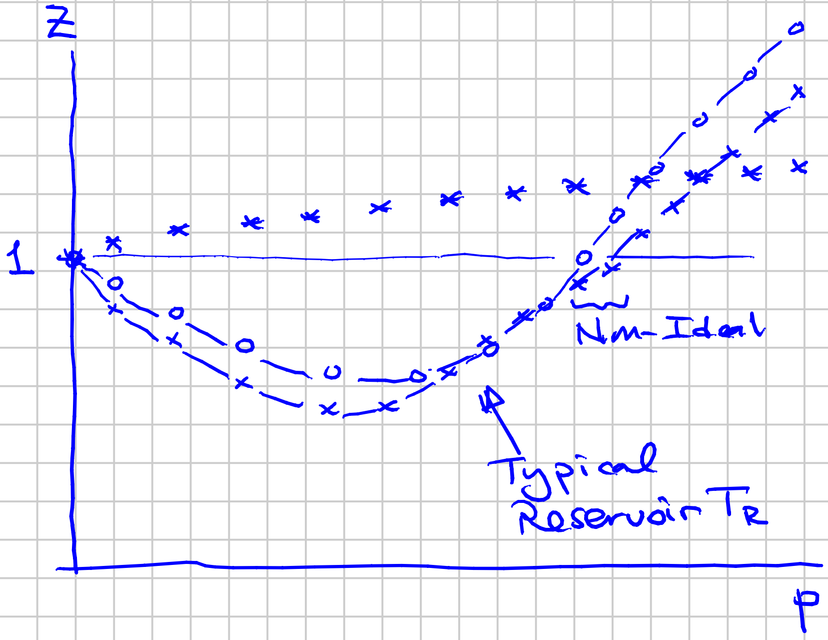
REAL GAS LAW

Deviation of p-V-T behavior of real systems (gas)
from "Ideal Gas Law" behavior ($Z = 1$)

$$Z \equiv \left(\frac{pV}{nRT}\right)$$

Measurements: Fixed n, Fixed Component (C_1)

	T(°C)	P(bara)	V(m³)	=> Z
o	200	1 ⋮ 100		
x	100	1 ⋮ 100		
*	400			



$$\underbrace{C_1 \quad N_2 \quad CO_2 \quad H_2S \quad C_2}_{\text{Reservoir Gases Consist of "Large" quantities}}$$

van der Waals (187x): p-V-T gases & liquids & dunno
"Theory of Corresponding States"

Reduced Variables:

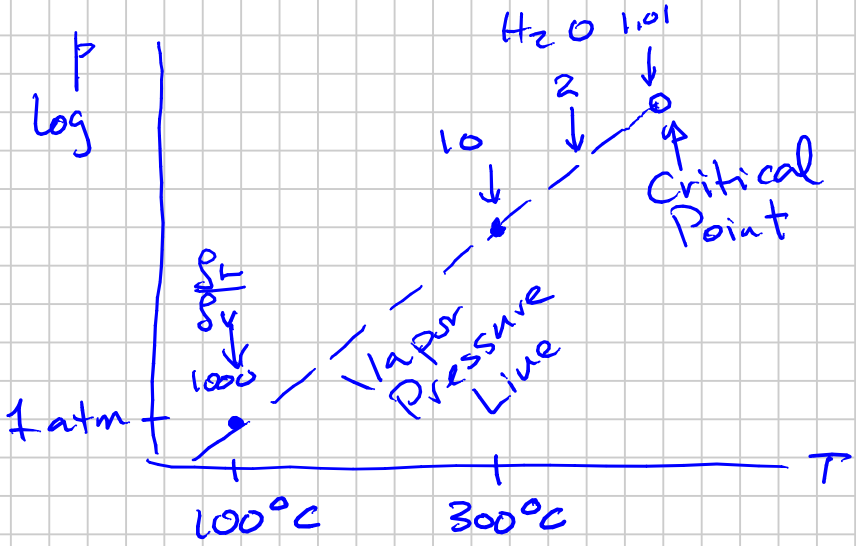
$$P_r \equiv \frac{p}{p_c}$$

$$T_r \equiv \frac{T}{T_c}$$

$$V_r \equiv \frac{V}{V_c}$$

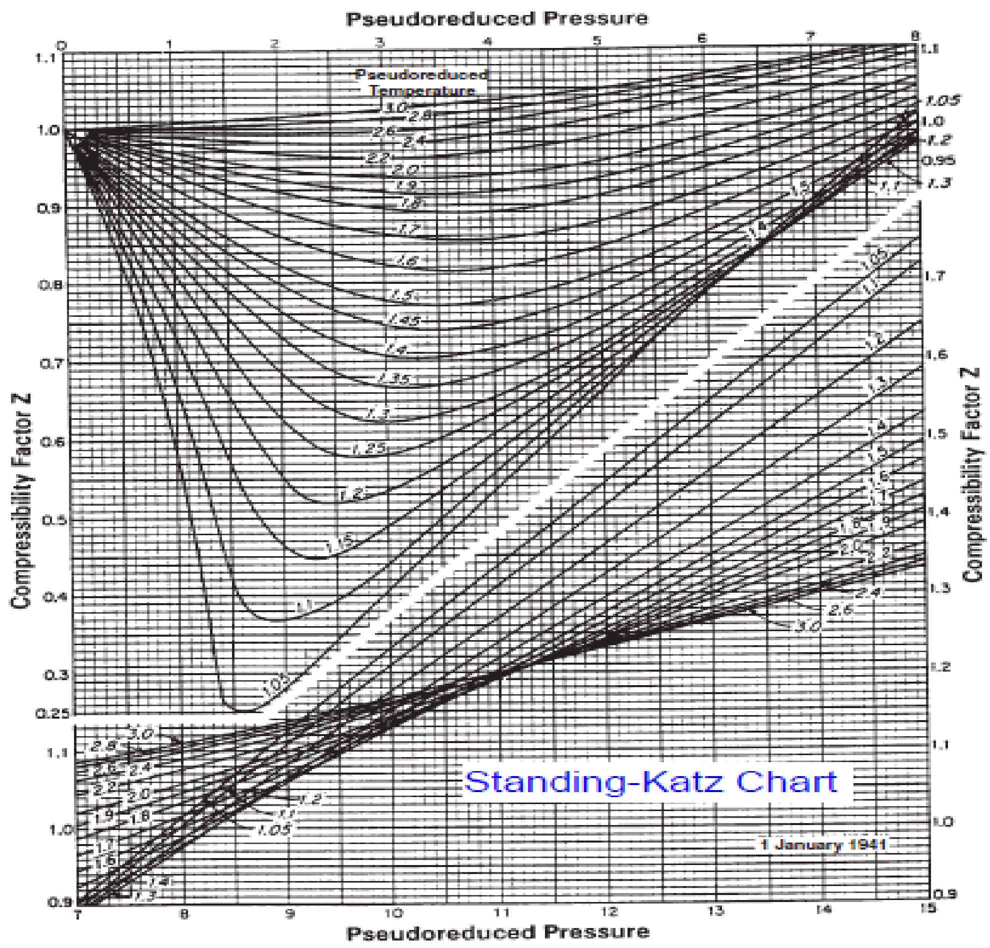
If any two gases have the same value of P_r & T_r then they will have the same $Z_g(T_r, P_r)$

(p_c, T_c) are the "critical" properties:



1940s : Donald Katz @ U. Michigan
Marshall B. Standing } $Z_g(T_r, p_r)$

All existing data +
new data for petroleum mixtures



Mixtures :

$*(H_2S, CO_2), (C_{7+})$ requires special treatment Ch.3

Average $\overline{T}_{pc} = \sum_{i=1}^{N: C_{7+}} y_i \cdot T_{ci}$

pseudo

Reservoir Gas Mole Fraction

$$\overline{p}_{pc} = \sum_{i=1}^{N} y_i \, p_{ci}$$

✓ $T_{pr} = \dfrac{T}{T_{pc}}$ 1.3 – 2.5

✓ $p_{pr} = \dfrac{p}{p_{pc}}$ 0 – 20

Lab measured – different for every reservoir

$$\text{Est.} \quad T_{c7+} = f(\overbrace{M_{7+}, \rho_{7+}})$$

$$p_{c7+} \quad f(\underbrace{M_{7+}, \rho_{7+}}_{C_{7+}\ \text{"characterization"}})$$

Matthews et al.

App. B: Example Calculations.

Real Gas Law :

$$pV = nRT \cdot \underbrace{z(p_r, T_r)}_{\text{SK Chart}}$$

Equations that fit the chart (Ch.3)

0.7 – 2

van der Waals

Donald Katz

Example Calculation : $\{Z_{gi}\}$

Troll Field ~ 45 Tcf $(45 \cdot 10^{12}$ scf$)$

$T_R = 71\,°C = 344\ K$

35.31 scf/Sm^3

$p_{Ri} = 158$ bara

$\sim 1.3 \cdot 10^{12}$ Sm^3

$T_{pc} = 360\,°R = 200\ K$

$p_{pc} = 670$ psia $= 46.2$ bara

14.5(0377)

$$T_{pr} = \frac{T_R}{T_{pc}} = \frac{344}{200} = 1.72$$

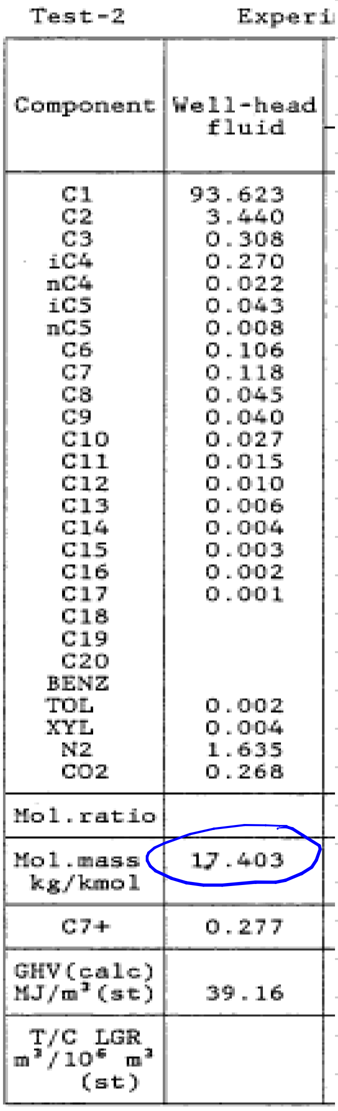
SK

$Z_g(T_{pr}, p_{pr})$

$$p_{pr} = \frac{p_{Ri}}{p_{pc}} = \frac{158}{46.2} = 3.41$$

$Z_{gi} \sim 0.85$

$$\boxed{(V_{pg})_{Troll} = \qquad\qquad m^3}$$



Test-2 Experi

Component	Well-head fluid
C1	93.623
C2	3.440
C3	0.308
iC4	0.270
nC4	0.022
iC5	0.043
nC5	0.008
C6	0.106
C7	0.118
C8	0.045
C9	0.040
C10	0.027
C11	0.015
C12	0.010
C13	0.006
C14	0.004
C15	0.003
C16	0.002
C17	0.001
C18	
C19	
C20	
BENZ	
TOL	0.002
XYL	0.004
N2	1.635
CO2	0.268
Mol.ratio	
Mol.mass kg/kmol	17.403
C7+	0.277
GHV(calc) MJ/m³(st)	39.16
T/C LGR m³/10⁶ m³ (st)	

Ideal Gases: $\rho_g = \frac{p_{sc} M}{R T_{sc}}$

$= \left(\frac{p_{sc}}{R T_{sc}}\right) M_g$ $\uparrow$

Eq. Sutton:

$$\boxed{\begin{matrix} T_{pc} \\ P_{pc} \end{matrix} = f(\gamma_g)} \quad *$$

Relative Density

Specific Gravity

$$\gamma_g = \frac{\rho_g}{\rho_a} = \frac{M_g}{M_{air}}$$

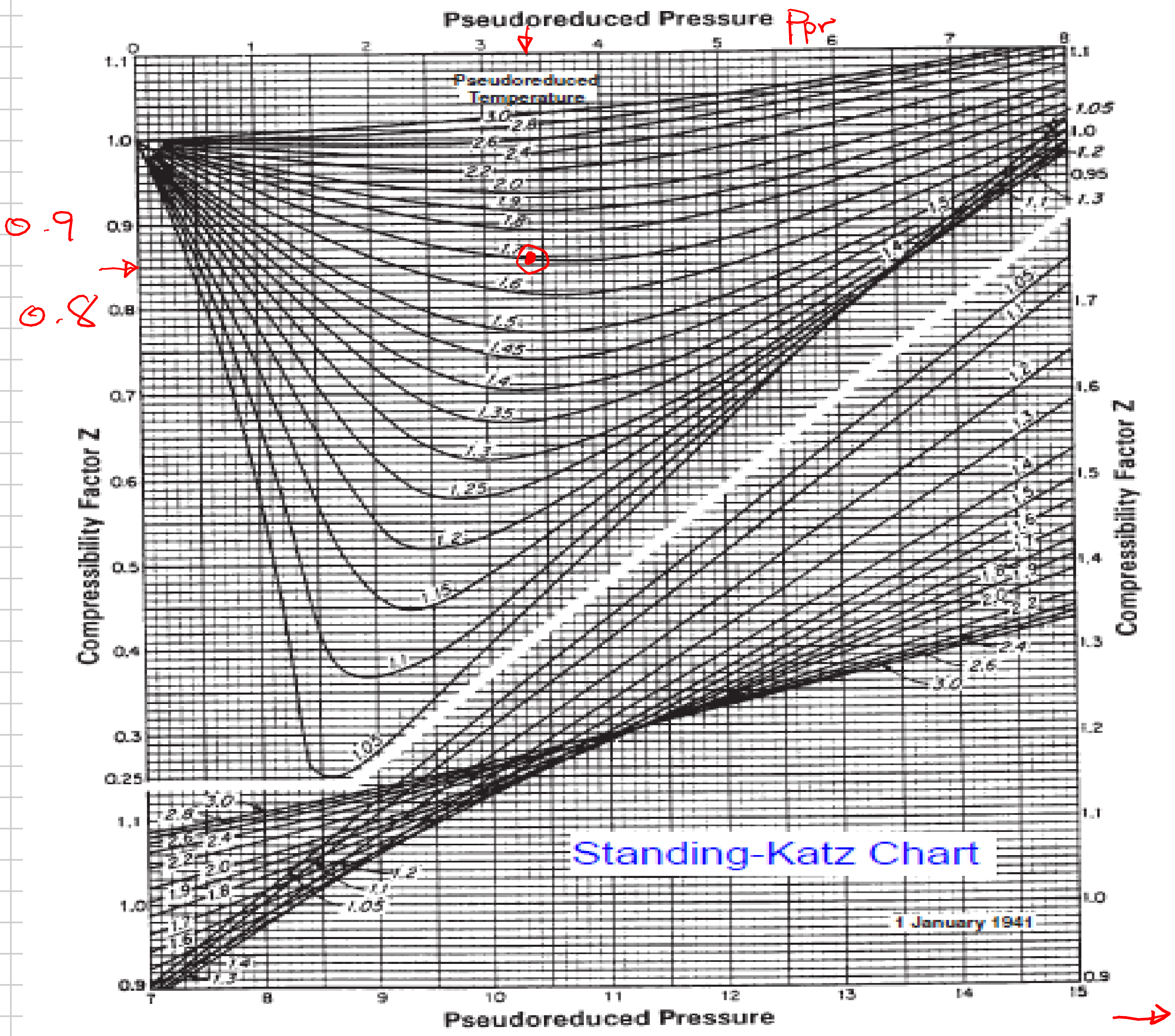
$M_g \simeq 17.4$

$$\gamma_g = \frac{17.4}{28.97} = 0.6$$

Fig. 3.7

$T_{pc} = 360$ °R

$P_{pc} = 670$ psia


Pseudoreduced Pressure
Ppr
0
1
2
3
4
5
6
7
8
Pseudoreduced Temperature
3.0
2.8
2.6
2.4
2.2
2.0
1.9
1.8
1.7
1.6
1.5
1.45
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
Compressibility Factor Z
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.25
1.05
1.2
0.95
1.3
1.7
1.6
1.5
1.4
Standing-Katz Chart
1 January 1941
7
8
9
10
11
12
13
14
15
Pseudoreduced Pressure

①

$$V_g = 45\cdot 10^{12}\ \text{scf} \xrightarrow{\hspace{2cm}} \times\frac{\text{Sm}^3}{35.31\ \text{Scf}}\quad 1.27\cdot 10^{12}\ \text{Sm}^3$$

$$\downarrow\quad p_{sc}V_g = n_g R\, T_{sc}$$

$$\underset{\text{lb-mol}}{n_g} = \frac{\overset{\text{ft}^3}{(45\cdot 10^{12})}\ \overset{\text{psia}}{(14.696)}}{(10.73)\ \underset{°\text{R}}{(520)}}$$

$$= \frac{45\cdot 10^{12}\ \text{scf}}{379\ \text{scf/lb-mole}}$$

$$=$$

$$\frac{1.27\cdot 10^{12}\ \text{Sm}^3}{23.68\ \frac{\text{Sm}^3}{\text{kg-mole}}}$$

$$= 5.36\cdot 10^{10}\ \text{kg-mole}$$

REAL GAS LAW:

$$p_{Ri}V_{Rg} = n_g R\, T_R \cdot Z_{gi}$$

$$\underset{\text{m}^3}{V_{Rg}} = \frac{\overset{\text{kg-moles}}{(5.36\cdot 10^{10})}(8314)\overset{\text{K}}{(344)}(0.85)}{\underset{\text{Pa}}{158\cdot 10^5}}$$

$$= 8.25\cdot 10^{9}\ \text{m}^3$$

$$V_{\substack{\text{Trondheim}\\ \text{Fjord}}} = 235\ (\text{km}^3) = 235\cdot 10^{9}\ \text{m}^3$$

$$\%\text{TFj} = \frac{8}{235} = 3\%$$

$$B_{gi} = \frac{V_g(p,T)}{V_{\bar{g}}} = \frac{8.25 \cdot 10^9 \ m^3}{1.27 \cdot 10^{12} \ Sm^3}$$

$$= 0.0065 \ m^3/Sm^3$$

$$\boxed{b_{gi} = 154 \ Sm^3/m^3}$$

PVT Fluid Behavior

Note Title 2013-09-05

Phase & Volumetric Behavior (++)

Gas

Oil

Water

(Solid)

Thermodynamics

Phase Equilibria

Chemical Equilibria

$$V = f(p, T, \overset{\text{composition}}{x_i})$$

Gas	y_i	
Oil	x_i	liquid (oil)
Water	x_{wi}	liquid (water)

$$x_i \equiv \frac{n_{ip}}{n_p}$$

$$y_i = \frac{n_{ig}}{n_g} = \frac{n_{ig}}{\sum_i n_{ig}}$$

$$v \equiv \frac{V}{n}$$

$$Z \equiv \frac{pV}{nRT} = \frac{pv}{RT}$$

$$\rho \equiv \frac{m}{V} = \frac{pM}{RTZ}$$

Mass : $\rho \cdot g$

Viscosity (μ) Pa·s ; cp *

Interfacial Tension (σ)

IFT **

(Surface) Tension

σ_{go} σ_{gw} σ_{ow}

* 1 mPa·s = 1 cp

** N/m

dynes/cm

$1\ \text{mN/m} \equiv 1\ \frac{\text{dyne}}{\text{cm}}$

Equation of State

Gas: $pV = nRT$ (Ideal)

$pV = nRTZ_g$ (Real)

$$\uparrow$$

$Z_g(p_r, T_r, y_i)$

Purely Volumetric

Oils : e.g. $c_o \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \overset{\sim}{=}$ constant

General EOS works for both G & O

Cubic EOS (other more complicated)

Ch. 4

$$p = \frac{RT}{v - b} - \frac{a}{[v^2]}$$

Repulsive   Attractive

vdW : $[\ ] = v^2$

* SRK : $[\ ] = v(v+b)$

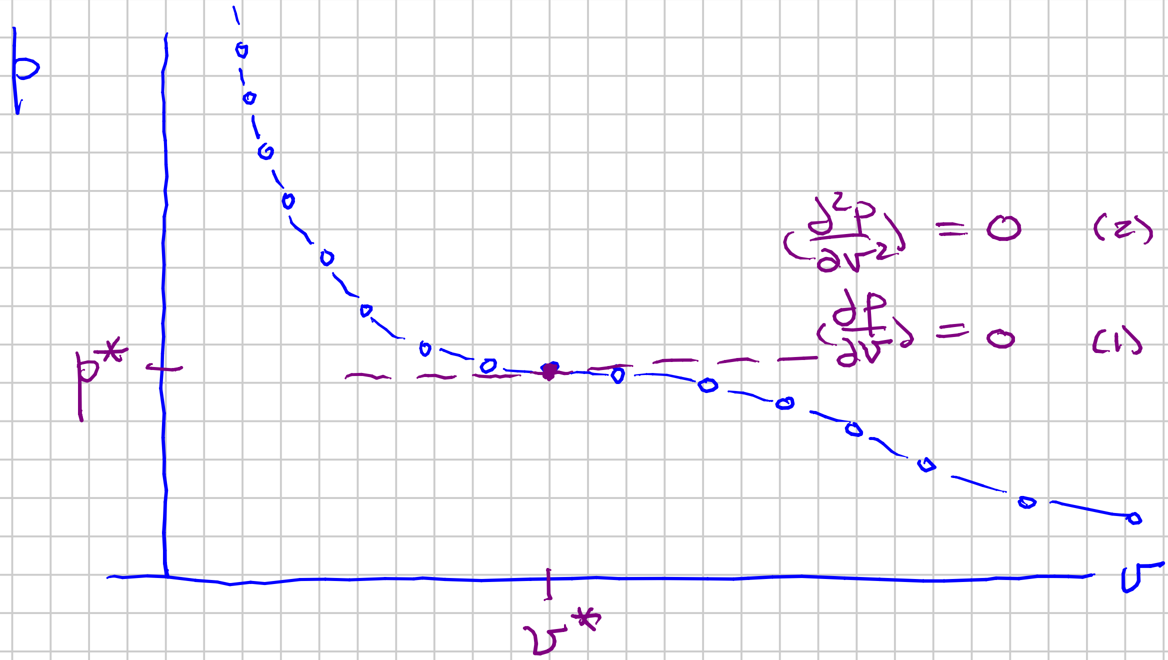
* PR : $[\ ] = v(v+b) + b(v-b)$

van der Waals

$v \to \infty$: $p = \frac{RT}{v}$ Ideal Gas

Need a&b to use this equation!

vdW : At some special temperature T^*



$T^* = T_c$

$p^* = p_c$

$$\Rightarrow \quad a_i = \Omega_a \frac{R^2 T_{ci}^2}{p_{ci}} \qquad \overset{vdW}{27/64} \qquad \Omega_a = \text{number "1"} \sim 0.4$$

$$b_i = \Omega_b \frac{R T_{ci}}{p_{ci}} \qquad 1/8 \qquad \Omega_b = \text{number "2"} \sim 0.1$$

Mixture: u_i (y_i x_i x_{wi}) phase p (g o w)
Phase

$$\bar{a}_p = \sum_{i=1}^{N} \sum_{j=1}^{N} u_i u_j (a_i a_j)^{1/2}$$

$$\bar{b}_p = \sum_{i=1}^{N} u_i b_i$$

g $\quad y_i$: $\bar{a}_g$ $\bar{b}_g$
o $\quad x_i$: $\bar{a}_o$ $\bar{b}_o$

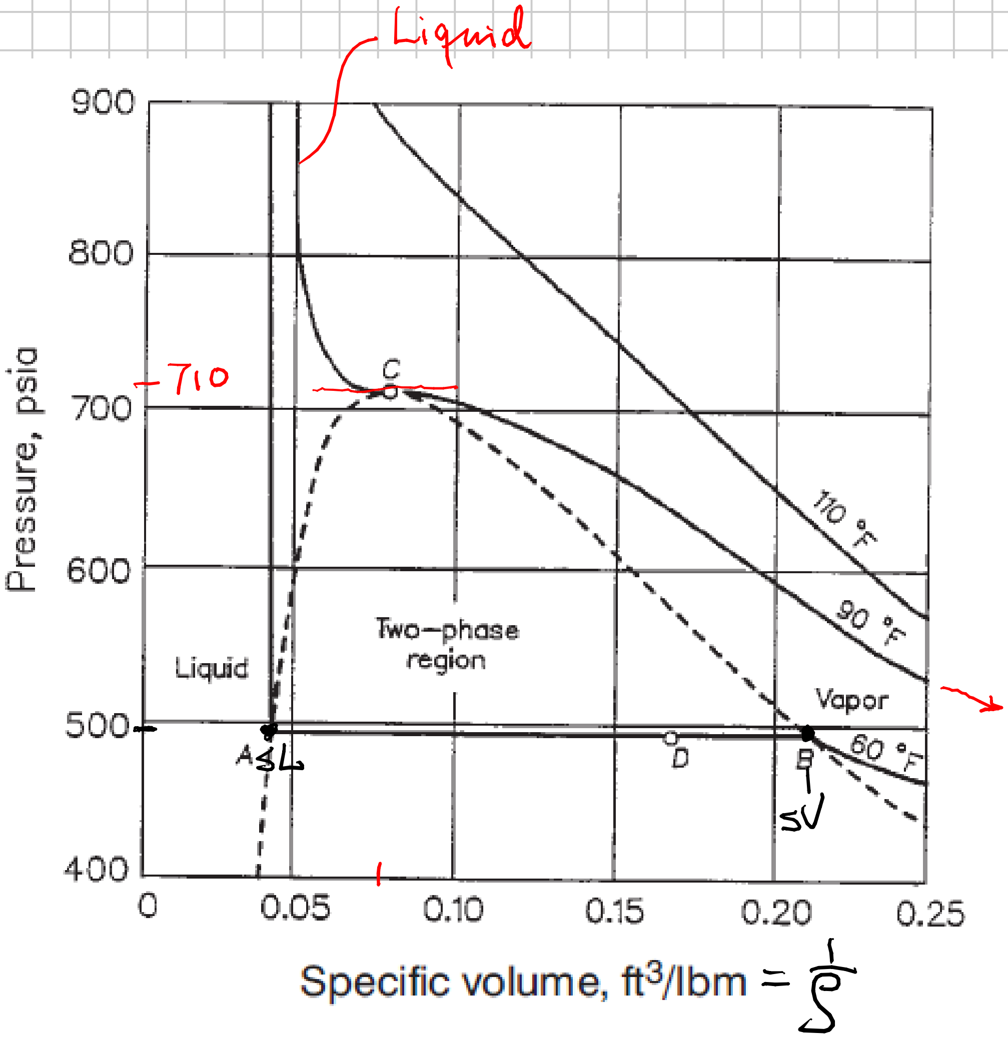


Fig. 2.7—p-V diagram for ethane at three temperatures (from Standing[26]).

$p_{vC_2}(T=60°F) =$ 490 psia

$\rho_{vs} \sim 5 \frac{lb}{ft^3}$

$\rho_{Ls} \sim 25 \frac{lb}{ft^3}$

→ Ideal Gas

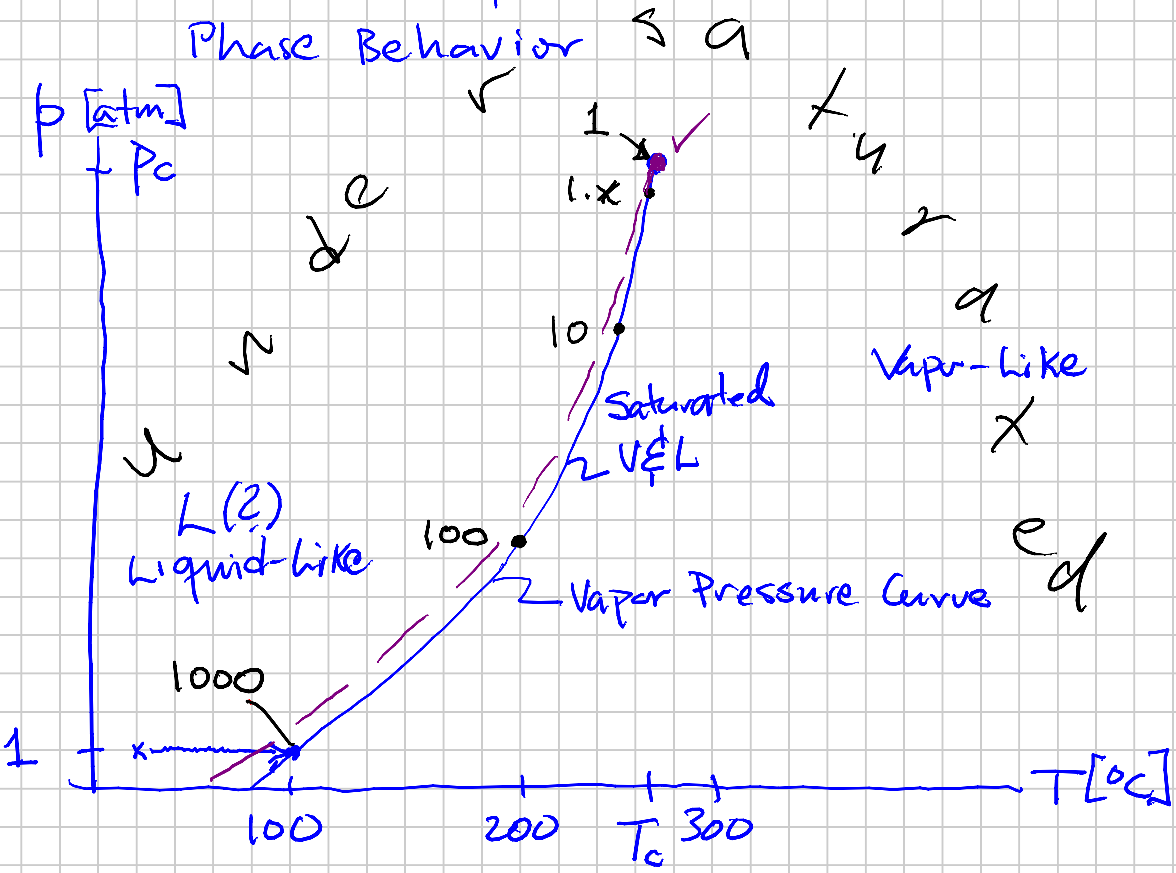
Single-Component Phase Equilibria

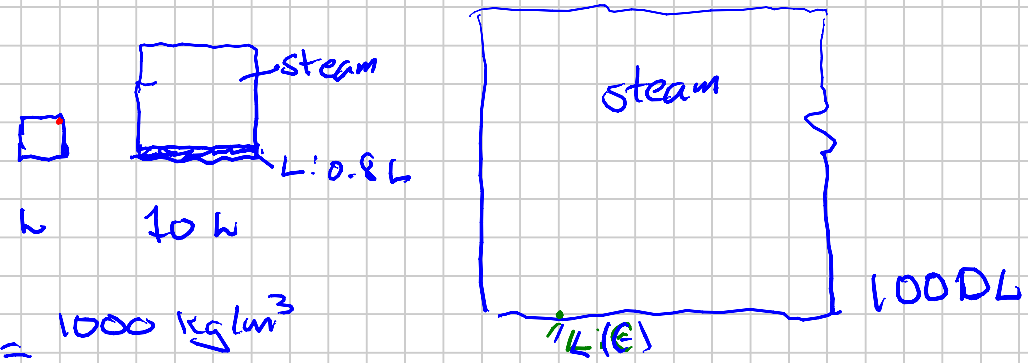
(Chemical Equilibria)

Phase Behavior

②

$\boxed{H_2O}$



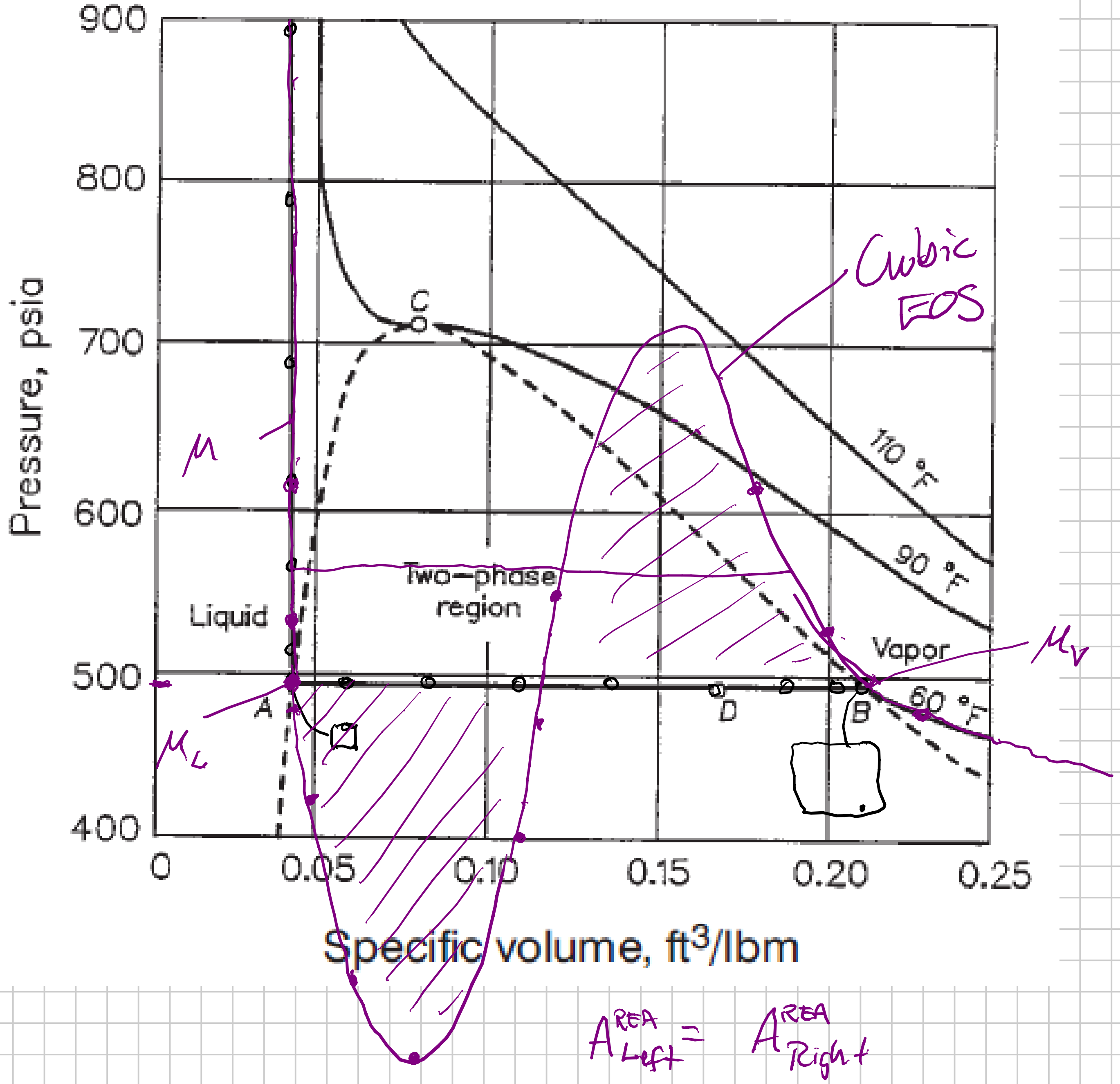


Liquid	ρ_L	1000 kg/m³
Steam	ρ_V	1 kg/m³

$$\frac{\rho_L}{\rho_V} = $$

$$p = \frac{RT}{v-b} - \frac{a}{v^2} = \frac{RTv^2 - a(v-b)}{v^3 - bv^2}$$

Cubic in Volume … $v^3 + \quad v^2 + \quad v + \quad = 0$



$A^{REA}_{Left} = A^{REA}_{Right}$

μ = chemical energy $= \int_0^p \left(v - \frac{RT}{p}\right) dp$

$\mu_L(p) = \mu_V(p) \qquad p = p_v$

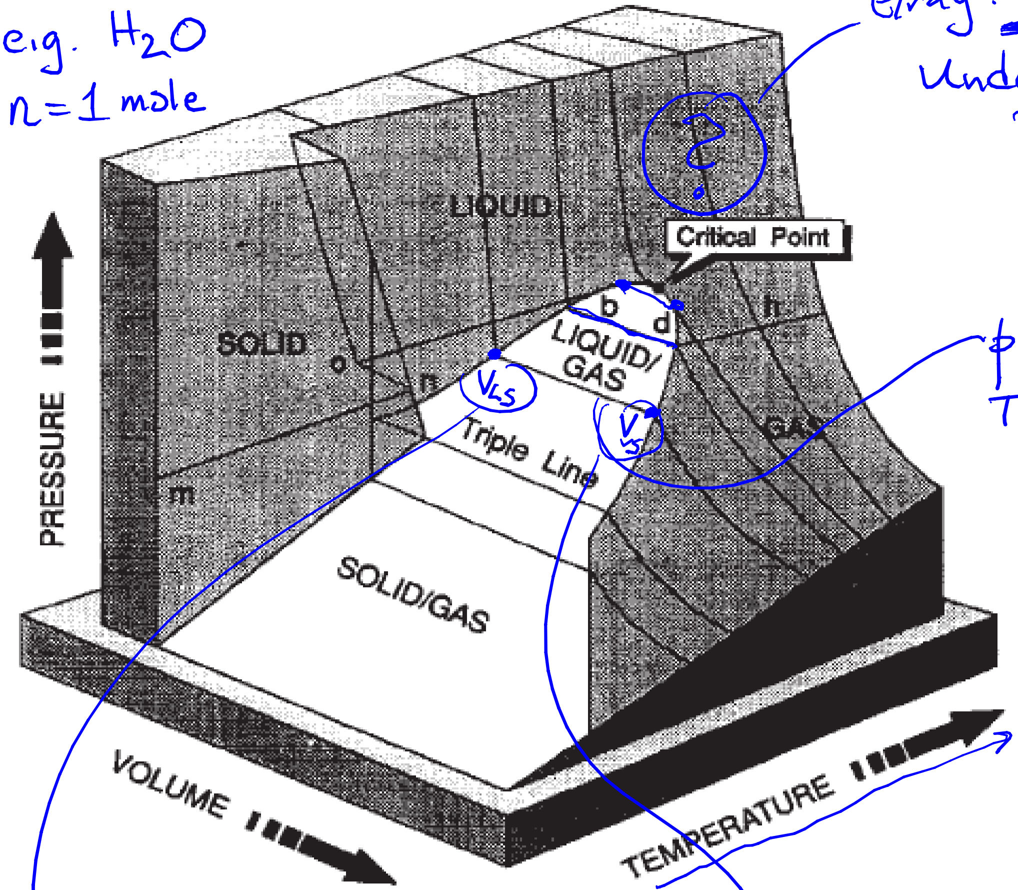
PURE COMPOUND

e.g. H_2O

$n = 1$ mole

Gray: Single Phase

Undersaturate Phase



$p = 1$ atm

$T = 100$ °C

212 °F

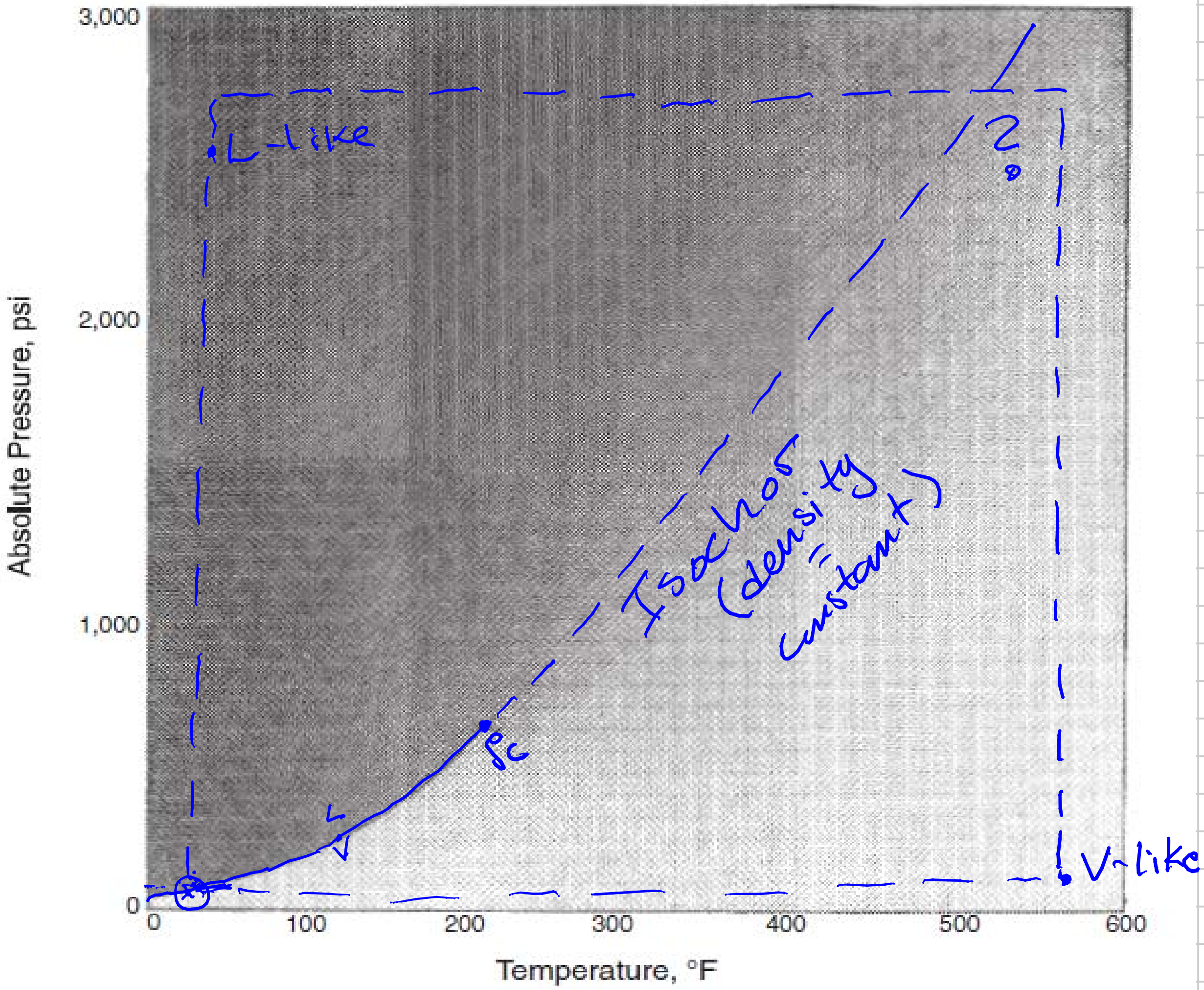
V_{sL} saturated liquid

1

V_{sV} = saturated vapor

1000

Propane

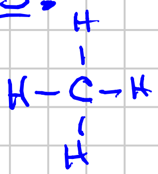
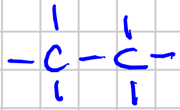
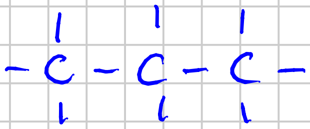
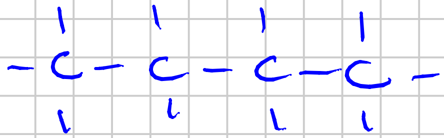


Degree of Shading $\propto \rho$

Compounds Making Up Petroleum Systems

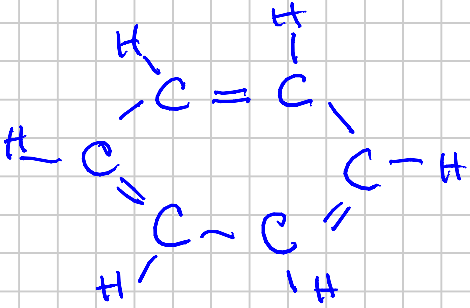
H_2O
N_2
CO_2
(H_2S)
} Non-Hydrocarbons

HYDROCARBONS:

C_1	CH_4		methane	
C_2	C_2H_6		ethane	C_nH_{2n+2}
C_3	C_3H_8		propane	
$i-C_4$	C_4H_{10} (isomer)		iso-butane	
$n-C_4$	C_4H_{10}		normal-butane	
$i-C_5$	C_5H_{12}		pentanes	
$n-C_5$				
$n-C_6$ ⋮	C_6H_{14}		"Paraffins" Alkanes	

Isomers with "straight" chains

Benzene C_6H_6



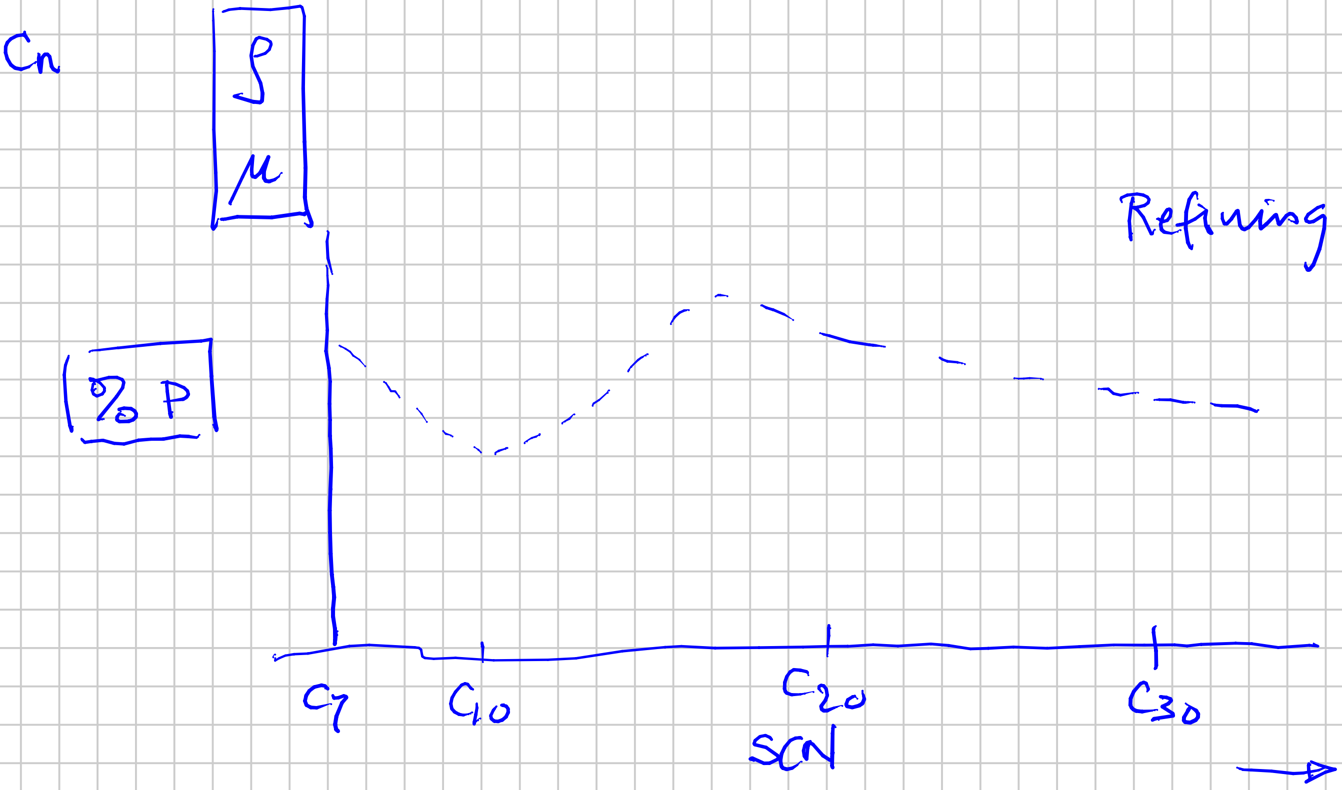
Cyclic
Ring
"AROMATICS"

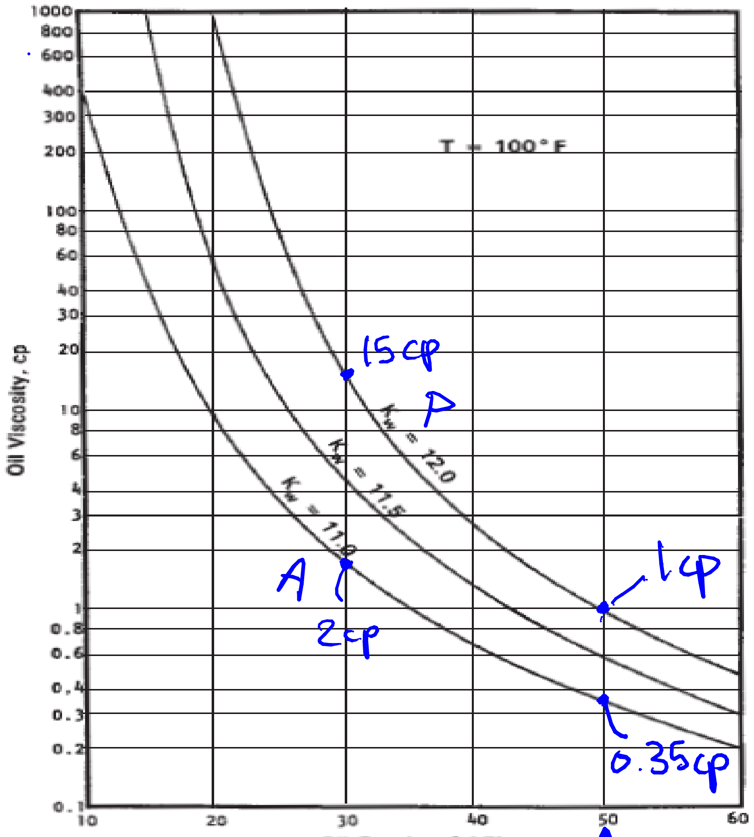
- Density much higher than ρ_{n-C_6}

875 kg/m^3 655 kg/m^3

Chemical Make-Up Character of HCs (PNA)

Paraffins Napthenes Aromatics





← ρ_L kg/m^3

Fig. 3.21—Dead-oil (stock-tank-oil) viscosities at 100°F for varying paraffinicity (from Ref. 33).

1000 750 700

C_{30+} SCN C_7

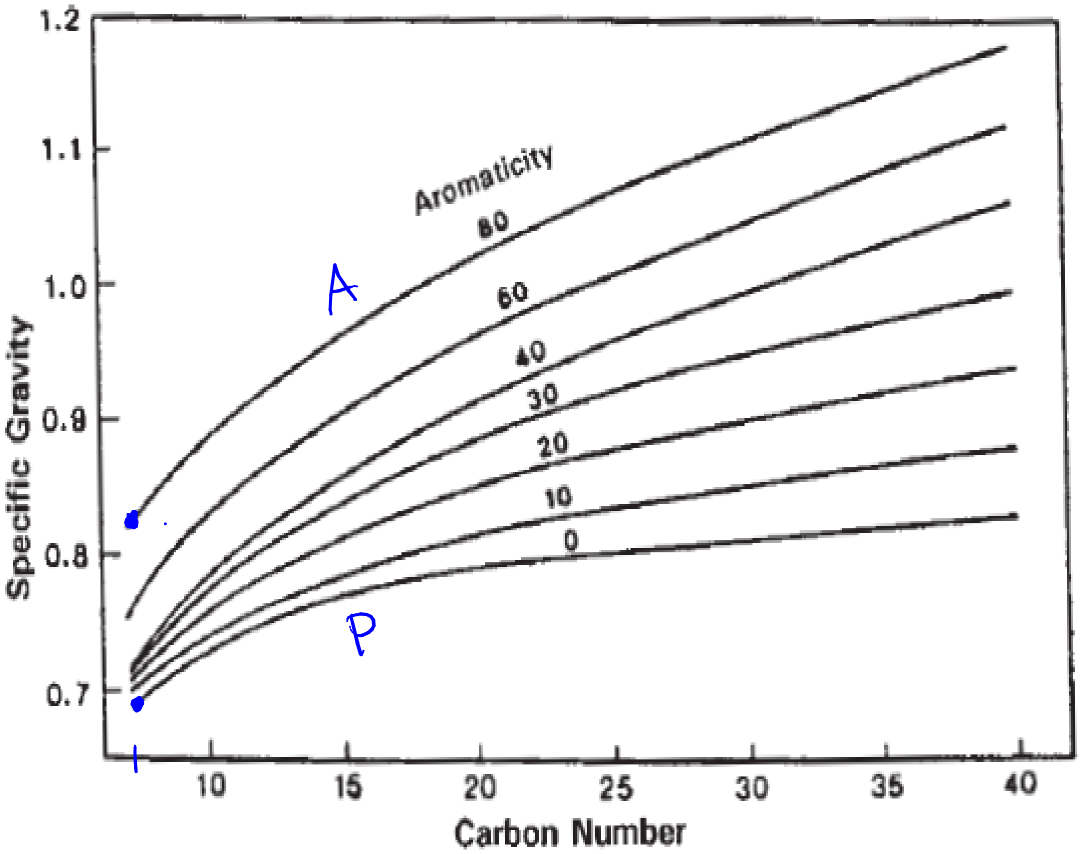
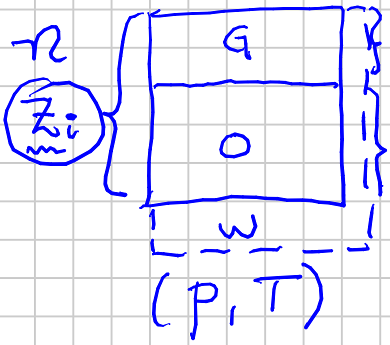


Fig. 5.15—Specific gravity vs. carbon number for constant values of the Yarborough aromaticity factor (after Yarborough[1]).

PHASE EQUILIBRIUM (K-values) K_i Equilibrium Ratios

GAS-OIL

Molar Compositions



$n_g \quad y_i = \frac{n_{ig}}{n_g}$

$n_o \quad x_i = \frac{n_{io}}{n_o}$

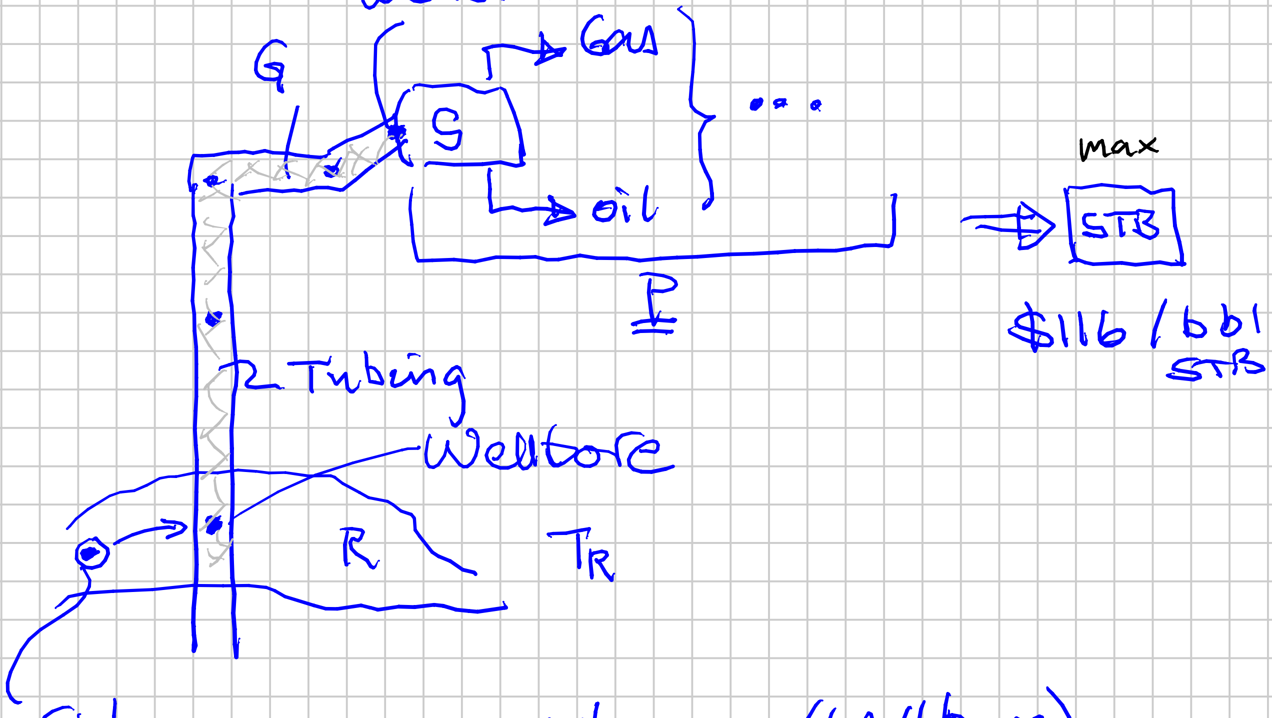
$$K_i \equiv \frac{y_i}{x_i}$$

$$\uparrow$$

$$\boxed{K_i(p, T, \underset{\sim}{z})}$$

Know @ Reservoir, Production Pipe,
Surface Gathering Lines,
Separator Unit

"Wellstream"



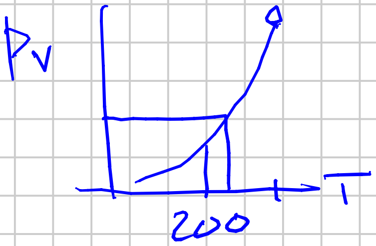
In-Situ Reservoir $\overset{=}{\neq}$ Wellstream (Wellbore)

Behavior of K_i

K_i represents (in lay terms) the relative preference of component i to "be" in the $\underbrace{\text{gas phase}}_{K_i > 1}$ or $\underbrace{\text{oil phase}}_{K_i < 1}$:

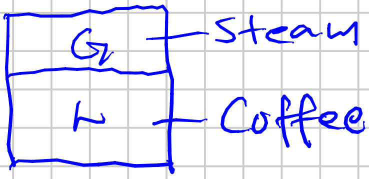
We can make more money by selling component i as part of the liquid phase (ultimate processed surface)

5-10 times the value



@$p_v(T)$

H_2O



$y_{H_2O} = 1$

$x_{H_2O} = 1$

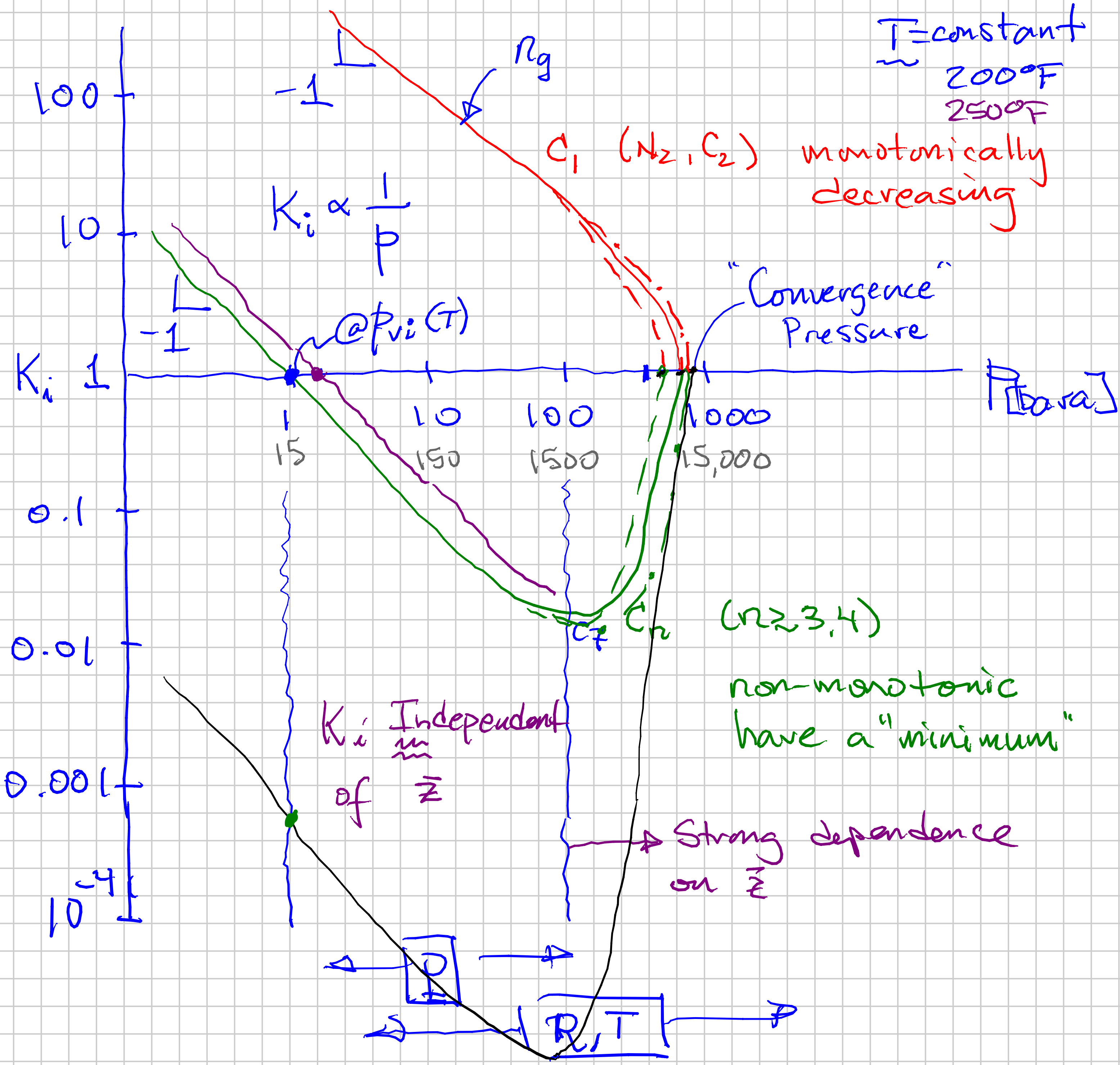
$K_{H_2O} = \frac{1}{1} = 1$

100°C

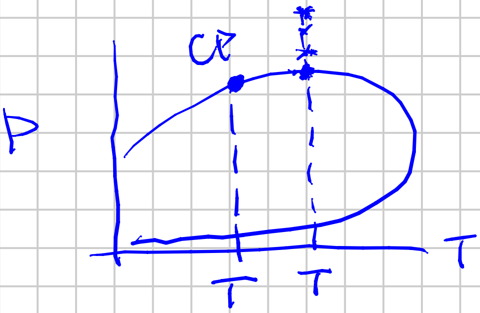
1 atm

$p_{vi}(T) \Rightarrow K_i(p \lesssim 100 \text{ bara}, T)$

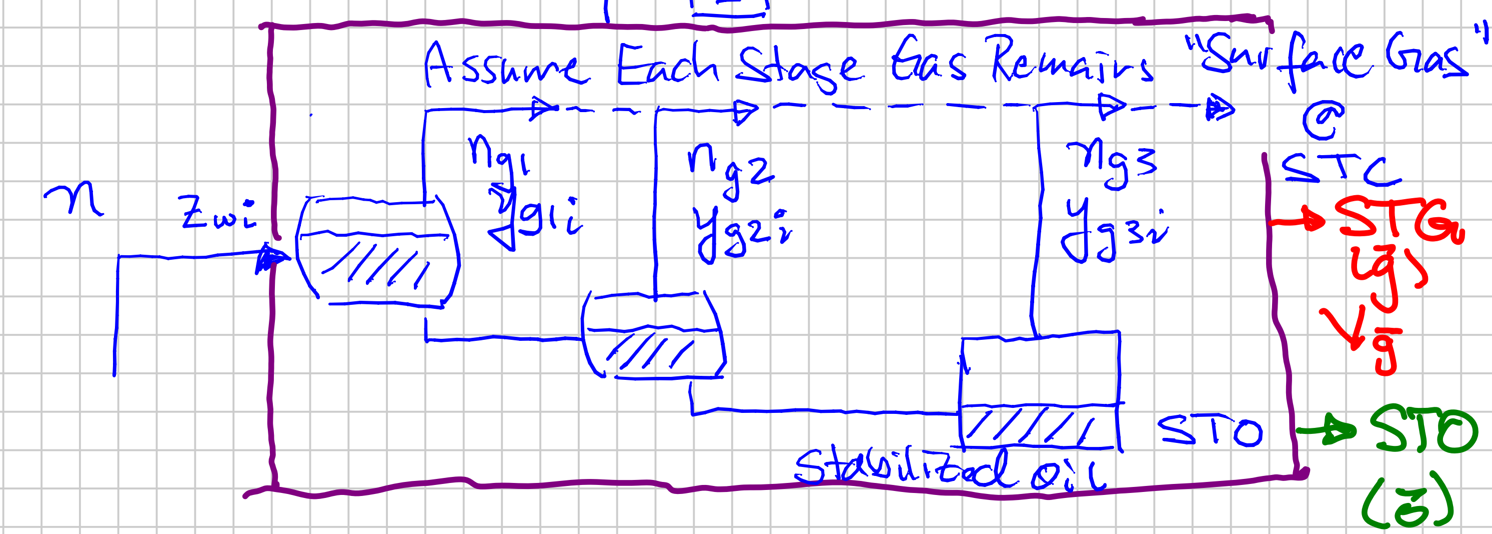
$$K_i \approx \frac{p_{vi}(T)}{p}$$



$K_{i\,min}$ @ ~ 50-120 bara



SURFACE PROCESSING "$\boxed{P}$"



$V_{\bar{o}}$ max

$n_{\bar{o}}$

Stage 1 $\left[\begin{Bmatrix} p_{sp1} \\ T_{sp1} \end{Bmatrix}\right.$ Stage 2 $\left.\begin{Bmatrix} p_{sp2} \\ T_{sp2} \end{Bmatrix}\right]$ Stage N $\begin{Bmatrix} p_{sp3} \\ T_{sp3} \end{Bmatrix}$

$K_{1i}(p,T)$ $K_{2i}(p,T)$ $K_{3i}(p,T)$

Controls the ultimate MOLES of i => STO

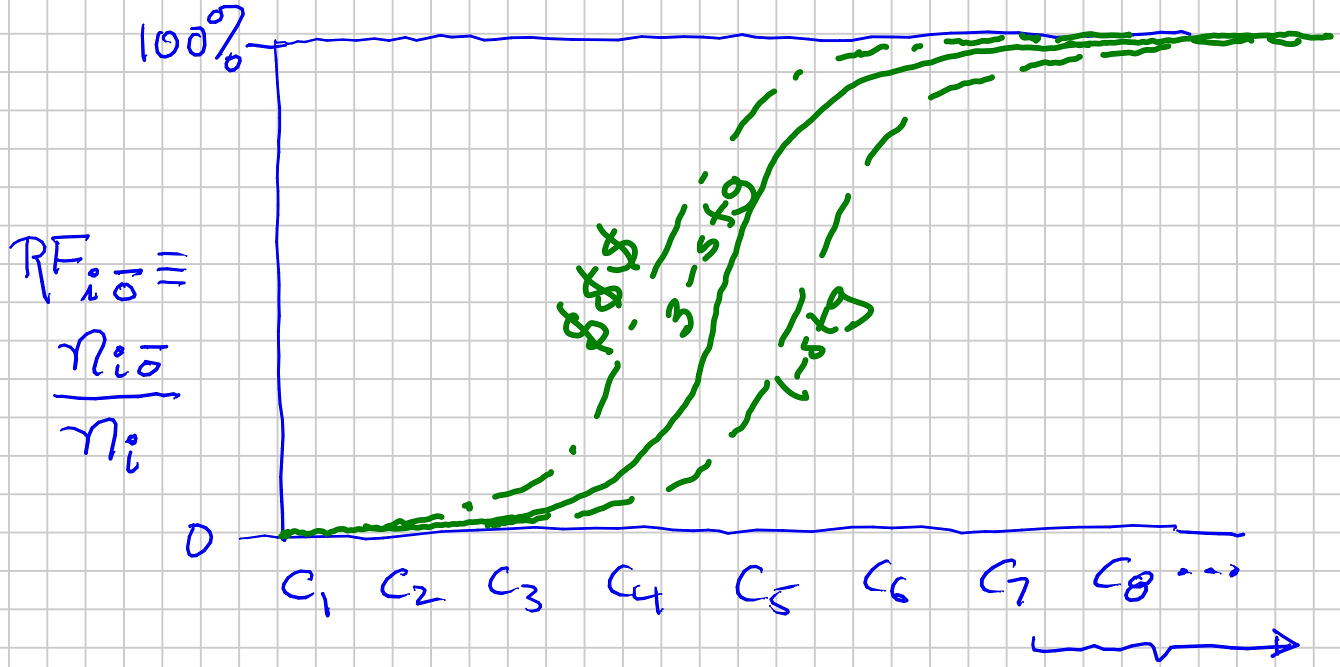
$$\begin{tikzcd}
K_i \arrow[d, "\bar{K}_{all}"'] \arrow[dr, bend left, "(K_i)"] & \\
(n_g) & y_i
\end{tikzcd}$$

$$n_{ig} = \underset{\substack{\uparrow \\ \max}}{\underline{(n_g)}} \cdot \underset{\substack{\uparrow \\ \max K_i}}{\underline{y_i}}$$

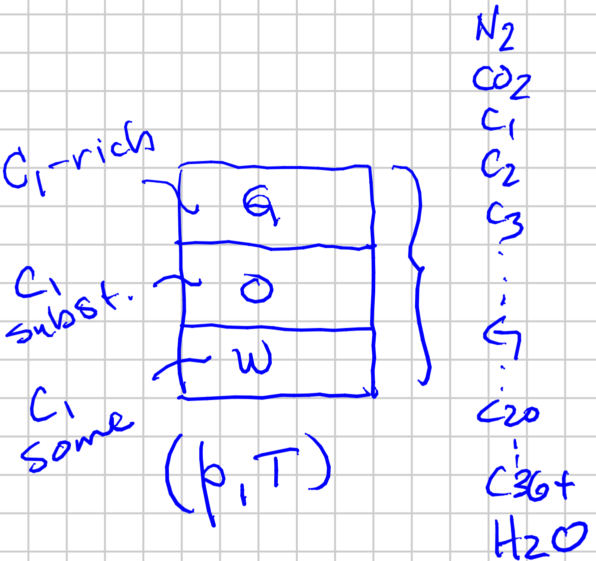
1-stg

$\underline{1949}$

$$V_{\bar{o}} \propto \left(1 - \sum_{k=1}^{3} n_{igk}\right)$$



Water – Gas-Oil Phase Equilibria



- Pore In-Situ
- Production Pipes
- Separator
- Ocean (Leakage Blowout)

In general

$$\text{Gas-Oil} \quad K_i(p, T, \vec{z}) \simeq K_i(p, T, \vec{z}^*)$$

Rigorous

No H_2O | H_2O Included

Engineering Guideline

⇒ We can ignore H_2O component and aqueous phase when doing gas-oil phase equilibria calculations.

$$K_{wgi} \equiv \frac{y_i}{x_{wi}} \underbrace{\gg}\, 1 \qquad \underbrace{\text{all } i \text{ except } H_2O}_{\substack{\text{Limited Solubility} \\ \text{in the Aqueos Phase}}}$$

$$K_{woi} \equiv \frac{x_i}{x_{wi}} \underbrace{\gg}\, 1$$

$$\underbrace{\ll}\, 1 \quad \text{for } H_2O \Rightarrow$$

CO_2

$K_{wgC_1} < K_{wgC_2}$ Methane

⋮

HCs

$\Big\downarrow$ Decreasing Solubility

H_2O-G-O

When is solubility important?

① Gas production ⇒ water production

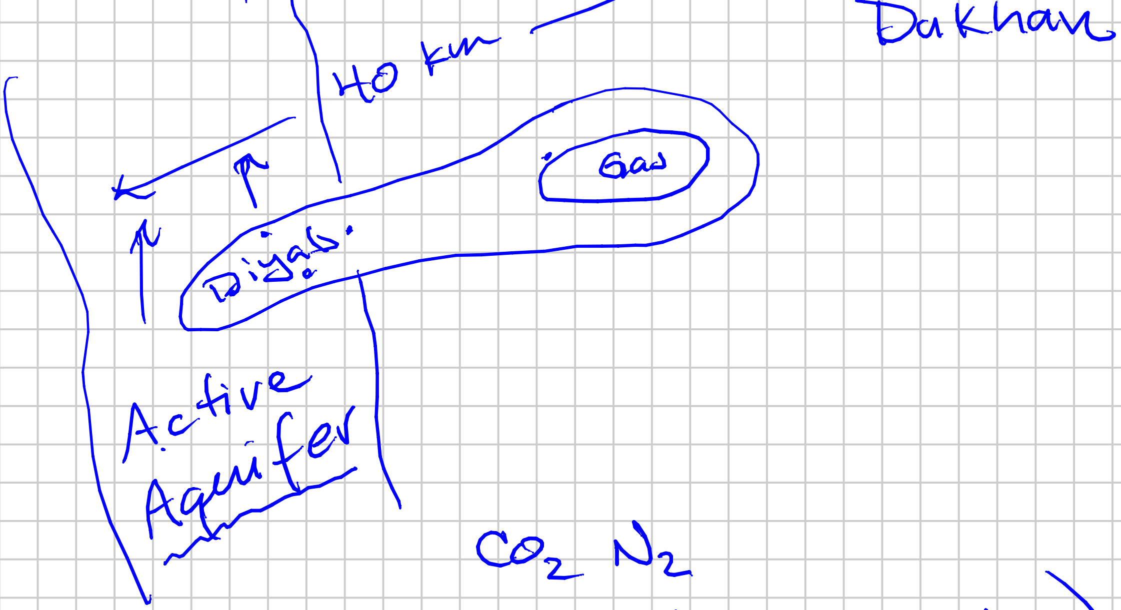
0.1–5 mol-% wellstream H_2O

→
- $T_R > 300°F$
- $p_{wf} < 500$ psia

② When oil & gas "sees" (mixes) with larger amounts of water that has no yet equilibrated ("seen") oil & gas yet.

- Injecting water (water flood)
- Offshore blowout situation

- Active Aquifer



CO_2 N_2

Gaseous Components (C_1 C_2 C_3 ; Aromatics)

PHASE EQULIBRIUM CALCULATIONS (& APPLICATIONS)

Note Title 2013-09-12

① Select Student Course Evaluation Group
'Reference' Groups

Lisbeth Hultmann

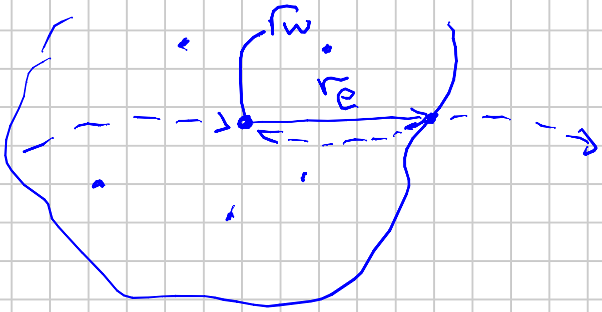
3 students:

1 - 5-yr Norsk : Carl Ivar Klovning
1 - 2-yr MS Norsk : Vinh Vuong Tran
1 - 2-yr MS International : Allan Katende

Darcy's Law:

$$v = \underbrace{\frac{k}{\mu}}_{\uparrow} \cdot \underbrace{\frac{\Delta p}{\ln(r_e/r_w)}}$$

"Well Spacing"



Higher v : fewer wells

	Quantity		Magnitude	# orders of Magnitude
God-Given	k	[md]	(10^{-4}) $(0.1-1) - 10{,}000$ shale Low	[5]
	μ	[cp]	$0.01 - 0.1 - 100$ Gas } oil	[5]
	Δp	[bar]	$10 - 1000$	[2-3]
	$\ln \frac{r_e}{r_w}$		$5 - 10$	
	$\frac{k}{\mu}$	"Mobility"		[→10]

The well spacing is (in reality) most dependent – for a given well type – on (k/μ)

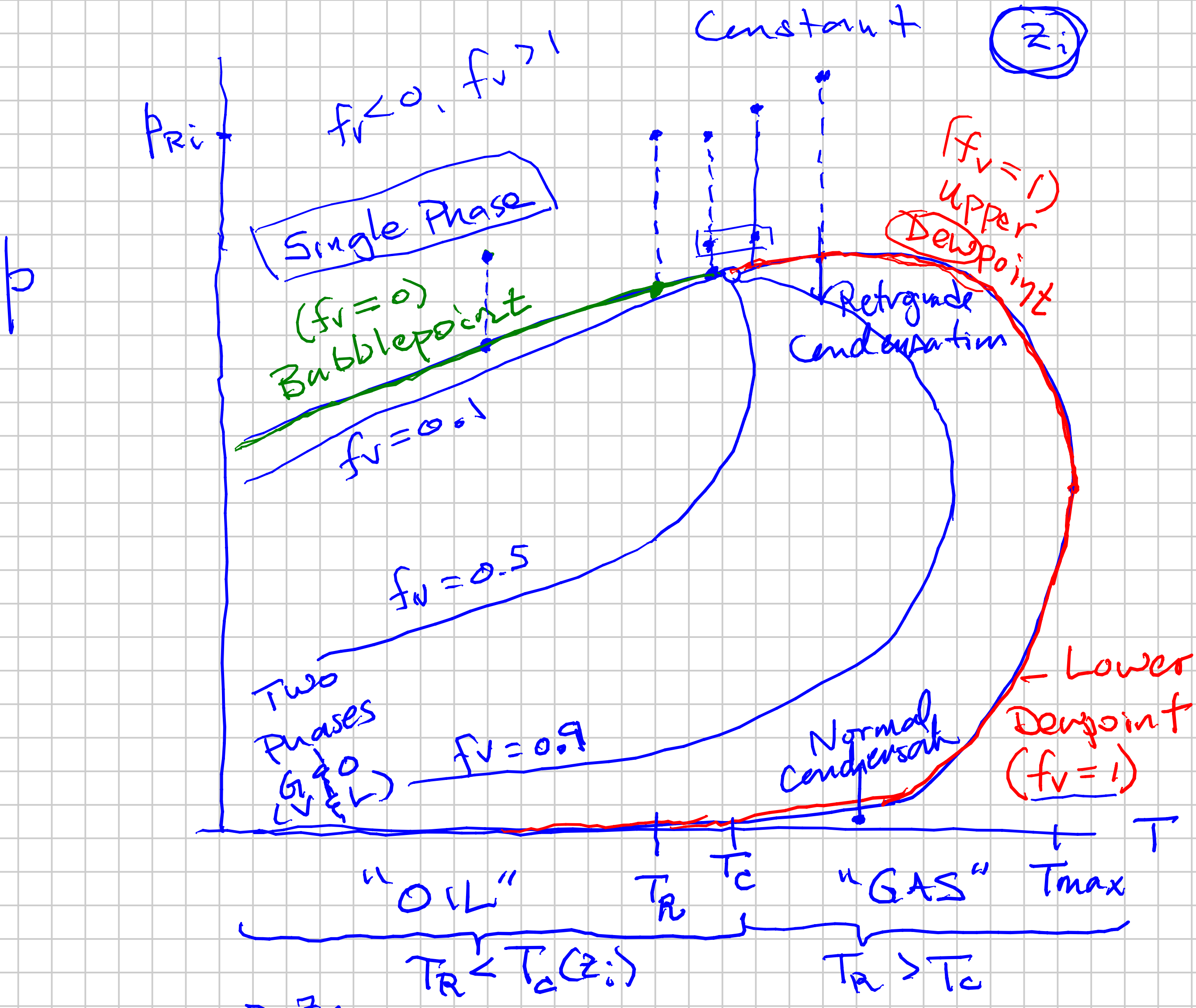
High — Lower Spacing (fewer wells)

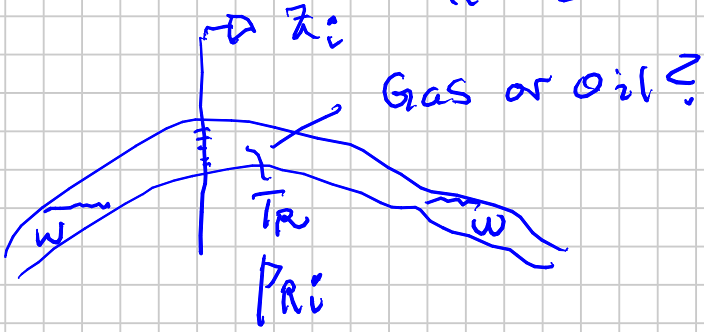
Low — Higher Spacing (more wells)
(More Dense)

i.e. How many wells (per area) are needed to "drain" within a "reasonable" period of time (20-30 yrs)

Formal / Scientific Fluid Type Definition (Reservoir)

Mixture z_i





$T_c < T_R < T_{max}$

Gas Condensate

"Retrograde" G.C.

$T_R > T_{max}$ (No Cond. inside Reservoir)

Wet Gas $\boxed{I}$ Dry Gas

Surface Condensation

No Surf. Condensation

% \$115/STB

Phase Equilibrium Calculations

Simplified "Isothermal FLASH"

n

z_i

$n_V \quad y_i \qquad n_{iV} = n_V \cdot y_i$

$n_L \quad x_i \qquad n_{iL} = n_L \, x_i$

(p, T)

Know : $\underbrace{z_i \quad (p, T)}_{\text{estimate}}$ $\Rightarrow$ $z_i \quad K_i(p,T)$

$$K_i = \frac{y_i}{x_i}$$

Requirement :

$$\sum z_i = 1$$

$$\left.\begin{aligned} n &= n_V + n_L \\ \Rightarrow \frac{n_i}{n} &= \frac{n_{iV}}{n} + \frac{n_{iL}}{n} \end{aligned}\right\} \text{Material Balances}$$

Definitions :

$$K_i \equiv \frac{y_i}{x_i}$$

$$\left\{\begin{aligned} f_V &\equiv \frac{n_V}{n} \\ z_i &\equiv \frac{n_i}{n} \end{aligned}\right. \qquad y_i = \frac{n_{iV}}{n_V} \qquad x_i = \frac{n_{iL}}{n_L}$$

$$z_i = f_V \, y_i + (1 - f_V) \, x_i$$ ⇐ Solve for x_i

Constraint : $\underbrace{\sum y_i = 1 = \sum x_i}$

$$\sum_i y_i - \sum_i x_i = \underbrace{\boxed{0 = \sum_i y_i - x_i}}_{\text{Constraint}} \qquad \text{MM}$$

Solving this equation $\Rightarrow \boxed{f_v \, , \, y_i \, , \, x_i}$

Muskat - McDowell 1949

$$K_i = \frac{y_i}{x_i} \Rightarrow y_i = K_i x_i$$

$$\sum y_i - x_i = \sum K_i x_i - x_i = \sum x_i (K_i - 1)$$

$$z_i = f_v K_i x_i + (1 - f_v) x_i$$

$$= f_v K_i x_i + x_i - f_v x_i$$

$$= x_i \left(f_v (K_i - 1) + 1 \right)$$

$$\Rightarrow x_i = \frac{z_i}{\left[f_v (K_i - 1) + 1 \right]}$$

$$0 = \sum y_i - x_i = \boxed{\sum_i \frac{z_i (K_i - 1)}{\left[f_v (K_i - 1) + 1 \right]} = 0} \quad h_v(f_v)$$

One unknown: f_v

Once f_v solved, calc. x_i & y_i

Rachford-Rice Equation $\boxed{\text{Ch. 4}}$

Muskat-McDowell: $c_i = \dfrac{1}{K_i - 1}$

(1949)

$$h(f_V) \quad \sum_{i}^{N} \frac{z_i}{f_V + c_i} = 0 \qquad ; \; c_i = \infty \quad K_i = 1$$

N-1 (N-2) solutions

Only <u>one</u> solution that yields PHYSICAL values of y_i & x_i $\underline{(\geq 0)}$

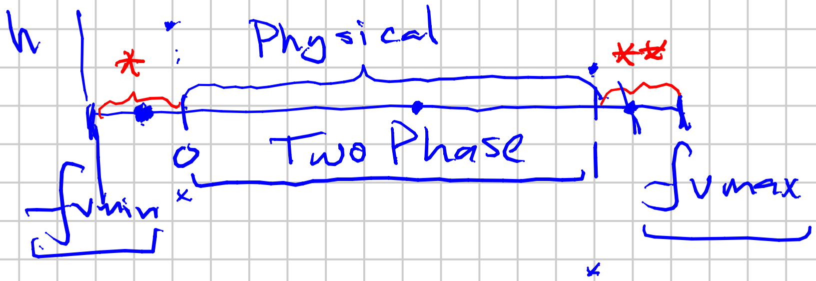
A priori we know which solution — i.e.

$$\underline{f_{V\min}} < f_V^* < \underline{f_{V\max}}$$

$$\uparrow \qquad\qquad \uparrow$$

$$\frac{1}{1-K_{max}} \qquad \frac{1}{1-K_{min}}$$

$$\uparrow \qquad\qquad \uparrow$$



We know K_i

$\Rightarrow$ K_{min}, K_{max} ✓

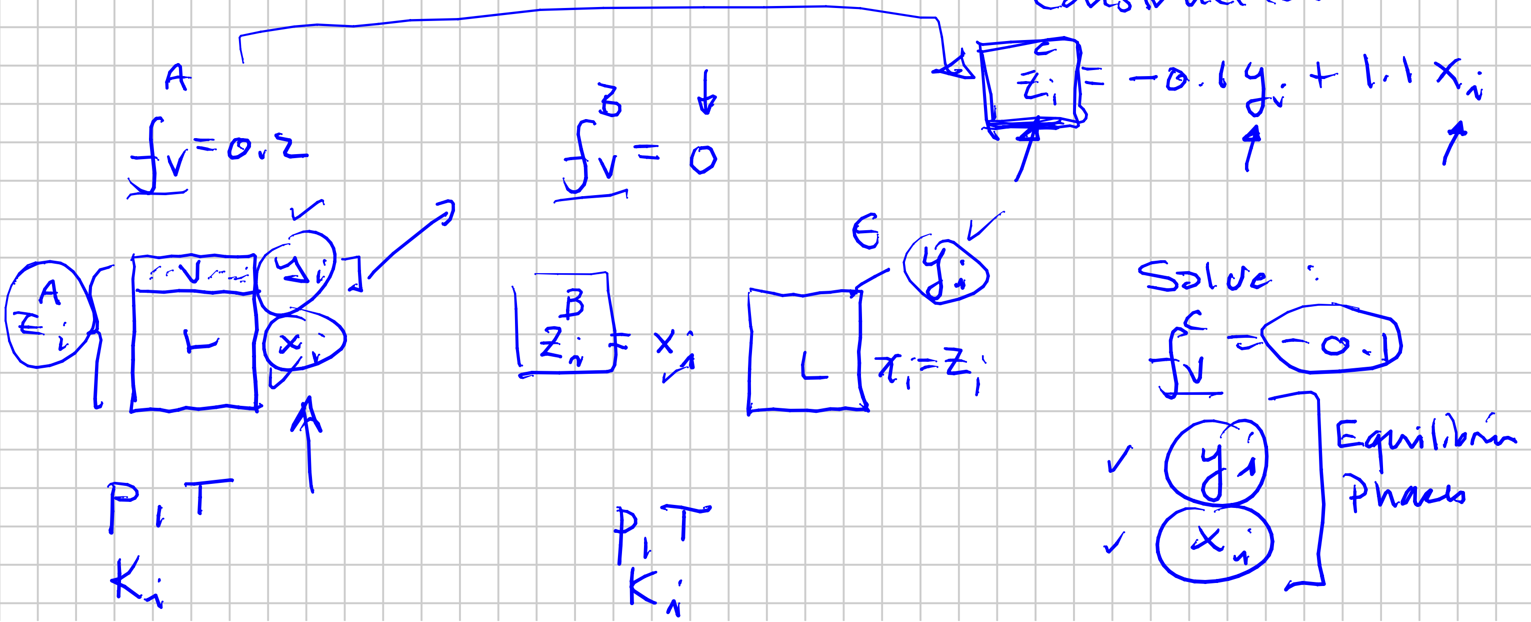
$f_{V\max}$ $f_{V\min}$ ✓

$h(f_V)$ is monotonic

* Single Phase "L-like"

** Single Phase "V-like"

Constructed



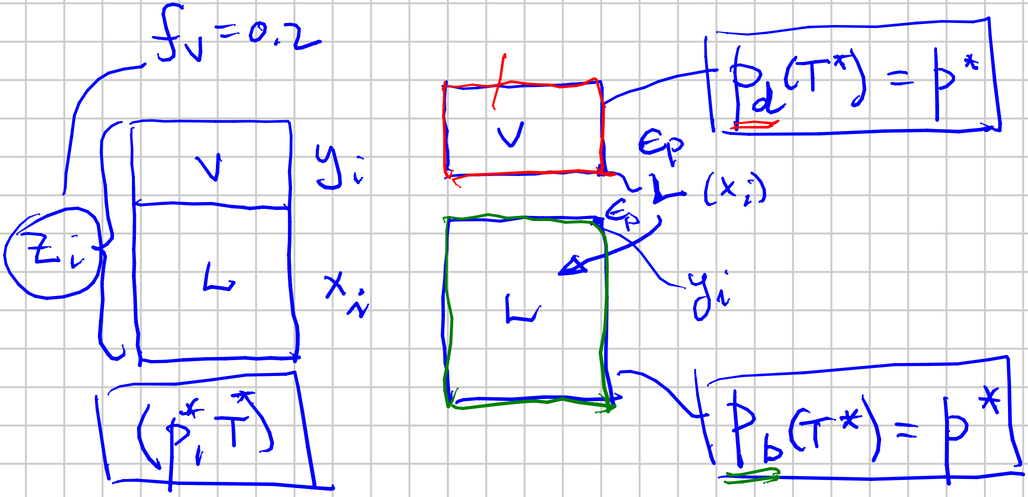
When two phases exist & are in equilibrium:

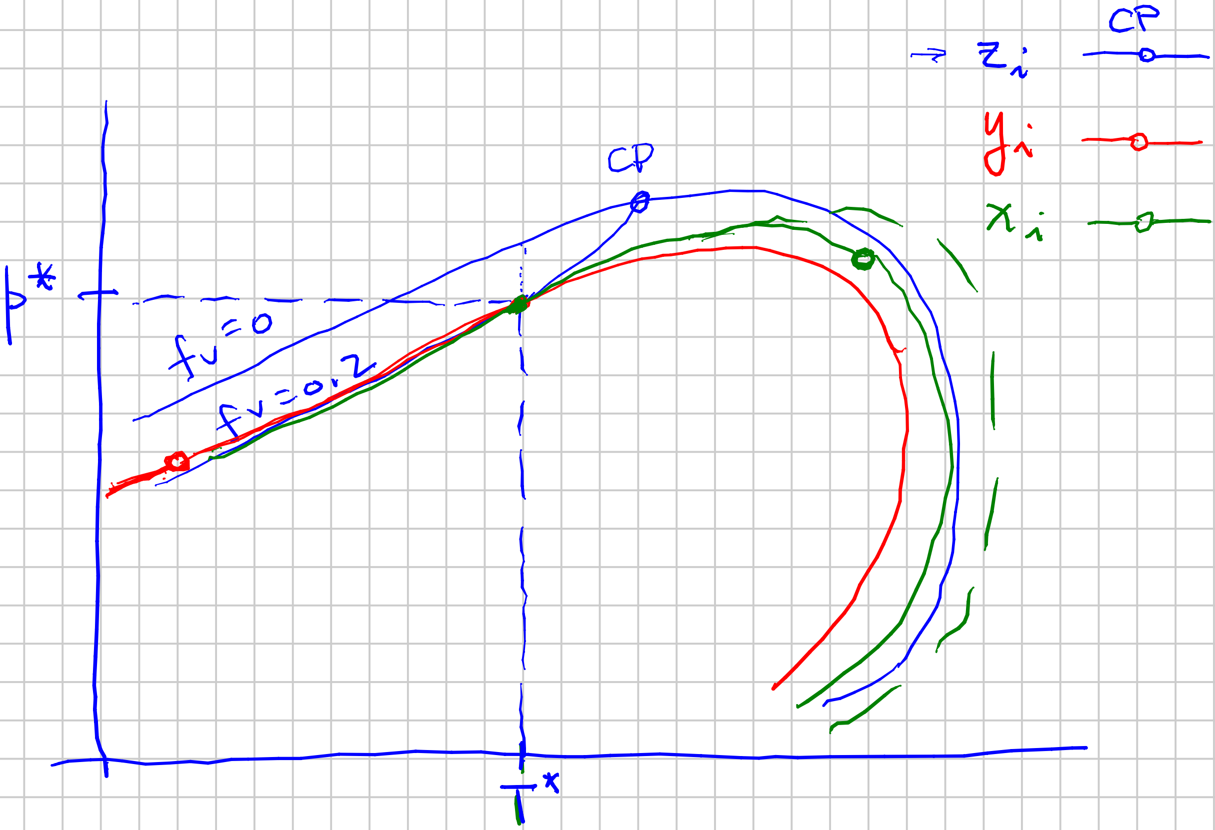
Gas (Vapor) Phase is "Saturated"

Oil (Liquid) Phase is "Saturated"

⇓

with the equilibrium phase(s)





ISOTHERMAL FLASH CALCULATION (Rachford-Rice)

PROBLEM Statement: (1) How many phases (V L)

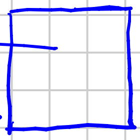
o g

: 1 or 2

Under-saturated

Saturated: 2 phases

$1-\epsilon$ ϵ



Overall Composition z_i Known

(p, T) Fixed, Known, Specified

(2) How much of each phase (moles or mole fraction)

n_V / n_L

$f_V \equiv \frac{n_V}{n}$

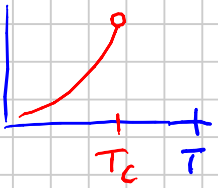
(3) Molar composition of each phase V: y_i

L: x_i

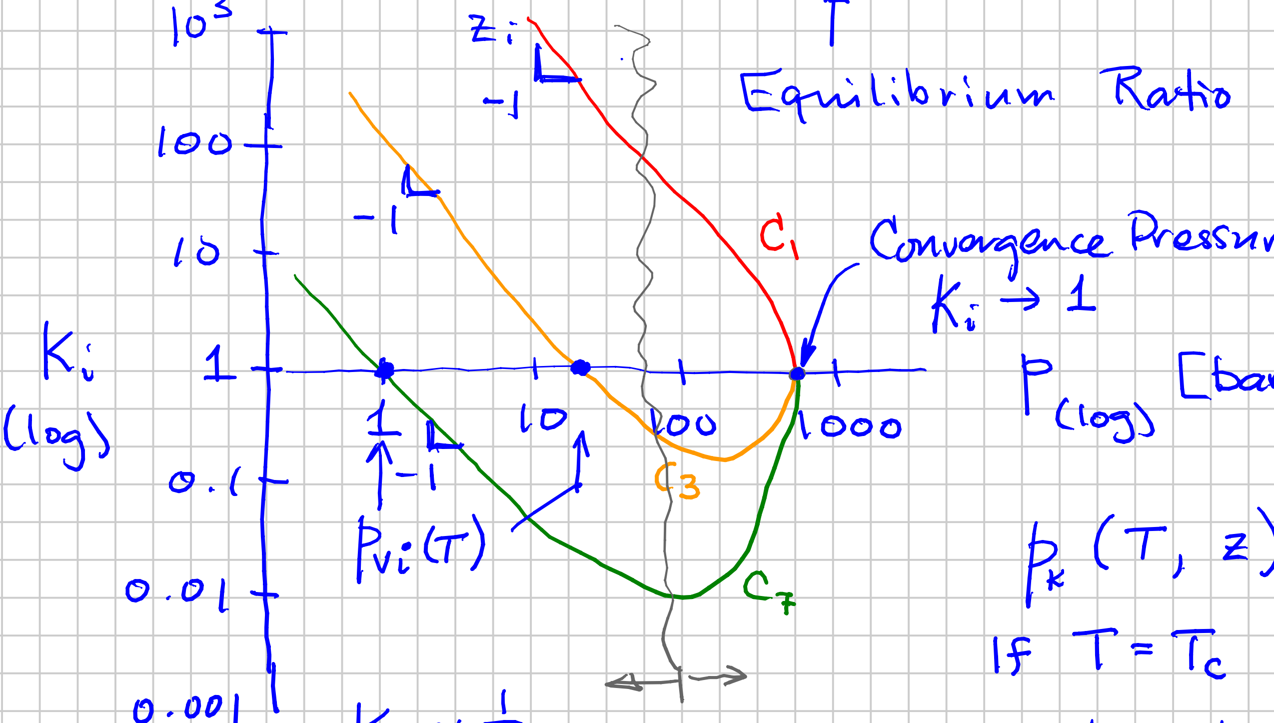
SOLUTION:

Requires estimate of $K_i = \frac{y_i}{x_i}$ @ (p, T, z_i)

Equilibrium Ratio



T = constant



$K_i \propto \frac{1}{p}$

$p_k(T, z)$

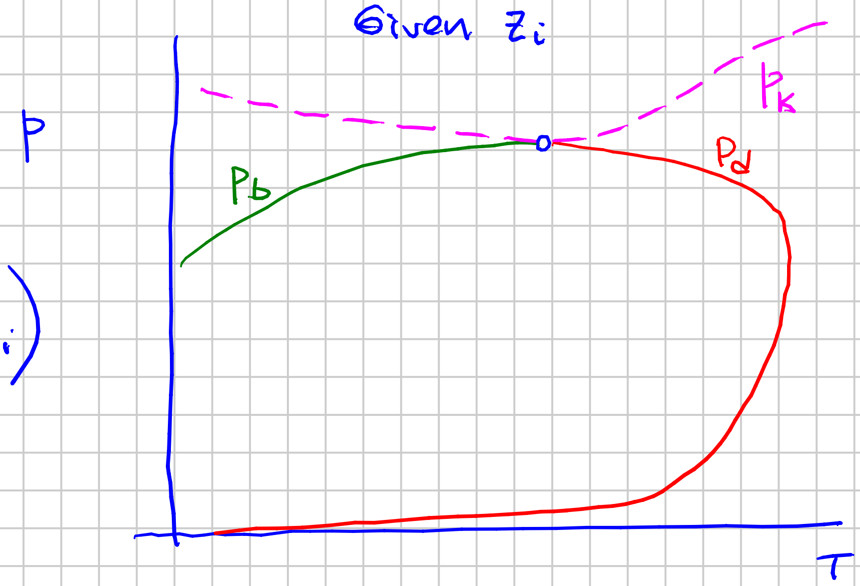
If $T = T_c$ of z_i

$\Rightarrow p_k = p_c$ of z_i

SPE PBM

* Modified Wilson Eq.

$$K_i(p, T, p_K ; \underline{\underline{p_{ci}}}, \underline{\underline{T_{ci}}}, \underline{\underline{\omega_i}})$$



Component Material Balance:

$$\begin{cases} n_i = n_{Li} + n_{vi} \longleftarrow \\ \text{Total Material Balance} \\ n = n_L + n_V \end{cases}$$

$$n = \sum_{i=1}^{N} n_i \qquad n_L = \sum_{i=1}^{N} n_{Li} \qquad n_V = \sum_{i=1}^{N} n_{Vi}$$

Define: $z_i \equiv \frac{n_i}{n}$ $\quad y_i \equiv \frac{n_{Vi}}{n_V}$ $\quad x_i \equiv \frac{n_{Li}}{n_L}$

$$f_V \equiv \frac{n_V}{n} \quad ; \quad f_L = \frac{n_L}{n} = 1 - f_V$$

Also show:

$$\sum z_i = 1 = \sum y_i = \sum x_i$$

$K_i \equiv \frac{y_i}{x_i}$ know

$$\left.\begin{array}{l} \boxed{z_i = f_V y_i + (1-f_V) x_i} \\ n_i = n_{Vi} + n_{Li} \end{array}\right\} \text{same thing}$$

$$K_i = y_i / x_i$$

$$\Rightarrow \quad y_i = K_i x_i$$

$$\underset{\checkmark}{z_i} = f_v \cdot (\underset{\checkmark}{K_i} x_i) + (1-f_v)\, x_i$$

Solve this for x_i

$$z_i = x_i \left[f_v K_i + (1-f_v) \right]$$

$$z_i = x_i \left[f_v (K_i - 1) + 1 \right]$$

✓ $$x_i = \frac{\overset{\checkmark}{z_i}}{\underset{\checkmark}{f_v}(\underset{\checkmark}{K_i}-1)+1}$$ ✓ ←

✓ $$y_i = K_i x_i = \frac{z_i K_i}{f_v (K_i - 1) + 1}$$ ✓ ←

1949: Muskat - McDowell

$$\sum y_i = 1 \qquad \sum x_i = 1$$

$$\sum y_i - \sum x_i = 1 - 1 = 0$$

$$\sum_{i=1}^{N} (y_i - x_i) = 0$$

"Rachford-Rice": 195x

$$h(f_v) \equiv \sum_{i=1}^{N} \left[\frac{\overset{\checkmark}{z_i}(\overset{\checkmark}{K_i}-1)}{f_v(\underset{\checkmark}{K_i}-1)+1} \right] = 0 \qquad (1)$$

if $f_v = \frac{-1}{K_i - 1}$

1949: M-M

$$c_i = \frac{1}{K_i - 1} \quad ; \quad c_i = 0 \text{ if } K_i = 1$$

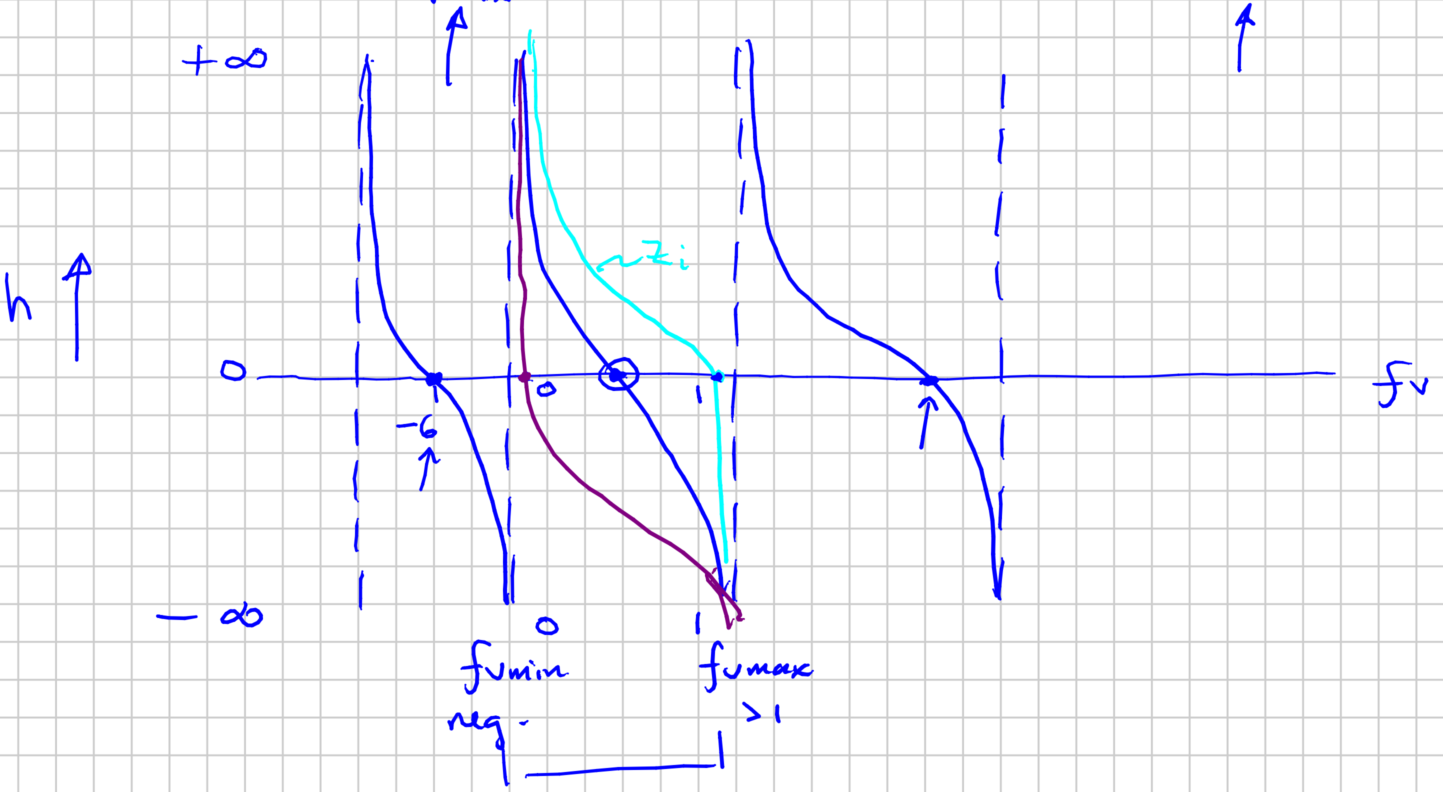
$$h(f_v) = \sum_{i=1}^{N} \left[\frac{z_i}{f_v + c_i} \right] = 0 \qquad (1')$$

(1) $h(f_v)$ is a monotonic function → NR

(2) N-1 solutions

- Only one of these yields $\underbrace{\text{physical solution}}_{\substack{x_i \geq 0 \\ y_i \geq 0}}$

$$0 > \frac{1}{1-K_{max}} = f_{v\,min} < f_v < f_{v\,max} = \frac{1}{1-K_{min}} > 1$$



: Solved f_v^*

L + V $\quad 0 < f_v^* < 1$: two phase solution

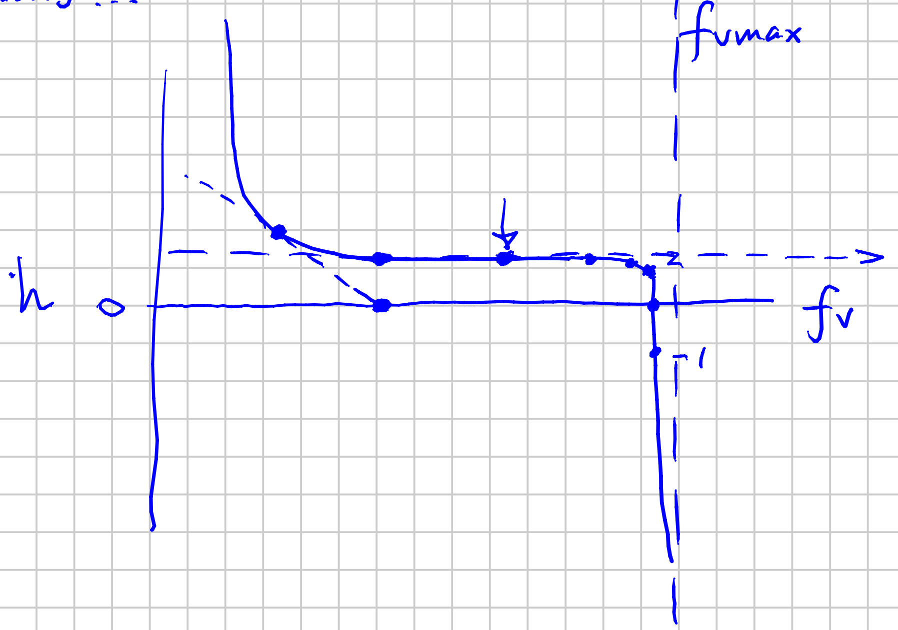
Liquid $\quad f_v^* = 0$
Vapor $\quad f_v^* = 1$ } : saturated single phase

Liquid-Like $\quad f_v^* < 0$
Vapor-Like $\quad f_v^* > 1$ } : undersaturated single phase

< 1% of calculations ...

$z_i \to \epsilon$

$K_i \to \epsilon$ or huge



Setup for Solution:

p, T, z_i Given

1. Estimate $K_i(p, T, \overset{\text{Guestimate / Estimate}}{p_K})$: Wilson Ch. 3 or 4

Guess $\boxed{f_v}$

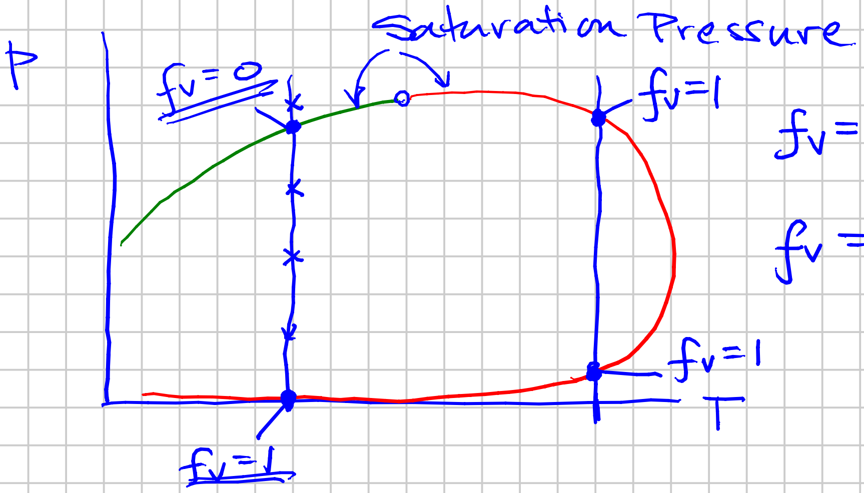
Calc $\boxed{f_{v\min}}$ $\boxed{f_{v\max}}$

2. Setup a Table

i	z_i	K_i	$c_i = \frac{1}{K_i - 1}$	Term$_i$	y_i	x_i
1						
2		K_{max}				
⋮						
N		K_{min}				

$\boxed{h}$ sum ← drive this to → 0 (ε)

Special Cases of Flash Calculation:



$f_v = 0$: Bubblepoint

$f_v = 1$: Dewpoint

Bubblepoint : $f_v = 0$

"Bubble"

$\Rightarrow \quad \sum y_i = 1$ equation

$$\boxed{h_{BP}(p_b) = 1 - \sum_{i=1}^{N} z_i (K_i(p_b)) = 0}$$

$$y_i = x_i K_i$$

$$y_i = z_i K_i(p ; \underbrace{T, p_K}_{\text{fixed}})$$

search

Dewpoint : x_i $\in$ phase $\quad f_v = 1 - \epsilon$

$$\sum x_i = 1$$

$$x_i = y_i / K_i$$

$$x_i = z_i / K_i(p ; T, p_K)$$

?

Fixed T, p_K

$$\boxed{h_{DP} = 1 - \sum x_i = 0 = 1 - \sum \frac{z_i}{K_i(p)}}$$

Two solutions:

Upper DP

Lower DP

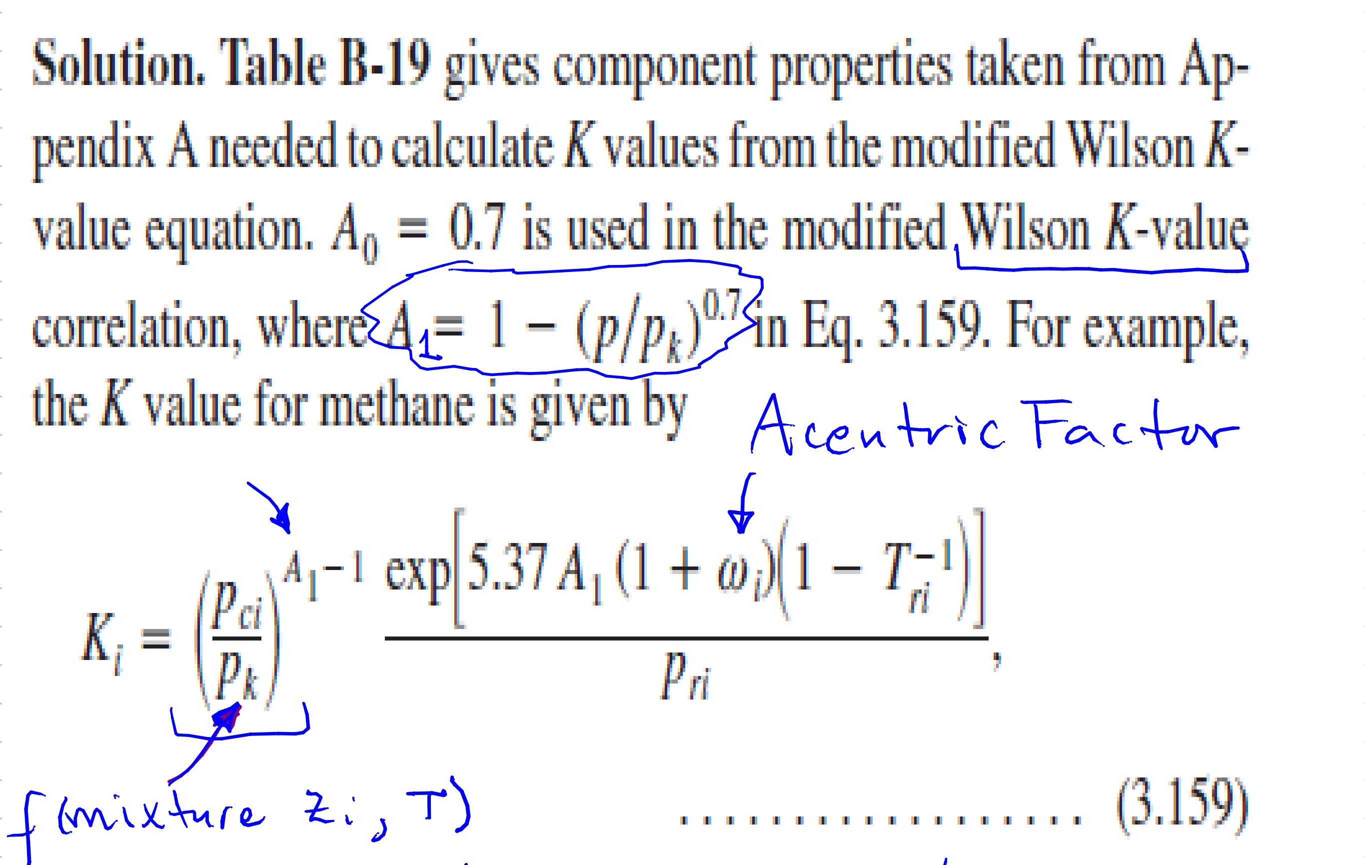
$$h(f_v) = \sum_i \frac{z_i}{f_v + c_i} = 0 \qquad MM$$

$$= \sum_i \frac{z_i(K_i-1)}{f_v(K_i-1)+1} = 0 \qquad RR$$

Given z_i $K_i(p, T, z)$

p_K = convergence pressure

p where all $K_i \to 1$

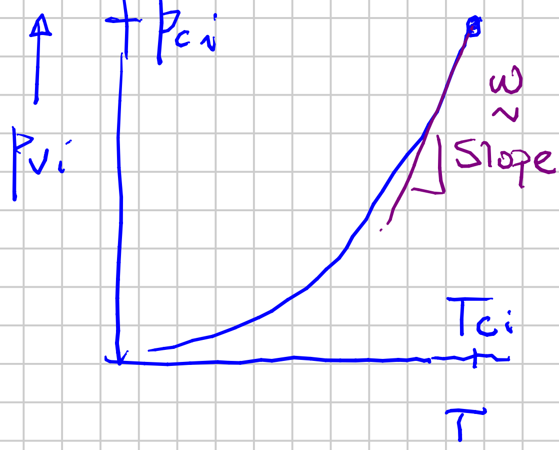


Solution. Table B-19 gives component properties taken from Appendix A needed to calculate K values from the modified Wilson K-value equation. $A_0 = 0.7$ is used in the modified Wilson K-value correlation, where $A_1 = 1 - (p/p_k)^{0.7}$ in Eq. 3.159. For example, the K value for methane is given by

$$K_i = \underbrace{\left(\frac{p_{ci}}{p_k}\right)}_{f(\text{mixture } z_i,\, T)}^{A_1-1} \frac{\exp\left[5.37 A_1 (1+\overset{\text{Acentric Factor}}{\omega_i})\left(1-T_{ri}^{-1}\right)\right]}{p_{ri}}, \qquad (3.159)$$

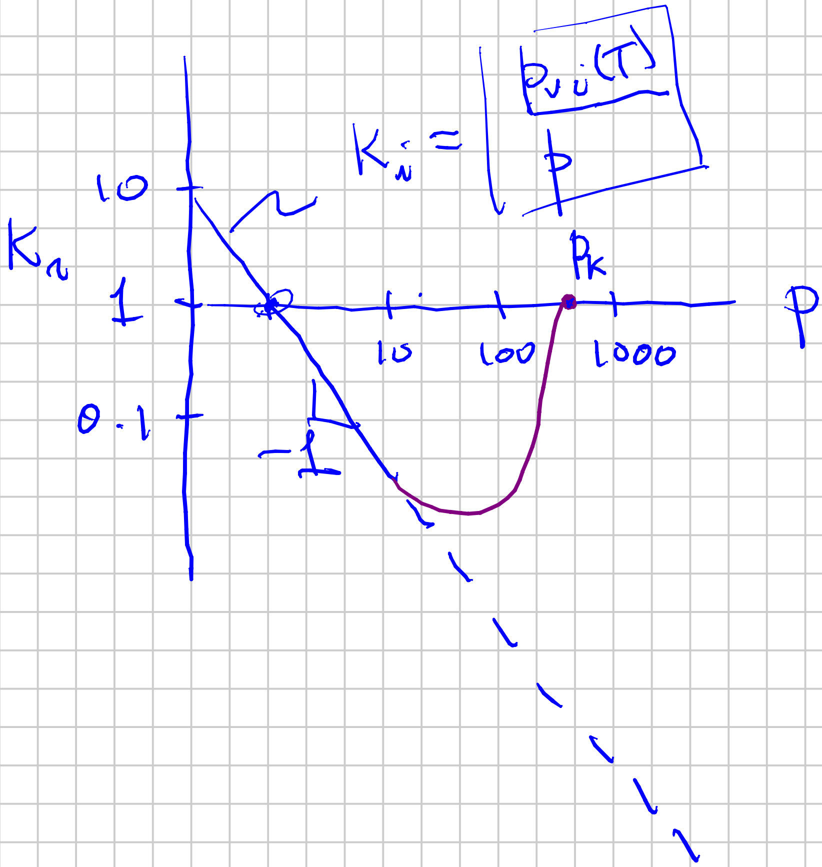
$$T_{ri} = \frac{T}{T_{ci}} \qquad p_{ri} = \frac{p}{p_{ci}}$$

Component i : T_{ci} p_{ci} ω_i



Original Wilson Eq.

$$K_i = \boxed{\frac{p_{vi}(T)}{p}}$$



SOLVER SOLUTION:

① Set Target Cell — $h = \sum \frac{z_i}{f_v + c_i}$

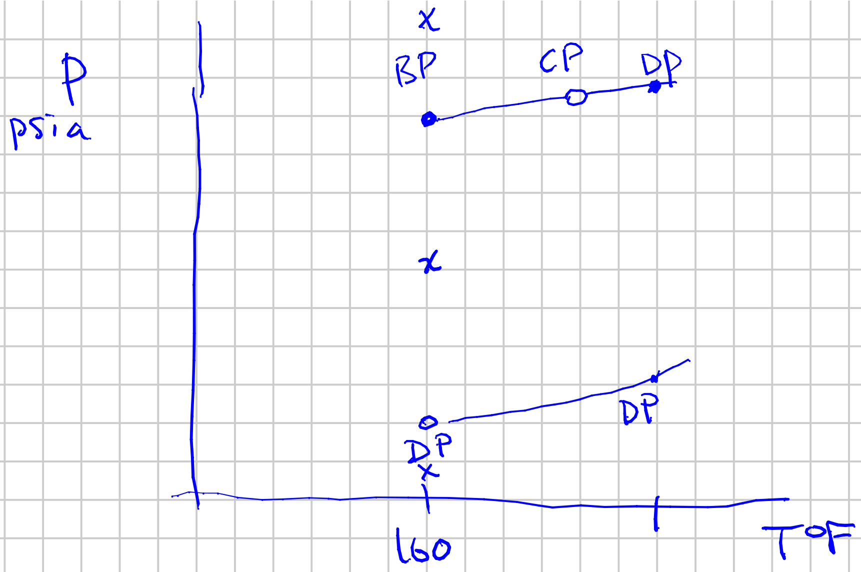
Options
- maximize
- minimize
- set a value = 0

② Set variable(s) to change to reach Target — f_v

③ Constrain the changes in variable(s) – $f_{vmin} < f_v < f_{vmax}$

④ Run Solver

* Try Solver using various initial estimates



Bubblepoint: Searching p where calculated $\sum y_i = 1$

$$K_i(p) = \frac{y_i}{x_i} = \frac{y_i}{z_i} \Rightarrow y_i = z_i K_i$$

$$\sum y_i = 1 = \sum z_i K_i(p)$$

Dewpoint: $\sum x_i = 1 = \sum z_i / K_i(p)$

$p = p_K \Rightarrow K_i = 1 \Rightarrow$ Automatically converge

DP $\sum x_i = 1$

BP $\sum y_i = 1$

SURFACE SEPARATION PROCESSING

Note Title 2013-09-20

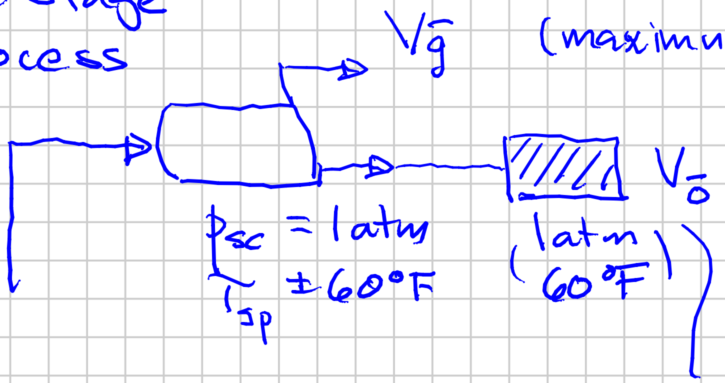
Maximize the resulting Stock Tank Oil Volume from a produced stream (well, manifold, pipeline)

Reason: Any component mole in STO has more value than that component mole being sold as part of the gas.

$\{C_3 - C_7\}$ ↑ Sales Gas (low $)

STO (high $)

Single Stage Process



(maximum)

(minimum)

minimum API

"Quality" STO: (API Gravity)

$$\rho_o \Rightarrow \gamma_o = \frac{\rho_o}{\rho_w}$$

Liquid Specific Gravity

~

Relative Density

$$\gamma_{API} = \frac{141.5}{\gamma_o} - 131.5$$

Lowest STO Density:

~650 kg/m^3

$\gamma_o = 0.65$

$$\gamma_{API} = \frac{141.5}{0.65} - 131.5$$

$= 86$ °API (Upper Limit)

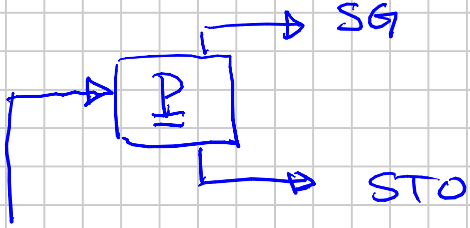
60 - 70

STOs:

10 ($/STB →) 70

i-Butane i-C_4

$n_{iC_4} = 1234$ kg-moles/D



35.31 scf/Sm³ gas
6.28 STB/Sm³ STO

Prices:

$2 – 10 /Msf $3.50/Mcf

$100/STB

$$n \rightarrow V_{\bar{g}} = 23.68 \frac{Sm^3}{kg\text{-}mole} \times n \qquad Sm^3\ SG$$

$$n \rightarrow V_{\bar{o}} = n \times M_{i\text{-}C_4} \frac{kg}{kg\text{-}mole} \times \frac{1}{(\rho_{i\text{-}C_4})_{\text{surface Liquid @ STC}}} \qquad Sm^3\ STO$$

App. A
560.7 kg/m³

Value as SG : $1234 \times 23.68 \times 35.31 \times \frac{1}{1000} \times 3.50 = \$3610$

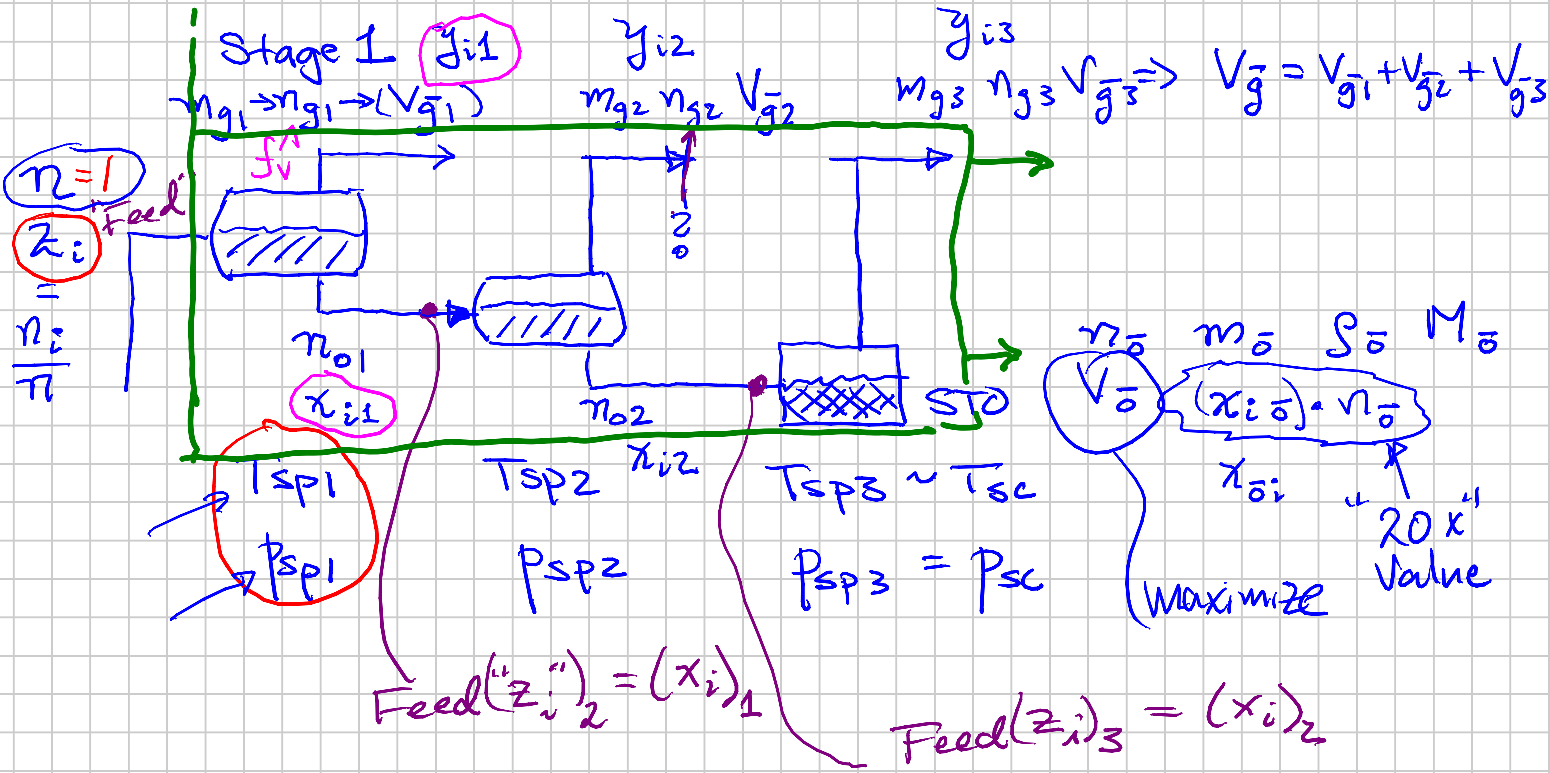
$$\cancel{kg\text{-}moles} \quad \frac{\cancel{Sm^3}}{\cancel{kg\text{-}mole}} \quad \frac{\cancel{scf}}{\cancel{Sm^3}} \quad \frac{\cancel{Msf}}{\cancel{scf}} \quad \frac{USD}{\cancel{Mscf}}$$

Value of STO : $1234 \times 58.12 \times \frac{1}{560.7} \times 6.28 \times 100 = \$80,300$

$$\cancel{kg\text{-}moles} \quad \frac{\cancel{kg}}{\cancel{kg\text{-}mole}} \quad \frac{\cancel{m^3}}{\cancel{kg}} \quad \frac{\cancel{bbl}}{\cancel{m^3}} \quad \frac{\$}{\cancel{bbl}}$$

$\boxed{22 \times}$

Traditional Multi-Stage Separator Process



Rachford-Rice (Muskat-McDowel) : FLASH Calculation

$$\underbrace{\boxed{z_i \quad (p, T) \Rightarrow K_i}}$$

$$\boxed{f_v = \frac{n_g}{n} \qquad y_i \checkmark \quad x_i \checkmark}$$

$$n_{g2} = n_{o1} \times (f_v)_2$$

Feed moles into Stage 2

$$n_{g3} = n_{o2} \times (f_v)_3$$

Assuming $n = 1$

Book-keeping of moles

$$n_{g1} = f_{v1}$$

$$n_{g2} = (1 - f_{v1}) \cdot (f_{v2}) =$$

$$n_{g3} = (1 - f_{v1})(1 - f_{v2})(f_{v3})$$

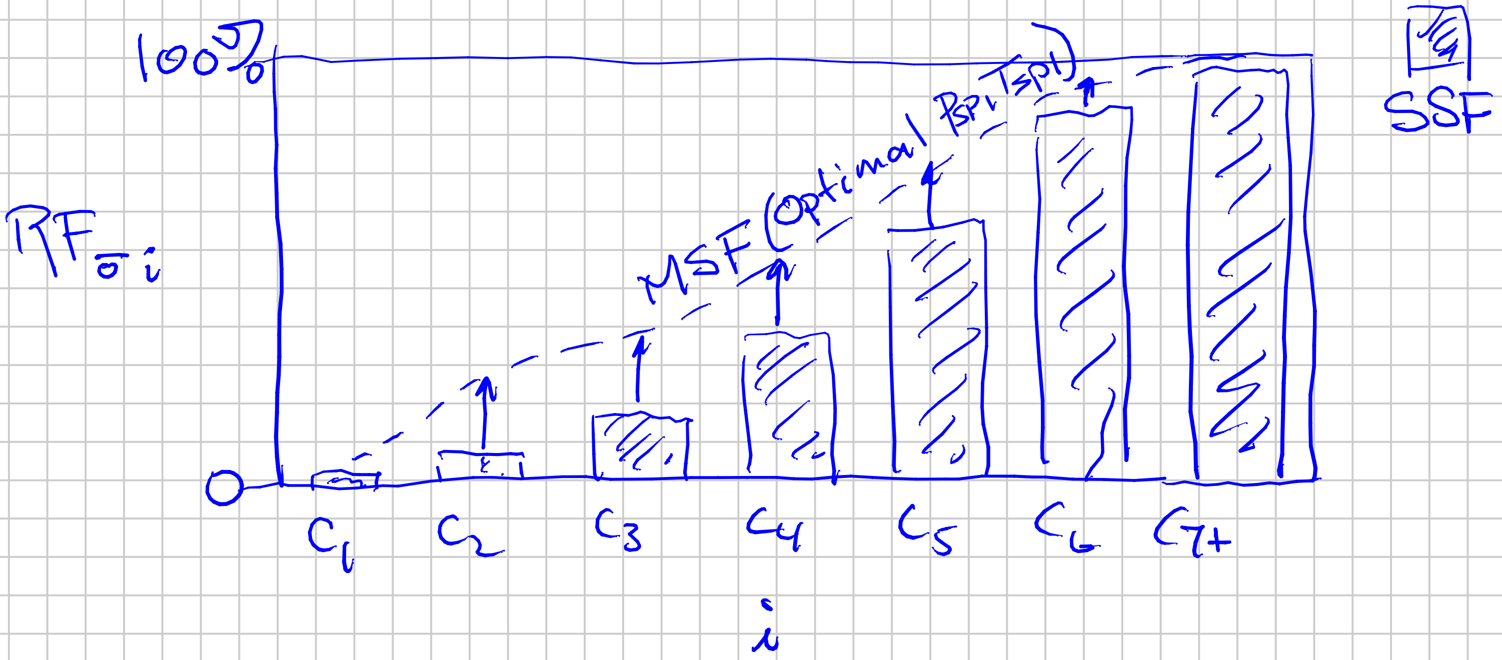
$$n_{o1} = (1 - f_{v1})$$

$$n_{o2} = (1 - f_{v1})(1 - f_{v2})$$

$$\rightarrow n_{o3} = (1 - f_{v1})(1 - f_{v2})(1 - f_{v3})$$

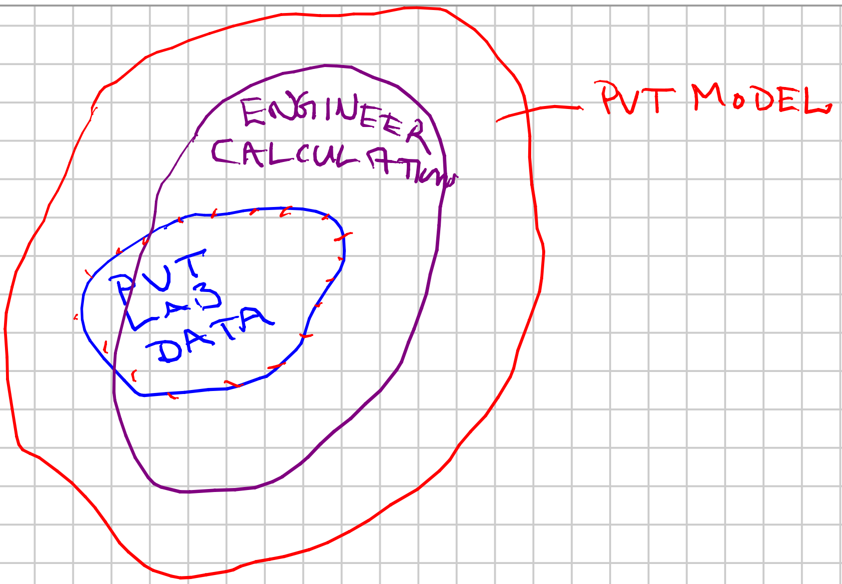
Component "STO" Recovery Factor $RF_{\bar{o}i}$

$$RF_{\bar{o}i} = \frac{n_{\bar{o}i}}{n_i} = \frac{n_{\bar{o}}\, x_{i3}}{n_i} \qquad \%$$



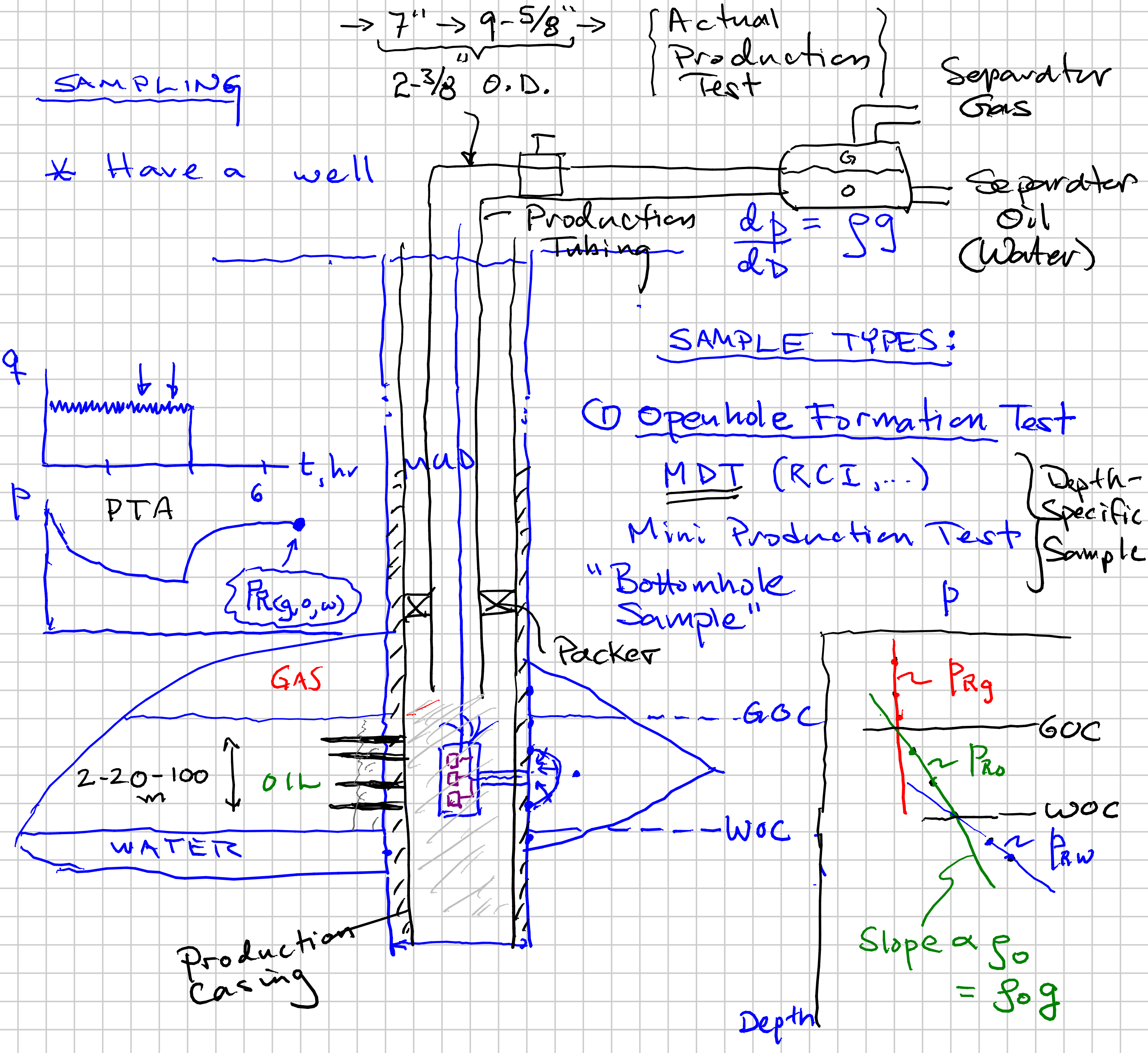
SAMPLING & PVT LABORATORY TESTS (CH. 6)

Note Title 2013-09-26



SAMPLING

* Have a well



SAMPLE TYPES:

① Openhole Formation Test

MDT (RCI, ...)

Mini Production Test

"Bottomhole Sample"

} Depth-Specific Sample

DURING CASED HOLE PRODUCTION TEST (Flowing/Shut-In)

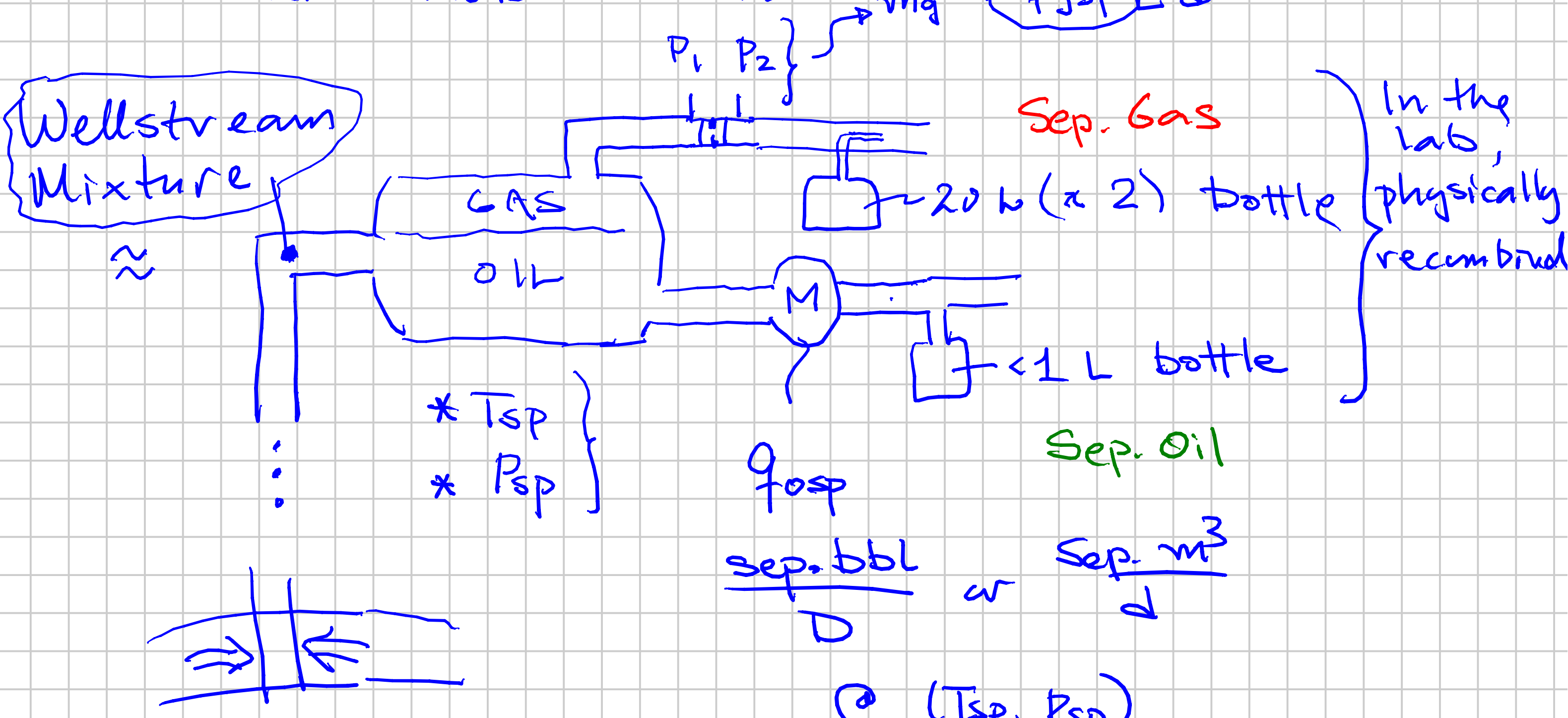
(2) Wireline Bottomhole Sampler
(usually located ~ at/above perforations)

(a) During Flow
(b) During Shut-In
} "Average" sample over the perf'd interval

ONLY Recommended for "Oil" reservoirs

whereas Openhole Formation BHS can be any kind of reservoir fluid (oil or gas/gas condensate)

(3) (Well Must be flowing)
SEPARATOR SAMPLE $\rightarrow \dot{m}_g \Rightarrow \boxed{\overset{T}{q_{gsp}}} \left[\frac{Sm^3}{d} \text{ or } \frac{Mscf}{D}\right]$



q_{osp} $\frac{\text{sep. bbl}}{D}$ or $\frac{\text{Sep. } m^3}{d}$

@ (T_{sp}, p_{sp})

<u>Separator Test GOR</u> $= \frac{q_{gsp}}{q_{osp}} \left[\frac{scf}{\text{sep. bbl}}, \frac{Sm^3}{\text{sep } m^3}\right]$

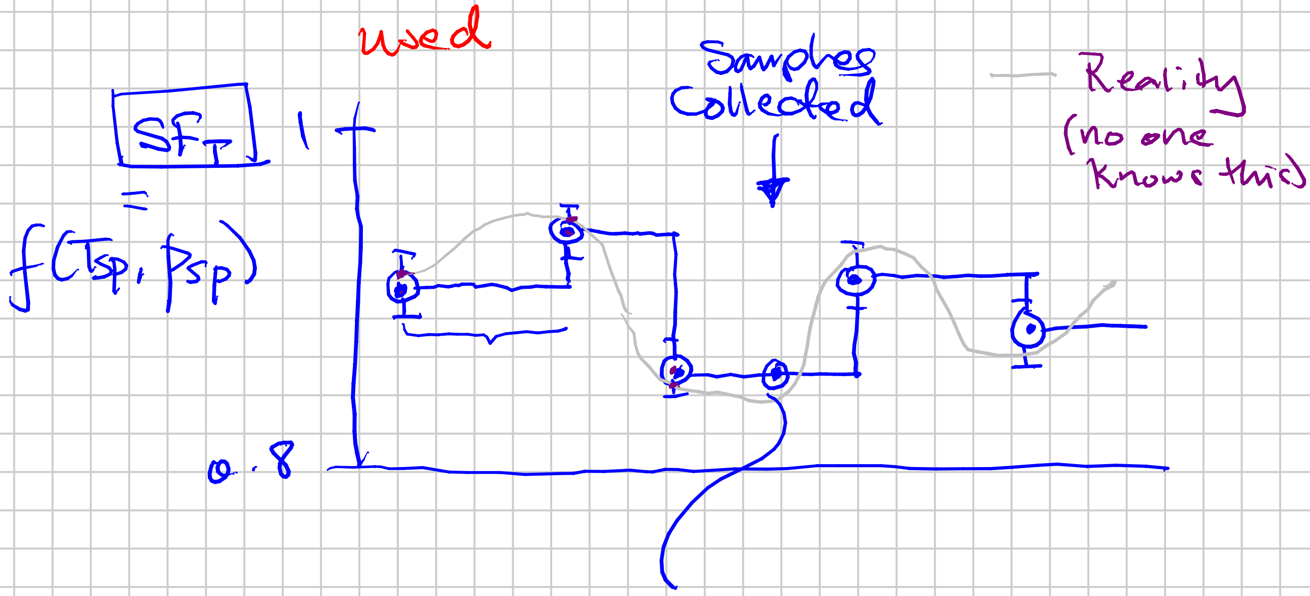
Beware: Test Company Report (xls) often convert the physically

measured q_{osp} [sep-bbl/D] into
a "Stock-Tank Oil" Rate $(q_{\bar{o}})_T$

How?
⇒ Is it made? $\underbrace{q_{osp} \text{ or } q_{\bar{o}}}_{2\% \to 30\%}$

ASK Testing Company
If $q_{\bar{o}}$ reported ⇒ Ask for the Test Shrinkage Factor

$$SF_T = \frac{q_{\bar{o}}}{q_{osp}}$$



$$\boxed{q_{osp}} = \frac{q_{\bar{o}T}}{SF_T}$$

PVT Lab needs

Lab will (slightly 1%-5%) q_{gsp}^{T} [scf/D]

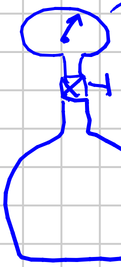
Gas Meter Eq.: (Z_g^T, M_g^T) approx.

Lab $\{ Z_g^{Lab} \quad M_g^{Lab} \} \Rightarrow q_{gsp}^{Corr}$

Lab will <u>Recombine</u> using $\left(\frac{q_{gsp}^{Correct}}{q_{osp}^{Correct}} \right) = R_{sp}$

Lab makes two QC tests of the sample bottles:

① Sep. Gas



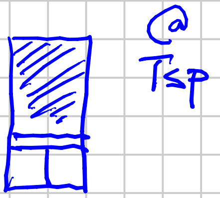
Check $p_{opening} \doteq p_{sp}$ >10% ?

Open Valve

Heat Gas Bottle to T_{sp}

Identify possible Leakage during transport

② Sep. Oil



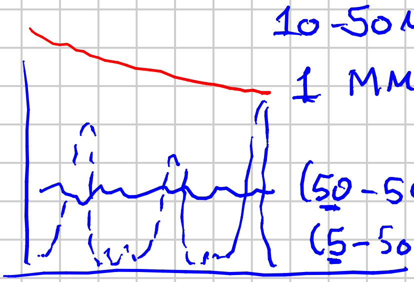
$(p_b) = p_{sp}$

Sep. Samples OK ✓ for reservoir oils & gas condensates

q_g

q_o

~ Stabilized



10-50 Mscf/D (off)
1 MMscf/D (on)

(50-5000⁺ STB/D
(5-5000 STB/D)

DETERMINING COMPOSITION OF THE SAMPLE(S)

$$M_i \left\{ \begin{array}{l} \text{weight (mass)-\%} \\ \text{molar-\%} \end{array} \right\} \text{ of } \begin{array}{l} N_2 \quad CO_2 \quad H_2S \\ C_1 \quad C_2 \cdots C_6 \cdots C_7 \quad C_8 \cdots \end{array}$$

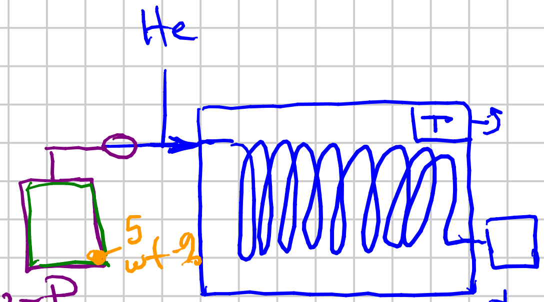
C_{N+}

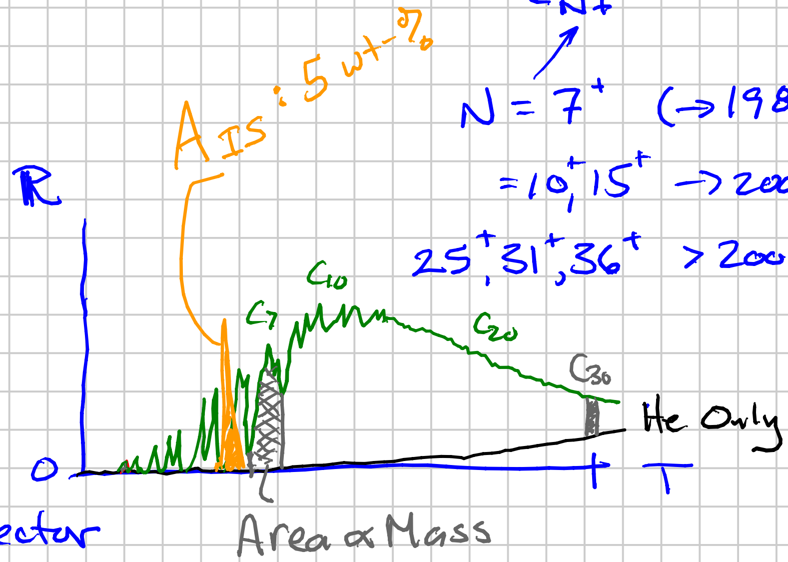
$N = 7^+$ (→1980)

$= 10^+, 15^+$ → 2000

$25^+, 31^+, 36^+$ > 2000

① GAS CHROMATOGRAPH





Low-P Sample:

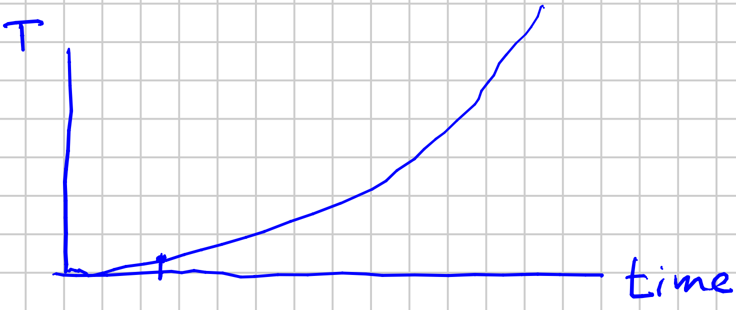
(1) atmospheric oil

(2) separator gas

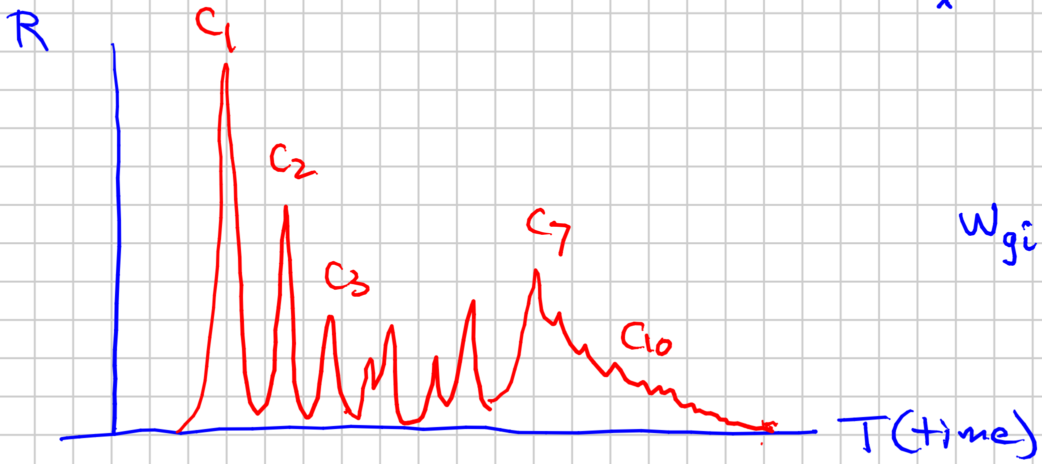
atmospheric gas

I.S.

i	A_i →	w_{oi}%
C_1		
C_2		$\sum_{i=1}^{N-1}$
⋮		
C_{N+}	?	$\boxed{100 - \Sigma}$



$$w_i \equiv \frac{m_i}{m}$$

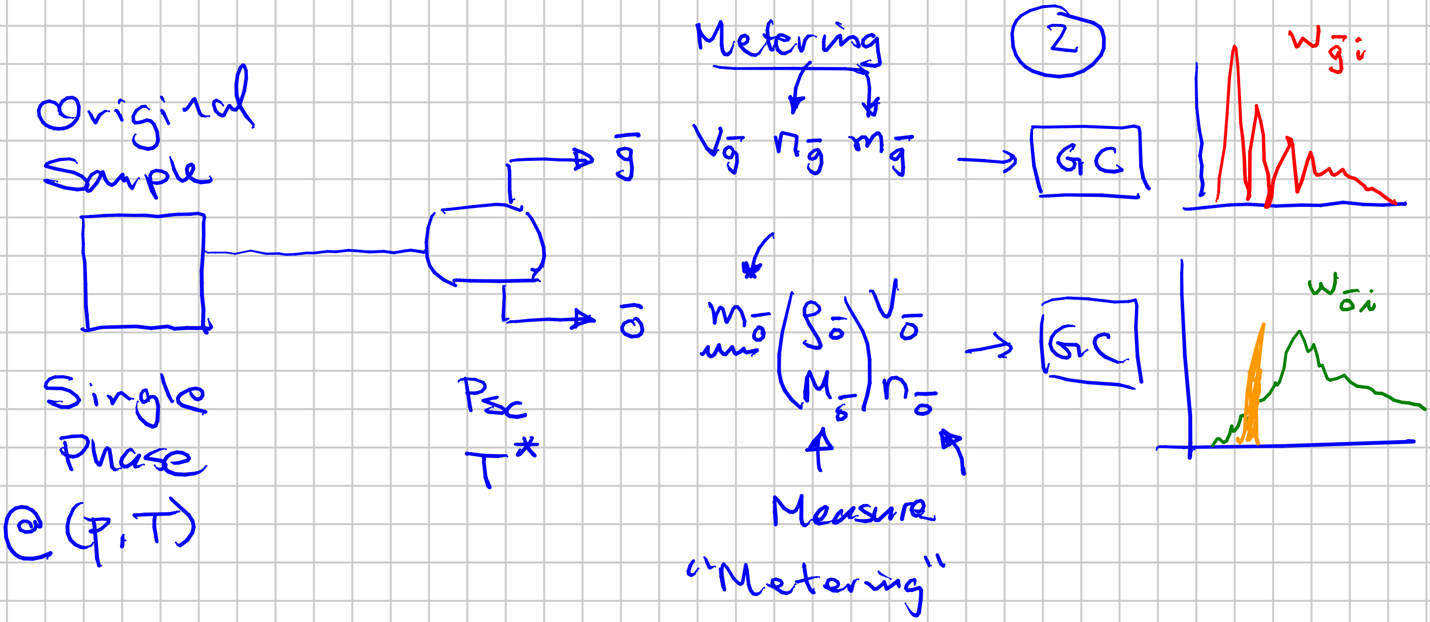


W_{gi}

"Pressurized" Samples

- Bottomhole Samples (all types)
- Separator oil

① Flash Equilibrium @ 1 atm T^* (~60°F)



③ Recombination $\bar g$ & $\bar o$ ⇒ original Sample

$$m_i = m_{\bar g} \cdot w_{\bar g i} + m_{\bar o}\, w_{\bar o i}$$

$$w_i = \frac{m_i}{\sum_{j=1}^{N_t} m_j}$$

④ Mole Fractions

$$n_i = w_i / M_i \qquad M_i\ ?$$

$$(x_i\,,\ z_i) = \frac{n_i}{\sum_{i}^{N_t} n_j} \quad ✓$$

$i =$ H_2S CO_2 N_2 ✓
$C_1 - C_5$

$\left.\begin{matrix} C_6 \\ C_7 \\ \vdots \end{matrix}\right\}$ Differ somewhat field to field

C_{N+}

$\{M_{N+}\}$ ↕ Large Variations Field to Field

Lab has "internal average"

M_i^{Lab} SCN $i = 7, \ldots, N-1$

(Ch. 5 Table 5.2 Katz & Firoozabadi)

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Lab measured $\boxed{M_{\bar{o}}}$ $\pm 2\text{-}10^{+}\%$

Cryoscopic Benzene (Freezing Point Depression)

↓

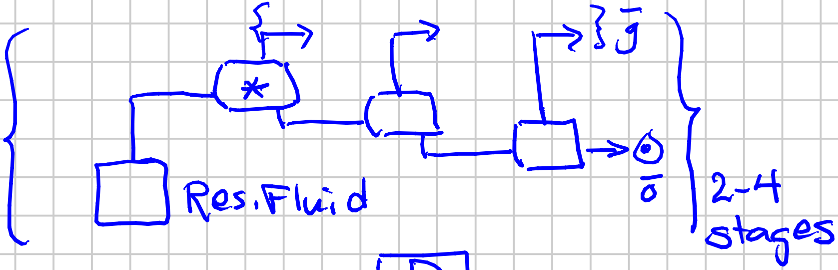
M

$$\overset{\checkmark}{M_{\bar{o}}} = \frac{\sum \overset{\checkmark}{w_i}}{\left(\sum_{i=1}^{N-1} \frac{\overset{\checkmark}{w_i}}{\underset{\checkmark}{M_i}}\right) + \frac{\overset{\checkmark}{w_{N+}}}{M_{N+}}} \longrightarrow \text{Back-Calculate } \underline{\underline{M_{N+}}}$$

Lab

PVT LAB TESTS (Ch. 6 ; App. D – oils)

Note Title 2013-09-27

PVT Laboratory Test	Reservoir Fluid Type: OIL	Reservoir Fluid Type: GAS CONDENSATE
*COMPOSITIONAL ANALYSIS (G.C.)		
– Bottomhole } $w_i \downarrow$	✓	✓
– Separator Recombined } z_i	✓	✓
– $M_{\bar{o}}$ $(\rho_{\bar{o}})$		
*CONSTANT COMPOSITION EXP (CCE) (MASS)	✓	✓
• $[\underline{p_s}$ (BP, DP), $\rho(p \geq p_s)]$ @T_R (@$T<T_R$)	✓	✓
→ • $V_g, V_o (p < p_s)$	(✓)	✓
⊛ MULTI-STAGE SEPARATOR TEST (SEP)	✓	(✓)
 Optimize: $\boxed{P}$		"Medium-Rich" G.C. $\frac{V_{\bar{o}}}{V_{\bar{g}}} \gtrsim 100 \frac{STB}{MMscf}$ $560 \frac{Sm^3}{10^6 Sm^3}$
* DEPLETION TESTS (@T_R)	✓	✓
$(V_g, V_o) = f(p < p_s)$, (ρ_g, ρ_o), (μ_g, μ_o) ↑ (y_i, x_i)		
⊛ DIFFERENTIAL LIBERATION (DL)	✓	–
* CONSTANT VOLUME DEPLETION (CVD)	(✓)	✓

"Compositions" $\quad$ pTx space ← for multiple samples

$$\left[\, p \,@\, T_R \qquad \underbrace{p \,@\, T_R \rightarrow T_{sc}}\,\right]$$

- Surface Process (Separators)
- Pipeflow

PVT Data Needed:

- Phases # ⇒ p_s
- Amount of each phase
  - mass
  - volume
  - mole
- Phase Properties
  - Density
  - Viscosity
  - Compositions → "Surface Process"

Each PVT Test:

- Procedure Used
- Data Measured
- Data Reported

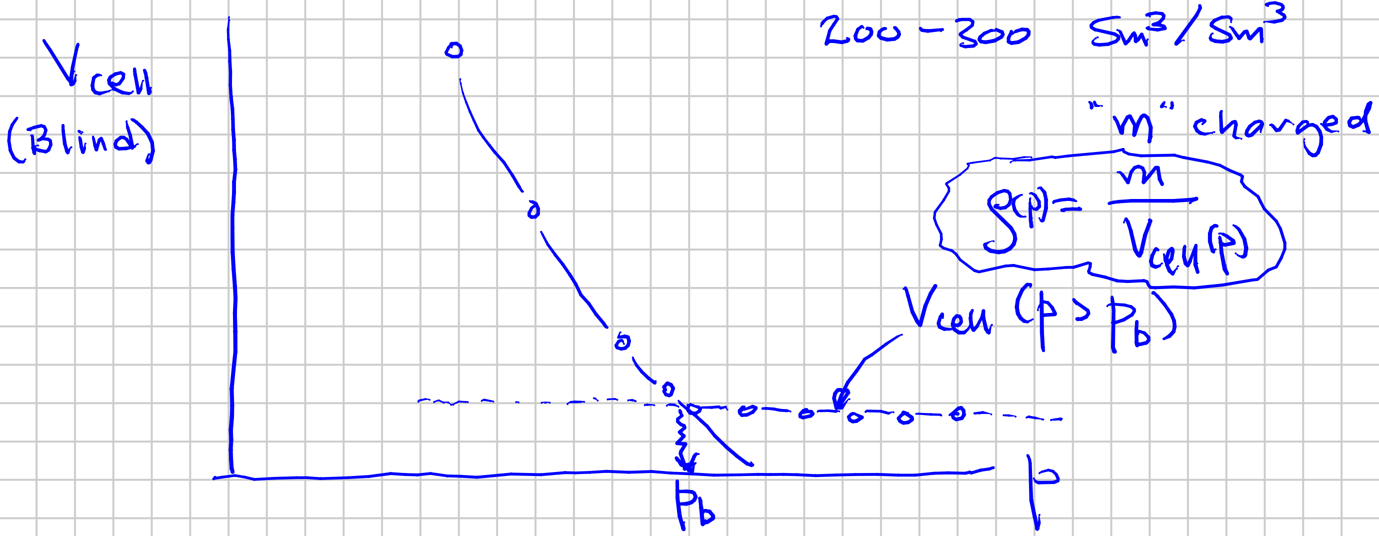
Data Measured / Data Reported:
- same sometimes
- different — " —

CCE: $\quad p_s(\underline{\underline{T_R}}\,;\,T) \qquad \rho(T_R,\, p > p_s)$

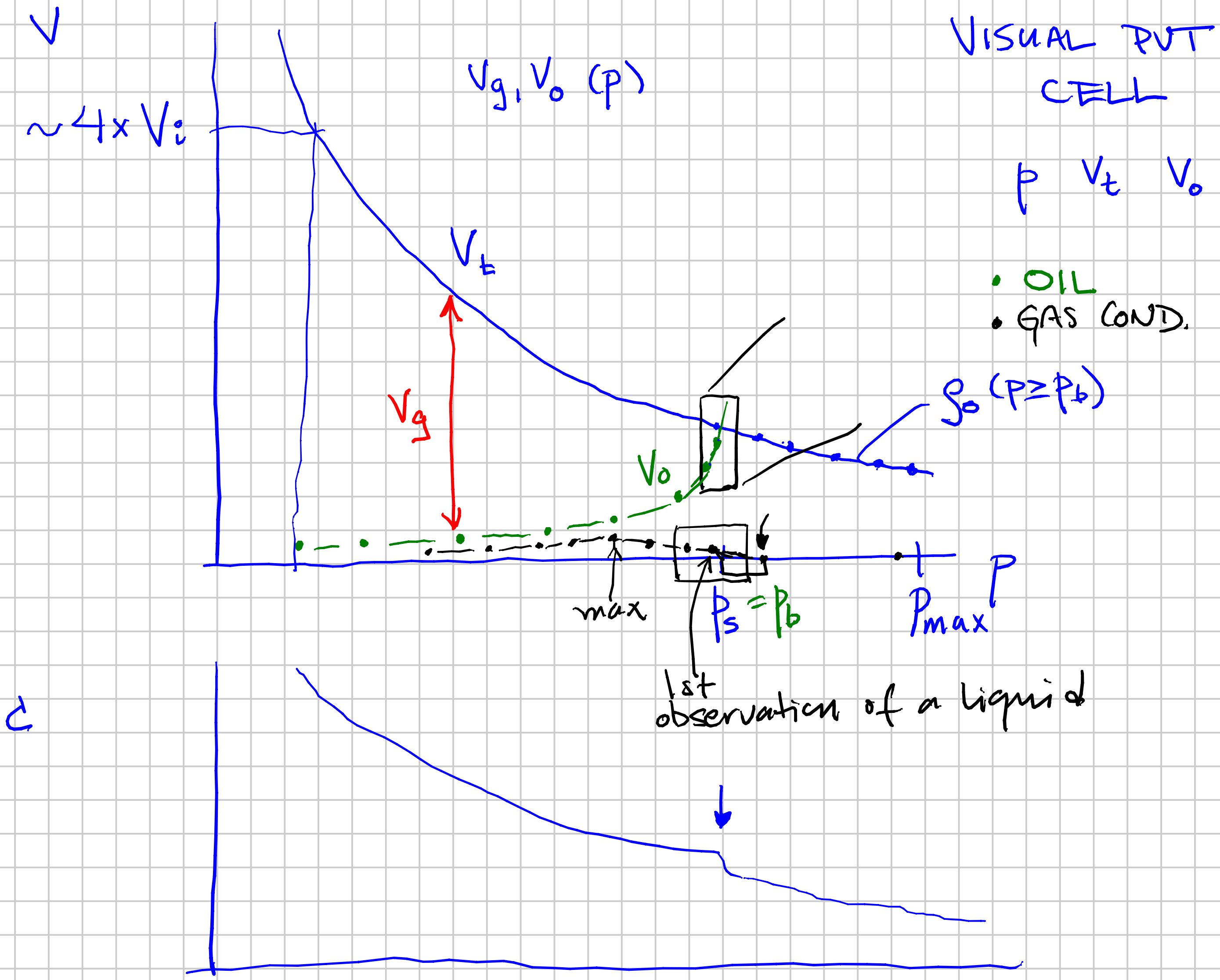
① OILS only – lower–medium GOR Oils

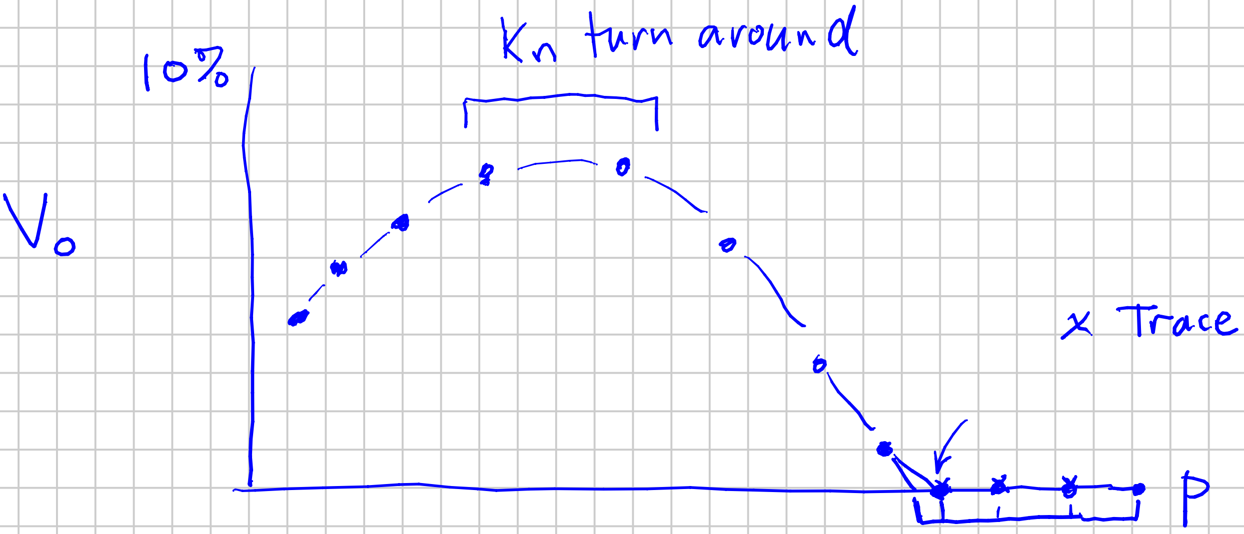
(GOR ≲ 1000–1500 scf/STB)

200–300 Sm^3/Sm^3



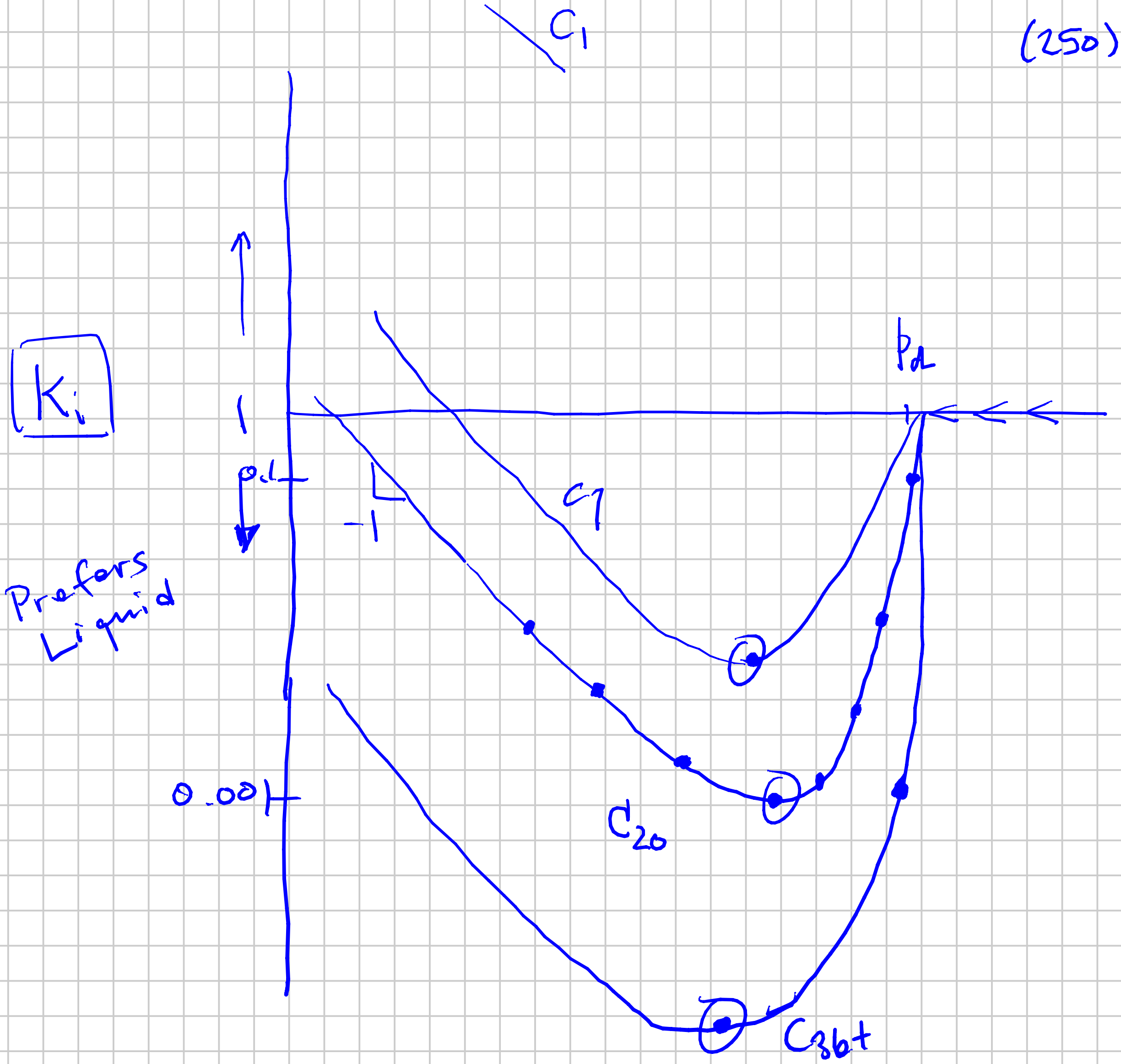
② GORs ≥ 1500 – 2000 scf/STB   OILS
300 – 400 Sm^3/Sm^3 ↓
GAS COND.





$P_d \pm$ 1-100 bar

(250) Vale



DEPLETION PVT TESTS

Note Title 2013-10-03

OIL TEST: Differential Liberation (DLE) Experiment

Ch. 6 & App. D (DLE & SEP ⇒ "Traditional" Black-Oil PVT properties)

Multi-Stage Sep Test

$\underline{B_o}$ $\underline{R_s}$ "$\underline{B_{gw}}$" ← wet gas FVF

Ch. 7 Black-Oil PVT Formulation

CVD: Gas Condensates & Volatile Oils ⇒ 1950s — 1980 • Traditional
\>1980s • Modified

Whitson-Torp

DLE: Only used for Oils* (CCE Test ⇒ $p_s = p_b$)

* Always for oils with GOR $\lesssim$ 2000 scf/STB
2500
375-450 Sm3/Sm3

{ CVD alternative GOR $\gtrsim$ 400-500 Sm3/Sm3 }

Purpose: $\underline{\text{Properties}}$ of Oil & Gas Phases

$= f(T_R, p \le p_b)$

→ ✓ρ_o ✓ρ_g (Z_g M_g or γ_g) ; V_o "Shrinkage" ✓ $\Delta V_o / V_o$

Does the depletion process <u>affect</u> (± 1-2%)

ρ's

μ's

ΔV's

$\underbrace{\qquad\qquad}$

No in most cases | "Some" for oils w/ GOR $\gtrsim$ 400 Sm³/Sm³

✓ $\left(\frac{\Delta V_g}{V_o}\right)$ released from the oil (dissolution)

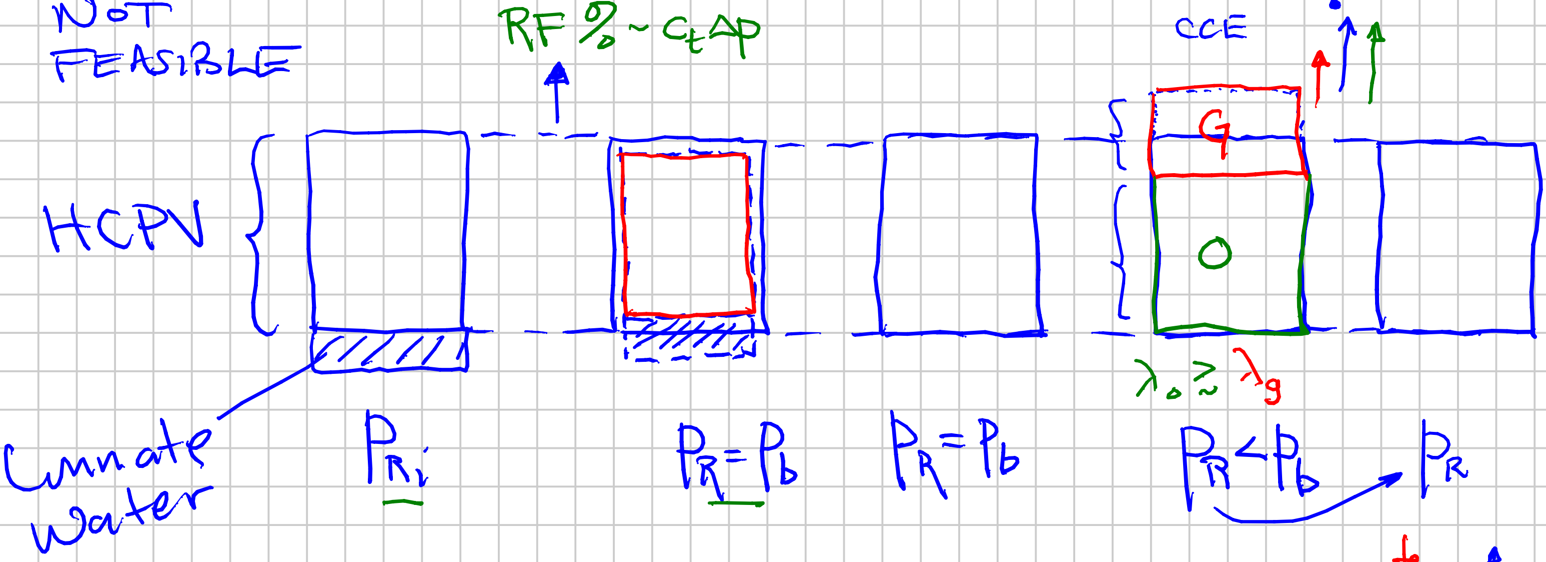
$\{y_i\}_{i \in C_3-C_6}$ Gas Liquids

μ_o μ_g

Option 1: Mimic actual Physical Depletion Process:

⇓

NOT FEASIBLE



Flow Out from Reservoir

$$\lambda_g = \frac{k_g}{\mu_g} = \frac{k\,\overset{\uparrow}{k_{rg}}(\overset{\uparrow}{S_g})}{\mu_g}$$

$$\lambda_o = \frac{k_o}{\mu_o} = \frac{k\,k_{ro}(\overset{\downarrow}{S_o})}{\mu_o}$$

$$S_o = (1 - S_g) - S_w$$

$$k_{rp} \propto S_p^{\,n_p} \qquad n_p \leftarrow 2.x - 4.x$$

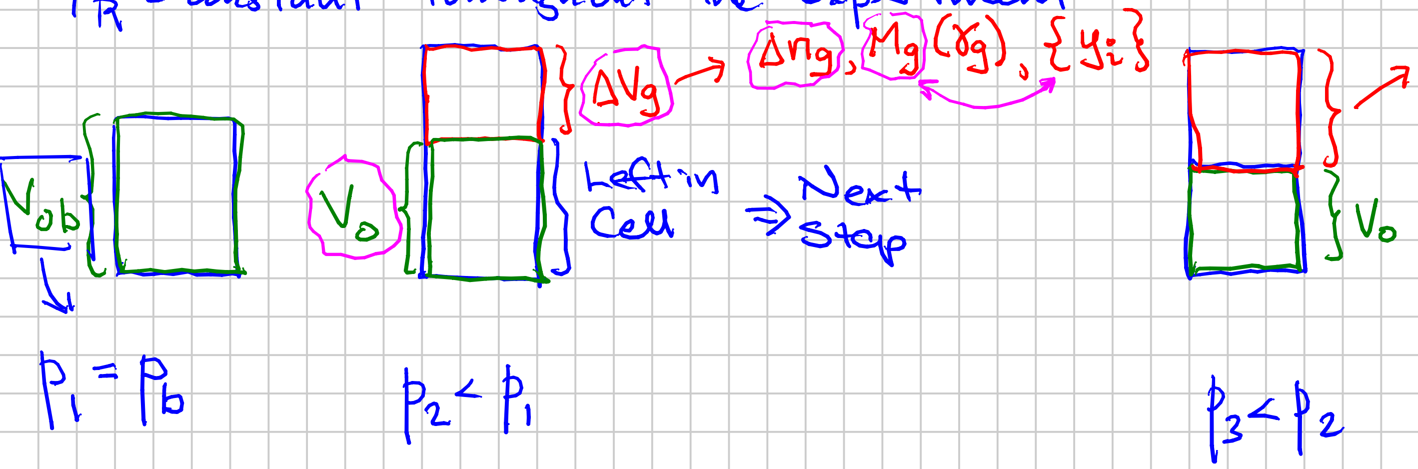
Option 2: (Actually Used)

Remove ALL Gas : Simplest Lab Procedure

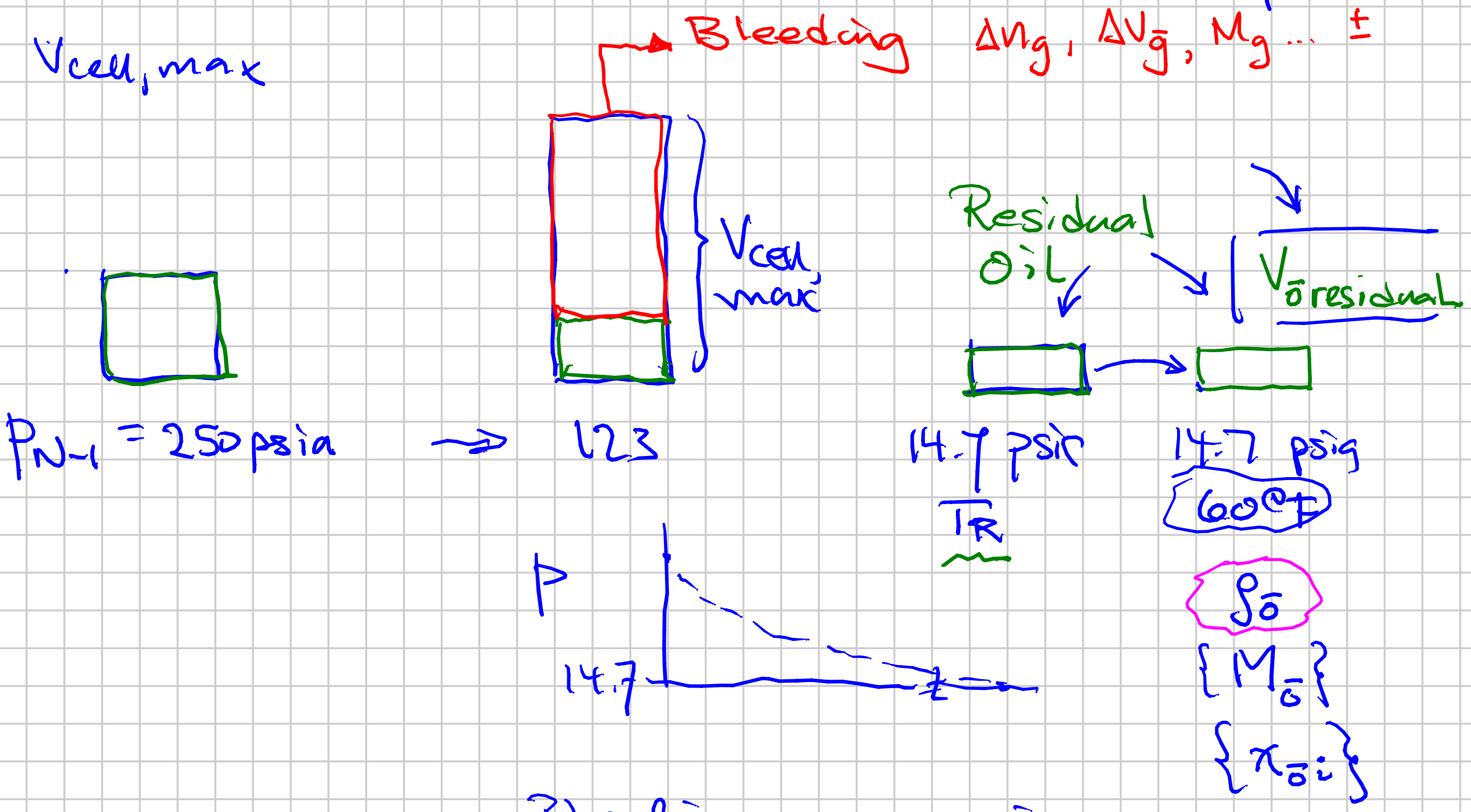
Gives accurate measurement of ALL properties without a visual PVT cell.

Measured $\gamma_g = \frac{M_g}{M_{air}}$

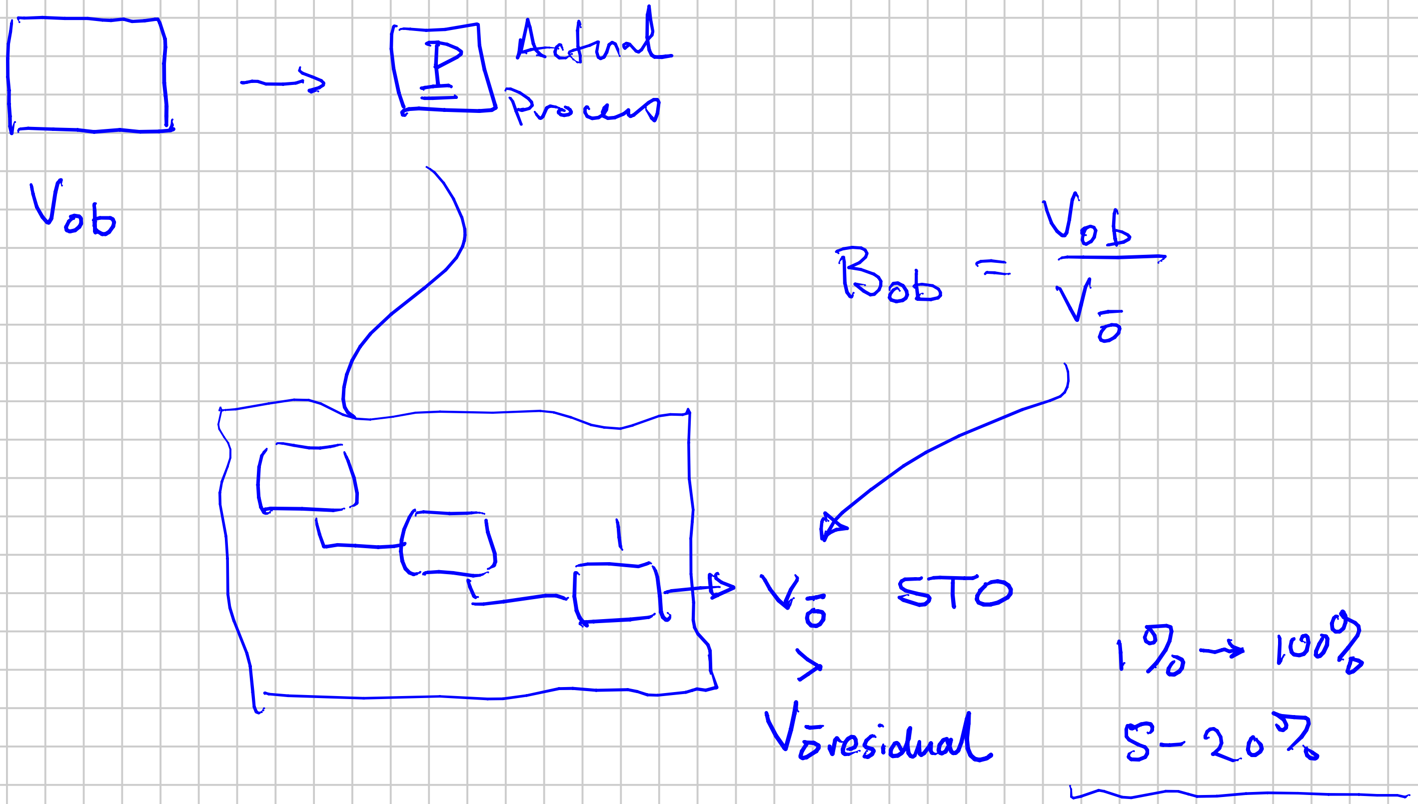
T_R = constant throughout the experiment



Next-to-Last Pressure $\left(250 \overset{450}{\underset{150}{}} \text{psia}\right)$ $\qquad \Delta V_g \propto \frac{\Delta n_g}{p}$



Bleeding process is not really an equilibrium process ~ only approximate.



$$B_{ob} = \frac{V_{ob}}{V_{\bar o}}$$

Lab Reports from DLE:

(1) $\dfrac{V_o(p)}{V_{\bar o\,residual}}$ "B_o" B_{od} NEVER to be used in engineering calculations with reservoir/prod/pipeline calculation

All text books

(2) Oil density: ρ_o

Calculated by a 'material balance'

± 1-2%

$$\rho_o(p_j) = \frac{m_o(p_j)}{V_o(p_j)} = \frac{\overset{\checkmark}{m}_{\bar o\,residual} + \sum_{k=N}^{j+1} \overset{\checkmark}{(\Delta n_{gk})} \overset{\checkmark}{M}_{gk}}{\underset{\checkmark}{V_o}(p_j)}$$

Maybe $\neq p_{DLE}$

(3) Oil Viscosity $\mu_o(p < p_b)$

Lab runs a separate, parallel DL test to get oil viscosities:

@ T_R, $p \leq p_b$

$\left\{ \begin{matrix} \Delta t \\ \rho_o \end{matrix} \right\} \rightarrow \mu_o$ ± 5-15%

GAS:

Reported:

④ $\Delta n_g \rightarrow \Delta V_{\bar{g}} \rightarrow \Delta R_{sd} = \dfrac{\Delta V_{\bar{g}}}{V_{\bar{o}residual}}$ and/or

"Solution" GOR

$$R_{sd}(p_j) = \frac{\sum_{k=N}^{j+1} \Delta V_{\bar{g}}}{V_{\bar{o}residual}}$$

Engineers "R_s" $\neq R_{sd}$

DON'T USE DIRECTLY!

Important Quantity:

$$\underbrace{R_{sd}(p_b) - R_{sd}(p)}_{\text{Evolved Gas from } p_b \rightarrow p}$$

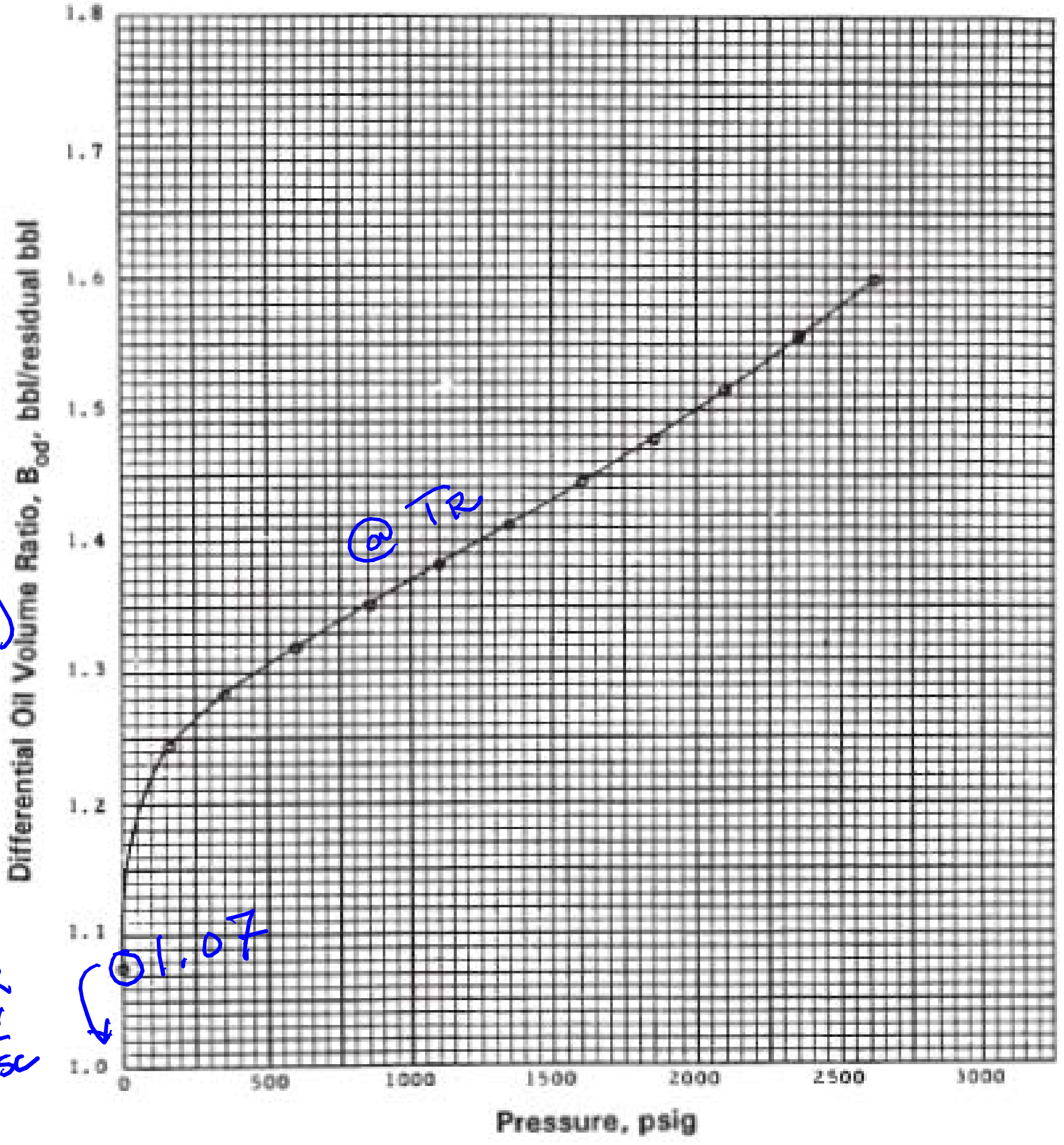
⑤ $$Z_g = \frac{p \cdot \Delta V_g}{\Delta n_g R T_R}$$

⑥ M_g or γ_g each removed gas

$$\left\{ ⑦ \quad \rho_g(p) = \frac{p M_g}{Z_g R T_R} = \frac{p \cdot M_g(p)}{Z_g(p)\, R\, T_R} \right\}$$

$M_g(p)$ changes because $y_i(p)$

$$\left\{ ⑧ \quad y_{DLE_i}(p) \right\}$$

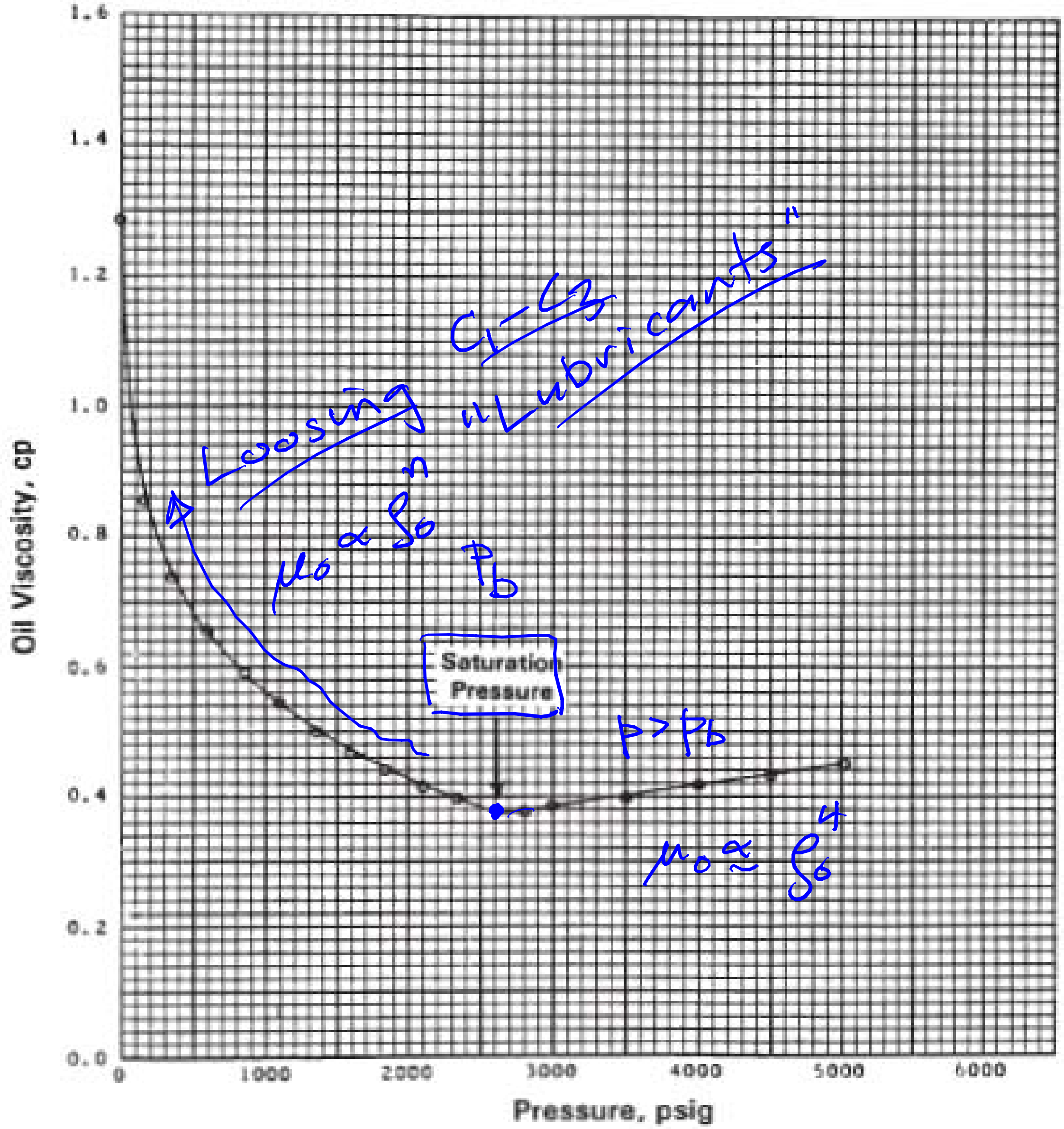


$$B_{od} = \frac{V_o(p)}{V_{\bar{o}res.}(T_{sc})}$$

@ T_R

$T_R \rightarrow T_{sc}$

Fig. 6.7B—DLE data for an oil sample from Good Oil Co. Well 4; differential oil FVF (relative volume), B_{od}.



Loosing $C_1 - C_3$ "Lubricants"

$\mu_o \propto \rho_o^n$

P_b

$p > p_b$

$\mu_o \overset{\propto}{\simeq} \rho_o^4$

Fig. 6.7C—DLE data for an oil sample from Good Oil Co. Well 4; oil viscosity, μ_o.

Core Lab

TABLE 6.11—DLE DATA FOR GOOD OIL CO. WELL 4 OIL SAMPLE

P	R_{sd}	B_{od}	B_{td}	ρ_o		B_{gw}	γ_g
			Differential Vaporization				
Pressure (psig)	Solution GOR (scf/bbl*)	Relative Oil Volume (RB/bbl*)	Relative Total Volume (RB/bbl*)	Oil Density (g/cm³)	Deviation Factor Z_g	Wet Gas FVF (RB/bbl*)	Incremental Gas Gravity
2,620	854	1.600	1.600	0.6562			
2,350	763	1.554	1.665	0.6655	0.846	0.00685	0.825
2,100	684	1.515	1.748	0.6731	0.851	0.00771	0.818
1.850	612	1.479	1.859	0.6808	0.859	0.00882	0.797
1,600	544	1.445	2.016	0.6889	0.872	0.01034	0.791
1,350	479	1.412	2.244	0.6969	0.887	0.01245	0.794
1,110	416	1.382	2.593	0.7044	0.903	0.01552	0.809
850	354	1.351	3.169	0.7121	0.922	0.02042	0.831
600	292	1.320	4.254	0.7198	0.941	0.02931	0.881
350	223	1.283	6.975	0.7291	0.965	0.05065	0.988
159	157	1.244	14.693	0.7382	0.984	0.10834	1.213
0	0	1.075		0.7892			2.039
→ @ 60°F		1.000**	?			?	

*Barrels of residual oil.

**At 60°F.

$$B_{td} \equiv \frac{V_t}{V_{\bar{o}res}} = \frac{V_o + \left(\sum \Delta V_g\right)_{p_b \to p}^{@p}}{V_{\bar{o}residual}}$$

$$B_{gw} \equiv \frac{V_g(p)}{V_{\bar{g}}}$$

assumption No surface condensation

$$\rightarrow B_{gw} = \frac{p_{sc}}{T_{sc}} \cdot \frac{T_R}{p} \cdot Z_g$$

$$\left(\sum_{p_b}^{p_j} \Delta V_g\right)_{@p} = \left[\sum_{k=2}^{j} \underbrace{\Delta n_{gk}(23.68)}_{\Delta V_{\bar{g}k}} \cdot \right] B_{gw}(p_j)$$

Pressure (psig)	Oil Viscosity (cp)	Calculated Gas Viscosity (cp)
5,000	0.450	
4,500	0.434	
4,000	0.418	
3,500	0.401	
3,000	0.385	
2,800	0.379	
2,620	0.373	
2,350	0.396	0.0191
2,100	0.417	0.0180
1,850	0.442	0.0169
1,600	0.469	0.0160
1,350	0.502	0.0151
1,100	0.542	0.0143
850	0.592	0.0135
600	0.654	0.0126
350	0.783	0.0121
159	0.855	0.0114
0	1.286	0.0093

$\underline{\mu_g\,(p<p_b)}$

Lee-Gonzalez

$$\mu_g = f(\underset{\uparrow}{\rho_g}, \underset{\uparrow}{T_R}) \quad \text{Ch. 3}$$

±5%

BLACK-OIL PVT (Ch. 7)

Flash Calculations

- Multi-stage separator test

z_{wi} (Wellstream) $\longrightarrow$ Sellable Volumetric Products

$\bar{g}$: surface gas

$\bar{o}$: stock-tank oil

"Black-Oil" PVT Formulation:

Uses two pseudo-components to describe the gas and oil phases @ (p,T)

INSTEAD of composition $(H_2\ CO_2\ C_1\ C_2, \ldots C_7 \ldots C_{35+})$

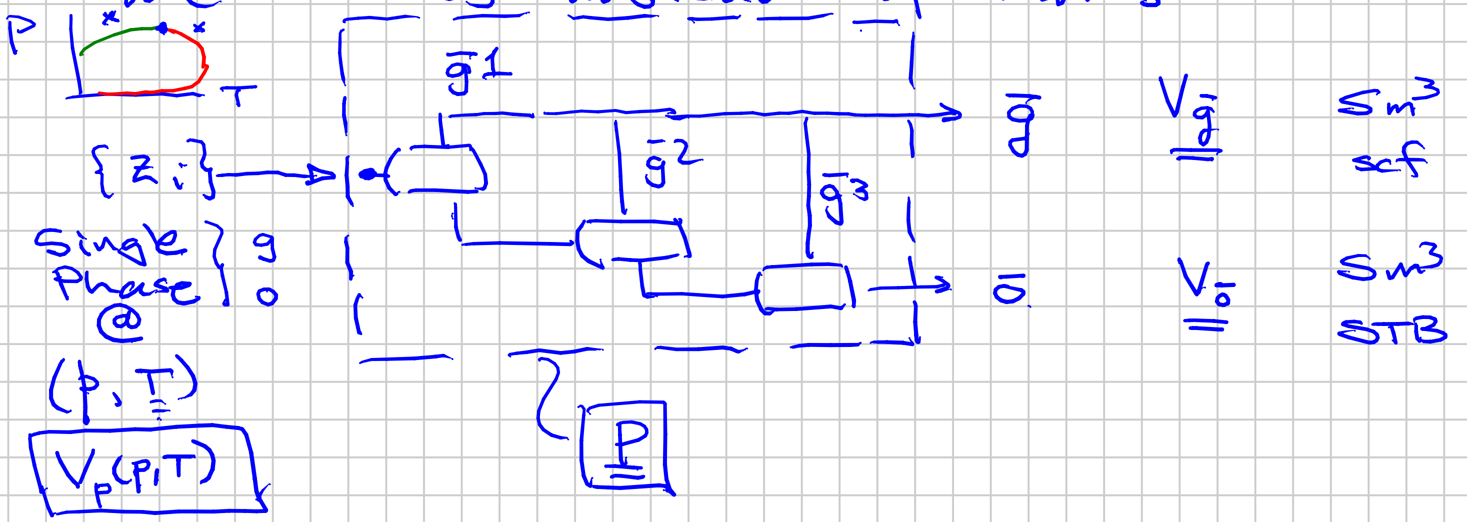
Two Pseudo-Components are:

"Surface Gas" $(\bar{g})$

"Surface Oil" $(\bar{o})$

$\Big\}$ results of a specific surface Process $\boxed{\underline{P}}$

use volumes instead of moles



$p \in (g,o)$

Black-Oil PVT Model uses
<u>VOLUME</u> <u>RATIOS</u> to define PVT phase & volumetric behavior

$\{f_g\ y_i\ x_i\}$ $\{n, m, \rho\}$: Compositional

① Surface Volume Ratios

$$R_s \equiv \frac{V_{\bar{g}o}}{V_{\bar{o}o}} \quad \underbrace{\text{for an } \underline{\underline{\text{oil}}}\ \underline{\text{phase}}}$$

"Solution GOR" (R_s)

$R_s \neq \frac{1}{r_s}$ ✓

$$r_s = \frac{V_{\bar{o}g}}{V_{\bar{g}g}} \quad \underbrace{\text{for a } \underline{\underline{\text{gas}}}\ \underline{\text{phase}}}$$

"Solution OGR" (r_s)

These quantities are the "<u>pseudo</u>" equivalents of $x_i : \underline{\underline{R_s}}$ and $y_i : \underline{\underline{r_s}}$

$i \in \{\bar{g}, \bar{o}\}$ $M_{\bar{o}o}$ $\rho_{\bar{o}o}$

$R_s \rightarrow x_i$

$$\frac{[\mathrm{Sm}^3]\ \underbrace{V_{\bar{g}o}} \times \frac{1}{23.68\ \mathrm{Sm}^3}\ \text{kg-mole}}{[\mathrm{Sm}^3]\ \underbrace{V_{\bar{o}o}}_{\substack{R_s\\(300)}} \times \rho_{\bar{o}o}/M_{\bar{o}o}} = \frac{n_{\bar{g}o}}{n_{\bar{o}o}} = 300\ \frac{200}{(850)(23.68)}$$

$$= 0.03$$

Ekofisk ~ $300\ \mathrm{Sm}^3/\mathrm{Sm}^3 = R_s$

$\rho_{\bar{o}o} \sim 850\ \mathrm{kg/m}^3$

$M_{\bar{o}o} \sim 200$ kg/kg-mole

$$x_{\bar{g}} \equiv \frac{n_{\bar{g}o}}{n_{\bar{g}o} + n_{\bar{o}o}} \cdot \frac{\frac{1}{n_{\bar{o}o}}}{\frac{1}{n_{\bar{o}o}}} \qquad x_{\bar{g}} = \frac{\frac{n_{\bar{g}o}}{n_{\bar{o}o}}}{\frac{n_{\bar{g}o}}{n_{\bar{o}o}} + \frac{n_{\bar{o}o}}{n_{\bar{o}o}}}$$

$$= \frac{\frac{n_{\bar{g}o}}{n_{\bar{o}o}}}{\frac{n_{\bar{g}o}}{n_{\bar{o}o}} + 1} = \frac{1}{1 + \left(\frac{n_{\bar{o}o}}{n_{\bar{g}o}}\right)}$$

$$\frac{n_{\bar{o}o}}{n_{\bar{g}o}} = \frac{1}{R_s} \, 23.68 \left(\rho_{\bar{o}o} / M_{\bar{o}o}\right)$$

$$\boxed{x_{\bar{g}} = \left\{ 1 + \frac{1}{R_s} \cdot 23.68 \left(\frac{\rho_{\bar{o}o}}{M_{\bar{o}o}} \right) \right\}^{-1}}$$

$$x_{\bar{o}} = 1 - x_{\bar{g}}$$

R_s [Sm^3/Sm^3]

ρ [kg/m^3]

M [kg/kg-mole]

$$\boxed{y_{\bar{g}} = \left\{ 1 + r_s \; 23.68 \left(\frac{\rho_{\bar{o}g}}{M_{\bar{o}g}} \right) \right\}^{-1}}$$

$$y_{\bar{o}} = 1 - y_{\bar{g}}$$

Note: surface oil from oil phase $\bar{o}o$
is physically NOT going to be
the surface oil from gas phase $\bar{o}g$

But in our use of Black-Oil PVT we
use the assumption that

$$\bar{o}o = \bar{o}g$$

$$\rho_{\bar{o}o} \cong \rho_{\bar{o}g}$$ Used Not particulary good

$$(M_{\bar{o}o} = M_{\bar{o}g})$$

$$\bar{g}o = \bar{g}g$$

$$\rho_{\bar{g}o} \cong \rho_{\bar{g}g}$$ Used so-so

Volume balance → 0 (ϵ)

- Still get a mass balance error ✓

$\Big\{$ $\rho_{\bar{o}o} \neq \rho_{\bar{o}g}$ and $\rho_{\bar{g}o} \neq \rho_{\bar{g}g}$

Affects the calculation accuracy of phase densities $\Big\}$

② (FORMATION) VOLUME FACTOR "FVF" (B)

$$B_p \equiv \frac{V_p(P,T)}{V_{\bar{p}p}}$$

$$B_o = \frac{V_o(P,T)}{V_{\bar{o}o}}$$

Oil FVF

1.0x – 3

(1.2 – 2) Most Oil Fields

$$b_o = \frac{1}{B_o} = \frac{V_{\bar{o}o}}{V_o(P,T)}$$

$\Rightarrow$ Shrikage Term

Shrinkage Factor $= 100\% \left(1 - \frac{1}{B_o}\right) = 100\% \left(1 - b_o\right)$

$B_o = 2 \quad b_o = \frac{1}{2} \quad SF = 50\%$

$B_o = 1.5 \quad b_o = \frac{2}{3} \quad SF = 33\%$

$B_o = 3 \quad b_o = \frac{1}{3} \quad SF = 67\%$

$B_o = 1.1 \quad b_o = 0.9 \quad SF = 10\%$

Gas FVF: B_g

$$B_g \equiv \frac{V_g(\underline{\underline{p}}, T)}{V_{\bar{g}g}}$$

$$b_g = \frac{1}{B_g} = \frac{V_{\bar{g}g}}{V_g(p,T)} = \text{gas expansion factor}$$

100 – 250

99% Text books assume $r_s = 0$

$$\left\| \quad B_{gw} = \frac{p_{sc}}{T_{sc}} \cdot \frac{T Z_{(p,T)}}{p} \right.$$

$: \; n_{\bar{g}g} = n_g$

$n_{\bar{o}g} = 0$

$$\text{True } B_{gd} = \frac{V_g(p,T)}{V_{\bar{g}g}}$$

| Gas Condensate Reservoirs

$$\boxed{B_{gd}} = \underbrace{\left[\frac{p_{sc}}{T_{sc}} \cdot \frac{TZ}{p} \right]}_{\text{Text Book Gas FVF } B_{gw}} \cdot \frac{1}{\left(1 - \frac{n_{\bar{o}g}}{n_g}\right)}$$

Use in your engineering !

$\frac{n_{\bar{g}g}}{n_g} \sim 0.85 - 0.99$

$\left(\frac{n_{\bar{o}g}}{n_g}\right) \sim 0.15 - 0.01$

Est.
~ $\underline{y_{6+}}$ or $\underline{y_{5+}}$

d: dry means that the surface gas is "dried" by removing surface oil $\bar{o}g$ from the surface gas

w: wet means that we assume all of the surface gas = reservoir gas ($n_{\bar{g}g} = n_g$)

wet surface gas because it still contains $\bar{o}g$

How to calculate oil phase and gas phase densities:

Know $\{R_s \quad B_o \quad r_s \quad B_{gd}\}$ @ $(\underline{\underline{p, T}})$

? $\rho_o(p,T)$ $\rho_g(p,T)$

Need for eng. calculations involving transport or hydrodynamics influence by gravity.

$$\left[\begin{array}{l} \rho_g(p,T) = \dfrac{\rho_{\bar{g}} + \rho_{\bar{o}} \cdot r_s(p,T)}{B_g(p,T)} \\[2em] \rho_o(p,T) = \dfrac{\rho_{\bar{o}} + \rho_{\bar{g}} \cdot R_s(p,T)}{B_o(p,T)} \end{array}\right]$$

Need $\boxed{\rho_{\bar g}}$ $\boxed{\rho_{\bar o}}$

~ TWO CONSTANTS ~

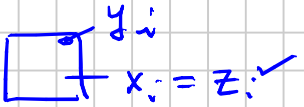
even though $\rho_{\bar g g}(p,T) \neq \rho_{\bar g o}(p,T)$

$$\rho_{\bar o o}(p,T) \neq \rho_{\bar o g}(p,T)$$

$K_i(p, T, p_K)$

$\overset{\uparrow}{?}$

p_b = Lab value 2620 psig



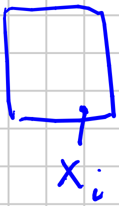
@ T

Bubblepoint Calc: $\sum y_i = 1$

$$y_i = K_i \cdot x_i = K_i z_i$$

$$\longrightarrow \quad h_{BP}(p_b)\Big|_{p_K} = 0 = 1 - \sum y_i = 1 - \sum z_i K_i(T, p_K, \underset{\substack{\uparrow \\ \text{know}}}{p_b})$$

Dewpoint: $\sum x_i = 1$



$$x_i = z_i / K_i$$

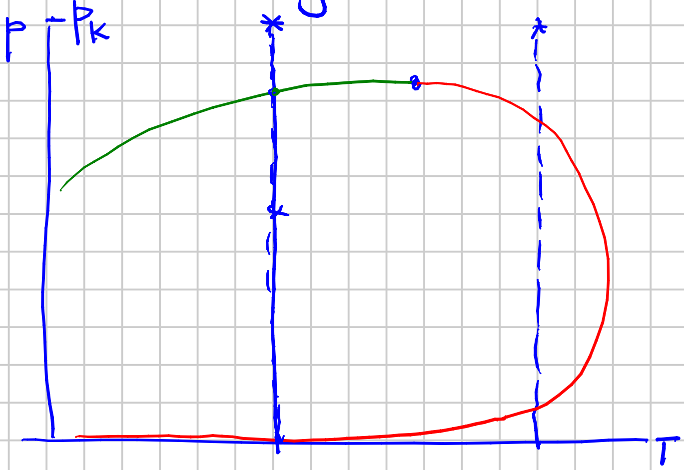
$$h_{DP} = 0 = 1 - \sum x_i = 1 - \sum z_i / K_i(T, p_K, p_d)$$

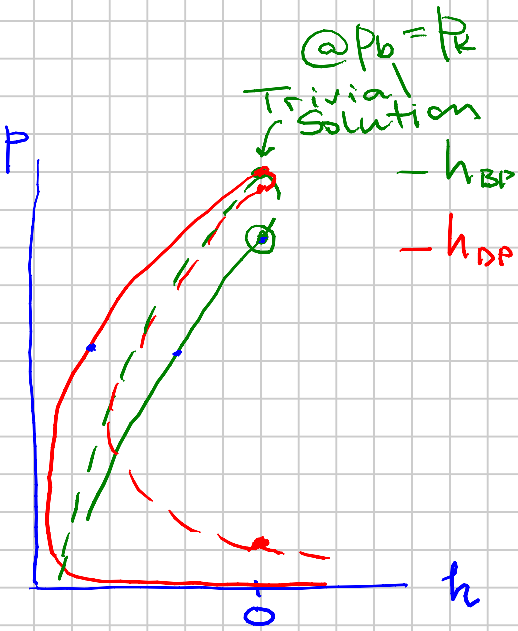
Know p_K, solve $p_b(T)$ or $p_d(T)$

p_b: Upper Sat.

p_d: Upper Sat / Lower Sat

Saturation Pressure Calculation (without knowing BP or DP):





Solve $h_{sp} = h_{BP}(p_s) \times h_{DP}(p_s) = 0$

Once found p_s, see if h_{BP} or h_{DP} that drove the $h_{sp} \to 0$

Black-Oil PVT Formulation (Ch. 7)

<u>Review:</u> Gas Phase: r_s – solution OGR $\sim y_i$

B_{gd} – gas FVF $\sim \rho_g$; expansion

Oil Phase: R_s – solution GOR $\sim x_i$

B_o – oil FVF $\sim \rho_o$; shrink

BO PVT are specific to a particular surface process $\underline{P}$

May be a strong dependence of BO PVT on the $\underline{P}$ used:

GOR $\gtrsim$ 200 Sm³/Sm³ measureable → large effect

< 100 little effect

$\boxed{\underline{P}}$ <u>Worst</u>

Single Stage Flash → ambient conditions

Best Recovery of i in $\bar{o}$

\# Multi-Stage Flash + GP

Applications of Black-Oil PVT:

To convert reservoir (p, T_R) Volumes
or @ any (p, T) Tubing, Flowlines etc.
to "surface" (sellable) gas $(\bar{g})$ and oil $(\bar{o})$

$$b_{gd} = \frac{V_{\bar{g}}}{V_g(p,T)} \sim 50 \text{ to } 250 \ \frac{Sm^3}{m^3 @ (p,T)} \quad \text{Expansion}$$

$$B_{gd} \sim 0.02 - 0.004 \ m^3 @ (p,T) / Sm^3$$

$$b_o = \frac{V_{\bar{o}}}{V_o(p,T)} \sim 0.9 \to 0.3 \quad \text{Shrinkage}$$

$$B_o \sim \underbrace{1.1 \to 3}_{\text{loss of mass into gas phase}} \quad m^3 @ (p,T) / Sm^3$$

RB / STB

bbl / STB

B_{gd}

$V_g \propto \frac{1}{p}$

condense (loose mass) into a liquid : 1-15% change in final surface gas volume

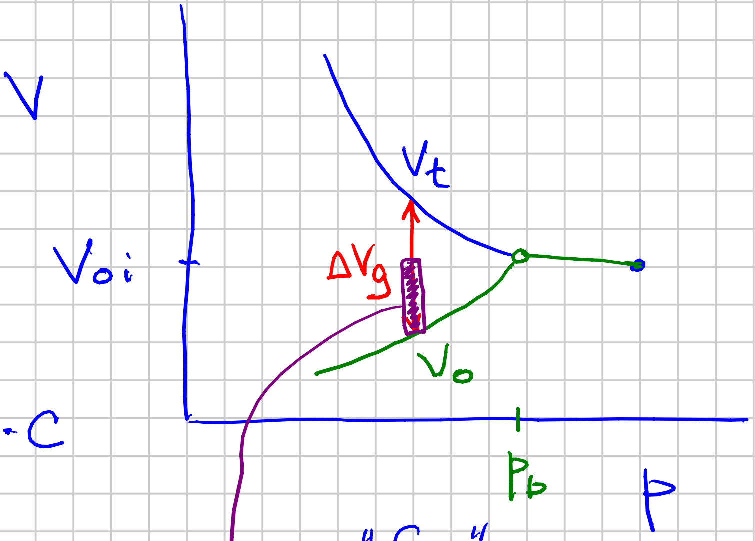
$R_s : \frac{V_{\bar{g}o}}{V_{\bar{o}o}}$ keep track of the (component surface) gas $\bar{g}o$ that is (still) in solution in the oil phase

$$\overbrace{\left[R_{si} - R_s(p) \right]}^{\Delta R_s} = \text{Liberated (Surface) Gas}$$

$$300 \ Sm^3/Sm^3 - 200 \ Sm^3/Sm^3$$

$$\underset{(p,T)}{\Delta V_g} = \left[\Delta R_s \times B_g(p,T)\right]$$

$$\frac{m^3}{Sm^3\,\bar{o}} \qquad \frac{\cancel{Sm^3}}{Sm^3\,\bar{o}} \qquad \frac{m^3}{\cancel{Sm^3}}$$



Solution OGR $r_s \propto \frac{\boxed{y_{5+}}}{(1-y_{5+})} \cdot C$

$r_s \propto \underset{mol\text{-}\%}{y_{C_{5+}}(p,T)}$

if you produce alotta "free" reservoir gas

$$\left(\frac{\$}{mol\text{-}\%}\right)_{\bar{o}} >> \left(\frac{\$}{mol\text{-}\%}\right)_{\bar{g}}$$

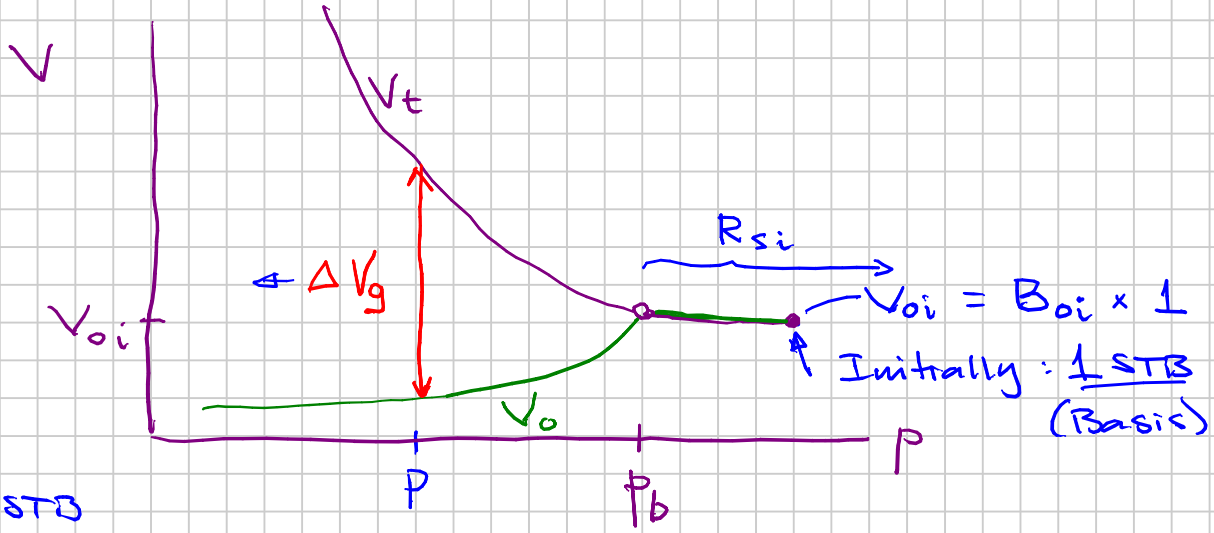
$\bar{o} = \bar{C}_{5+}$

\$115 / bbl

\$ 3/Mscf

$$\boxed{\begin{matrix}\Delta V_{\bar{o}} \\ Sm^3\,\bar{o}\end{matrix}} = \underbrace{0.6(\Delta V_g) \times \frac{1}{B_{gd}}}_{\cancel{Sm^3\,\bar{g}}} \times r_s \qquad \frac{Sm^3\,\bar{o}}{\cancel{Sm^3\,\bar{g}}}$$

$$\Delta V_{\bar{o}g} = 0.6 \cdot \Delta R_s \cdot r_s$$



Basis : 1 STB

$$\Delta V_g(p) = \overbrace{\underbrace{\left(R_{si} - R_s(p)\right)}_{\text{Liberated Gas}} B_{gd}}$$

Liberated Gas ~~scf~~ Sm^3 $\frac{ft^3}{\cancel{scf}}$

$$\Delta V_{\bar{o}g} = r_s(p)\left(R_{si} - R_s(p)\right) = r_s(p)\ \Delta V_g\ \frac{1}{B_{gd}(p)}$$

condensate $\frac{STB}{scf}$ $\frac{V_{\bar{o}g}}{V_{\bar{g}g}}$

Black Oil PVT Properties

vary w/ pressure @ T = const

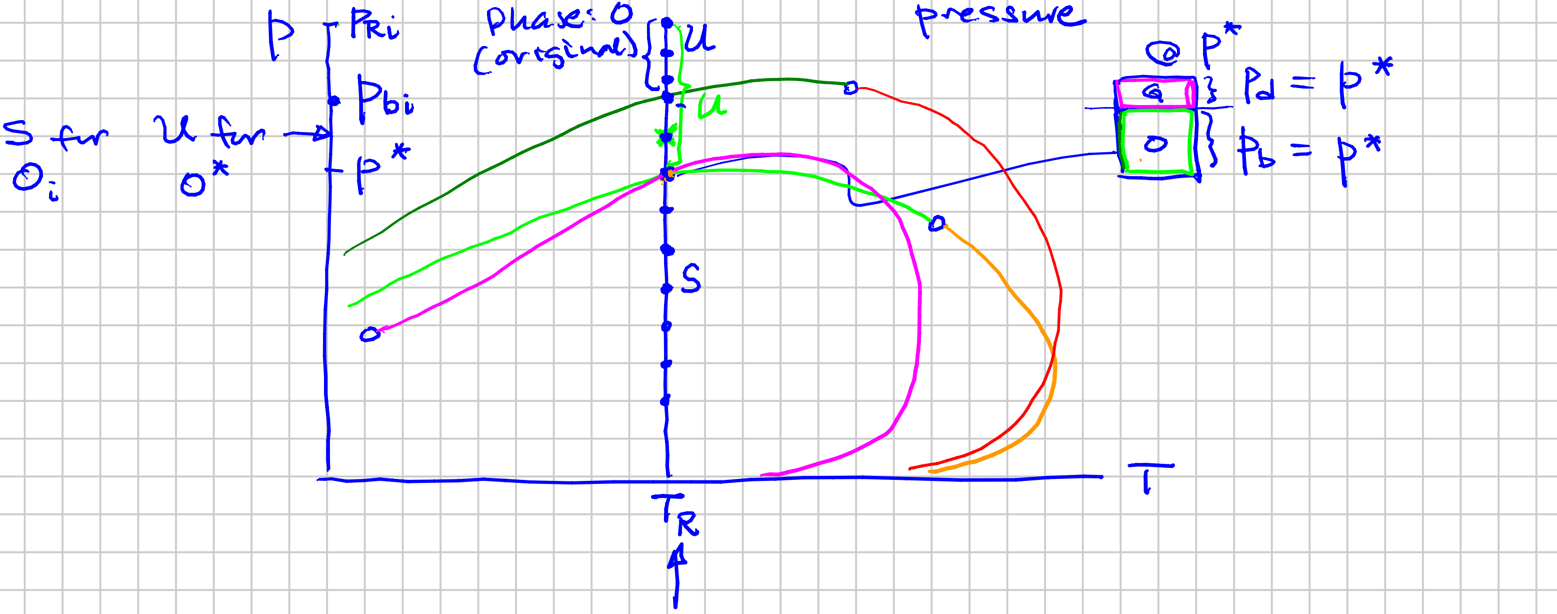
OIL: R_s B_o μ_o

GAS: r_s B_{gd} μ_g

} Saturated & Undersaturated

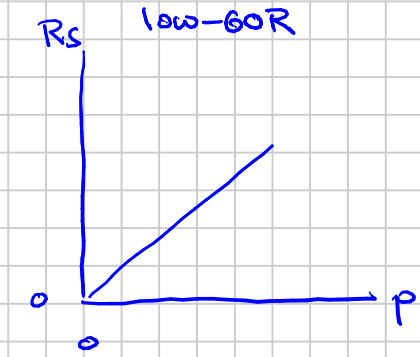
- "2-phases"
- phase is at its saturation pressure

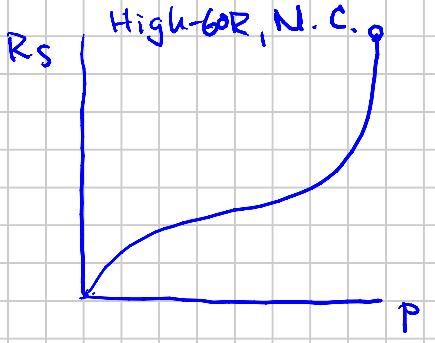
phase is @ $p > p_s$

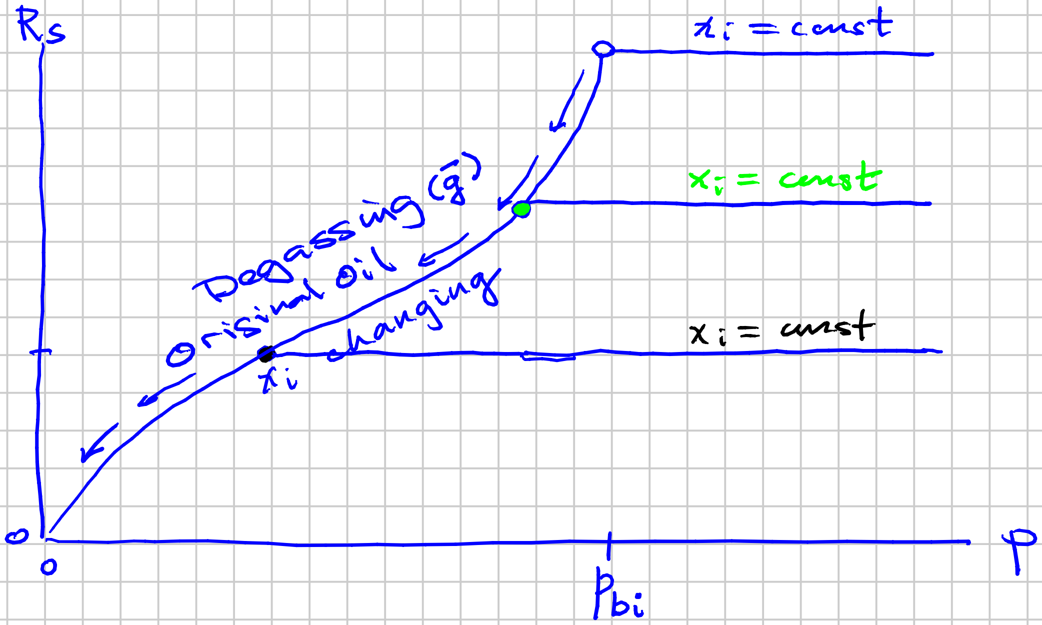


OIL PHASE BO PVT: R_s B_o μ_o

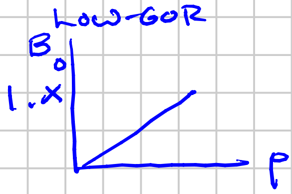
(x_i) (V_o, ρ_o)

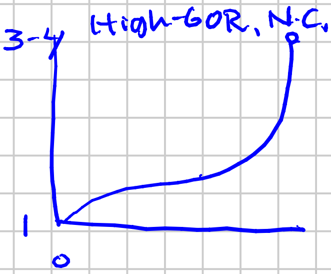


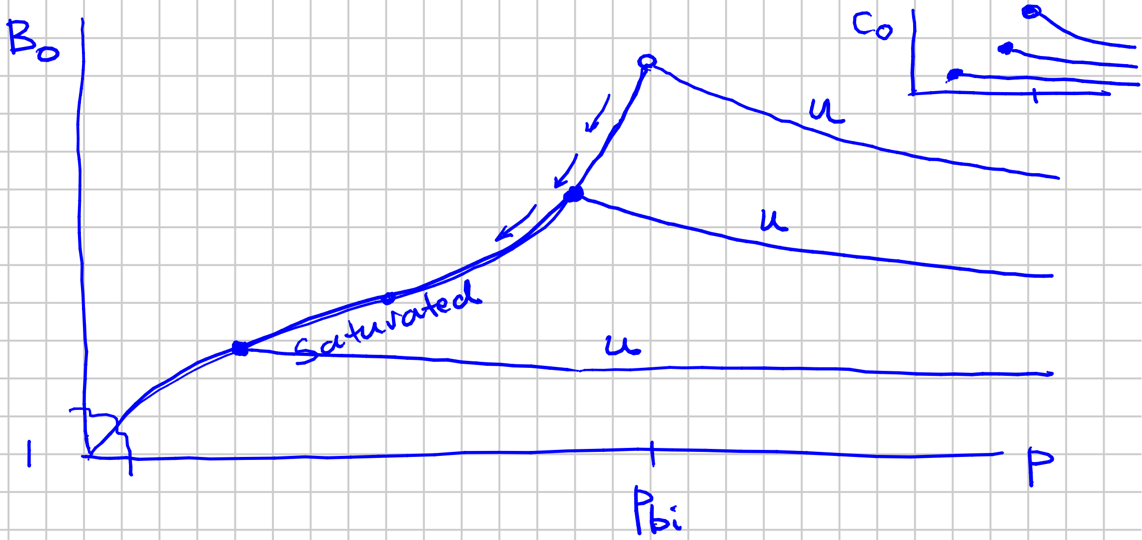




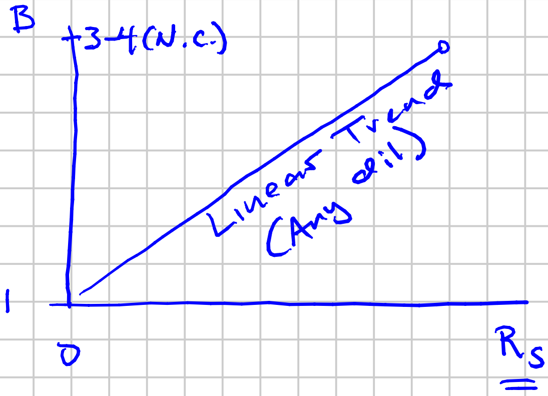
Bo







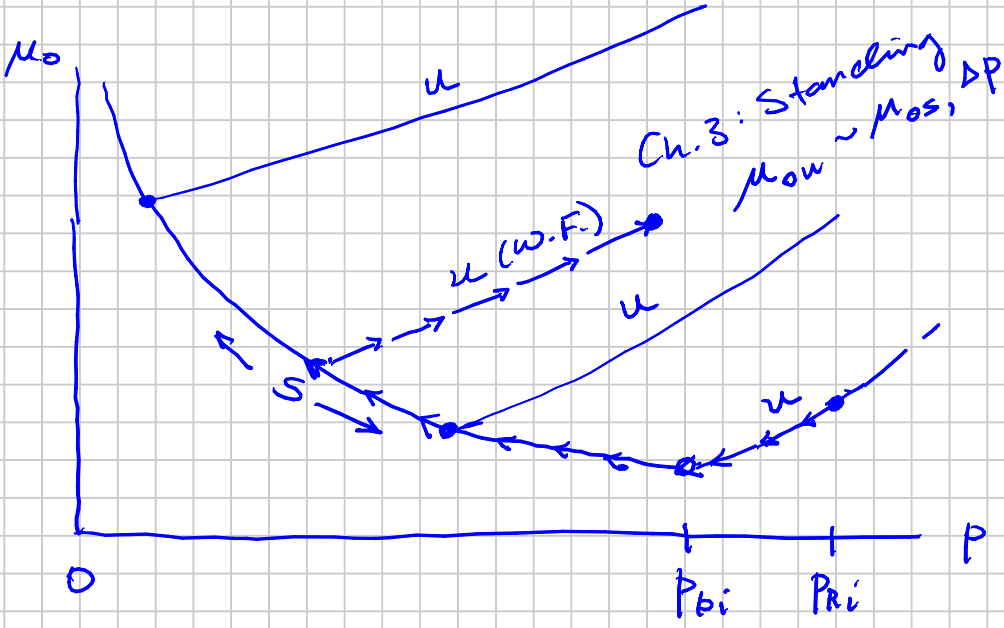
Saturated Bo vs Rs



$\mu_o \propto \rho_o^{n \sim 3-4}$

$\rho_o \propto \frac{1}{B_o}$

$\mu_o \sim \left(\frac{1}{B_o}\right)^n$



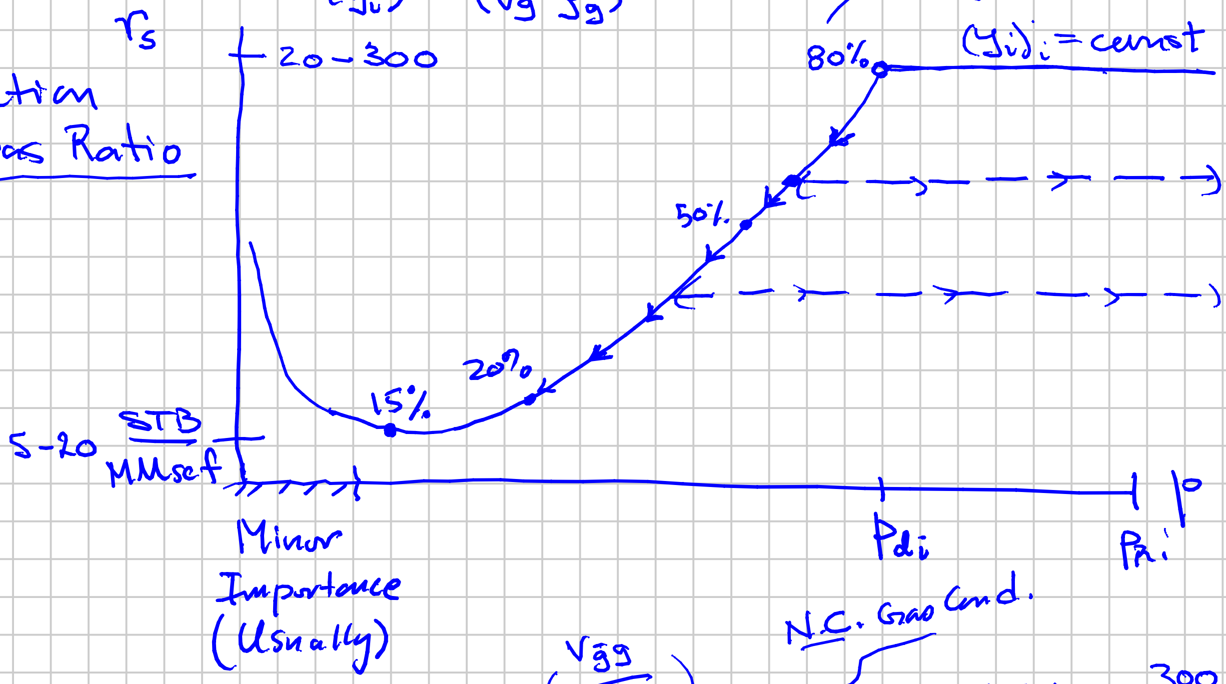
Ch. 3: Standing

$\mu_{ow} \sim \mu_{os}, \Delta P$

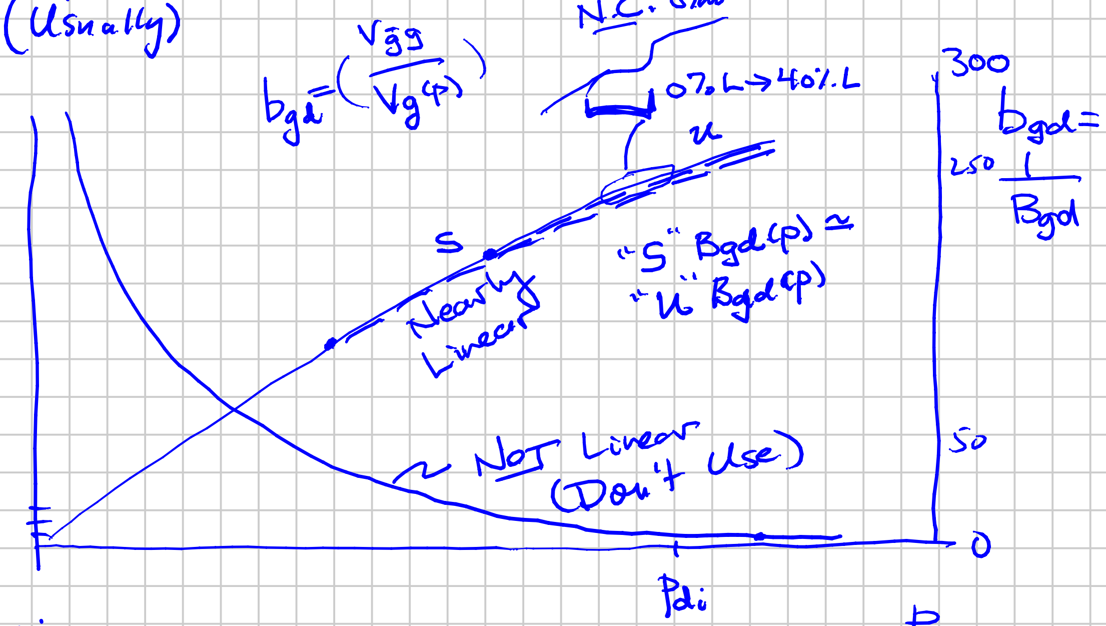
GAS BO PVT r_s (y_i) B_{gd} (V_g ρ_g) μ_g

% Revenue from r_g (condensate)

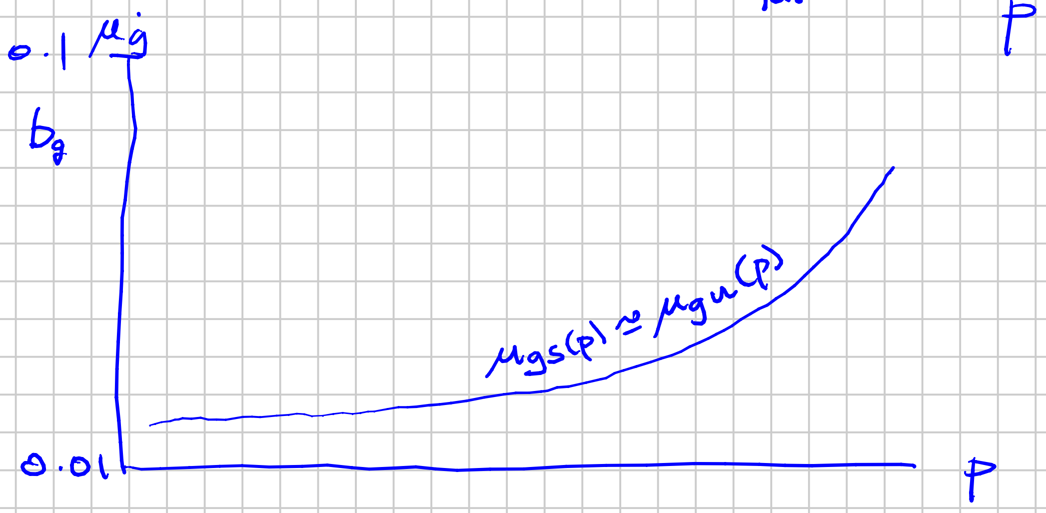
Solution Oil-Gas Ratio



$B_{gd} \propto \frac{1}{p}$



$\mu_g \propto \rho_g^m \propto b_g$



HAPPY HALLOWEEN

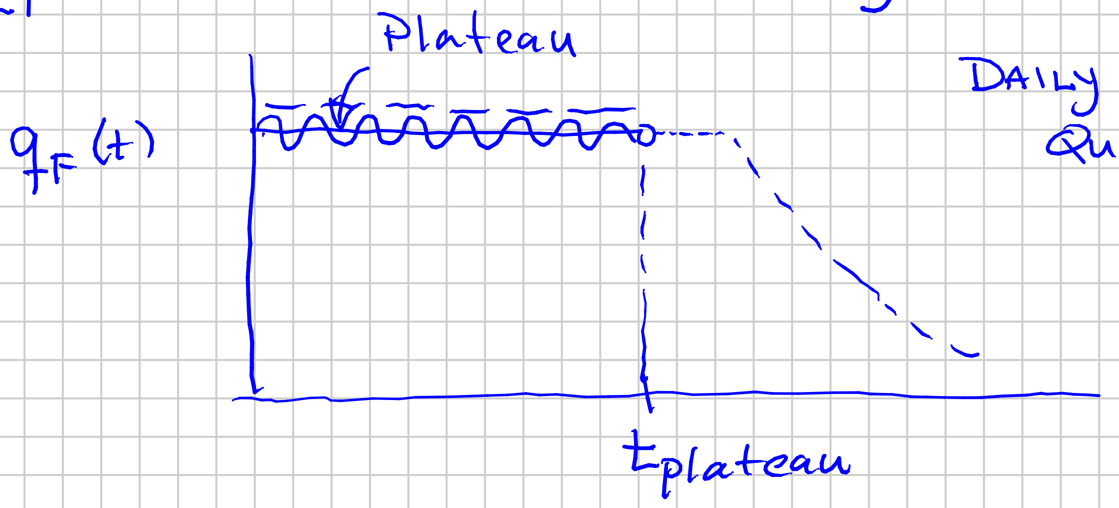
RESERVOIR MECHANICS (OF GAS RESERVOIRS)
(+ flow in tubing) of Pressure Depletion

- Volumetric Material Balance
- Reservoir (Flow) Rate Equation
  (+ Tubing Rate Equation)

} Combine ⇒ $q(t)$

Sales Product $\bar{g}$ ($\bar{o}$)

$\left[q_w(t) \right.$: Production Forecasting



DAILY CONTRACT QUOTA DCQ

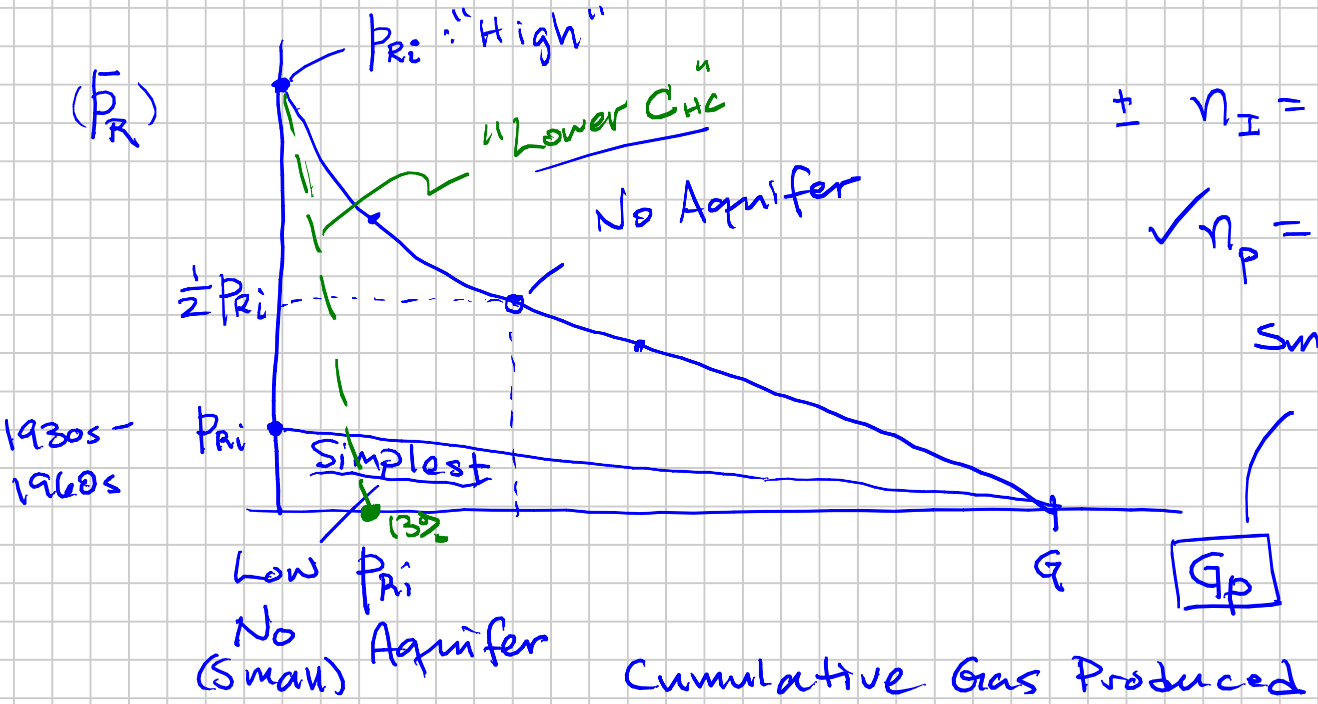
$$q_F = \sum_{w=1}^{N_w} q_w(t) < \begin{matrix} N_w \cdot q_w \\ \\ N_w(t) \cdot \underline{\underline{q_{wmax}}}(t) \end{matrix}$$

Constant for $t_{plat.}$ ↑

↑ Cost Item

Volumetric Material Balance (MB)

$c_g = \frac{1}{p} - \left(\frac{1}{Z}\frac{dZ}{dp}\right)$



(Q_p)

± $n_I = G/23.68$

✓ $n_p = G_p/23.68$

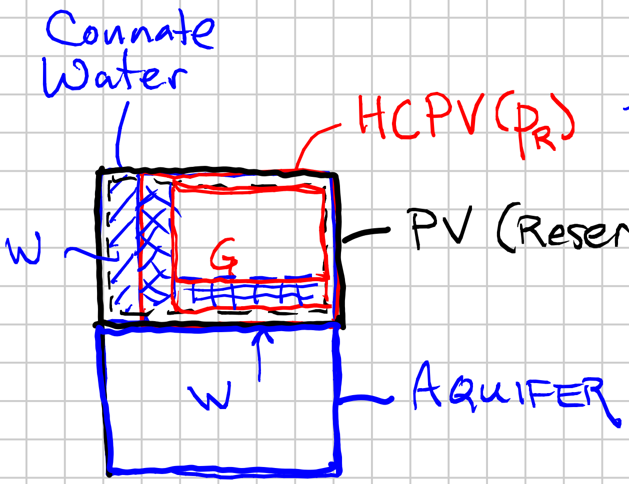
Sm^3

Gas on oiL

G = Initial Gas In Place (Sm^3 or scf)

Bcf 10^9 scf

Tcf 10^{12} scf

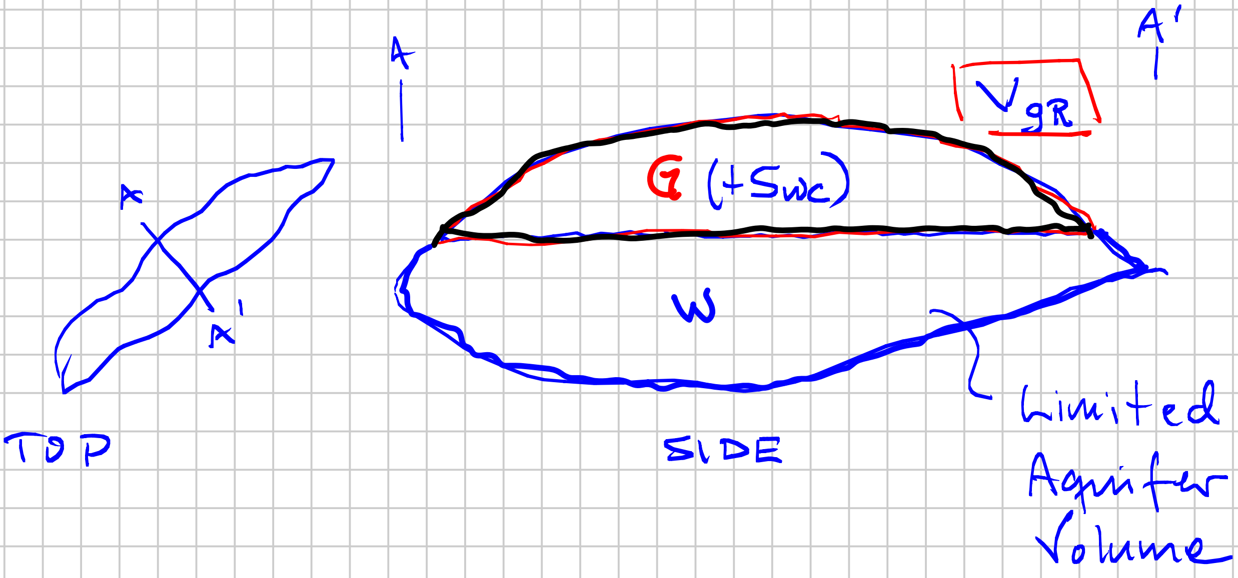


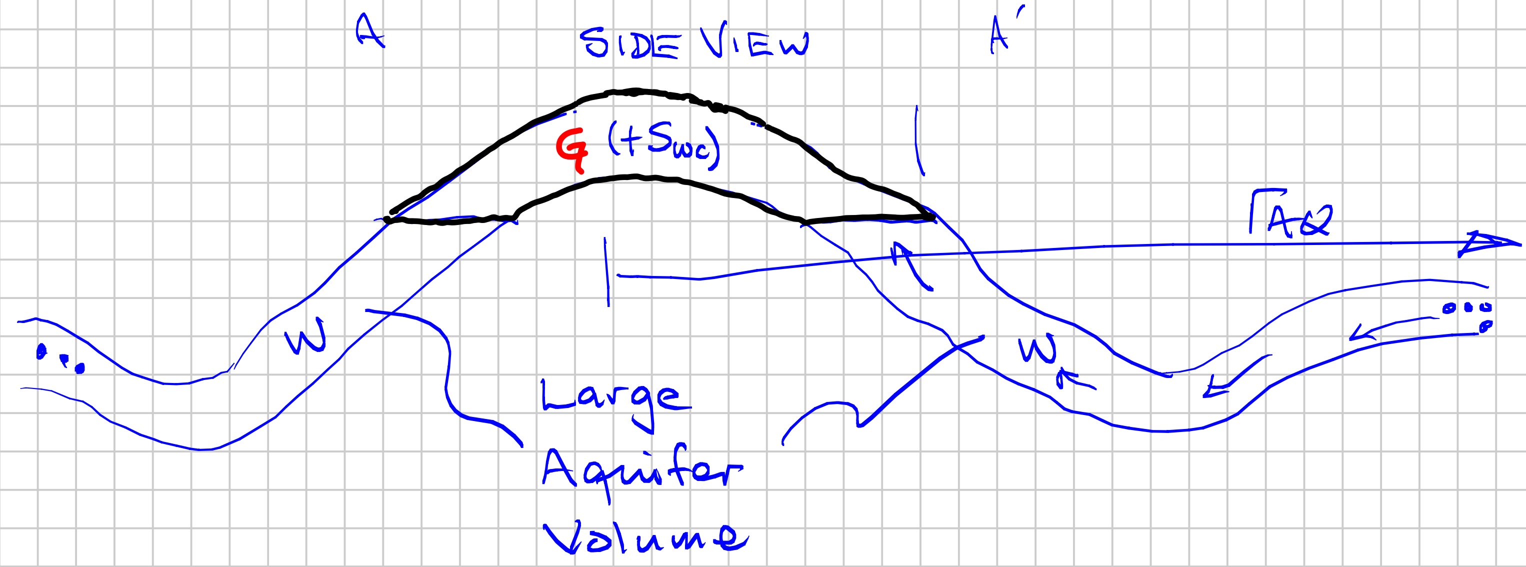
Real Gas Law: EOS

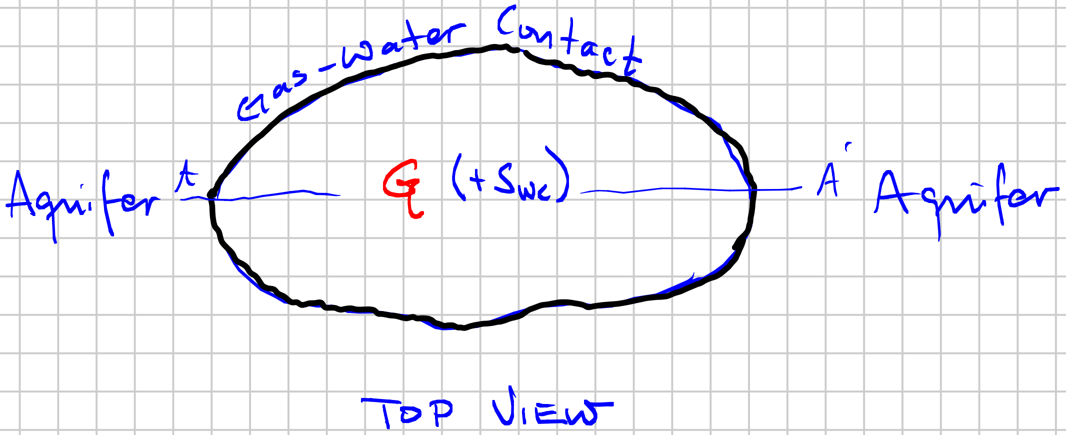
$$\bar{p}_R V_{gR} = n_{gR} R T_R Z_{gR}(p_R, T_R)$$

n_{gR} = Current gas moles in Reservoir

$= n_I - n_p$







Derive

Gas Material Balances:

Simplest Approximation: $\left[p_R = p_{Ri} \left(1 - \frac{n_p}{n_I}\right) \right] \Rightarrow p_R = p_{Ri} \left(1 - \frac{G_p}{G}\right)$

Assumptions: (1) Ideal Gas Law $p_R V_{gR} = n_R R T_R$

$z_{gR} = 1$

(2) HCPV = constant

$\left\{ \begin{array}{lll} \cdot & c_w \sim 0 & \text{(water)} \\ \cdot & c_f \sim 0 & \text{(pore)} \end{array} \right.$

$c_w \cdot p_{Ri} << 1$

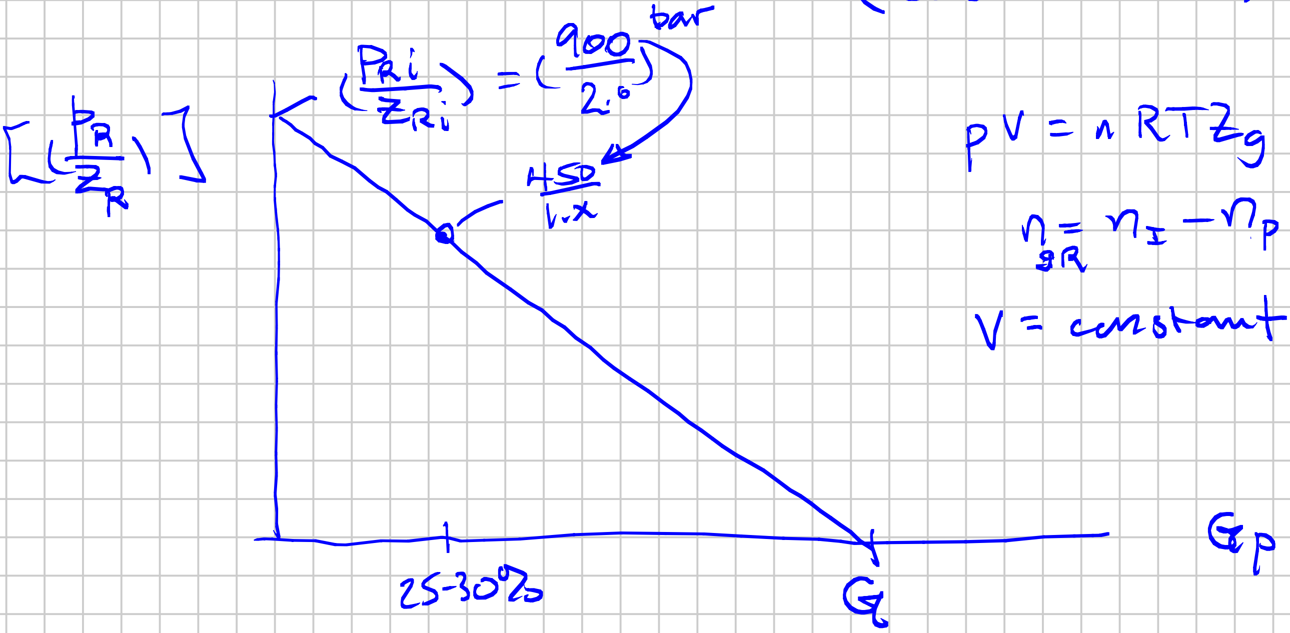
$c_f \cdot p_{Ri} << 1$

$\left(\frac{V_{Aq}}{V_{pR}} \right) (c_w + c_f)\, p_{Ri} << 1$

Next Simplest : Real Gas Law

HCPV = constant

$$\frac{p_R}{Z_R} = \left(\frac{p_{Ri}}{Z_{Ri}}\right)\left(1 - \frac{G_p}{G}\right) \quad : \left(\begin{array}{l}\text{Straight-Line}\\ \text{Gas M.B.}\end{array}\right)$$



$pV = nRTZ_g$

$n_{gR} = n_I - n_p$

$V = \text{constant}$

$c_o \sim 10 \cdot 10^{-6} \; \frac{\text{vol/vol}}{\text{psi}}$

$\left[\frac{\Delta V_o}{V_{oi}}\right] = c_o (p_{Ri}) = 10 \cdot 10^{-6} \, (900 \cdot 14.5) = 0.13$ 13%

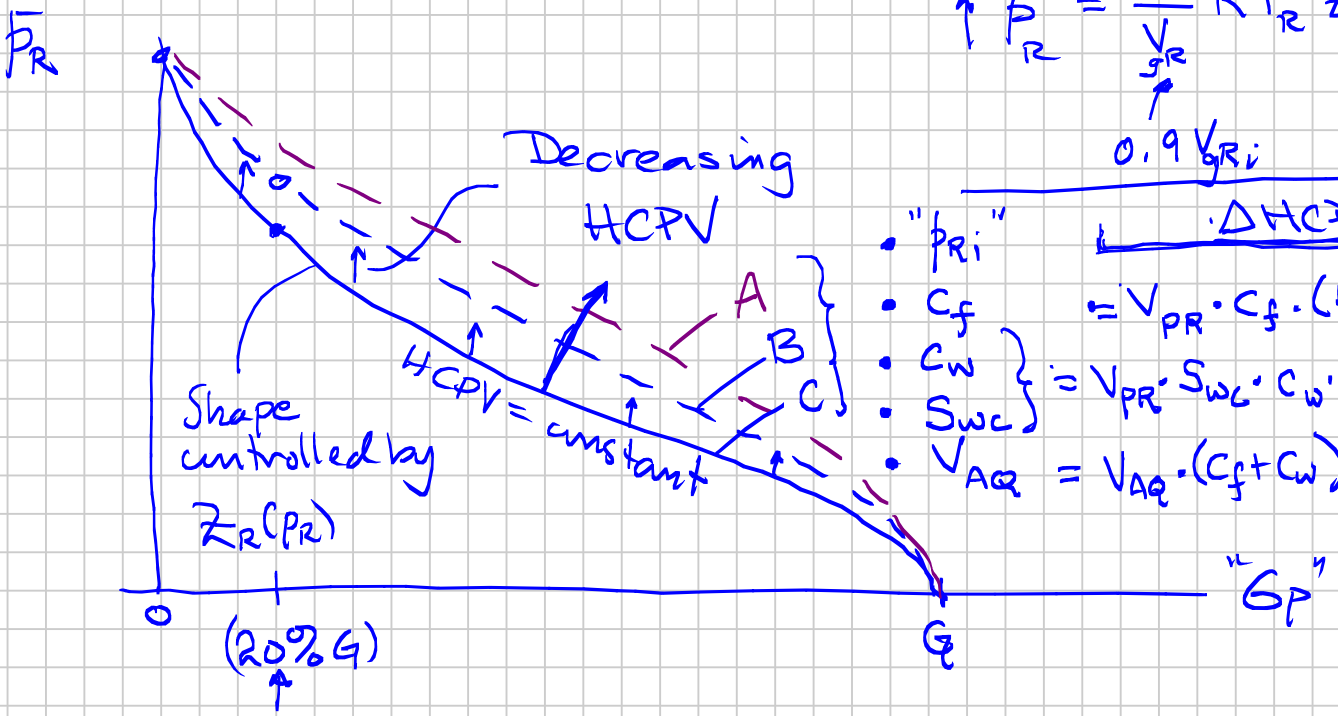
10 000

10^{-6} 10^{4}

$pV = nRTZ$

$\uparrow p_R = \frac{\overset{\checkmark}{n_R}}{V_{gR}} R \overset{\checkmark}{T_R} Z_R$

$\uparrow 0.9\, V_{gRi}$



ΔHCPV

- "p_{Ri}"
- c_f $= V_{pR} \cdot c_f \cdot (p_{Ri} - p_R)$
- c_w
- S_{wc} $\Big\} = V_{pR} \cdot S_{wc} \cdot c_w \cdot (\Delta p_R)$
- $V_{AQ} = V_{AQ} \cdot (c_f + c_w) \cdot (\Delta p_R)$ ↑

n_{gR}

$$\text{“}\ \overset{\uparrow}{q_g} = \underset{\substack{\uparrow \\ \text{Same}}}{C}\ \overset{\uparrow}{(}\, p_R - \underset{\substack{\uparrow \\ \text{same}}}{p_{wf}}\,)\ \text{”}$$

$\boxed{+}$

Effect of HCPV ↓ (p_R)

⇒ Higher Gas Rate @
Level of Recovery
Gp

Generalized Gas M.B. (Fetkovich Reese Whitson)
SPE 22921

$$\frac{p_R}{Z_R}\underbrace{\left[1-\bar{c}_e(p_{Ri}-p_R)\right]}_{\bar{c}_e:\ \uparrow} = \frac{p_{Ri}}{Z_{Ri}}\left(1-\frac{G_p}{G}\right)$$

Cumulative
Effective
Total
Compressibility

"Pot"
Aquifer
$p_{Aq} \simeq p_{Rg}$

External
Water
Sources

ⓐ Aquifer
ⓑ Non-Net Pay
(NNP)

{External Water}

$$M = \frac{V_{w\,External}}{V_{pR}}$$

$$c_e = \frac{\overset{\checkmark}{c_f} + \overset{\checkmark}{c_w}\, \overset{\checkmark}{S_{wc}} + \overbrace{\overset{\checkmark}{M}(c_f + c_w)}}{1-S_{wc}}$$

Assumption
$p_{NNP} \simeq p_{Rg}$

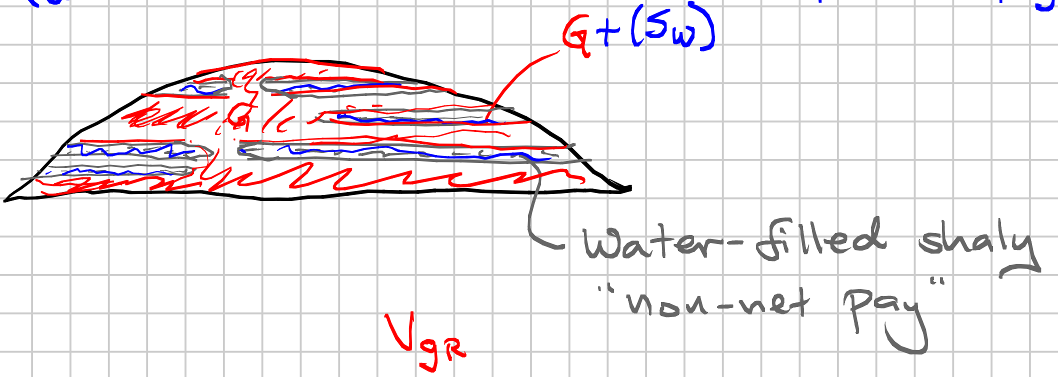
p_{Ri} ↑ ⇒ More important
ALL terms

↓

c_{gi} can be "smaller"

~

c_f, c_w



$$NGR = \frac{V_{gR}}{V_{gR}}$$

c_f : $3-30 \cdot 10^{-6}$ 1/psi × 14.5 $\frac{psi}{bar}$ ⇒ $45-450 \cdot 10^{-6} \frac{1}{bar}$

c_w : $3-5 \cdot 10^{-6}$ 1/psi

S_{wc} : 0.05-0.5 1/psi

$\frac{V_{AQ}}{V_{pR}} = M_{AQ}$: 0.2-5

$p_{AQ} \sim p_{Rg}$

$$M_{NNP} = \left\{ \frac{\bar{\phi}_{NNP}}{\bar{\phi}_R} \cdot \frac{1-NGR}{NGR} \right\}$$ Log Analyst & Geologist

NGR
- 0.9+ (high-k sands)
- 0.2-0.5 (low-k dirty sands)

Non-Ideal (Not accounted for in FRW MB):

① Larger-Huge Aquifers ($\underline{M_{AQ}}$: 5-100+) r_{AQ}

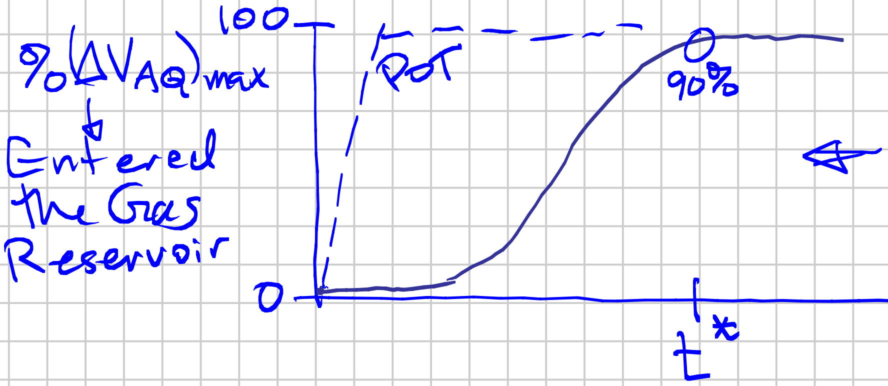
Lower-Moderate $\underline{k_{AQ}}$

1 - 100 md

$p_{AQ} > p_{Rg}$

Lag time in p_{AQ}

Behavior of a "Transient" Aquifer



$$t^* \propto \frac{k}{A_{AQ}} \sim \frac{k_{AQ}}{r_{AQ}^2}$$

"τ_{AQ}"

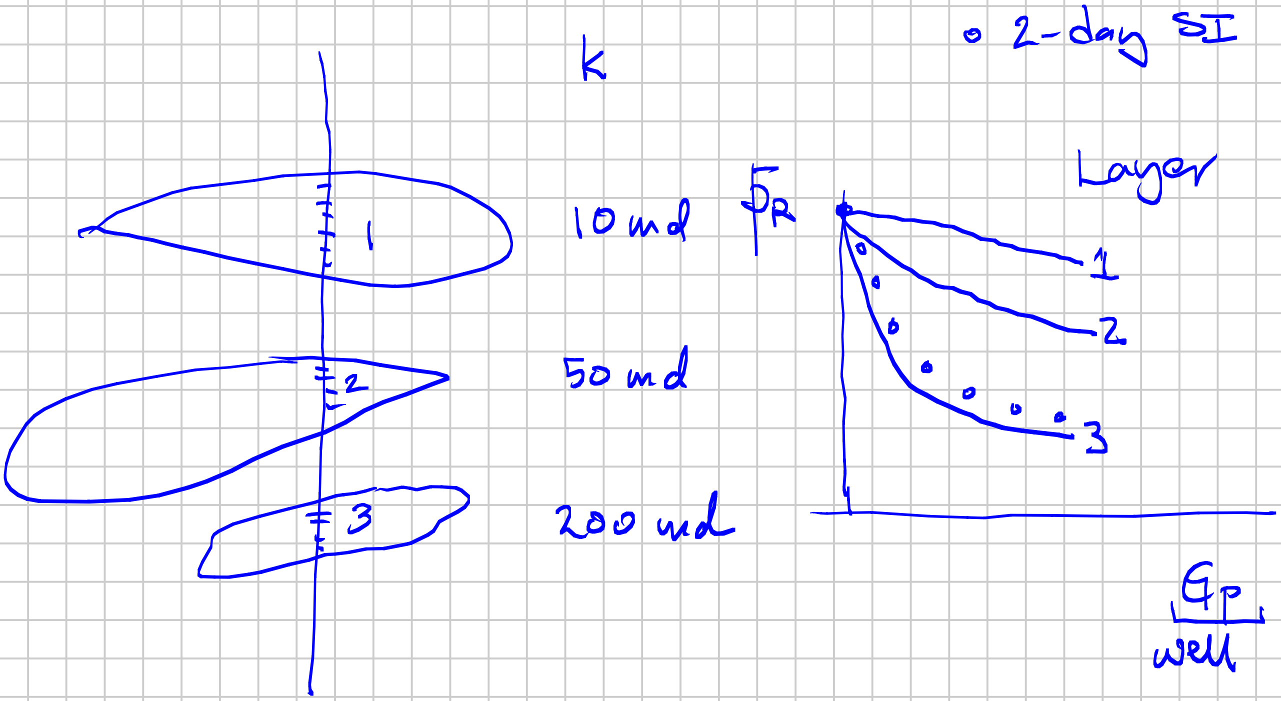
$$(t^*)_{\text{Pot AQ}} \leq 6 \text{ mo.}$$

- Rigorous : Method of Superposition, Van Everdingen – Hurst

Approximations:

- Schilthius
- Fetkovich simplified

② 2-or more non-communicating (in R) reservoir units produced by same well



Volumetric Material Balance

$$\bar{p}_R(t) = f(G_p) = f(N_p) = f(Q_p)$$

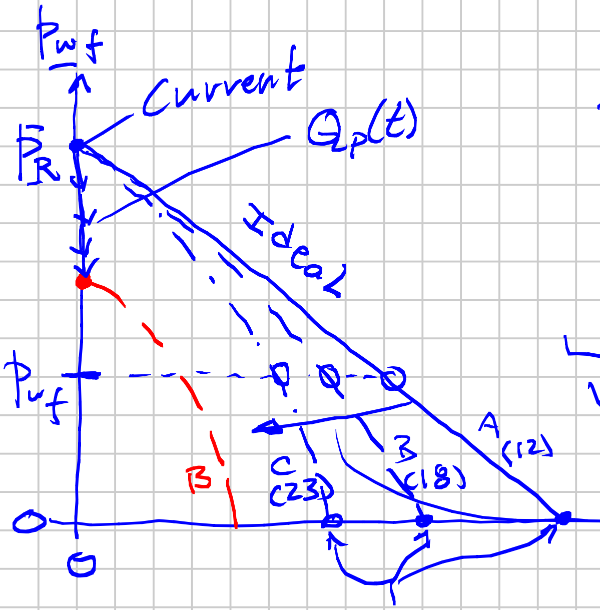
Gas | Oil | General

Reservoir Rate Equation

$$q_{\bar{g}}, \; q_{\bar{o}} = f(\bar{p}_R, p_{wf})$$

Oil: (Gas)
Inflow Performance Relationship (IPR)

Gas:
Deliverability



Flowing
Bottomhole
Pressure
FBHP
p@Wellbore
WE HAVE
CONTROL

Non-Ideal Behavior

- non-Darcy ("turbulent-like")
- PVT (ρ, μ) vary w/p
  $\rho_g = \frac{pM}{RTZ}$
- Multi-Phase Flow

Theoretical
Absolute
Openflow
Potential
} q_{max} (AOFP)

$$\frac{dQ_p}{dt} = q$$

$$\underbrace{\begin{matrix} Q_p \\ G_p \\ N_p \end{matrix} = \int_0^t q(t)\, dt}$$

$\Rightarrow$ $q(t)$, $Q_p(t)$, $p_R(t)$

Generalized Gas Material Balance ("Pot" Aquifer Assmp)

$p_{Rg} \sim p_{RAQ}$

$$\left(\frac{p_R}{Z_{gR(p)}}\right)\underbrace{\left[1 - \bar{c}_e\left(p_{Ri} - p_R\right)\right]}_{} = \left(\frac{p_{Ri}}{Z_{gi}}\right)\left(1 - \frac{G_p}{G}\right)$$

$$\bar{c}_e = \frac{c_f + c_w S_{wc} + M\left(c_f + c_w\right)}{1 - S_{wc}}$$

$S_{wc} = S_{wi}$

$\uparrow$

Initial = Irreducible

$\lambda_w = 0$

$$M_{AQ} = \frac{V_{AQ}}{V_{pR}}$$

$V_{AQ} \leftarrow$ External $S_w = 1$ aquifer

$V_{pR} \leftarrow$ {Pore Volume containing gas + S_{wc}}

$(1 - NGR)$ = fraction of V_{pR} not containing gas (shale, dirty sand)

$$M_{NNP} \simeq \frac{\bar{\phi}_{NNP}}{\bar{\phi}_R} \cdot \frac{1 - NGR}{NGR}$$

$$\left\{\begin{array}{l} c_w = 3\text{-}5 \cdot 10^{-6} \ 1/\text{psi} \\ c_f = 3\text{-}20^{+} \cdot 10^{-6} \ 1/\text{psi} \end{array}\right\}$$

$M_{AQ} = 0.5 - 5$

$NGR \sim 1 - 0.5 \ (0.3)$

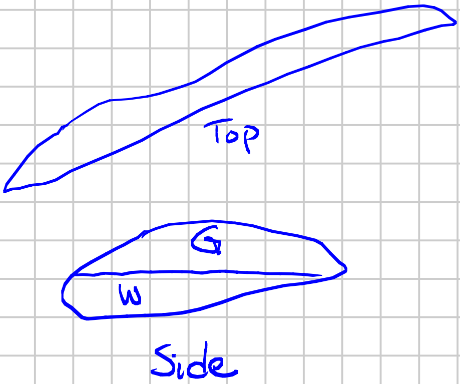
$$\left[c_g = \frac{1}{p} - \frac{1}{Z}\frac{dZ}{dp}\right]$$

$$c_g \sim \frac{1}{p}$$

$$\underline{c_w, c_f} \sim \underline{c_g \text{ (high } p)}$$

c_g (1000 psi) $= 0.001 = 1000 \cdot 10^{-6}$

$V_{PR} = 10^9$ bbl



$p_{Ri} = 11850$ psia

$T_R = 240\,°F$

$c_w = 3 \cdot 10^{-6}$ psi^{-1}

$c_f = \boxed{\underline{5}} / 10 \cdot 10^{-6}$ psi^{-1} (uncertainty in data)

$S_{wc} = S_{wi} = 0.22$

$\phi = 0.2$

$M_{AQ} = \boxed{\underline{0}} - 1$

$NGR = \boxed{\underline{1}} - 0$

① Change in HCPV $p_{Ri} \rightarrow 10{,}000$ psia ?

② Gas Recovery Factor G_p/G :

- $M_{AQ} = 0$, $M_{NNP} = 0$ (no "external" water, AQ or NNP)
- 1 0.9 (max — " —)
- $c_f = 10 \cdot 10^{-6}$ instead of $5 \cdot 10^{-6}$

$$\underline{HCPV}_i = V_{PR} \cdot \underbrace{(1 - S_{wc})}_{S_{gi}} = 10^9 (1 - 0.22) = 780 \cdot 10^6 \text{ bbl}$$

$g \qquad g \ \& \ S_{wc}$

$$\left.\begin{array}{r} -\ 9.25 \\ -\ 1.20 \\ -\ 14.8 \end{array}\right\} 3\%$$

$\overline{755 \cdot 10^6}$

HCPV = $755 \cdot 10^6$
@10,000

ΔV_p ? $\qquad \underbrace{11850 \rightarrow 10\,000}_{\Delta p_R}$

$$c_f = \frac{1}{V_i} \cdot \frac{\Delta V_p}{\Delta p}$$

assumes ~ constant

$$\frac{\Delta V_{PR}}{V_{PRi}} = \overset{\downarrow}{c_f} \cdot \Delta p_R$$

% change from V_{PRi} →

$$= \underset{1/psi}{5\cdot10^{-6}} \, (\underbrace{11850-10000}_{psi})$$

$$= 0.00925 = 0.9\,\% \sim 1\,\% = 9.25\cdot10^{6}\ bbL$$

$$0.00925\,(10^9)$$

Connate Water Expansion :

$$c_w = \frac{1}{V_{cwi}} \cdot \frac{\Delta V_w}{\Delta p}$$

$c_w \sim const$

↑ Initial Connate water volume

$$\Delta V_w = c_w\, V_{cwi}\, \Delta p$$

$$= 3\cdot10^{-6}\,(V_{PR}\cdot S_{wc})\,(1850)$$

$$= 1.2\cdot10^{6}\ bbL$$

$M_{AQ} = 1$

$$c_w = \frac{1}{V_{AQi}} \cdot \frac{(\Delta V_{AQ})_w}{\Delta p} =$$

Aq Water expanding

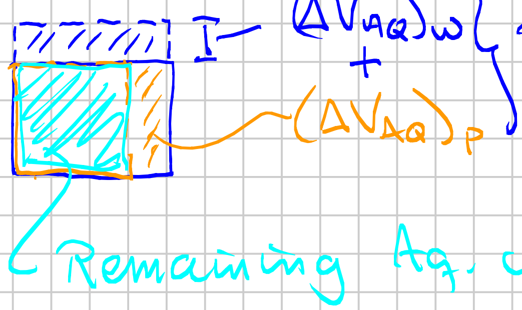
$\Delta V_w =$

$$c_f = \frac{1}{V_{AQi}} \cdot \frac{(\Delta V_{AQ})_p}{\Delta p}$$

Aq. PV shrunk

$$(\Delta V_w)_{AQ\to R} = (\Delta V_{AQ})_w + (\Delta V_{AQ})_p$$

$$= (M\cdot V_{PR})(c_w + c_f)(\Delta p_R)$$



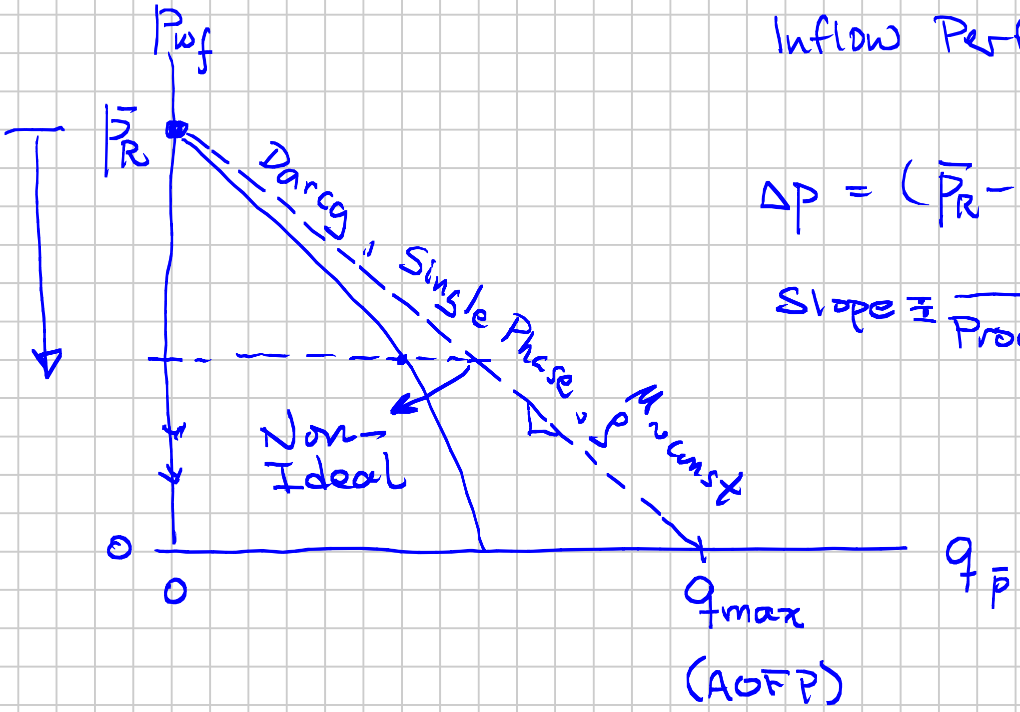
$$\left.\begin{array}{c}(\Delta V_{AQ})_w\\ +\\ (\Delta V_{AQ})_p\end{array}\right\} \rightarrow = 14.8\cdot10^{6}\ bbL$$

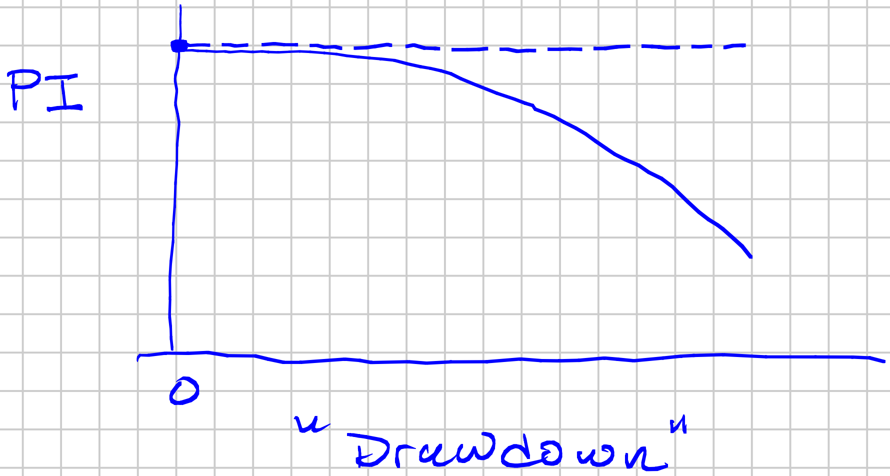
RESERVOIR RATE EQUATION : $q(p_{wf})$

Inflow Performance Relation (IPR)

$\Delta p = (\bar{P}_R - P_{wf}) =$ "Drawdown"

$\text{Slope} \mathrel{\overset{?}{=}} \dfrac{1}{\text{Productivity Index}}$





Darcy's Law : 1850's (d'Arcy)

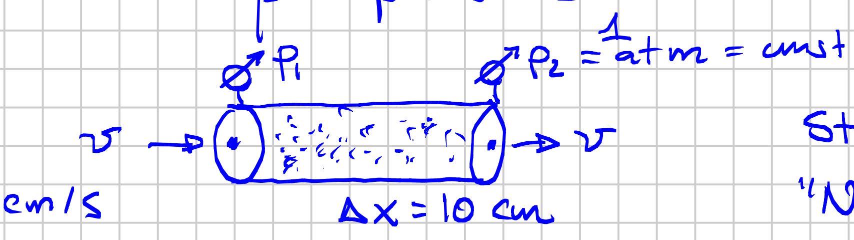
$$v = C\frac{\Delta p}{\Delta s} = C\frac{dp}{ds}$$

s = distance flowed

$1\,\frac{cm}{s} = v \equiv \dfrac{\Delta x}{\Delta t} = \dfrac{10\ cm}{}$

p = pressure

$\Rightarrow \Delta t = 10\ s$



Steady State : SS

"No Accumulation"

v	P_2	P_1	ΔP
1	1	2	1
2	1	3	2
4	1	5	4

Mass In = Mass out

$$|\underline{C}| = v \frac{\Delta s}{\Delta p} = 1 \ \frac{10}{1}$$

$$= 10$$

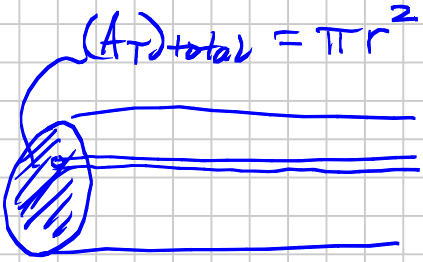
"Mobility" $\lambda \equiv \left(\frac{k}{\mu}\right)$ Rock / Fluid

$$v = \frac{k}{\underset{\rightarrow}{\mu}} \frac{dp}{ds}$$

Muskat Botset
1930s

$$\left[\text{Darcy velocity} \equiv \frac{q}{(A_\perp)_{total}}\right]$$
Bulk

Bundle of small capillaries


$(A_T)_{total} = \pi r^2$

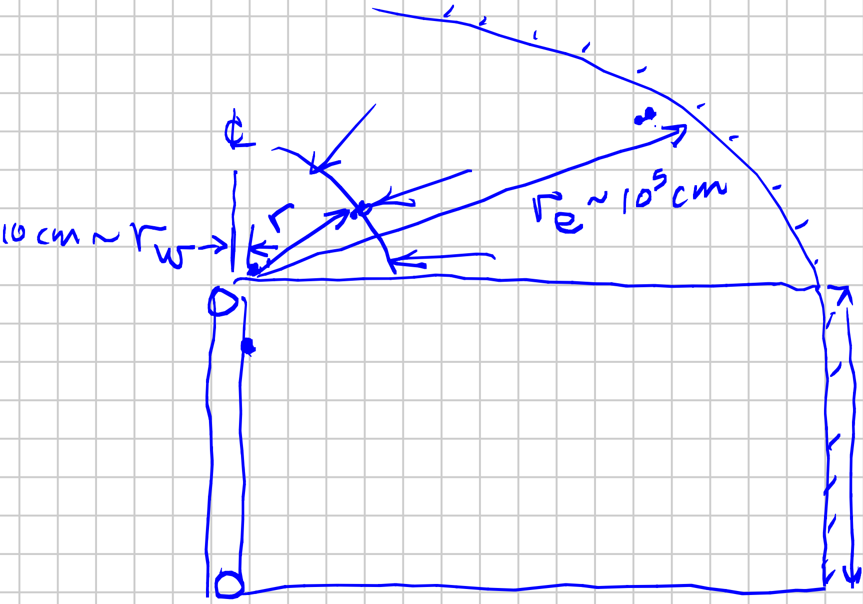
$d \sim 100\ \mu m$

$A_{pores} = 0.2\ A_{total}$

Physical "pore" velocity

$$\underline{v_p} = v / \phi$$

Very Important When Injecting (Water &/or Gas)
Enhanced Oil Recovery (EOR)



Radial - Cylindrical
Flow Geometry

$(A_\perp) = h \cdot 2\pi r$

$A_w = h \cdot 2\pi r_w$

$A_e = h \cdot 2\pi r_e$

$\Rightarrow \boxed{v = \frac{1}{r}}$ Steady State "everywhere"

$\frac{dp}{ds} = \frac{dp}{dr} = \frac{\mu}{k} v$ $\qquad$ $v_{rw} >> v_{re}$

$\frac{\Delta p (atm)}{cm} \propto v$

Units:

v · [cm/s]

s [cm]

p [atm] $\qquad$ 1.0135 atm / bar

μ [cp] $\qquad$ ≡ [mPa·s]

⇒ k [D] $\qquad$ "darcy" $\qquad$ $\{L^2\}$

Example: $\Delta p = 1$ atm

$\Delta x = 10$ cm

$\mu = 1$ cp $\qquad$ $v = \frac{k}{\mu} \frac{\Delta p}{\Delta x}$

$v = 1$ cm/s

$$\Rightarrow k = \frac{v \mu}{(\Delta p / \Delta x)} \qquad [D]$$

$$= \frac{(1)(1)}{(1/10)} = 10 \; D \sim$$ Maximum God-given oil & Gas

SPE: 1 md $= 10^{-3}$ D

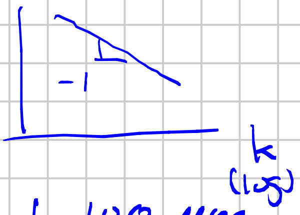
1000 md $= 1$ D

When Darcy's law doesn't work?

At "high" velocity

~1930 Forchheimer (Austrian, U. Graz)

Lindquist (1933) ← $n = 2$



Muskat (1930s) $\beta \sim \frac{1}{k}$

$$\frac{dp}{ds} = \frac{\mu}{k} v + (\rho\beta) v^2$$

Fluid ↗ ↑ Rock

Turbulence
Interial
High-Velocity
non-Darcy
"Turbulent"-Like

1–100 μm ↓

$$Re = \frac{\rho v \bar{d}_p}{\mu}$$

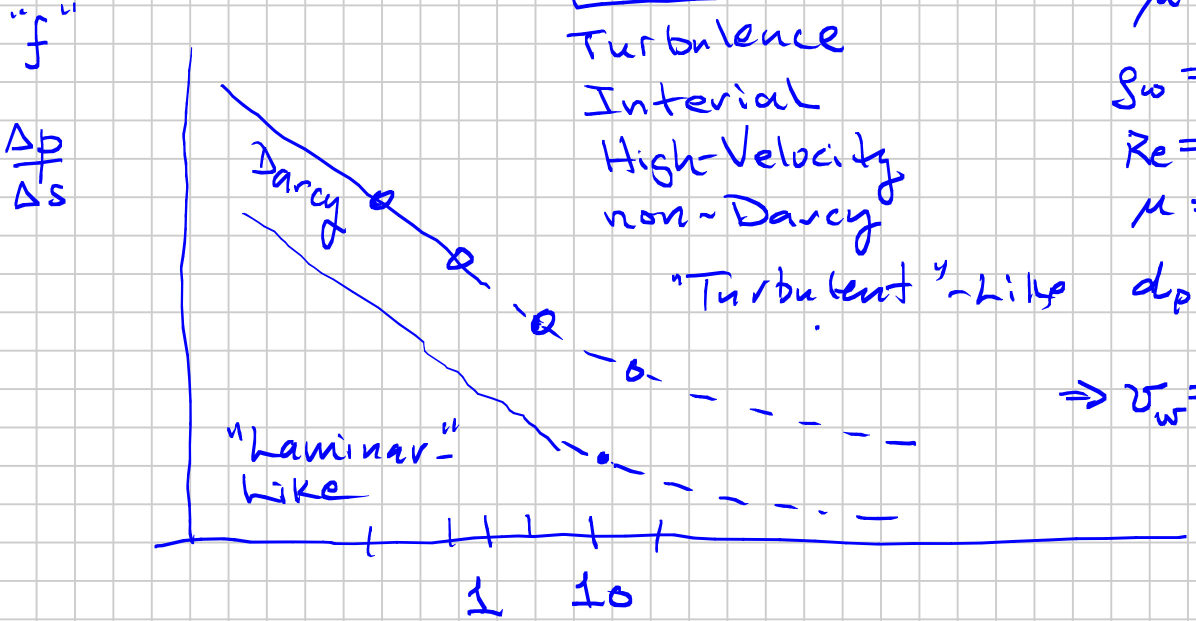
$\rho_w = 1000\ kg/m^3$
$Re = 10$
$\mu = 10^{-3}\ Pa \cdot s$
$d_p = 10 \cdot 10^{-6}\ m$

$$\Rightarrow v_w = \frac{Re\,\mu}{\rho\, d_p} = \frac{10(10^{-3})}{10^3(10^{-5})}$$

$v_w = 1\ m/s$

Gas: $\mu_g \sim 0.01$
$\rho_g \sim 100\ kg/m^3$

$$v_g = \frac{10(10^{-5})}{10^2(10^{-5})} = 0.1\ m/s$$

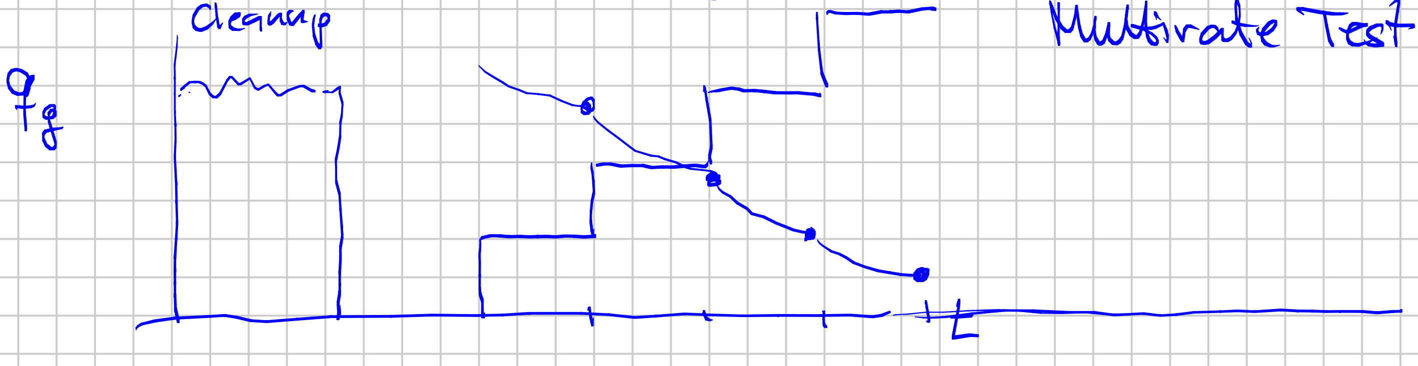


Onset of non-Darcy is "earlier" (more likely) for gas than oil.

1974 Mike Fetkovich
(1972)

Isochronal Testing of OIL WELLS

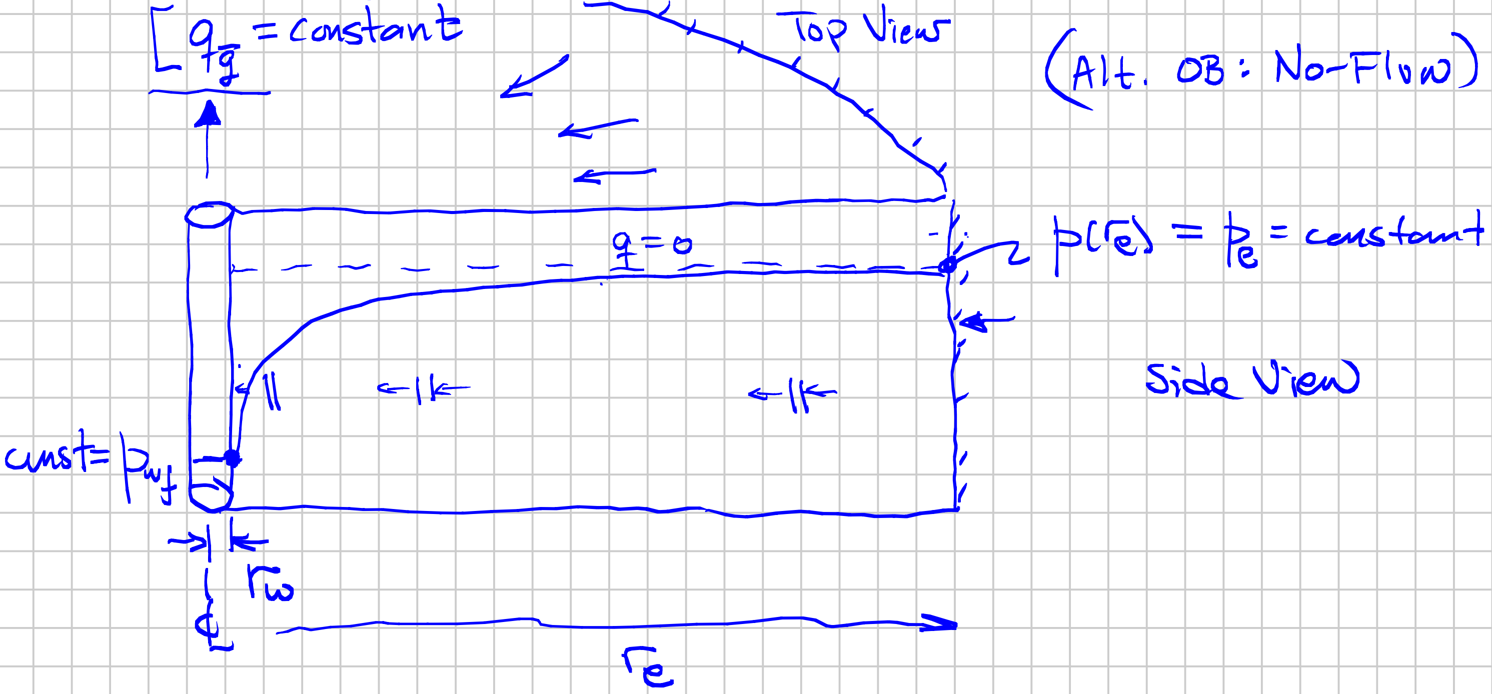
Multirate Test



Gas Rate Equation with Darcy (... extend to non-D)

Vertical Well

Radial-Cylindrical



(Alt. OB: No-Flow)

Steady State Condition: No Accumulation

$$\dot m = \left(\frac{dm}{dt}\right) = \text{constant at } \underline{\text{ALL}} \text{ radii}$$

$\Delta m / \Delta t$

$$\underset{\substack{\uparrow \\ \text{STC} \\ 1\text{ atm} \\ 60°\text{F}}}{q_{\bar g}} = \underset{\substack{\downarrow \\ \text{kg/s}}}{\dot m_g} / \rho_{\bar g}$$

$$\rho_{\bar g} = \frac{p_{sc} M_g}{R T_{sc}}$$

$$\underline{q_{\bar g} \cdot \rho_{\bar g}} = \dot m_g = \overbrace{\underset{\substack{\uparrow \\ \text{Darcy}}}{v_{(r)}} A_{\perp}^{(r)}}^{q(r) = \Delta V/\Delta t} \cdot \rho_g^{(r)} = \underset{\substack{\uparrow \\ \text{Darcy}}}{v} \cdot 2\pi h r \cdot \underline{\frac{p M_g}{R T_R Z_g}}$$

$$q_{\bar{g}} = v \cdot 2\pi h r \cdot \frac{p \cancel{M_g}}{\cancel{R} T_R Z_g} \cdot \frac{\cancel{R} T_{sc}}{p_{sc} \cancel{M_g}}$$

$$= \left(2\pi h \frac{T_{sc}}{T_R p_{sc}}\right) \underset{\substack{\uparrow \\ \text{Darcy's Law}}}{v} \cdot r \cdot \frac{p}{Z_g}$$

$$= \left(\qquad \right) \frac{k}{\mu} \frac{dp}{dr}\, r\, \frac{p}{Z_g}$$

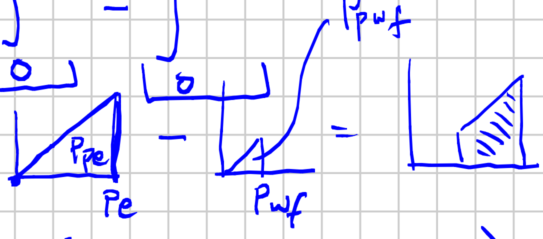
$$q_{\bar{g}} = \left(\frac{2\pi kh}{T_R(p_{sc}/T_{sc})}\right)\left(\frac{r}{dr}\right) \frac{p}{\mu_g Z_g}\, dp \quad = \quad \text{constant}$$

$$q_{\bar{g}} \cdot \int_{r_w}^{r_e} \frac{1}{r}\, dr = \left(\quad \right) \int_{p_{wf}}^{p_e} \frac{p}{\mu_g Z_g}\, dp$$

$$q_{\bar{g}} \cdot \ln r_e/r_w = \left(\quad \right) \int_{p_{wf}}^{p_e} \frac{p}{\mu_g Z_g}\, dp$$

$$\Rightarrow q_{\bar{g}} = \underbrace{\frac{\overset{\downarrow}{2\pi kh}}{T_R(p_{sc}/T_{sc})\left[\ln(r_e/r_w)\right]}}_{\text{Constant: "PI"}} \underbrace{\left[\int_{p_{wf}}^{p_e} \frac{p}{\mu_g Z_g}\, dp\right]}_{\int_0^{p_e} - \int_0^{p_{wf}}}$$

$$p_p = \int_0^p \frac{p}{\mu Z}\, dp$$



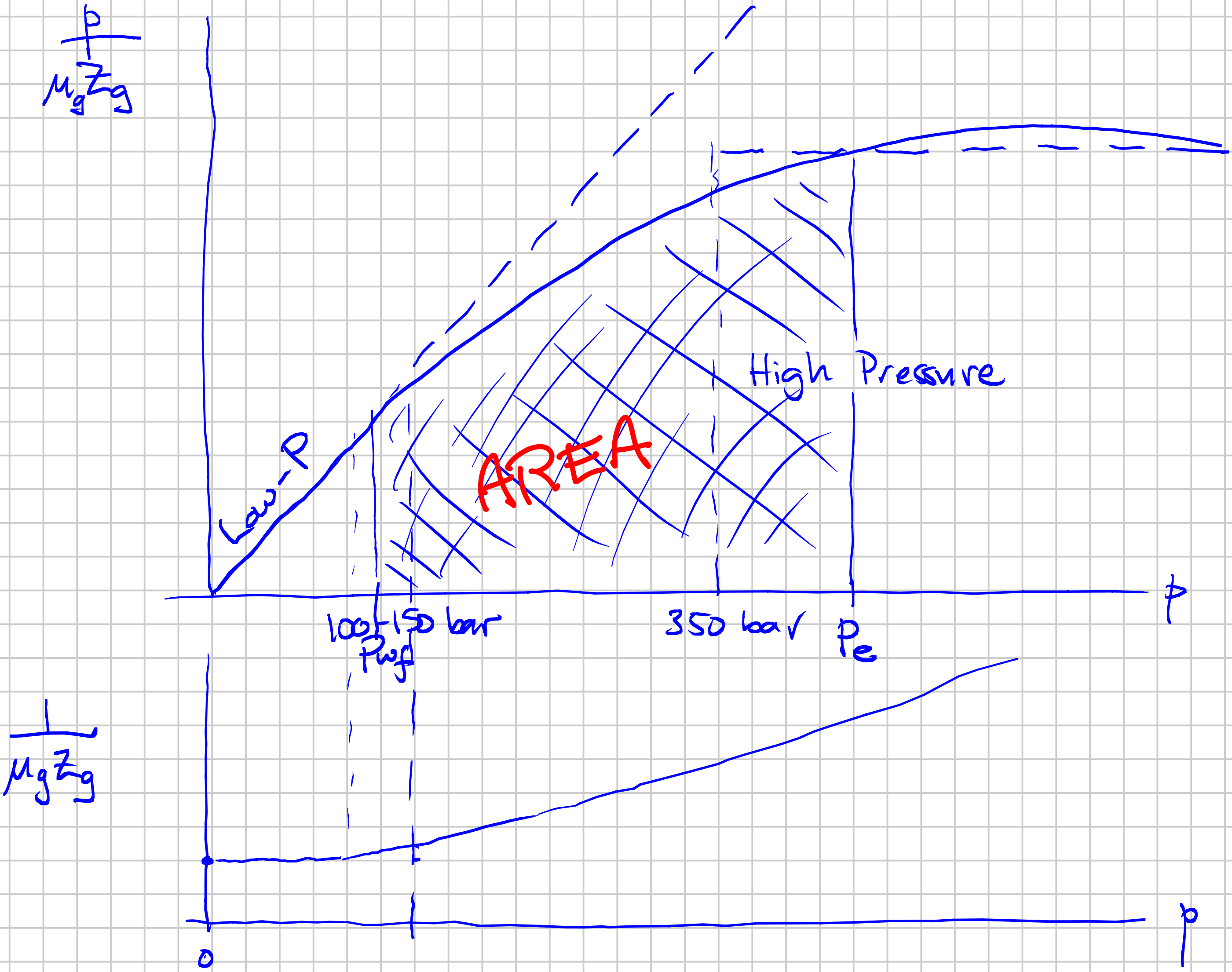
Constant: "PI"

["kh"]

["Skin"]

(Damage – those damn drilling engineers)

(Stimulation – thanks completion engineers)

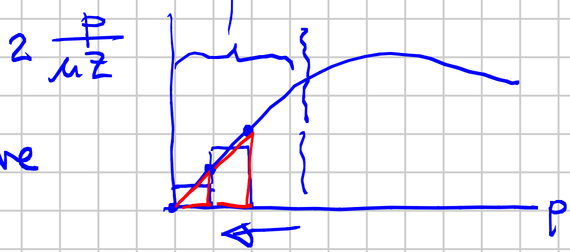


$$q_g = PI \cdot AREA$$

$p_p = (\ )p^2$

TAM

197x : Al-Hussainy, Ramey, Crawford @ U. Texas ?



Gas Pseudopressure : $m(p)$ Before

p_p Now

$\underline{p}$ $\underline{\mu}$ $\underline{Z}$ $\underline{p/\mu Z}$ → $\boxed{p_p}$

$$m(p) = \underline{\underline{p_p}} \equiv 2 \cdot \int_0^p \frac{p}{\mu Z}\,dp$$

0

$A_1 + A_2 + A_3$
$A_1 + A_2$
A_1

↓

Calculate ONCE, use $\boxed{p_p}$ instead of p in all Flow Eqs.

Turns gas into a "slightly incompressible" liquid

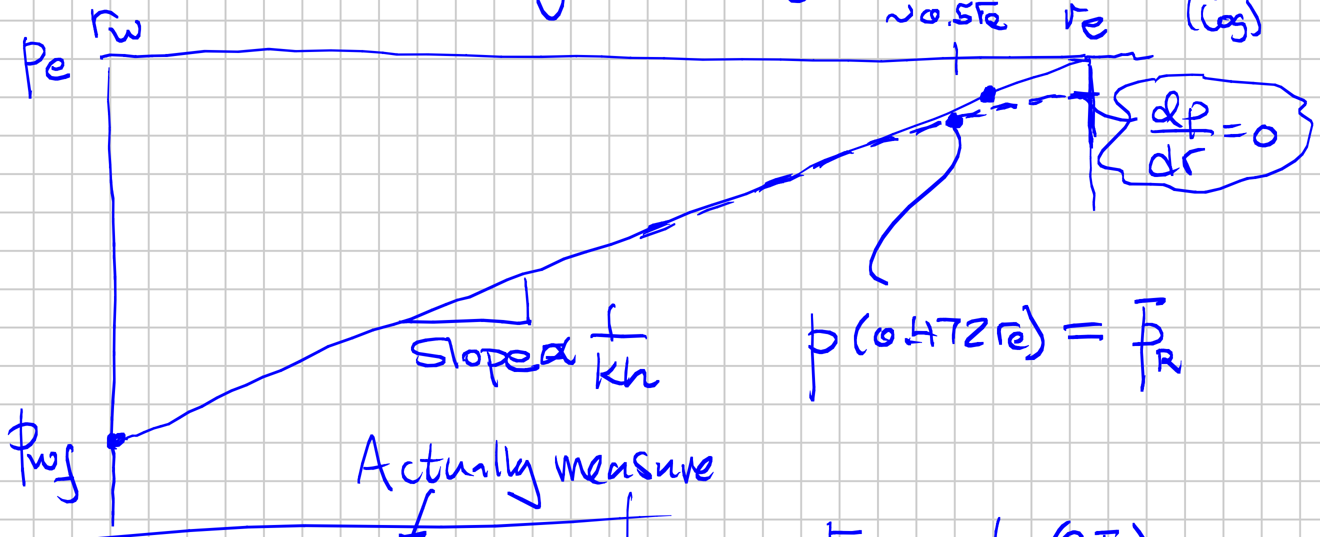
$$\ln\frac{0.6r_e}{r_w} = \ln\frac{r_e}{r_w} - \frac{1}{2}$$

$\boxed{\text{Const-pressure O.B.}}$ Steady State Gas Darcy

$$\Rightarrow \quad q_{\bar g} = \frac{\pi k h (\overset{\overset{p_{PR}}{\downarrow}}{p_{pe}} - p_{pwf})}{T_R (p_{sc}/T_{sc}) \ln \underbrace{\boxed{r_e}}_{0.6r_e}/r_w}$$

$$\Rightarrow \quad p \propto \ln r$$

No-Flow Outer Boundary $\left(\frac{dp}{dr}\right)_{r_e} = 0 \Rightarrow$



$$p(0.472r_e) = \bar p_R$$

Actually measure

$$\boxed{q_{\bar g} = \frac{\pi k h (\overset{\downarrow}{\bar p_{PR}} - p_{wf})}{T_R (p_{sc}/T_{sc}) \ln \frac{0.47r_e}{r_w}}}$$

$$\bar p_{PR} = p_p(@\bar p_R)$$

$\uparrow$ Longterm Shut-In

$$\ln\frac{0.47r_e}{r_w} = \left[\ln\frac{r_e}{r_w} - 0.75\right]$$

$$-\frac{3}{4}$$

Forchheimer Flow Equation

$$q_{\bar g} = \frac{\pi k h (\bar p_{PR} - p_{pwf})}{T_R (p_{sc}/T_{sc}) \left[\ln\frac{r_e}{r_w} - \frac{3}{4} + \underbrace{D q_{\bar g}} + \underset{\overset{\uparrow}{\text{skin}}}{s}\right]}$$

$s>0$: damage
$s<0$ stimul.

$D \propto \beta$

$$\underset{\substack{\uparrow\\ kh,\,D}}{B\, q_{fg}^2} + \underset{\substack{\uparrow\\ kh,\,s}}{A\, q_{fg}} - (p_{pR} - p_{pwf}) = 0$$

Low-Pressure:

$$p/\mu Z = 2\left(\frac{1}{\mu Z}\right)^{\overset{1\text{ atm}}{\circ}} \cdot p = \frac{2}{\mu_g^\circ} \cdot p$$

$$p_p = \frac{1}{\mu_g^\circ} p^2$$

$$q_g = \frac{\pi k h}{T_R \left[\ln r_e/r_w\right]\left(\frac{p_{sc}}{T_{sc}}\right)} \cdot \underbrace{2\int_{p_{wf}}^{p_e} \frac{p}{\mu Z}\, dp}_{\frac{1}{\mu_g^\circ}(p_e^2 - p_{wf}^2)}$$

$$q_{\bar g} = \frac{\pi k h (p_e^2 - p_{wf}^2)}{T_R \mu_g^\circ \left[\ln r_e/r_w\right]}$$

Low-Pressure Steady State Rate Eq.

$$q_{\bar g} = \frac{\pi k h (\bar{p}_R^2 - p_{wf}^2)}{T_R \mu_g^\circ \left[\ln \frac{r_e}{r_w} - 3/4\right]\left(\frac{p_{sc}}{T_{sc}}\right)}$$

No-Flow O.B. @ r_e

Add Skin (s): damage (s>0) or stimulation (s<0)
& non-Darcy "$D q_g$"

$\underbrace{S > +5 \rightarrow +50}$ $\underbrace{S \sim -4}$

$$q_{\bar g} = \frac{\pi k h (\bar{p}_R^2 - p_{wf}^2)}{T_R \mu_g^\circ \left[\ln \frac{r_e}{r_w} - \frac{3}{4} + \underbrace{(s + D q_{fg})}\right]\left(\frac{p_{sc}}{T_{sc}}\right)}; \quad \bar{p}_R \lesssim 100\text{–}150 \text{ bar}$$

$$q_{\bar{g}} = \frac{\pi k h \left(F_{PR} - P_{pwf} \right)}{T_R \left[\underbrace{\ln \frac{r_e}{r_w} - \frac{3}{4}}_{8} + \underbrace{s + D q_g} \right] \left(\frac{P_{sc}}{T_{sc}} \right)}$$

$$\frac{p}{\mu_g Z_g} = \frac{1}{\mu_g B_g} T_R (P_{sc} / T_{sc}) \qquad B_g = \frac{P_{sc}}{T_{sc}} \frac{T_R Z}{p}$$

$$\int \frac{p}{\mu Z} = T_R (P_{sc} / T_{sc}) \int \frac{1}{\mu_g B_g}$$

$\ln \frac{r_e}{r_w}$

(1) Pseudo Steady State Assumption

□ @ Any r

$t = 0 \quad p_{wf} = P_{Ri} \Rightarrow$ SS Flow $\dot{m}_{in} = \dot{m}_{out}$

takes time

$$t_{pss} \propto \left(\frac{\overset{\checkmark}{r_e^2}}{\underset{\checkmark}{k}} \right)$$

$0 < t < t_{pss}$

$$\ln \frac{r_e}{r_w} \rightarrow p_D(t_D)$$

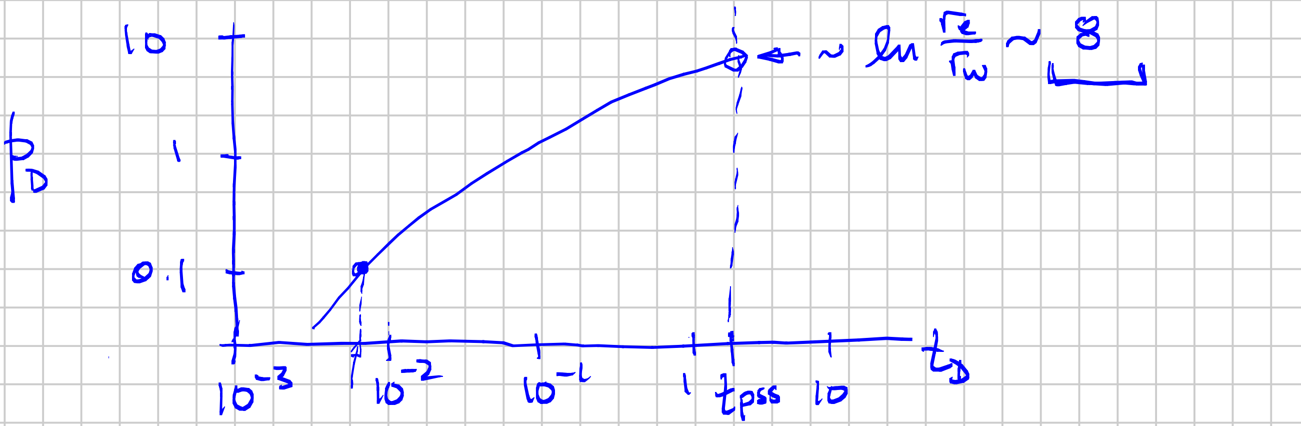
$$\nearrow$$

$$(P_{Ri} - P_{wf}(t))$$

Pressure Transient Analysis
(PTA)

Well Testing

(G-10)



Fixed

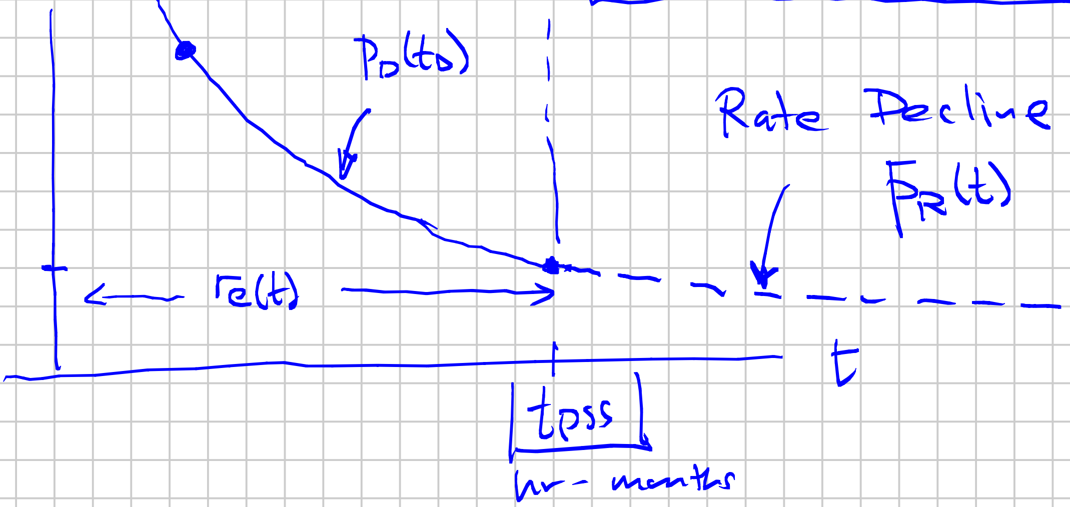
$$q_g = \frac{\pi k h \left(P_{PRi} - \overset{\downarrow}{P_{pwf}}\right)}{\underbrace{\left(\frac{p_{sc}}{T_{sc}}\right) T_R \cdot \left[p_D(t) \right]}}$$

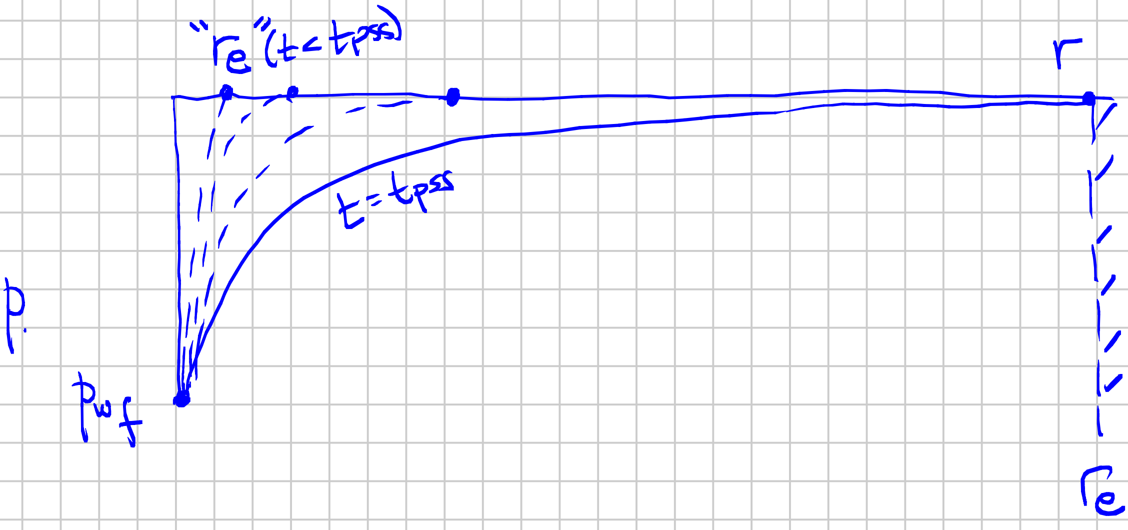
Fix

$$q_g = \frac{\pi k h \left(\bar{P}_{PR}(t) - \overset{\downarrow}{P_{pwf}}\right)}{\underbrace{\left(\frac{p_{sc}}{T_{sc}}\right) T_R \left[\ln \frac{r_e}{r_w} \right]}}$$

$$p_D(t) = \ln \frac{r_e(t)}{r_w}$$

PSS : q_g





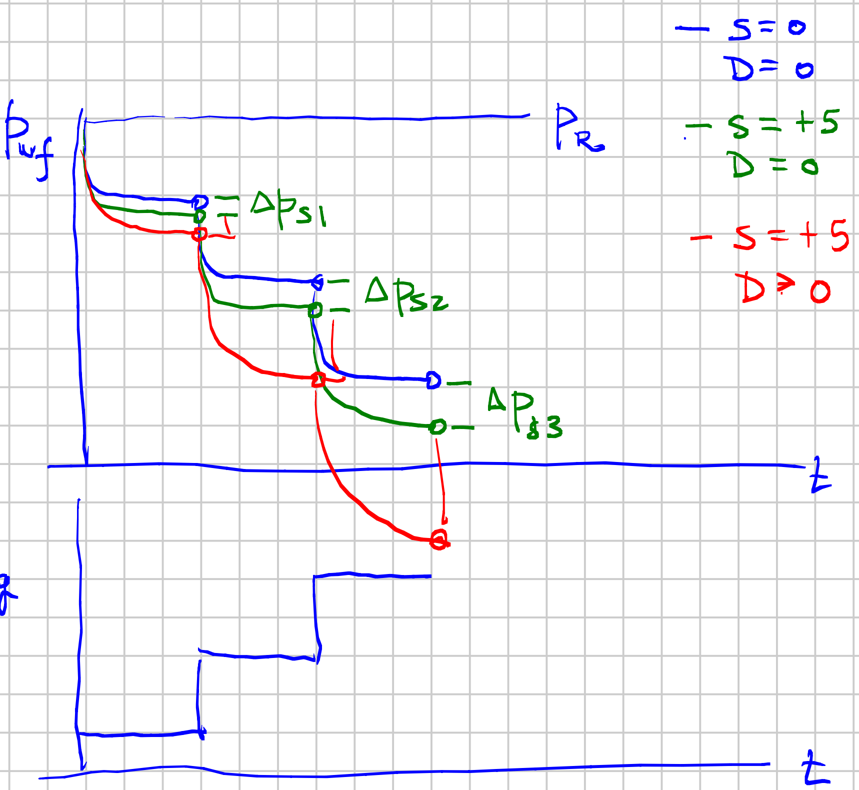
$$+\beta_g v^2$$

$$(2) \quad \left[\underbrace{(s + D q_g)} \right]$$

Rate-Dependent Skin

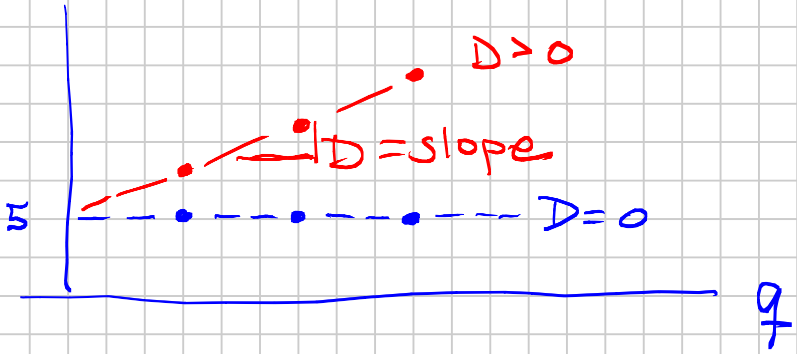
PTA : $\underbrace{\Delta p, q}$

Calc "Skin"

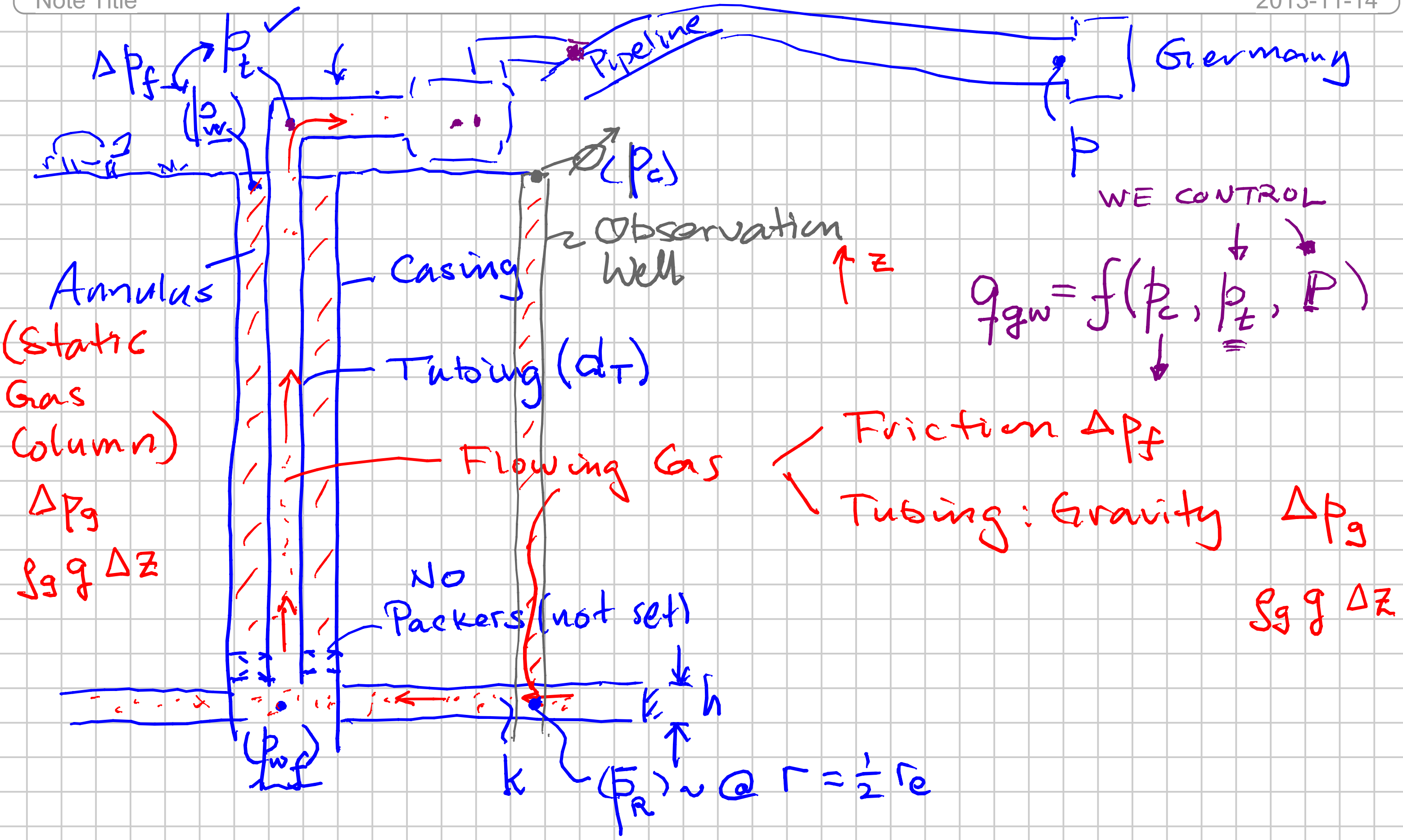


$\Delta p_s = S \cdot q$

$\text{``}S\text{''} \sim \frac{\Delta p}{q}$



Fetkovich Multi-Point Testing of Gas Wells



$$q_{gw} = f(p_c, p_t, P)$$

p_{wf} = flowing bottomhole (wellbore) pressure

p_t = flowing tubinghead pressure

p_w = flowing pressure at annulus "wellhead"

$(\bar{p}_R) \sim p_c$ = "wellhead" (long-time, stabilized) shut-in press.
= volumetric average reservoir pressure expressed @ surface (with a static gas column)

= $p_w = p_t$ during a shut-in ($q_g = 0$)

M.B. $\quad \overset{\checkmark}{p_{Ri}} \rightarrow \overset{\checkmark}{p_R^{(t)}}$

$\quad G_p^{(t)}$ ✓

$$\frac{\overset{\checkmark}{p_R}}{Z_g} = \frac{\overset{\checkmark}{p_{Ri}}}{Z_{gi}}\left(1 - \frac{\overset{\checkmark}{G_p}}{G}\right)$$

? ±

Bottomhole-to-Surface-to-BH Pressure Conversion for a $\underbrace{\text{static gas column}}$

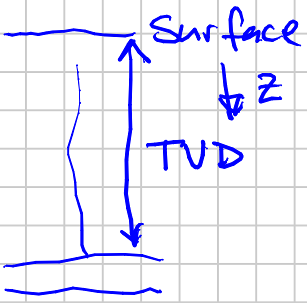
$$\rho_g g = \frac{dp}{dz}$$

z : true vertical directi

$$\rho_g = \left(\frac{M}{TR}\right)\frac{p}{z} \simeq \left(\frac{M}{\bar{T} R \bar{Z}_g}\right) p$$

Assume • $T(z) \sim$ constant or that $\bar{T}$ is ok.

• $Z_g(z) \sim \bar{Z}_g$



$$\frac{dp}{dz} = g \cdot \left(\frac{M}{\bar{T} R \bar{Z}_g}\right) p$$

$$\int_{P_{BH}}^{P_s} \frac{1}{p}\, dp = \frac{Mg}{R \bar{T} \bar{Z}_g} \int_{TVD}^{0} dz$$

$$\ln \frac{P_{BH}}{P_s} = \frac{Mg}{R \bar{T} \bar{Z}_g} \cdot TVD$$

$$\left(\frac{P_{BH}}{P_s}\right) = \text{constant} = \exp\left(\underbrace{\frac{Mg\,TVD}{R \bar{T} \bar{Z}_g}}_{(S/2)}\right)$$

$$\frac{P_{BH}^2}{P_s^2} = \exp\left(\underbrace{\frac{2 \cdot \overset{\checkmark}{M} g\, \overset{\checkmark}{TVD}}{R \underset{\checkmark}{\bar{T}}\, \underset{\checkmark}{\bar{Z}_g}}}_{\text{"}S\text{"}}\right)$$

"S" "static gas column"

BH pressures →

Surface → "equivalent"

$$\frac{p_R}{p_c} \simeq \frac{p_{wf}}{p_w} = e^{S/2} \cong \text{constant}$$

TVD (ft)   T (°R) $\bar{Z}_g$

$$S = \underbrace{0.0375}_{\frac{2}{R} \times \text{units}}\ GH/T_a\ Z_a$$

$$\gamma_g = \frac{M_g}{M_{air}}$$

Turbulent Gas Through Pipe

- Vertical (with "gravity" removed)
- Horizontal
- Tubing
- Pipeline

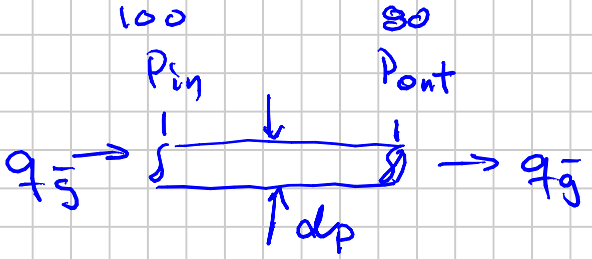
$$\rho_g \propto p$$

$$q_g = C_p\,(p_{in}^2 - p_{out}^2)^{0.5}$$

1st order "right"

Surface Gas Volume

$\dot{m}_g$



$$C_p \propto d_p^{2.6}$$

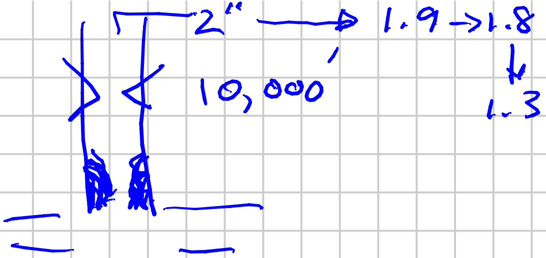
"Tubing Flow: Rate Equation"

$$q_g = C_T \cdot \overbrace{(p_w^2 - p_t^2)}^{\text{Due to } \Delta p_f}{}^{0.5}$$

M.i.F: "C_T"

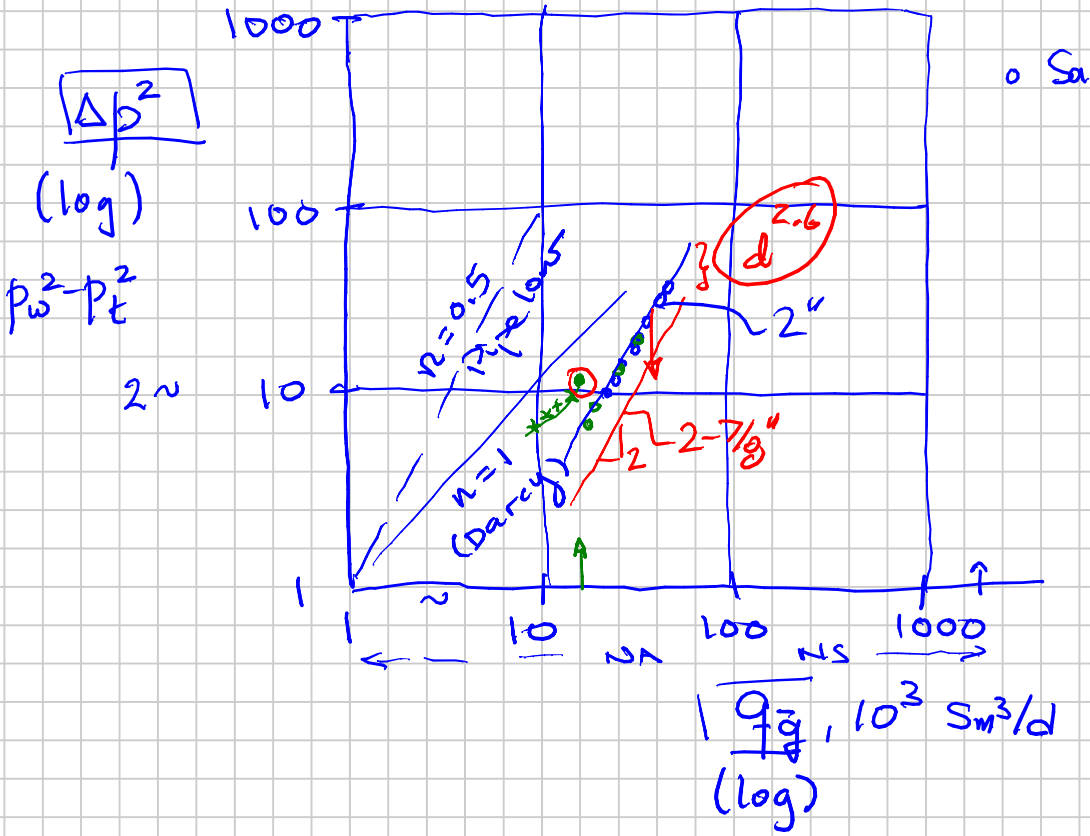
p_{wf} with 'gravity' removed

$$C_T \propto \text{"}\underbrace{d_T}\text{"}^{2.6}$$



Graphical Representation of Rate Equation

BACKPRESSURE PLOT



o Sally Joe #3

$= 1/n$

Slope $= 1 = 1/$exponent

$$q_g = C(\Delta p^2)^1$$

$$q_g = C_T(\Delta p^2)^{0.5}$$

$$\text{"}\Delta p^2\text{"} = (p_w^2 - p_t^2)$$

in

$$\cancel{\Delta p^2 \neq (p_w - p_t)^2}$$

t	q_g ✓	p_w ✓	p_t ✓	p_c
				✓
				✓
				✓

$\underbrace{q_g \quad p_w \quad p_t}_{\text{Tubing}}$

Tubing

Performance

Annual Shut In

$(p_c^2 - p_w^2)$?

Reservoir Rate Eq. ("low-p" form)

$$q_g = C_R(p_R^2 - p_{wf}^2) \qquad \text{Darcy}$$

$$\rightarrow \underbrace{p_R^2 - p_{wf}^2}_{\text{Reservoir Pressure Drop}} = \overset{\text{Darcy}}{A_R\, q_g} + \overset{\text{non-Darcy}}{B_R\, q_g^2}$$

Forchheimer

$$\underbrace{\frac{T_R\, \mu Z}{\pi\, kh}\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s\right]}_{\frac{1}{C_R}} \qquad D\,\frac{T_R\, \mu Z}{\pi\, kh}$$

Convert BH pressures in the Reservoir rate equation into surface-equivalent p's

$$\frac{p_{BH}^2}{p_s^2} = \exp(S) = e^S = \text{const}$$

$$\begin{cases} p_R^2 = p_c^2 \cdot e^S \\ p_{wf}^2 = p_w^2 \cdot e^S \end{cases}$$

$$(p_c^2 - p_w^2) = \underbrace{e^{-S} A_R}\, q_g + \underbrace{e^{-S} B_R}\, q_g^2$$

$$\rightarrow \underset{\uparrow}{p_c}^2 - \underset{\uparrow}{p_w}^2 = A_R'\, q_g + B_R'\, q_g^2 \qquad Ⓡ$$

surface-datum

$$(p_w^2 - p_t^2) = \left(\frac{1}{C_T}\right)^2 q_g^2$$

$$q_g = C_T\left(p_w^2 - p_t^2\right)^{1/2} \qquad Ⓣ$$

RHS

$$\boxed{p_c^2 - p_t^2 = A_R'\, q_g + \underbrace{\left(B_R' + \frac{1}{C_T^2}\right)}\, q_g^2}$$

↑ "Starting" pressure in R

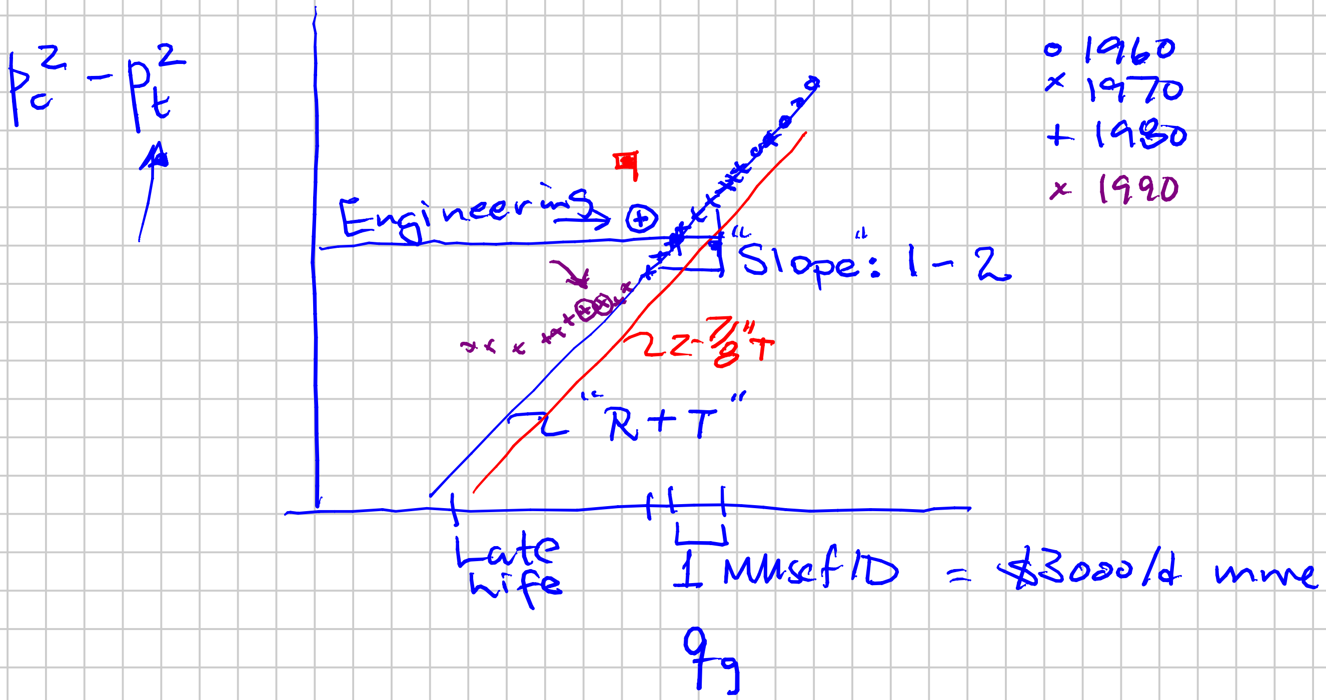
↑ "Sales" (End) pressure @ Surface

↑ Non-Darcy & Tubing Friction

"β" (D)

Wellhead (Gas) Deliverability Equation

One variable (only) changing with time: $p_c(t)$



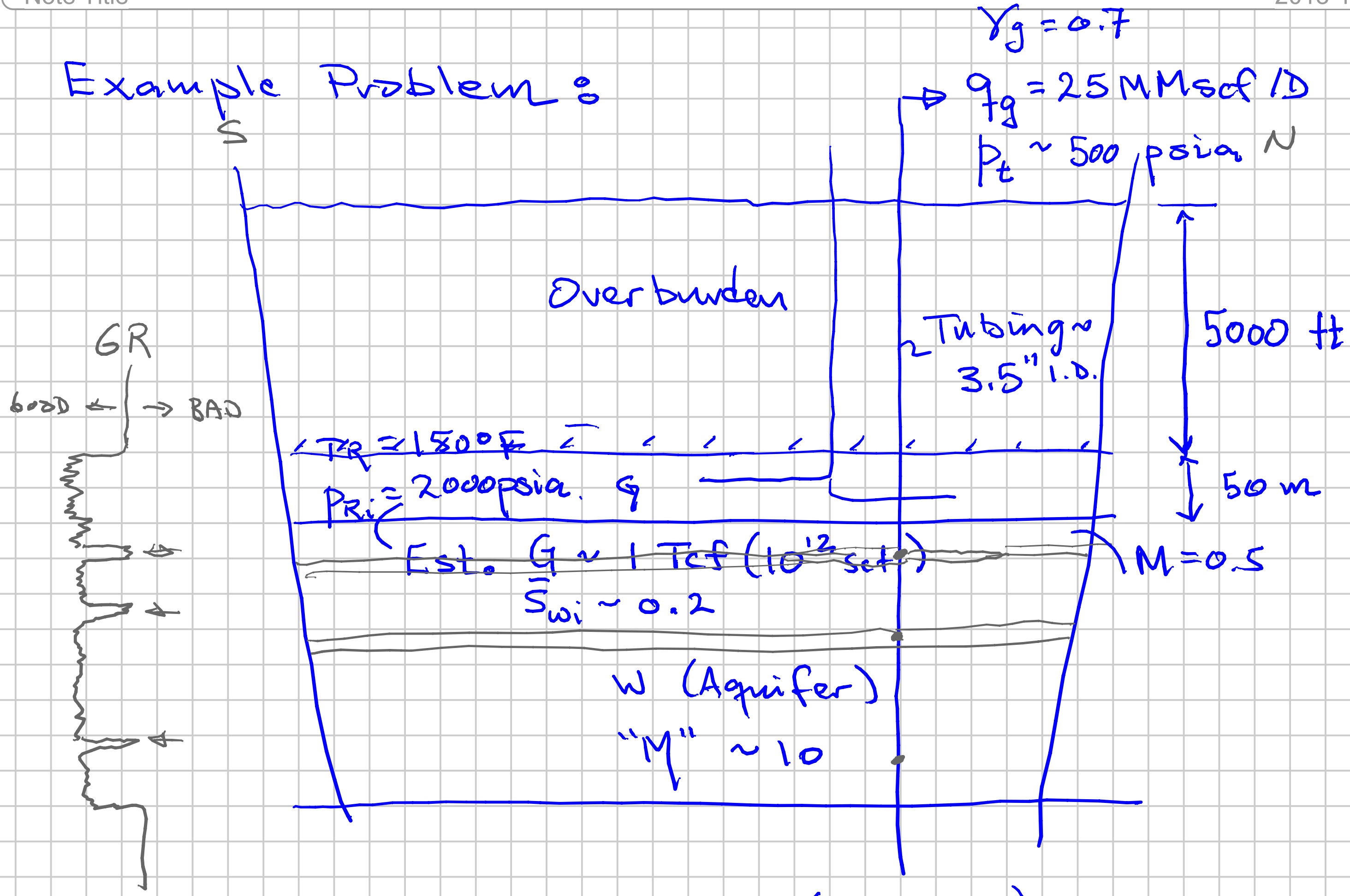
$n \sim 1$: "R" main restriction to flow

⇒ $n \sim 0.5$: "T" main restriction

Combine Material Balance & Rate Equations

Note Title 2013-11-15

Example Problem:



Assumptions: $c_w \sim 3.5\cdot10^{-6}$ psi^{-1}
$c_f \sim 7.0\cdot10^{-6}$ psi^{-1} } M.B.

Production Test: $C_R \sim 20$ scf/D/psia^2 | PTA:
k ~ 20 md
s ≳ "+10"
$p_{wf}(t)$

$$q_g = C_R\left(p_c^2 - p_t^2\right)$$

↑ surface datum

$$q_g = C_T\left(p_w^2 - p_t^2\right)^{0.5}$$

$C_T \sim 25000$ scf/D/psia

Measured: q_g P_t P_c $P_{wf} \rightarrow P_w$

Market ✓

Buyer : q_{gF} ~ constant $\underline{DCQ}$ $q_{gF} = \frac{0.055 \cdot 10^{12}}{365}$ $\frac{scf}{D}$

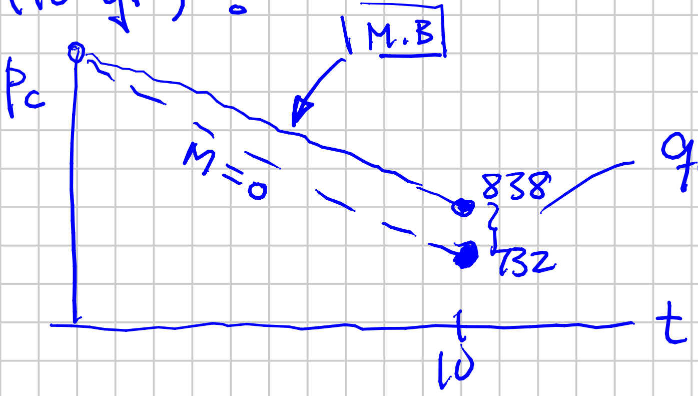
$t_{plateau}$

5.5% IGIP/year , 10 years

$p_{t\,min} = p_{\substack{pipeline\\inlet}}$ ~ 500 psia

N_w (today) ? $\cong$ 6 $\underbrace{(d_T \uparrow ,\ S \downarrow)}$ $\longrightarrow$ N_w' (today)

N_w (10 yr) ?



Wellhead Backpressure

q_{gw} : ?

$$\boxed{B q_{gw}^2 + A q_{gw} - \underbrace{(p_c^2 - p_t^2)} = 0}$$

$A = \frac{1}{C_R}$ ✓

$B = \frac{1}{C_T^2}$ ✓

p_t = 500 psia ✓

p_c = ?

Mat. Bal. Eq.

$$\boxed{\frac{p_R}{Z_R}\left(\underbrace{1 - c_e(p_{Ri} - p_R)}\right) = \frac{p_{Ri}}{Z_{Ri}}\left(1 - \frac{G_p}{G}\right)}$$

'Class' simplification $Z_R \sim 1$

Ignore "c_e" effect

$$\frac{p_R}{e^{S/2}} \simeq \frac{p_{Ri}}{e^{S/2}}\left(1 - \frac{G_p}{G}\right)$$

$$\boxed{p_c \simeq p_{ci}\left(1 - \frac{G_p}{G}\right)}$$

"Back of the envelope"

$e^{S/2} \sim 1.23$

$\frac{G_p}{G}$? = 10 yr × 0.055/yr = 0.55

$p_c \sim \frac{2000}{1.23}(1 - 0.55) = \underline{732}$ psia

$$\Rightarrow q_g(10\,yr) \sim 5\cdot 10^6\ scf/D = \frac{1}{5}\,q_g(today)$$

$$\Rightarrow N_w = 5\times 6 = 30 \text{ wells}$$

$$P_c = \frac{732}{\left(1 - c_e\left(\frac{2000}{1.23} - 732\right)\right)} = \frac{732}{0.874} = \underline{838 \text{ psia}}$$

$$N_w = \frac{0.055\cdot 10^{12}}{365\,(7.3\cdot 10^6)} = 21$$

$$\left.\begin{array}{lll} d_T & \text{change} & 3.5'' \rightarrow 6''\,\text{I.D.}\,(7''\,\text{O.D.}) \\ s & \text{change} & +10 \rightarrow 0 \rightarrow -4 \\ & & \quad\$ \end{array}\right\} N_w$$

C_R

$$(C_T)_{New} = (C_T)_{old} \times \frac{(d_T)^{2.6}_{New}}{(d_T)^{2.6}_{old}} = 25000\left(\frac{6}{3.5}\right)^{2.6}$$

$$= 100\,000$$

$$(C_R)_{new} = \frac{kh\,(T_{sc}/p_{sc})}{T_R(\mu Z)\left[\underbrace{\ln\frac{r_e}{r_w} - \frac{3}{4}}_{8} + \underset{\overset{\uparrow}{+10 \rightarrow 0 \rightarrow -4}}{s}\right]}$$

$$r_e = \left(\frac{A_{field}}{N_w}\right)^{1/2}$$

$$(C_R)_{new} = (C_R)_{old} \times \frac{[8+10]_{old}}{[8+s]_{new}}$$

$$= 20 \times \frac{18}{8 + S_{new}}$$

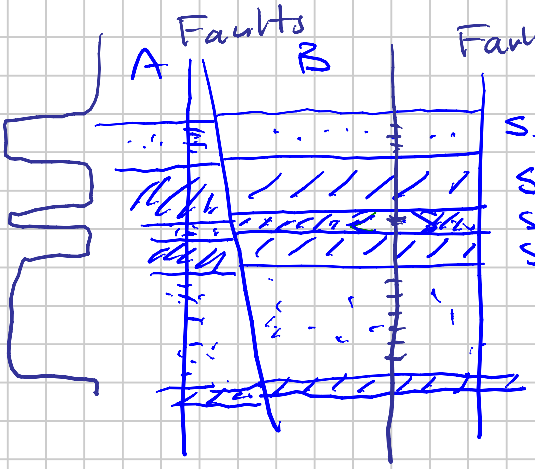
Note Title 2013-11-22

XXX

Thasher, Thomas COP SPE paper ~ 2008

Brittania Field $P_{Ri} = 2000$

$P_{wf} = 500$ psia for 1 year



	@ 1year		C_R
SS_1	1700	30'	3
Shale			
SS_2	500	10'	
Shale			
SS_3	1900	60'	6
Shale			
		100'	10

Fetkovich 5-10

Layered No-Crossflow (LNX)

No flow between reservoirs layers (units) except in the wellbore (usually during shutin)

SS2 depletes faster:

$$\left\{\begin{array}{ll} (1) & k_2 > k_1 > k_3 \\ (2) & s_2 < s_1 < s_3 \end{array}\right\}$$

$$\underbrace{\left\{\begin{array}{l} s_1 = s_2 = s_3 \\ k_1 = k_2 = k_3 \end{array}\right\}}$$

Same areal extent all layers $A_1 = A_2 = A_3$

$\Rightarrow \underbrace{P_{R1} = P_{R2} = P_{R3}}$

(3) $A_2 < A_1 < A_3$

Uniform Pressure Depletion Regimes

$$\boxed{\left(\frac{C_{R2}}{V_{g2}}\right) > \left(\frac{C_{R1}}{V_{g1}}\right) > \left(\frac{C_{R3}}{V_{g3}}\right)}$$

G_2 G_1 G_3

$$\boxed{\frac{C_{R1}}{G_1} \simeq \frac{C_{R2}}{G_2} \simeq \frac{C_{R3}}{G_3}}$$

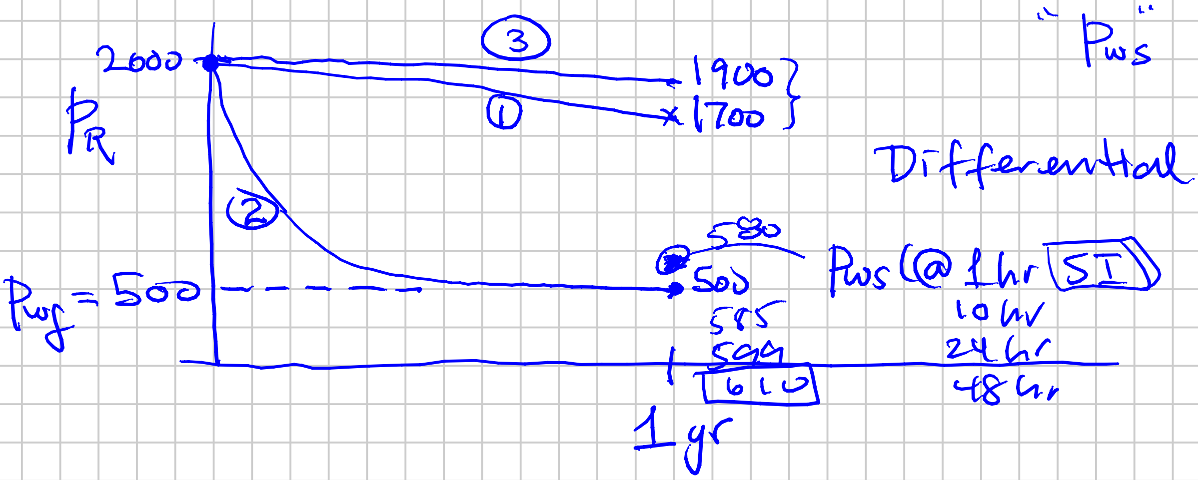
$\left(\frac{C_R}{G}\right)$ = Voidage Ratio :

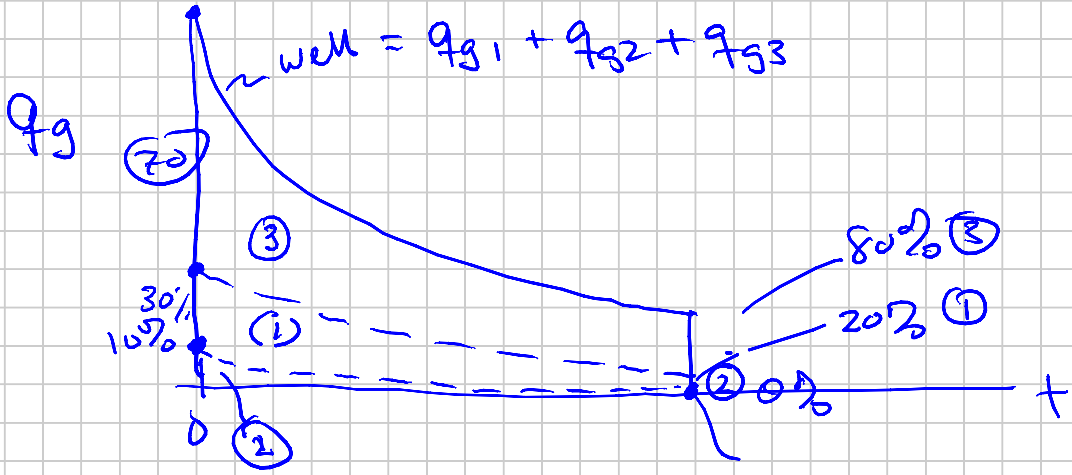
↑ Faster Depletes

↓ Slower Depletes

"P_{ws}"

Differential Depletion





$$q_g = \underline{0} = \left.\begin{array}{l} C_{R1}(p_{R1}^2 - p_{ws}^2) + \\ C_{R2}(p_{R2}^2 - p_{ws}^2) + \\ C_{R3}(p_{R3}^2 - p_{ws}^2) \end{array}\right\} \Rightarrow p_{ws} > p_{layer} \text{ most depleted}$$

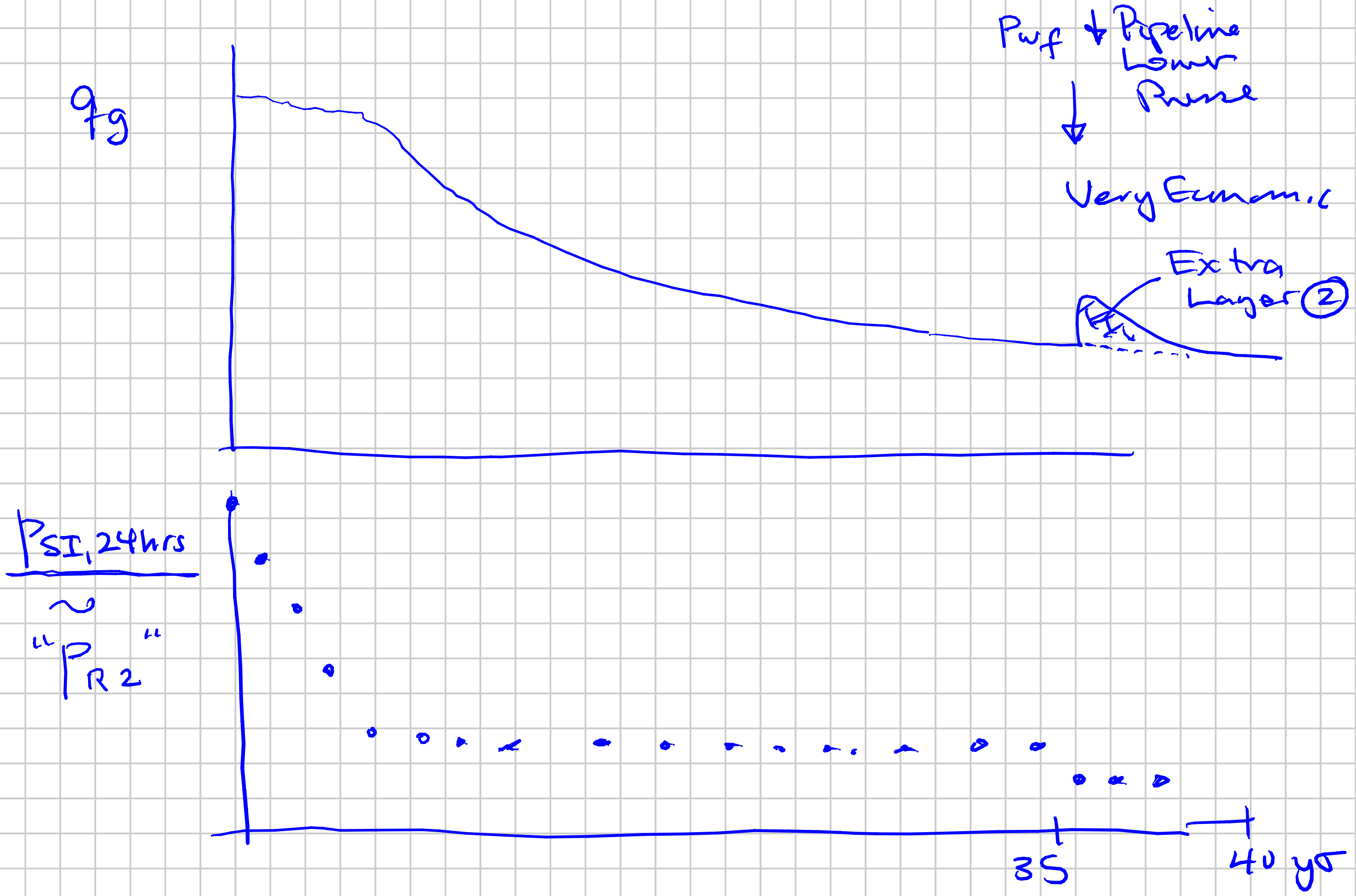
SI Pressures in LNX fields (wells) will <u>often</u> be lower (much lower) than $\boxed{\bar{P}_R}$ of the entire gas volume

$$\uparrow$$

$0.3(1700) + 0.1(500) + 0.6(1900)$

You don't "see" this pressure, ever

$P_{SI} \sim P_{layer}$, most depleted


q_g
Pwf + Pipeline Lower Pressure
Very Economic
Extra Layer ②
P_SI,24hrs
~
"P_R2"
35
40 yr