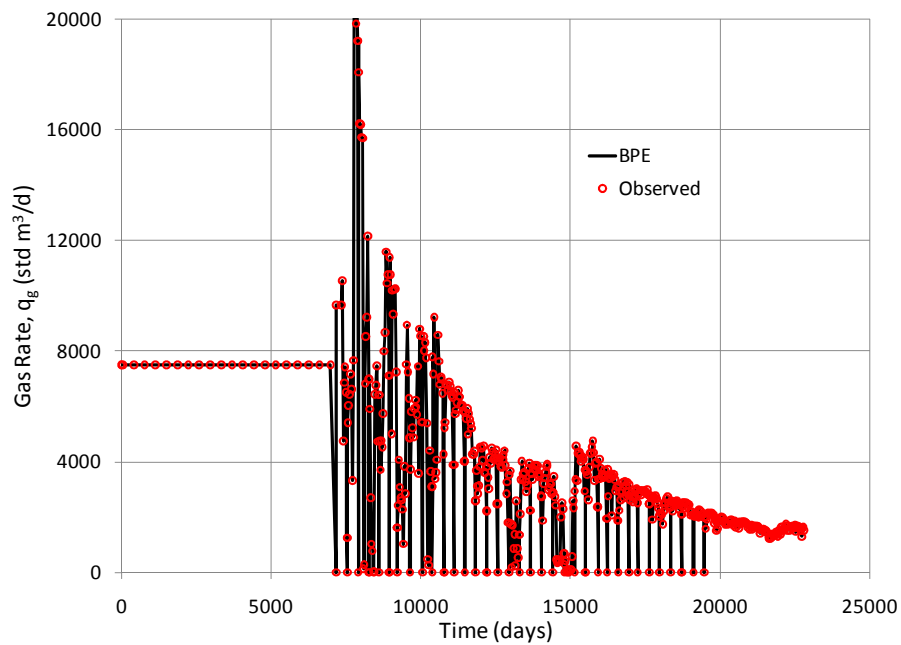
Fig 6. – Production gas rates for the Buf #3 well in the Guymon-Hugoton field. Production rate prior to 1967 is assumed constant.
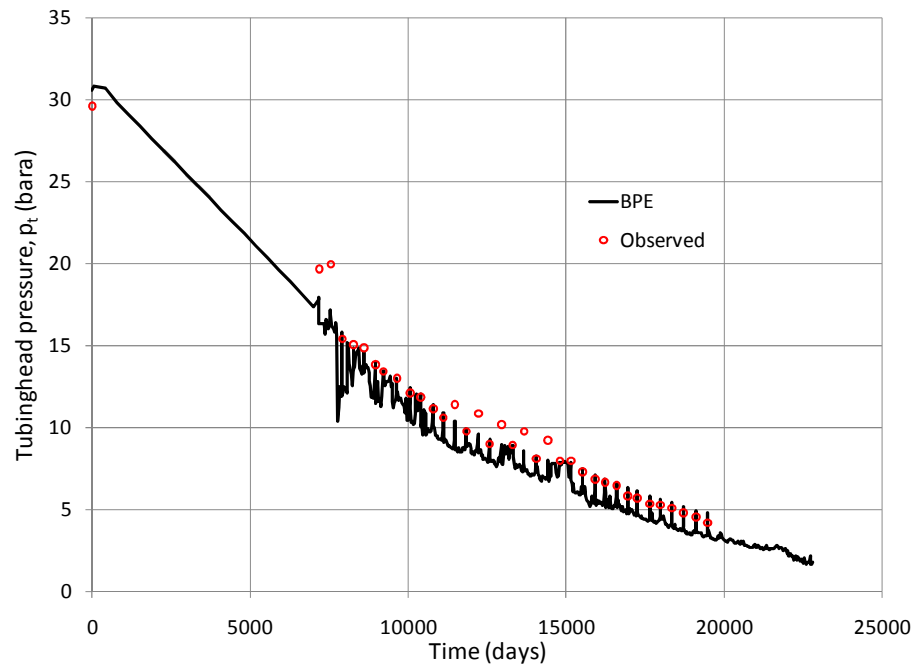
Fig. 7 – Tubinghead pressures calculated by the BPE vs. measured pressures for the Buf #3 well in the Guymon-Hugoton field.
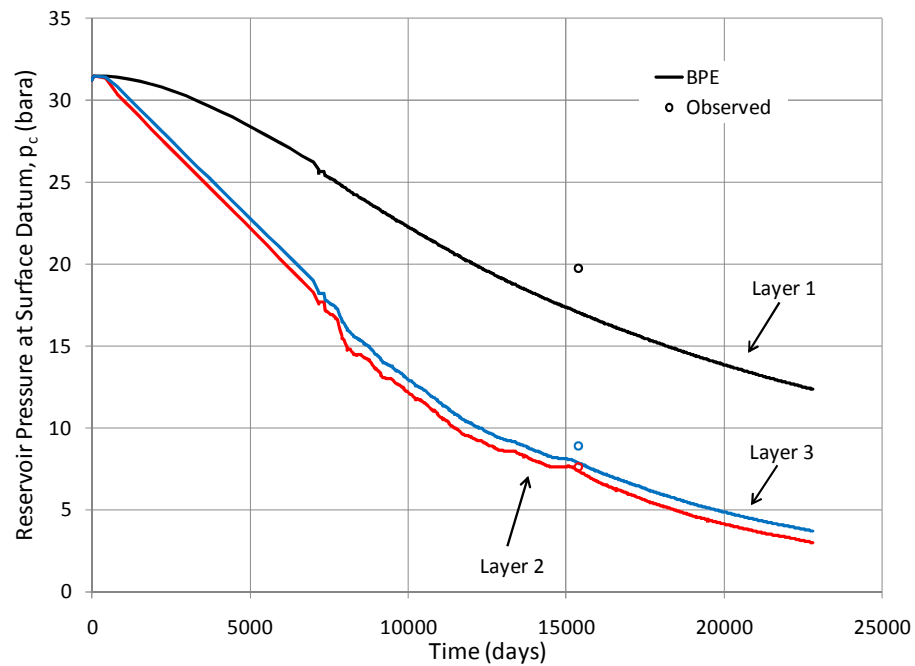
Fig 8. – Layer pressures calculated by the BPE for the Buf #3 well in the Guymon-Hugoton field. The observed data represents a layer pressure test conducted on the well in 1989.
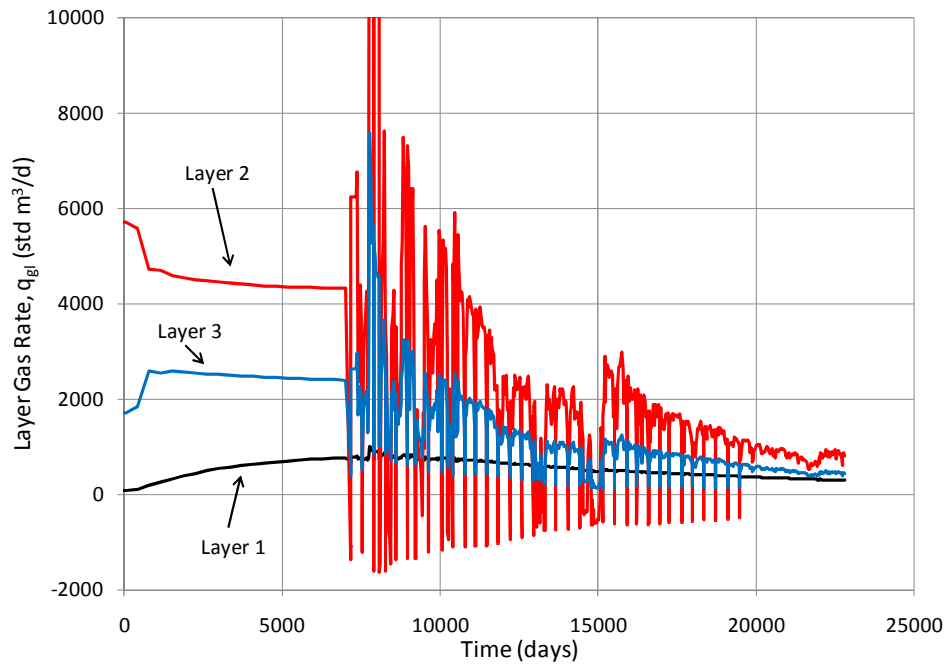
Fig. 9 – Layer gas rates calculated by the BPE for the Buf #3 well in the Guymore-Hugoton field. No observed data is available.
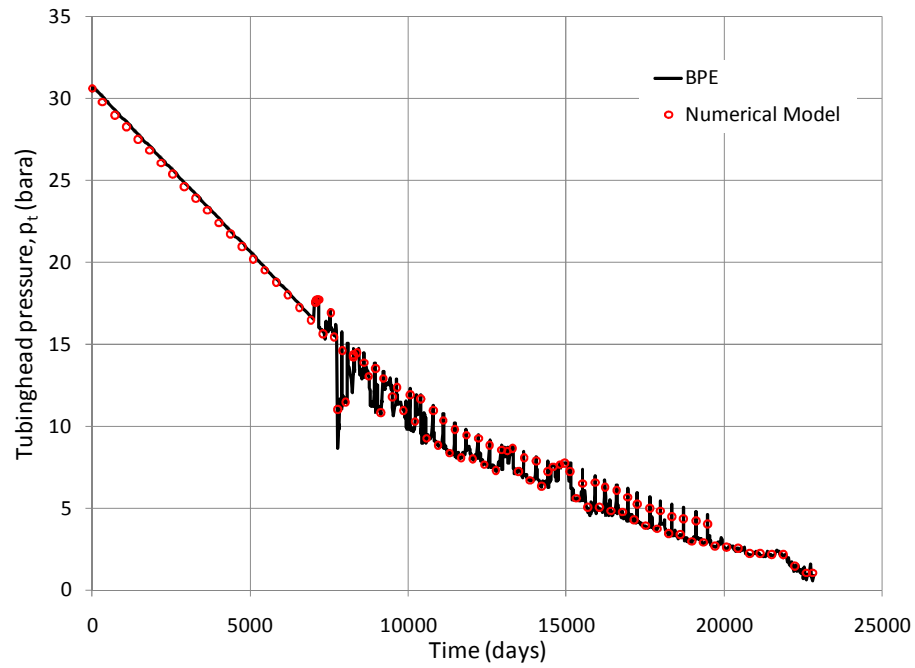
Fig 10. – Tubinghead pressure for the numerical model based on the Buf #3 well, and the BPE predictions.
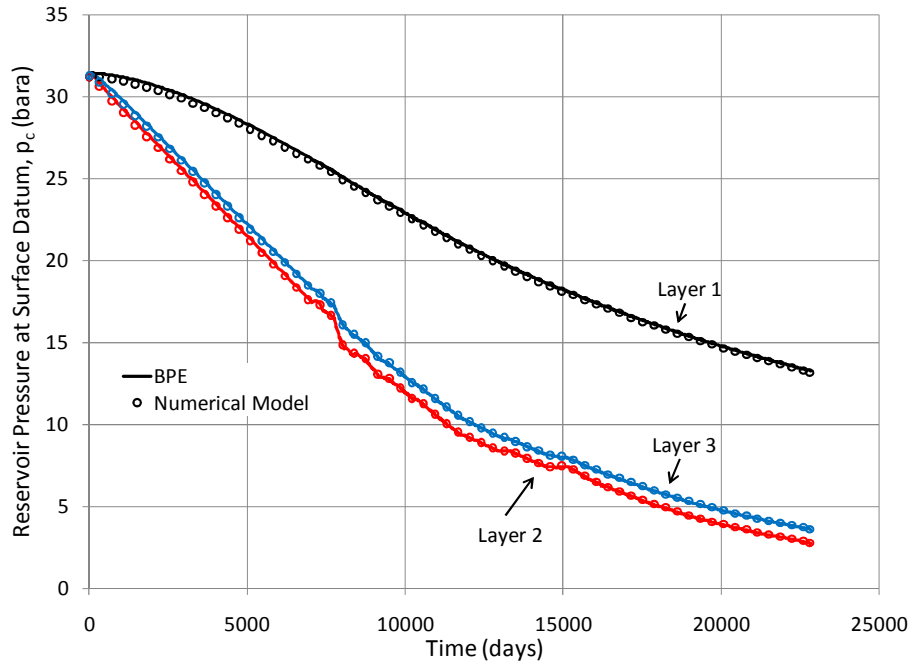
Fig. 11 – Layer pressures for the numerical model based on the Buf #3 well, and the pressures predicted by the BPE.
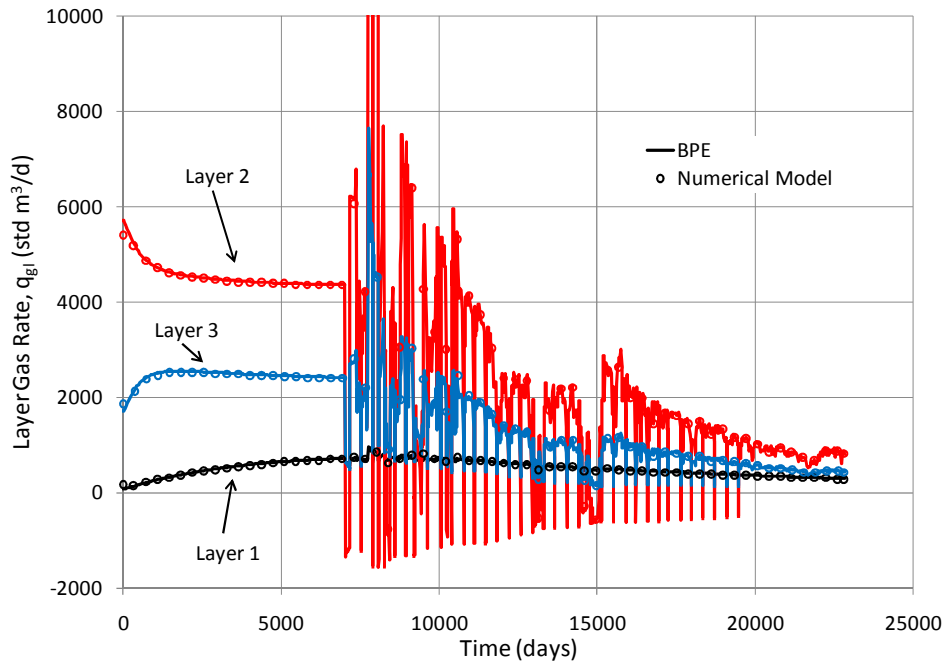
Fig 12. – Individual layer gas rates for the numerical model based on the Buf #3 well, and the BPE predicted rates.
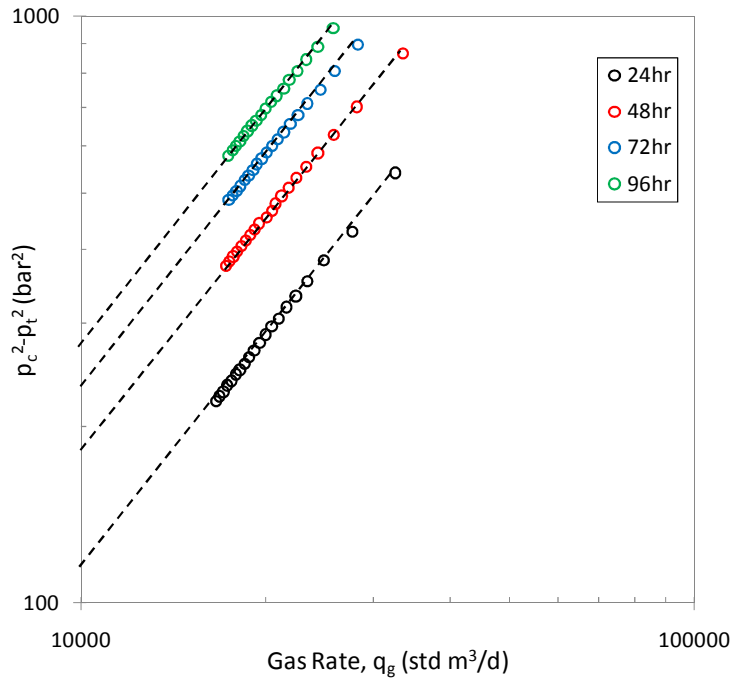
Fig. 13 – Backpressure plot using different shut-in periods. The tests corresponding to a specific shut-in duration all fall on a straight line on the log-log plot.
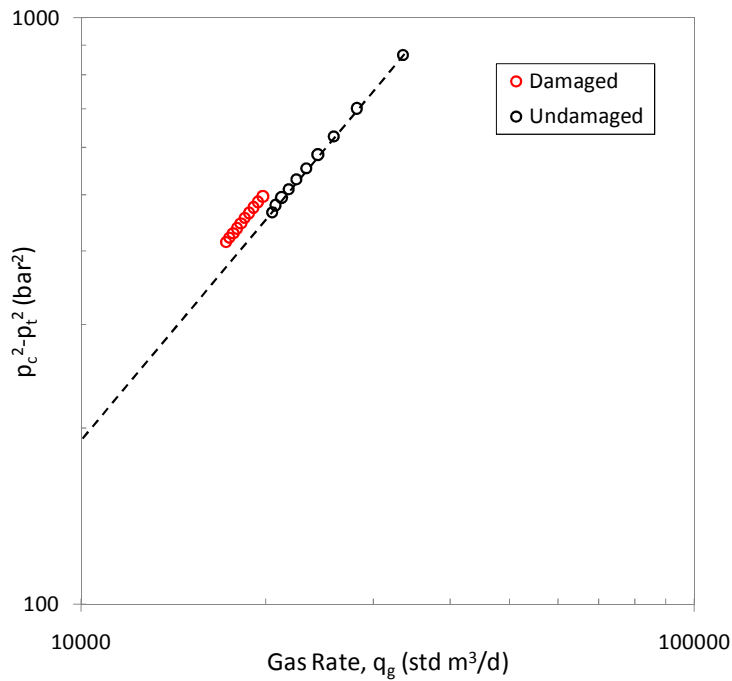
Fig 14. – A large positive skin factor (+20) was introduced in the model after 10 years, to show how the performance of the well deviates from the established backpressure curve when the formation is damaged.
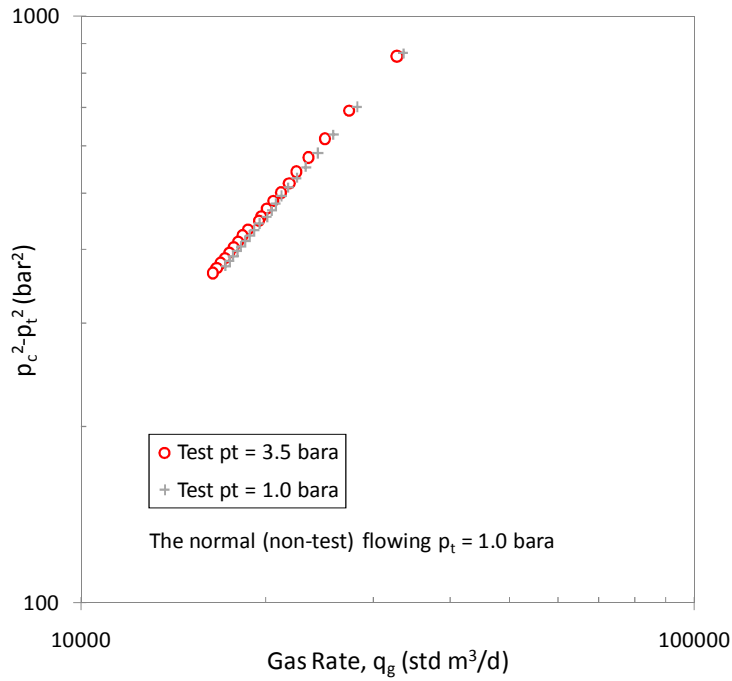
Fig. 15 – Backpressure plot of a two layer model where the wellhead pressure during the post shut-in flow period was higher than the normal flowing pressure.
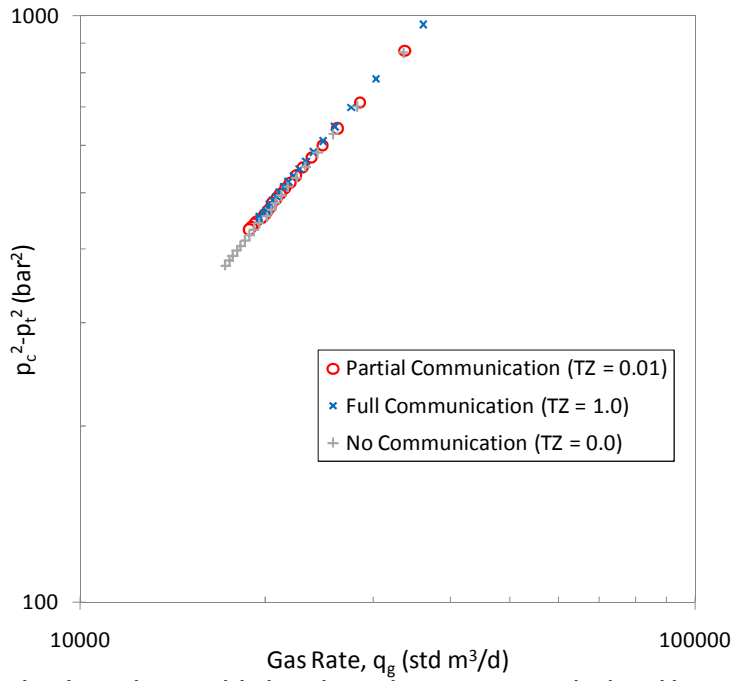

Fig 16. – Backpressure plot of a two layer model where the two layers are communicating with varying degrees throughout the reservoir.
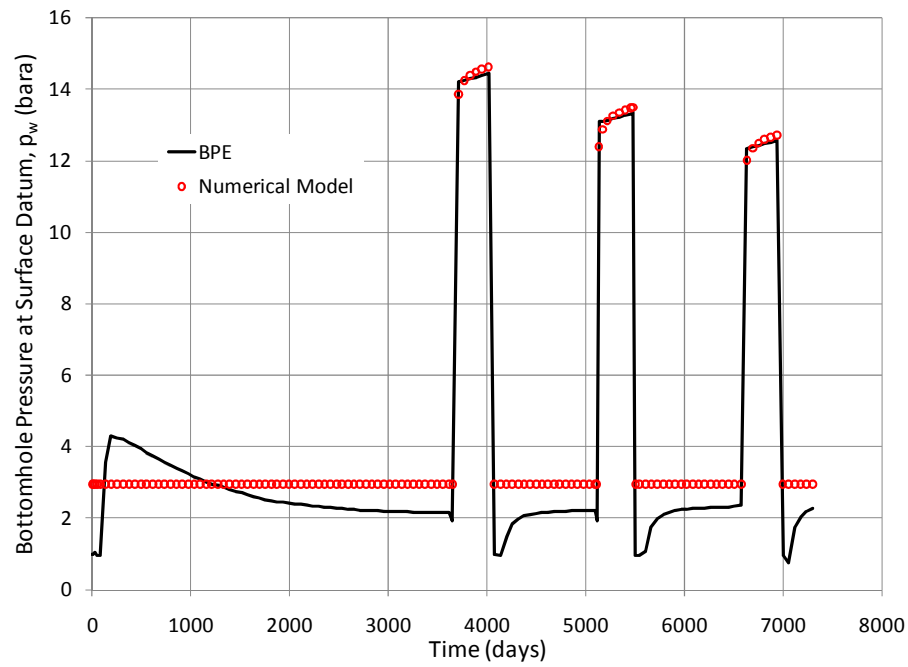
Fig. 17 – Bottomhole pressure prediction for the two-layer model based on Fetkovich et al. (1990) with permeabilities of 10 and 0.1 md for the high- and low-perm layers respectively.
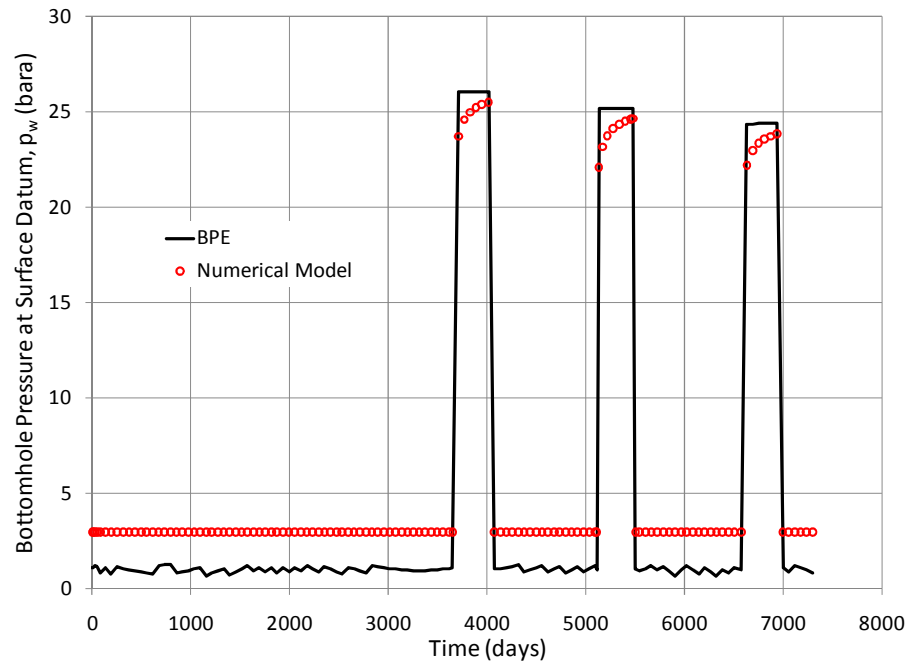

Fig 18. – Bottomhole pressure prediction for the two-layer model based on Fetkovich et al. (1990) with permeabilities of 1 and 0.01 md for the high- and low-perm layers respectively.
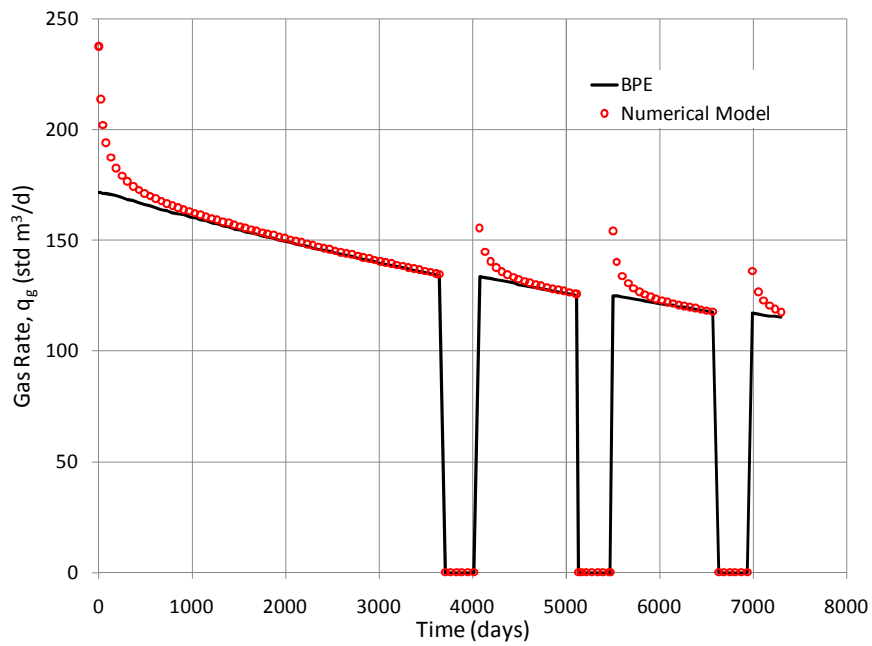
Fig. 19 – Gas rates for the two-layer model based on Fetkovich et al. (1990) with permeabilities of 1 and 0.01 md for the high- and low-perm layers respectively. The BPE is no longer able to predict the transients in the gas rate.