CURTIS HAYS WHITSON – TPG 4145

Note Title 2010-08-24

(Natural)

GAS PVT (Pressure-Volume-Temperature)

- Density ρ (mass)
- Viscosity μ
- Isothermal Compressibility $c \equiv -\frac{1}{V}\left(\frac{dV}{dp}\right)_{T,n} = \frac{1}{\rho}\left(\frac{d\rho}{dp}\right)_{T}$
- Component Make-up of gas (Amounts – moles or mass)

  $\{CO_2 \quad N_2 \quad H_2S\} \quad C_1 \quad C_2 \quad C_3 \quad iC_4 \quad nC_4 \quad iC_5 \quad nC_5 \quad C_{6s} \ldots C_{n+}$

- $M_i \equiv \frac{m_i}{n_i}$ e.g. $M_{C_1} = 16.04 \frac{g}{mol} = 16.04 \frac{g}{gmol}$

  SI : kg

  SI : "gmol"

  $= 16.04 \frac{kg}{kmol} = \frac{kg}{kgmol}$

"e-notes" Self-Topical Learning Aids

2-page ppt + linked files

REAL GAS LAW (EQUATION OF STATE)

$pV = n \cdot RTZ$

Z : Deviation from ideal gas behavior caused by crowding

IDEAL GAS LAW

$pV = nRT$ $(Z = 1)$ $\frac{V}{n} = \frac{RT_{sc}}{p_{sc}}$

$22.4 = \frac{V}{n}$ $\left[\frac{L}{gmol}\right]$ @ T = 20°C

Girls: Sm³/kgmol

Boys: scf/lbmol

ft³

} @ Standard Conditions

1 atm, 60°F

1.0135 bar, 15.56°C

14.696 psia , 60 °F

$R = 8.314$

m^3 kg·mol K kPa

GIRLS : $P_{sc} = 1.01325$ bar $= 1.0135 \cdot 10^5$ Pa ✓ kg-mol

$T_{sc} = 15.56\,°C + 273.15 = 288.71$ K ✓ m^3

$R = 8314.3$

$$\frac{V_{gsc}}{n} = \frac{V_{\bar{g}}}{n} = \frac{R\,T_{sc}}{P_{sc}} = \frac{8314.3(288.71)}{1.0135 \cdot 10^5} = 23.67\ \frac{Sm^3}{kg\text{-}mol}$$

BOYS: $P_{sc} = 14.696$ psia

$T_{sc} = 60\,°F + 459.67 = 519.67\ °R$ lb-mol ft3

$R = 10.73146$

$$\frac{V_{\bar{g}}}{n} = \frac{10.7315(519.67)}{14.696} = 379.7\ \frac{scf}{lb\text{-}mol}$$

Gas Z-factor

$0.7 \lessapprox Z_g \lessapprox 2$

- Predict / Calculate Z_g within ~ 2-3% knowing gas "makeup"
  M_g y_i (molar composition)
  - Standing-Katz Chart
    - Equation fits the SK Chart (Hall-Yarborough) BWR EOS

- van der Waals – All fluids behave alike in terms of reduced variables

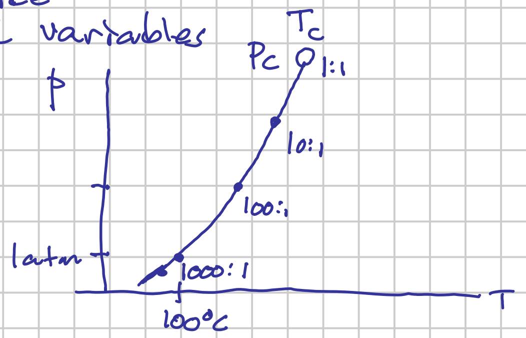
"Theory of Corresponding States" (187x)

$$T_r \equiv \frac{T}{T_c}$$

$$P_r \equiv \frac{P}{P_c}$$

$$Z_{C_1}(T_r = 1.5, P_r = 10)$$

$$= Z_{N_2}(T_r = 1.5, P_r = 10)$$



$Z(T_r, p_r)$: Pure Components & Mixtures

Gas Mixture : $T_{pc} \overset{\sim}{=} \sum y_i T_{ci}$

$$p_{pc} \overset{\sim}{=} \sum y_i p_{ci}$$

$$\boxed{pV = nRTZ_{(p,T,y)}} \qquad \frac{n}{V} = \frac{p}{RTZ}$$

$$\rho_g = \frac{m_g}{V_g} = \frac{n_g M_g}{V_g} = \underline{\boxed{\frac{pM_g}{RTZ_g} = \rho_g}}$$

$$m_g = n_g \times M_g$$

$\mu_g = f(\rho_g)$ ±5% by corelation

Specific Gravity $\gamma_g \equiv \frac{\rho_g}{\rho_{air}} = \frac{M_g}{M_{air}}$

GAS Z-FACTOR & PVT PROPERTIES

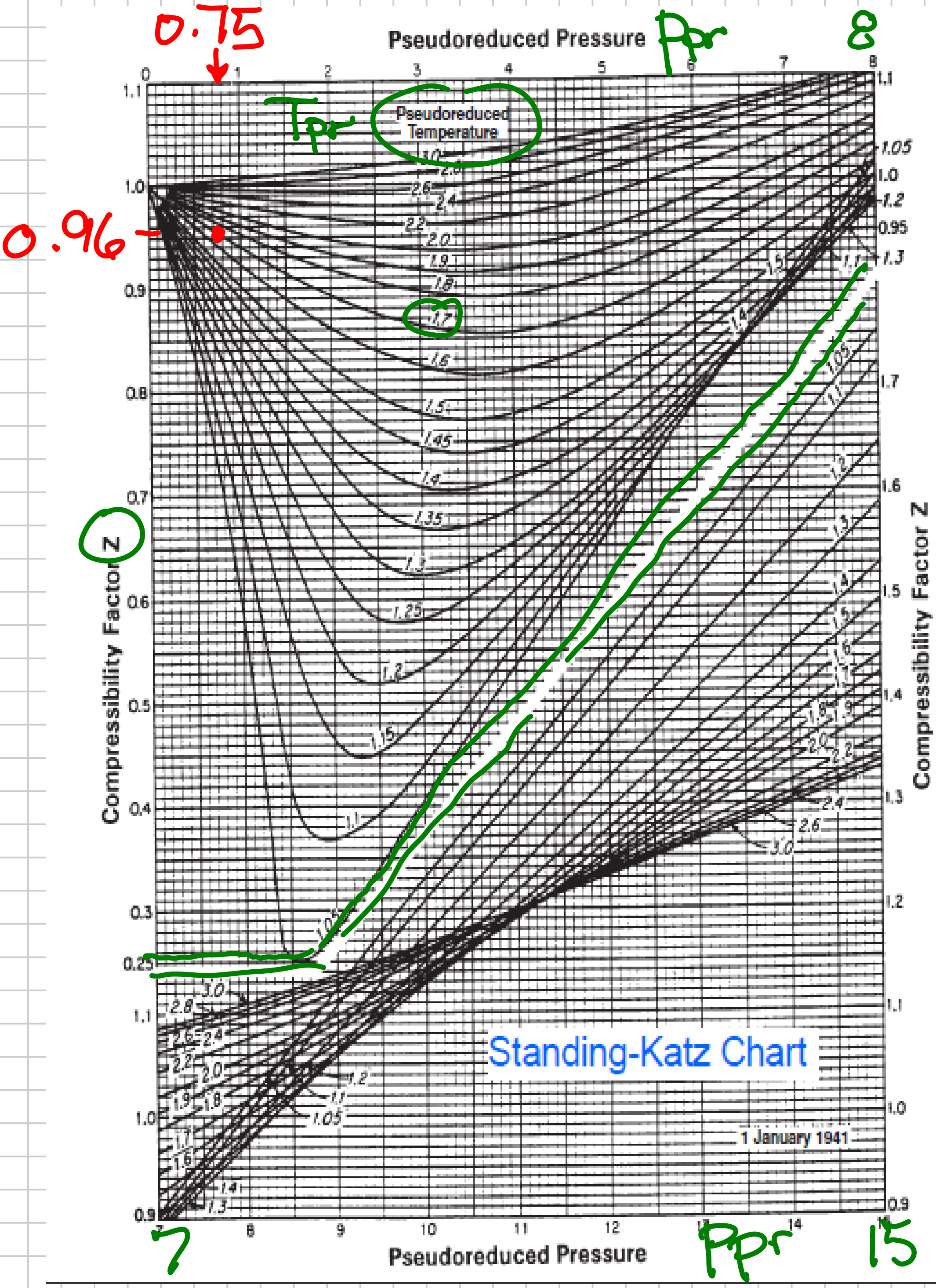
$$pV = nRT\,Z(p, T, y_i)$$

↑ molar compositions of the gas

$$Z(p_r, T_r)$$

$$p_r = p/p_c$$

$$T_r = T/T_c$$ [absolute units K, °R)



$$T_K = T_C + 273.15$$

$$T_R = T_F + 459.67$$

$$T_F = 1.8T_C + 32$$

$$T_C = (T_F - 32)\frac{5}{9}$$

Gas Mixtures

T_{pc} p_{pc}

$$\boxed{\begin{aligned} T_{pc} &= \sum y_i T_{ci} \\ p_{pc} &= \sum y_i p_{ci} \end{aligned}}$$

Estimate EQ.

$$\left.\begin{aligned} &p_{cC_{7+}} \\ &T_{cC_{7+}} \end{aligned}\right\} \begin{aligned} &M_{7+} \\ &\gamma_{7+} \end{aligned}$$

@ s.c.

Liquid $\gamma_{7+} \equiv \dfrac{(\rho_{7+})_{s.c.}}{(\rho_w)_{s.c.}}$

Troll C_{7+} $M_{7+} \sim 115$ $(C_7\ C_8\ C_9)$

Smørbukk C_{7+} $M_{7+} \sim 190$ $(C_7 \ldots C_{40} \ldots C_{100})$

Calculating $Z_g(T_R, p_R)$ given Gas (T_{pc}, p_{pc})

T_R: Reservoir 30°C — 300°C

p_R: 20 – 1200 bar

$$0.7 < Z_g < 2$$

Example: 92 mol-% C_1

8 mol % N_2

$$\left.\begin{array}{l} T_{pc} = 185.5\ \text{K} \\ T_R = 311.15\ \text{K} \end{array}\right\} T_{pr} = 1.678$$

Noblin Well

Atoka formation

$$\left.\begin{array}{l} p_{pc} = 45.08\ \text{bar} \\ p_{Ri} = 500\ \text{psia} \times \dfrac{\text{bara}}{14.50377\ \text{psia}} \\ \quad = 34.5\ \text{bara} \end{array}\right\} p_{pr} = 0.76$$

psi = pounds per sq. inch

a : absolute

g : gauge = absolute − ambient

(s.c.)

What is Z_{gi} ?

$T_{pr} = 1.7$

$P_{pr} = 0.75$

Why do I care ?

- Initial Gas In Place "G" (IGIP) (OGIP) (GIIP)

↑ Sellable "Surface" @ Standard Condition

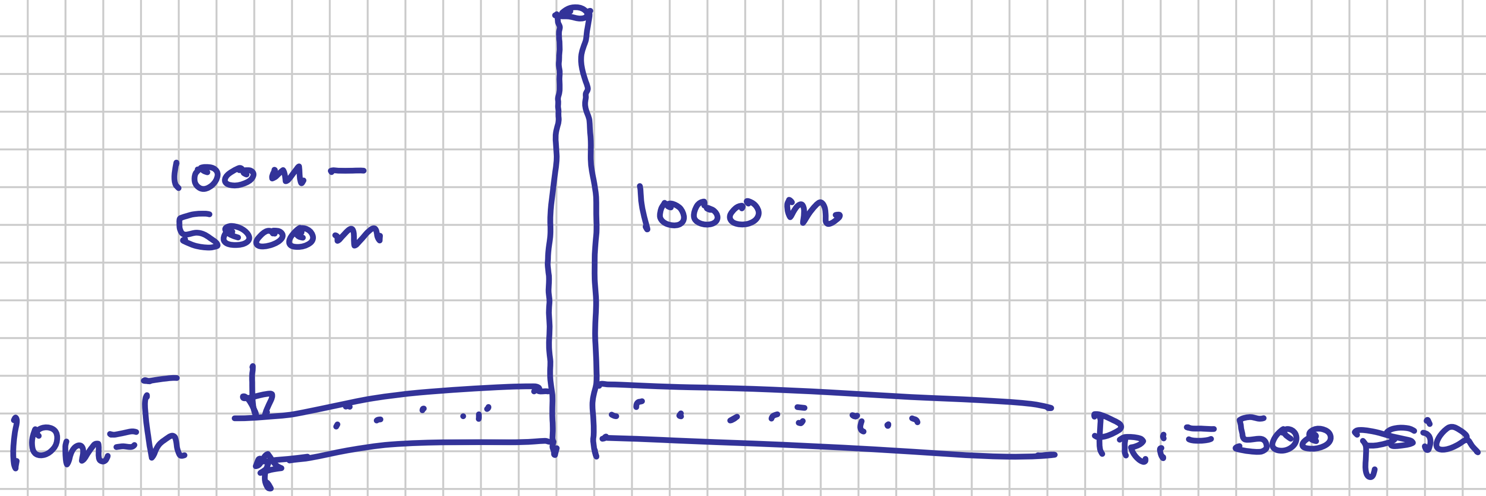
SPE

www.spe.org

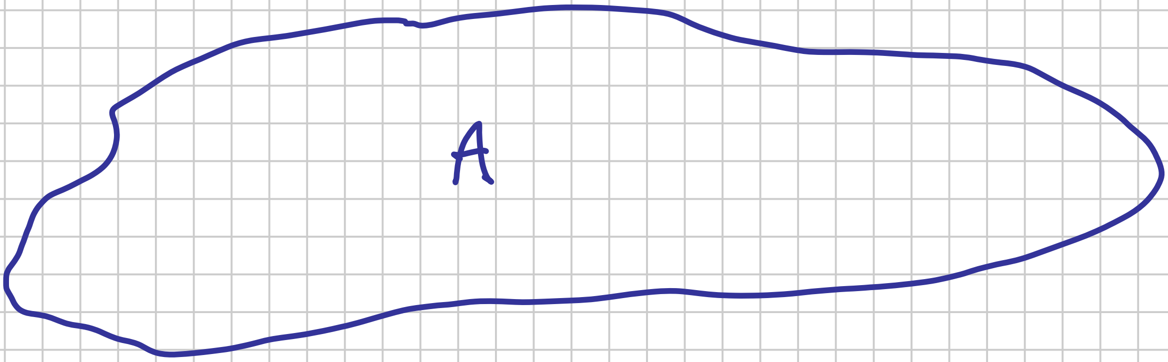
Nomenclature

Find the reservoir
- Seismic
- Maps etc
- Drill a well

} \$1m – 100m



TOP Sketch (Areal)



Porosity $\frac{V_{pores}}{V_{rock}}$ ϕ 0.05 - 0.45

S_w Water Saturation $= \frac{V_{pw}}{V_p}$ 0.05 - 0.45

Hydrocarbon Pore Volume HCPV

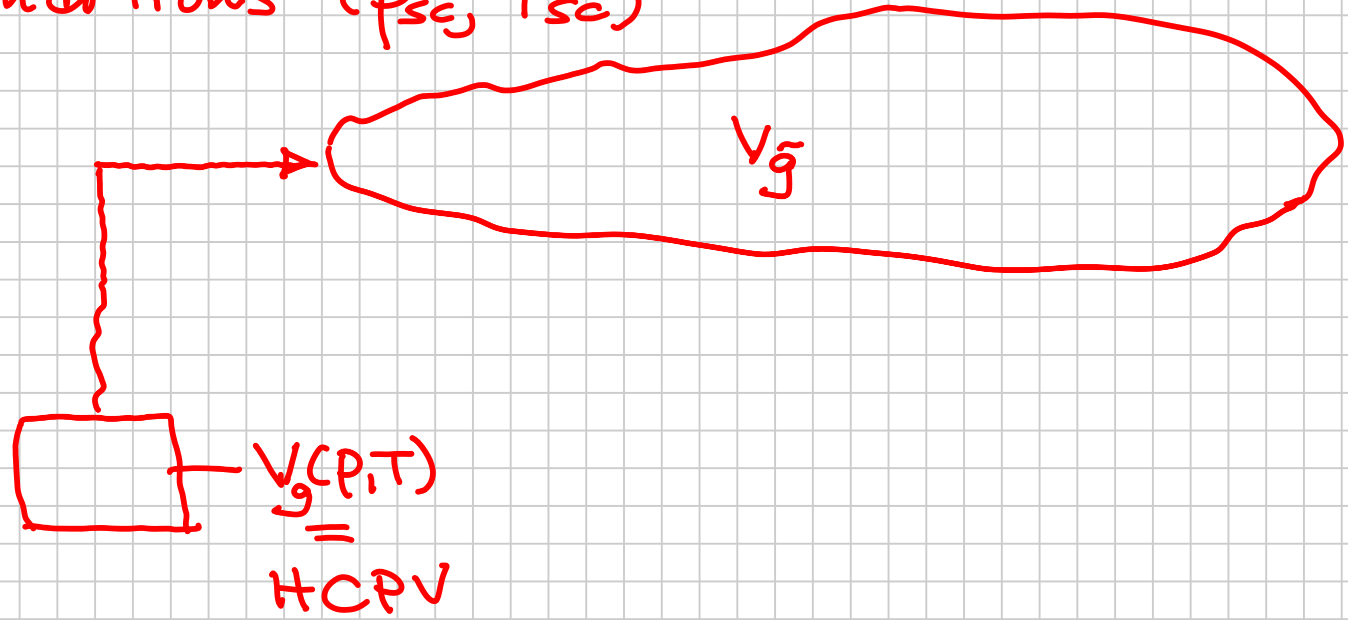
$$\underbrace{HC}_{\text{"g"}}PV = \underbrace{\underbrace{\underbrace{\bar{h}\,A}_{V_{rock}}\,\bar{\phi}}_{V_{pore}}\,(\underbrace{1-S_w}_{HC})}_{V_{pHC} = V_{gR}}$$

We don't sell gas @ "R" conditions

sell gas @ s.c. 1 atm

15.56 °C

Conversion of gas volume at (p, T) to surface gas volume at standard conditions (p_{sc}, T_{sc})



$\underbrace{\text{Gas Formation Volume Factor}}_{\text{FVF}}$ $B_g \equiv \dfrac{V_{gR}}{V_{\bar{g}}}$

$$B_g = \frac{V_g(p,T)}{V_g(p_{sc}, T_{sc})}$$

Inverse FVF (Expansion Factor) $b_g \equiv \dfrac{1}{B_g} = \dfrac{V_{\bar{g}}}{V_g}$

$$B_g = \frac{\dfrac{n_g \cancel{R}\, T_R\, Z_R}{p_R}}{\dfrac{n_g \cancel{R}\, T_{sc}\, Z_{sc}}{p_{sc}}}$$

⊙ Assume that n_g is the same @ R and @ S

⊙ $z_{gsc} \simeq 1$

$$V_g = \frac{n_g R T Z_g}{P}$$

$$\boxed{B_g = \frac{P_{sc}}{T_{sc}} \cdot \frac{T_R Z_{gR}}{P_R}}$$

$$\underset{\uparrow}{\left[\frac{m^3}{Sm^3}\right]} \quad \underset{\uparrow}{\left[\frac{ft^3}{scf}\right]} \quad \frac{bbl}{scf}$$

$$\frac{bbl}{Mcf}$$

bbl = barrel
42 gallons
$5.6146\ ft^3/bbl$
$6.28\ bbl/m^3$
~160 L/bbl

$$35.31\ \frac{scf}{Sm^3}$$

$$\frac{USD}{MCF} \quad 2.5-6.0 \quad (4)$$

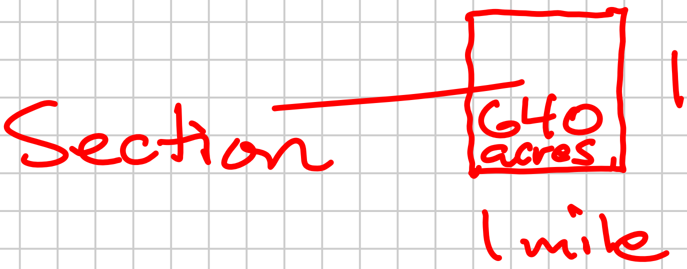
$\text{Mscf} \quad \underline{\text{Mcf}} \quad \text{MCF} = \underline{10^3\ \text{scf}}$

$\text{MMscf} \quad \text{MMcf} \quad \text{MMCF} = 10^6\ \text{scf}$

$\text{bcf} = 10^9\ \text{scf}$

$\text{Tcf} = 10^{12}\ \text{scf}$

Noblin Well : 100 acres



1 mile ≅ 1600 m

$$1 \text{ acre} = 43560 \text{ ft}^2$$

$$\sim 4356 \text{ m}^2$$

$$h = 10 \text{ m}$$

$$IGIP = \frac{[h \, A \, \phi \, (1 - S_w)]}{B_{gi}}$$

$$B_{gi} = \frac{HCPV_g}{IGIP}$$

Geofolks & A well

bottom line #

$$B_{gi} = \frac{1.0135 \text{ bara}}{(15.56 + 273.15)} \cdot \frac{(38+273)(0.96)}{(34.5 \text{ bara})}$$

$$= 0.0304 \frac{m^3}{Sm^3} \qquad \frac{ft^3}{scf}$$

$\phi \sim 0.12 \quad S_w \sim 0.3$

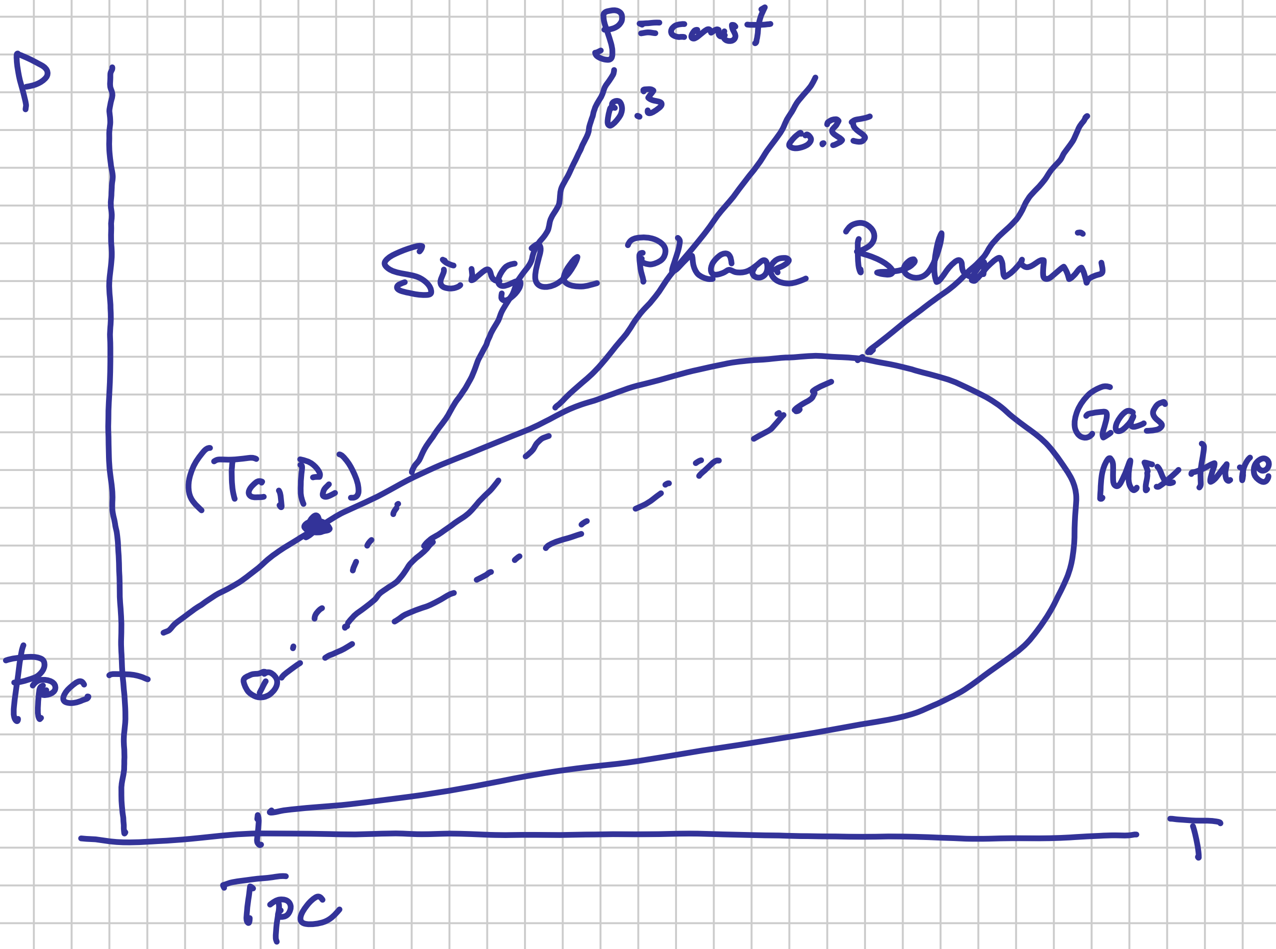
$$IGIP = \frac{(10\,\cancel{m})\left(3.28\,\frac{ft}{\cancel{m}}\right)(100\,\cancel{acre})(43560\,ft^2/\cancel{acre})(0.12)(1-0.3)}{0.0304\ ft^3/scf}$$

$$= 395 \cdot 10^6\ \cancel{scf} \times \frac{Mscf}{1000\,\cancel{scf}}$$

$$= 395 \cdot 10^3\ Mscf$$

Value : \$4.25/Mscf

$= \$1.6\ m \quad = 10\ m\ NOK$


ρ = const
0.3
0.35
P
Single Phase Behavior
Gas
Mixture
(Tc, Pc)
Ppc
T
Tpc

GAS PVT – Continued

Note Title 2010-08-31

* CHW gone next week –
Self-Study & Problems

- Solve Problems 1 & 2 in SPEPBM (App. B)
  - Embed your solution in the Gas-PVT Excel file, as a separate sheet(s)
    OR (more challenge to you)
  - Extract VBA code from Gas-PVT Excel file into your own solution Excel file for these to SPEPBM problems.

- Read (1 or 2 times) the Fetkovich paper on "Multi-Rate Testing of Gas Wells" (download from Its Learning)

GAS PVT

$Z(p_{pr}, T_{pr})$ key to all (most) PVT properties

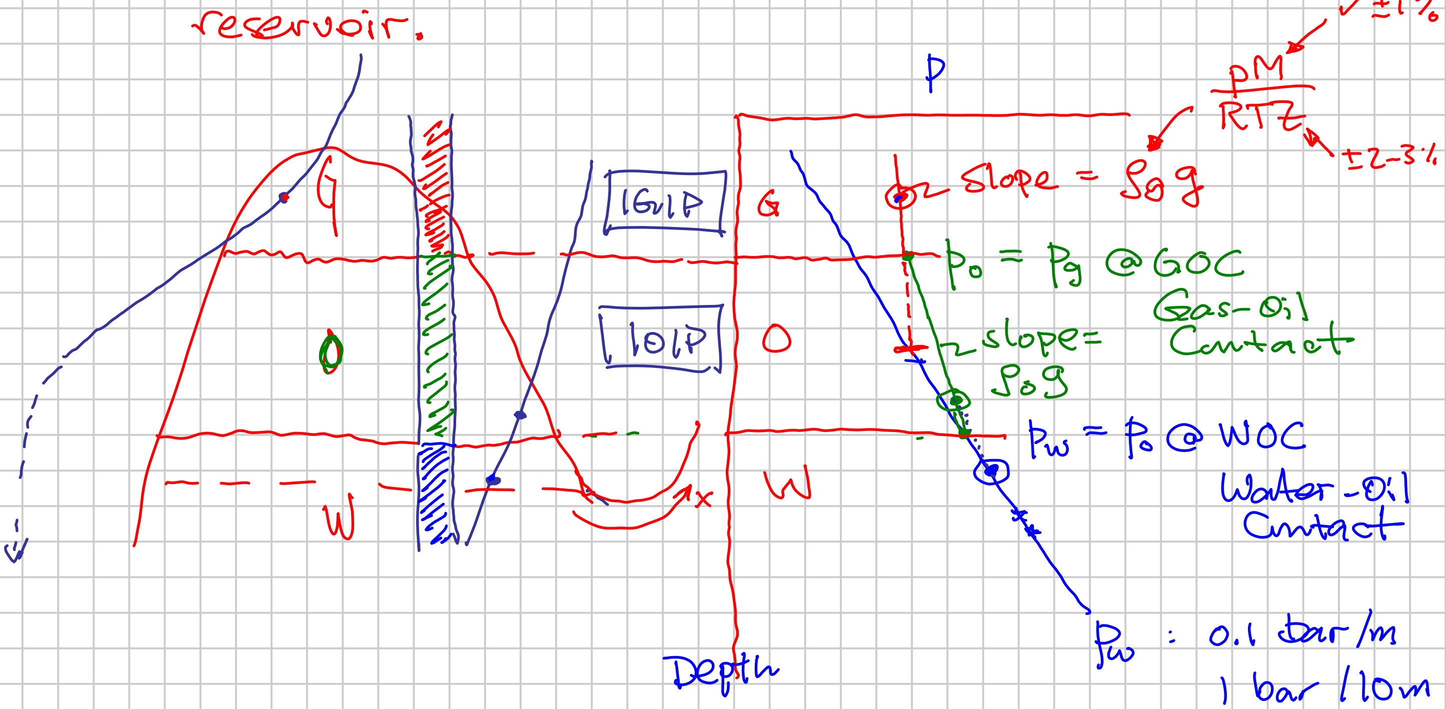
$$\rho_g = \frac{m_g}{V_g} = \frac{n_g M_g}{V_g} = \boxed{\frac{p M_g}{R T Z_g} = \rho_g}$$

$$pV = nRTZ \Rightarrow \frac{n}{V} = \frac{p}{RTZ}$$

How to use ρg ?

ρg ↑ $g = 9.81\ m/s^2$

① Defining the phase contacts in a reservoir.



p_w : 0.1 bar/m

1 bar/10m

=

$\rho_w \cdot g$

$1000\ \frac{kg}{m^3} \cdot 9.8\ \frac{m}{s^2}$

$10^4\ \frac{Pa}{m} \cdot \frac{bar}{10^5\ Pa}$

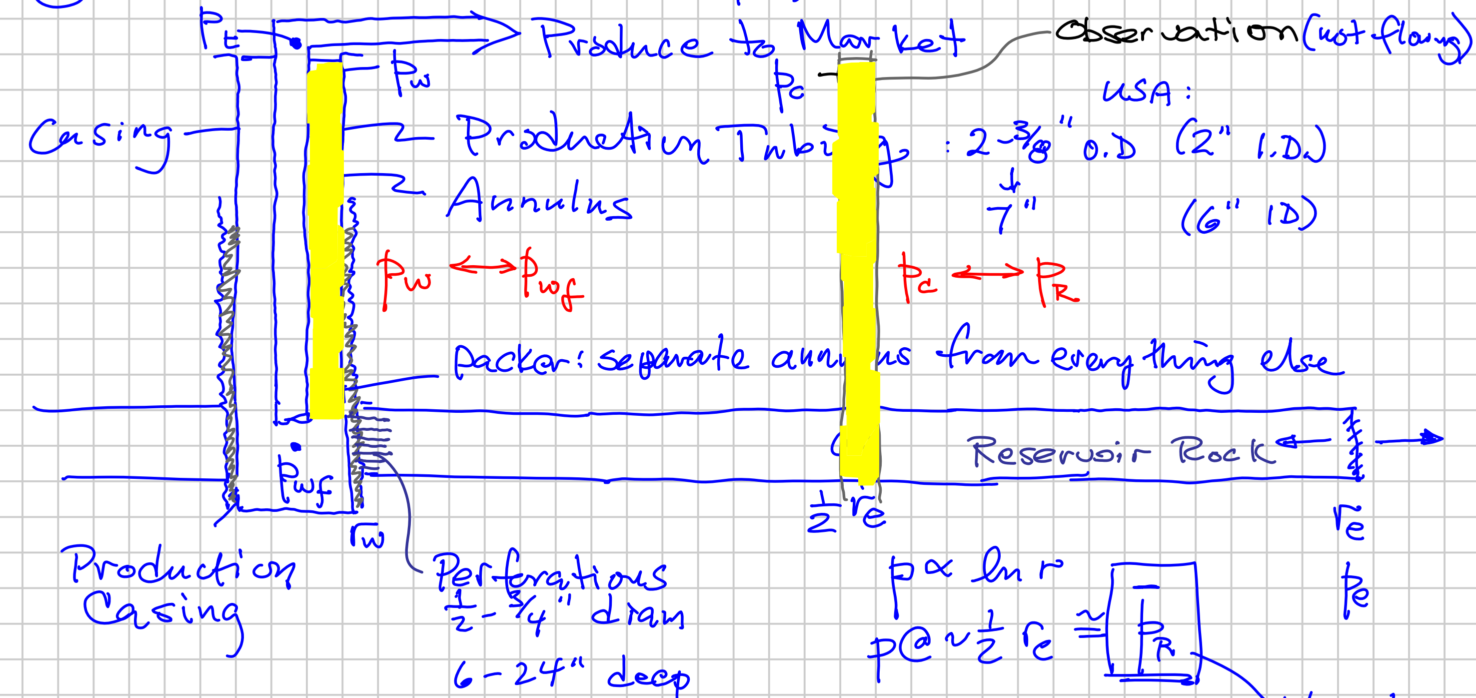
= 0.1 bar/m

Not-lectured Comment

$$\underbrace{p_o(\rho_o, D) - p_w(\rho_w, D)}_{\text{Always True}} = P_{cow}(S_w^{@D}, \underbrace{\sigma_{ow}^{@D}, \dots}_{\updownarrow \pm ?})$$

② Static Gas Column (in Pipe)

1 in(") ≡ 2.54 cm



p_{wf} = flowing bottomhole pressure (BHP)
$p @ r_w$

@ Surface
p_t = flowing tubing pressure @ Surface
p_w = static annulus surface pressure while the well is flowing.
p_c = static shutin pressure of an observation well @ $\sim \frac{1}{2} r_e$, or long-term shutin value of p_w in our producing well.
(p_t)

$\{\bar{p}_R, p_{wf}\}$ @ Reservoir Depth

$\downarrow z$

$$\frac{dp}{dz} = \rho g$$

Gas :
$$\rho_g \cong \underbrace{\left(\frac{M_g}{R\bar{T}\bar{Z}_g}\right)}_{\text{Constant } (\alpha)} p = \alpha p$$

$T(z)$, $Z_g(z)$ are approximated by averages $\bar{T}$, $\bar{Z}_g$

$$dp = \alpha \, p \, g \, dz_{TVD}$$

TVD = True Vertical Depth (H) ?

$$\int_{p_w,\, p_c}^{p_{wf},\, p_R} \frac{1}{p} \, dp = \alpha g \int_0 dz$$

$$\ln \frac{p_R}{p_c} = \alpha g \cdot TVD$$

$$\boxed{\frac{p_{wf}}{p_w} \approx \frac{p_R}{p_c}} = \exp\Big[\underbrace{\alpha g \, TVD}_{= S/2}\Big] = \underbrace{e^{S/2}}_{\text{Constant}}$$

$$\left(\frac{p_R}{p_c}\right)^2 = \underbrace{e^{S}}_{\text{Constant}} \; ; \; S = 2 \cdot \alpha \cdot g \, TVD$$

$$= 2 \frac{M_g}{R \bar{T} \bar{Z}} g \cdot TVD$$

$$\downarrow$$

- M_g
- TVD

$\bar{T} \; \bar{Z}_g$

Why do we need the translation R ↔ S pressures

= Read Fetkovich

- Pressure drop in the reservoir (Darcy's law)

  $(p_R \rightarrow p_{wf}) \propto q_g$

- Pressure drop in the prod. tubing
  - Gravity : $p_{wf}^2 - p_w^2$ $p_{wf}^2 / p_w^2 = \text{const} = e^S$
  - Friction : $(p_w^2 - p_t^2) \propto q_g$

(R+T)

↳ Composite the total pressure drop

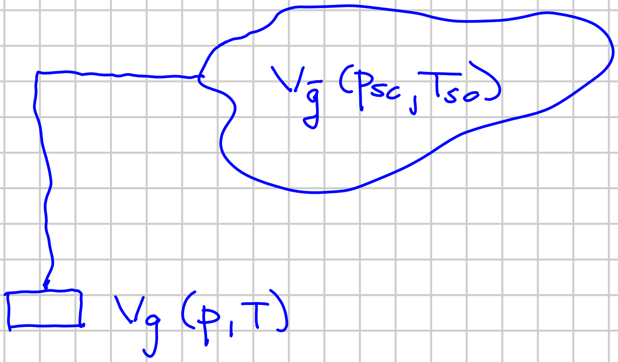
$$\begin{tikzcd} p_R \arrow[r] \arrow[d, squiggly] \arrow[dr, phantom, "\text{“S”}"] & p_{wf} \arrow[r] \arrow[d, squiggly] & p_t & \text{vs} & q_g \\ p_c \arrow[r] & p_w \arrow[r] & p_t & \text{vs} & q_g \end{tikzcd}$$

Surface Datum @ $\{ p_c \rightarrow p_w \rightarrow p_t \}$ vs q_g

R + T

"Nodal Analysis"

Gas Formation Volume Factor (FVF) B_g



$$B_g = \frac{V_g(p,T)}{V_{\bar g}(p_{sc},T_{sc})}$$

T_{sc} 15.56°C (60°F)

p_{sc} 1atm 1.0135 bara
14.696 psia

Real Gas Law: $pV = nRTZ$

$$\Rightarrow B_g = \underbrace{\left(\frac{p_{sc}}{T_{sc}}\right)}_{\text{constant}} \frac{T_R Z_R}{p_R}$$

$$B_g \propto \frac{Z}{p} \quad \tilde{\propto} \quad \frac{1}{p}$$

$$E_g = b_g \equiv \frac{1}{B_g} \propto \left(\frac{p}{Z}\right) \quad \tilde{\propto} \quad p \qquad \frac{Sm^3}{m^3}$$

$\sim 25-350$

Check Curtis:

Noblin: $p_{Ri} = 350$ psia
$T_R = 100$°F
$\sim C_1$

$$b_{gi} = 23 \quad \frac{Sm^3}{m^3}$$

Kristin: $p_{Ri} \sim 900$ bara
$T_R \sim 180$°C
$M_g \simeq 25$

$$b_{gi} = 340 \quad \frac{Sm^3}{m^3}$$

Gas Recovery Factor Calculations

← Cumulative (surface) gas produced

$$RF = \frac{G_p}{G}$$

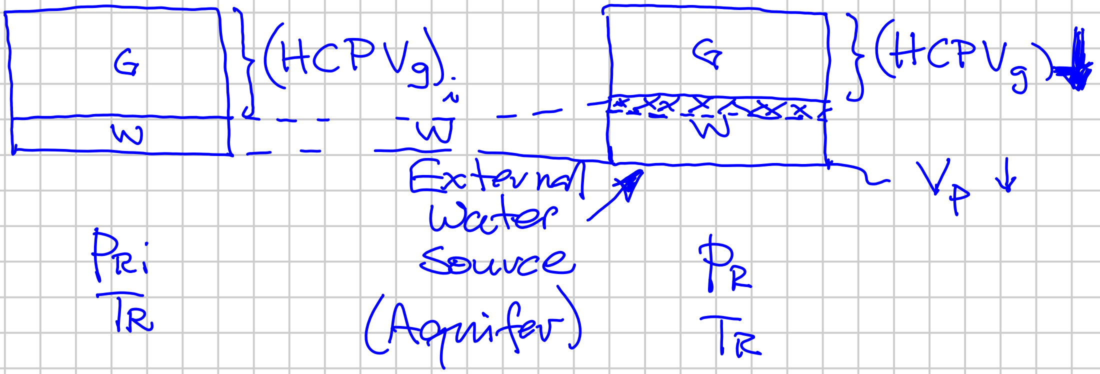
: B_{gi}, B_g @ current state of Reservoir where we want RF

Z_{Ri} Z_{gR}

IGIP (surface gas)

"Gas Material Balance"

$$\underbrace{P_R} \longleftrightarrow \underbrace{G_p}$$



The general Gas Material Balance Calculation

Know:

- $HCPV_i$

Current • HCPV = $HCPV_i$ − Pore Volume Reduction (small) − V_w Encroached into $HCPV_i$

$\left\{ \begin{matrix} P_R \\ \text{or} \\ G_p \end{matrix} \right\}$

Get:

(a) $P_R \rightarrow \underline{\underline{G_p}}$?

(b) $\underline{G_p} \rightarrow P_R$

Sm^3

$$G_p = \left(n_{gi} - n_{gR}\right) \times 23.67 = n_{gp} \times 23.67$$

$n_{gp} = G_p / 23.67$ kg-mol

Sm^3 ⇒ $n_{gR} = n_{gi} - n_{gp}$

① Calculate moles of gas initially in place

$$\checkmark\, n_{gi} = \frac{\overset{\checkmark}{p_{Ri}}\ \overset{\checkmark}{HCPV_{gi}}}{\underset{\checkmark}{R}\ \underset{\checkmark}{T_R}\ \underset{\checkmark}{Z_{Ri}}} \qquad G_i = 23.67 \cdot n_{gi} \qquad pV = nRTZ$$

② Calculate moles of gas currently in place

$$\checkmark\, n_{gR} = \frac{p_R\ \overset{\checkmark}{HCPV_g}}{R\ \underset{\checkmark}{T_R}\ Z_R}$$

(b) ⇒ Solve for (p_R/Z_R)

$$(p/Z)_R = n_{gR}\,\frac{RT_R}{HCPV_g}$$

→ Solve by trial-and-error for p_R

Remember: $\dfrac{V_{\bar{g}}}{n_g} = 23.67\ \dfrac{Sm^3}{Kg\text{-}mole}$

GAS MATERIAL BALANCE

Note Title 2010-09-01

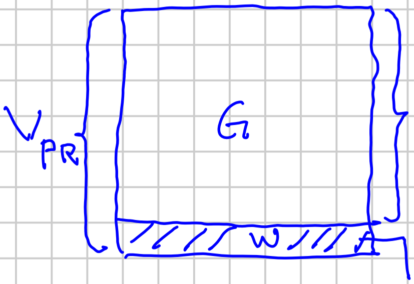
$\bar{p}_R \leftrightarrow G_p$ [surface gas volumes]

Volumetric Average Reservoir Pressure

Cumulative Gas Produced

Simplifying Assumptions

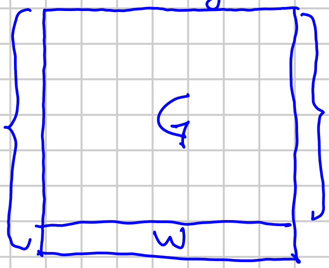
- $HCPV_g(t) = \text{constant} = HCPV_{gi}$



$HCPV = V_{pgi}$
$= V_{pR} \cdot (1-S_{wc})$

p_{Ri}
T_R

Connate Water
S_{wc}
S_{wi}



$HCPV_g = HCPV_{gi}$

$p_R < p_{Ri}$
T_R

Produced
G_p Sm^3

$$n_{gi} = \underbrace{\overbrace{\boxed{V_{pgi} \cdot b_{gi}}}^{IGIP\ (G)}}_{Sm^3} \cdot \left(\frac{\text{kg-mole}}{23.67\ \cancel{Sm^3}} \right)$$

$\cancel{m^3}$ $\quad$ $Sm^3/\cancel{m^3}$

kg-moles

$\cancel{Sm^3}$

$$\frac{p_{sc}}{R\,T_{sc}}$$

$$b_{gi} = \frac{1}{B_{gi}} = \underbrace{\left(\frac{T_{sc}}{p_{sc}}\right) \cdot \frac{p_{Ri}}{T_R Z_{gi}}}$$

Gas PVT : $Z_{gi}(p_{Ri}, T_R, y_i)$ Initial Gas Comp.

$V_{pgi}\ (HCPV_i)$: Estimate from geological data and well log data.

t

$$\underbrace{\bar{A} \quad \bar{h}_g \quad \bar{\phi} \quad \bar{S}_{wc}}_{\text{spatial variation}}$$

(x, y, z)

Remaining $n_{gR} = V_{pgi} \cdot b_{gR} / 23.67$

$\uparrow$

@ p_R

$$b_{gR} = \frac{T_{sc}}{p_{sc}} \cdot \frac{p_R}{T_R Z_{gR}}$$

$$p_R V = n R T Z_{gR}$$

$$\frac{p_R}{Z_{gR}} = \frac{n_{gR} R T_R}{V_{pgi}} = \frac{(\cancel{V_{pgi}}\, b_{gR}) R T_R}{\cancel{V_{pgi}}\,(23.67)}$$

$$23.67 = \frac{R T_{sc}}{p_{sc}}$$

$$\frac{p_R}{Z_{gR}} = \frac{p_{Ri}}{Z_{gi}}\left(1 - \frac{n_{gp}}{n_{gi}}\right)$$

$$\frac{n_{gp}}{n_{gi}} = \frac{G_p / 23.67}{G / 23.67}$$

$$\boxed{\left(\frac{p_R}{Z_R}\right) = \left(\frac{p_{Ri}}{Z_{gi}}\right)\left(1 - \frac{G_p}{G}\right)}$$

Straight-line Material Balance

$HCPV_g$ = constant

$\frac{G_p}{G} = RF_g$ (Recovery Factor)

0.6 – 0.95

Observations: Rules of Thumb

- Low P_{Ri} reservoirs

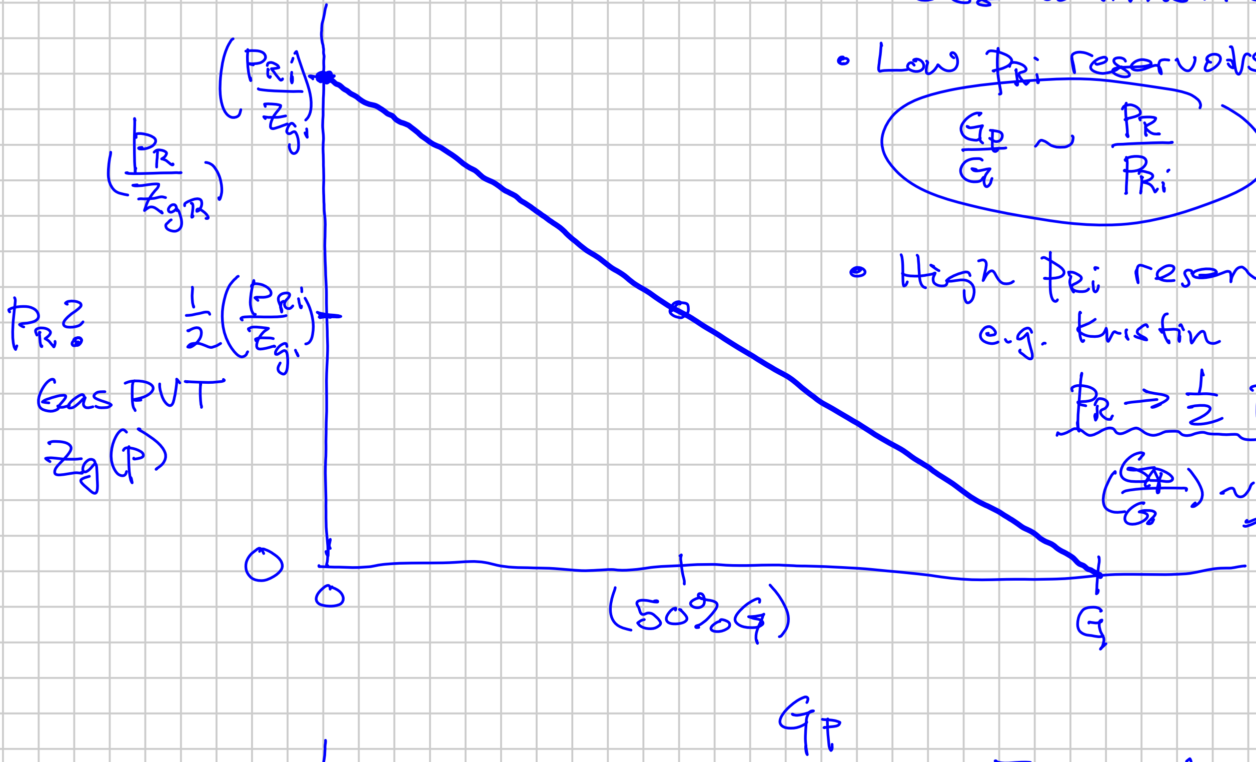
$$\frac{G_p}{G} \sim \frac{P_R}{P_{Ri}}$$

- High P_{Ri} reservoirs

e.g. Kristin $P_{Ri} \sim 900$ bar

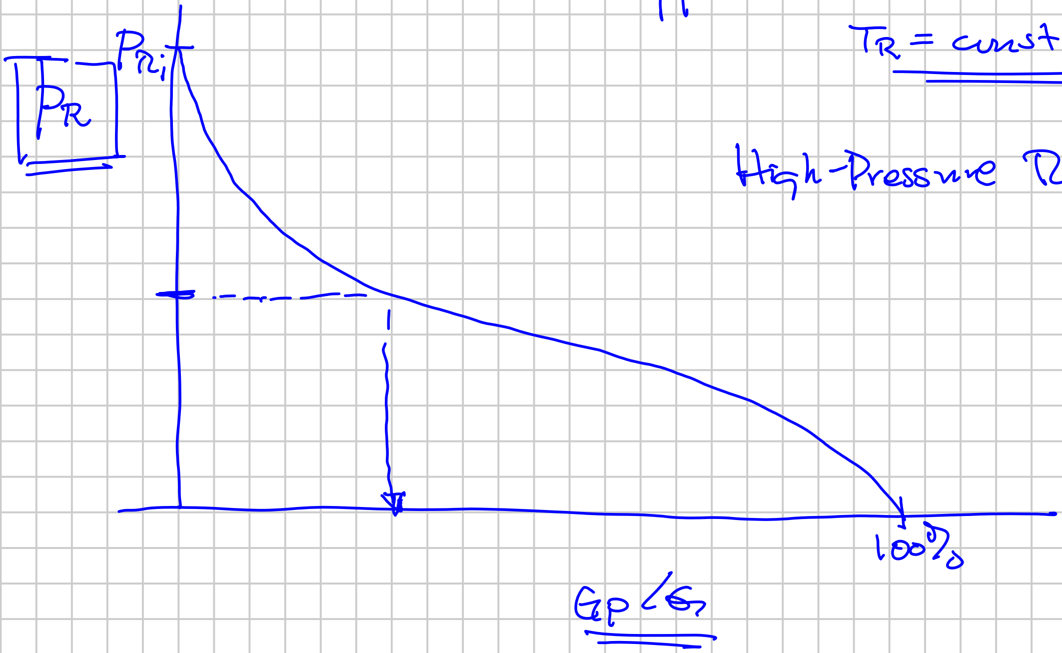
$P_R \to \frac{1}{2} P_{Ri}$ 450

$\left(\frac{G_p}{G}\right) \sim 25\%$



T_R = constant

High-Pressure Reservoir



Influence of Water Encroachment on Gas M.B.

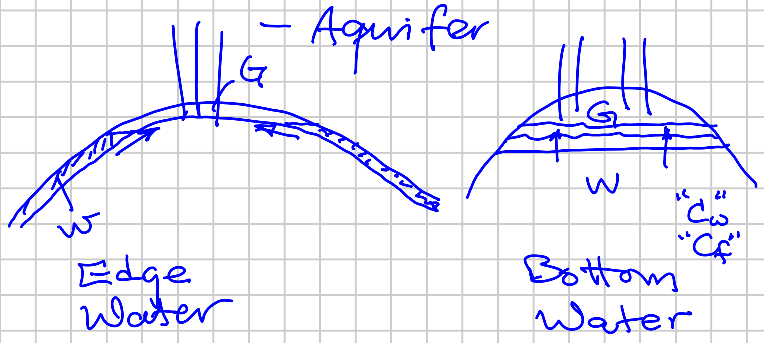
$\boxed{HCPV_g \text{ getting smaller}}$ as f(t)

- V_{PR} gets smaller, c_f = rock formation compressibility
- Connate water expands $c_w \equiv -\frac{1}{V_w}\left(\frac{dV_w}{dp}\right)$
- Water encroachment
  - Aquifer

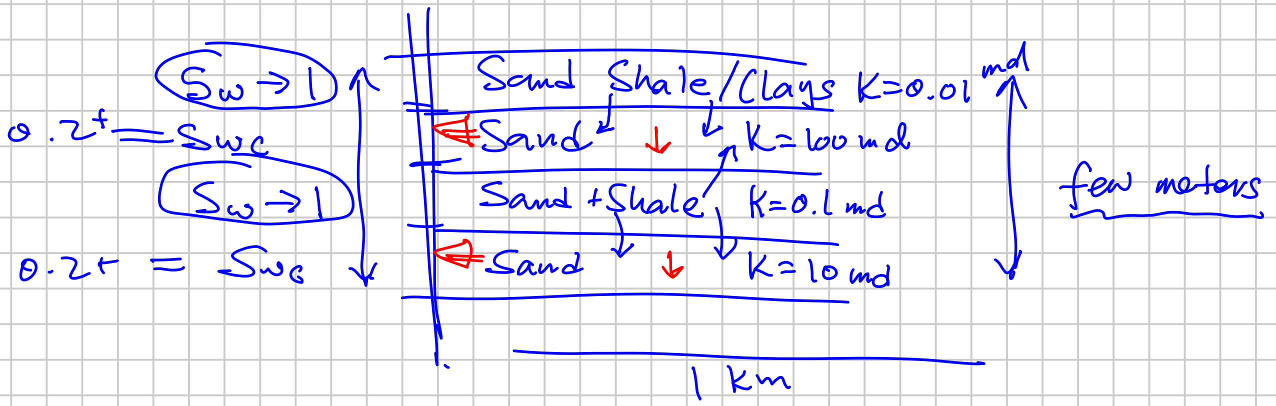
$\sim 3\text{-}5 \cdot 10^{-6} \frac{vol/vol}{psi}$

$\times \frac{14.5 \; psi}{bar}$

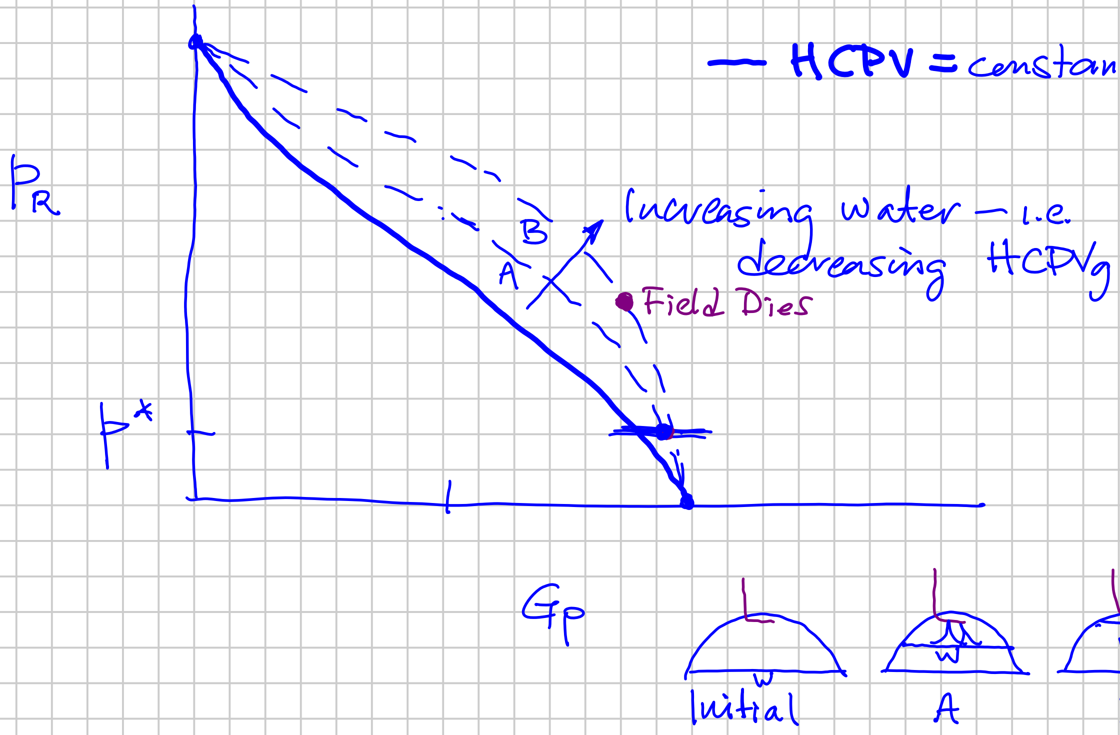
$\sim 50 \cdot 10^{-6} \frac{vol/vol}{bar}$

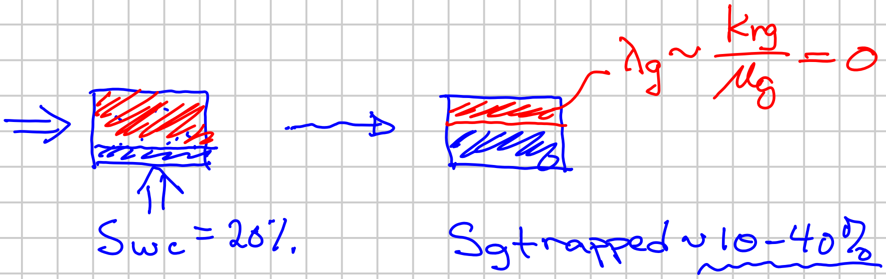


- Adjacent (interbedded) water(



— **HCPV = constant**





Model to quantify the effect of <u>all</u> factors changing $HCPV_g$ over time:

"Pot" Aquifer (extended) Fetkovich

$$\frac{p_R}{z}\left[1 - \underset{\uparrow}{c_e}\left(\underset{\uparrow}{p_{Ri}} - \overset{\downarrow}{p_R}\right)\right] = \left(\frac{p_{Ri}}{z_i}\right)\left(1 - \frac{G_p}{G}\right)$$

constant

$$c_e = \frac{\overset{\checkmark}{c_w}\,\overset{\checkmark}{S_{wc}} + \overset{\checkmark}{c_f} + \overset{\checkmark}{M}\left(c_w + \overset{\checkmark}{c_f}\right)}{1 - S_{wc}}$$

$$M = \frac{V_{AQ}}{V_{pR}} \sim 0.5 - 5 \;\ldots$$

Assumption: $(\Delta p_R)_g$ felt "<u>instantaneously</u>" in the associated water volume defined by M

GAS MATERIAL BALANCE

Note Title 5/26/2007

GAS PVT EQ. – REAL GAS LAW

$$pV = nRTZ$$

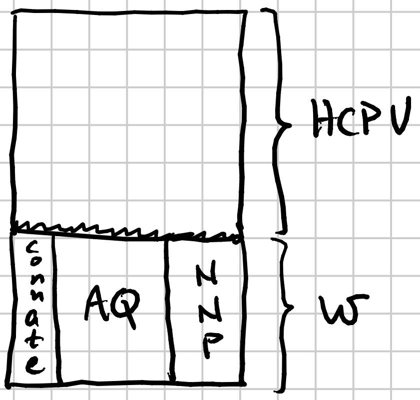
Assumptions:

V = HCPV = constant ⊛

$Z = f(p, T)$

$T = T_R =$ const.

$n_R = n_i - n_p$



"Pot Aquifer"
Small AQ
High k

Introduce: Surface Gas Volumes

$$V_{gsc} = \underbrace{\frac{RT_{sc}}{p_{sc}}}_{\substack{379 \text{ scf/lbmol} \\ 23.64 \text{ Sm}^3/\text{kgmol}}} \cdot n$$

"s.c." 1 atm (14.7 psia)
60 °F (15.56 °C)

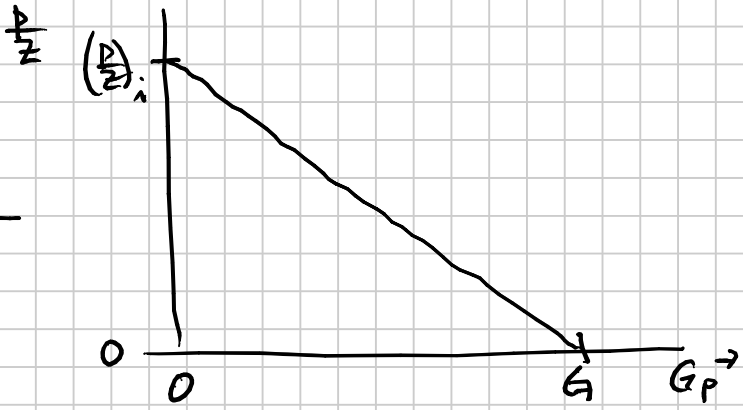
(G) IGIP $= n_i \cdot 379$

(Gp) Cum. Gas Prod. $= n_p \cdot 379$

⇒ Straight-Line Gas. M.B

$$\boxed{\left(\frac{p}{Z}\right) = \left(\frac{p}{Z}\right)_i \left(1 - \frac{G_p}{G}\right)}$$

RF



$$\frac{G_p}{G} = 1 - \frac{(p/Z)}{(p/Z)_i}$$

Pot Aquifer Model

$$\frac{p}{Z}\left[\underbrace{1-\bar{c}_e(p_i-p)}\right] = \left(\frac{p}{Z}\right)_i\left(1-\frac{G_p}{G}\right)$$

$$\bar{c}_e = \frac{c_f + c_w S_{wc} + \overset{\checkmark}{M}(c_f+c_w)}{1-S_{wc}}$$

$$\left.\begin{array}{l} S_{wc} \sim 0.25 \\ c_f \sim c_w \sim 5\cdot10^{-6}\ \text{psi}^{-1} \\ M \sim \dfrac{V_{Aq}}{V_{PHC}} \sim 1-5 \quad (\text{Pot Aq.}) \end{array}\right\} c_e =$$

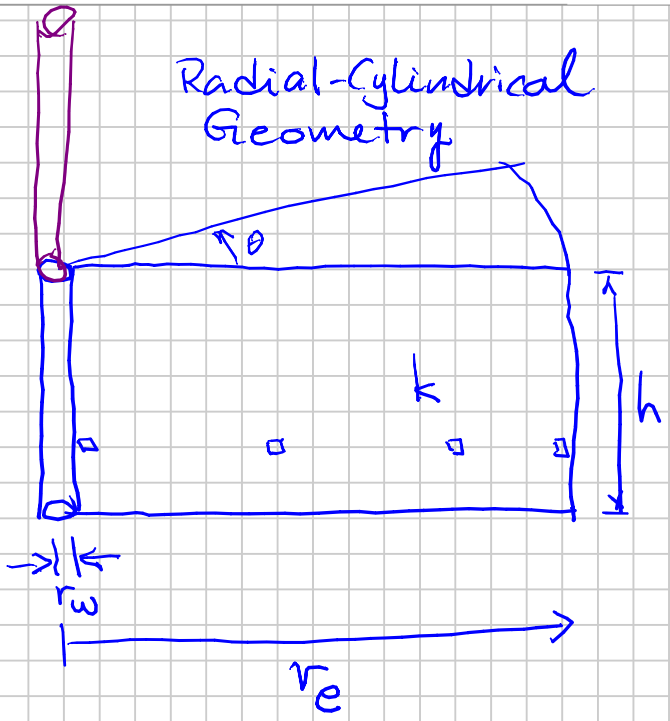
$$\frac{G_p}{G} = 1 - \frac{(p/Z)\left[1-c_e(p_i-p)\right]}{(p/Z)_i}$$

GAS RATE EQUATION

Note Title 2010-09-14

Darcy's Law

$$v = \frac{k}{\mu}\cdot\frac{dp}{dr} \quad \|$$



- Gas Flow Only
- Steady-State Flow
  - Mass or Molar rate is same everywhere
- O.B.: $p_e \equiv p @ r = r_e = \text{const.}$

Molar Rate q_m

$$q_m = v A_\perp \rho_m \Rightarrow v_r = \frac{q_m}{2\pi h r}\cdot\frac{1}{\rho_m} = \qquad \frac{1}{r}$$

$$A_\perp = 2\pi h r$$

$$\rho_m = \frac{n}{V} = \frac{p}{RT_R Z}$$

Real Gas Law

$$q_m = \frac{k}{\mu}\frac{dp}{dr}\cdot 2\pi h r\cdot\frac{p}{RTZ} \qquad \left[\frac{kg\text{-}mol}{s}\right]; T = T_R$$

Ideal Gas Law:

$$\frac{V_g}{n} = \frac{RT_{sc}}{p_{sc}} \qquad \frac{Sm^3}{kg\text{-}moL} \qquad 23.67$$

$$q_g\left[\frac{Sm^3}{s}\right] = \frac{RT_{sc}}{p_{sc}}\cdot q_m = \text{constant} : \underline{\underline{SS}}$$

$$\underset{\uparrow}{q_g} = \frac{k}{\mu}\,\frac{dp}{dr}\,2\pi h r\,\frac{p}{\cancel{R}TZ}\,\frac{\cancel{R}T_{sc}}{p_{sc}}$$

$$= \underbrace{2\pi\left(\frac{T_{sc}}{p_{sc}}\right)}_{\text{Numerical const.}}\cdot\underbrace{\left(\frac{kh}{T}\right)}_{\text{\textcircled{R} specific}}(r)\left(\frac{p}{\mu Z}\right)\frac{dp}{dr}$$

$$\underset{\substack{\uparrow\\ \text{Const.}}}{q_g}\int_{r_w}^{r_e}\frac{1}{r}\,dr = \underbrace{2\pi\left(\frac{T_{sc}}{p_{sc}}\right)\left(\frac{kh}{T}\right)}_{\text{const. for a given WELL}}\int_{p_{wf}}^{p_e}\frac{p}{\mu Z}\,dp$$

$$q_g\cdot\underbrace{\ln\frac{r_e}{r_w}}_{\substack{7-10\\ \text{worldwide}}}$$

$$\underset{\substack{\uparrow\\ \text{flowing}}}{p_{wf}} \equiv p @ r_w$$

$$\underset{\substack{\uparrow\\ \text{Shut-in }(q=0)}}{p_{ws}} \equiv p @ r_w$$

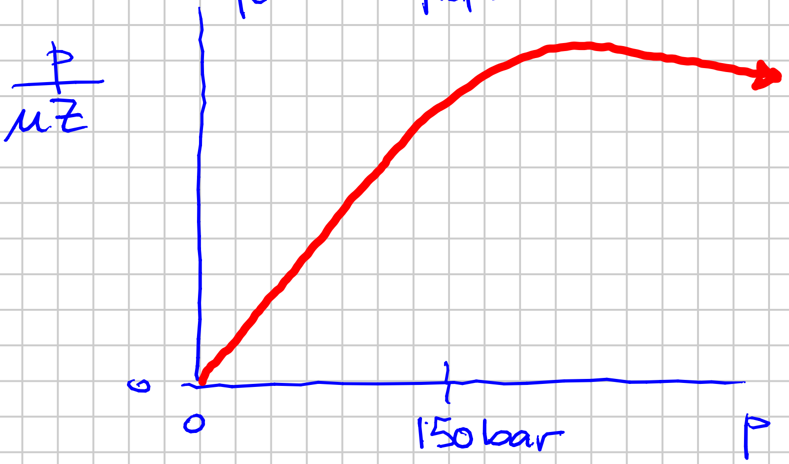
$$q_g = \pi\left(\frac{T_{sc}}{p_{sc}}\right)\frac{kh}{T\cdot\ln r_e/r_w}\,\underbrace{2\int_{p_{wf}}^{p_e}\frac{p}{\mu Z}\,dp}_{m(p_e)-m(p_{wf})}$$

196x:

Gas Pseudopressure Function:

Al-Hussainy, Ramey, Crawford

( )



$$m = 2\int_0^{p} \frac{p}{\mu z}\,dp \quad ; \quad \int_{p_{wf}}^{p_e} = \int_0^{p_e} - \int_0^{p_{wf}}$$

Consistency w/ historical equation using p^2

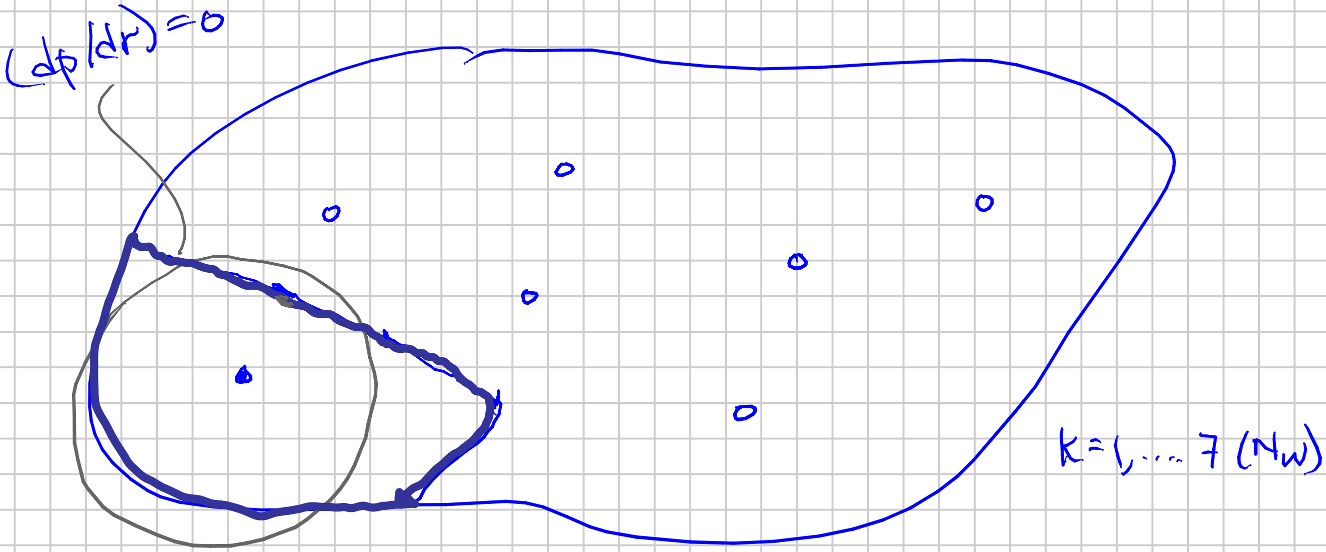
$$m(p_e) - m(p_{wf})$$

m(p): known function for a specific gas reservoir ; y_i, T_R are invariant

$$q_g = \pi\left(\frac{T_{sc}}{p_{sc}}\right)\frac{kh}{T\cdot\ln r_e/r_w}\cdot\left[m(p_e) - m(p_{wf})\right]$$

- O.B. Condition: ~~$p_e = p@r=r_e$~~ $=$ const

$$\left(\frac{dp}{dr}\right)_{r=r_e} = 0 \quad \text{(No-Flow)}$$



$K(x,y,z)$ = homogeneous

q_{wk} = const.

$$\underline{\Rightarrow A_{wk}} = \text{constant} = (A_{Field} / N_w) = \pi \tilde{r}_{ek}^2$$

$$q_{wk} \neq \text{constant}$$

$$\frac{A_{wk}}{A_{Field}} = \frac{q_{wk}}{\sum_{j=1}^{N_w} q_{wj}}$$

No-Flow O.B. Condition

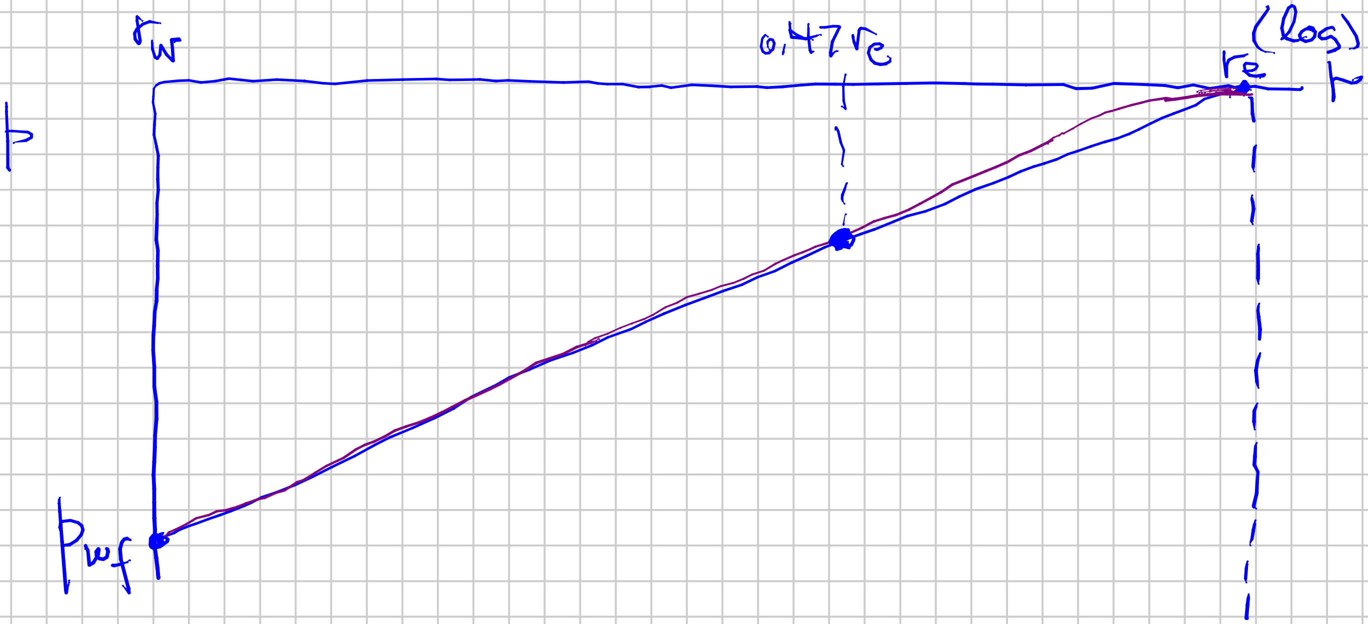
– $\bar{p}_R$ = Volumetric Average Pressure $r_w \to r_e$ $= \dfrac{\int p(r)\cdot r\,dr}{\int r\,dr}$

Well Testing (Pressure Transient Analysis PTA)

$$q_g = \pi\left(\frac{T_{sc}}{p_{sc}}\right)\frac{kh}{T\left[\underbrace{\ln\left(\frac{0.47 r_e}{r_w}\right)}_{\left[\ln\left(\frac{r_e}{r_w}\right)-3/4\right]}\right]}\left[m(\bar{p}_R) - m(p_{wf})\right]$$

$$-\frac{dp}{dr}\Big|_{r_e} = 0$$

$$\bar{p}_R \simeq p\left(r = \tfrac{1}{2} r_e\right)$$



O.B. Condition $p_e = \text{const}$

$\bar{p}_R$ p_e

$$\boxed{\ln \frac{0.6\, r_e}{r_w}} = \ln \frac{r_e}{r_w}$$

$$\ln \frac{r_e}{r_w} - \frac{1}{2}$$

Skin Effect – Near wellbore Δp deviation from that expected from Darcy's law
(s)
↑
dimensionless

"1–20"
$S \sim <1 \rightarrow +100$

$S > 0$

$$\Delta p_s = \frac{q}{kh} \cdot s$$

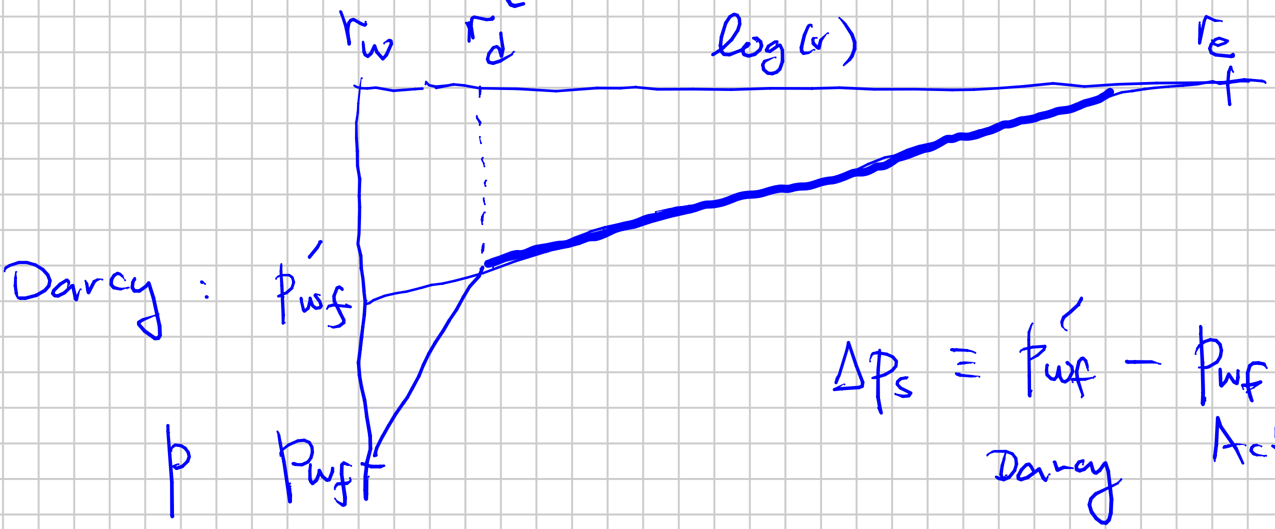
* Damage ($k_d < k$ @ $r_w \rightarrow r_d$)

$\frac{k_d}{k} = 0.5 - 0.01$; 0.2–2 m

– Perforated h (h_p) < h

– High Velocities ("Turbulence")

– ⋮ 2+ other bad effects



$$\Delta p_s = \underset{\text{Darcy}}{p'_{wf}} - \underset{\text{Actual}}{p_{wf}}$$

$r_f \sim 10\,m - 200\,m$

$\ln \frac{r_f}{r_w}$

$r_w \sim 0.1\,m$

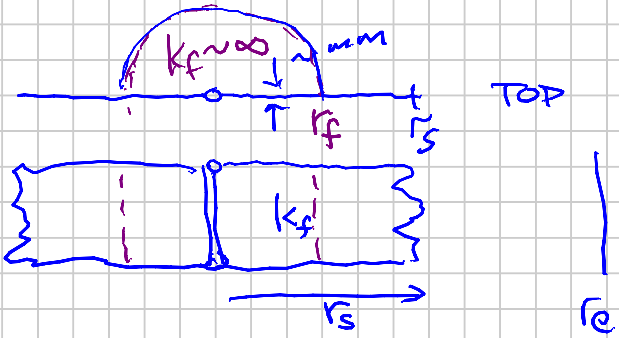
$S_f \sim -\ln \frac{100}{0.1}$

$= -4^+$

Typical stimulation skin level

- Stimulation (e.g. Fraction)

$k_s > k \qquad r_w \rightarrow r_s$



$s = -\ln \frac{r_f}{r_w}$

No Δp $\quad r_f \rightarrow r_w$

$\frac{r_f}{r_s} \sim \frac{1}{2}$

O.B. #1

$$q_g = \pi \left(\frac{T_{sc}}{p_{sc}}\right) \frac{kh}{T\left[\ln(r_e/r_w) + s\right]} \left[m(p_e) - m(p_{wf})\right]$$

O.B. #2

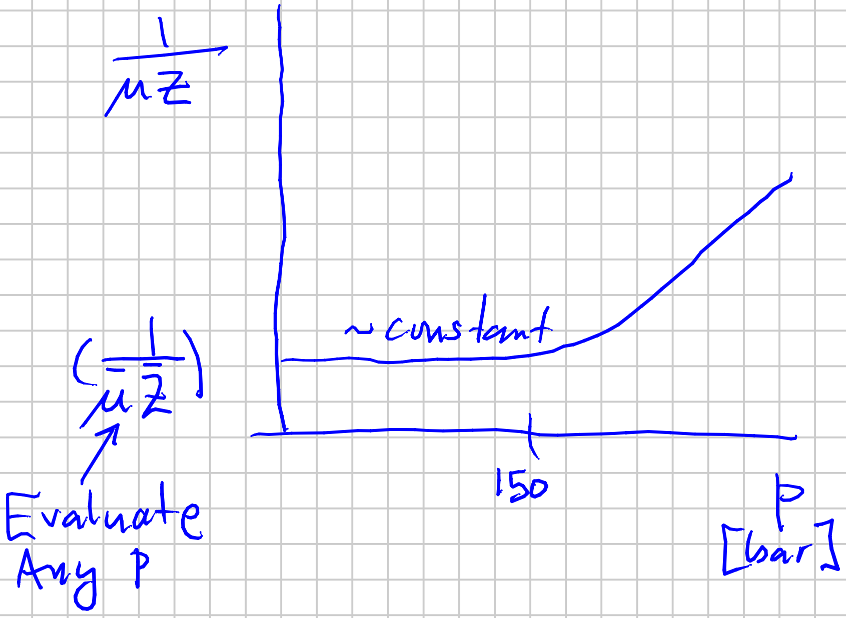
$$q_g = \pi \left(\frac{T_{sc}}{p_{sc}}\right) \frac{kh}{T\left[\ln \frac{r_e}{r_w} - \frac{3}{4} + s\right]} \left[m(\bar{p}_R) - m(p_{wf})\right]$$

* * *

Low-Pressure Approximation for $\bar{p}_R \lesssim 150-200$ bar

$$2\int_0^p \frac{p}{\mu Z}\,dp \cong \frac{1}{(\mu Z)_R} \cdot 2\int_0^p p\,dp$$

$$m(p) = \frac{1}{(\mu Z)_R} p^2$$

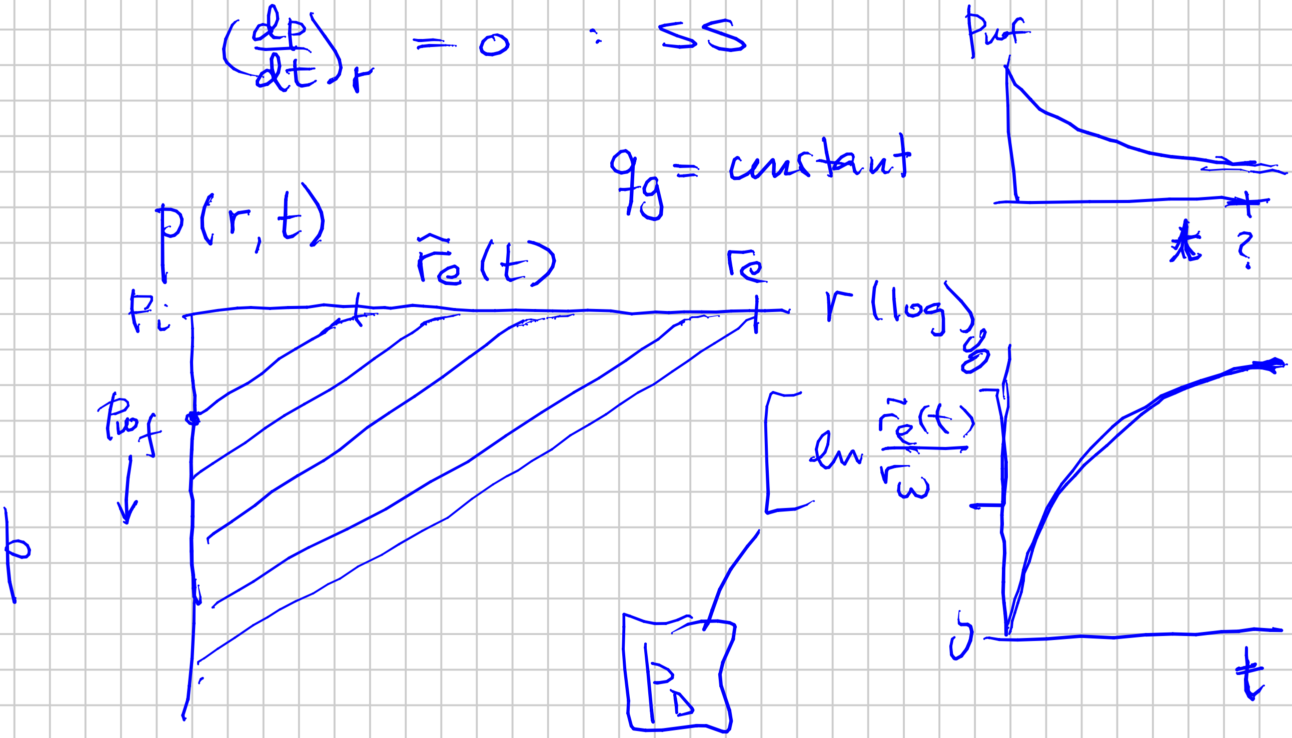


$$q_g = \pi \left(\frac{T_{sc}}{p_{sc}}\right) \frac{kh}{T\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s\right](\mu Z)_R} \left(p_R^2 - p_{wf}^2\right)$$

what if SS assumption is not valid

$$\left(\frac{dp}{dt}\right)_r = 0 \quad : \quad SS$$

$$q_g = \text{constant}$$



$$q_g = \pi \left(\frac{T_{sc}}{p_{sc}}\right) \frac{kh}{T\left[\underbrace{p_D(t)} + s\right]} \left[m(p_{Ri}) - m(p_{wf})\right]$$

$$p_D \cong \frac{1}{2}\left[\ln t\right]$$

$$\ln \frac{r_e(t)}{r_w} \propto \ln t \qquad t \to 0 \ldots t_{ss}$$

$$\frac{\left(p_{Ri} - p(r_e, t_{ss})\right)}{p_{Ri}} \sim 0.5\%$$

Until SS conditions are reached

Skin : High Velocities

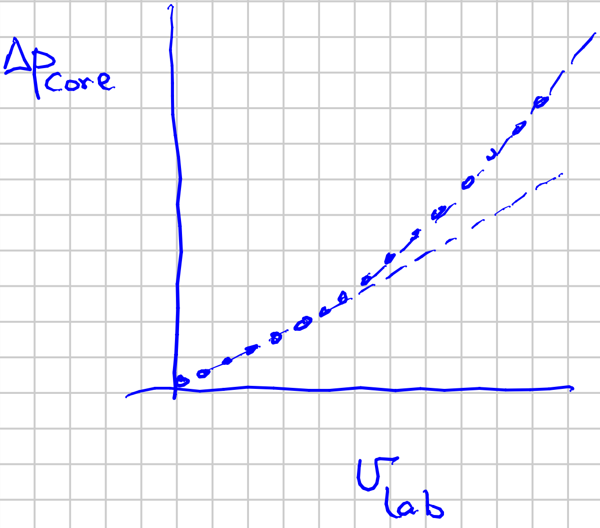
Forcheimmer

$$\frac{dp}{dr} = \frac{\mu}{k} v + \beta \rho v^2$$

β = rock property $[L^{-1}]$

ρ = mass density

Muskat 193x's



$$\boxed{q_g = \underbrace{\pi \left(\frac{T_{sc}}{p_{sc}}\right)} \frac{kh}{T\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s_t\right]} \left[m(\bar{p}_R) - m(p_{wf})\right]}$$

$$S_t = S + D q_g$$

Damage
Geometric
Stimulation
⋮

$$\boxed{\beta \propto \frac{1}{k}}$$

$$D \propto \beta$$

$$D = (\text{stuff})\,\beta$$

Solve for rate q_g given p_{wf}

$$\underset{\substack{\uparrow\\ \text{non-Darcy}\\ kh,\,D}}{B q_g^2} + \underset{\substack{\uparrow\\ \text{Darcy}\\ kh,\,s}}{A q_g} - \left(m(p_R) - m(p_{wf})\right) = 0$$

Task: r [m]

Input k [md]
h [m]
T_R [K]
μ [cp]
p [bar]

Output q_g [Sm^3/d]

$$q_g = \left[\quad\right] \frac{kh\,\Delta m}{T_R\left[\ln\frac{r_e}{r_w} + s\right]}$$

$$\left[\frac{bar^2}{cp}\right]$$

$$m(p) \equiv 2\int \frac{p}{\mu z}\,dp$$

scf/D °R psia ft cp

$$\underset{[Sm^3/s]}{q_g} = \underbrace{\pi\left(\frac{T_{sc}}{p_{sc}}\right)}\frac{kh}{T\cdot \ln(r_e/r_w)+s}\,\overbrace{2\int_{p_{wf}}^{p_e}\frac{p}{\mu z}\,dp}$$

Task:

Units

Input

r [m]

k [md] $1\,D = 10^{-12}\,m^2 = 1\,(\mu m)^2$

h [m]

T_R [K]

μ [cp] $1\,cp = 1\,mPa\cdot s$

p [bar] $1\,bar = 10^5\,Pa$

Our preferred units

(in Norway)

Output q_g [Sm^3/d]

T_{sc} = 15.56 °C = 60 °F = 273.15 + 15.56 = ______ K

p_{sc} = 1.0135 bar

$$q_g = \pi\left(\frac{T_{sc}}{p_{sc}}\right)\frac{kh}{T\cdot \underbrace{\ln(r_e/r_w)+s}}\,2\int_{p_{wf}}^{p_e}\frac{p}{\mu z}\,dp$$

$$\frac{Sm^3}{s} = \pi \left(\frac{T_{sc}[K]}{p_{sc}[\cancel{bar}]\cdot 10^5 \frac{Pa}{\cancel{bar}}} \right) \frac{\left\{ k[\cancel{md}] \cdot \frac{10^{-3}\cancel{D}}{\cancel{md}} \cdot \frac{10^{-12}\, m^2}{\cancel{D}} \right\}}{T_R[K]}$$

$$\cdot \frac{\left\{ p^2[\cancel{bar^2}] \times \left(10^{10} \frac{Pa^2}{\cancel{bar^2}}\right) \right\}}{\left\{ \mu[\cancel{cp}] \cdot \frac{\cancel{mPa\cdot s}}{\cancel{cp}} \cdot \frac{Pa\cdot s}{10^3\, \cancel{mPa\cdot s}} \right\}}$$

$$= \pi \frac{(273.15+15.56)}{(1.0135)10^5} \cdot \frac{(10^{-3})(10^{-12})}{} \cdot \frac{(10^{10})}{(10^{-3})}$$

$$\cdot \frac{k[md]\, h[m]}{T_R[K]\left[\ln r_e/r_w + s\right]} \quad \Delta m \left[\frac{bar^2}{cp}\right]$$

$$\frac{Sm^3}{s} = \left\{ 8.949 \cdot 10^{-5} \frac{k[md]\, h[m]}{T_R[K]\left[\ln r_e/r_w + s\right]} \quad \Delta m \left[\frac{bar^2}{cp}\right] \right\}$$

$$\times \frac{60s}{min} \times \frac{60min}{hr} \times \frac{24hr}{D}$$

$$\boxed{q_g\left[\frac{Sm^3}{D}\right] = 7.73 \frac{k[md]\, h[m]}{T_R[K]\left[\ln r_e/r_w + s\right]} \quad \Delta m \left[\frac{bar^2}{cp}\right]}$$

↑

Excel Gas-PVT
file

Fetkovich Constant:

$$\frac{1}{1424} = 0.000703$$

Curtis
0.703

$$q_g\left[\frac{Mscf}{D}\right]$$

k[md]
h[ft]
T_R[°R]

p[psia]
μ[cp]

$$q_g\left[\frac{scf}{D}\right]$$

k[md]
h[ft]
T_R[°R]

p[psia]
μ[cp]

Low-p Eq.

$$q_g\left[\frac{Sm^3}{d}\right] = 7.73\,\frac{k[md]\,h[m]\,\left[p_R^2 - p_{wf}^2\right]\;\left[bar^2\right]}{T_R[K]\,\left(\mu_g[cp]\,Z\right)\left[\ln\frac{r_e}{r_w}+s\right]}$$

@ any p; $p_R \lesssim 150$ bar

Gas Rate Eq. IN RESERVOIR (into the wellbore @ p_{wf})

Gas Rate Eq. IN TUBING

@same elevation

$$q_g = C_T \underbrace{\left(p_w^2 - p_t^2\right)^{0.5}}_{\text{ONLY FRICTION LOSSES}}$$

Tubing Constant

$C_T(H, D)$

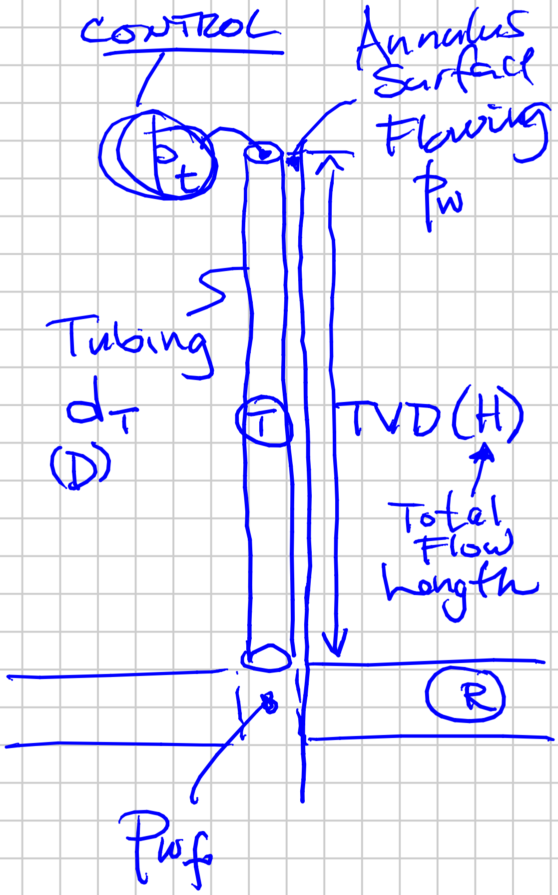
$$\frac{p_{wf}}{p_w} = \exp\left[S/2\right]$$

Not skin

TVD, (H) $\bar{Z}\,\bar{T}$

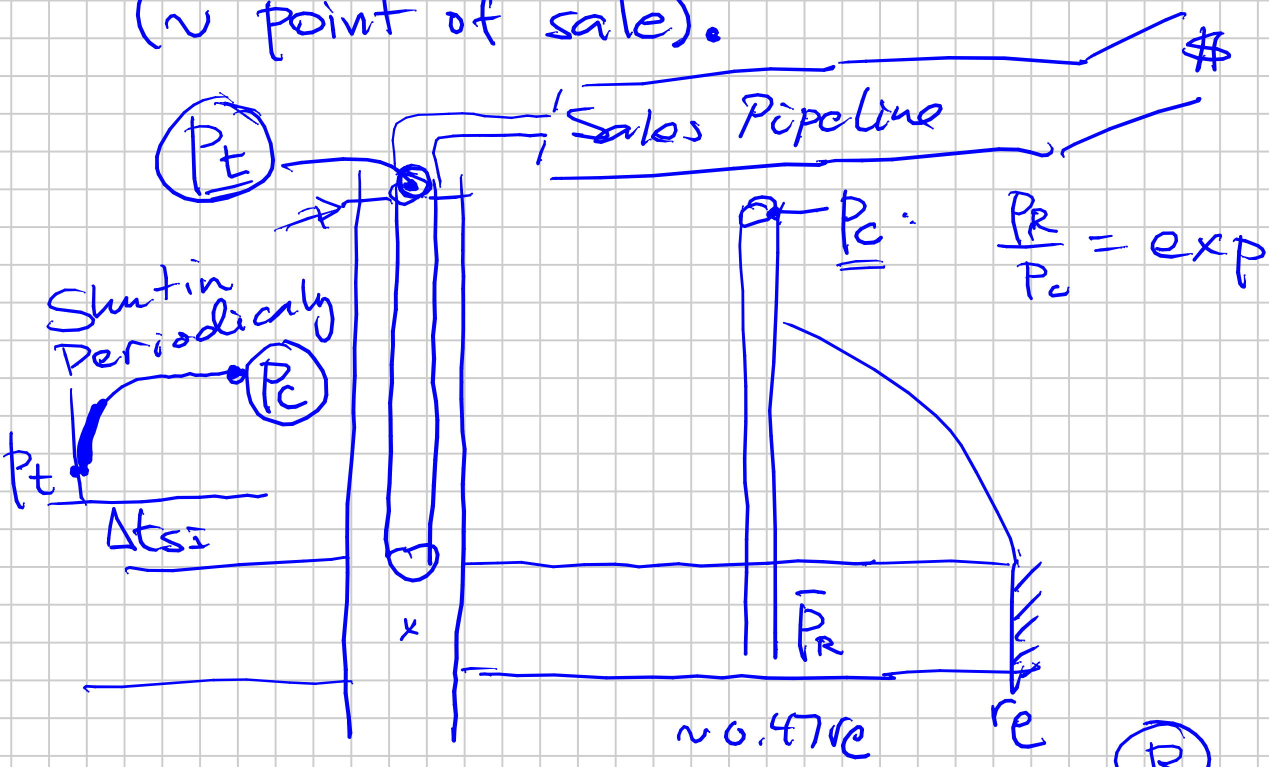
M_g

1st Order Correct for any gas flow in pipe



"WELLHEAD BACKPRESSURE" EQUATION

Combines the $\Delta p_R + \Delta p_T \Rightarrow$ Total Δp from R (e.g. @ r_e) to top of tubing (~ point of sale).



$$\frac{p_R}{p_c} = \exp(S/2)$$

p_R : Reservoir Gas Rate Eq.

Ⓡ		Ⓣ	
p_{wf}	q_g	p_w	p_t

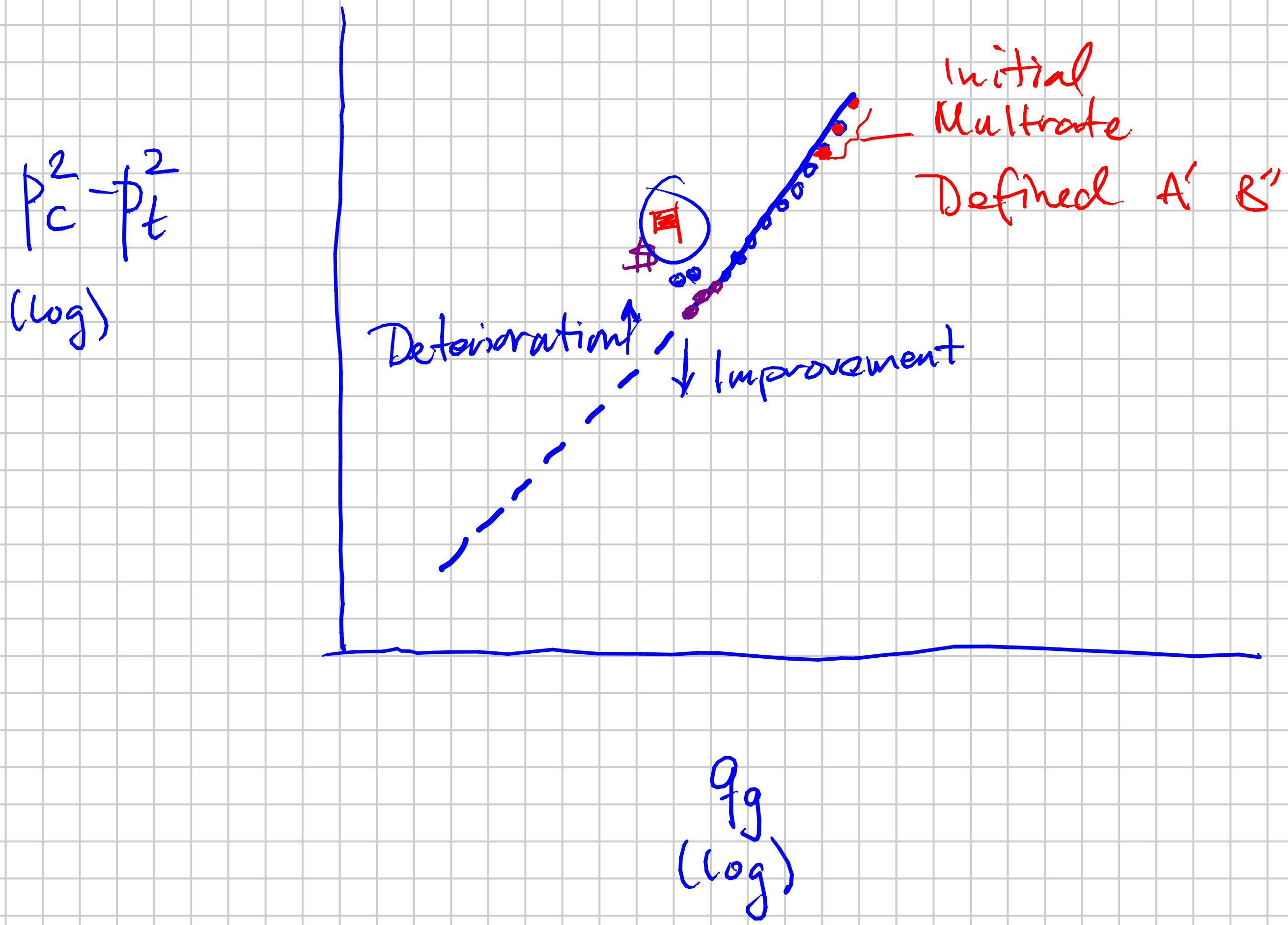
$$Ⓡ \quad \frac{p_R^2}{e^S} - \frac{p_{wf}^2}{e^S} = \left(\frac{\overset{\overset{kh,D}{\downarrow}}{B}}{e^S}\right) q_g^2 + \left(\frac{\overset{\overset{kh,s}{\downarrow}}{A}}{e^S}\right) q_g$$

$$(p_c^2 - p_w^2) = B' q_g^2 + A' q_g \quad : Ⓡ\ \Delta p$$

$$Ⓣ \quad \underline{(p_w^2 - p_t^2)} = \underline{\frac{1}{C_T^2} q_g^2} \quad : Ⓣ\ \Delta p$$

$$(p_c^2 - p_t^2) = \left(\underset{\uparrow}{B'} + \frac{1}{\underset{\uparrow}{C_T^2}}\right) q_g^2 + A' q_g \quad : \text{TOTAL } \Delta p$$

$$(p_c^2 - p_t^2) = \underset{\underset{(R,T)}{\uparrow}}{B''} q_g^2 + \underset{\underset{(R)}{\uparrow}}{A'} q_g$$

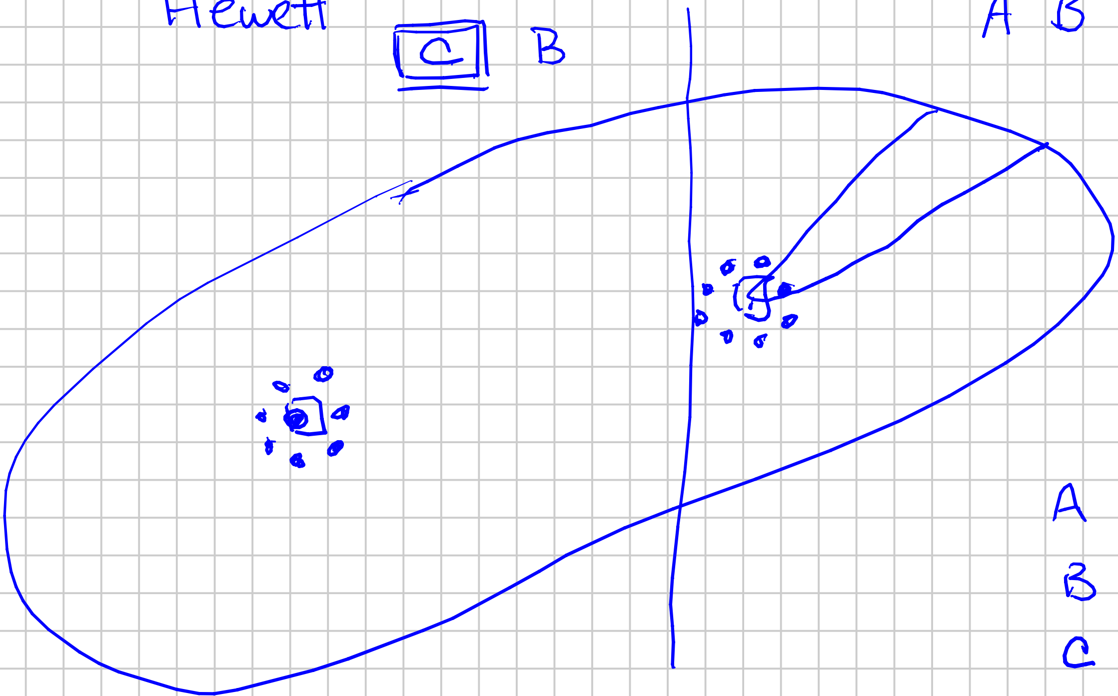


$$\frac{\Delta p_R}{\Delta p} \sim \frac{\Delta p_T}{\Delta p}$$

Less Control

More Control

Hewett



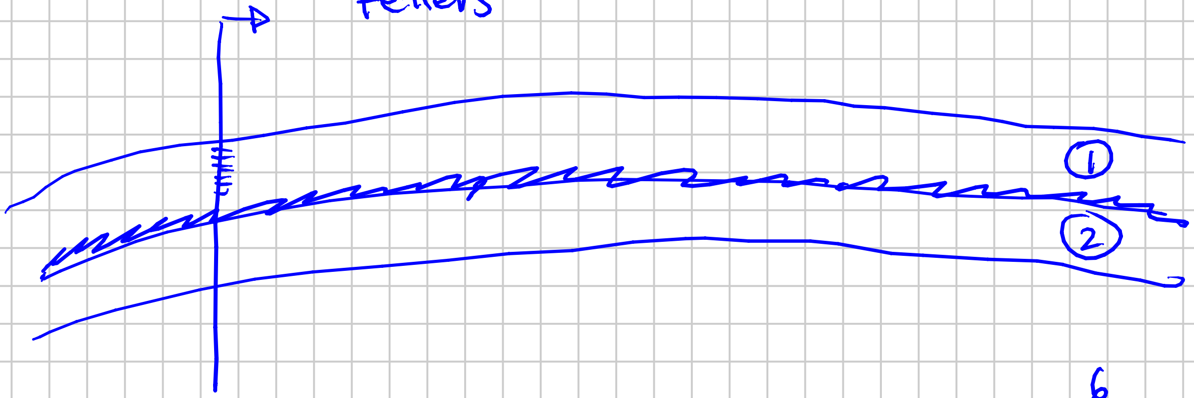
% q_g^{Test}

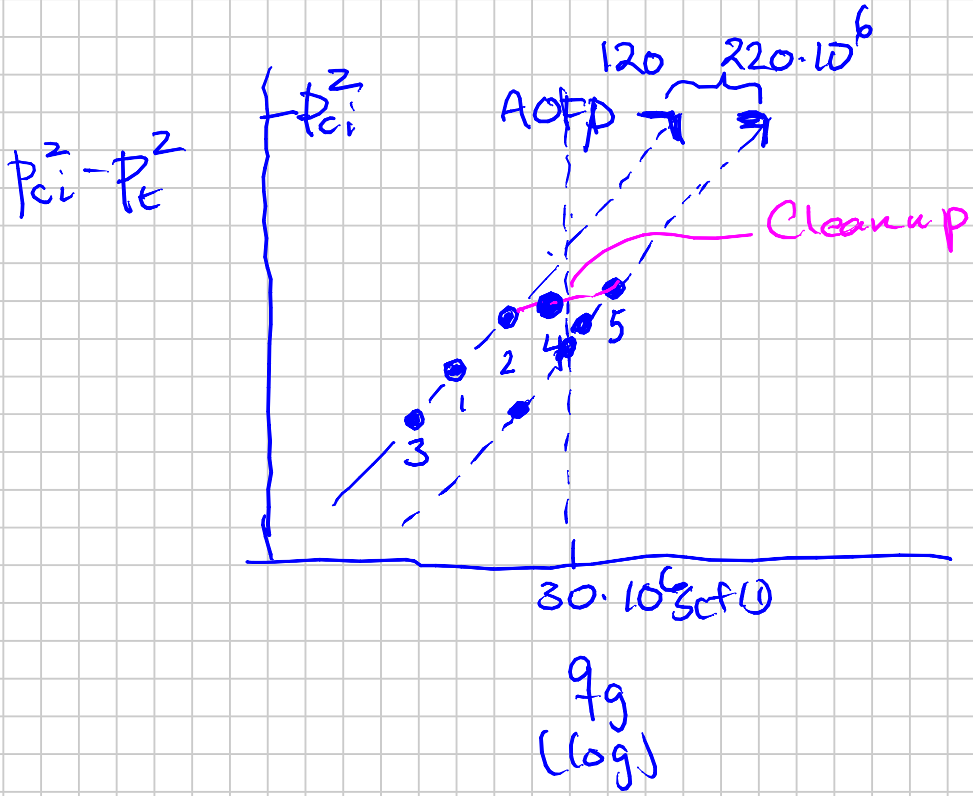
A

B

C

Fetters





NOBLIN GAS WELL MODEL "PROBLEM"

Note Title 2010-09-21

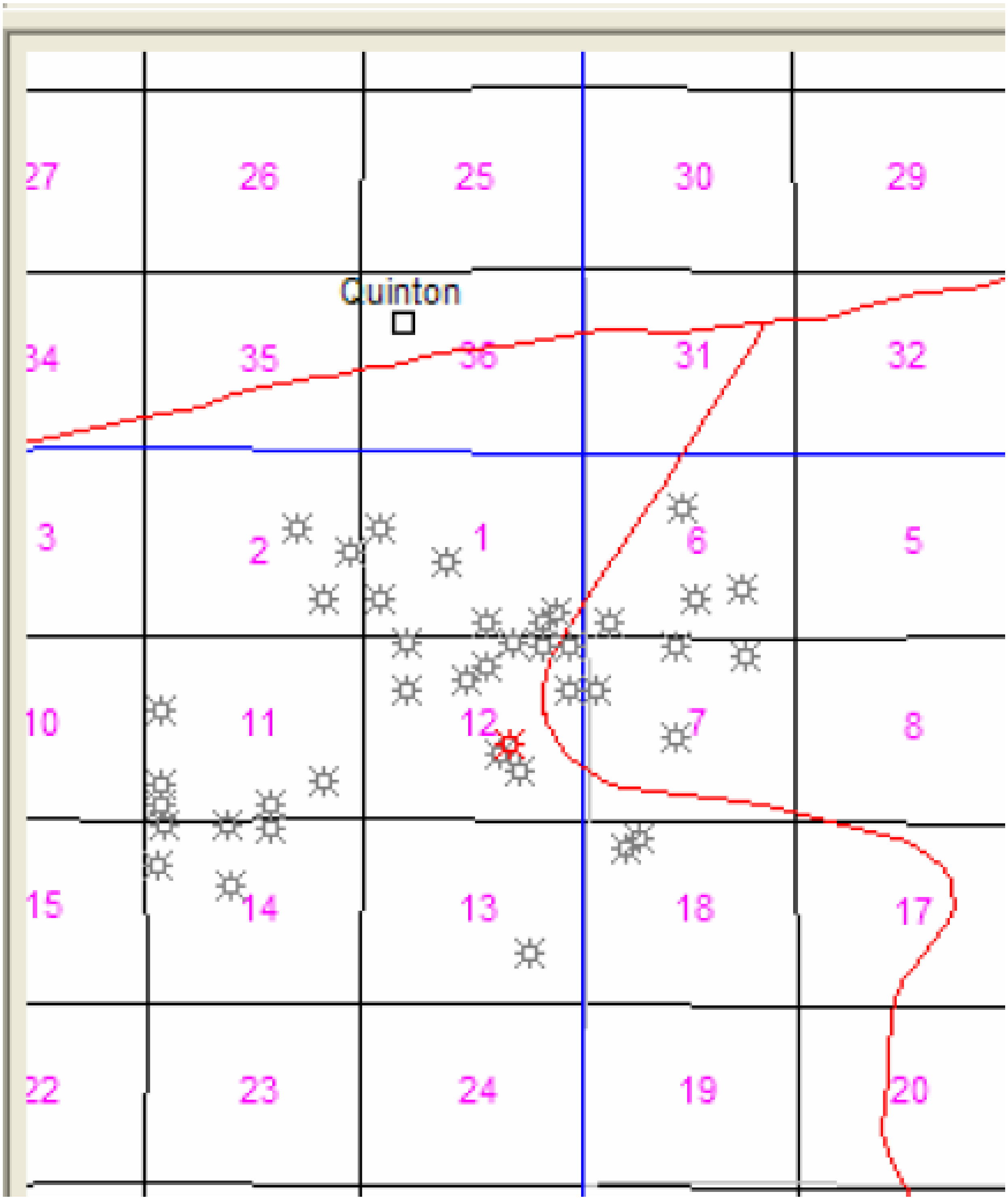
GOAL:

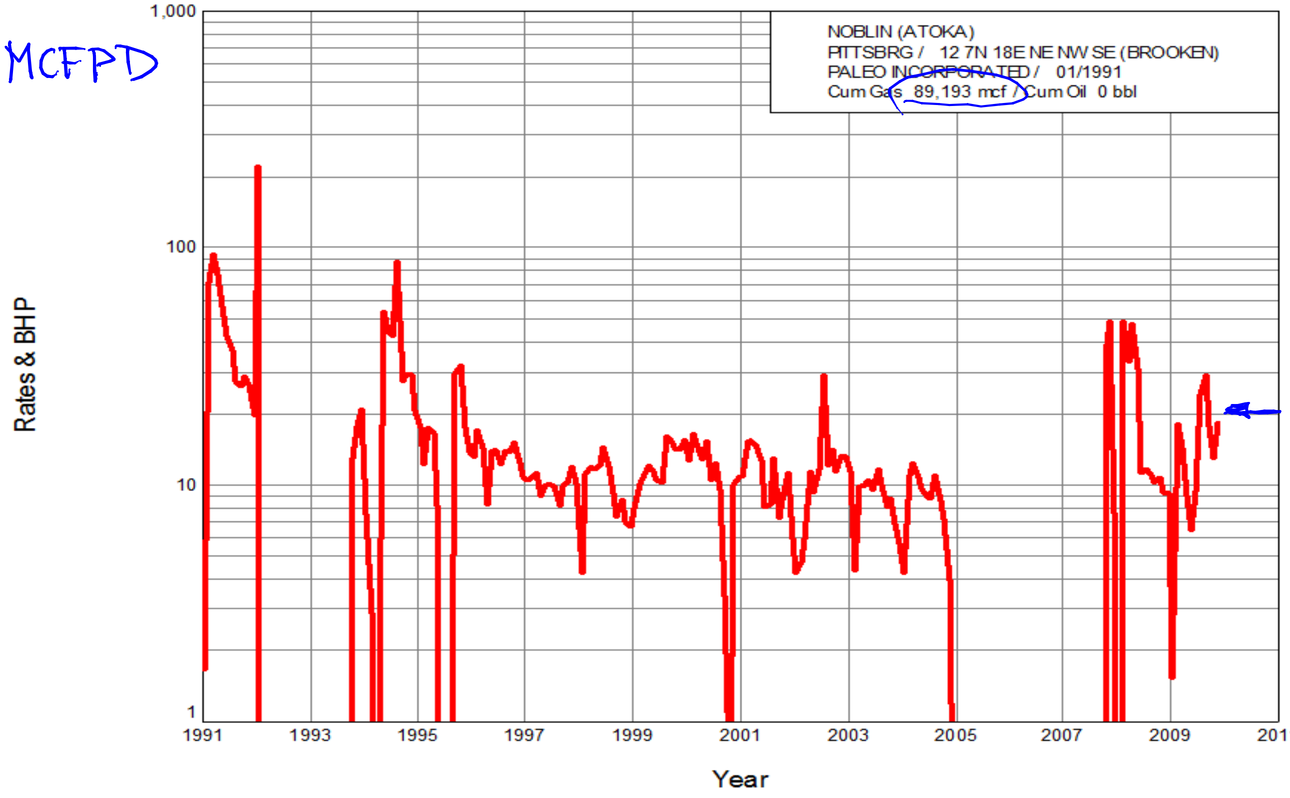
- Understand/orient about Noblin (gas) well in Oklahoma – part of the NTNU/IOC research project ROOKIE

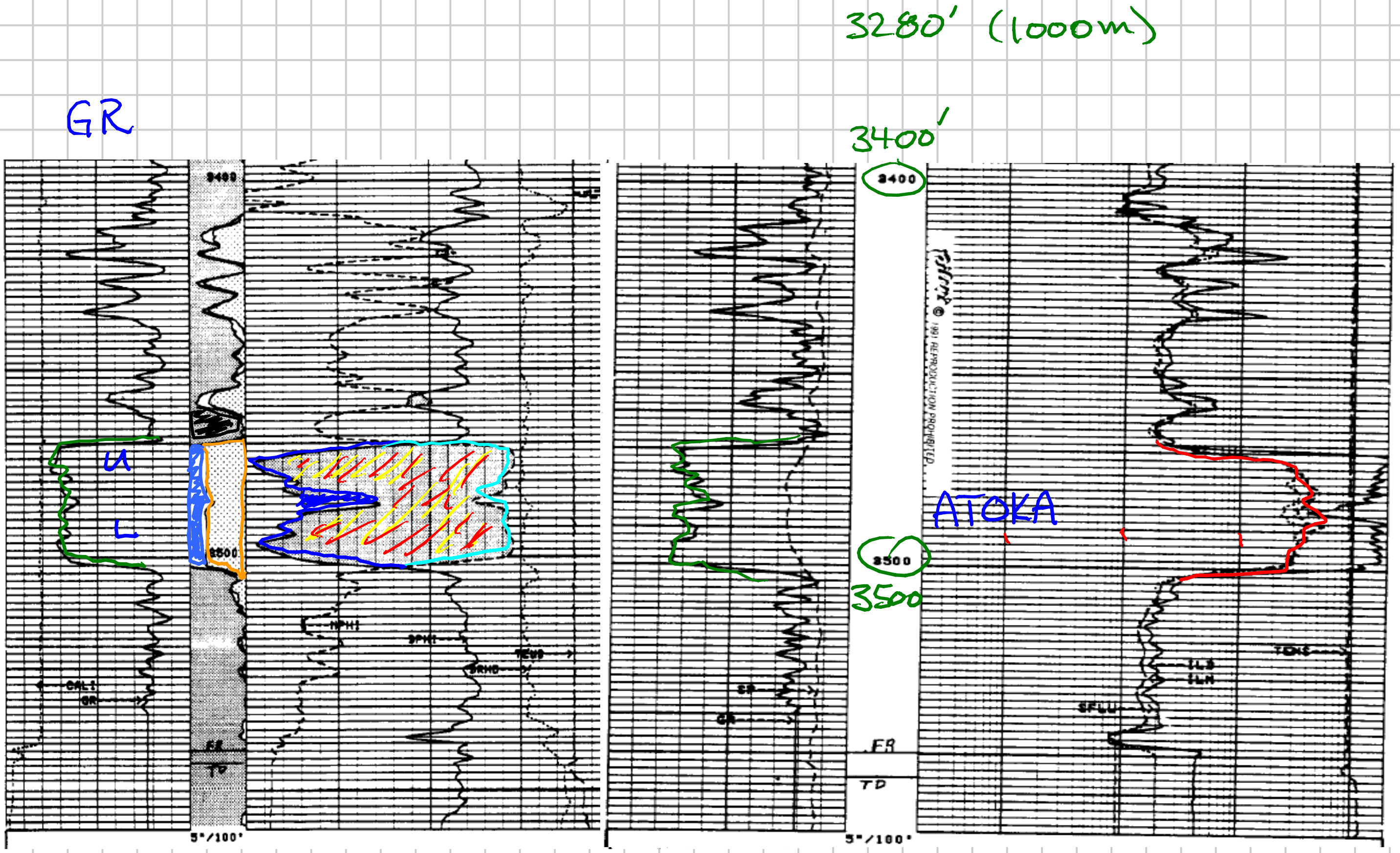
  Remote Operations in Oklahoma Intended for Education

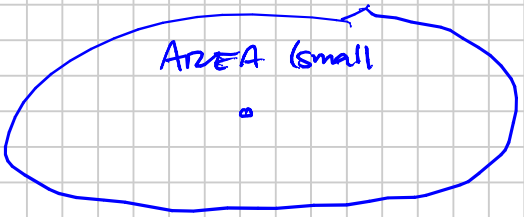
- Build a simple model of the Noblin well using basic theory and models
- Use model to compare & understand actual historical production performance
- Use model to design production test(s) of the Noblin well
  - To verify model
  - To optimize production
  - To recommend operation changes and/or well intervention.

CROW CREEK
NOBLIN
12, 7N-18E, NE NW SE
LATIMER CO.

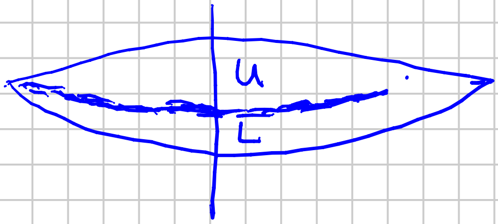



MCFPD
1,000
100
10
1
Rates & BHP
NOBLIN (ATOKA)
PITTSBRG / 12 7N 18E NE NW SE (BROOKEN)
PALEO INCORPORATED / 01/1991
Cum Gas 89,193 mcf / Cum Oil 0 bbl
1991
1993
1995
1997
1999
2001
2003
2005
2007
2009
2011
Year

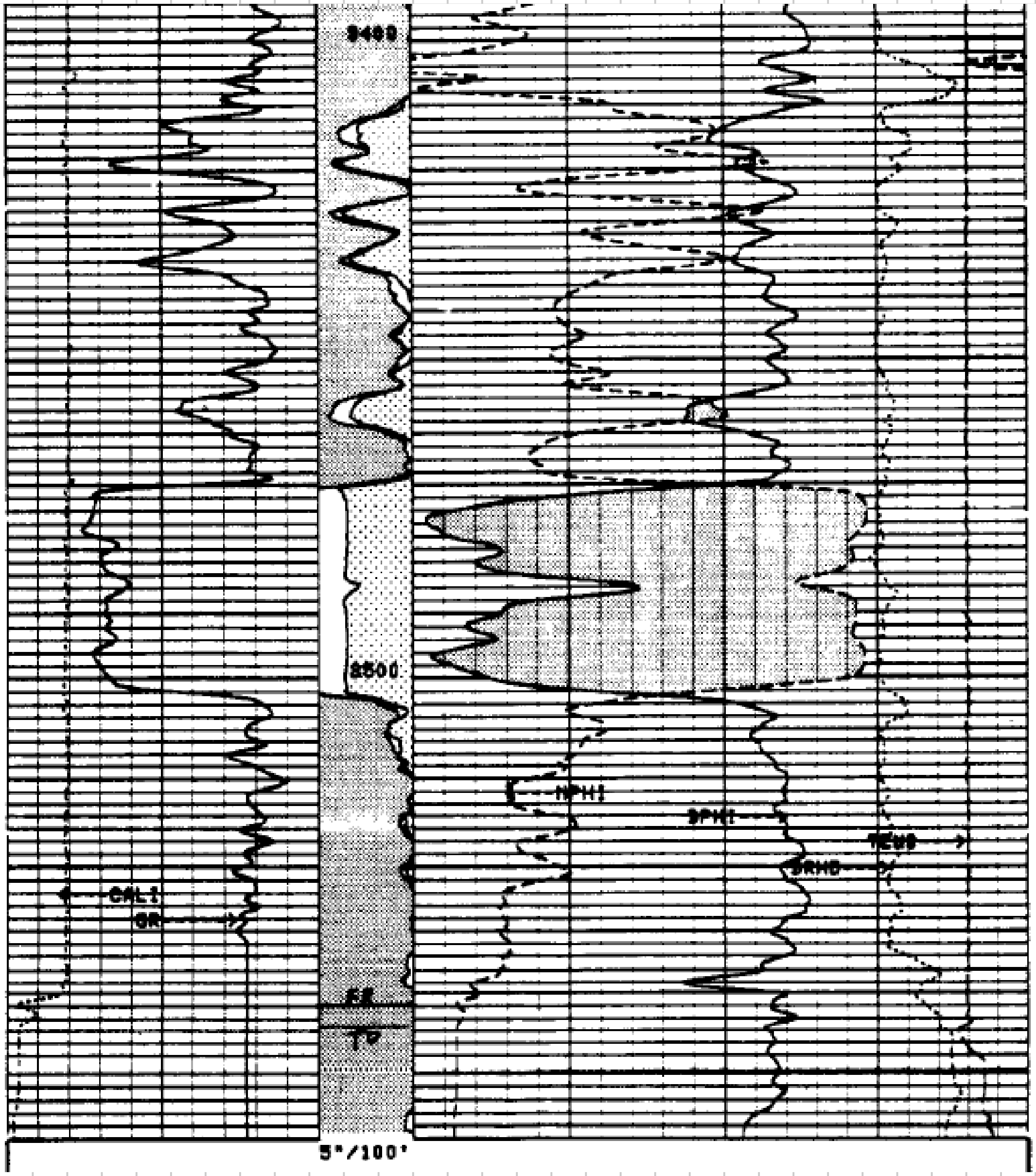

3280' (1000m)
GR
3400'
3400
U
L
3500
3500
ATOKA
CALI
GR
NPHI
DPHI
DRHO
TENS
SP
GR
FR
TD
5"/100'
5"/100'
Petroleum Information © 1991 REPRODUCTION PROHIBITED.
ILD
ILM
SFLU
TENS



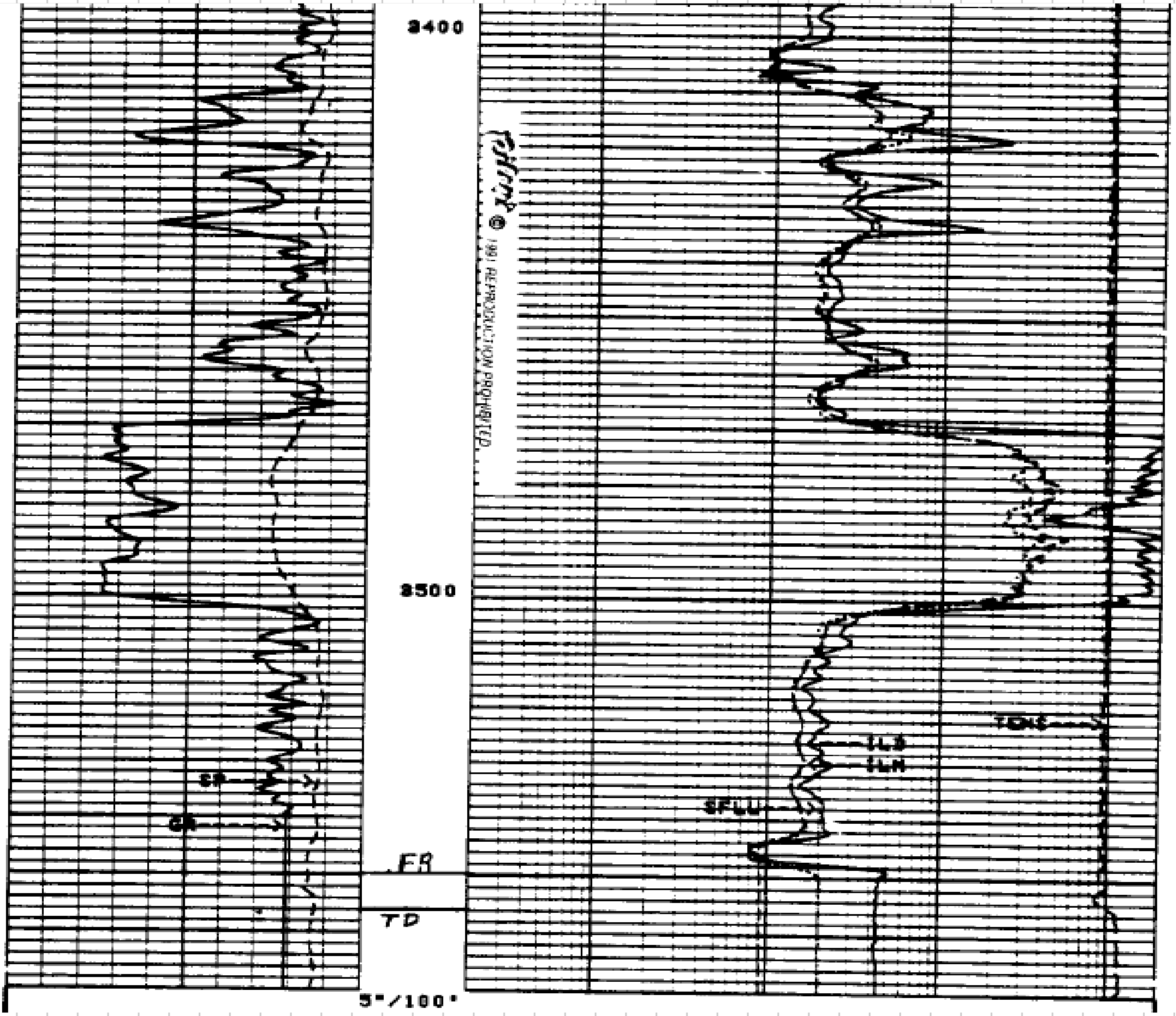
Top View



Side View



Middle Atoka
3470- 3502


3400
3500
SP
GR
FR
TD
SFLU
ILD
ILM
TENS
5"/100'
Riley's © 1991 REPRODUCTION PROHIBITED

PI/Dwights PLUS on CD Well Summary Report

General Information

State	: OKLAHOMA	Final Status	: GAS
County	: PITTSBURG	Drill Total Depth	: 3622
Field	: QUINTON DISTRICT	Log Total Depth	:
Operator Name	: SAMSON RESOURCES CO	True Vertical Depth	:
Lease Name	: NOBLIN		
Well Number	: 2-12	Spud Date	: MAR 28, 1990
API Number	: 35121215230000	Comp Date	: MAY 19, 1990
Regulatory API	:		
Init Lahee Class	: D	Hole Direction	: VERTICAL
Final Lahee Class	: DG	Reference Elevation	:
Permit Number	:	Ground Elevation	: 610
Geologic Province	: ARKOMA BASIN	KB Elevation	:
Formation at TD	: 409PSLV	PENNSYLVANIAN	
Oldest Age Pen	: 403	PENNSYLVANIAN ATOKAN/BEND/POTTSVILLIAN	
Producing Formation	: 403ATOKM	ATOKA MIDDLE	
Township	: 7 N	Section	: 12 SEC
Range	: 18 E	Spot	: NE NW SE
Base Meridian	: INDIAN		

Additional Location Information

Footage Location	: 2055 FSL 910 FWL CONGRESS QTR		
Latitude	: 35.0928600	Latitude (Bot)	:
Longitude	: -95.3545800	Longitude (Bot)	:
Lat./Long. Source	: TB		
Location Narrative	: Type	Remark	
	LOCATION	2 3/4 MI S & 1/2 MI E QUINTON, OK	

Initial Potential Tests

Test	Top Form	Base Form	Top Depth	Base Depth	Choke	GOR	Oil Grav	Prod Method	Test Method
001	403ATOKM	403ATOKM	3470	3502	48/64			PERF	FLOWING

IP Volume

Test	Amount	Oil Unit	Desc	Amount	Cond Unit	Desc	Amount	Gas Unit	Desc	Amount	Wtr Unit	Desc
001							1150	MCFD				UW

$q_g \propto \frac{1}{\ln\frac{r_e}{r_w}}$

p_t p_c

IP Pressure

Test	FTP	SITP	FCP	SICP
001	65	420		

q_g Initially : Flush Production

$p_D \sim 1-2$ for 1-2 days

IP Perforation

Test	Top	Base	Type	Method	Top Form	Base Form	Status	Count	Density	Per
001	3470	3502		PERF	403ATOKM	403ATOKM				

Production Tests

$q_g \propto \frac{1}{p_D(t)}$

Test	Top Form	Base Form	Top Depth	Base Depth	Choke	GOR	Oil Grav	Prod Method	Test Method
001	403ATOKM	403ATOKM	3470	3502				PERF	UNDESIGNATED

1 month

$p_D \sim 8$

Production Volume

Test	Amount	Oil Unit	Desc	Amount	Cond Unit	Desc	Amount	Gas Unit	Desc	Amount	Wtr Unit	Desc
001												

Production Treatment

Test	Top	Base	Volume	Meas	Amount	T/P	PSI	Inj	Type	Nbr	Agent	Add
001	3470	3502	3600	GAL					ACID			

Production Perforation

PI/Dwights PLUS on CD Well Summary Report

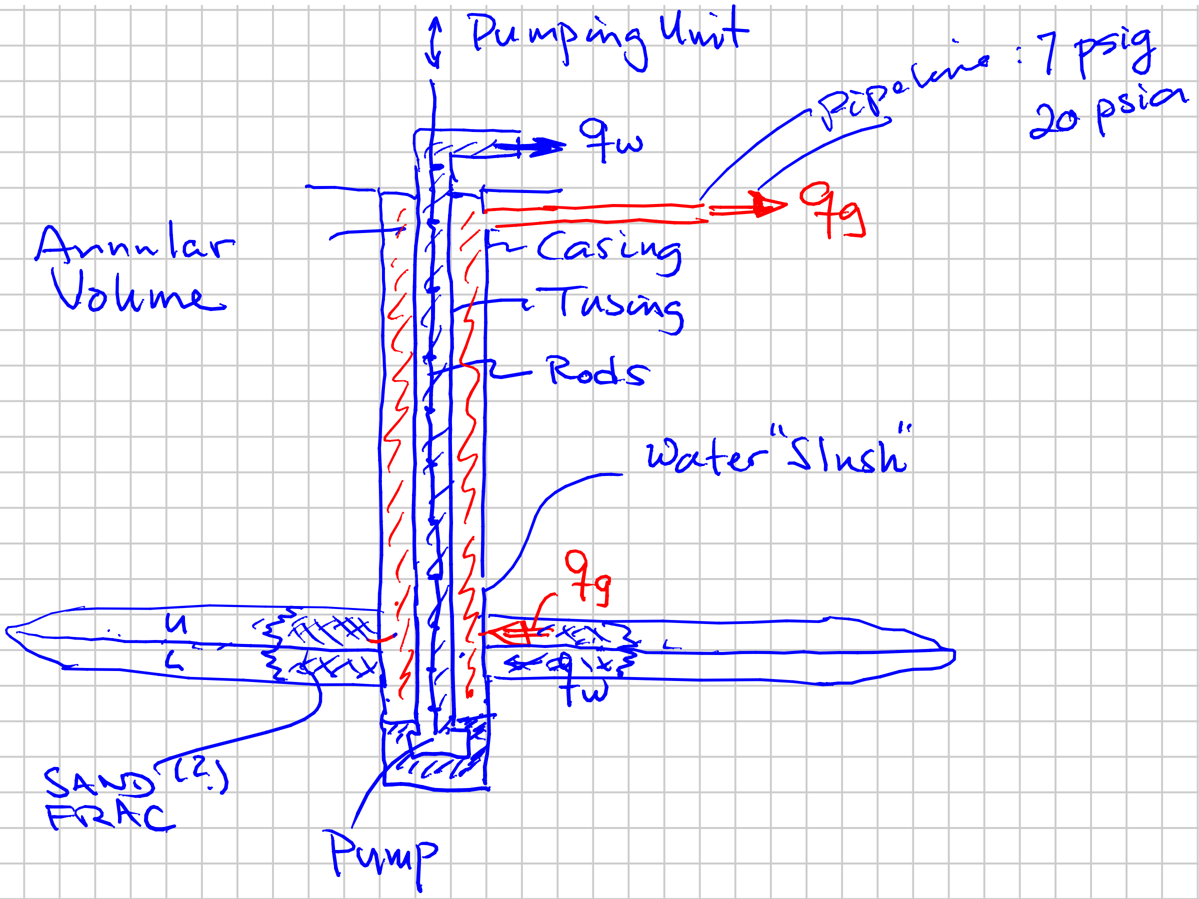
Test	TopBase		Type	Method	Top Form	Base Form	Status	Count	Density	Per
001	3470	3502		PERF	403ATOKM	403ATOKM				

------------------- **Formations** ----------------------------

Form Code	Form Name	Top Depth	Top TVD	Base Depth	Base TVD	Source	Lithology
404BOCHU	BOOCH UPPER	761				LOG	
404BOCHL	BOOCH LOWER	938				LOG	
404HRSR	HARTSHORNE	1680				LOG	
403ATKLM	ATOKA /LM/	3410				LOG	SS

------------------- **Casing Data** ----------------------------

Size	Base Depth	Cement	Unit
8 5/8 IN	502	150	SACK
4 1/2 IN	3606	400	SACK



MODELING

- Material Balance Eq.

Assume • $Z(p)$ negligible : $Z_g = 1$
• No Water Influx

$$\boxed{P_c = P_{ci}\left(1 - \frac{G_p}{G_i}\right)}$$

G_1
G_2

Two Reservoir Unit Model:

Upper (1)

Lower (2)

- Reservoir Rate Eq.

Assume: • $D = 0$
• $m(p) \sim p^2$

$$\boxed{q_g = C_R'(P_c^2 - P_w^2)}$$

C_{R1}
C_{R2}

$$\left(\frac{p_{wf}}{p_w}\right) = \exp(S/2)$$

- Annulus ("Pseudo-Tubing") Rate Eq.

$$\underbrace{P_{Ri} \Rightarrow P_{ci}}_{\exp(\text{"}S\text{"}/2)} = 420 \text{ psig} = 435 \text{ psia}$$

$$\boxed{q_g = \underset{\uparrow}{C_T}(p_w^2 - p_t^2)^{0.5}}$$

Annulus Flow : Equivalent Hydraulic Diameter

$$\text{"}d_t\text{"} = \left[\frac{A_{annulus}}{(\pi/4)}\right]^{0.5}$$

$$\pi\left(\frac{d_t}{2}\right)^2 \cong A_{annulus}$$

Calculation Procedure:

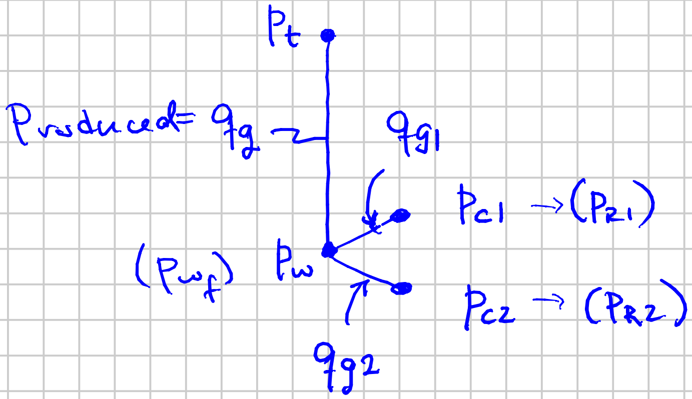
$t = 0 \quad P_R = P_{Ri}$

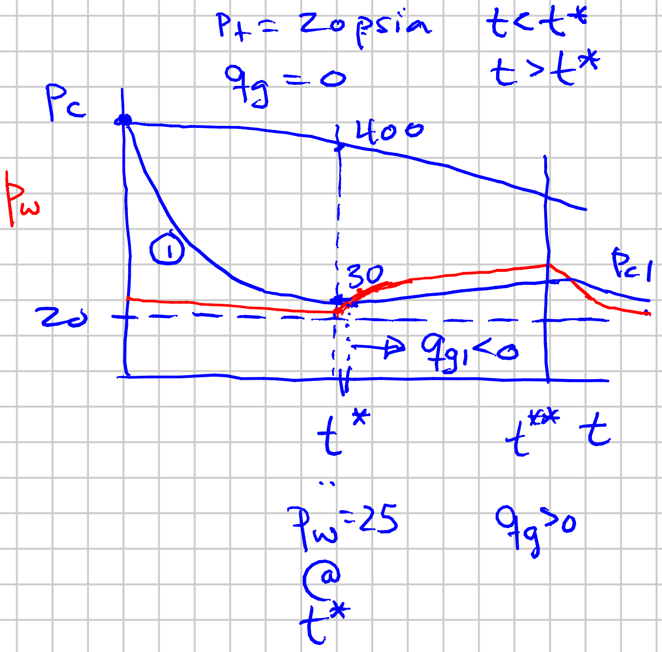
(1) Specify p_t
- Calculate q_g produced out of well
- Calculate $p_w, p_{wf}, p_c, p_R = f(t)$

(2) Specify q_g produced out of well

- Calculate $p_t(t)$
- Calculate $p_w, p_{wf}, p_c, p_R = f(t)$

(3) Hybrid : p_t specified or q_g specified





$q_g \geq 0$

q_{g1} ±

q_{g2} ±

If $V_{wellbore} \sim 0$ ("Zero" Wellbore Storage" - DTA)

$$q_g = q_{g1} + q_{g2}$$

For Δt : q_g, q_{g1}, q_{g2} are constants

$$\Delta G_{p1} = \Delta t \cdot q_{g1}$$

ΔG_p = change in cum. gas produced

$$\Delta G_{p2} = \Delta t \cdot q_{g2}$$

Update @ end of Δt : $G_{p1}^{end} = G_{p1}^{start} + \Delta G_{p1}^{\Delta t}$

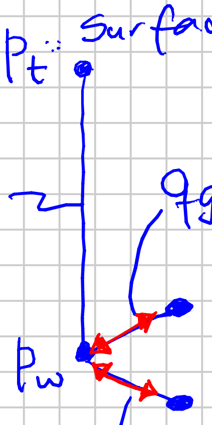
$$G_{p2}^{end} = G_{p1}^{start} + \Delta G_p^{\Delta t}$$

$\Delta G_{p1}, \Delta G_{p2}$ ±

But

$$\Delta G_{p1} + \Delta G_{p2} = \Delta G_p \geq 0$$

p_t : Surface Pressure @ Point of "Sales"



$(q_{g1} + q_{g2}) = q_g \geq 0$ ✓

$C_T (p_w^2 - p_t^2)^{0.5} \doteq$ ✓

calc. $\nearrow$ assume (p_{wf}) $\nearrow$

$q_{g1} (\pm) = C_{R1} (p_{c1}^2 - p_w^2)$ ✓ ✓ ✓

p_{c1} (p_{R1})

p_{c2} (p_{R2})

$q_{g2} (\pm) = C_{R2} (p_{c2}^2 - p_w^2)$

Don't Know

$\rightarrow C_{R1}$ G_1 ✓

$\rightarrow C_{R2}$ G_2 ✓

$\left.\right\}$ $G_1 + G_2 > G_p$ 2010 $\sim 3 \cdot 10^6 \, Sm^3$

$$\frac{p_{R1}}{p_{c1}} = \frac{p_{R2}}{p_{c2}} = \frac{p_{wf}}{p_w} = \text{cnst}$$

1.08

$C_T \simeq 4000 \,(Sm^3/d)/bar^2$

Know:

C_T , $\exp(S/2)$

$p_{c1_i} = p_{c2_i} = 420$ psig $= 435$ psia $= 30$ bara

guess: 10 md
5 m

$$C'_{R_\ell} = \frac{7.7 \quad k_\ell h_\ell}{e^{s/2} T_R \left[\underbrace{\ln \frac{r_e}{r_w}} - \frac{3}{4} + s \right] (\mu_{gi} z_{gi})} = \frac{7.7 \, (10)(5)}{311 \, (5)(1.08)(0.012)(1)}$$

$$= \frac{0.25}{0.012} = 21 \quad \frac{Sm^3/d}{bar^2}$$

$(273 + 38)$
311 K

Stimulation
$s < 0$
$s = -2$

$$\underbrace{\ln\frac{r_e}{r_w} - \frac{3}{4}}$$

$r_w = 0.1$ m (~ 8" hole)

$r_e = 200$ m

$$\ln\left(\frac{200}{0.1}\right) - \frac{3}{4} - 2 = 5$$

* Given q_g

Guess p_t

$q_g = C_T(p_w^2 - p_t^2)^{0.5}$

Calc $p_w = \left[\frac{q_g^2}{C_T^2} + p_t^2\right]^{1/2}$

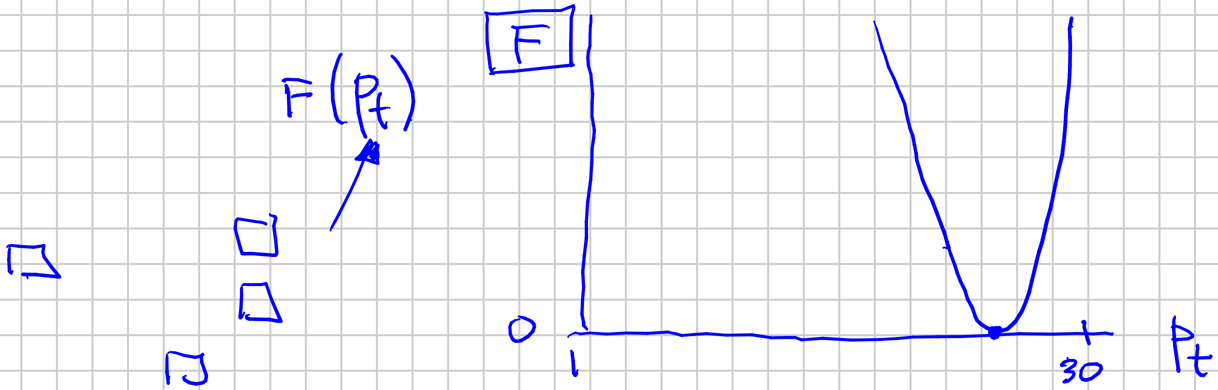
$$\boxed{p_{c\,\text{new}\atop T.S.} = p_{ci}\left(1 - \left(\frac{G_p}{G}\right)_{\text{end previous T.S}}\right)}$$

Sum of Squares function

$$F_{SSQ} = F = \sum \underbrace{\left(\frac{q_m - q_{input}}{q_{input}}\right)}_{\text{deviation}}^2$$

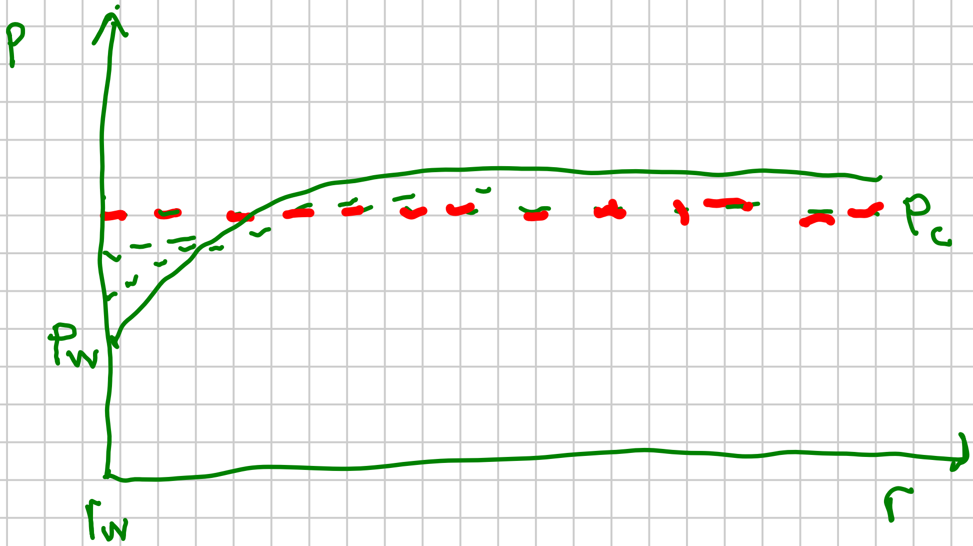
Change p_t to get $F = 0$

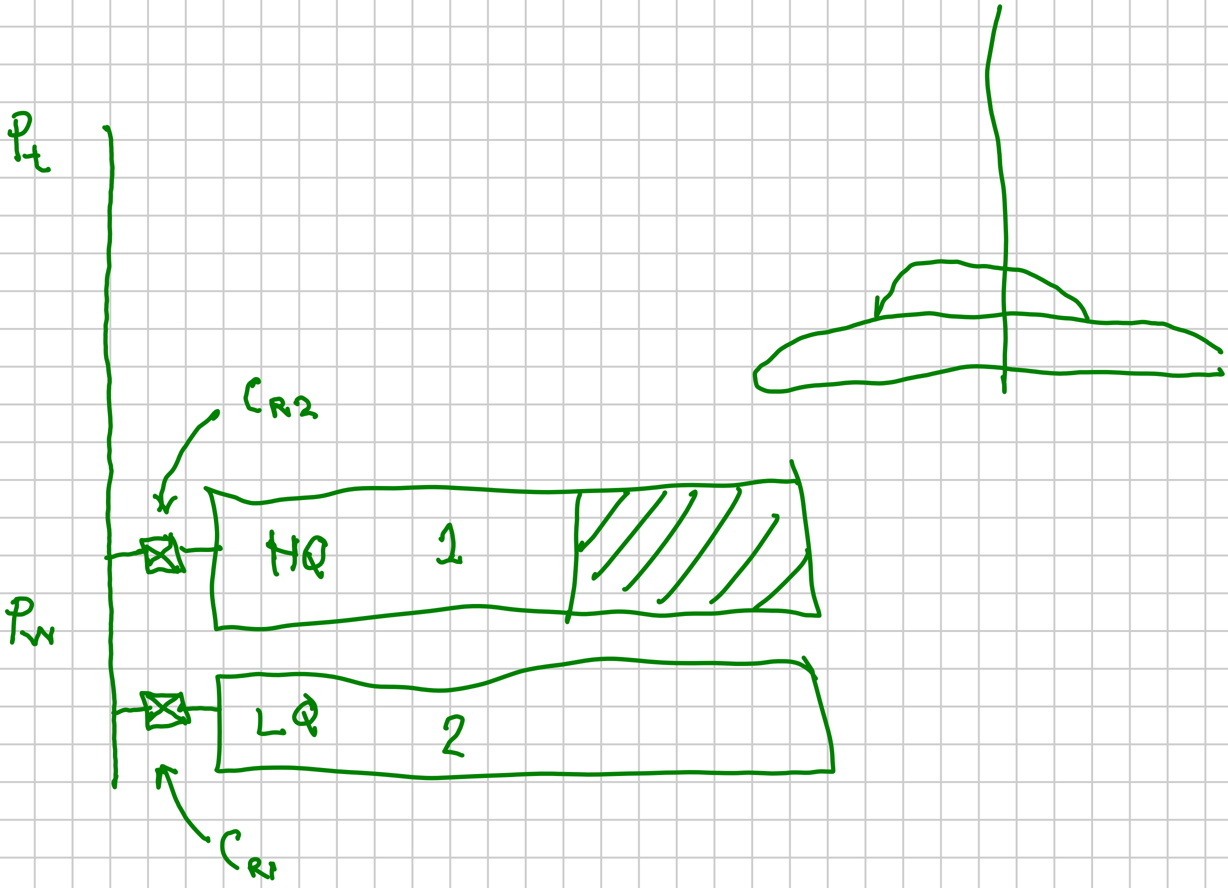
minimize



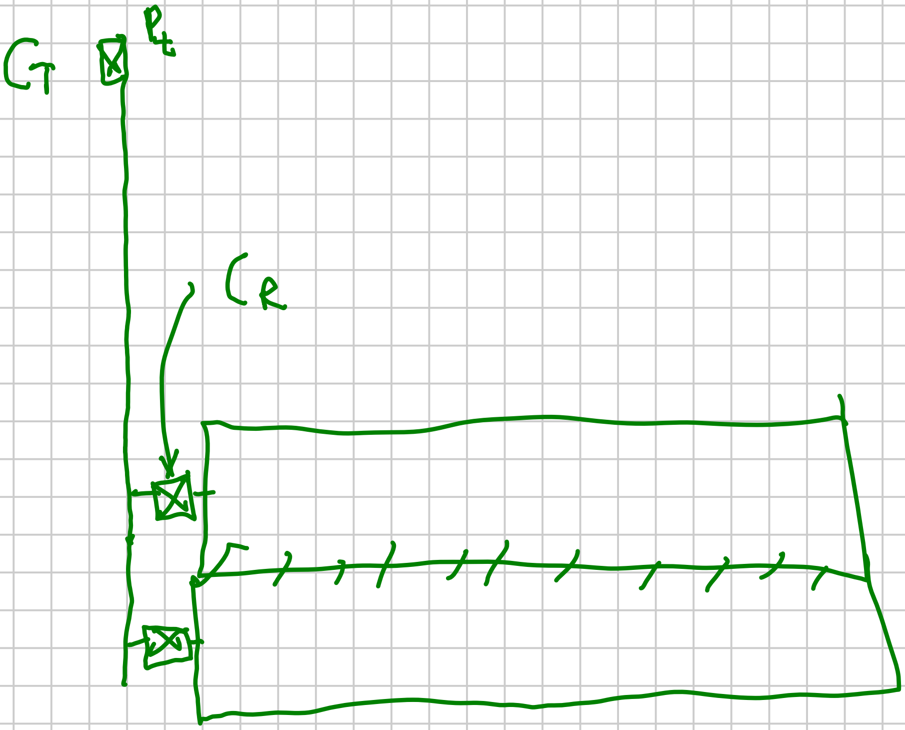
Specify p_t

- Guess q_g ↕
- Calc. p_w (Tubing Eq)
- Calc. q_{g1}, q_{g2}
- Check $q_g(\text{Guess}) = q_{g1} + q_{g2}$ ✓


p
p_c
p_w
r_w
r


p_t
p_w
C_{R2}
HQ
1
LQ
2
C_{R1}

The Model



Fitting the model

- q_{tg} @ initial test (32564)

- $$\left.\begin{array}{l} q_{tg} = 5800\ m^3/d \\ P_w = 8\ \text{bara} \end{array}\right\}\ \text{in jan 2008}$$

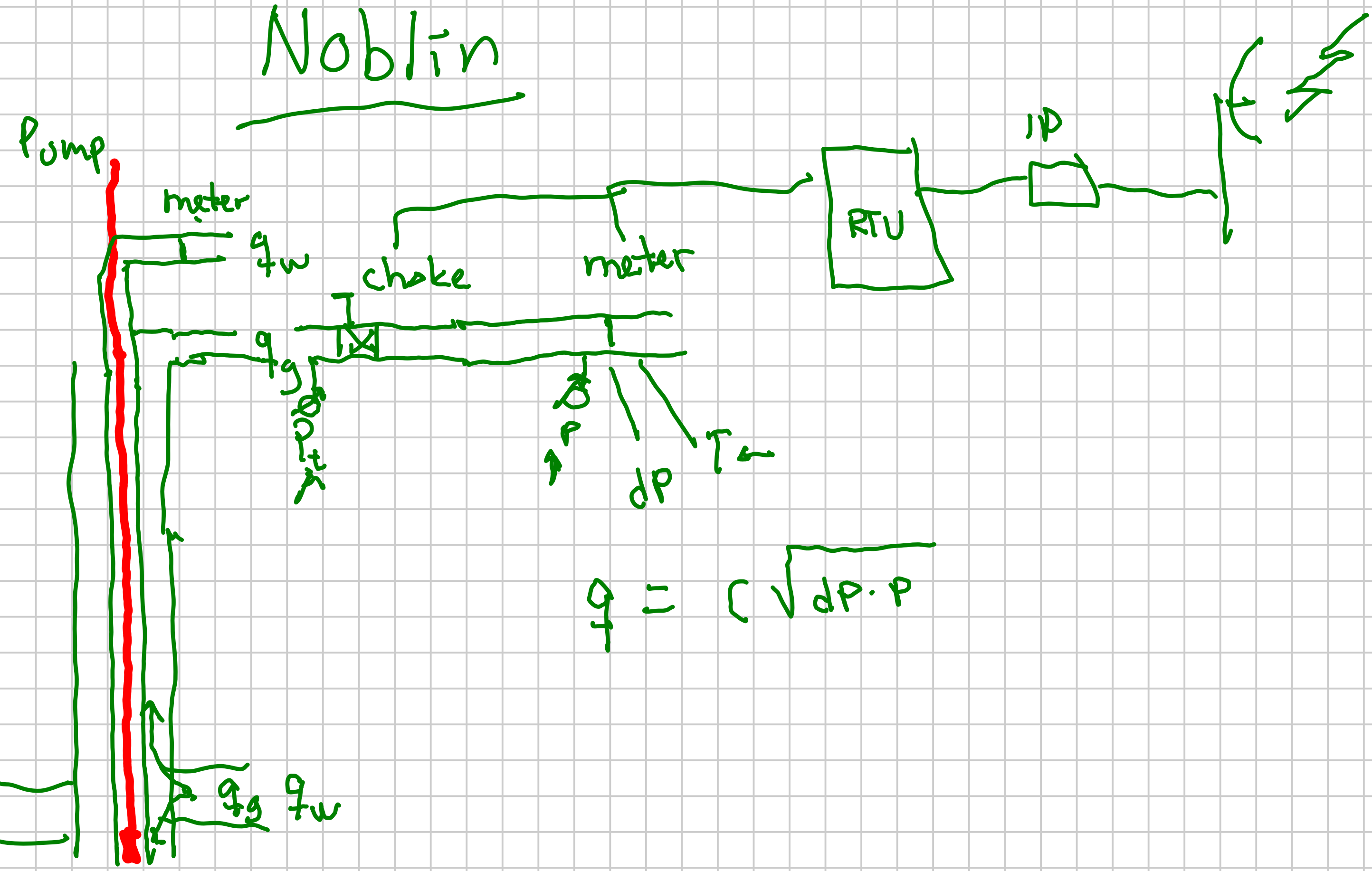
- $q_{tg} \leq 550\ m^3/d$ present

$P_t = 4.5$ bara after a week of SI

$$\Downarrow$$

changing : $\boxed{C_{R1}\ ,\ C_{R2}\ ,\ \sigma_1\ ,\ \sigma_2}$

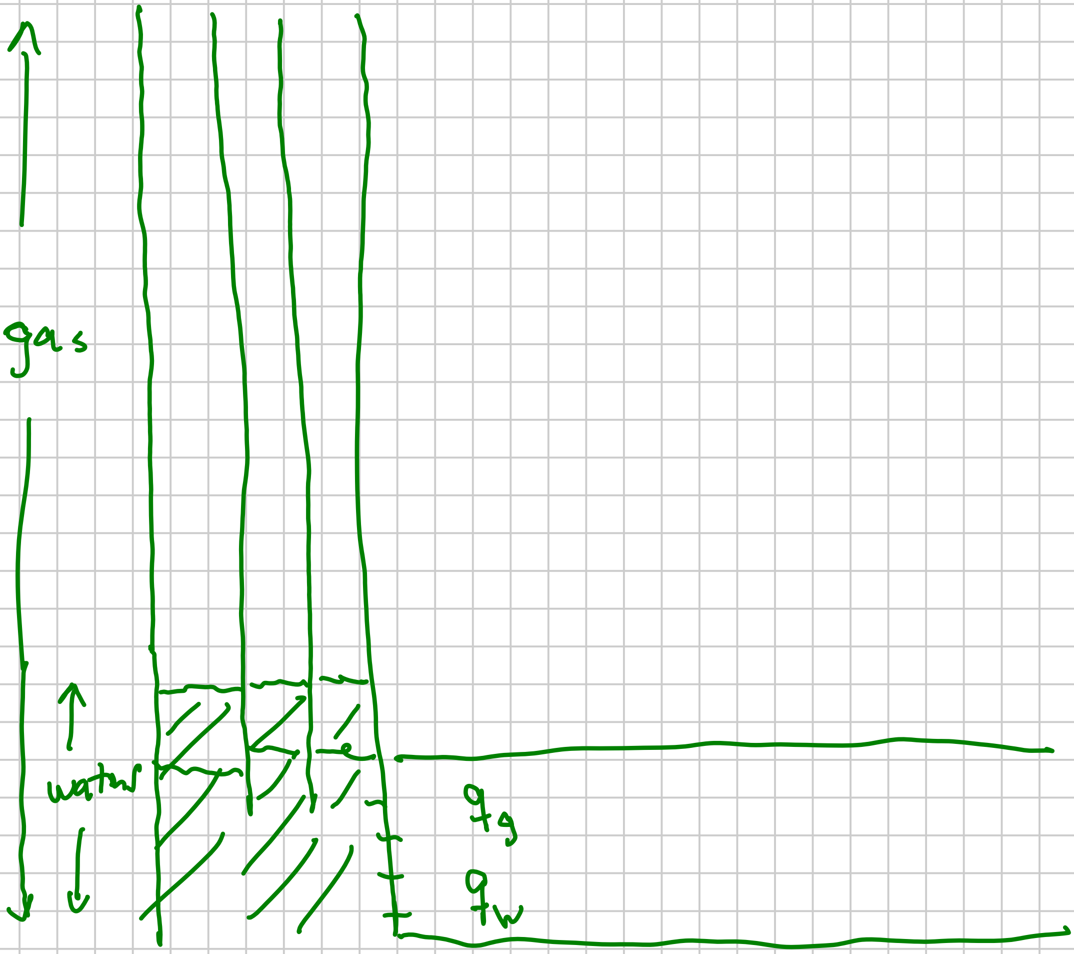
Noblin



$$q = C\sqrt{dP \cdot P}$$

Rookie GUI

folk.ntnu.no/kjetiljo/RookieGUI-kjetil


gas
water
q_g
q_w

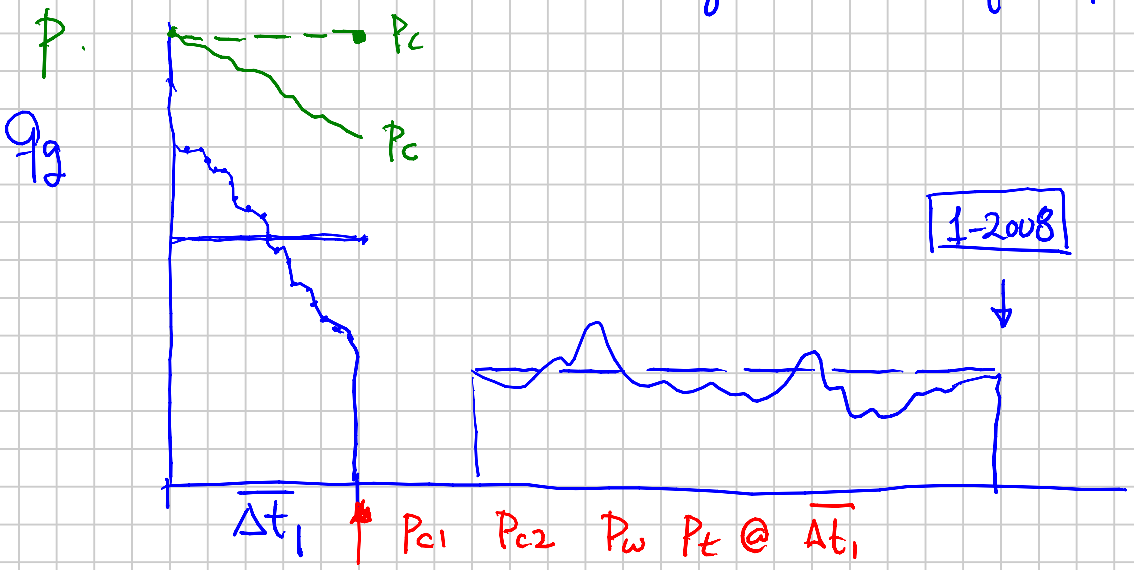
Computational Issue:

① $\Delta t \cdot q_{g_\ell} > (G_\ell - G_{p_\ell})$ = remaining gas

q_{g_ℓ} ↑ Calc'd $>$ ↑ Not allowed

$$q_{g_\ell} = \min\left(\underbrace{C_{R_\ell}(p_{c_\ell}^2 - p_w^2)}_{\checkmark}, \underbrace{\left(\frac{G_\ell - G_{p_\ell}}{\Delta t}\right)}_{\text{This rate will empty the layer}}\right)$$

② Reduce the number of time steps (i.e. variables p_t)



$$\bar{q}_g \cdot \overline{\Delta t}_1 = \sum_{i=1}^{34} q_{g_i} \Delta t_i$$

Check:

① $C_{R1} = 11$ $C_{R2} = 31$ $C_{R1}+C_{R2} = 42$

(21, 21)

$G_1 = 1\cdot10^6$ $G_2 = 3\cdot10^6$ $G_1+G_2 = 4\cdot10^6$

$[\Delta t = 1\ \text{mo}]$

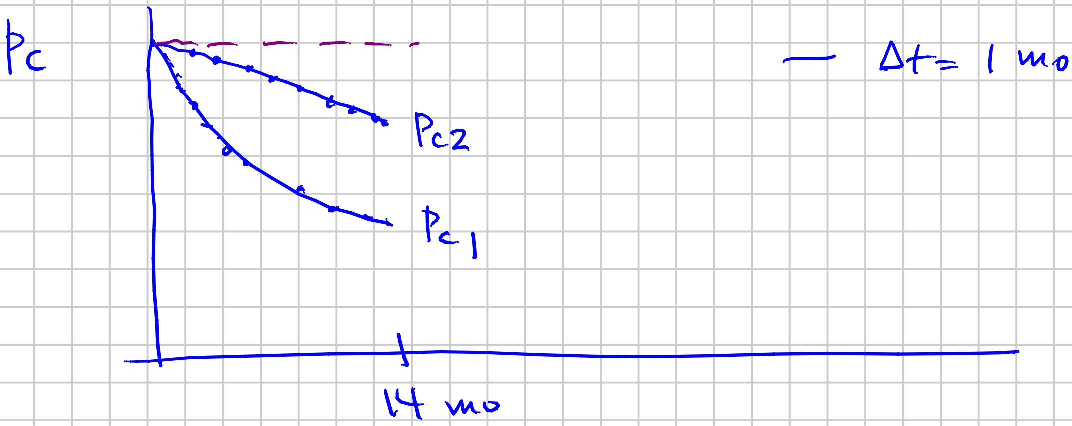
@ 14 mo $P_{c1} = 8.58$ bara $P_w = 4.09$ bara

$P_{c2} = 17.63$ bara $P_t = 3.89$ bara

$[\Delta t = 7\ \text{mo}]$ $P_{c1} = 6.75$ X

$P_{c2} = 27.24$

Bad Idea



- Decouple the pipe-flow (annulus) rate equation

⊙ Solve directly for p_w ?

- Then calculate p_t from p_w & q_g

Given (Historical rates)

↓

$$q_g = q_{g1} + q_{g2}$$

$$\underset{\checkmark}{C_{R1}} \left(\underset{\checkmark}{p_{c1}^2} - p_w^2\right) + \underset{\checkmark}{C_{R2}} \left(\underset{\checkmark}{p_{c2}^2} - p_{wf}^2\right)$$

What do we not know?

$$C_{R1} p_{c1}^2 - p_w^2 \left(C_{R1} + C_{R2}\right) + C_{R2}\, p_c^2 = q_g$$

$$\underbrace{\frac{\left(C_{R1} p_{c1}^2 + C_{R2} p_{c2}^2\right)}{\left(C_{R1} + C_{R2}\right)}} - p_w^2 = \frac{q_g}{\left(C_{R1} + C_{R2}\right)}$$

$$\boxed{\left(\bar{p}_c^{\,2} - p_w^2\right) C_R = q_g}$$

$$C_R = C_{R1} + C_{R2}$$

$$\bar{p}_c^{\,2} = \left(\frac{C_{R1}\, p_{c1}^2 + C_{R2}\, p_c^2}{C_{R1} + C_{R2}}\right)$$

$$p_w = \underbrace{\left\{ \bar{p}_c^{\,2} - \frac{\overset{\checkmark}{q_g}}{C_R} \right\}^{1/2}}$$

✓

$$p_t = \left[p_w^2 - \left(\frac{q_g}{C_t} \right)^2 \right]^{1/2}$$

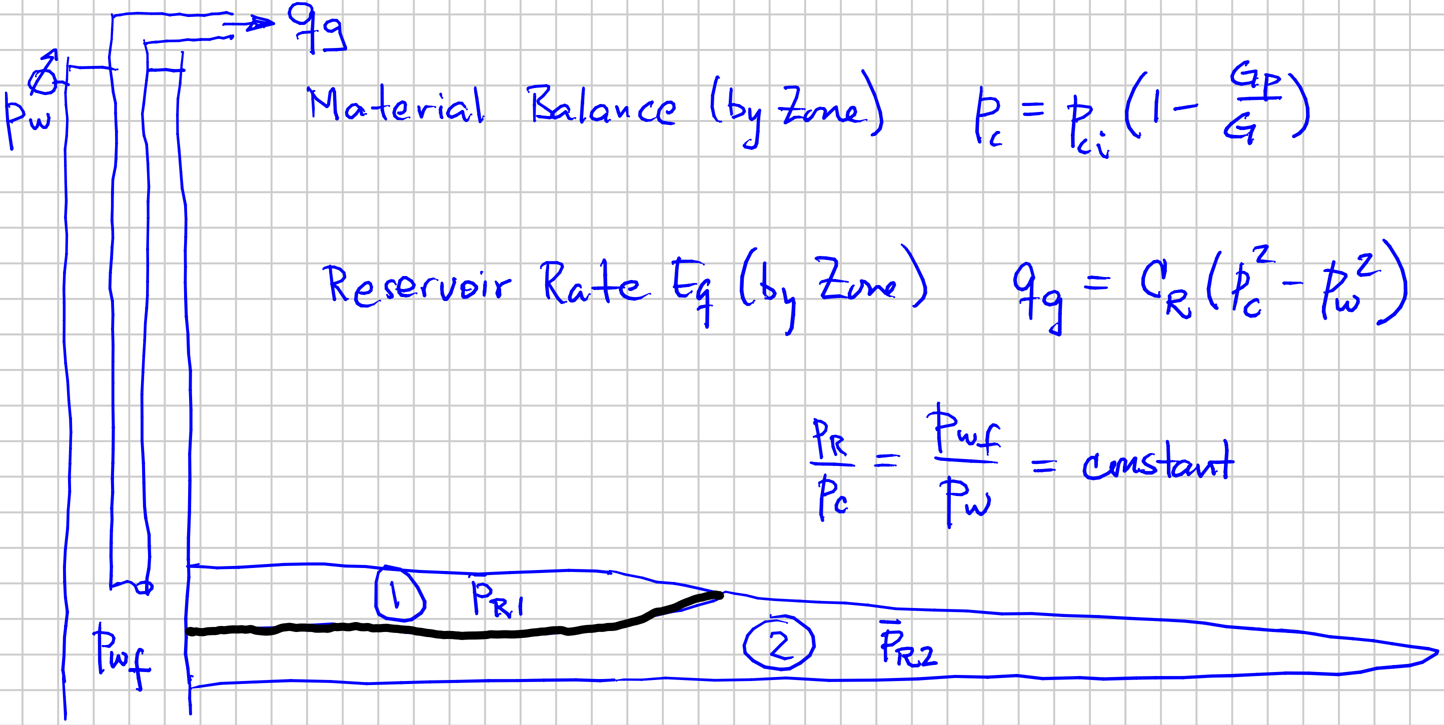
$$q_g = C_t \left(p_w^2 - p_t^2 \right)^{1/2}$$

Direct Solution for p_t

given q_g and previous p_{c1}, p_{c2}

Simplified Two-Zone Gas Well/Reservoir Model

Note Title 2010-10-05



Material Balance (by Zone) $p_c = p_{ci}\left(1 - \frac{G_p}{G}\right)$

Reservoir Rate Eq (by Zone) $q_g = C_R\left(p_c^2 - p_w^2\right)$

$$\frac{p_R}{p_c} = \frac{p_{wf}}{p_w} = \text{constant}$$

Initial	Zone 1	Zone 2
p_{ci} [bara]	30	30
$C_R \left[\frac{Sm^3/d}{bar^2}\right]$	31	11
G [Sm^3]	$1 \cdot 10^6$	$3 \cdot 10^6$

Current	1	2	Well
p_c [bara]	20	28	
G_p	✓	✓	✓RF %
q_g (p_w=0)	✓	✓	✓

(a) If Shut-In (long-time, forever)

p_c	✓	✓

(b) If $\underline{|\ \Delta t = 30 \text{ days}\ |}$

(i) — Maximum physical average gas rate by zone $p_w = 0$

(ii) Shut-in the well

maximum physical average gas rate by zone

Solution (Structure)

① $RF_\ell = \frac{G_{p\ell}}{G_\ell}$ ℓ = Layer (zone)

$$\overset{\checkmark}{p_{c_\ell}} = \overset{\checkmark}{p_{ci_\ell}}\left(1 - \overset{\checkmark \text{ Solve}}{RF_\ell}\right)$$

Given : $(p_{ci})_1 = (p_{ci})_2 = 30$ bara

Current : $p_{c1} = 20$ bara

$p_{c2} = 28$ bara

$$RF_{total} = \frac{G_{p\,total}}{G_{total}} = \frac{(\overset{\checkmark}{G_{p1}} + \overset{\checkmark}{G_{p2}})_{current}}{\underset{\checkmark}{G_{1i}} + \underset{\checkmark}{G_{2i}}}$$

$$G_{p1} = \underbrace{\left(\frac{G_{p1}}{G_1}\right)}_{\underset{\checkmark}{RF_1}} \times \underset{\checkmark}{G_{1i}} \qquad \underline{Sm^3}$$

$$G_{p2} = \underset{\checkmark}{RF_2} \times \underset{\checkmark}{G_{2i}} \qquad Sm^3$$

② $q_{fg}(P_w = 0) \equiv$ AOFP

$$\underset{\text{Calc.}}{q_{fg\ell}} = \underset{\checkmark}{C_{R\ell}} \left(\underset{\checkmark}{P_{c\ell}^2} - \underset{\substack{\checkmark \\ =0}}{P_w^2} \right)$$

AOFP $q_g = q_{fg1}(P_w=0) + q_{fg2}(P_w=0)$

$$= C_{R1} P_{c1}^2 + C_{e2} P_{c2}^2$$

③ Physics: $V_{p1} = \text{constant}$

$V_{p2} = \text{constant}$

$$\overset{\checkmark}{P_i} V = \overset{\checkmark}{n}\,\overset{\checkmark}{R}\,\overset{\checkmark}{T} \Rightarrow V_{pg} \text{ calc.}$$

$$n \uparrow = \frac{G^{\checkmark}}{\underbrace{\left(\frac{RT_{sc}}{P_{sc}}\right)}}$$

$$V_{pg1} = \left(\frac{n_i R T_R}{P_i}\right)_1 = (\qquad)\cdot \frac{\overset{\checkmark}{G_1}}{\underset{\checkmark}{P_{c1i}}}$$

$$V_{pg2} = \left(\frac{n_i R T_R}{P_i}\right)_2 \cdot (\ )\cdot \frac{\overset{\checkmark}{G_2}}{P_{c2}}$$

$$\underset{\text{Initial}}{(\qquad)\frac{G_1}{P_{c1i}}} = \underset{\text{Current}}{(\qquad)\frac{\overset{\checkmark}{\left[\left(\overset{\checkmark}{G_{1i}} - \overset{\checkmark}{G_{p1}}\right)\right]}^{\text{Left in R}}}{P_{c1}\checkmark}} = \underset{\text{After } \infty \text{ SI}}{(\quad)\frac{\overbrace{\left(G_{1i} - G_{p1}^*\right)}}{P_{cV}}}$$

$$= (\quad)\frac{(G_{2i} - G_{p2}^*)}{p_{cv}}$$

$$p_{cv} = \bar{p}_c^* \;?$$ "Volumetric Average Reservoir Pressure"

$$G_{p1}^*, \; G_{p2}^* \;?$$

$$\underbrace{(G_{1i} - G_{p1}^*)}_{\text{Gas remaining ① @ } \infty}, \; \underbrace{(G_{2i} - G_{p2}^*)}_{\text{Gas remaining ② @ } \infty} \;?$$

sum =

$$(G - G_p)$$

$$\checkmark \qquad \checkmark$$

$$(G_{1i} + G_{2i}) \qquad (G_{p1} + G_{p2})$$

$$\checkmark \qquad \checkmark$$

Algebra:

$$(V_1 + V_2) = (V_1 + V_2)$$

Let in layer 2 after SI ∞

$$\overset{\checkmark}{\frac{G_1}{p_{c1}}} + \overset{\checkmark}{\frac{G_2}{p_{c2}}} = \frac{G_1^*}{p_{cv}} + \frac{G_2^*}{p_{cv}} \quad p_{cv} \rightsquigarrow p_{cv}$$

$$\checkmark \qquad \checkmark$$

$$\checkmark \; \boxed{(G - G_p)}$$

$$= \frac{(G_1^* + G_2^*)}{\boxed{p_{cv}}} = \frac{[(G_1 - G_{p1}) + (G_2 - G_{p2})]}{\boxed{p_{cv}}}$$

$$\boxed{p_v = \frac{(G - G_p)}{\frac{G_1}{P_{c1i}} + \frac{G_2}{P_{c2i}}}}$$

④ You can't produce during Δt (in a zone) more gas than is in that zone

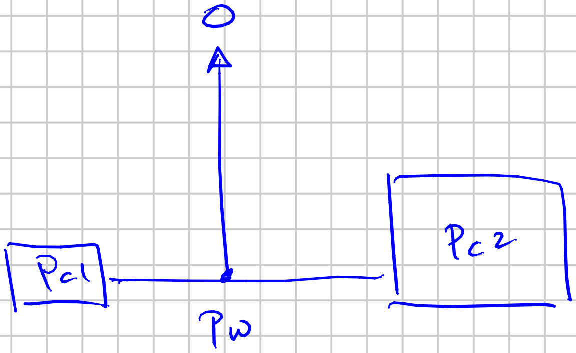
remaining at start Δt

Check $q_{g1\,max} = \left[\frac{(G_{1i} - G_{p1})}{\Delta t} \right]$ ✓ use

vs $>$

$= C_{R1} \cdot p_{c1}^{2}$ ok when $P_{c1} = 20$

⑤



Δt=30days

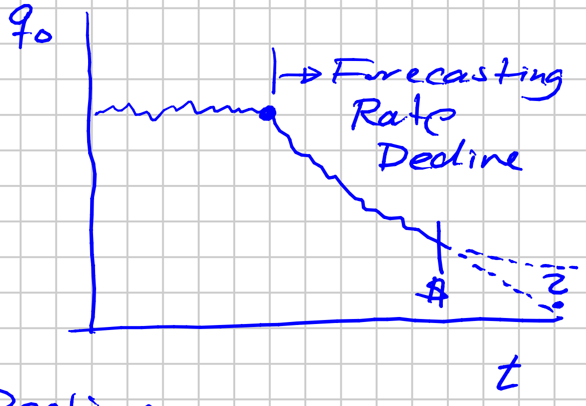
Find Pw $\quad q_{g2} = -q_{g1}$

$\Delta t \, q_{g2} \sim q_{g1} \, \Delta t$

Short Introduction to Decline Curve Analysis (DCA)

History:

Arp's (195x?) : Historical Data
- Individual Wells
- Fields



Empirical Data Fitting

@Start of Decline

$$q = \frac{q_i}{[1 + bDt]^{1/b}}$$

Hyperbolic

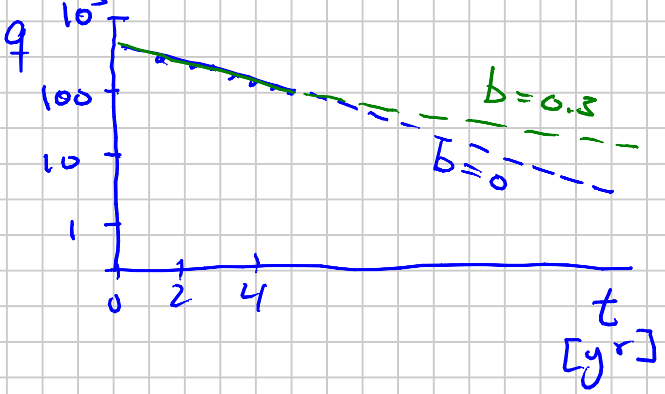
b & D & q_i are constants

$0 \le b \le 1$

$b = 0$:

$$q = q_i \exp(-Dt)$$

Exponential Decline



b = 1 : DON'T EVER USE (Harmonic)

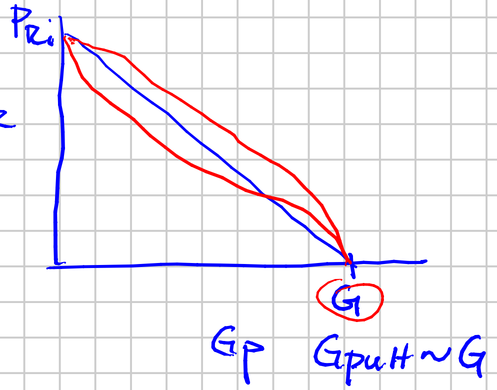
pre-Fetkovich : Most used b=0

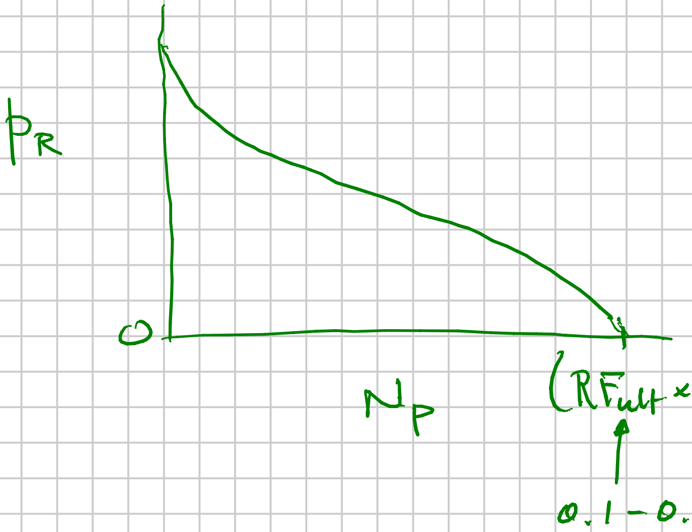
post-Fetkovich : 197x

Showed that Arp's Eq. "falls out" of basic physics in reservoir engineering

$$\begin{cases} q_g = \boxed{C_g(p_R^2 - P_{wf}^2)} \\ q_o = C_o(P_R - P_{wf}) \text{ or } \boxed{C_o'(P_R^2 - P_{wf}^2)} \end{cases}$$

$$p_R = f(G_p ; N_p)$$





Oil Solution Gas Drive

General Observations from Fetkovich

$$q_{gi} = \underbrace{\frac{7.6\,kh \times (\overset{P_R \text{ at start of decline}}{P_R^2} - \overset{\text{const}: P_R \to P_{wf} \text{ at } \infty \text{ time}}{P_{wf,ult}^2})}{T_R(\overline{\mu Z})\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s\right]}}_{\text{Productivity Index}}$$

"C_R"

- $P_{wf} \approx$ constant during decline

- $$\boxed{D = \underbrace{\left(\frac{q_{gi}}{G_{pult}}\right)}_{\text{"Rate of Decline"}} \cdot \frac{1}{1-b}}$$: $\underline{G_{pult}} = \int_0^\infty q_g(t)\,dt$

$Q_{ult} = \int_0^\infty q\,dt$

$Q = G_p$ or N_p = cumulative production

- b = 0 is the exception, not the rule

SINGLE ZONE

b could be estimated

Gas : $0 < b < \underline{0.5}$

$\Leftarrow \left(\frac{P_{wf,ult}}{P_{Ri}}\right)$

1 → Strong Water Drive

0 → Normal Gas Prod.

Oil : $0 < b < 0.4$

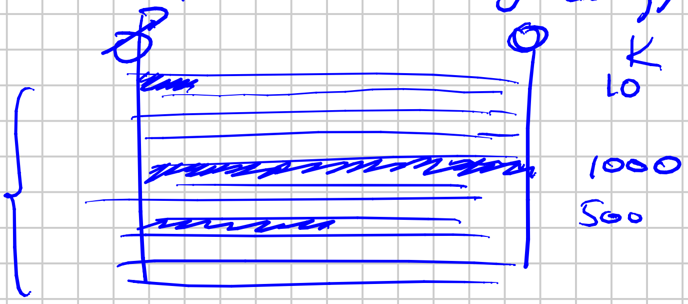
$\uparrow$ 0 → $\left(\frac{P_b}{P_{Ri}}\right)$ → $\uparrow$ 1

Normal Solution Gas Drive

$\underline{b \sim 0.5}$

Water Drive

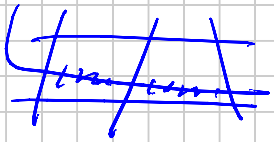
Muskat (Analytically)



"Normal" Permeability Variation

- Multi-Zone Wells

(Layered No-Crossflow) LNX



$1 > \underline{\underline{b}} > 0.5$

Very Common

$q_1(t) + q_2(t)$

$$q(t) = q_{i1} \cdot \exp(-D_1 t) + q_{i2} \exp(-D t)$$

$b = 0$ $b = 0$

$$\approx \frac{q_i}{[1 + bDt]^{1/b}}$$

$b > 0.5$

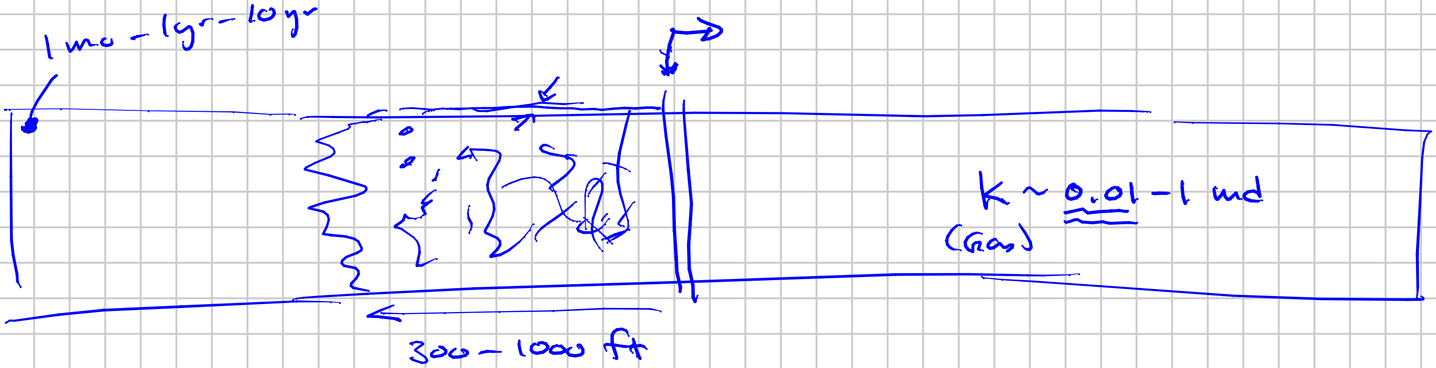
$\bar{b} > 0.5$

$q_i = q_{i1} + q_{i2}$

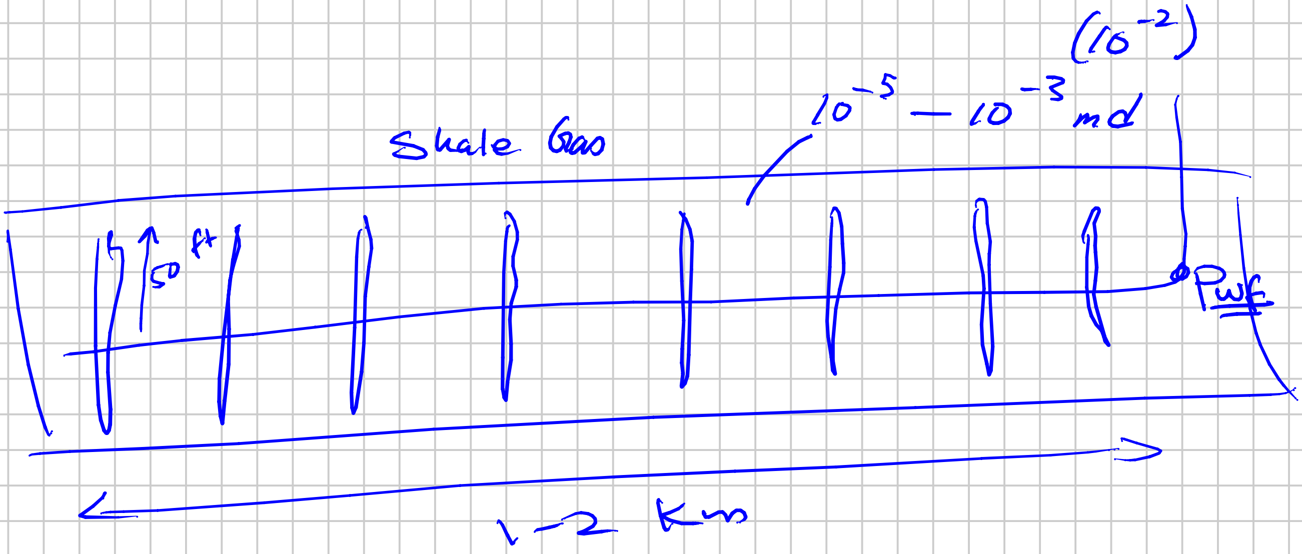
- q_g

  $p_R(Q)$ } Steady State Assumption

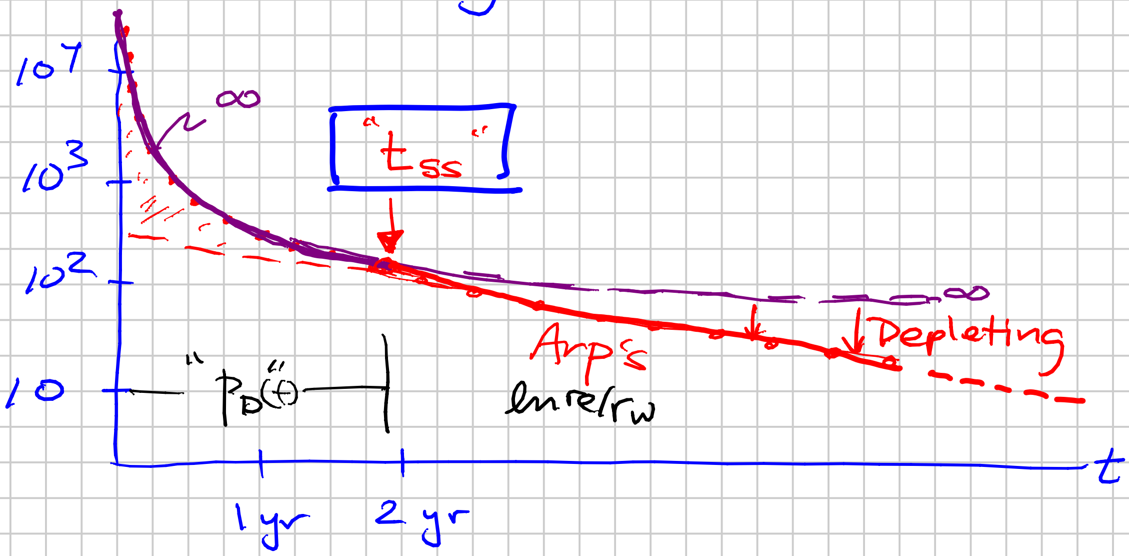
$$\left(\frac{k}{\mu}\right) \gtrsim 100 \ \frac{md}{cp} \quad ✓$$



p_{wf} = const (low)



$q(t)$: Infinite-Acting Period (1mo – 3yr)



Transient (pre-SS) behavior

$$p_D \leftrightarrow \ln \frac{\boxed{r_e(t)}}{r_w}$$

$$\boxed{\begin{aligned} p_c &= p_{ci}\left(1-\frac{G_p}{G}\right) \\ q_g &= C_R\left(p_c^2-p_w^2\right) \\ q_g &= \frac{dG_p}{dt} \\ &\Downarrow \end{aligned}}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} q_o &= q_{oi}\, e^{-Dt} \\ D &= \frac{q_{oi}}{N_{ult}} \end{aligned}}$$

Single-Phase Oil

$$p_R = p_{Ri}\left(1-\frac{N_p}{N_{ult}}\right)$$

$$q_o = C\left(p_R - p_{wf}\right)$$

$$q_o = \frac{dN_p}{dt}$$

$$\frac{dN_p}{dt} = C\left(p_R - p_{wf}\right)$$

$$p_R = p_{Ri} - \frac{p_{Ri}}{N_{ult}} \cdot N_p$$

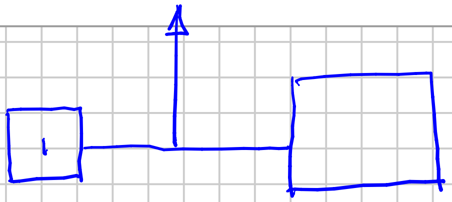
$$\frac{dN_p}{dt} = C p_{Ri} - \frac{C p_{Ri}}{N_{ult}} N_p - C p_{wf}$$

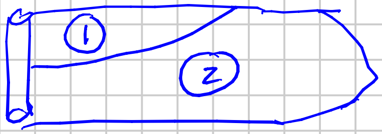
$$= \boxed{C\left(p_{Ri} - p_{wf}\right)}^{=q_{oi}} - \boxed{\frac{C p_{Ri}}{N_{ult}}} N_p$$

$$\frac{dN_p}{dt} = q_{oi} - \frac{C p_{Ri}}{N_{ult}} N_p$$

$$\frac{dN_p}{q_{oi} - \frac{C p_{Ri}}{N_{ult}} N_p} = dt$$

④





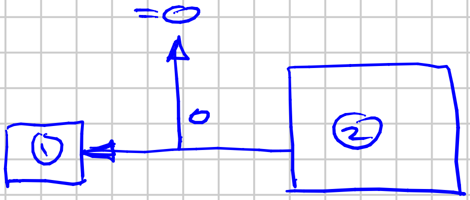
Δt

M.B. $q_{gmax,1} = \frac{G_1}{\Delta t} = \frac{(G_{1i} - G_{p1})}{\Delta t} \Rightarrow$ Solve $\underline{\underline{p_w}} > 0$

$<$

$$AOFP_1 = C_{R1} \cdot (p_{c1}^2 - \overset{0}{\cancel{p_w^2}}) = C_{R1} \cdot p_{c1}^2$$

⑤



Δt

Layer 1 & Layer 2

$$q_{gxmax} \leq \frac{\Delta G_x}{\Delta t}$$

$(p_{c1}^2 - p_w^2) C_{R1} = q_{gxmax}$

Physical: $\boxed{\begin{matrix} > 0 \\ \leq p_{cv} \end{matrix}}$

$\Delta G_x : p_{c1} = p_{c2} = \underline{\underline{p_{cv}}}$

$\Delta t = \infty$

$- \quad p_{c2} > p_w > p_{c1}$

$$C_{R2}(p_{c2}^2 - p_w^2) = C_{R1}(p_w^2 - p_{c1}^2)$$

$\Rightarrow p_w(\Delta t)$:

Arps DCA:

$$q_g = \frac{q_{gi}}{\left[1 + bDt\right]^{1/b}}$$

Assumptions

① $q_g = C_R\left(p_c^2 - p_w^2\right)^{n}$

② $p_c = p_{ci}\left(1 - \frac{G_p}{G_i}\right)$

Fetkovich: 1973 Eq. 30

$$\boxed{b = 0.5}$$

$p_w = 0$: $q_{gi} = C_R\, p_c^2$ ← at start of decline

$$D = \frac{q_i}{G} \cdot \left(\frac{1}{1-b}\right)$$

Gas in place at start decline

If p_w = constant > 0

$$\underline{q_{gi}} = C_R\left(p_c^2 - p_w^2\right)$$

↑ at start of decline

G used in calc D would be the total gas produced at ∞ time when

$p_c \to p_w$

at start of decline

$$p_c = p_c\left(1 - \frac{G_p}{G}\right)$$

p_c ↓ p_w

max produced $p_c \to p_w$

Gas in place at start of decline

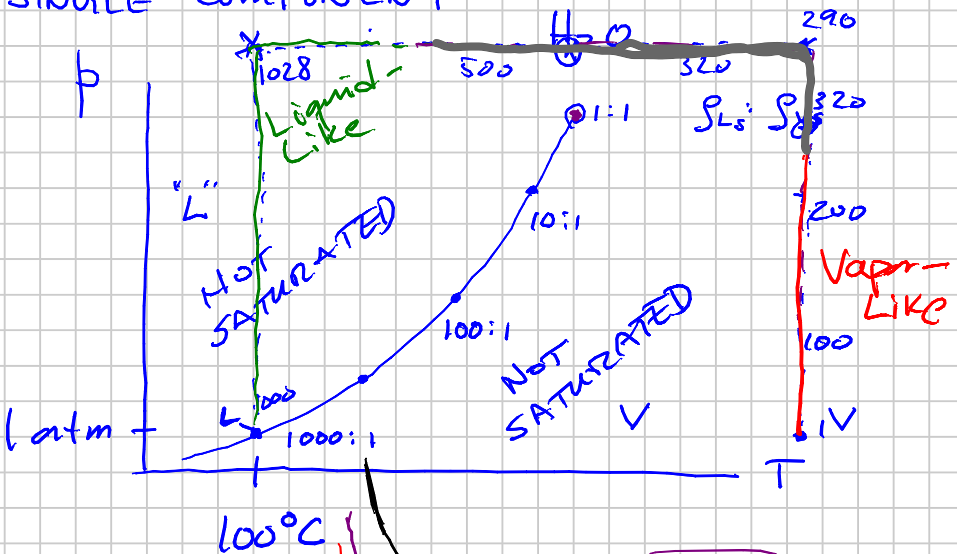
$$p_w = p_c\left(1 - \frac{\overset{\swarrow\ \text{Solve}}{G_p^*}}{G}\right)$$

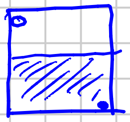
$$D = \frac{q_{gi}}{G_p^*}\,\frac{1}{1-b}$$

RESERVOIR FLUID PHASE & VOLUMETRIC BEHAVIOR (Ch. 2)

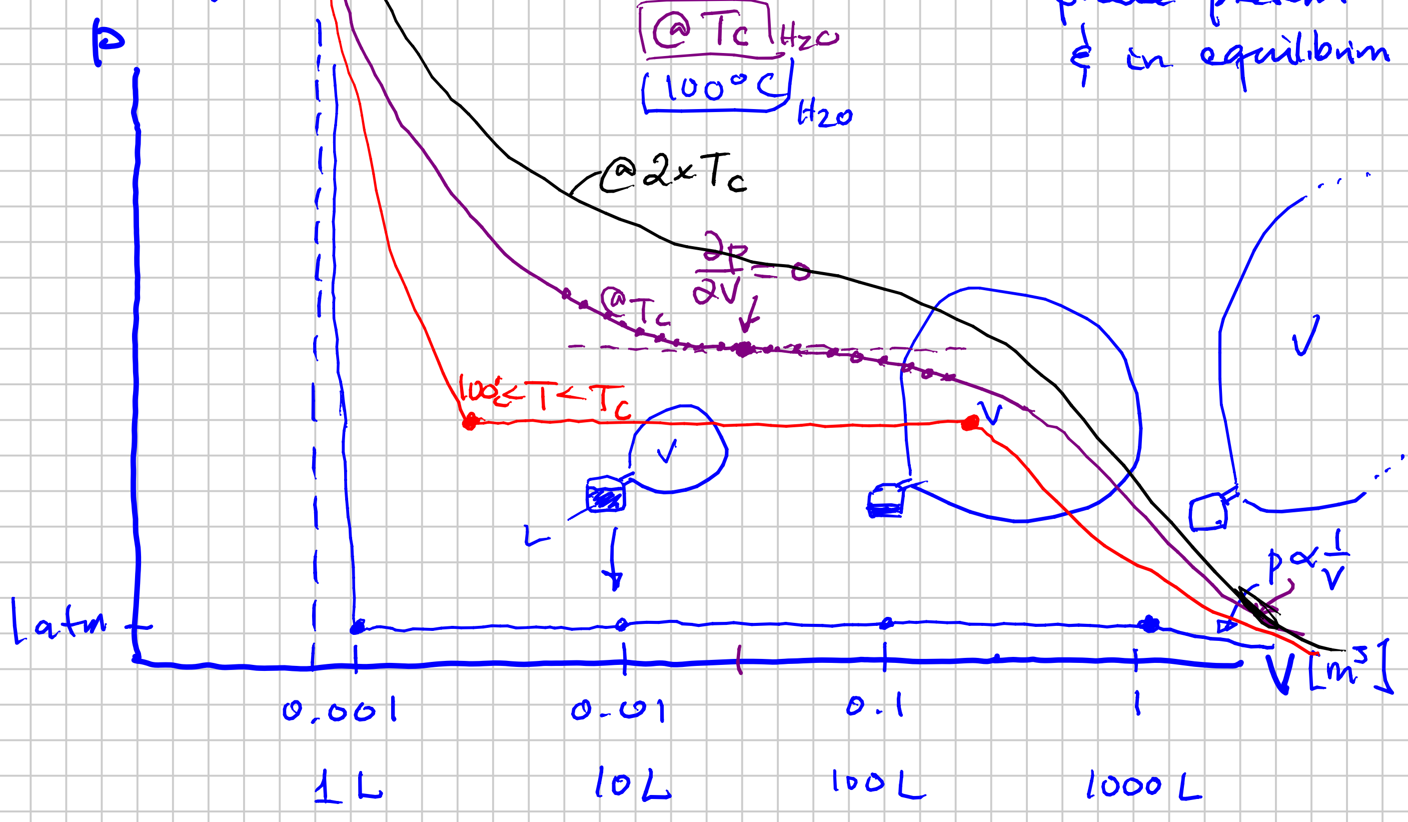
- Single Component (elementary building block)
- Binary System (has same features)
- Ternary Systems (sometimes a good surrogate)
- Actual Res. Fluids

SINGLE COMPONENT

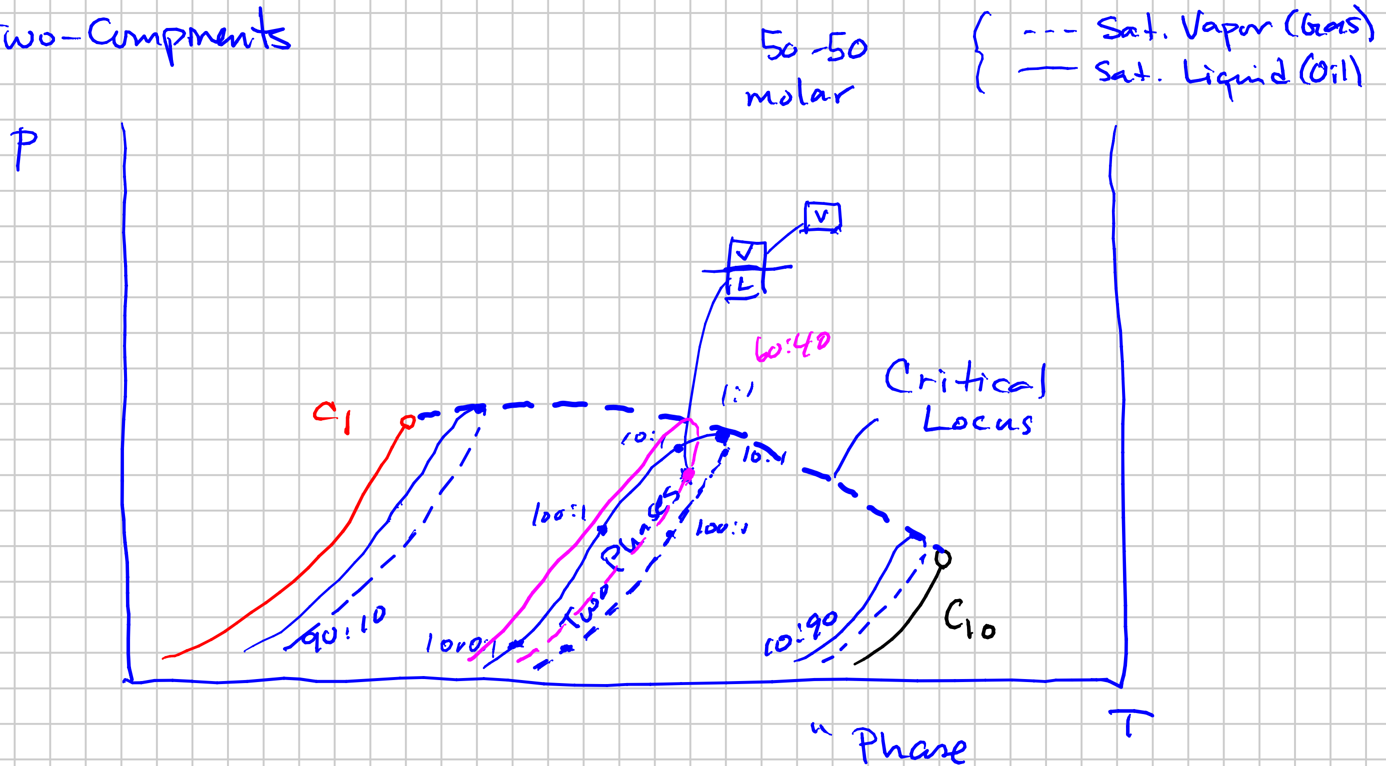




"Saturated Phase" is a phase that has another phase present & in equilibrium



Two-Components



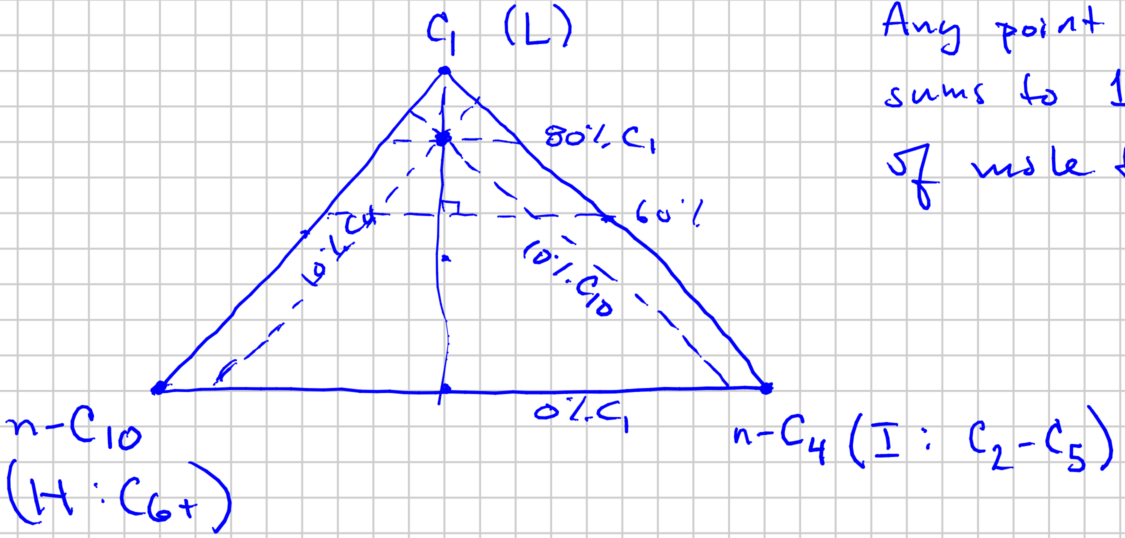
"Phase Envelope"

Along the saturated phase diagram, and inside, you have two saturated phases

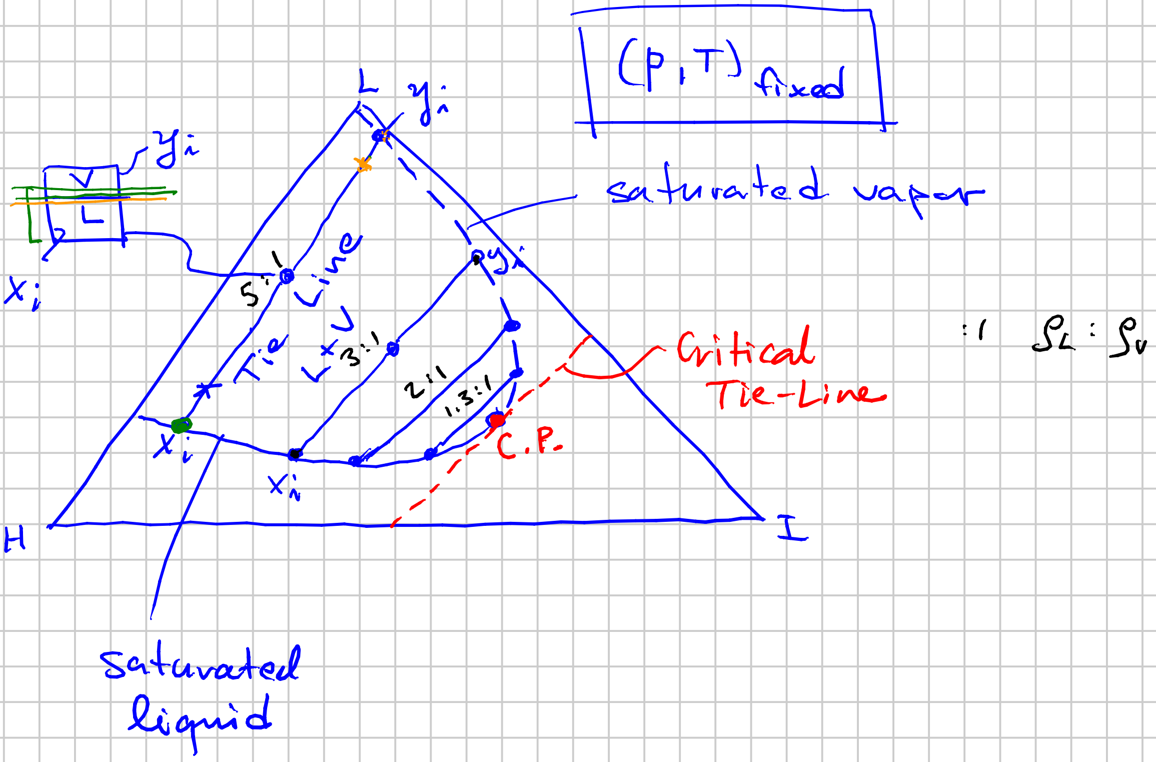
A saturated vapor is at ITS "dewpoint"

A saturated liquid is at ITS "bubblepoint"

Ternary System



Any point inside, along sums to 1 in terms of mole fraction.



Used mostly to describe mechanisms achieving miscible displacement of oil by gas. 1950s-1980s (1985)

RESERVOIR FLUID PHASE BEHAVIOR

Note Title

2010-10-13

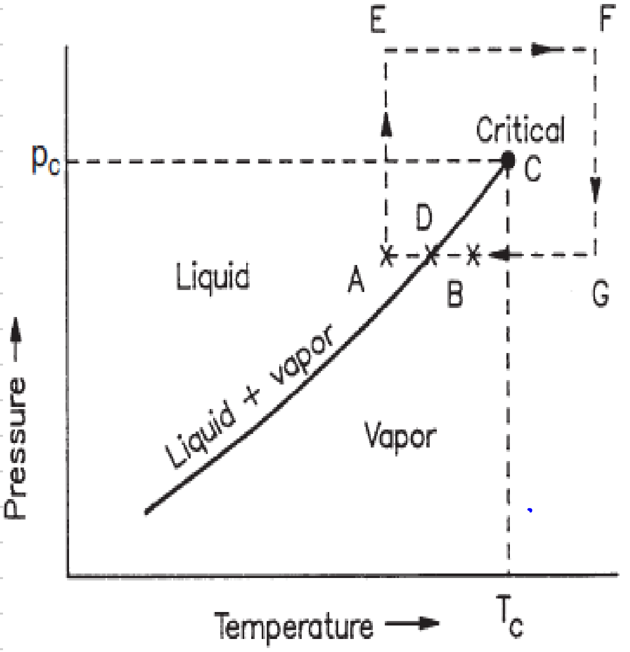
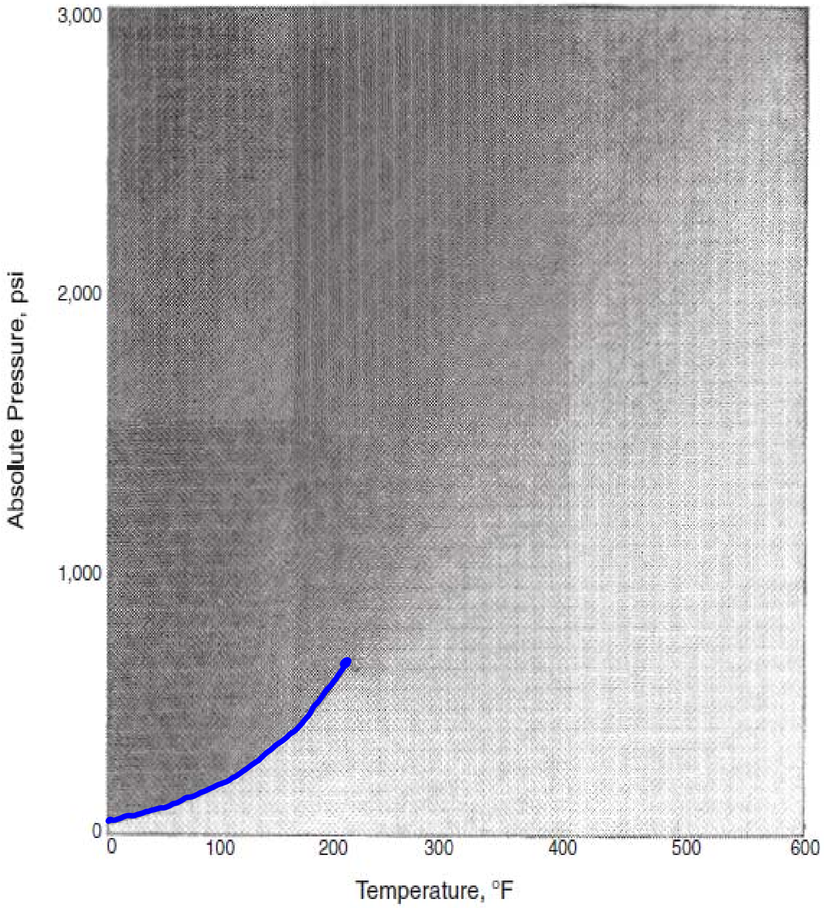


Fig. 2.4—*p-T* diagram for a single component in the region of vapor/liquid behavior near the critical point (p_c = critical pressure and T_c = critical temperature).



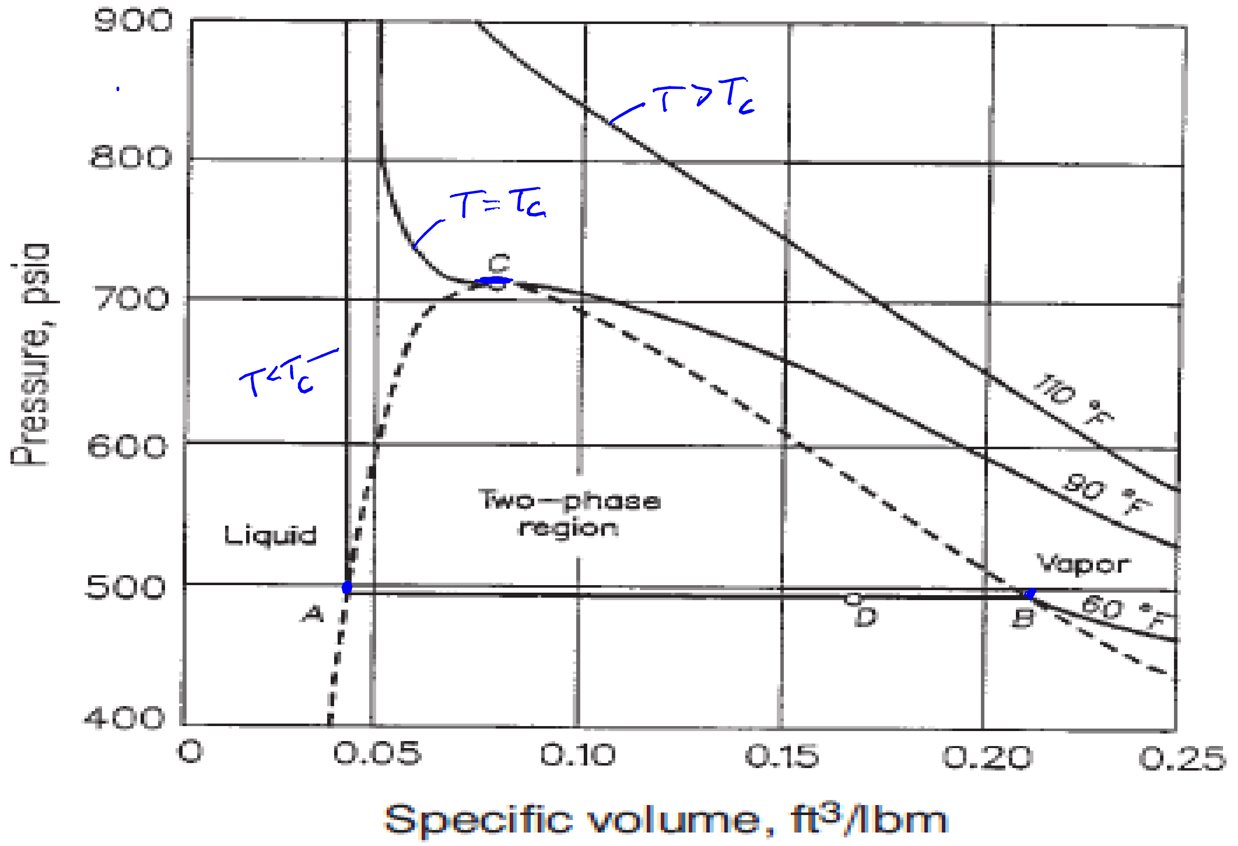


Fig. 2.7—p-V diagram for ethane at three temperatures (from Standing[26]).

Gas Mixtures : $Z\left(\frac{P}{P_{pc}}, \frac{T}{T_{pc}}\right)$
$\qquad\qquad\qquad\nearrow\qquad\nearrow$

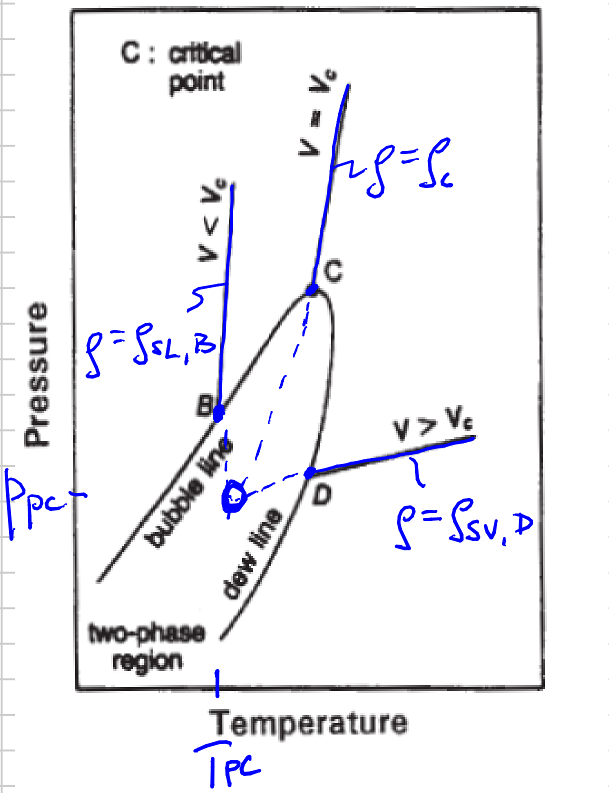


Fig. 2.8—Qualitative *p-T* and *p-V* plots for pure flui

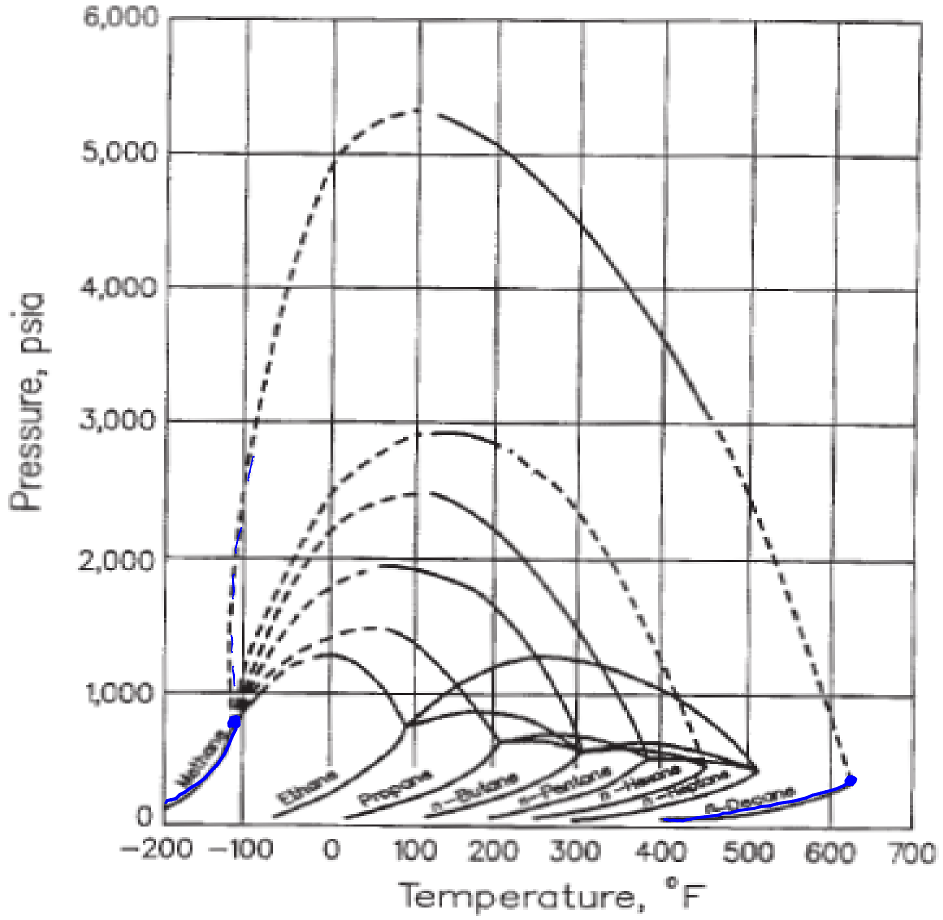


Fig. 2.11—*p-T* diagram for various hydrocarbon binaries illustrating the effects of molecular-weight differences on critical-point loci (after Brown *et al.*[31]).

Phase Behavior (p-T Phase Diagram) of Actual Reservoir Fluids (100s → 1000s)

$\{N_2 \quad CO_2 \quad H_2S\}$ non-HC compounds

$\underline{H_2O}$ Ch. 9

(12 → 50)

... $\underset{\sim}{C_{7+}}$ C_{30+}

$\{C_1 \quad C_2 \quad C_3 \quad \underbrace{i\text{-}C_4 \quad n\text{-}C_4 \quad i\text{-}C_5 \quad n\text{-}C_5}\}$

Light & Intermediate HCs

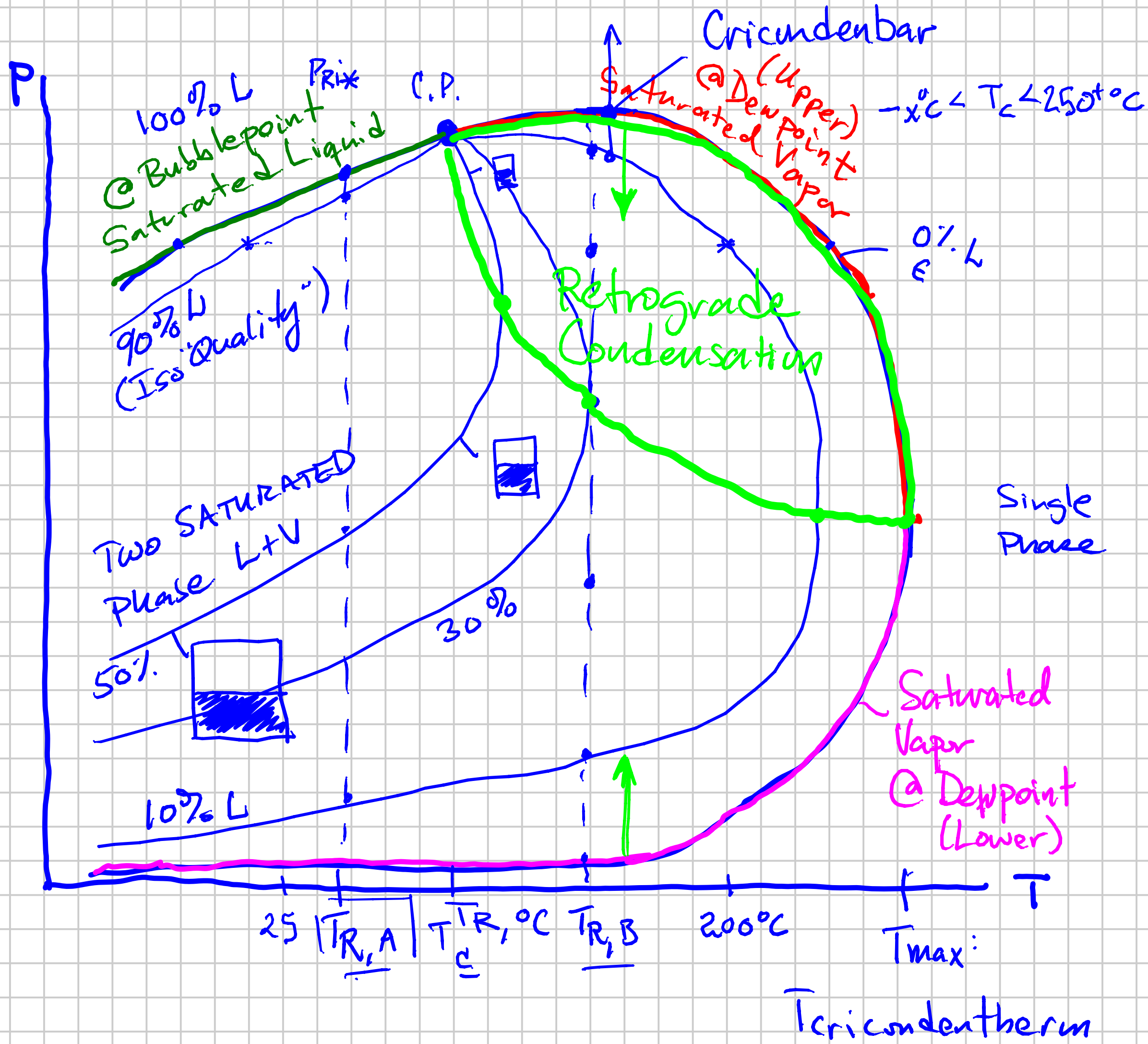
⟹ significant portion are sold as "gas"

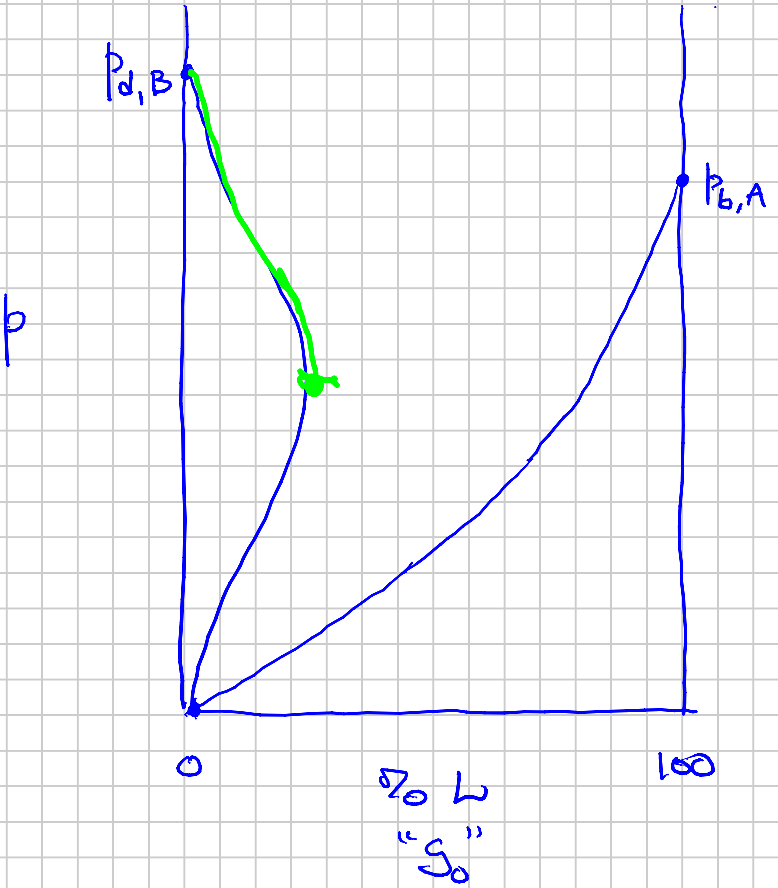
$\{C_6 \quad C_7 \quad C_8 \ldots \quad C_{20} \quad C_{40} \quad C_{50} \quad C_{100s} \ldots\}$ {Asphaltenes}

M ~ 1400

5000 → 20000

Sold as "oil"


Cricondenbar
(Upper)
@ Dewpoint
Saturated Vapor
-x°C < T_c < 250+°C
P
100% L
P_R,i*
C.P.
@ Bubblepoint
Saturated Liquid
0% L
ε
Retrograde
Condensation
90% L
(Iso "Quality")
Two SATURATED
Phase L+V
50%
30%
10% L
Single
Phase
Saturated
Vapor
@ Dewpoint
(Lower)
T
25
T_R,A
T_c
T_R, °C
T_R,B
200°C
Tmax:
Tcricondentherm


P_d,B
P_b,A
P
0
100
% L
"S_o"

$$q_o \propto \underbrace{\left(\frac{k_{ro}(S_o)}{\mu_o}\right)}_{\text{Mobility}}$$

DEFINING RESERVOIR FLUID TYPES

OIL & GAS

$T_R > T_{\text{cricondentherm}}$

DRY GAS

$>90\%\ C_1$ $>99\%\ C_1$-C_5

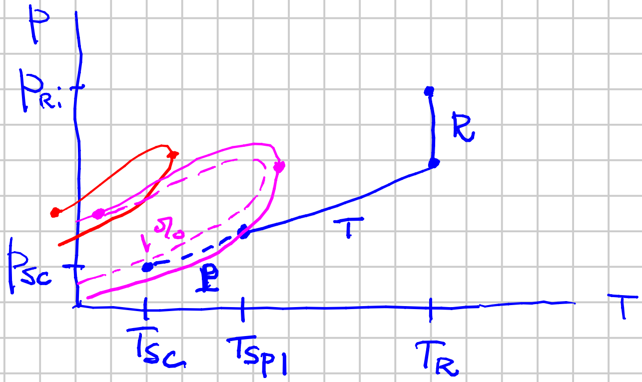
Zero (~No) Surface Oil

GOR = ∞

WET GAS

Some/little surface oil

GOR ≳ 20000 Sm³/Sm³

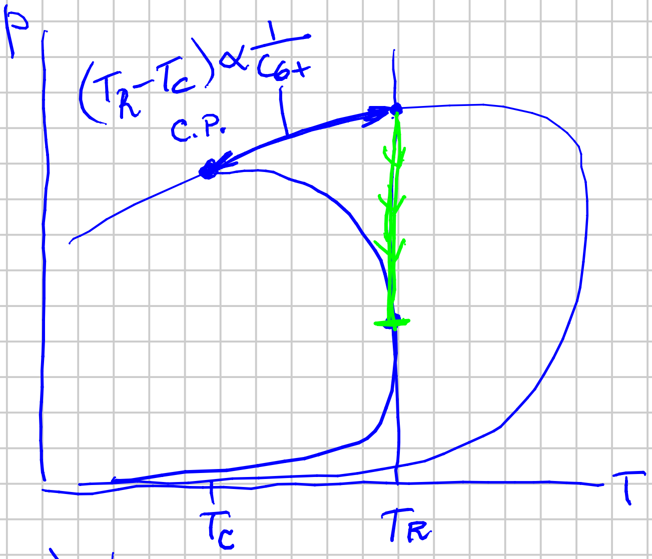


GAS CONDENSATE

$T_c < T_R < T_{cric}$

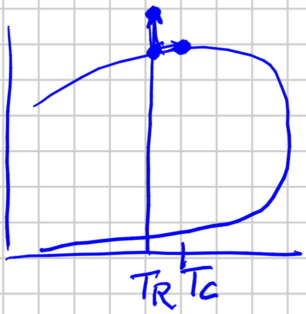
mol-% $C_{6+} \lesssim 15$

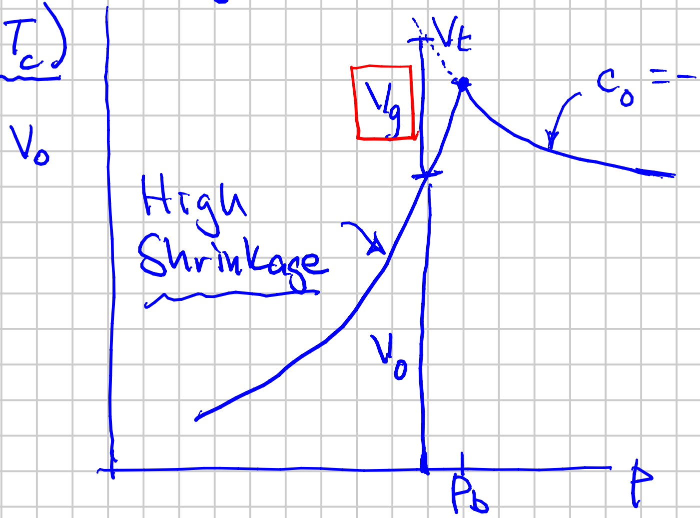
~400 - 10,000 Sm³/Sm³



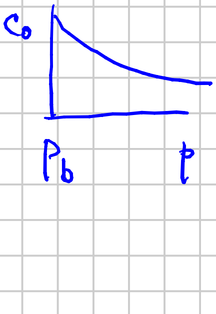
VOLATILE OIL $\underbrace{(T_R \sim> T_c)}$

$(T_R < T_c)$: Any Oil

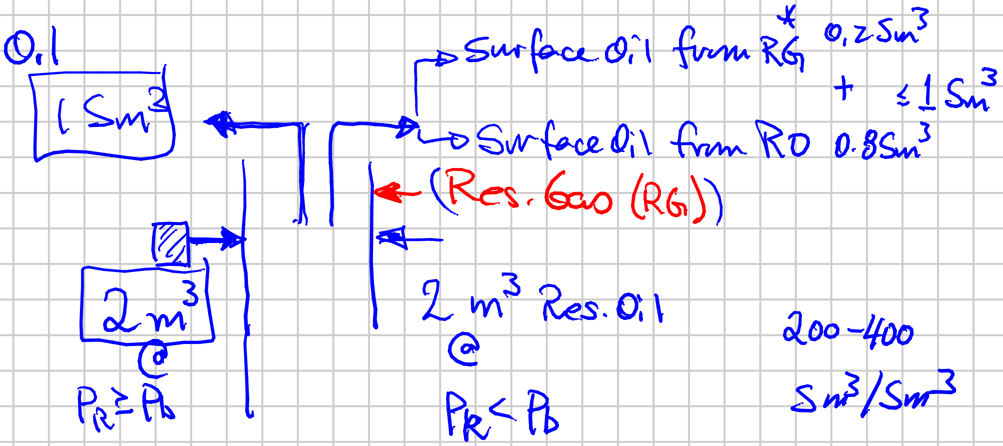




$$c_o = -\frac{1}{V_o}\frac{dV_o}{dp} = f(p)$$



Volatile Oil

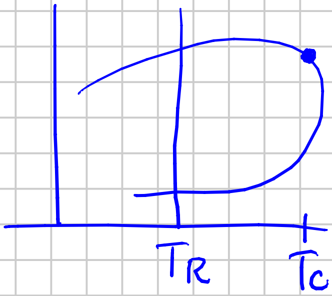


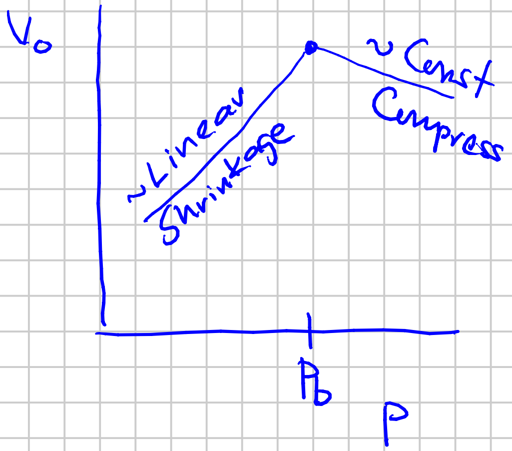
200–400 Sm³/Sm³

* Volatile Oil reservoirs, the carry capacity of surface oil in reservoir gas is important.

BLACK OIL

$T_R << T_c$





20–
<200 Sm³/Sm³

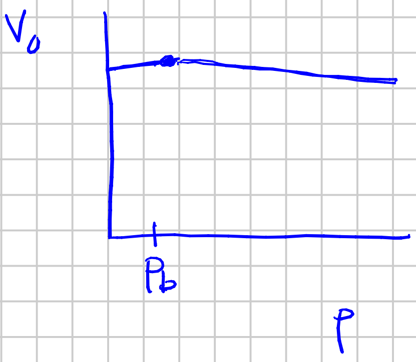
DEAD (HEAVY) OIL

- Little to no "surface gas" in the reservoir oil
- Surface oil is heavier (>900 kg/m³)

⓪ Higher oil viscosity (10 cp → 10,000 cp)

$$q_o \propto \frac{k}{\mu_o}$$

<20 Sm³/Sm³

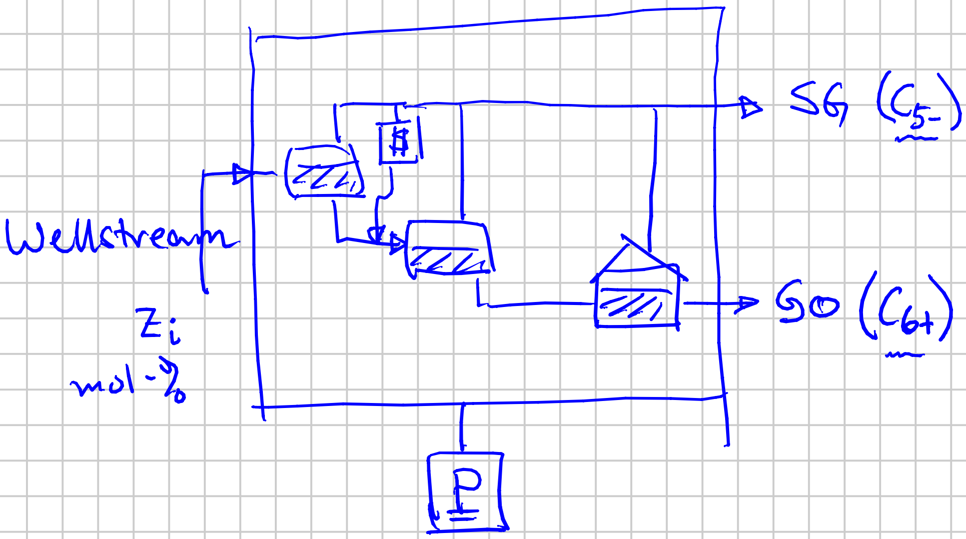


$T_R << T_c$

Characteristic Used (Sometimes) in Defining Reservoir Fluid Type

Surface Gas-Oil Ratio GOR

Simplified Surface Process



$$GOR = \frac{q_g}{q_o}$$

$$\boxed{\underset{\left[\frac{Sm^3}{Sm^3}\right]}{GOR} = \frac{(1-z_{6+})}{z_{6+}} \cdot \frac{23.67}{\left(\frac{M}{\rho}\right)_{6+}}}$$

ρ $[kg/m^3]$

z_{6+} mole fraction

19.10.2010

professor Michael Golan
step in for Prof. Curtis Whitson

Phase equilibrium
in surface processing

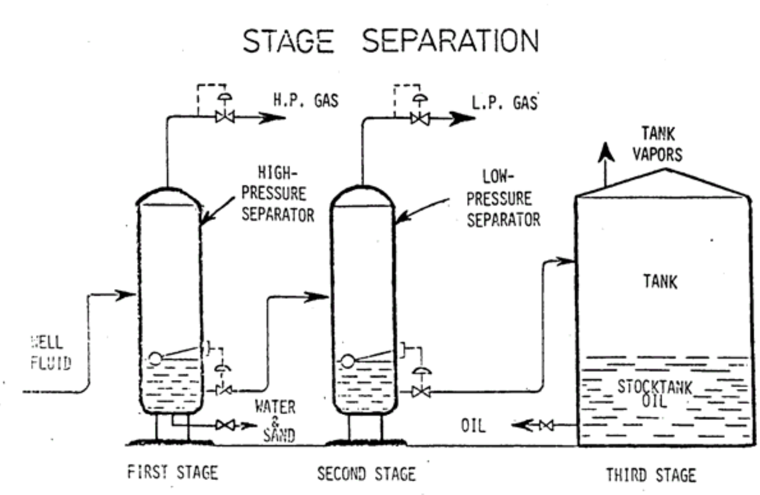
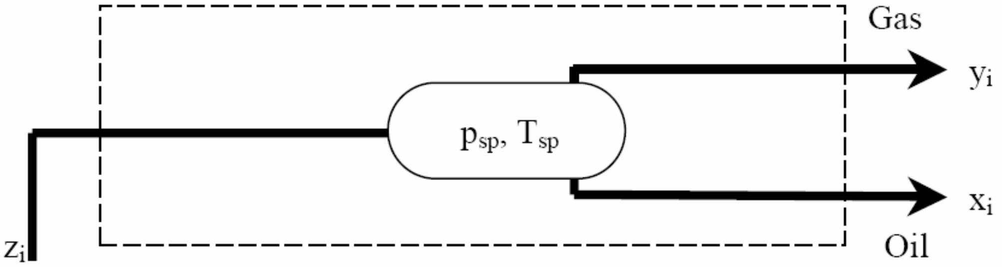
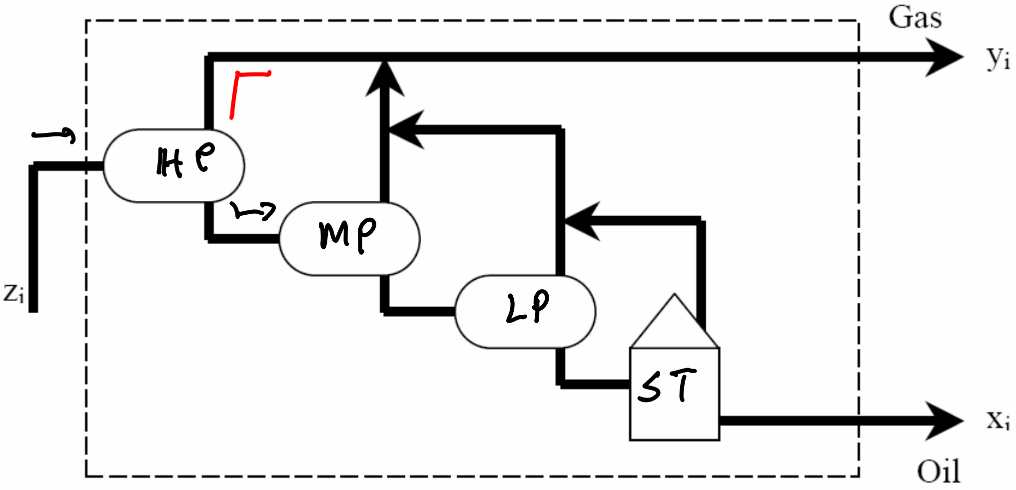


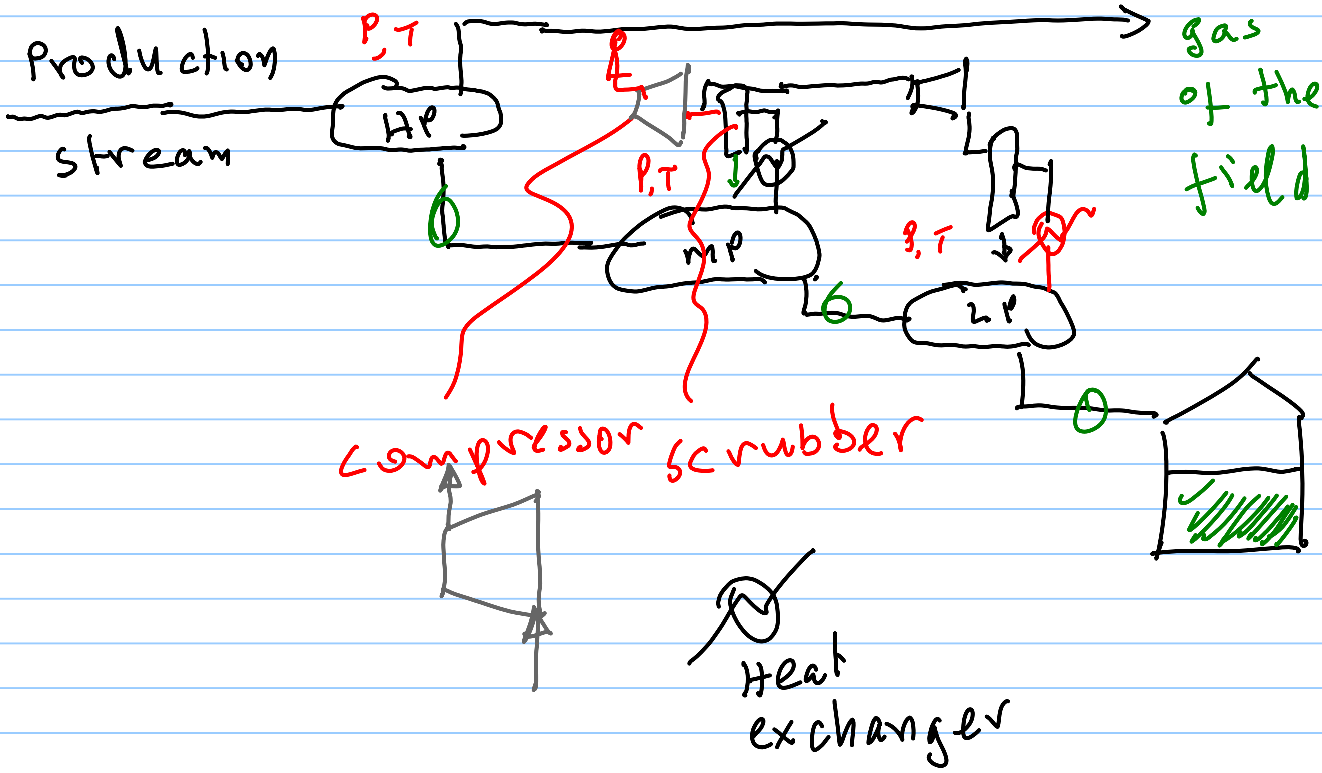
Figure 12 Stage separation

Case A *Single stage separation (Flash Separation)*

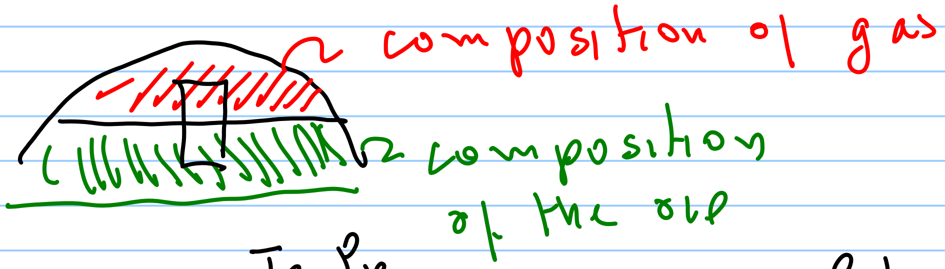


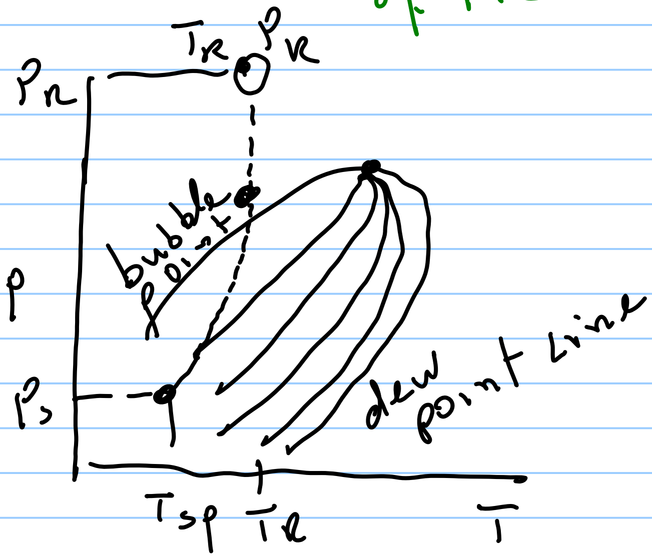
Case B *Multi-stage separation*

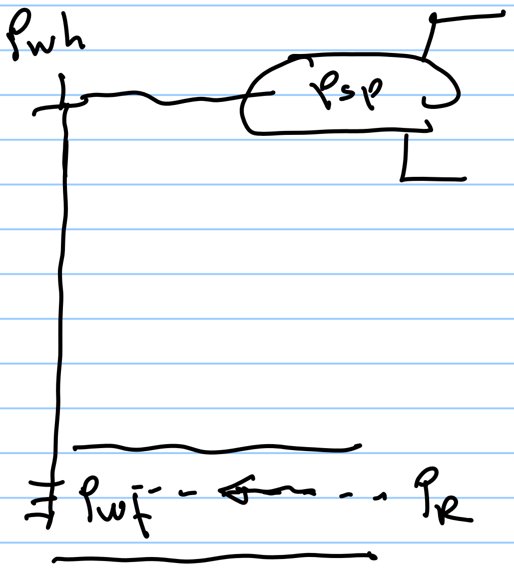


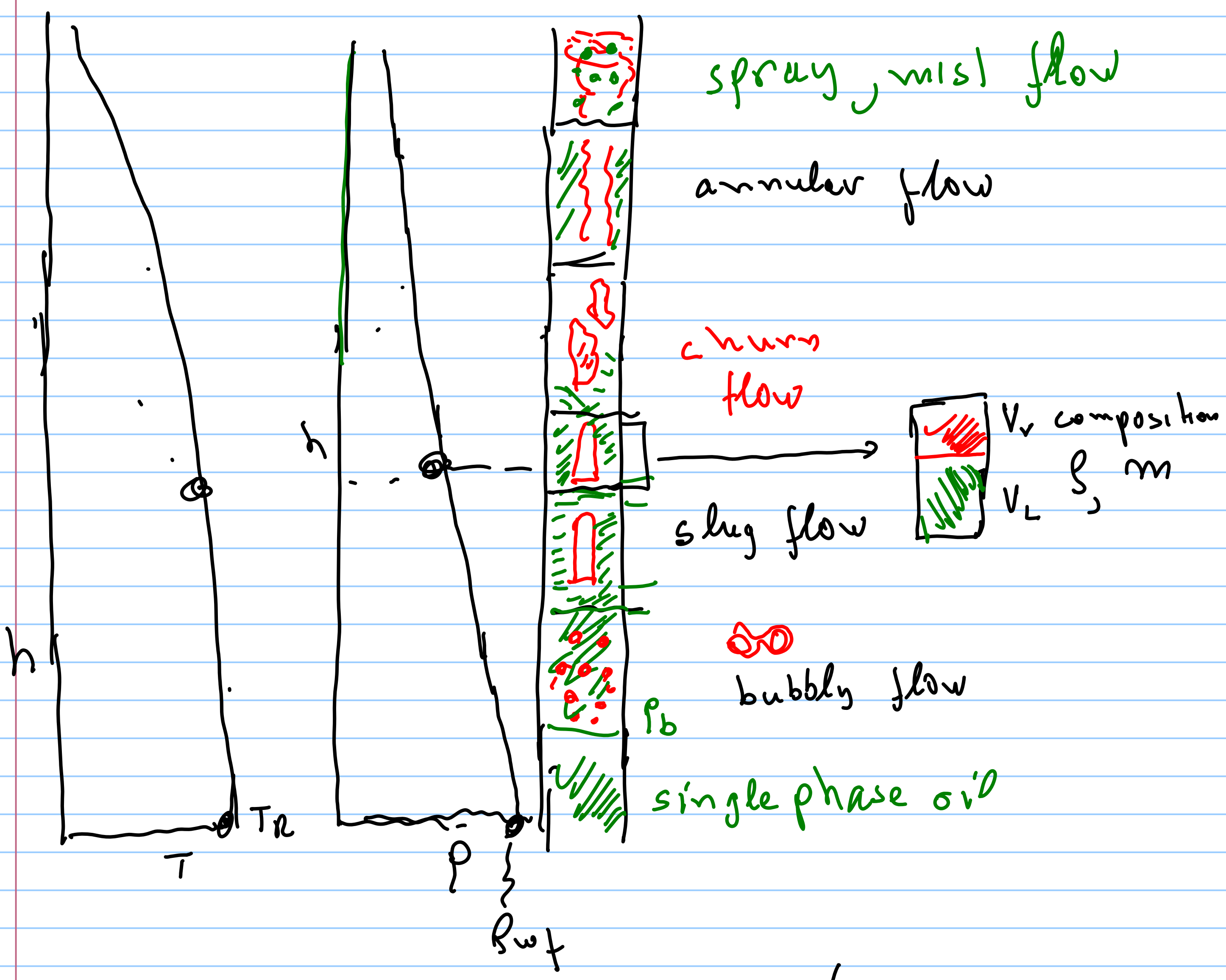


phase equilibrium in the reservoir







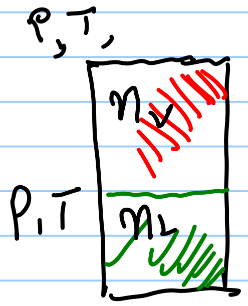


for each point in the pipe (tubing)
given p, T wish to calculate

V_G ρ_G m_G M_G μ_G composition gas
V_L ρ_L m_L m_L μ_L composition liquid
(molecular
composition

calculate VLE (vapor-Liquid equilibrium)

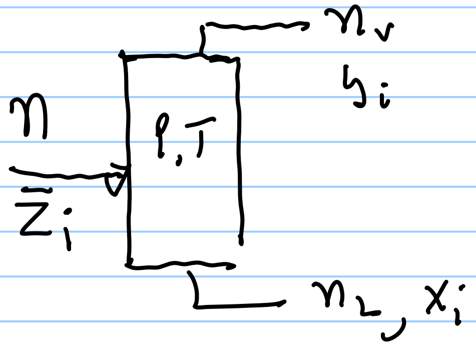
Flash calculation



n = total number of moles in the tank

$$n_{mixture} = n_v + n_L \qquad (1)$$

(conservation of material)



we know the molecular composition of the mixture given as mole fraction

$$\frac{\text{moles of comp } i}{\text{Total moles of the mixture}} = \text{mole fraction}$$

	z_i	y_i	x_i
c_1	0.95	x	x
c_2	0.04	x	x
c_3	0.01	x	x
	Σ 1.0	1.0	1.0

$n \times (\text{mol fraction})_i =$

conservation of spices

$$n \cdot z_i = n_v y_i + n_L x_i \quad \Big| : n \qquad (2)$$

$$(n_v + n_L) z_i = n_v y_i + n_L x_i$$

Defining = β or $F_v = \frac{n_v}{n_v + n_L} = \frac{n_v}{n}$ (3)

$$z_i = F_v y_i + (1 - F_v) x_i \qquad (2a)$$

unknows

given z_i → $\left.\begin{matrix} x_i & y_i \\ n_L & n_L \end{matrix}\right\}$ $2N+2$ unknowns

Equlibrium ration, K factor
partition quantity

$$K_i = \frac{y_i}{x_i} \qquad (4)$$

Reduce the number of unknown to $N+1$
(N = number of components in the mixture)

constraints (5)

$$\sum x_i = \sum y_i = \sum z_i = 1.0$$

$$\sum x_i - \sum y_i = 0$$

Low pressure and Temperature

$K = f(P, T)$ ((Typical surface processing < 100 bar))

in Reservoir conditions

$K = f(P, T, \text{composition})$

Solution

1949-50 Musket - McDowell (Gulf oil)
(Foot note)

1956 Rachford-Rice Exxon Research

$$\sum y_i - \sum x_i = 0$$

$$\sum (y_i - x_i) = 0$$

<u>main condition</u>

Development

$$z_i = F_v y_i + (1 - F_v) x_i \qquad (1)$$

✓ $$y_i = K_i x_i \qquad (2)$$

$$z_i = F_v K_i x_i + (1 - F_x) x_i \qquad (3)$$

$$z_i = x_i (F_v K_i + 1 - F_v) \qquad (4)$$

$$z_i = x_i [F_v (K_i - 1) + 1] \qquad (5)$$

✓ $$x_i = \frac{z_i}{[F_v (K_i - 1) + 1]} \quad \Big| \times (K_i - 1) \qquad (6)$$

$$x_i (K_i - 1) = \frac{z_i (K_i - 1)}{[F_v (K_i - 1) + 1]} \qquad (7)$$

$$\boxed{\sum x_i (K_i - 1)} = \sum \frac{z_i (K_i - 1)}{[F_v (K_i - 1) + 1]} \qquad (8)$$

However

$$\sum y_i - \sum x_i = \underline{\sum (y_i - x_i)} = 0 \qquad (9)$$

$$\sum (K_i x_i - x_i) = 0 \qquad (9_b)$$

or

$$\boxed{\sum x_i (K_i - 1) = 0} \qquad (9_c)$$

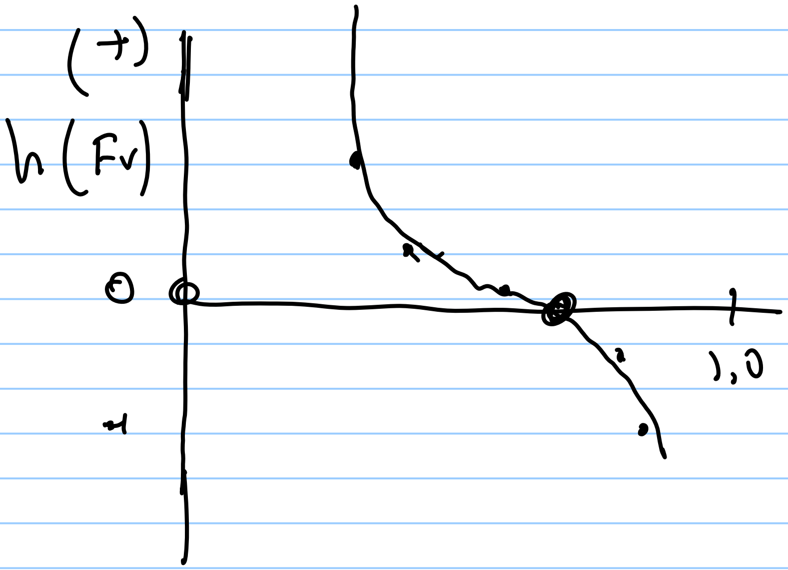
Then

$$h(F_v) = \sum_{i=1}^{N} \frac{z_i (K_i - 1)}{[F_v (K_i - 1) + 1]} = 0$$

Rachford Rice

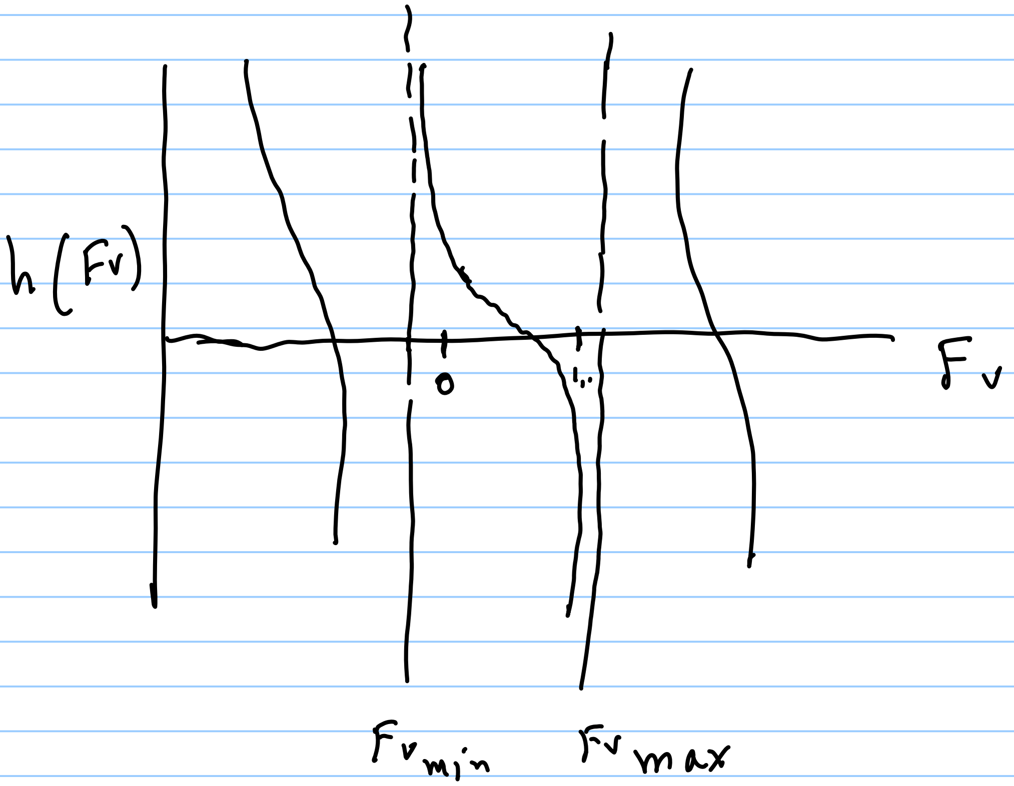
in manual calculation (1950 - 1980)

component z_i		K_i	Assume $F_v = 0,5$	F_v	F_v	x_i	y_i
C_1	x		$\frac{z_i(K_i-1)}{[F_v(K_i-1)+1]}$				
C_2	x						
C_3	x		↗				
⋮	x		x				
			x				
Σ	1,0		$\Sigma \cdot \boxed{X}$				



In Exell → drive $h(F_v) \to 0$ by changing F_v

Problems: multiple solution



only one solution when $Fv > Fv_{min}$

$Fv < Fv_{max}$

gauranty that all mole fractions will be positive

$$Fv_{min} = \frac{1}{1-K_{i_{max}}} = -\frac{1}{K_{i_{max}}-1} = -C_{i_{max}} = -MAX(C_i)$$

$$Fv_{max} = \frac{1}{1-K_{i_{min}}} = -\frac{1}{K_{i_{min}}-1} = -C_{i_{min}} = -MIN(C_i)$$

muskal & McDowell

Difine $C_i = \frac{1}{K_i-1}$ //

$$\text{"term"} = \frac{z_i(K_i-1)}{\underbrace{[Fv(K_i-1)+1]}} = \frac{z_i}{F_v+\frac{1}{K_i-1}} = \frac{z_i}{(F_v+C_i)} ..$$

as $\sum \frac{z_i(K_i-1)}{[F_v(K_i-1)+1]} = 0$ Then $\sum$ Term $= 0$

old manual calculations

component	z_i	K_i
c_1		
⋮		
N		

assume F_v

$\frac{z_i(K_i-1)}{[F_v(K_i-1)+1]}$

×
×
×
×

$\sum$ ☐

n_v	n_L	y_i	x_i

$(\sum \text{Term})^2$ ☐ ← solve $= 0$

P ☐ bara
T ☐ °C, °K
P_k ☐

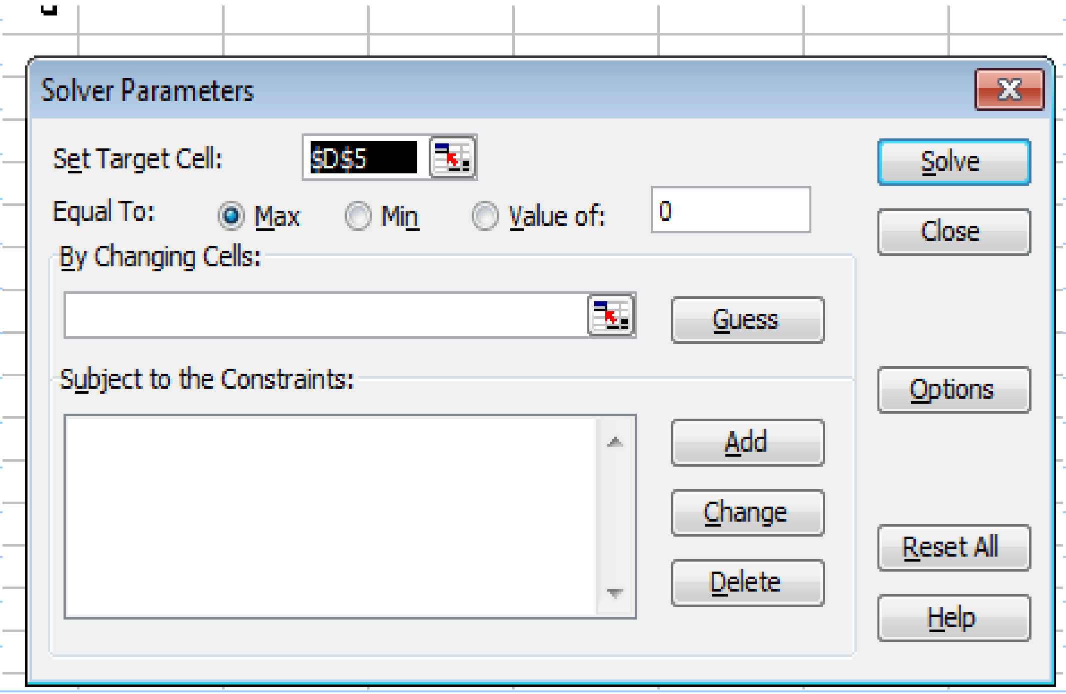
$F_v =$ ☐ e.g assume 0.5

$F_{v\max} = -\min(c_i)$

$F_{v\min} = -\max(c_i)$

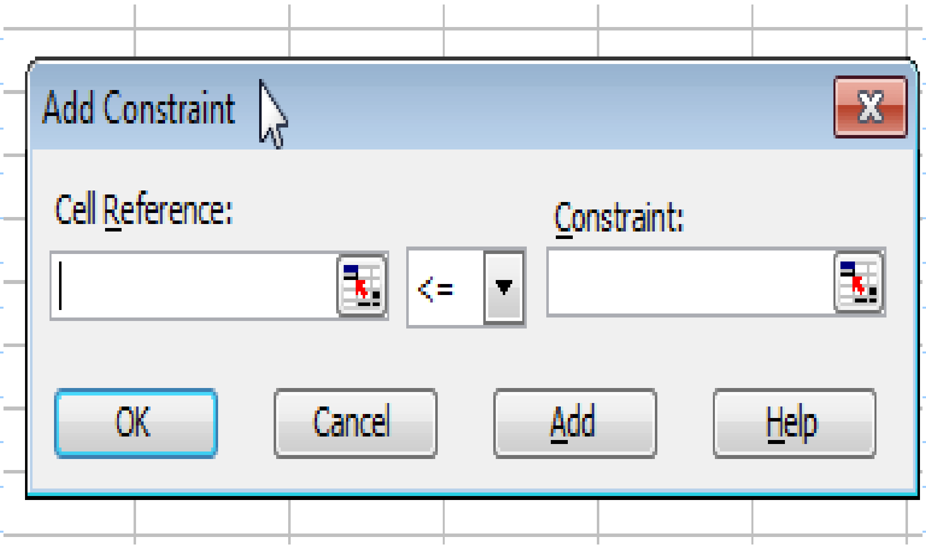
comp	z_i	$K_i(P,T,z_i)$	$c_i = \frac{1}{K_i-1}$	"Term" $f(F_v)$
c_1	☐	○	○	○
c_2	○	○	○	○
⋮	○	○	○	○
	○	○	○	○
	○	○	○	○
	○	○	○	○
			○	○
C_N				○
	$\sum 1.0$			$\sum$ ☐ $\sum$ term

solver



Set Target Cell: $D$5

Equal To: ● Max ○ Min ○ Value of: 0



<=

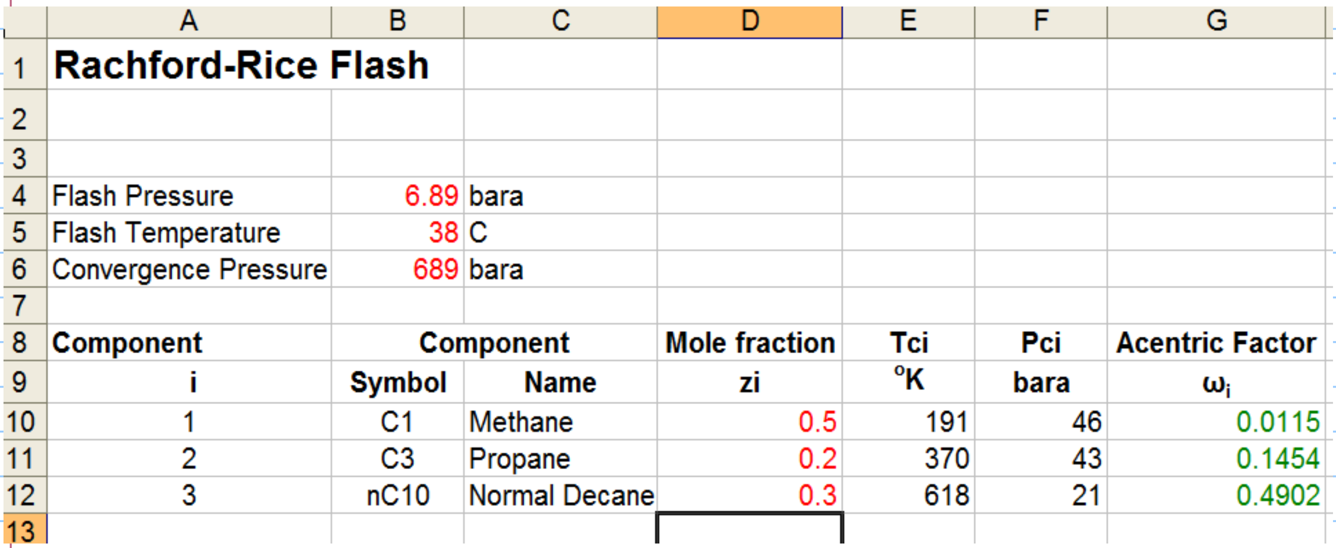
For quick conversion

solving for $(\Sigma \text{ Term})^2 = 0$ Function

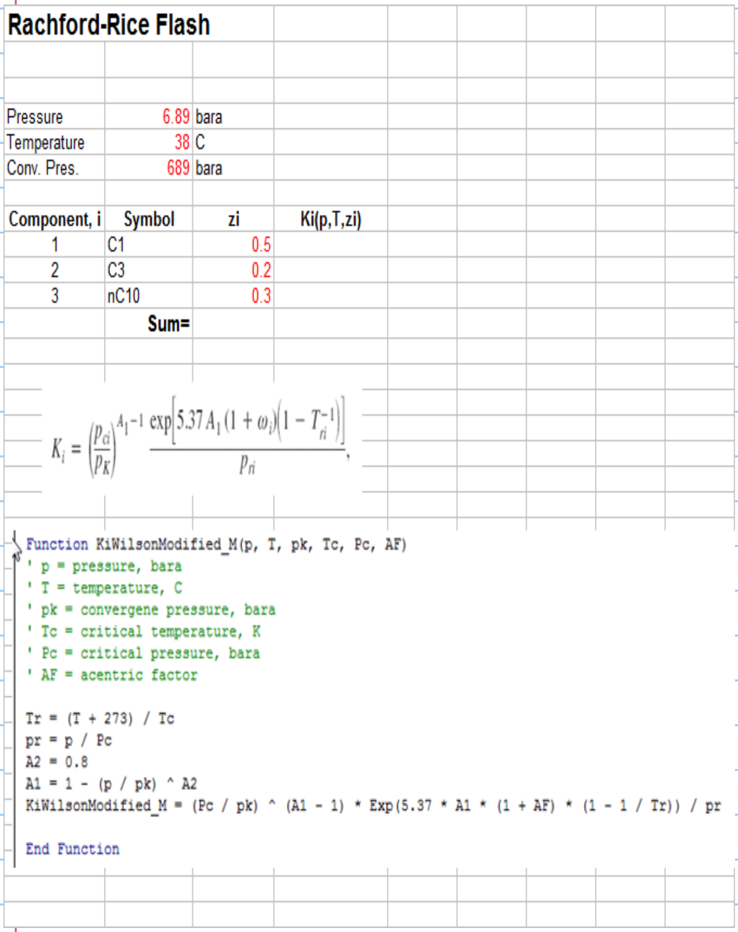
buy changing Fv independent variable

subject to

$Fv_{max} > Fv > Fv_{min}$ constaints



	A	B	C	D	E	F	G
1	**Rachford-Rice Flash**						
2							
3							
4	Flash Pressure	6.89	bara				
5	Flash Temperature	38	C				
6	Convergence Pressure	689	bara				
7							
8	**Component**	**Component**		**Mole fraction**	**Tci**	**Pci**	**Acentric Factor**
9	**i**	**Symbol**	**Name**	**zi**	**°K**	**bara**	**ω_i**
10	1	C1	Methane	0.5	191	46	0.0115
11	2	C3	Propane	0.2	370	43	0.1454
12	3	nC10	Normal Decane	0.3	618	21	0.4902
13							



Rachford-Rice Flash

Pressure	6.89	bara	
Temperature	38	C	
Conv. Pres.	689	bara	
Component, i	**Symbol**	**zi**	**Ki(p,T,zi)**
1	C1	0.5	
2	C3	0.2	
3	nC10	0.3	
	Sum=		

$$K_i = \left(\frac{p_{ci}}{p_K}\right)^{A_1-1} \frac{\exp\left[5.37 A_1 (1+\omega_i)\left(1-T_{ri}^{-1}\right)\right]}{p_{ri}},$$

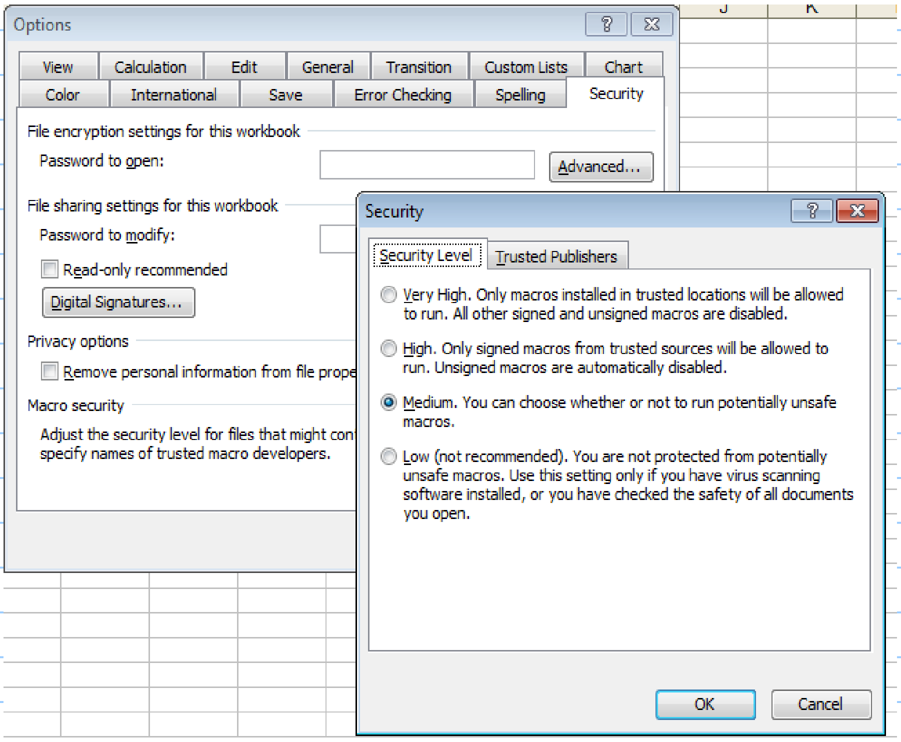
```
Function KiWilsonModified_M(p, T, pk, Tc, Pc, AF)
' p = pressure, bara
' T = temperature, C
' pk = convergene pressure, bara
' Tc = critical temperature, K
' Pc = critical pressure, bara
' AF = acentric factor

Tr = (T + 273) / Tc
pr = p / Pc
A2 = 0.8
A1 = 1 - (p / pk) ^ A2
KiWilsonModified_M = (Pc / pk) ^ (A1 - 1) * Exp(5.37 * A1 * (1 + AF) * (1 - 1 / Tr)) / pr

End Function
```

UDF in Excel (user defined function)

Reduce security to midium (Excel 2003)



File encryption settings for this workbook

Password to open:

File sharing settings for this workbook

Password to modify:

☐ Read-only recommended

Privacy options

☐ Remove personal information from file prope

Macro security

Adjust the security level for files that might con specify names of trusted macro developers.

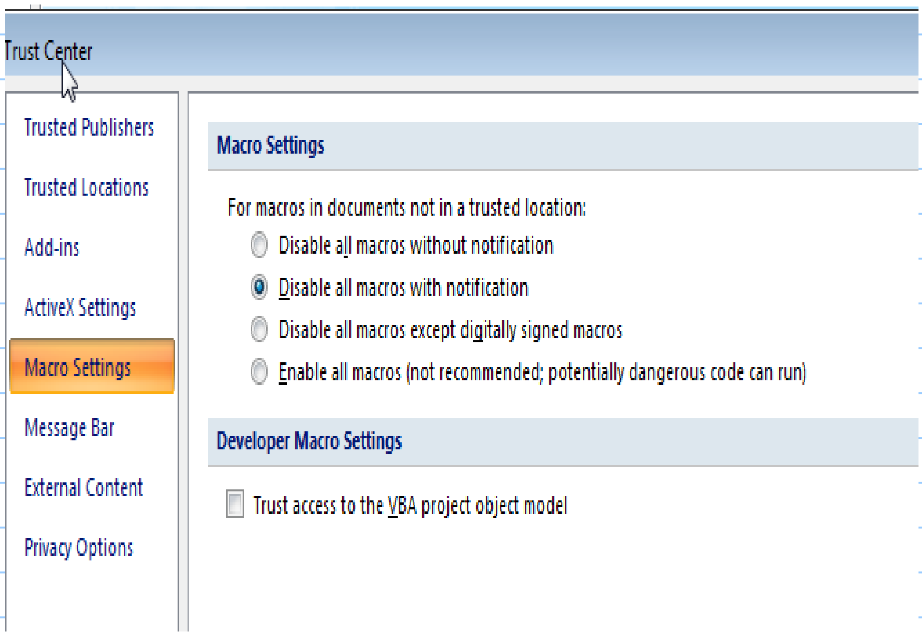
○ Very High. Only macros installed in trusted locations will be allowed to run. All other signed and unsigned macros are disabled.

○ High. Only signed macros from trusted sources will be allowed to run. Unsigned macros are automatically disabled.

● Medium. You can choose whether or not to run potentially unsafe macros.

○ Low (not recommended). You are not protected from potentially unsafe macros. Use this setting only if you have virus scanning software installed, or you have checked the safety of all documents you open.

Excel 2007



Macro Settings

For macros in documents not in a trusted location:

○ Disable all macros without notification

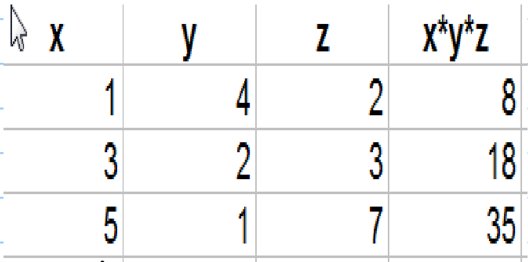
● Disable all macros with notification

○ Disable all macros except digitally signed macros

○ Enable all macros (not recommended; potentially dangerous code can run)

Developer Macro Settings

☐ Trust access to the VBA project object model



x	y	z	x*y*z
1	4	2	8
3	2	3	18
5	1	7	35

Alt F11 - open editor

insert modul

Alt F11 - return to Excel sheet

VLE - Flash calculation

we need to solve

$$h(F_v) = \sum_{i=1}^{i=N} (y_i - x_i) = \sum_{i=1}^{i=N} \frac{z_i (K_i - 1)}{\underbrace{1 + F_v (K_i - 1)}_{\text{RC Term}}} = \sum \frac{z_i}{\underbrace{F_v + C_i}_{\text{MM Term}}} = 0$$

$K_i = \frac{y_i}{z_i} = f(P, T, z_i)$

z_i = inflow stream composition

$F_v = \frac{n_v}{n_v + n_L}$

N = number of components in the stream

$C_i = \frac{1}{K_i - 1}$

subject to (constraints)

$$F_{v\,min} = \frac{1}{1 - K_{i\,max}} = -C_{i\,max} = -MAX(C_i)$$

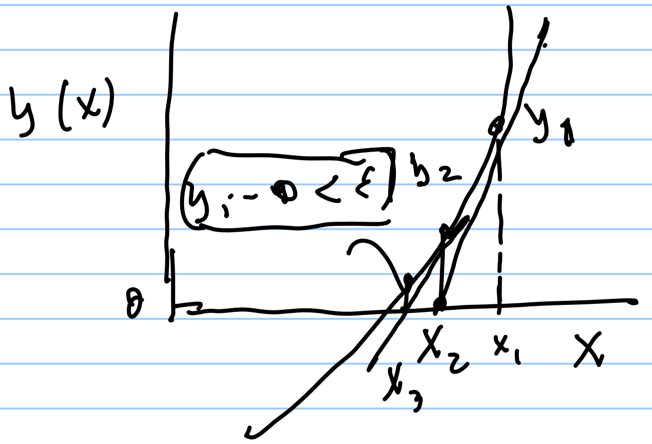
$$F_{v\,max} = \frac{1}{1 - K_{i\,min}} = -C_{i\,min} = -MIN(C_i)$$

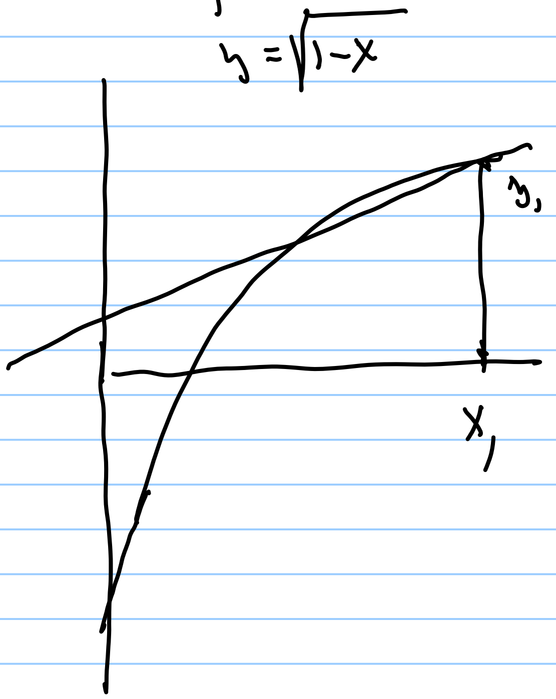
with numerical tricks

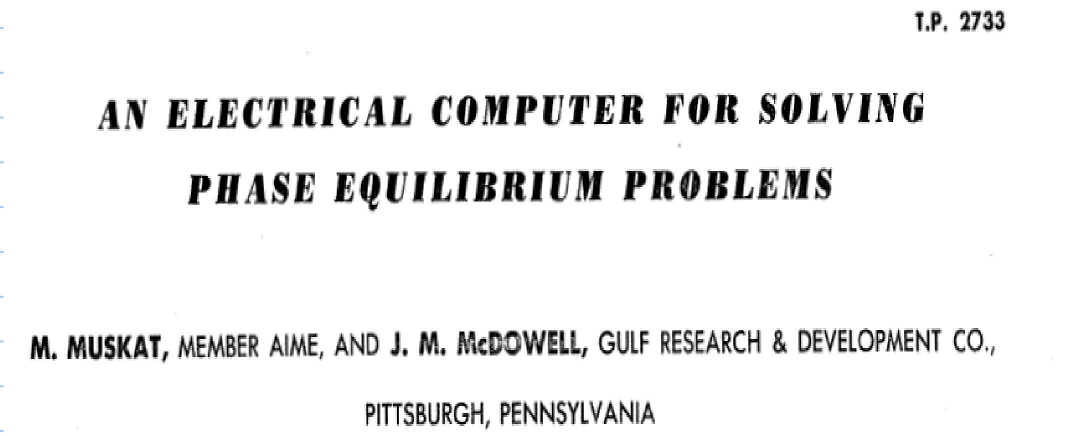
if $K_i = 1.0$, then C_i = "no term"

if C_i = "no term", then $\frac{z_i}{F_v + C_i} = 0$

N-R solution (graphical explanation)



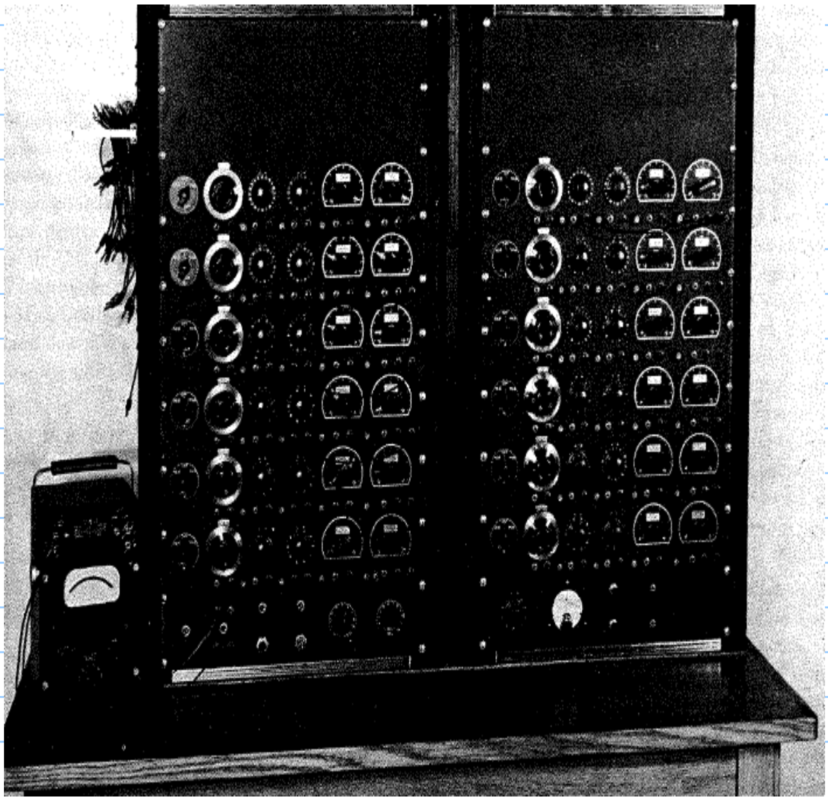


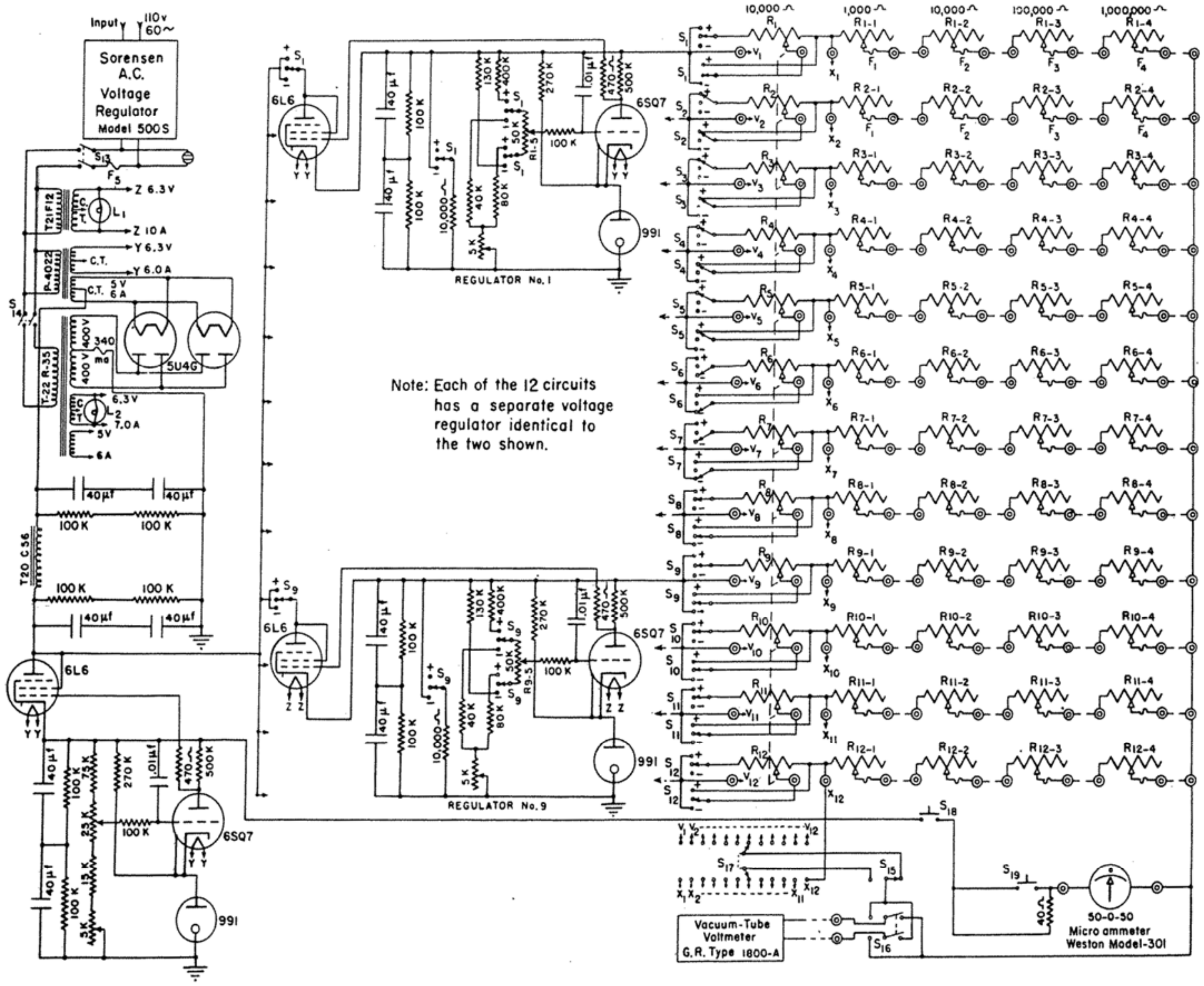


T.P. 2733

AN ELECTRICAL COMPUTER FOR SOLVING PHASE EQUILIBRIUM PROBLEMS

M. MUSKAT, MEMBER AIME, AND J. M. McDOWELL, GULF RESEARCH & DEVELOPMENT CO.,
PITTSBURGH, PENNSYLVANIA





Note: Each of the 12 circuits has a separate voltage regulator identical to the two shown.

$$\Sigma \frac{n_j}{c_j + n_g} = 0 \quad . \quad . \quad . \quad . \quad (6)$$

In Eq. (6) the voltages applied to the separate component analogs are given directly by the total composition

---

* It may be noted here that Eq. (6) is itself of interest as providing a means for substantially reducing the time required for direct numerical solution of the equilibrium equations, as generally used in the form of Eqs. (3), provided the K_j are converted into the corresponding c_j by preliminary calculations.

3.6.1 Hoffman *et al.* Method. Hoffman *et al.*[99] propose a method for correlating K values that has received widespread application.

$$K_i = \frac{10^{\left(A_0 + A_1 F_i\right)}}{p}$$

$$\text{or } \log K_i p = A_0 + A_1 F_i, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.155)$$

$$\text{where } F_i = \frac{1/T_{bi} - 1/T}{1/T_{bi} - 1/T_{ci}} \log\left(p_{ci}/p_{sc}\right); \quad \ldots\ldots\ldots\ldots \quad (3.156)$$

T_c = critical temperature; p_c = pressure; T_b = normal boiling point; p_{sc} = pressure at standard conditions; and A_1 and A_0 = slope and intercept, respectively, of the plot $\log(K_i p)$ vs. F_i.

Whitson and Torp[100] suggest a generalized form of the Hoffman *et al.*[99] equation in terms of convergence pressure and acentric factor.

$$K_i = \left(\frac{p_{ci}}{p_K}\right)^{A_1 - 1} \frac{\exp\left[5.37\, A_1\, (1 + \omega_i)\left(1 - T_{ri}^{-1}\right)\right]}{p_{ri}},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.159)$$

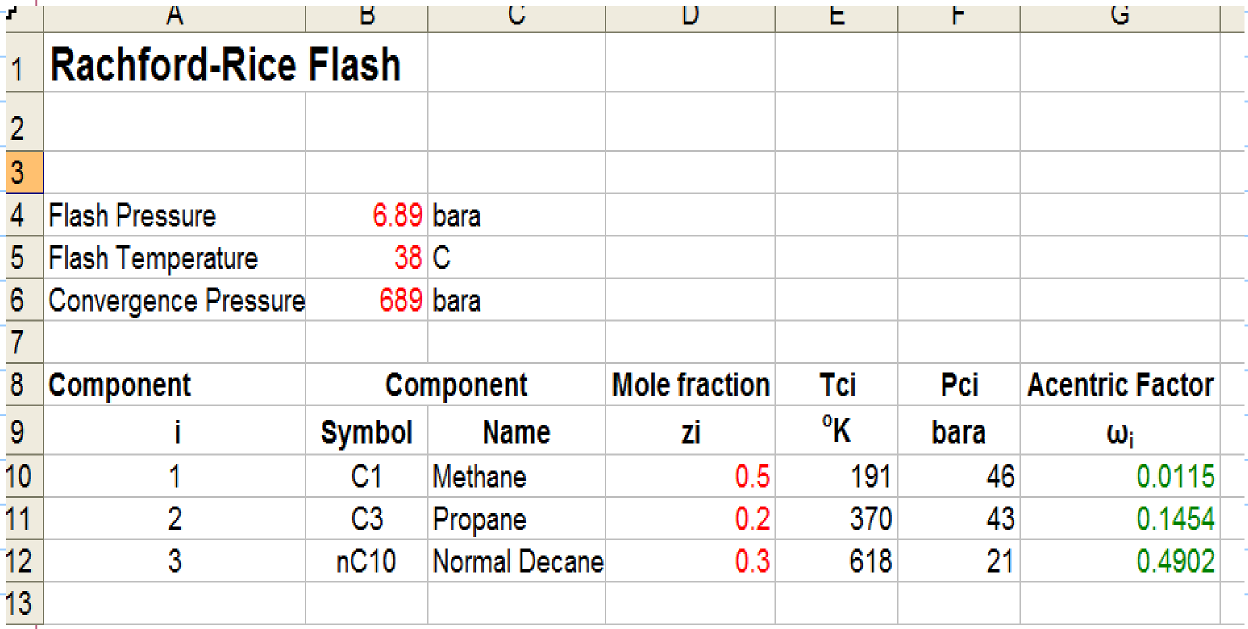
where A_1 = a function of pressure, with $A_1 = 1$ at $p = p_{sc}$ and $A_1 = 0$ at $p = p_K$. The key characteristics of K values vs. pressure and temperature are correctly predicted by Eq. 3.159, where the following pressure dependence for A_1 is suggested.

$$A_1 = 1 - (p/p_K)^{A_2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.160)$$

where A_2 ranges from 0.5 to 0.8 and pressures p and p_K are given in psig. Canfield[105] also suggests a simple K-value correlation based on convergence pressure.

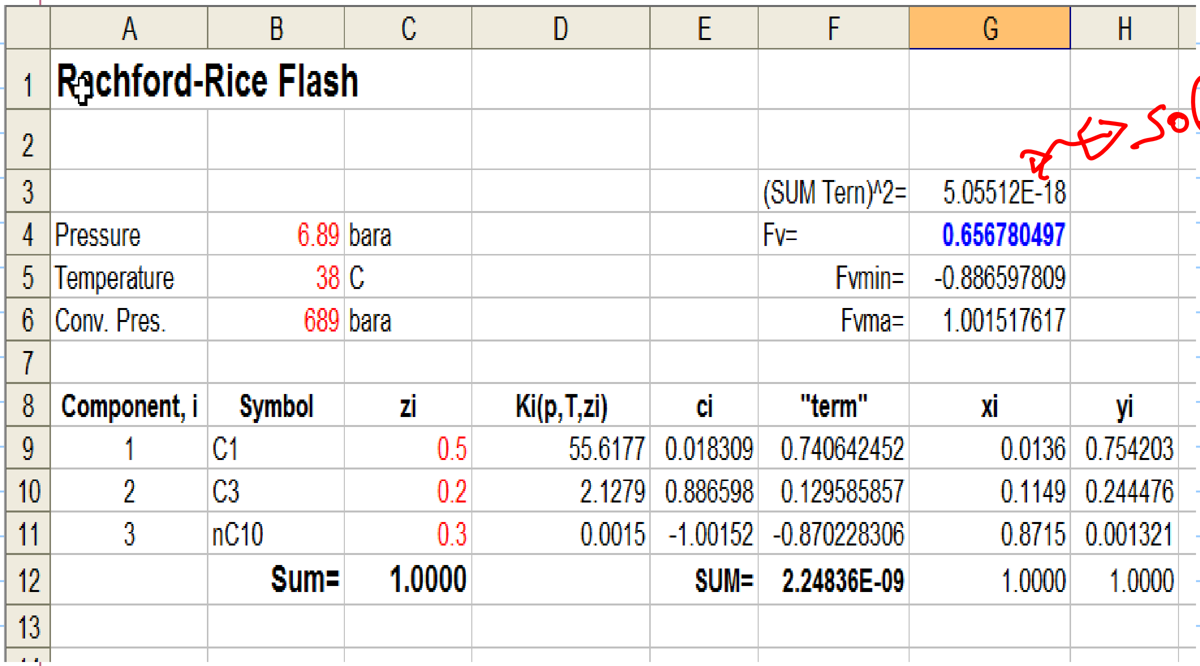
$$x_i = \frac{z_i}{\left[F_v\left(K_i-1\right)+1\right]}$$

Data (given)



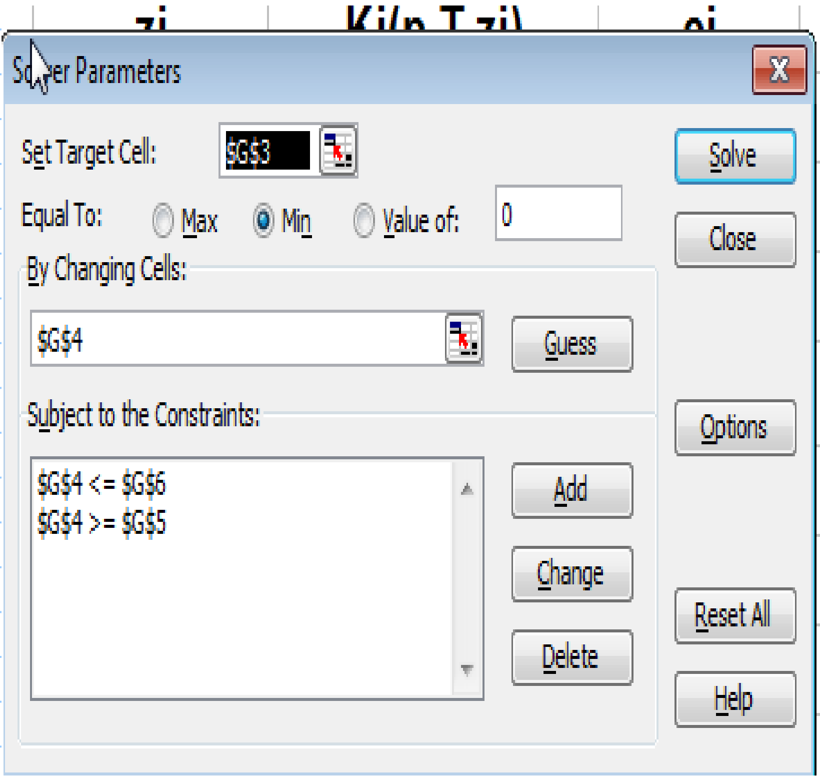
	A	B	C	D	E	F	G
1	**Rachford-Rice Flash**						
2							
3							
4	Flash Pressure	6.89	bara				
5	Flash Temperature	38	C				
6	Convergence Pressure	689	bara				
7							
8	**Component**	**Component**		**Mole fraction**	**Tci**	**Pci**	**Acentric Factor**
9	**i**	**Symbol**	**Name**	**zi**	**°K**	**bara**	**ω_i**
10	1	C1	Methane	0.5	191	46	0.0115
11	2	C3	Propane	0.2	370	43	0.1454
12	3	nC10	Normal Decane	0.3	618	21	0.4902
13							

solution



solver

	A	B	C	D	E	F	G	H
1	**Rachford-Rice Flash**							
2								
3						(SUM Tern)^2=	5.05512E-18	
4	Pressure	6.89	bara			Fv=	**0.656780497**	
5	Temperature	38	C			Fvmin=	-0.886597809	
6	Conv. Pres.	689	bara			Fvma=	1.001517617	
7								
8	**Component, i**	**Symbol**	**zi**	**Ki(p,T,zi)**	**ci**	**"term"**	**xi**	**yi**
9	1	C1	0.5	55.6177	0.018309	0.740642452	0.0136	0.754203
10	2	C3	0.2	2.1279	0.886598	0.129585857	0.1149	0.244476
11	3	nC10	0.3	0.0015	-1.00152	-0.870228306	0.8715	0.001321
12		**Sum=**	**1.0000**		**SUM=**	**2.24836E-09**	1.0000	1.0000
13								



Set Target Cell: $G$3

Equal To: ○ Max ● Min ○ Value of: 0

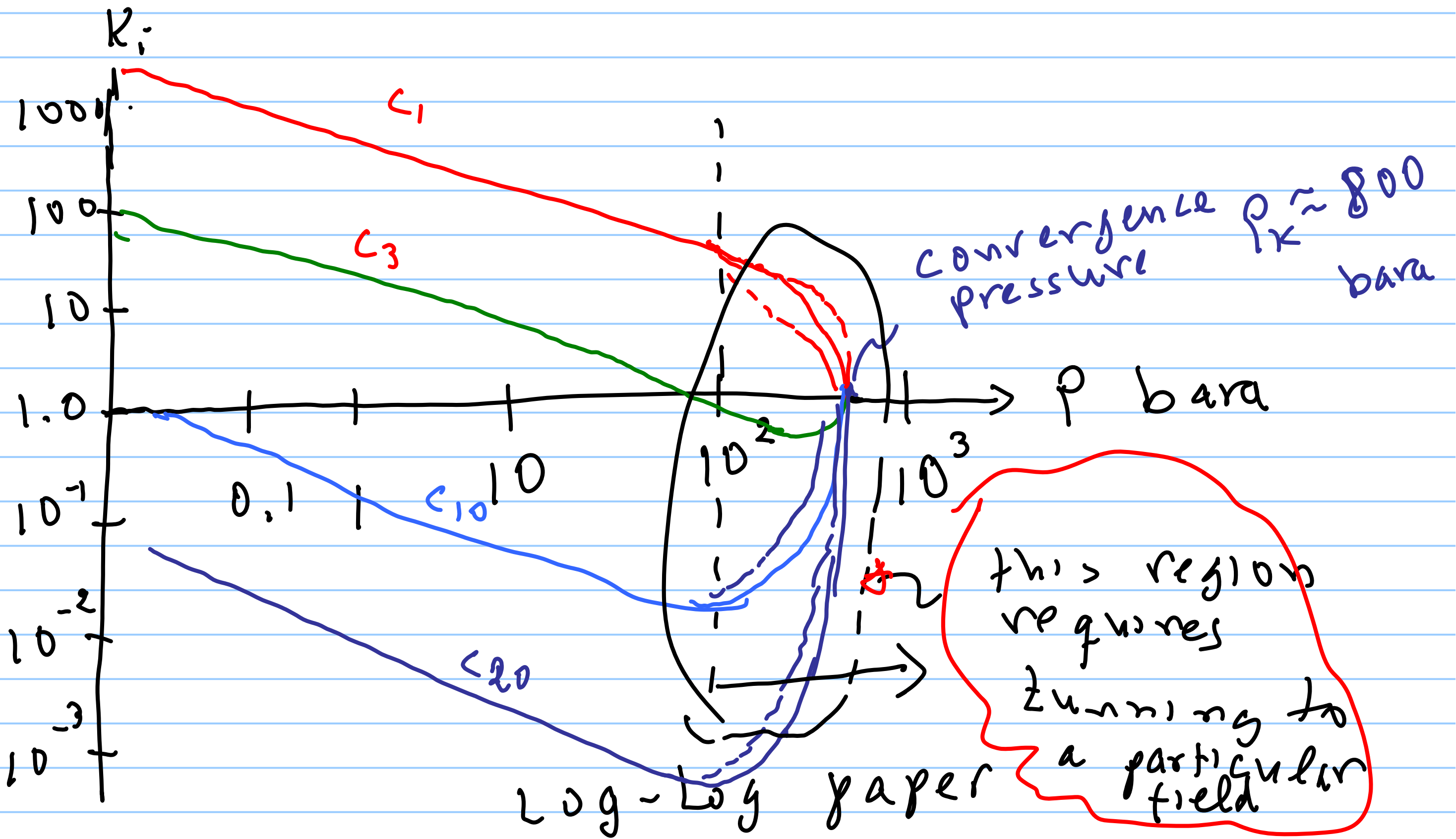
By Changing Cells: $G$4

Subject to the Constraints:
$G$4 <= $G$6
$G$4 >= $G$5

solver menue

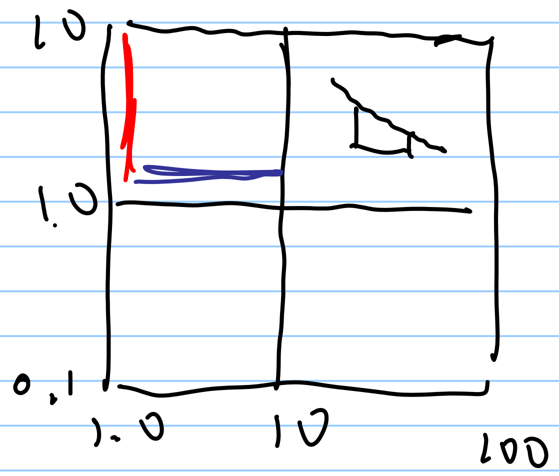
Laboratory measurements of K_i

Example $T = 100\ °C$ log-log plot



at the convergence pressure all lines converges to $K_i = 1.0$

Proper Log-Log paper



the K_i vs p on log-log pape have a slop of -1

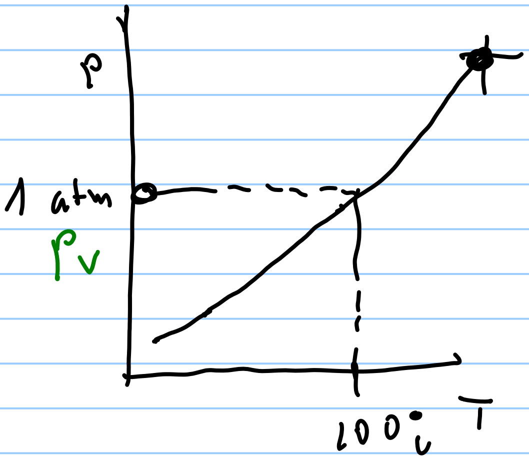
$$K_i = \frac{a}{p}$$

$$\log K_i = \log a - \log p$$

$K_i = 1.0$

$\log K_i = 0$

$p = a$



normal vapor pressure (water)

for single component

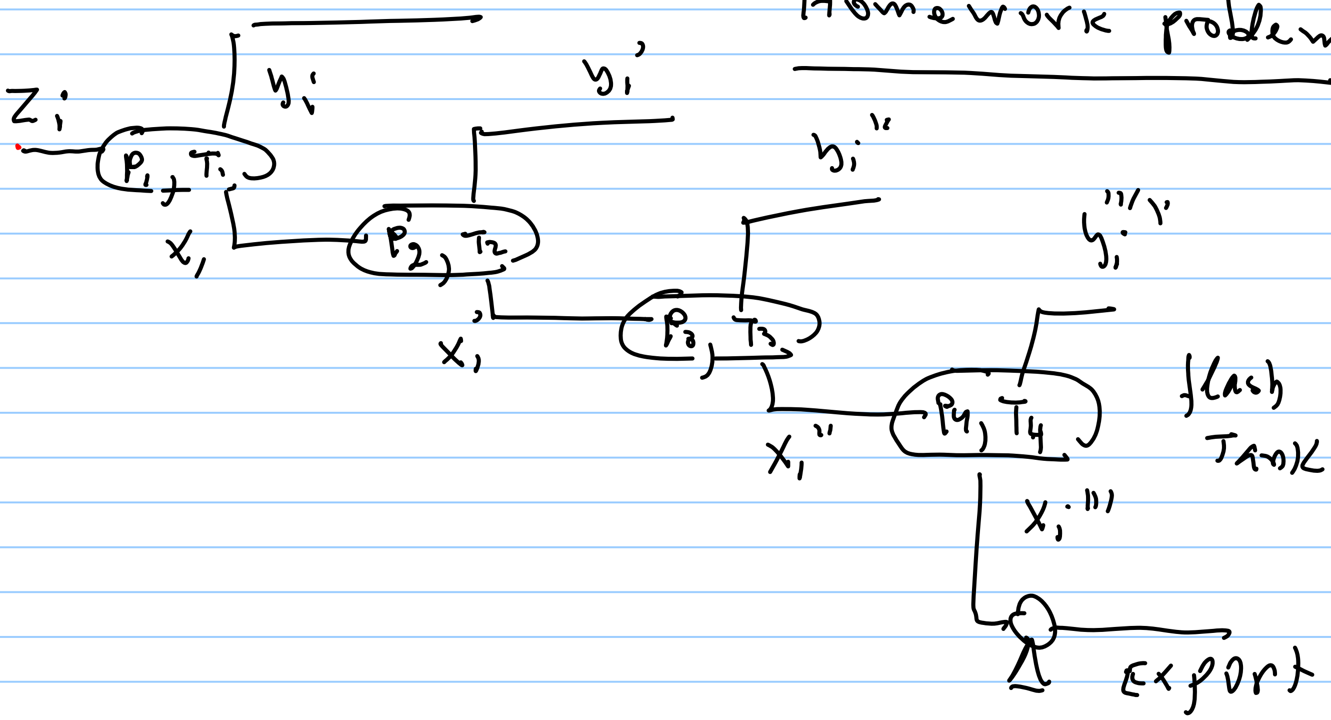
$y_i = x_i = \underline{1}$ $K_i = 1.0$

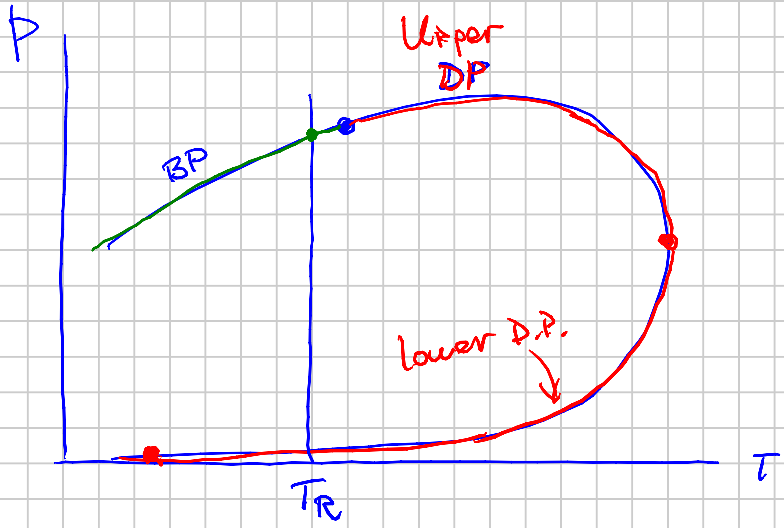
single component

for the straight line part

$$\boxed{K_i = \frac{P_{vi}(T)}{p}}$$

Homework problem





For the reservoir mixture with composition z_i

$GOR(z_i)$

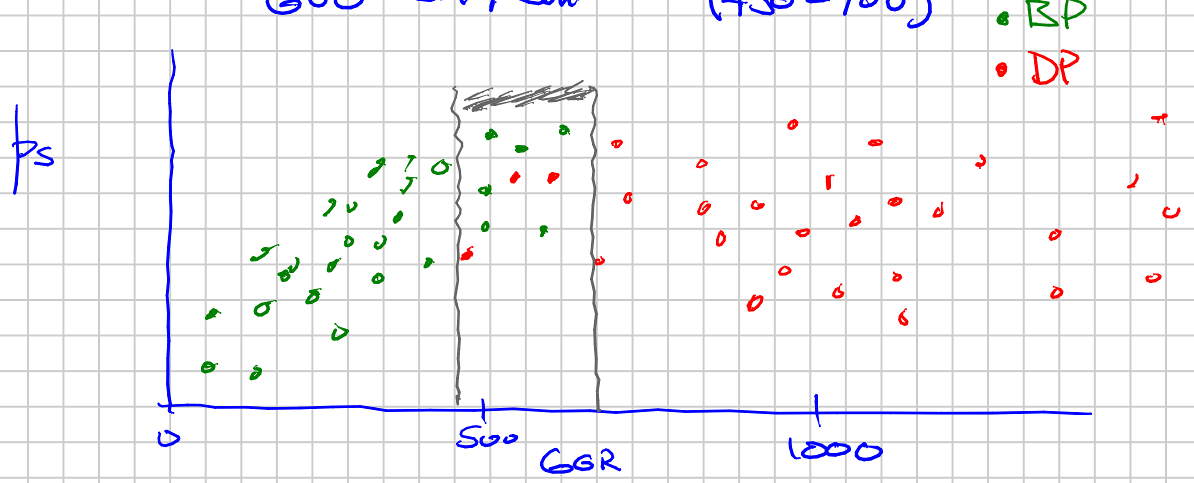
OIL : $T_R < T_c$: B.P. p_b @ T_R $< GOR^*$

GAS : $T_R > T_c$: D.P. p_d @ T_R $> GOR^*$

$$GOR \approx \frac{(1-z_{6+})}{z_{6+}} \cdot \frac{\overbrace{(RT_{sc}/p_{sc})}^{23.67\ Sm^3/kg\text{-}mole}}{(M/\underset{\uparrow\ kg/m^3}{\rho})_{6+}} \qquad \left[\frac{Sm^3}{Sm^3}\right]$$

GOR^* : Dividing B.P. − D.P.

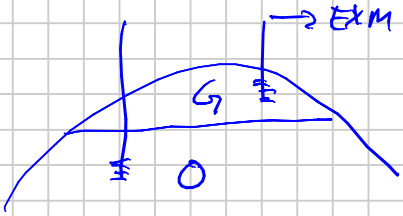
~ 600 Sm^3/Sm^3 (450−700)



OK/TX : GOR* ~ 10,000 scf/STB = 1800 Sm^3/Sm^3

“Gas” or “Oil” – Who Cares?

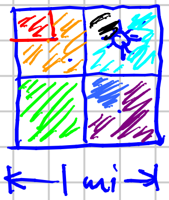
(– Tax)



– Middle East (& elsewhere)
Ownership can be linked to “Gas Reservoir” OK
“Oil Reservoir” X

– Domestic USA
“Well Spacing” = # Wells per 1 mi × 1 mi Section

“Gas Well”
640 acre spacing
⇒ 1 well/section



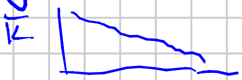
“Oil Well”
80 acre 8 wells/section
160 acre 4 wells/section

} Ownership & Cost & Revenues

“Ideal Spacing” (Depletion)
As few wells as needed to recover as much as economically possible (Gas: 60-90%, Oil: 15-25%), in a “prudent” time (10-30 yr).

Two properties determine ~ how fast a well can drain a given area.

– k – ≥ 10 md



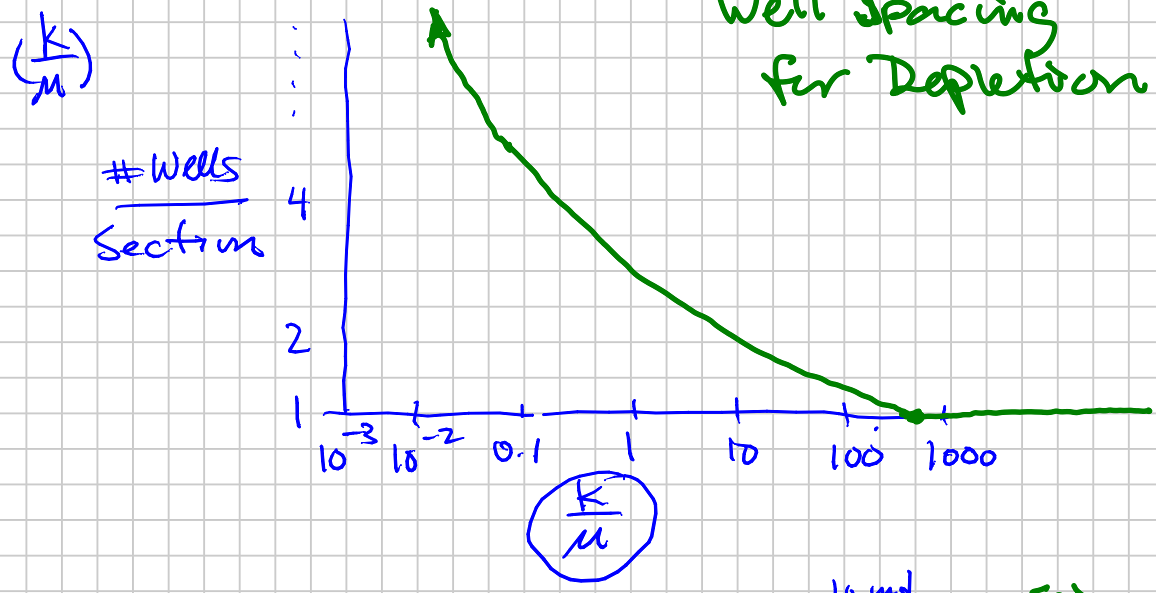
– μ – $\mu_g \sim 0.02 - 0.04$ cp

$\mu_o \sim \underrightarrow{0.2 - 200}^{++}$

} drain 1 section w/ 1 well ~ 10-20 years

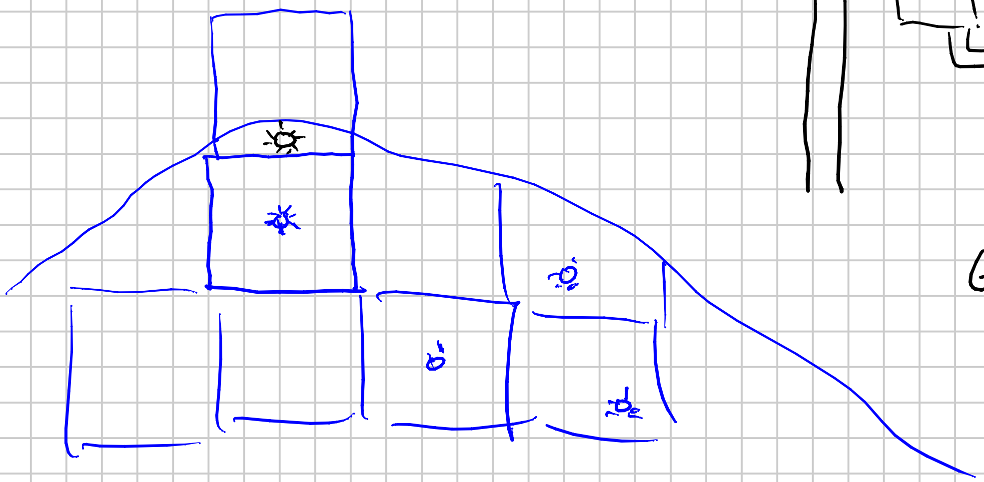
8
9
4
↑

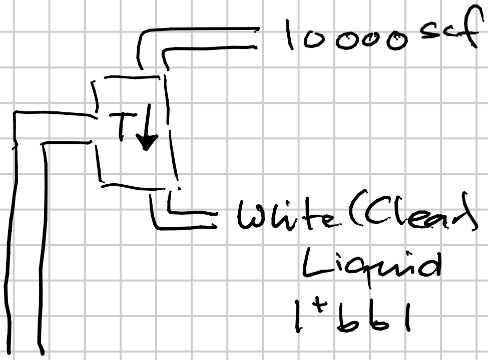
Well Spacing for Depletion



$\frac{10\text{ md}}{0.02\text{ cp}} = 500$

White Oilers in Texas 1980s



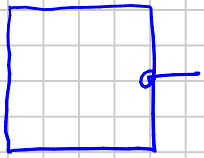


GOR ≤ 10,000

4 wells/section

Saturation Pressure Calculation

Saturated ⇒ Two-Phases



Appearing Phase = "Incipient" Phase

$y_i : x_i$ are in equilibrium!

$z_i = x_i$: Bubblepoint ($F_g = 0$) ⇒ y_i is ∈ "appearing" gas phase

$= y_i$: Dewpoint ($F_g = 1$) ⇒ x_i is ∈ "appearing" oil phase

$$K_i = y_i / x_i$$

$$z_i = F_g \, y_i + (1 - F_g)\, x_i$$

$$F_g \equiv \frac{n_g}{n_g + n_o}$$

$$K_i\left(p, T, \text{"}\underline{\underline{z_i}}\text{"} : p_K = \text{convergence pressure}\right)$$

$$p_K(T, z)$$

Bubblepoint Calc: $x_i \equiv z_i$

$$\sum y_i = 1$$

$$y_i = K_i \cdot x_i = x_i \cdot K_i = z_i K_i(p, T, p_K)$$

$$\sum z_i K_i = 1$$

$$h = 0 = 1 - \sum z_i K_i(p, T, p_K)$$

For a given T and z_i
we know (or can find) p_K

$$\boxed{h_b = 1 - \sum z_i K_i(\underset{\uparrow}{p_b})_{T,p_K} = 0}$$

Find p_b to drive $h \to 0$

Dewpoint: $y_i = z_i$

$$\sum x_i = 1 \qquad x_i = y_i / K_i = z_i / K_i$$

$$\sum \frac{z_i}{K_i} = 1$$

$$\boxed{h_d = 1 - \sum \frac{z_i}{K_i(\underset{\uparrow}{p_d})_{T,p_K}} = 0} \qquad \text{fixed } \underbrace{T \,\&\, z_i}_{p_K}$$

Dewpoint Press
that drives $h_d \to 0$

e.g. Modified Wilson Eq. $K_i(p\,;\,T, p_K)$

* * *

You don't (necessarily) know if you have
a bubblepoint <u>OR</u> an upperdewpoint

- One will converge to $p_s = p_K$ (Trivial Solution)
$K_i = 1$

*

- Other will converge to a "valid" solution where $p_s < p_k$ and $K_i \neq 1$

* Exception is if $\underbrace{p_s = p_c}_{K_i = 1}$ → i.e $T = T_c$

				$h_b \times h_d$
B.P.	$\underline{h_b = 0}$	$\underline{h_d \neq 0}$	@ solved p_b	0
D.P.	$h_d = 0$	$h_b \neq 0$	@ solved p_d	0
C.P.	$h_d = h_b = 0$		@ solved $p_s = p_k$	0

$\boxed{h(p_s) = 0}$ solve for p_s

then ask: BP
DP
CP

$h_b(p)$

$h_d(p)$

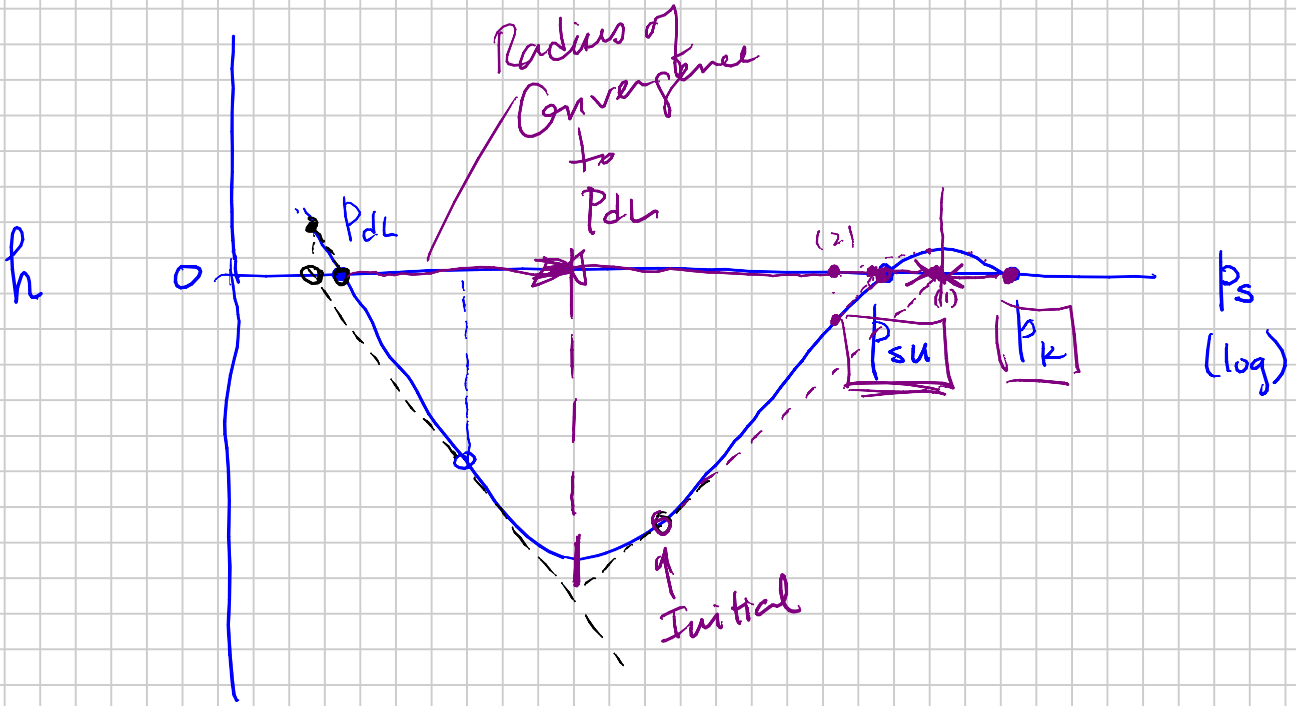
$$h(p_s) = h_b(p) \times h_d(p) = 0$$

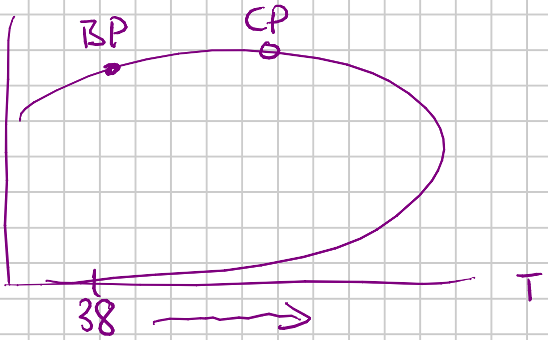
After solution

if $h_b(p_s) = 0 \Rightarrow$ BP

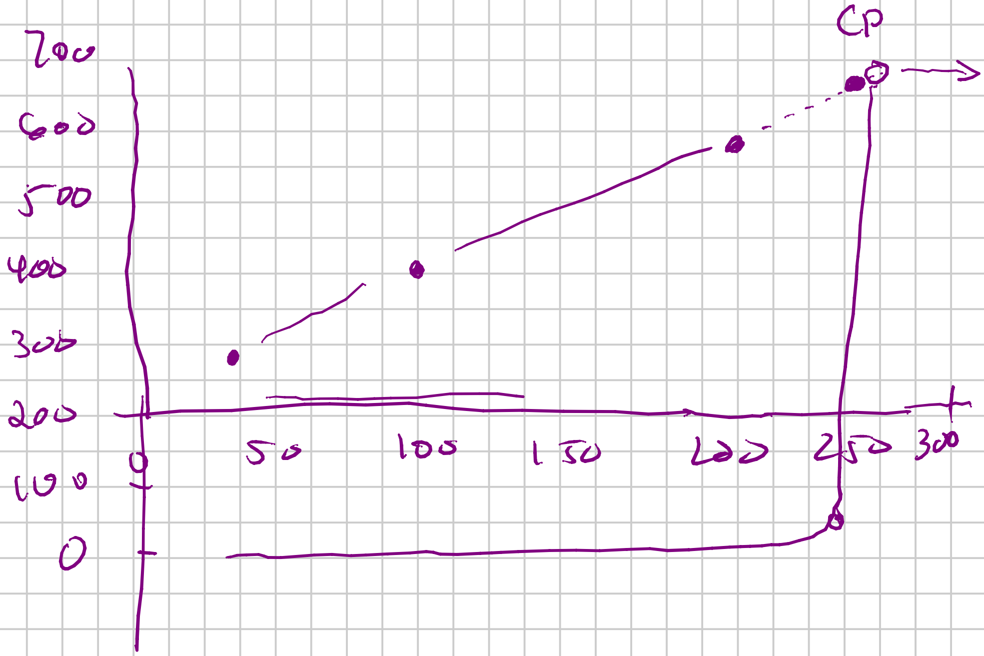
if $h_d(p_s) = 0 \Rightarrow$ DP

if $h_b(p_s) = h_d(p_s) = 0 \Rightarrow$ CP





	T (°C)	P_s (bara)	Sat Type
	38	267.8	BP
	100	401.0	BP
	200	579.6	BP
Tc ↕	250	675.9	
	300		



SUMMARY SATURATION PRESSURE CALCULATION

Bubblepoint: $\sum y_i = 1 \Rightarrow h_b = 1 - \sum z_i K_i(p)_{T, p_K}$

Dewpoint: $\sum' x_i = 1 \Rightarrow h_d = 1 - \sum z_i / K_i(p)_{T, p_K}$

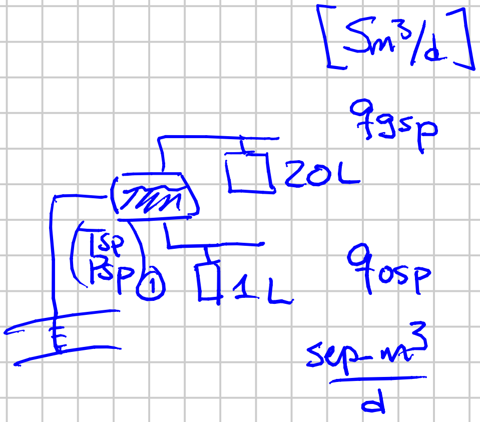
Ch. 6 – Laboratory PVT Experiments (all)

Ch. 7 – Black Oil PVT Model (part)

} 3 weeks

OILS PVT Tests

- Sample from the Well
  - Surface Separator Samples
  - Traditional Bottomhole (cased-hole)
  - Openhole Formation Tester Samples
    MDT
    RCI



Lab: QC
Recombine in Separator Test

$GOR_{sp} = \frac{q_{gsp}}{q_{osp}}$

⇓

Wellstream

PVT Cell ~ 300-500 mL

Heat the Cell ~ T_R

① Blind Cell / OK for Oils $GOR \lesssim 200\ Sm^3/Sm^3$

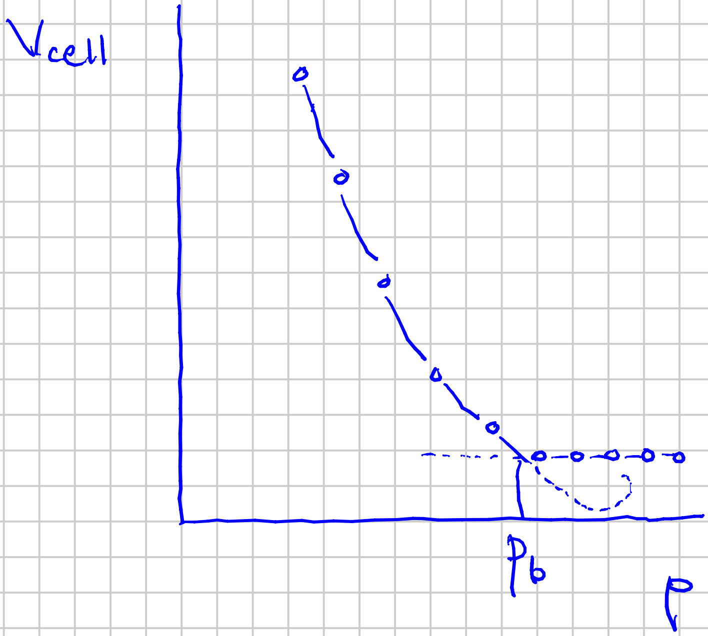
② Windowed Cell

Constant Composition Expansion (CCE)

Constant Mass Expansion (CME)

$\underline{P}$	$\underline{V_{cell}}$
$\underline{p_b}$	

$\Big\} \Rightarrow c_o \sim \text{const}$

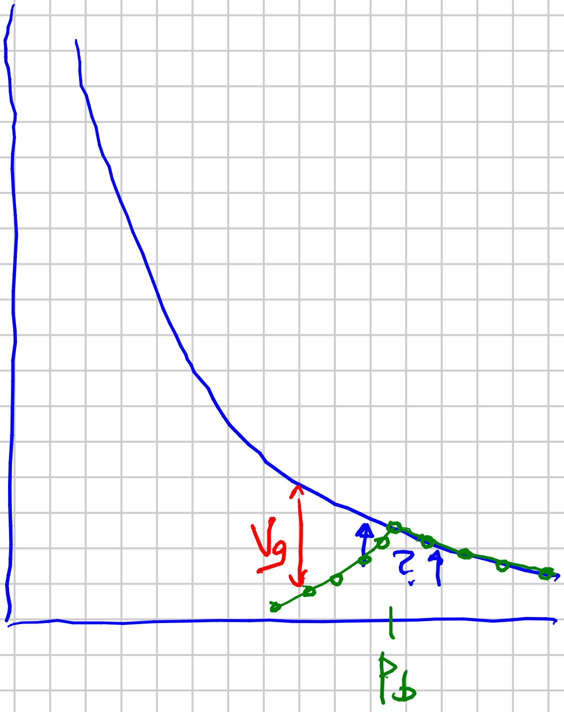


$GOR \gtrsim 250\ Sm^3/Sm^3$

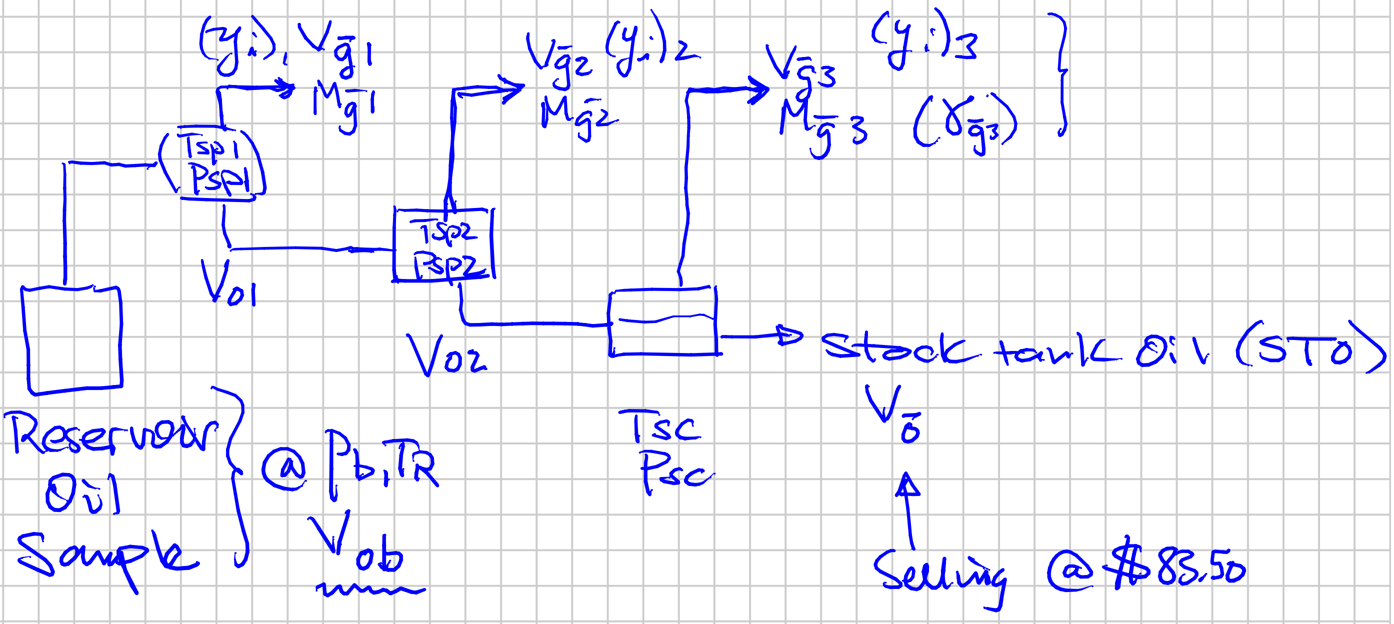
Windowed Cell

P	V_{cell}	V_o	V_o/V_{cell}
⋮			ρ_o
$\boxed{p_b}$			
⋮			

$\Big\} \Rightarrow c_o(p)$



Multi-Stage Separator Test (SEP)



$\bar{g}$: S.C. gas
$\bar{o}$: STO
} @ Tsc, Psc

REPORTED (KEY) DATA

0.92 → 0.35

$$\underset{@\,P_b}{B_{ob}} = \frac{V_{ob}}{V_{\bar{o}}} > 1$$

1:02-3

; Shrinkage Factor

$$b_{ob} = \frac{1}{B_{ob}} = \frac{V_{\bar{o}}}{V_{ob}}$$

Oil Formation Volume Factor (B_o)
"FVF"

Total Solution Gas-Oil Ratio

$$R_s = \frac{\sum_{i=1}^{N_s} V_{\bar{g}}}{V_{\bar{o}}}$$

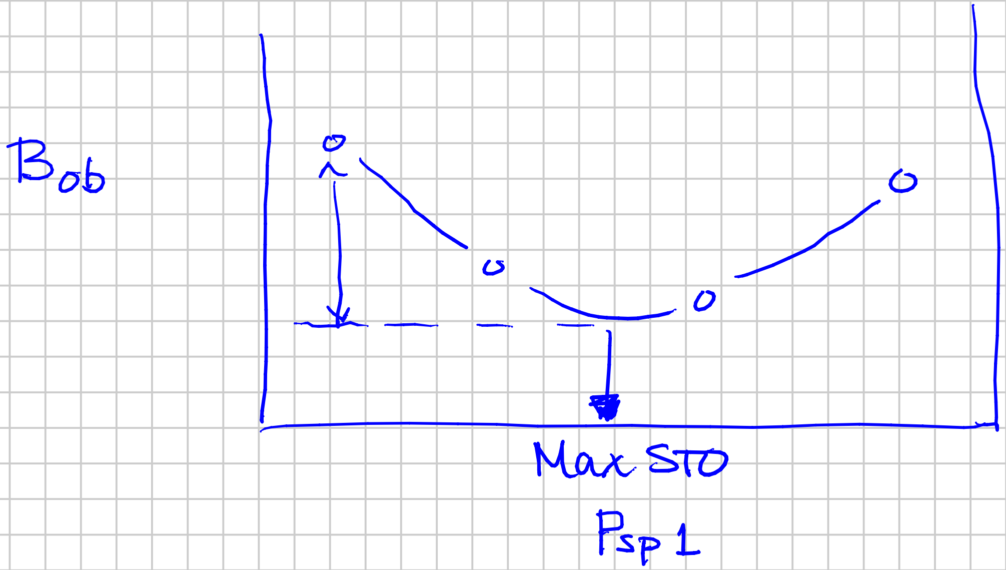
Total Surface Gas MW ($\gamma_{\bar{g}}$)

$$\bar{\gamma}_{\bar{g}} = \frac{\sum V_{\bar{g}i} \cdot \gamma_{\bar{g}i}}{\sum V_{\bar{g}i}}$$

$\gamma_{\bar{o}}$ usually expressed as API Gravity

$$\gamma_{API} = \frac{141.5}{\gamma_{\bar{o}}} - 131.5$$

3-4 SEP Tests



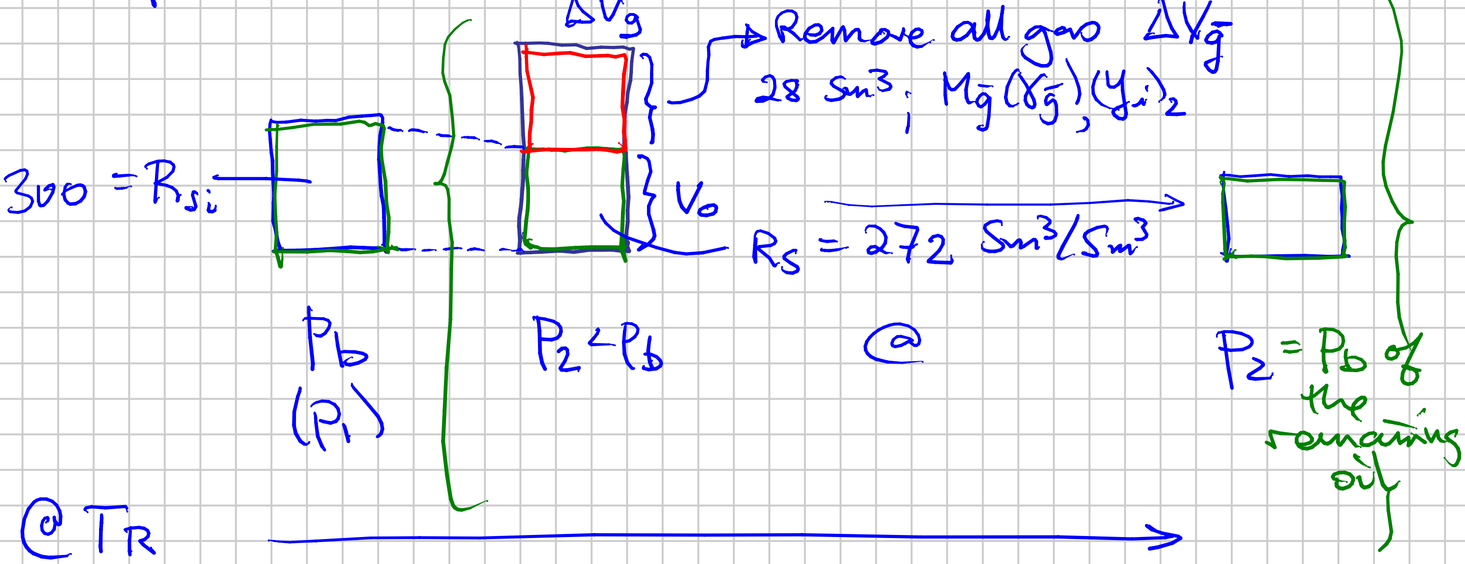
How B_o and R_s change as p_R drops below the initial bubblepoint.

- Calc. q_o & q_g from Darcy
- Material Balance – $p_R = f(N_p)$ ← Cum. Oil Prod.
- $IOIP = \dfrac{\text{"}HCPV_o\text{"}}{B_{oi}}$

$$\underset{\text{(Solution)}}{IGIP} = IOIP \times R_{si}$$

$$\left.\boxed{\begin{matrix} B_o(p<P_b) \\ R_s(p<P_b) \end{matrix}}\right\} \text{Important for RF and Flow}$$

Laboratory conducts an experiment at $p < P_b$: Depletion Test for Oils

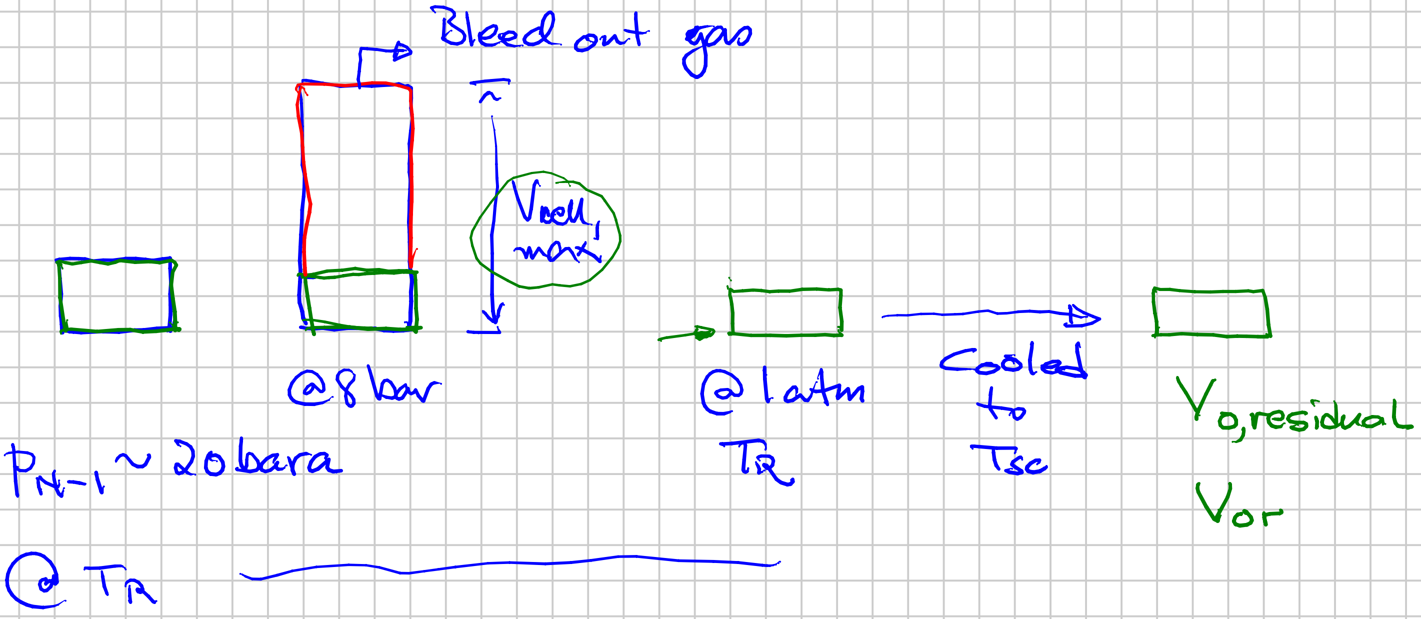


Repeat ~ 7–12 stages

Differential Liberation Experiment (DLE)

Differential Flash Liberation (DFL)

Last stage of Depletion is special



REPORTED AS LAB DATA:

$$B_{od} \equiv \left(\frac{V_o}{V_{or}}\right) \qquad \left(\frac{\Delta V_g}{V_{or}}\right) \equiv \Delta R_{sd}$$

May depend on how the last stage is conducted (high T_R, high API)

$$\left(\begin{array}{l} \text{Actual } \boxed{B_o(p)} \text{ and } \boxed{R_s(p)} \text{ can be estimated} \\ \text{from } \underline{B_{od}(p)} \text{ and } \underline{R_{sd}(p)} \text{ together with} \\ \underline{\text{SEP test}} \quad B_{ob} \ \& \ R_{sb} \end{array} \right)$$

$1.85 \qquad 300 \ Sm^3/Sm^3$ — e.g. Ekofisk

@ $p_b \sim 380$ bar

For many oils:

$$\underset{\uparrow}{B_o(p)} \cong B_{od}(p) \times \frac{B_{ob}^{SEP}}{B_{od,b}}$$

$$\underset{\uparrow}{R_s(p)} \cong R_{sb}^{SEP} - \left(R_{sd,b} - R_{sd}(p)\right) \times \frac{B_{ob}^{SEP}}{B_{od,b}}$$

Ch. 6 : App. D

(Black OiL)

GAS CONDENSATE PVT TESTS & INTRO TO G.C. Bo PVT

Note Title 2010-11-02

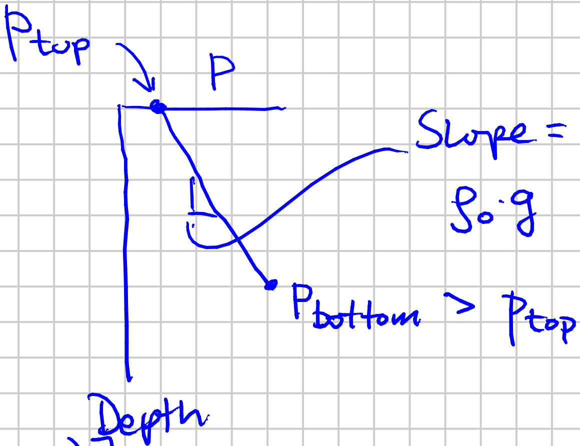
Ch. 6 : Gas Condensate PVT Tests
(+ optional reading : Whitson & Torp
"Evaluating Constant Volume Depletion Data"
found in e-notes : GC-Papers)

- Extra study support material
  GC-PVT e-note

www.ted.com

$$v_{x,y} = \frac{k}{\mu} \cdot \frac{\Delta p}{\Delta y}$$

(x,y)



$$v_z = \frac{k}{\mu} \frac{\left[\overbrace{(p_1 + \rho g z_1)}^{\Phi} - (p_2 + \rho g z_2)\right]}{\Delta z}$$

$$= \frac{k}{\mu} \frac{(p_1 - p_2) + \rho g (z_1 - z_2)}{\Delta z} = \frac{k}{\mu} \cdot \frac{\overbrace{(\Delta p + \Delta z\, \overset{\text{Density}}{\rho} g)}^{\Delta \phi}}{\Delta z}$$

Static, $v_z = 0$, $\Delta p = -\Delta z\, \rho g$

v_{gz} ↑

v_{oz} ↓

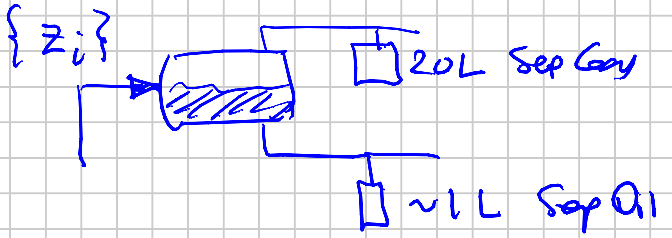
$$\underline{\overset{\downarrow}{k_z(\rho_o-\rho_g)g}} \propto \left\{ \text{Strength of vertical segregation} \right\}$$

GAS CONDENSATE PVT TESTS

- Ch. 6
- GC-PVT e-note
- Whitson-Torp paper in PVT-Papers directory

Collect Samples:

① Surface Separator Sampling ~ same as for oils



q_{gsp}

q_{osp}

Lab: Physical Recombination → z_i

Charge (Fill) HP PVT Cell ~100 cc ⟸ {Wellstream Mixture}

② Openhole (Formation Tester) Sample

MDT
RCI

Samples → PVT Cell

Saturation Pressure = Dewpoint ⇒ Gas Condensate

Special: GOR ~ 400 – 600 $\frac{Sm^3}{Sm^3}$ from a test

may not know if Gas (Dewpoint) or Oil (B.P.)

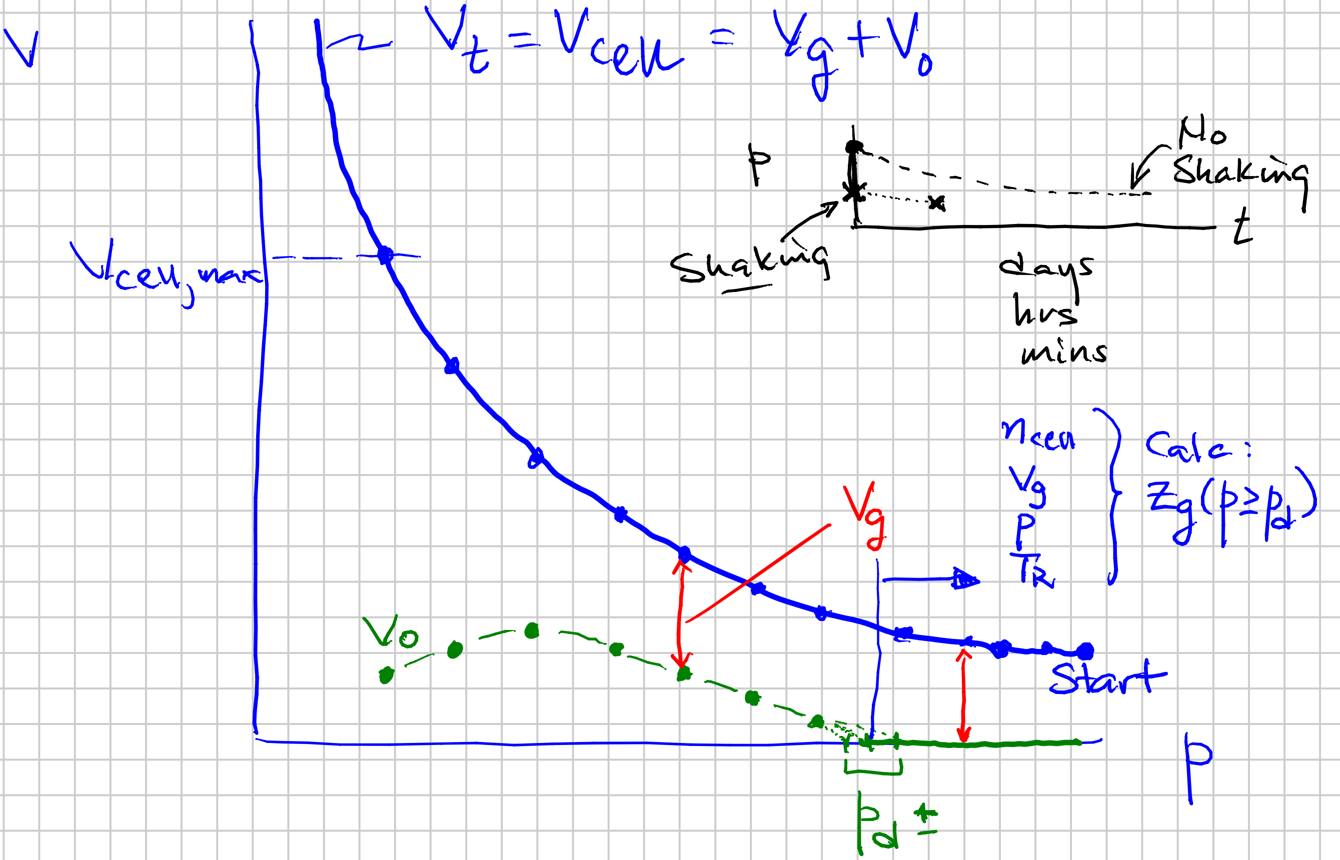
Windowed PVT Cell: @ T_R

(1) Constant Composition Expansion (CCE) (CME)

- Saturation Pressure
- Saturation Type (BP or DP)
- ρ_g or Z_g for $p > p_d$
- V_o $(p < p_d)$ "relative oil volume" "liquid dropout curve"

$\uparrow$

$\propto (1-F_V)$ in Rachford-Rice: $K_i(p)$



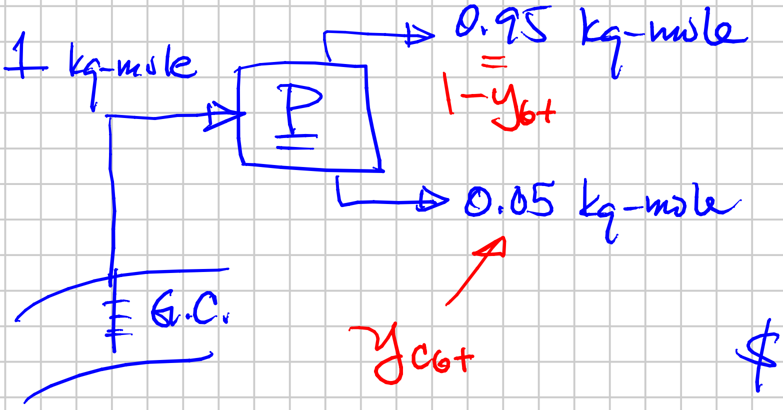
(2) CONSTANT VOLUME DEPLETION TEST

⊛ Provide properties of the equilibrium gas as p drops below the dewpoint. $y_i ; i \in \{C_{6+}\}$ ρ_g Z_g

- Amount of condensate that drops out from the original gas $V_o (p < p_d)$

- Emulate (approximate) the behavior of a gas condensate reservoir developed by "depletion" $p < p_d$.

- Indirectly (by material balance) the properties of condensate liquid
  x_i ρ_o M_o



Sales Gas $3/Mscf } 23.67 $Sm^3/kg\text{-}mole$

Sales Oil (Condensate)

$USD 85/bbl

$$\left\{ \begin{array}{l} M_o \sim 130 \ kg/kg\text{-}mole \\ \rho_o \sim 700 \ kg/m^3 \end{array} \right.$$

$$35.31 \ \frac{scf}{Sm^3}$$

$$6.28 \ \frac{bbl}{Sm^3}$$

% Total Revenue from Sales Oil ?

$$R_g = 0.95\ \cancel{\text{kg-moles}} \times 23.67\ \frac{\cancel{\text{Sm}^3}}{\cancel{\text{kg-mole}}} \times \frac{35.31\ \cancel{\text{scf}}}{\cancel{\text{Sm}^3}} \times \frac{\cancel{\text{Mscf}}}{10^3\ \cancel{\text{scf}}}$$

$$\times\ 3\ \frac{\text{USD}}{\cancel{\text{Mscf}}} = \underline{\underline{2.38\ \text{USD}}}$$

$$R_o = 0.05\ \cancel{\text{kg-mole}} \times \frac{130\ \cancel{\text{kg}}}{\cancel{\text{kg-mole}}} \times \frac{\cancel{\text{Sm}^3}}{700\ \cancel{\text{kg}}} \times \frac{6.28\ \cancel{\text{bbl}}}{\cancel{\text{Sm}^3}}$$

$$\times\ 85\ \frac{\text{USD}}{\cancel{\text{bbl}}} = \underline{\underline{4.96\ \text{USD}}}$$

$$(\%)_{\bar{o}} = 67\% \qquad 2/3$$

$\bar{g} \sim C_{5-}$ or C_{4-}

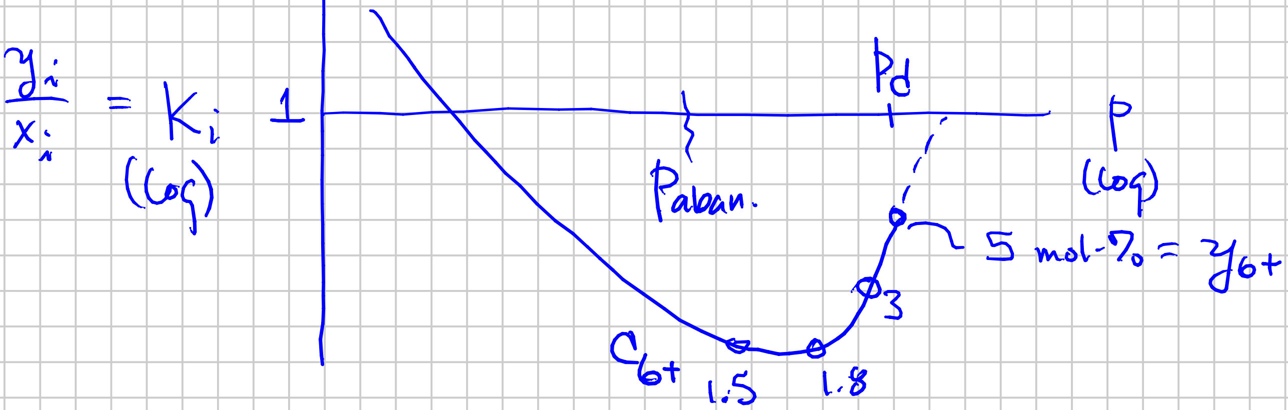
$\bar{o} \sim C_{6+}$ or C_{5+}

$\uparrow$ Less Efficient Surface Process

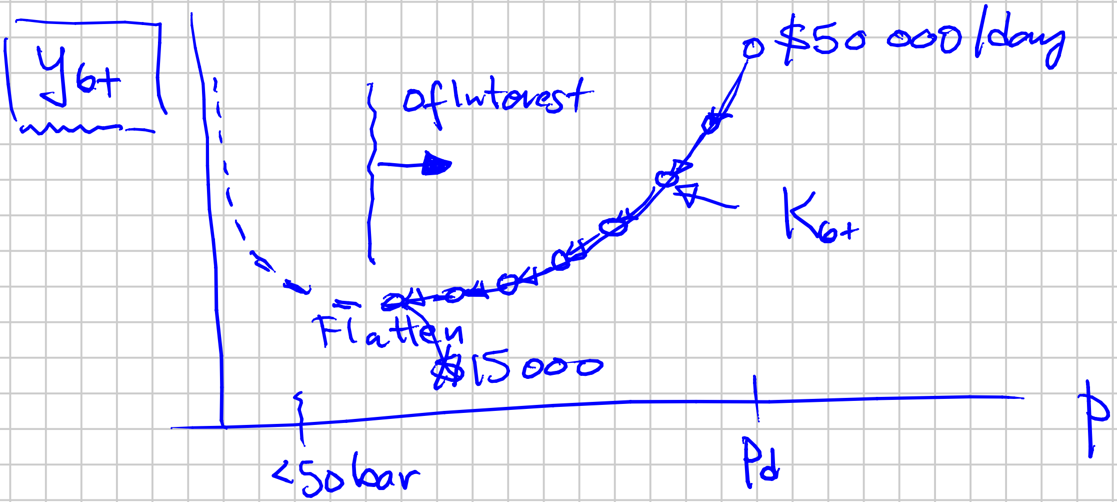
$\uparrow$ More Efficient

Carrying capacity (of condensate) of reservoir gas $= f(\,?\,)$

$y_{6+}(p)$: Lab CVD provides this!



yi/xi = Ki
(log)
1
Pd
P
(log)
Paban.
5 mol-% = y6+
3
C6+
1.5
1.8



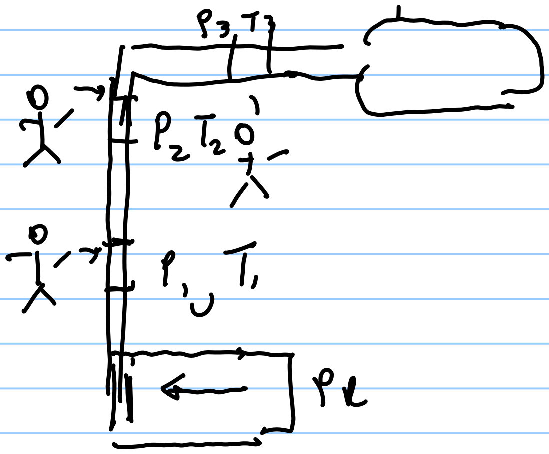
y6+
of Interest
$50 000/day
K6+
Flatten
$15 000
<50 bar
Pd
P

TPG 4145

Black oil Formulation

How different engineering disciplines address from streams

constant rate flow



- mechanical Eng $\dot m$ kg/s
- chemical Eng kg-mole/s

$\dot n = \frac{\dot m}{M}$ $\frac{kg/s}{kg/mole}$

- pet Eng m^3/s volume

steady state $= \frac{\partial}{\partial t} = 0$ no time changes

changes along the system: $\frac{\partial}{\partial x} \neq 0$

In dry gas stream Volume at standard conditions is related to molar flow rate

$$pV = nZRT$$

$$v = n \cdot \frac{ZRT}{p}$$

$$V_{sc} = n \cdot \frac{\overset{1.0}{Z_{sc}} R T_{sc}}{p_{sc}}$$

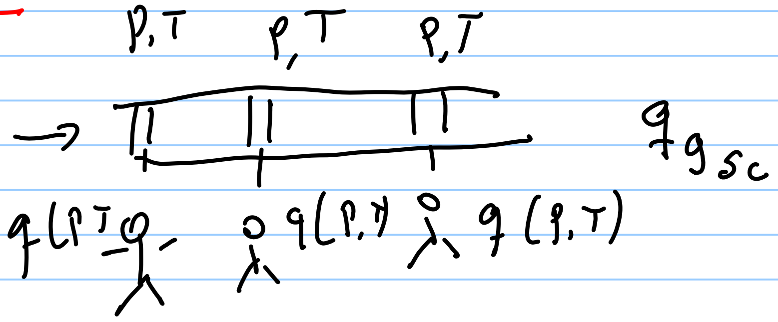
≈ constant

$\boxed{V_{sc} = 379 \cdot n}$ scf/lb mole °R

CFU

$\boxed{V_{sc} = 23.6 \cdot n}$ Sm³/kg-mole °K

SI



$q_{\tilde g}$ → standard condition T = 15.56 °C p = 1 ata = 1.013 bara

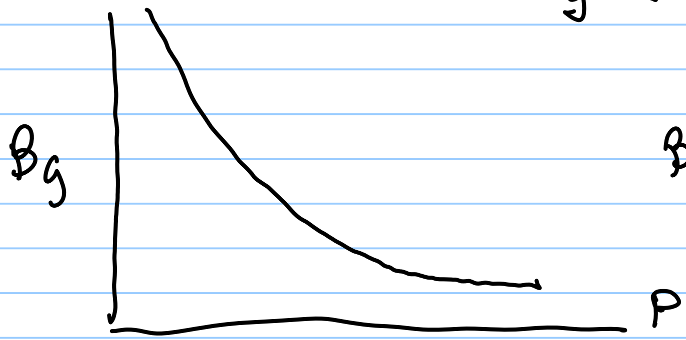
q_g (p, T)

Relationship between $V_{\bar{g}}$ (standard conditions) and $V_g(P,T)$ at any conditions

$$B_g = \frac{V_g(P,T)}{V_{\bar{g}}(P_{sc},T_{sc})} = \text{gas volume factor} \simeq \frac{\frac{nRT}{P}\cdot Z}{\frac{nRT_{sc}}{P_s}} = \frac{P_{sc}}{T_{sc}}\frac{TZ}{P}$$

A particular case of B_g of interest to reservoir Eng:

$$B_g = \frac{V_g(P_R,T_R)}{V_{\bar{g}}(P_{sc},T_{sc})} = \text{gas formation volume factor}$$

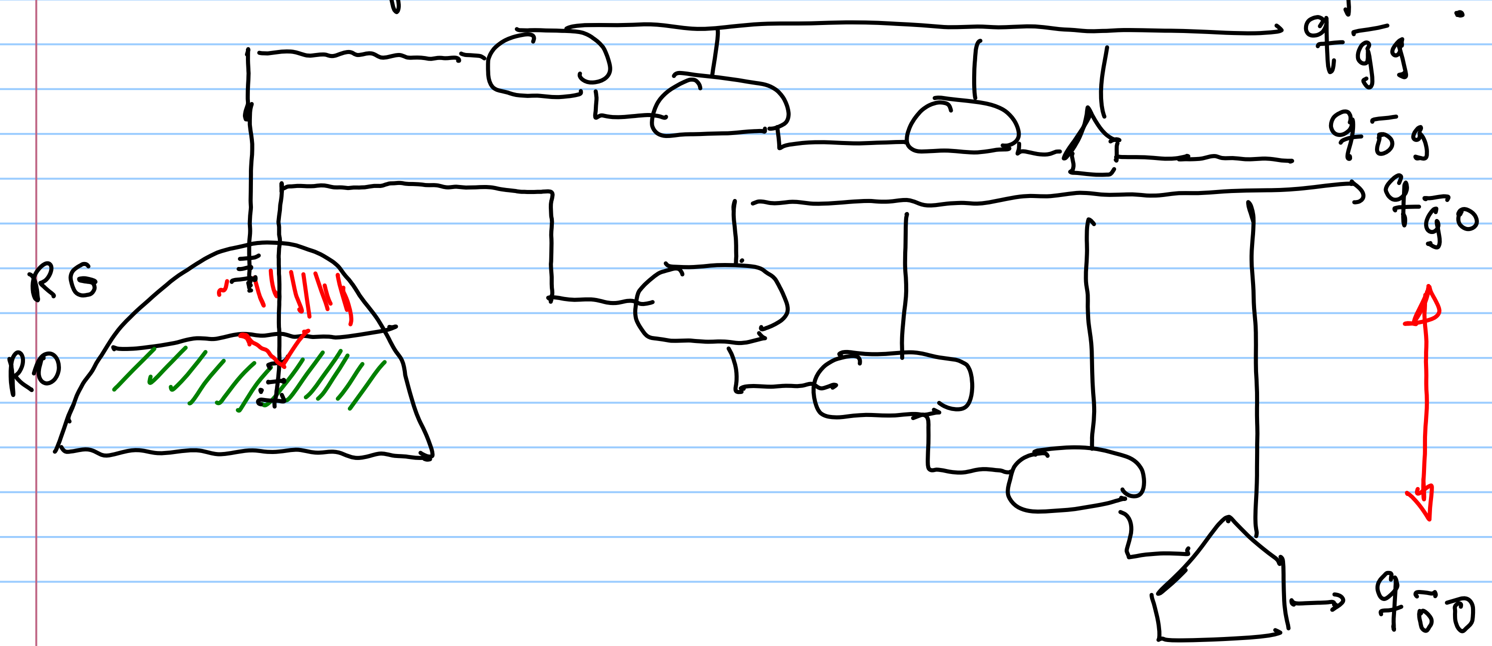


$$B_g \propto \frac{1}{P}$$

Relationship between IGIP (Sm^3) and Initial HC in Res condition

$$\text{Initial gas in place}, |Sm^3| = \frac{\text{Initial reservoir gas volume}, |m^3|}{B_g \, |m^3/Sm^3|}$$

what products we obtain on the surface?



Four surface Products

$q_{\bar{o}o}$ → surface oil from oil Reservoir

$q_{\bar{g}o}$ → surface gas from oil Reservoir

$q_{\bar{g}g}$ → surface gas from gas Reservoir

$q_{\bar{o}g}$ → surface oil from gas Reservoir

$q_{\bar{g}} = q_{\bar{g}o} + q_{\bar{g}g}$ Total surface gas

$q_{\bar{o}} = q_{\bar{o}o} + q_{\bar{o}g}$ Total surface oil

definition

1) GOR = Gas oil ration
production gas oil ratio $= \dfrac{q_{\bar{g}o} + q_{\bar{g}g}}{q_{\bar{o}o} + \underbrace{q_{\bar{o}g}}_{\text{often negligible}}} = R_p$

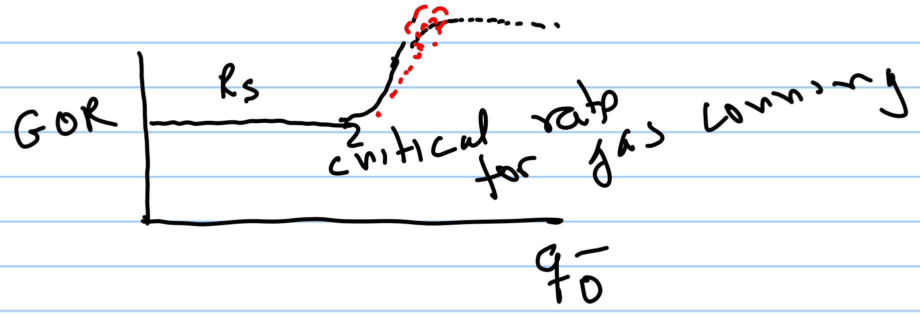
~~GOR~~ R_p
~~BF~~ — F_R

note
use only on charicter for parameter

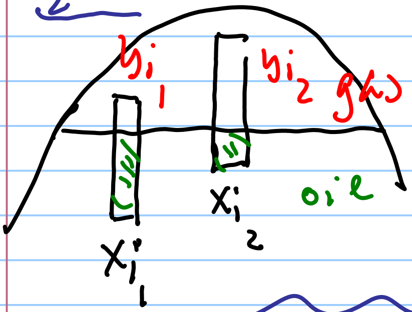
2) $\dfrac{q_{\bar{g}o}}{q_{\bar{o}o}} = R_s$ → solution gas oil ratio $\dfrac{Sm^3 \text{ gas}}{Sm^3 \text{ oil}}$

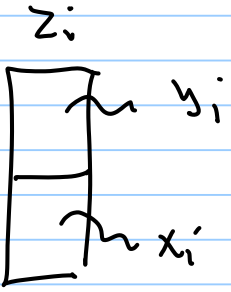
3) $\dfrac{q_{\bar{o}g}}{q_{\bar{g}g}} = r_s$ condensate gas ratio

r_v, R_v
careful



Quiz: Two samples





$x_{i_1} = x_{i_2}$

$y_{i_1} = y_{i_2}$

$K_{i_1} = K_{i_2}$

what is the same and what is different

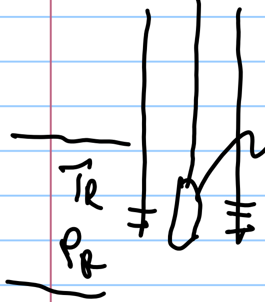
$$B_g(p,T) = q_g(p,T)/q_{\bar{g}g}$$

$$R_s(p,T) = q_{\bar{g}o}/q_{\bar{o}o}$$

$$r_s(p,T) \quad q_{\bar{o}g}/q_{\bar{g}g}$$

$$B_o = \frac{q_o(p,T)}{q_{\bar{o}o}}$$

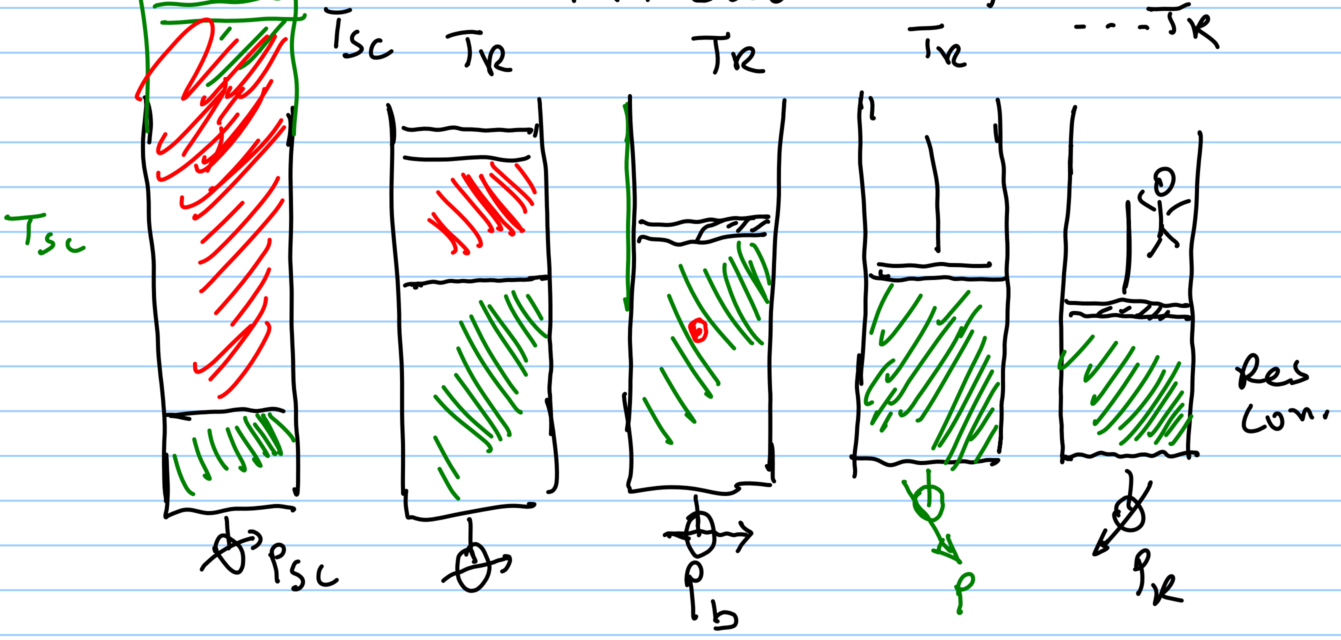
Demonstrating the meaning of these parameter through a CCE PVT experiment



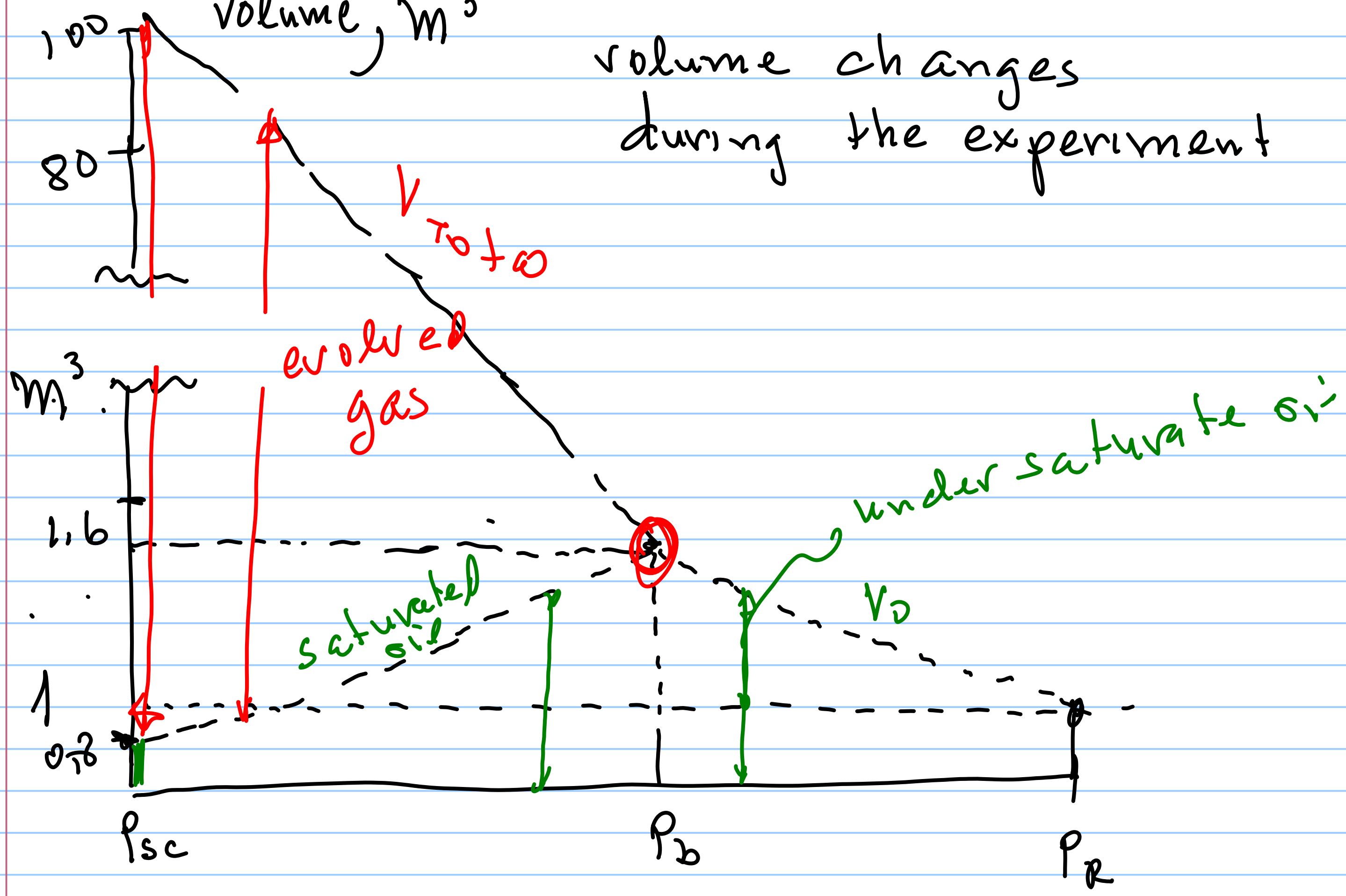
CCE = constant composition expansion

← sampling in a well

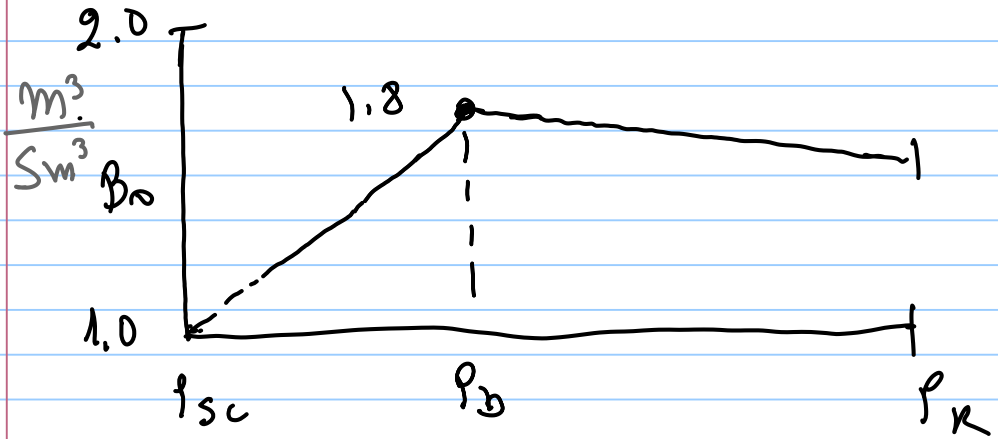
PVT Laboratory

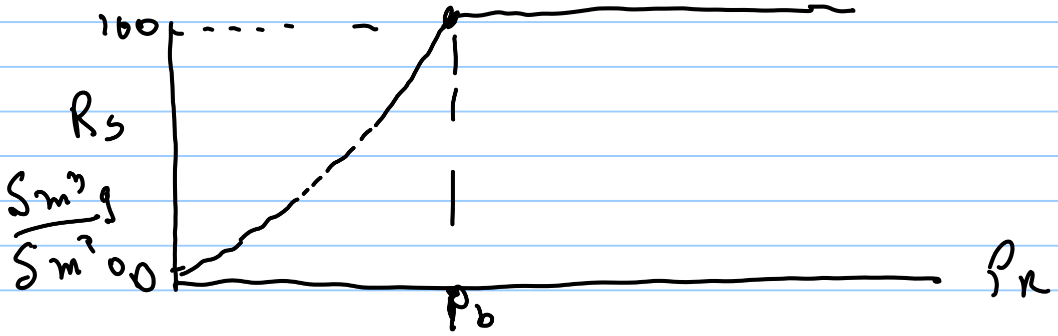


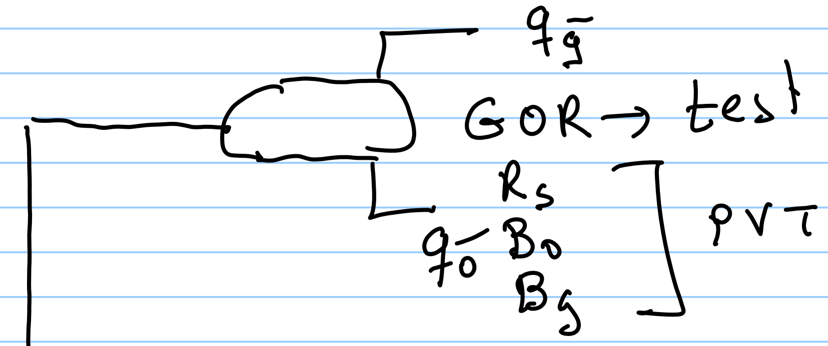
volume changes during the experiment



$$B_o(p,T)=\frac{V_o(p,T)}{V_{\bar{o}o}(p_{sc},T_{sc})}$$







Test GOR, $q_{\bar o}$, $q_{\bar g}$

P_1, T_1

$$q_o(p,T) = B_o(p,T)\cdot q_{\bar o}$$

$$q_g(p,T) = \underbrace{\left[R_p - R_s(p,T)\right]}_{\text{amount of free gas Sm}^3\text{ o/o}} q_{\bar o}\cdot B_g(p,T)$$

free

Total amount of free gas

Total amount of free gas in local conditions

$$\bar u = \frac{q}{A}$$ average flow velocity

u_o

u_g

$u_g - u_o = u_s$

assume $u_s = 0$

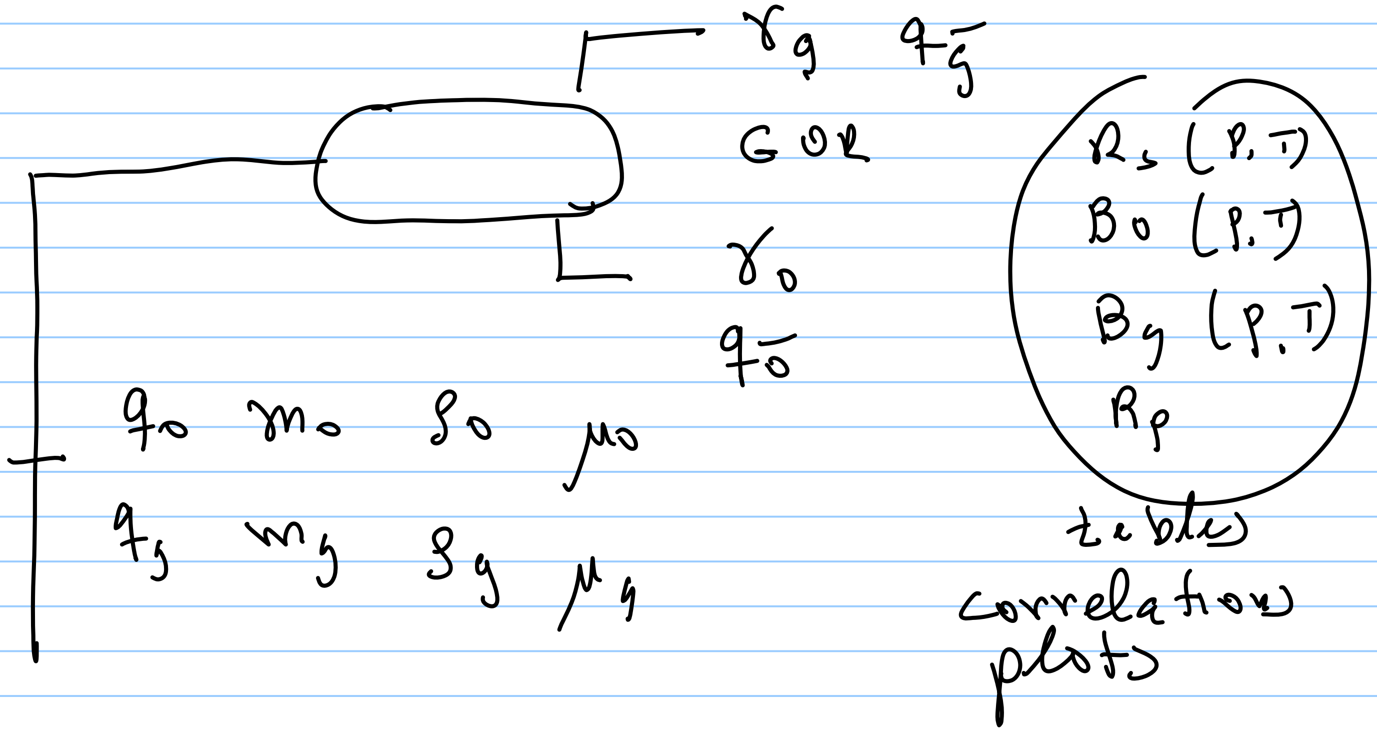
$$m_o(p,T) = \left[\rho_{\bar o} + R_s(p,T)\,\gamma_g\cdot\rho_{air_{sc}}\right]V_{\bar o}$$

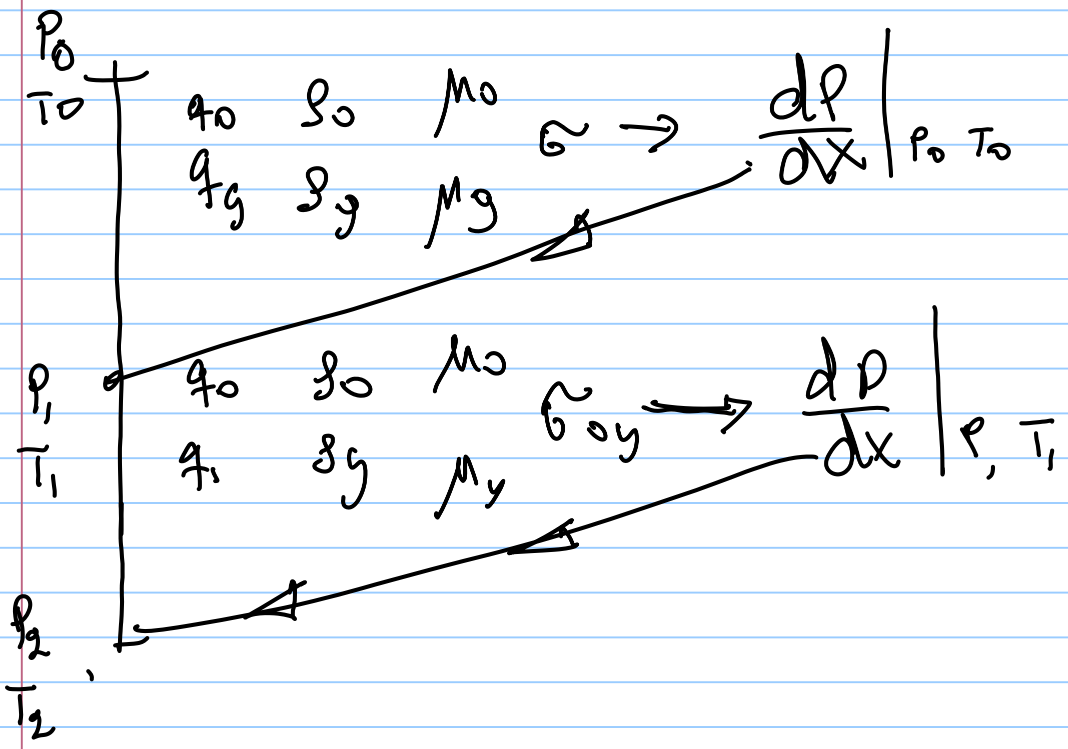
$$m_g(p,T) = \left[R_p - R_s(p,T)\,B_g(p,T)\cdot\rho(p,T)\right]\cdot V_{\bar o}$$

$$\rho_o(p,T) = \frac{m_o(p,T)}{V_o(p,T)} = \frac{\rho_{\bar o} + R_s(p,T)\,\gamma_g\cdot\rho_{air_{sc}}}{B_o}$$

$$\rho_g(p,T) = \frac{m_g(p,T)}{V_g(p,T)} = \frac{M_g}{R}\frac{p}{ZT} = \gamma_g\frac{M_{air}}{R}\frac{p}{ZT} = 28.97\,\gamma_g\frac{p}{ZRT}$$

How do we use Local properties of the oil and gas to calculate pressure loss in the tubing





I am attaching a more organized equations to calculate local conditions in a flow stream from Black oil parameters and surface gravities of oil and gas

Local volumes, mass and densities of oil and gas based on surface gravities and black oil parameters

Metric

$$q_o(p,T) = B_o(p,T)\cdot q_{\bar{o}}$$

1) local oil rate

$$\left|\frac{m^3_{oil}}{d}\right| = \left|\frac{m^3}{\cancel{Sm^3}}\right| \left|\frac{\cancel{Sm^3}_{oil}}{d}\right|.$$

$$q_{g\,dissolved}(p,T) = R_s(p,T)\cdot B_g \cdot q_{\bar{o}}$$

2) local dissolved gas rate

$$\left|\frac{m^3}{d}\right| \qquad \left|\frac{\cancel{Sm^3}_{gas}}{\cancel{Sm^3}_{oil}}\right| \left|\frac{m^3_{gas}}{\cancel{Sm^3}_{gas}}\right| \left|\frac{\cancel{Sm^3}}{oil}\right|$$

$$q_{g\,free\,gas}(p,T) = \left[R_p - R_s(p,T)\right] B_g \cdot q_{\bar{o}}$$

3) local free gas rate

$$\left|\frac{m^3_{gas}}{d}\right| \qquad \underbrace{\left|\frac{\cancel{Sm^3}_{gas}}{\cancel{Sm^3}_{oil}}\right|}_{\text{free gas at }(p,T)} \left|\frac{m^3_{gas}}{\cancel{Sm^3}_{gas}}\right| \left|\cancel{Sm^3}_{oil}\right|$$

free gas at (p,T)

$$\frac{Sm^3\ gas}{Sm^3\ oil}$$

free gas at local conditions per 1 Sm^3 oil

Total free gas at local condition

4) Local oil mass

$$m_o(p,T) = \left[\rho_{\bar{o}o} + R_s(p,T)\,\rho_{\bar{g}o}\right]\cdot V_{\bar{o}}$$

$$\left.\frac{\rho_{\bar{g}d}}{\cancel{\rho_{air}}}\right|_{sc}\cdot \cancel{\rho_{air}}_{sc} = \rho_{\bar{g}d}$$

$$m_o(p,T) = \left[\rho_{\bar{o}} + R_s(p,T)\cdot \underbrace{\gamma_{\bar{g}o_d}\cdot \rho_{air_{sc}}}\right]\cdot V_{\bar{o}}$$

$$|kg| \qquad \left|\frac{kg}{\cancel{Sm^3}_{oil}}\right| + \left|\frac{\cancel{Sm^3}_{gas}}{\cancel{Sm^3}_{oil}}\right|\cdot\left|\frac{kg}{\cancel{Sm^3}_{gas}}\right|\ \left|\cancel{Sm^3}_{oil}\right|$$

$$\rho_{air_{sc}} = M_{air}\frac{p}{ZRT} = 28.97\cdot\frac{p}{ZRT} =$$

$$= 28.97\cdot\frac{1\ bara}{1\cdot(0.08314)\cdot(273+15.6)} = 1.2074\ \left.\frac{kg}{m^3}\right|_{sc}$$

(water = 1000 kg/m³)

5) Local gas mass

$$m_g(p,T) = \left\{\left[P_R - R_s(p,T)\right] B_g(p,T)\cdot \rho_g(p,T)\right\} V_{\bar{o}}$$

$$|kg| \qquad \left|\frac{\cancel{Sm^3}_{gas}}{\cancel{Sm^3}_{oil}}\right|\cdot\left|\frac{\cancel{m^3}_{gas}}{\cancel{Sm^3}_{gas}}\right|\ \left|\frac{kg}{\cancel{m^3}_{gas}}\right|\ \left|\cancel{Sm^3}_{oil}\right|$$

$$\rho_{gas} = 28.97\cdot\gamma_{\bar{g}o}\cdot\frac{p\ bara}{Z\cdot(0.08314)(273+{}^\circ C)}$$

6) Local oil density.

$$\rho_o(p,T) = \frac{m_o(p,T)}{V_o(p,T)} = \frac{\left[\rho_{\bar{o}} + R_s(p,T)\cdot\gamma_{\bar{g}o_d}\cdot\rho_{air_{sc}}\right]V_{\bar{o}}}{B_o\cdot V_{\bar{o}}} =$$

$$\left|\frac{kg}{m^3}\right|$$

$$\left|\frac{kg}{\cancel{Sm^3}_{oil}}\right| + \left|\frac{\cancel{Sm^3}_{gas}}{\cancel{Sm^3}_{oil}}\right|\cdot\left|\frac{kg}{\cancel{Sm^3}_{gas}}\right|$$

$$= \frac{\rho_{\bar{o}} + R_s(p,T)\cdot\gamma_{\bar{g}o_d}\cdot\rho_{air_{sc}}}{B_o} = \left|\frac{kg}{m^3}\right|$$

$$\left|\frac{m^3\ oil}{\cancel{S}m^3\ oil}\right|$$

7 Local gas density

$$\rho_g(p,T) = M_g\frac{p}{ZRT} = 28.97\,\gamma_g\frac{p}{ZRT} = 28.97\,\gamma_g\frac{\overset{bara}{p}}{Z(0.08314)\cdot \underset{(273.+°C)}{T}}$$

8. Local gas gravity

$$\gamma_g(p,T) = \gamma_{g_{sc}}\left[\frac{R_p - \gamma_{\bar{g}d}\cdot R_s(p,T)}{R_p - R_s(p,T)}\right]$$

19/11/2010 comments of the separation exercise

we solve multi-stage separation problem using flash calculation → Rachford Rice R-R.

given
$z_i\ (P, T), M_i, K_i \rightarrow$ obtain $\rightarrow F_v = \dfrac{n_g}{n_o + n_g}, \ y_i, \ x_i$

$F_L = (1 - F_v)$

from y_i and x_i we can calculate!
molecular weight of the gas and oil phases

M_g, M_o - using simple mixing rule $\bar{M}_g = \sum M_i y_i$

$\bar{M}_o = \sum M_i x_i$

we can calculate the mass of each stream

mass, W

$W_{ig} = \dfrac{y_i M_i}{\sum y_i M_i}$ weight fraction

F_v

$W_{io} = \dfrac{x_i M_i}{\sum x_i M_i}$

F_L

calculating Densities

$\rho_g = \dfrac{M_g}{ZRT} \cdot P$ 1 mole → 23.6 Sm^3

$\rho_o = \dfrac{M_o}{V_o} \simeq \dfrac{\sum x_i M_i}{\sum x_i \hat{V}_i} \approx \dfrac{\sum x_i M_i}{\sum \dfrac{x_i M_i}{\rho_i}}$

$\hat{V}_i$ Partial molar volume $= \hat{V}_i = \dfrac{\partial V}{\partial n_i}$

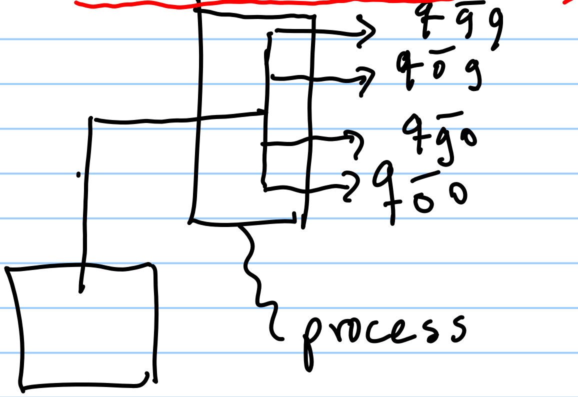
$n \rightarrow m \rightarrow V$

The above coment are made in connection with the homework separation exercise

Effect of fluid properties - The course

- mobility - flow - stream splitting] volumetric properties, flow properties
- storage -
- Thermal properties] not in this course
- flow assurance -

BO Formulation



$$B_g = \frac{V_g(p,T)}{V_{\bar g g}}$$

$$r_s = \frac{V_{\bar o g}}{V_{\bar g g}}$$

] gas reservo. gas cap

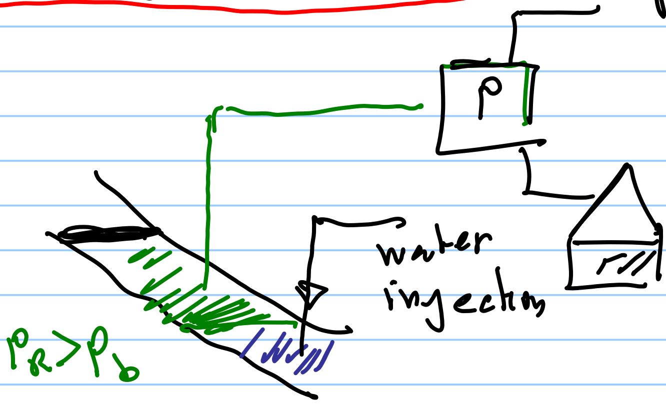
$$R_s = \frac{V_{\bar g o}(p,T)}{V_{\bar o o}}$$

$$B_o = \frac{V_o(p,T)}{V_{\bar o o}} \quad \frac{m^3}{Sm^3}$$

] Oil Reservoir

How do we use the Bo parameter?

1. Voidage/withdrawal replacement



Example

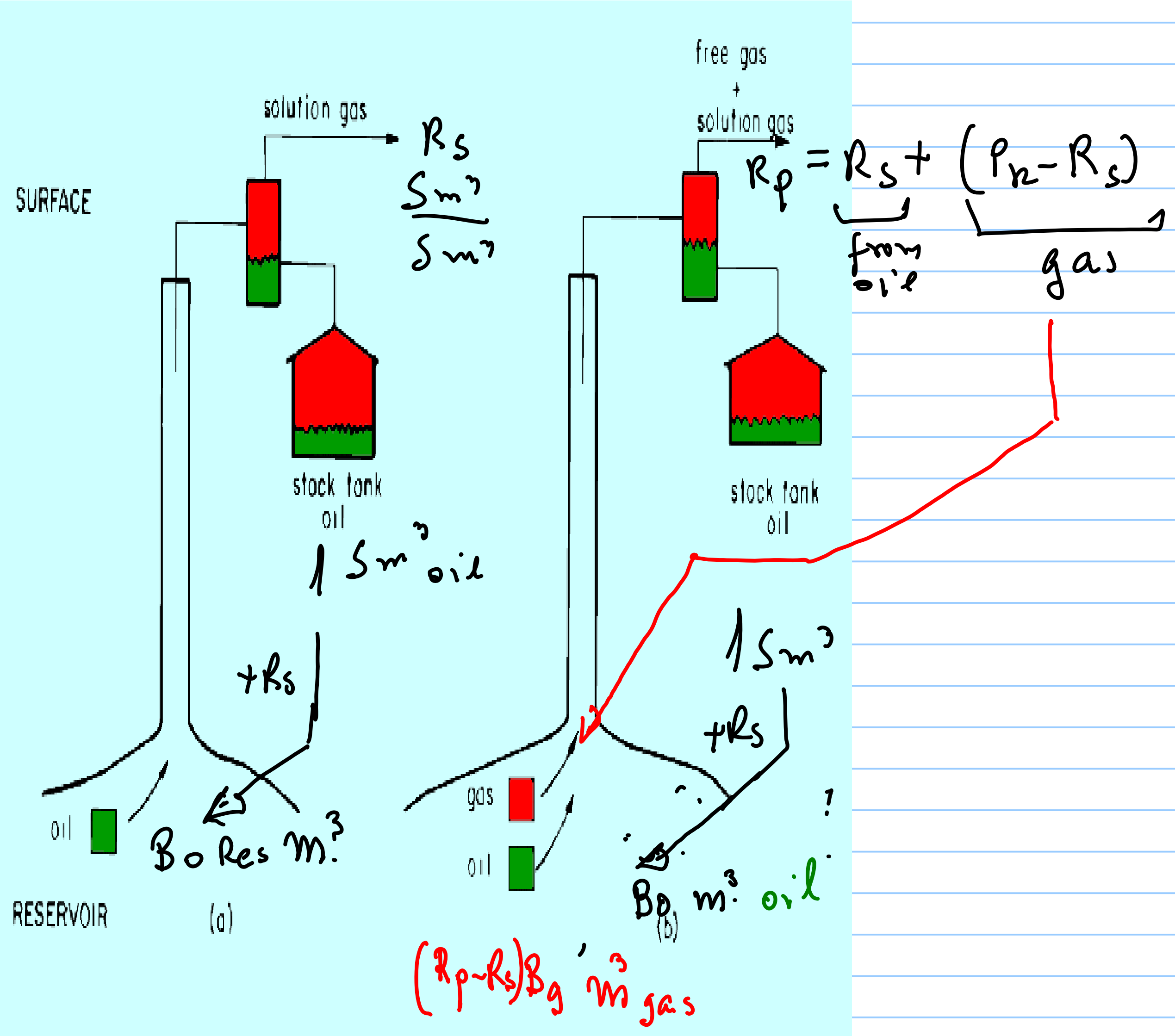
100.000 STB/D

$B_o(p_R, T_R) = 1.4$

Draugen, Shell

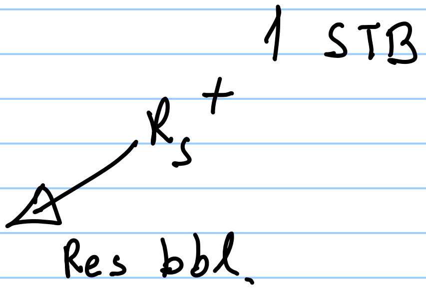
Assume water incompressible

$$q_{w_{\text{needed}}} \rightarrow q_{\bar o o} \times B_o(p_R, T_R)$$



observation

1 STB (surface oil), when combined with the dissolved gas, $R_s \frac{scf}{STB}$ and compressed to reservoir condition will give $1 \times B_o$ Res bbl



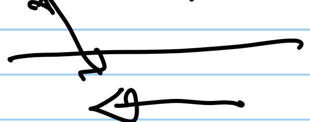
② oil and gas in place + material balance calc.

$$\frac{\text{Initial Gas pore volume } m^3}{B_g \frac{m}{Sm^3}} = \text{Initial gas in place } Sm^3$$

$$\frac{\text{Initial oil pore volume } m^3}{B_o \ \frac{m^3}{Sm^3}} = \text{Initial oil in place } Sm^3$$

③ Flow conditions - mobility in the reservoir

Reservoir



Reservoir flow

$V_o \quad S_o \quad k_{ro} - \lambda_o$

$V_g \quad S_g \quad k_{rg} - \lambda_g$

Pipe flow

$V_o \rightarrow$ slippage $\quad \rho_o \ \mu_o$

$V_g \quad U_g - U_o = U_s \quad \rho_g \ \mu_o$

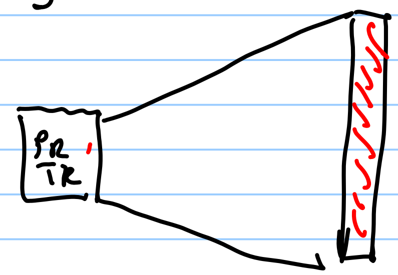
σ_{og} - pressure losses

④ classical use of BO formulation in reservoir simulation

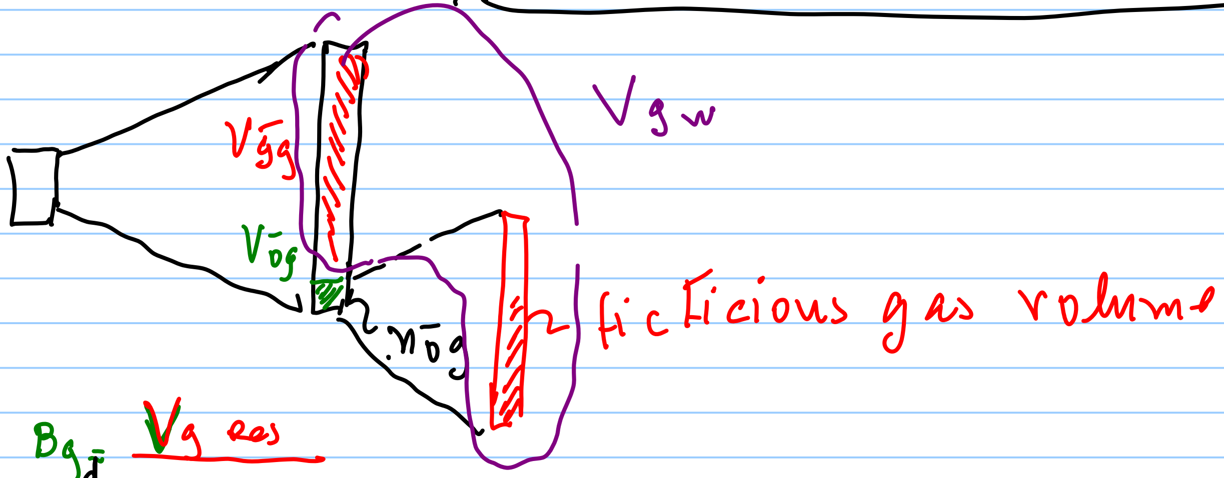
⑤ modern approach - Two component model

$\bar{g}, \quad \bar{o} \quad \boxed{q_{\bar{g}} \quad q_{\bar{o}}}$

$B_g =$ Discussion of B_g



$$\boxed{B_g = \frac{p_{sc}}{T_{sc}} \cdot \frac{TZ}{p} \to \text{ingnoring } r_s \quad r_s = 0 \quad V_{\bar{o}g} = 0}$$



$$B_{gd} = \frac{V_{g\,res}}{V_{\bar{g}d}}$$

$$B_{gw} = \frac{V_{g\,res}}{V_{\bar{g}w}}$$

Traditionally, when using $B_g = \frac{p_{sc}}{T_{sc}} \frac{TZ}{p}$ we assume that all the gas phase in the reservoir or in the stream, becomes surface gas product, $\underbrace{V_{\bar{o}g}}_{p_{sc},\, T_{sc}} \to 0$

sometimes we used the B_g also for cases where $r_s > 0$. Thus we have two products from the reservoir gas $\bar{g}g$ $\bar{o}g$

we combine the two products into a ficticious surface gas equivalent → Wet Gas

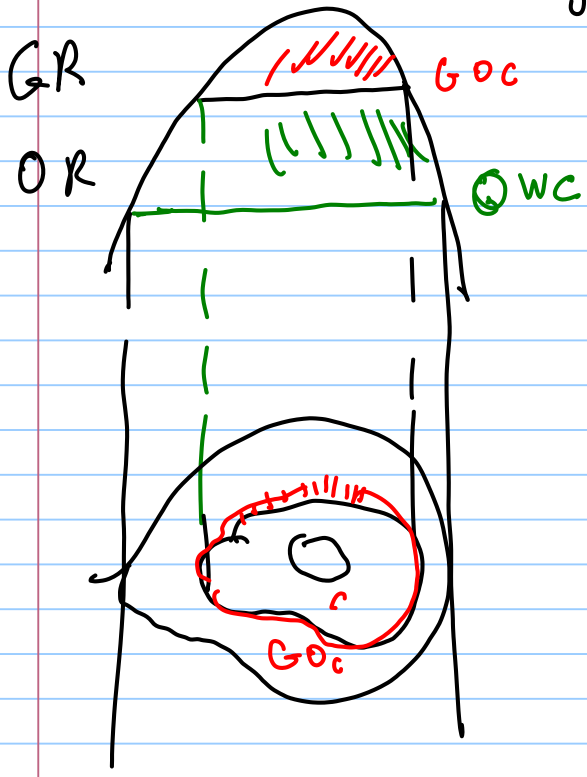
$$V_{gw} = \underbrace{V_{\bar{g}g}}_{dry} + \underbrace{V_{\bar{o}g} \cdot \frac{\rho_{\bar{o}}}{M_{\bar{o}}}}_{n_{\bar{o}g}} \cdot \underbrace{\frac{R\,T_{sc}}{p_s}}_{23.68\ \frac{Sm^3}{Kg\text{-}mol.}}$$

$$B_{gw} = \frac{V_g}{V_{\bar{g}w}} = \frac{p_{sc}}{T_{sc}} \cdot \frac{TZ}{p}$$

$$B_{\bar{g}d} = \frac{V_g}{V_{\bar{g}g}} = B_{gw} \cdot \frac{1}{y_{\bar{g}}}$$

$$y_{\bar{g}} = \frac{V_{\bar{g}g}}{V_{\bar{g}w}} = f(r_s,\ \text{Process}) \qquad 0.99 - 0.88$$

Example – use of B.O parameters to calculate Hydrocarbon in Place



$$\begin{array}{lcc} V_{bg}\cdot\phi_g\cdot\bar{S}_g = HCPV_g \rightarrow & G_g & N_g \\ & + & + \\ V_{bo}\cdot\phi_o\cdot S_o = HCPV_o \rightarrow & G_o & N_o \\ & G & N \end{array}$$

G = original gas in place, G_p = cummulative gas production

N = original oil in place

$G_p/G = FR_g$ $\frac{N_p}{N} = FR_{oil}$

N_p = cummulative oil production

Initial conditions p_{Ri}, T_R

$G_g = \frac{HCPV_g}{B_{gdi}}$ p_{Ri}, T_R	$N_g = G_g \cdot r_{si} = \frac{HCPV_G}{B_{gdi}/r_{si}}$ production start
$G_o = N_o \cdot R_{si}$	$N_o = \frac{HCPV_o}{B_{oi}}$
G	N

Ekofisk Twin

calculate

1) $IGIP_{RG}$
2) $IOIP_{RG}$
3) $IOIP_{RO}$
4) $IGIP_{RO}$

① — ② — ③ — ④

$HCPV_{RG} = 10^9\ m^3$ (giga)

$HCPV_{RO} = 10^9\ m^3$

G and G geologist geophysicist

PVT data

gas cap	$B_{gd} = \frac{V_g}{V_{\bar{g}g}} = 0.004 \frac{m^3}{Sm^3}$	$r_s = 562 \frac{Sm^3}{10^6 Sm^3}$ $\frac{V_{\bar{o}g}}{V_{\bar{g}g}}$
oil zone	$B_o = \frac{V_o}{V_{\bar{o}o}} = 1.7 \frac{m^3}{Sm^3}$	$R_s = 300 \frac{Sm^3}{Sm^3}$ $\frac{V_{\bar{g}o}}{V_{\bar{o}o}}$

All PVT data depend on the surface process. In this case (Ekofisk Twin)

separator	T, °F	P psia
HP sep	150	1000
MP sep	80	265
S. Tank	60	14.7

① $IGIP_{RG} = \frac{HCPV_{RG}}{B_{gdi}} = \frac{10^9 \cancel{m^3}}{0.004 \frac{\cancel{m^3}}{Sm^3}} \equiv 250\times10^9\ Sm^3$

② $IOIP_{RG} = IGIP_{RG} \cdot r_{si} = 250\times10^9\ Sm^3 \times 562\times10^6 \frac{Sm^3}{Sm^3}$

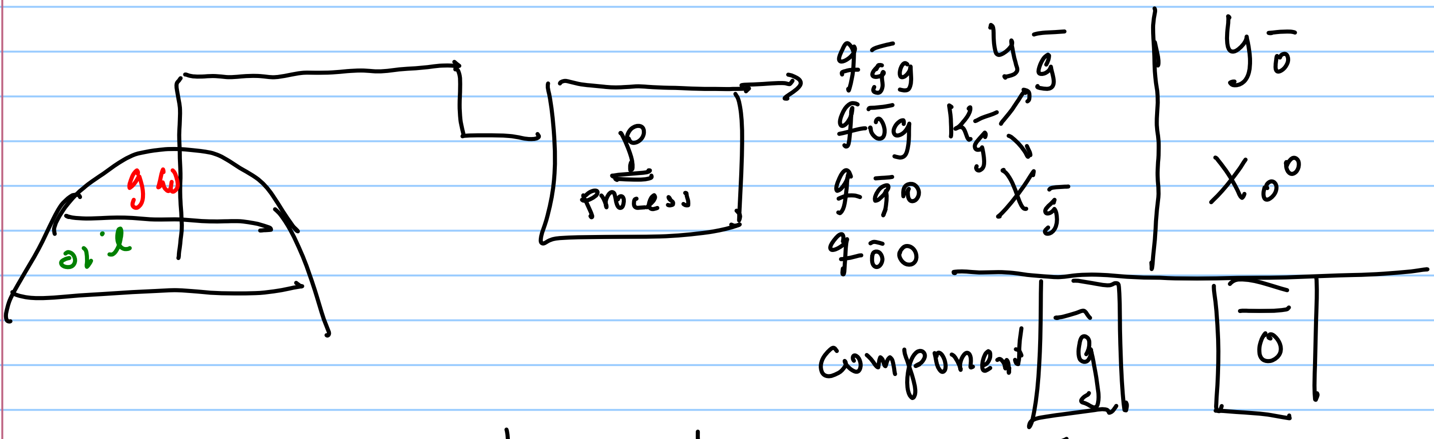
③ $= IOIP_{RO} = \frac{HCPV_{RO}}{B_{oi}} = \frac{10^6\ m^3}{1.7\ m^3/Sm^3} = 588\times10^6\ Sm^3$

④ $= IGIP_{RO} = IOIP_{RO} \cdot R_{si} = \frac{10^9}{1.7} Sm^3 \cdot 300 \frac{Sm^3}{Sm^3} =$

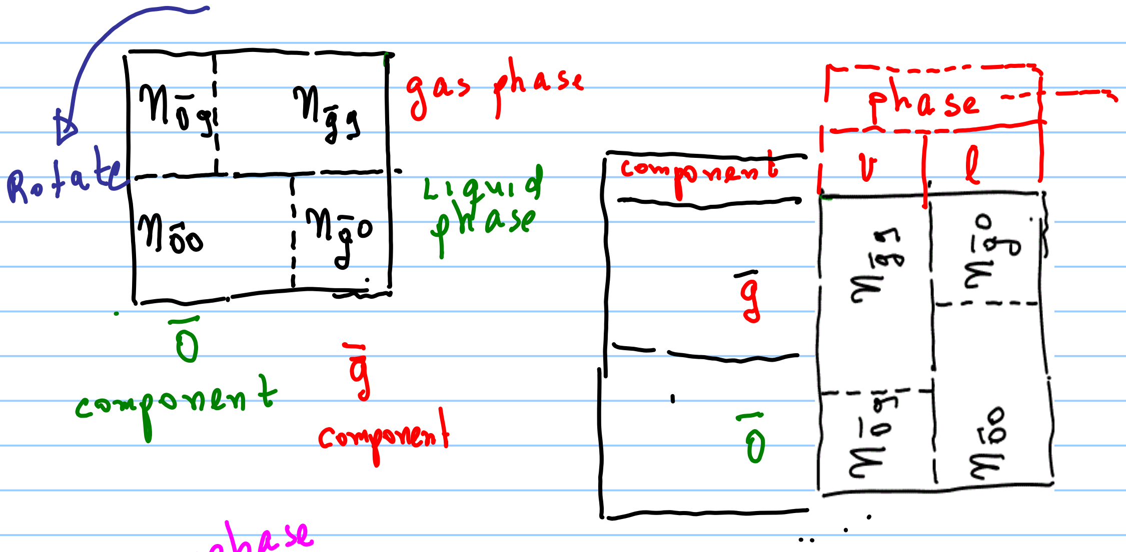
$1.765\times10^{11}\ Sm^3$

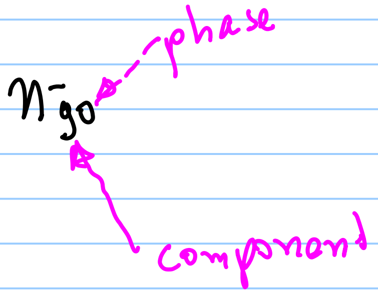
$= 6.23$ TCF Terra Trilliono

Black oil formulation → to be used in compositional Res simulator



Two component, surface gas $\bar{g}$ and surface oil $\bar{o}$





We will organize the components and the partition between oil and gas in a table identical to the one used in multi-component flash calculation.

given p T $F_v = \boxed{}$

component	z_i mole fraction	$K_i = \frac{y_i}{x_i}$	y_i mole fraction vapor	x_i mole fraction Liquid
$\bar{g}$ surface gas	$z_{\bar{g}}$	$\frac{y_{\bar{g}}}{x_{\bar{g}}}$	$y_{\bar{g}}$	$x_{\bar{g}}$
$\bar{o}$ surface oil	$z_{\bar{o}}$	$\frac{y_{\bar{o}}}{x_{\bar{o}}}$	$y_{\bar{o}}$	$x_{\bar{o}}$
	$\sum z_i = 1.0$		$\sum = 1.0$	$\sum = 1.0$

Two components $\bar{o}$, $\bar{g}$ partitioned between vapore phase and Liquid phase

Expressing the components z_i, y_i, x_i as mole fraction as done in flash calculation.

$$F_v = \frac{n_g}{n_g + n_o} \qquad F_l = (1 - F_G) = \frac{n_o}{n_g + n_o}$$

$$n_g = n_{\bar{g}g} + n_{\bar{o}g} \qquad n_o = n_{\bar{g}o} + n_{\bar{o}o}$$

Comp	z_i	K_i	y_i	x_i
$\bar{g}$	$z_{\bar{g}} = \frac{n_{\bar{g}}}{n_{\bar{g}} + n_{\bar{o}}}$	$K_{\bar{g}} = \frac{y_{\bar{g}}}{x_{\bar{g}}}$	$y_{\bar{g}} = \frac{n_{\bar{g}g}}{n_{\bar{g}g} + n_{\bar{o}g}}$	$x_{\bar{g}} = \frac{n_{\bar{g}o}}{n_{\bar{g}o} + n_{\bar{o}o}}$
$\bar{o}$	$z_{\bar{o}} = \frac{n_{\bar{o}}}{n_{\bar{g}} + n_{\bar{o}}}$	$K_{\bar{o}} = \frac{y_{\bar{o}}}{x_{\bar{o}}}$	$y_{\bar{o}} = \frac{n_{\bar{o}g}}{n_{\bar{g}g} + n_{\bar{o}g}}$	$x_{\bar{o}} = \frac{n_{\bar{o}o}}{n_{\bar{g}o} + n_{\bar{o}o}}$
	$\sum = 1.0$		$\sum = 1.0$	$\sum = 1.0$

The component of the stream z_i and the partition components y_i and x_i (all expressed in mole fraction) are related to the black oil propertis, GOR, R_s and r_s.

These relationships will be developed below

$$GOR = \frac{V_{\bar g}}{V_{\bar o}} = \frac{m_{\bar g}/\rho_{\bar g}}{m_{\bar o}/\rho_{\bar o}} = \frac{n_{\bar g}\cdot M_{\bar g}/\rho_{\bar g}}{n_{\bar o} M_{\bar o}/\rho_{\bar o}}$$

rearranging the expression yields:

$$\frac{n_{\bar g}}{n_{\bar o}} = \frac{GOR}{\overbrace{\underbrace{\underset{\substack{\downarrow\\0.1}}{\frac{M_{\bar g}}{M_{\bar o}}}\cdot\underset{\substack{\downarrow\\1000}}{\frac{\rho_{\bar o}}{\rho_{\bar g}}}}_{\phi\,\simeq\,100}}} = \;= \frac{GOR}{\phi}$$

or

$$n_{\bar o} = \frac{n_{\bar g}\cdot\phi}{GOR}$$

$$n_{\bar g} = \frac{n_{\bar o}\,GOR}{O}$$

substitution $n_{\bar o}$ and $n_{\bar g}$ in the expression of $Z_{\bar g}$ and $Z_{\bar o}$ =

$$Z_{\bar g} = \;= \frac{n_{\bar g}}{n_{\bar g}+n_{\bar o}} = \frac{GOR}{GOR+\phi}$$

and $$Z_{\bar o} = \left(1 - Z_{\bar g}\right) = \frac{n_{\bar o}}{n_{\bar o}+n_{\bar g}} = \frac{\phi}{GOR+\phi}.$$

by analogy

$$X_{\bar g} = \frac{n_{\bar g o}}{n_{\bar g o} + n_{\bar o o}} = \frac{R_s}{R_s + \phi}$$

It can be shown (try to develop yourself to check the professor's results

$$Z_{\bar g} = \frac{V_{\bar g}/V_{\bar o}}{V_{\bar g}/V_{o} + \phi} = \frac{GOR}{GOR + \phi} \qquad Z_{\bar o} = 1 - Z_{\bar g} = \frac{\phi}{GOR + \phi}$$

$$X_{\bar g} = \frac{V_{\bar g o}/V_{\bar o o}}{V_{\bar g o}/V_{\bar o o} + \phi} = \frac{R_s}{R_s + \phi} \qquad X_{\bar o} = 1 - X_{\bar g} = \frac{\phi}{R_s + \phi}$$

$$y_{\bar g} = \frac{V_{\bar g g}/V_{\bar o g}}{V_{\bar g g}/V_{\bar o g} + \phi} = \frac{r_s^{-1}}{r_s^{-1} + \phi} \qquad y_{\bar o} = 1 - y_{\bar g} = \frac{\phi}{r_s^{-1} + \phi}$$

$$K_{\bar g} = \frac{y_{\bar g}}{x_{\bar g}} = \left(\frac{r_s^{-1}}{R_s}\right)\left(\frac{R_s + \phi}{r_s^{-1} + \phi}\right) \qquad K_{\bar o} = \frac{y_{\bar o}}{x_{\bar o}} = \frac{R_s + \phi}{r_s^{-1} + \phi}$$

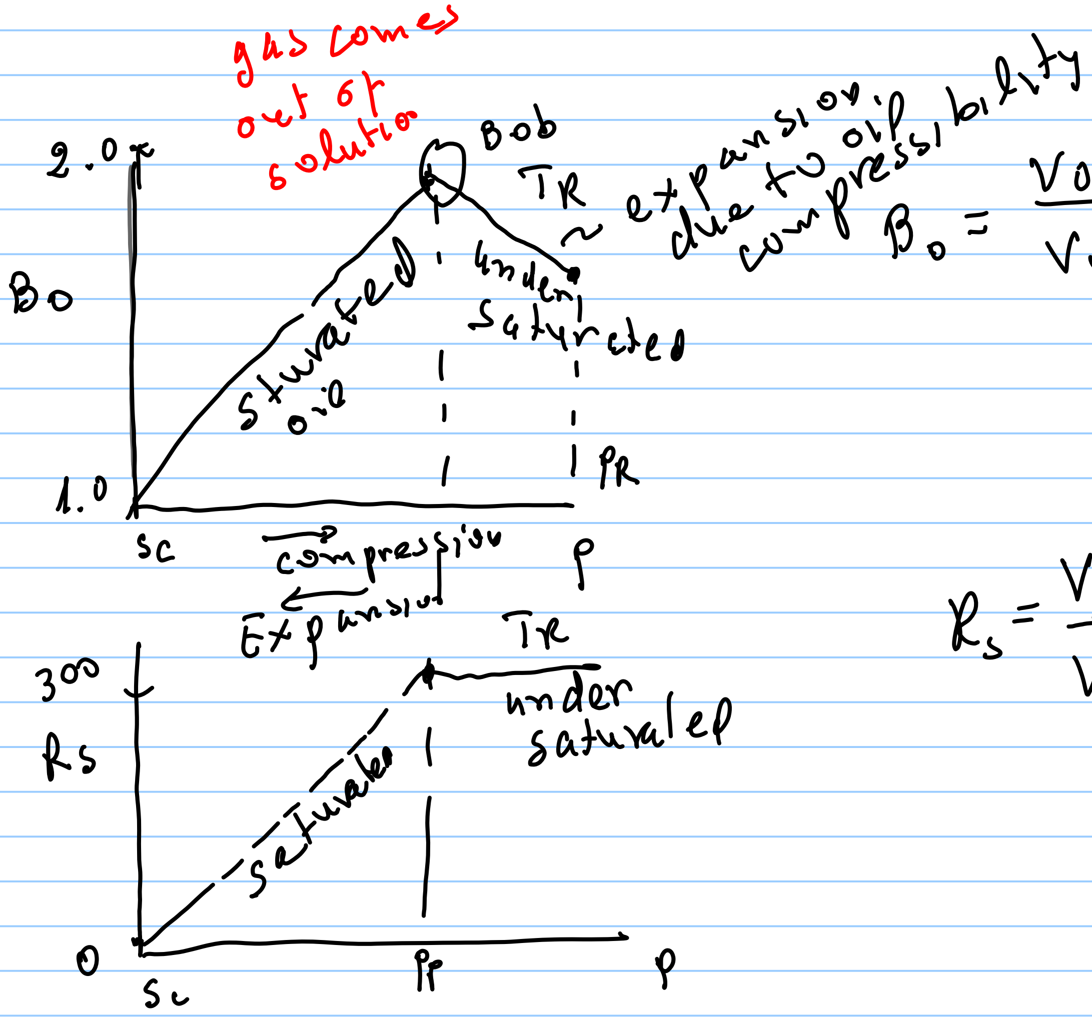
$$z_i = l x_i + v y_i \qquad l + v = 1$$

Note: Some confusion could be eliminated by calling the components $\bar g$ (surface gas) and $\bar o$ (surface oil) and at the same time calling the partitioned phases v (vapor) and l (liquid).

chp 3 in the phase behavior SPE book addresses "oil and gas properties an correlations." It gives answers to the question regarding how to convert stream information from mole fraction to mass (weight fraction) and volumes/densities.

Interesting source on oil and gas properties is in the booklet by M. B. Standing "Volumetric and phase behavior of oil field hydrocarbon system" published 1952, reprinted by SPE in 1977.

changes of PVT properties with reservoir pressure

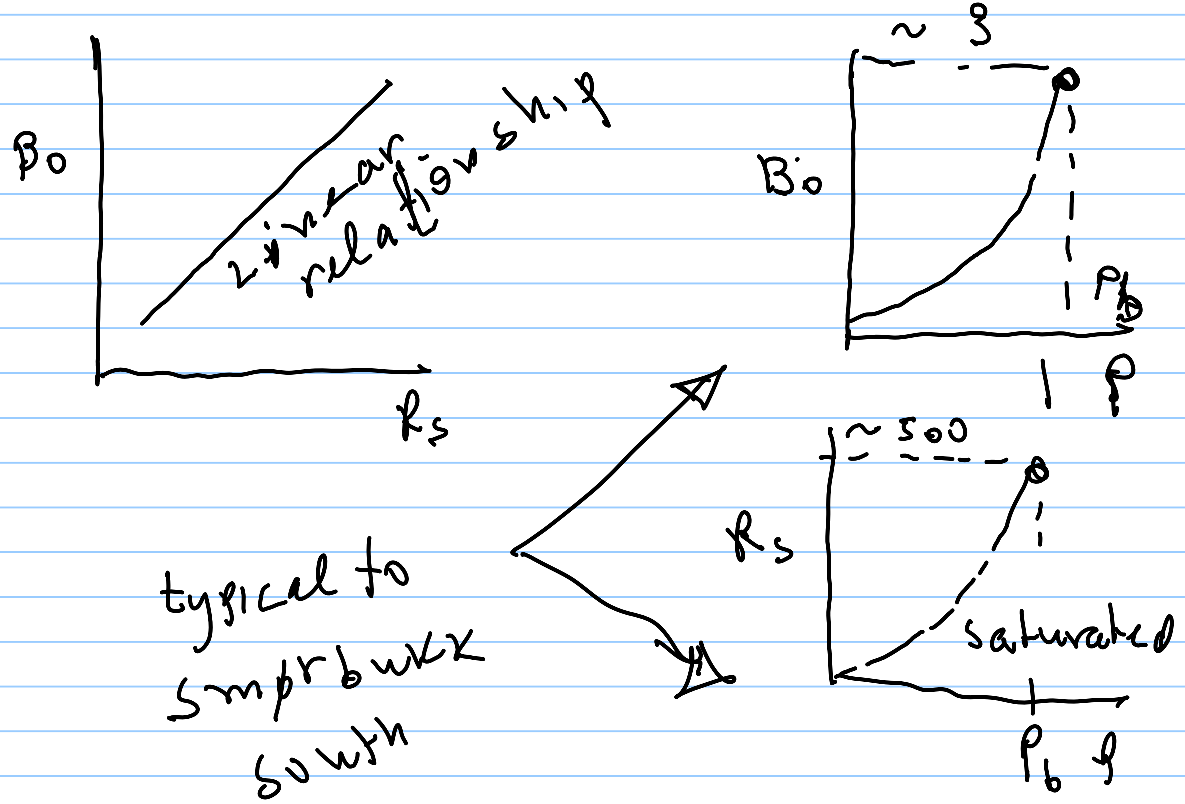


$$B_o = \frac{V_o(p)}{V_{\bar{o}o}(s.c)}$$

$$R_s = \frac{V_g \text{ dissolved } (p)}{V_{\bar{o}o}(sc)}$$

The parameters B_o and R_s of an oil system show normally linear relationship in the saturated region.

An Example from the reservoir shows that even if $R_s(p)$ and $B_o(p)$ are highly non-Linear the shapes of these curves are similar giving Linear B_o vs R_s plot

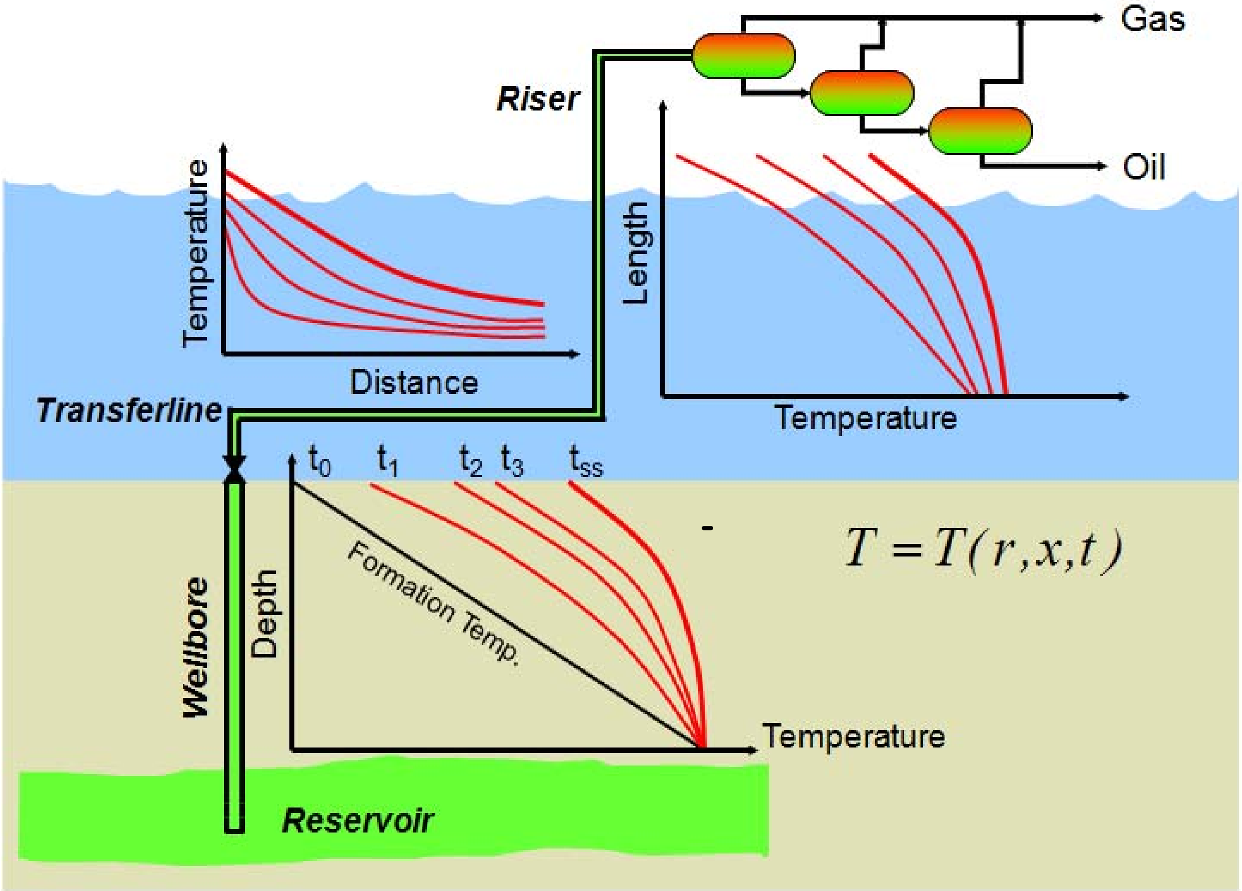


In reservoir engineering the plots and the tables of black oil properties are given for reservoir temperature, T_R.

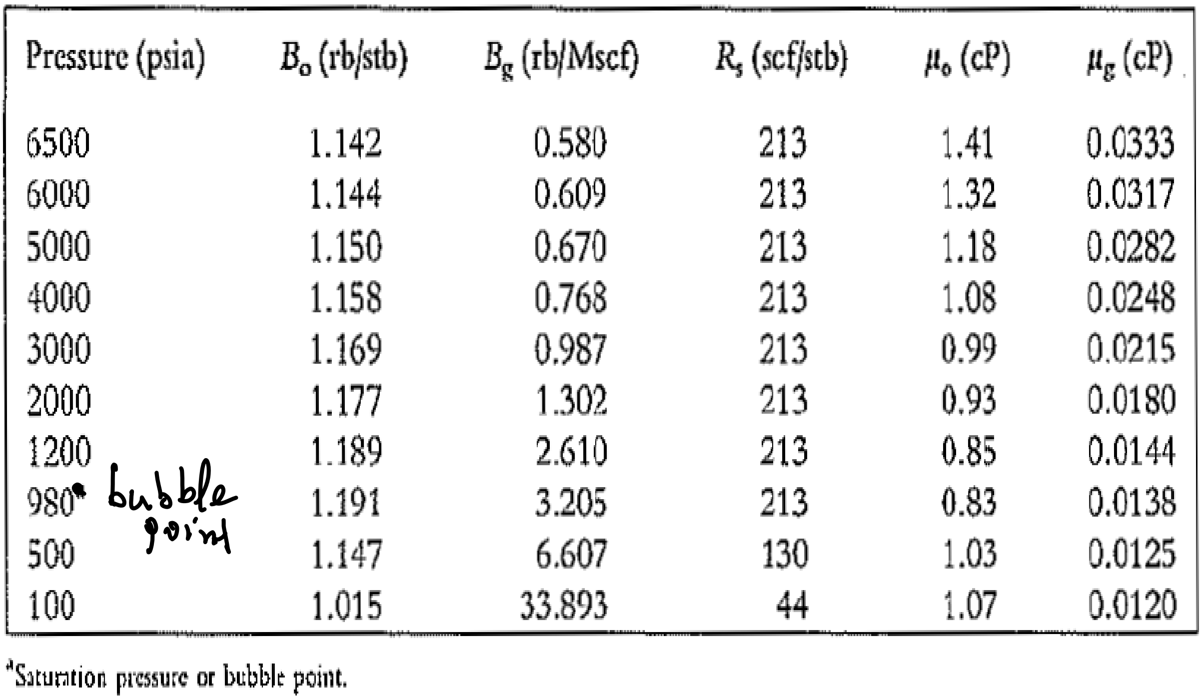
when using black oil propertis to determine the phases in the production system (wellbore, flowlines), the properties have to account for the variation in temperature

Description of problem

wellstream temperature profile



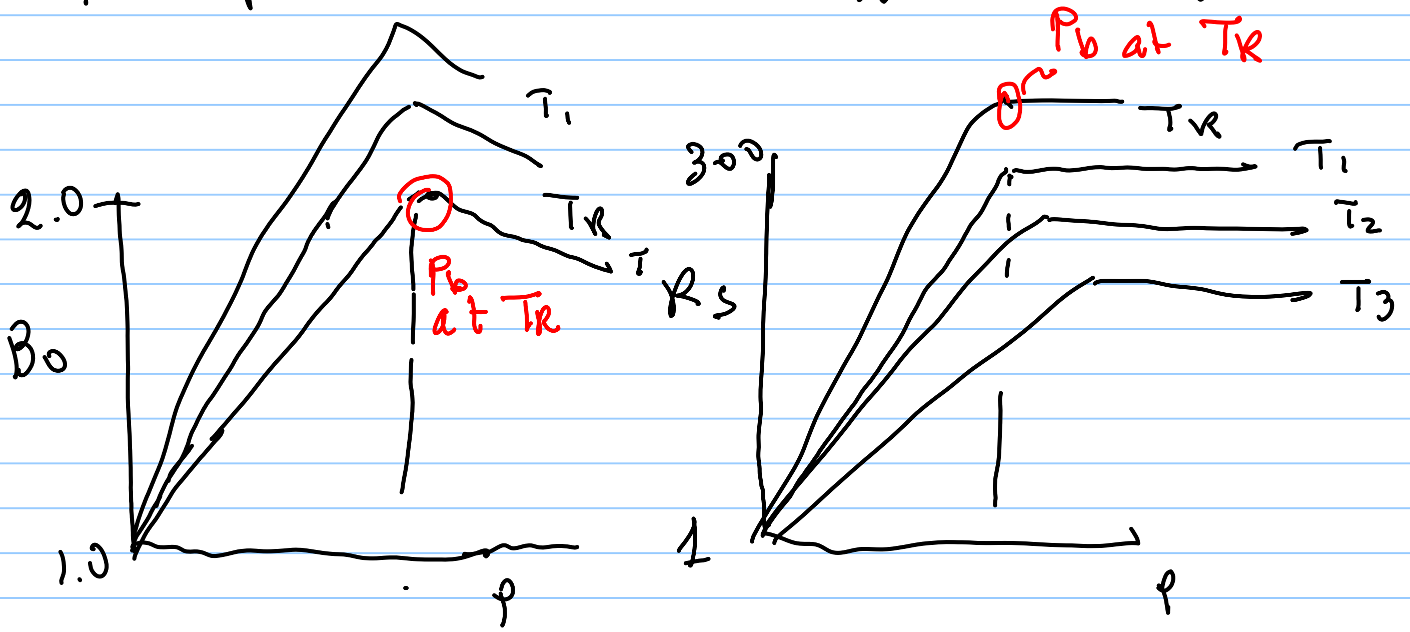
Typical black oil table for TR



Pressure (psia)	B_o (rb/stb)	B_g (rb/Mscf)	R_s (scf/stb)	μ_o (cP)	μ_g (cP)
6500	1.142	0.580	213	1.41	0.0333
6000	1.144	0.609	213	1.32	0.0317
5000	1.150	0.670	213	1.18	0.0282
4000	1.158	0.768	213	1.08	0.0248
3000	1.169	0.987	213	0.99	0.0215
2000	1.177	1.302	213	0.93	0.0180
1200	1.189	2.610	213	0.85	0.0144
→ 980* bubble point	1.191	3.205	213	0.83	0.0138
500	1.147	6.607	130	1.03	0.0125
100	1.015	33.893	44	1.07	0.0120

*Saturation pressure or bubble point.

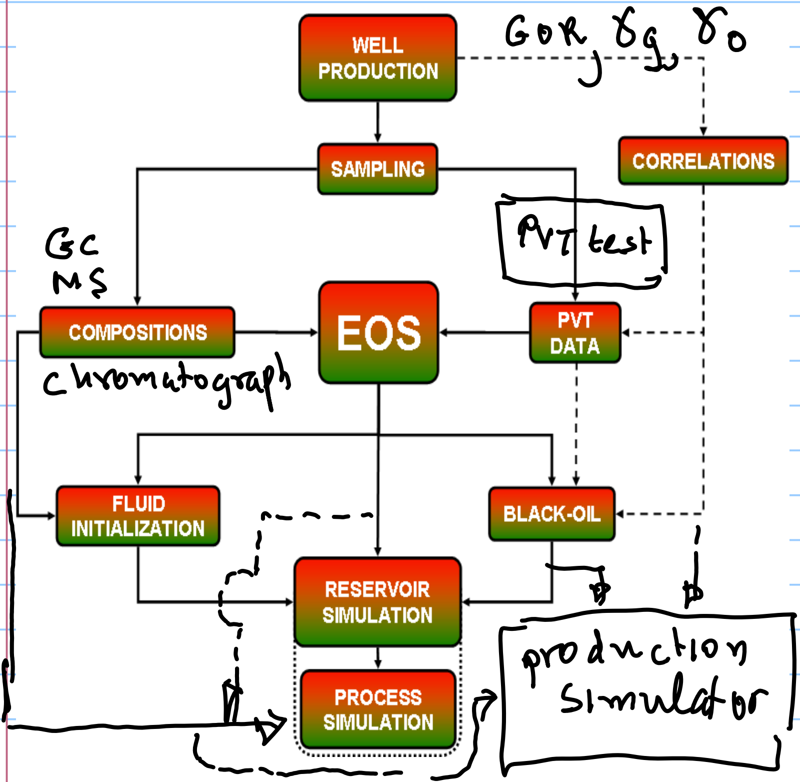
plot of Bo and Rs for different temps.



Black oil information presented

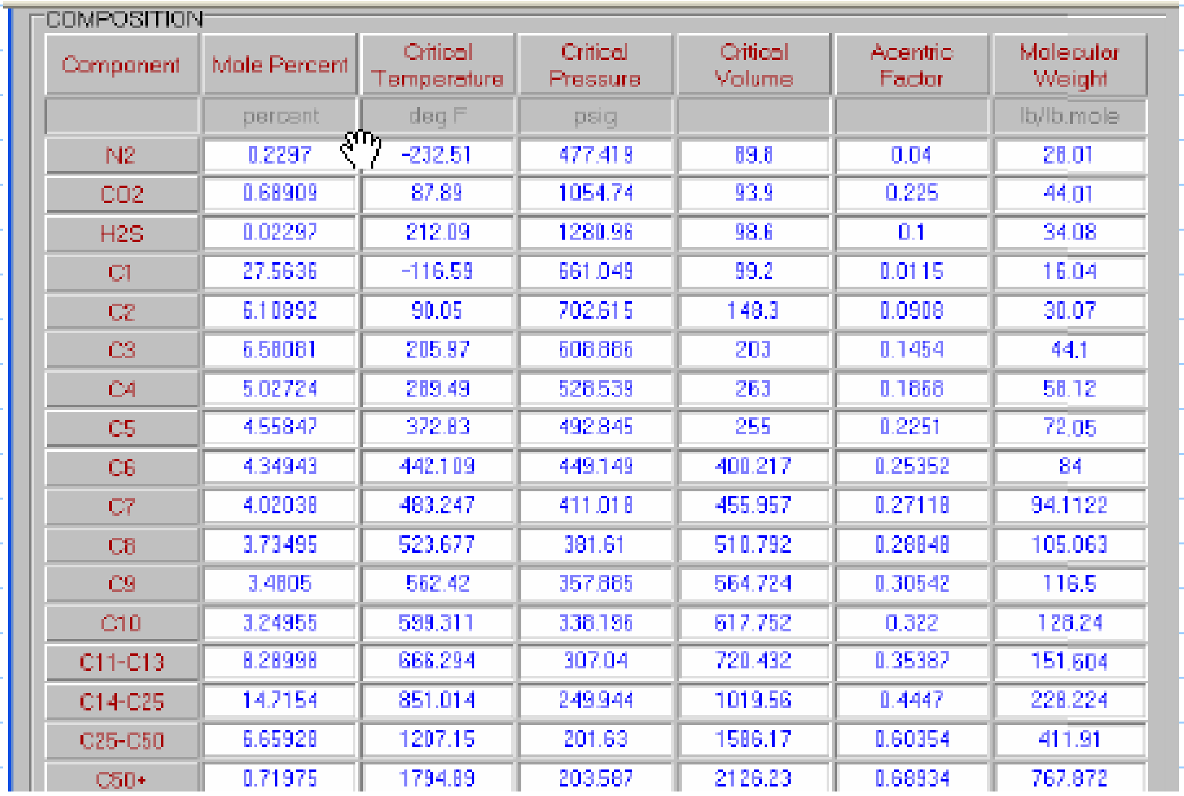
1. Table
2. plot
3. correlations

The figure below illustrated the position of the black oil information relative to the overall information body of fluid properties.



Production simulators e.g PROSPER, PipeSim, Pipeflo, gives the option of B.O. and compositional flow input.

Example of compositional input of stream info:

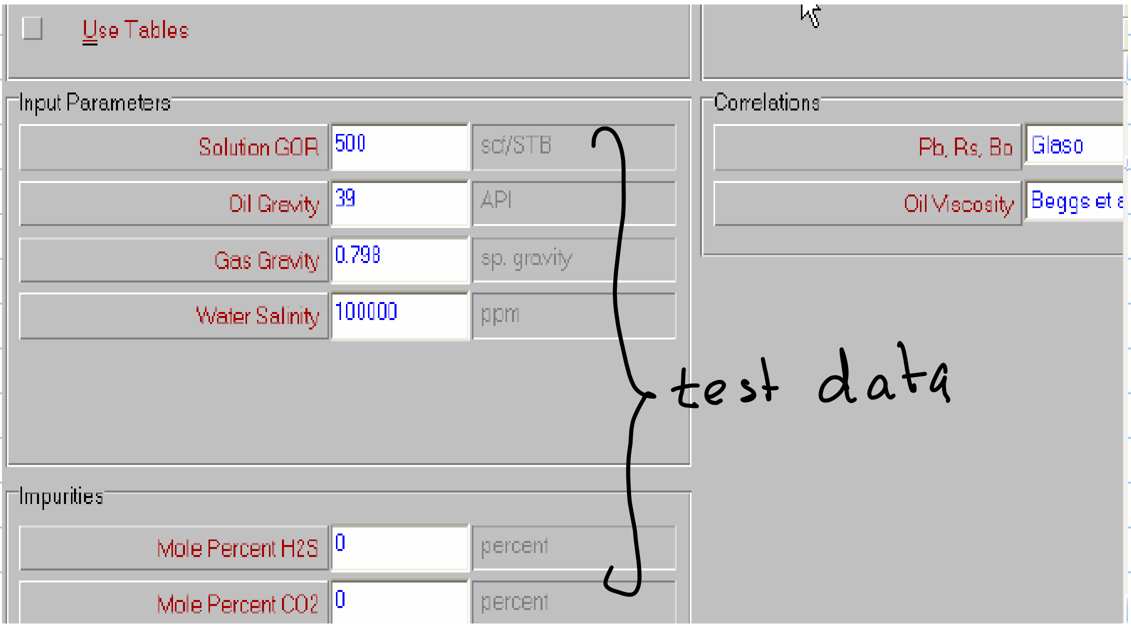


COMPOSITION

Component	Mole Percent	Critical Temperature	Critical Pressure	Critical Volume	Acentric Factor	Molecular Weight
	percent	deg F	psig			lb/lb.mole
N2	0.2297	-232.51	477.419	89.8	0.04	28.01
CO2	0.68909	87.89	1054.74	93.9	0.225	44.01
H2S	0.02297	212.09	1280.96	98.6	0.1	34.08
C1	27.5636	-116.59	661.049	99.2	0.0115	16.04
C2	6.10892	90.05	702.615	148.3	0.0908	30.07
C3	6.58061	205.97	608.866	203	0.1454	44.1
C4	5.02724	289.49	528.539	263	0.1868	58.12
C5	4.55847	372.83	492.845	255	0.2251	72.05
C6	4.34943	442.109	449.149	400.217	0.25352	84
C7	4.02038	483.247	411.018	455.957	0.27118	94.1122
C8	3.73495	523.677	381.61	510.792	0.28848	105.063
C9	3.4805	562.42	357.885	564.724	0.30542	116.5
C10	3.24955	599.311	338.196	617.752	0.322	128.24
C11-C13	8.28998	666.294	307.04	720.432	0.35387	151.604
C14-C25	14.7154	851.014	249.944	1019.56	0.4447	228.224
C25-C50	6.65928	1207.15	201.63	1586.17	0.60354	411.91
C50+	0.71975	1794.89	203.587	2126.23	0.68934	767.872

$\Sigma = 100\%$

Example of test data input (GOR, γ_o, γ_g) and use of black oil correlations in production simulator



☐ Use Tables

Input Parameters

Solution GOR	500	scf/STB
Oil Gravity	39	API
Gas Gravity	0.798	sp. gravity
Water Salinity	100000	ppm

Impurities

Mole Percent H2S	0	percent
Mole Percent CO2	0	percent

Correlations

Pb, Rs, Bo	Glaso
Oil Viscosity	Beggs et a

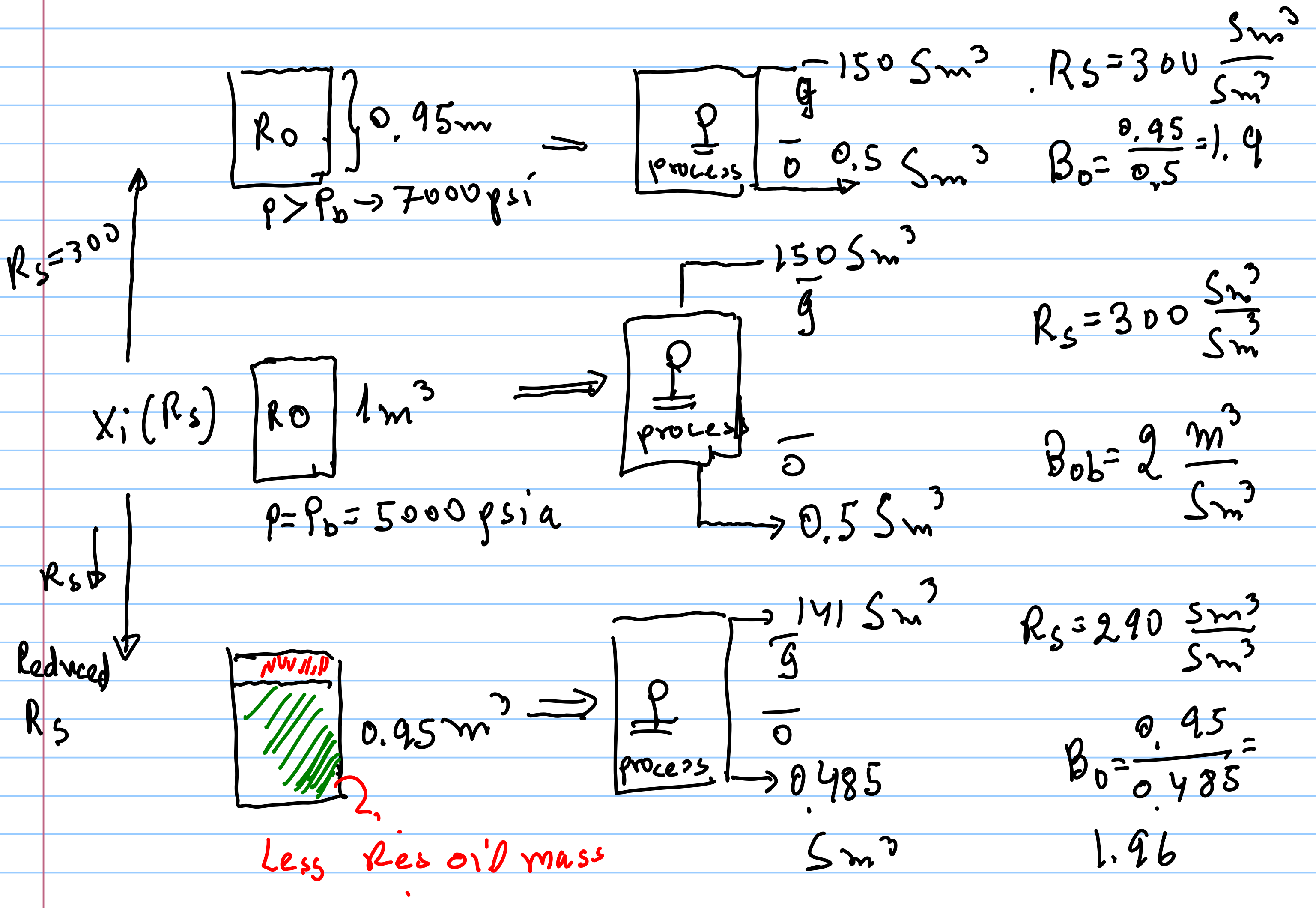
→ Correlations

test data

correlations to determine black oil properties from production tests are listed in Chp 3.
They are field specific and in "normal" oil field are ok for production calculations

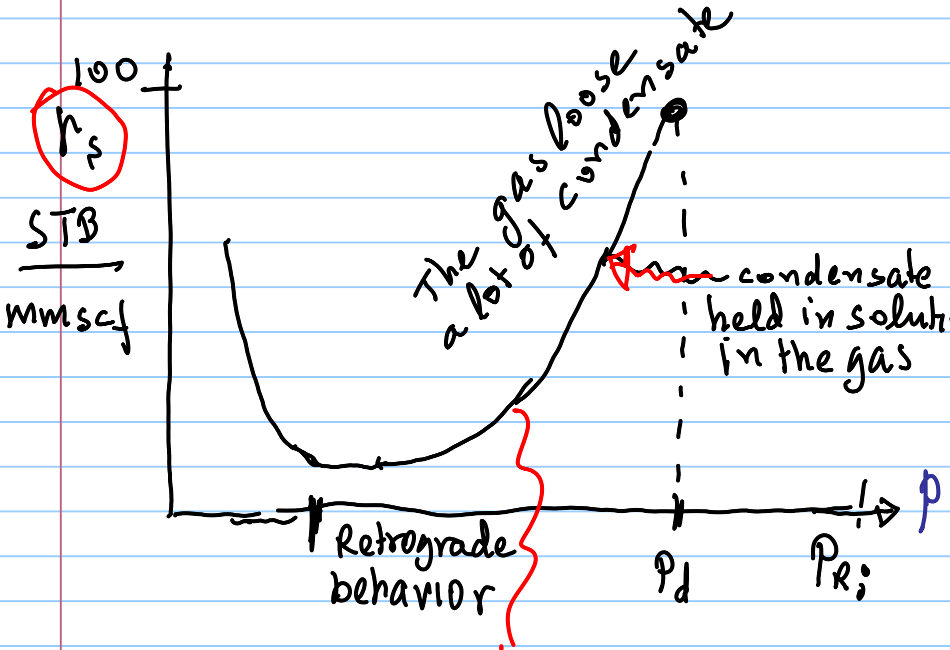
The correlations Were not discussed, so far, in the course

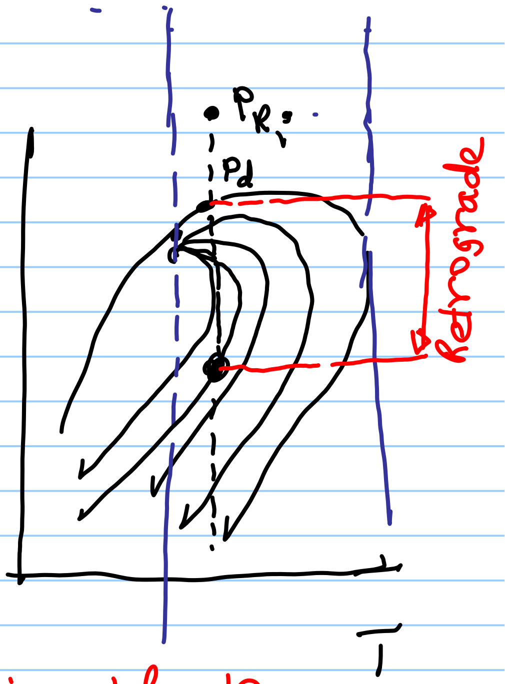
Example to demonstrate changes in B.O. parameters with changing reservoir pressure above, at, and below the bubble point.



Reservoir GAS black oil PVT

solution condensat-gas ratio, $r_s = \frac{Sm^3 \text{ condensate}}{Sm^3 \text{ gas}}$

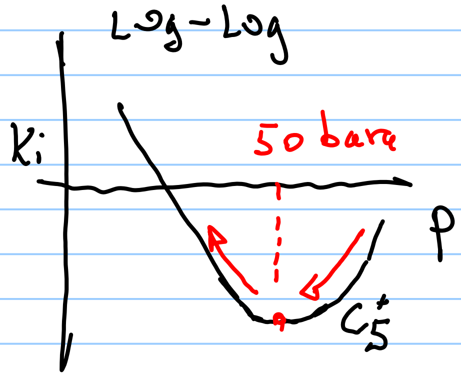




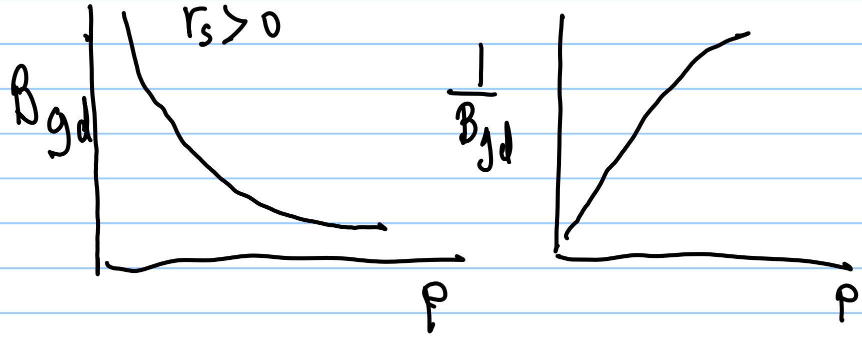
as the pressured is lowered, the gas is able to hold less liquid in solution and the liquid drops as liquid phase (and most likely to be captured by the capillary forces of the pores,

The retrograde behavior above can best be explained by the P-T diagram and the equi-quality lines that isothermal pressure reduction is crossing

The Log-Log plot of Ki vs p for C_5 shows that as pressure declines, Ki is reduced indicating increase in Liquid fraction, from a certain pressure (e.g 50 bara) this trend is inversed



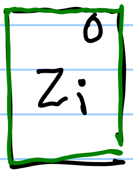
Pressure changes in B_{gd} $(r_s > 0)$



The plot of $\frac{1}{B_{gd}}$ is close to linear and it is usful for approximate interpolations and careful extrapolations

Effect of reservoir pressure decline on Reservoir oil

$\lambda_o = \frac{k_{ro}}{\mu_o}$ → mobility



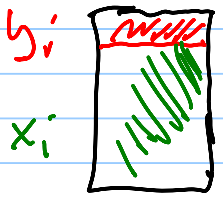
P_{R_i}

$\rightarrow \lambda_o = \frac{k_{ro}}{\mu_o} = 1$ ← reference value

$q_o \propto \lambda_o$

Oil production at a given draw down is proportional to the mobility factor λ_o

gas not move before S_g is 2%–10%.



$\lambda_g = 0$

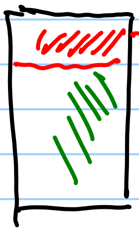
$\lambda_o = 0.9$

$P_R < P_{sat}$

Hon } At this range, the average mixture in the reservoir, $\bar{Z}_{iR}$ becomes lighter as free gas increses $\{C_1 - C_4\}\uparrow$

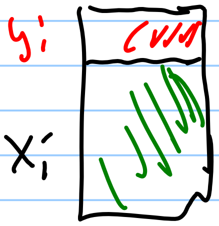
GOR ↓

At the same time, production GOR decreases



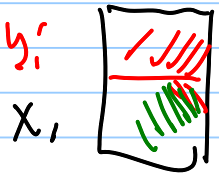
$\rightarrow \lambda_g = 0.1$

$\lambda_o = 0.5$



$\lambda_g = 0.2$

$\lambda_o = 0.2$



$\lambda_g = 1.0$

$\lambda_o = 0.05$

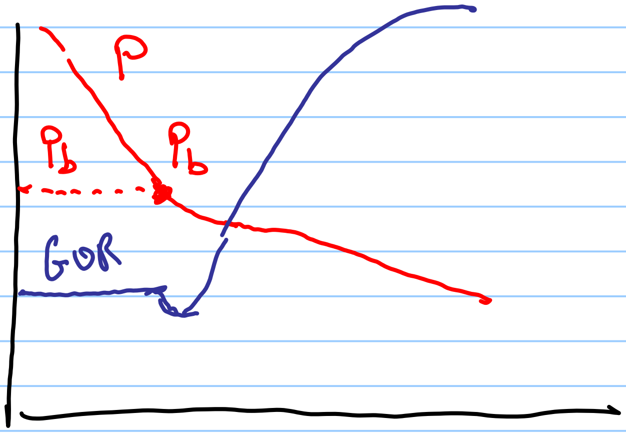
oil mobility decreases

gas mobility increases

Z_{iR} becomes "heavier" as it losses the light components to gas which high mobility and flow at increasing higher rate.

GOR increses

The general GOR Trend



$\frac{N_p}{N}$

An interesting observation (made by Curtis !!)

In can be shown that while the total composition changes during the depletion process due to pressure and phase mobility changes, the compositions $x_i(p)$ and $y_i(p)$ change very little.

Thus, though the gas, or the vapore fraction F_v in the total mixture increases, and the total composition $z_i(p)$ changes, x_i and y_i remain the same.

This observation can be explained by the fact that both $z_i(p)$ and $K_i(p)$ change with pressure, but their pressure dependecy, within the range of the depletion process results in little change in $x_i(p)$, $y_i(p)$

Material on fluid properties and Black oil correlations (used at the course TPG 4145

1. Yellow Data book - M. B. Standing (1974)
   1994- Revision by C. H. Whitson
2. Chapter 3 - Phase behaviour SPE Mo 20

In previous lectures we have introduced and used 4 Black oil parameters .

Black oil → Properties B_o, R_s, B_g, r_s

$$\begin{tikzcd} & B_g \arrow[dl] \arrow[dr, dash] & \\ B_{gw} & & B_{gd} \end{tikzcd}$$

One more useful parameter derived from the others is Total FVF: (3.70)

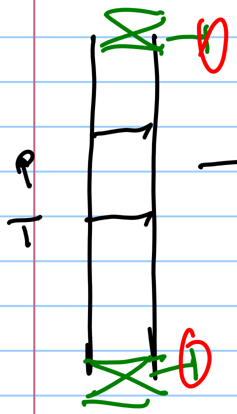
$$B_t = \frac{(V_o + V_g)\big|_{p,T}}{V_{o_{sc}}} = \frac{V_o + \bar{V}_g}{V_{\bar{o}}}$$

This parameter uses a conversion factor when stated in CFU

metric	CFU
V_o m³ liquid	V_o bbl
V_g m³ gas	V_g f³
	conversion $5.614 \frac{ft}{bbl}$

why B_T is important in production engineering?
It is used to calculate phase void fraction

given pressure P and temperature, T, in a shot tubing segment, then:



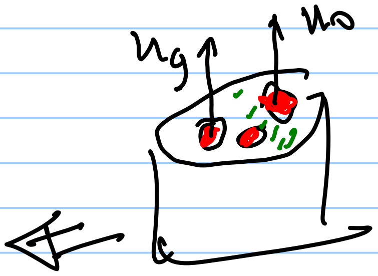
$$\frac{q_o}{q_T}\Big|_{P,T} = \frac{q_o}{q_o+q_g}\Big|_{P,T}$$

in no-slip condition

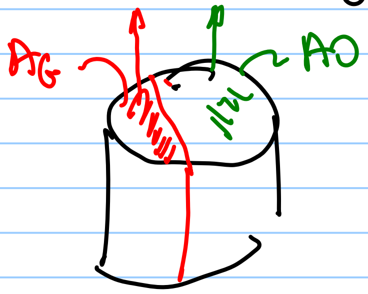
$$\bar{u}_o = \bar{u}_g$$

$$\bar{u}_o = \frac{q_o}{A_o} \qquad \bar{u}_g = \frac{q_g}{A_G}$$

$$\frac{q_o}{q_T} = \frac{A_o}{A_T} = \frac{A_o}{A_o+A_g}$$



slip velocity
$u_s = u_G - u_o$



$$\frac{q_o}{q_T}\Big|_{P,T} = \frac{B_o}{B_T}\Big|_{P,T} = \frac{B_o}{B_o+\underline{(GOR-R_s)B_g}}$$

oil free gas → in metric units

$$\frac{q_o}{q_T} = \frac{B_o}{B_T} = \frac{B_o}{\underbrace{B_o}_{Bbl} + \frac{(GOR-R_s)\,B_g}{5.614}}$$

$\frac{\text{bbl}\ (P,T)}{\text{bbl}.\ (P,T)}$ → in CFU

$B_o \left|\frac{bbl}{STB}\right|$

GOR, R_s |scf/STB|

$B_g \left|\frac{ft^3}{scf}\right|$

$5.614 \left|\frac{ft^3}{bbl}\right|$

unit equation ⇐

$$\frac{B_o\left|\frac{bbl}{\cancel{STB}}\right|}{B_o\left|\frac{bbl}{\cancel{STB}}\right| + \frac{(GOR-R_s)\left|\frac{\cancel{scf}}{\cancel{STB}}\right|\cdot B_g\left|\frac{\cancel{ft^3}}{\cancel{scf}}\right|}{5.614\left|\frac{\cancel{ft^3}}{bbl}\right|}}$$

correlation – Reservoir oil

Two regions – undersaturated $p > p_b$

saturated $p < p_b$

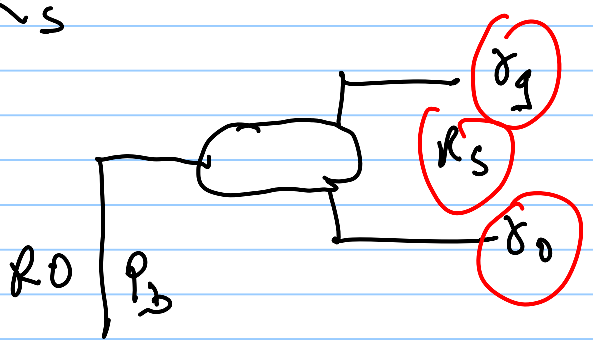
Three useful correlation will be introduced

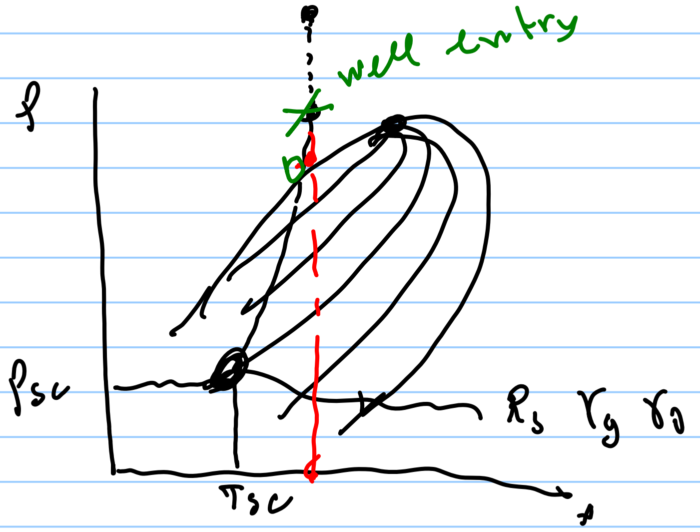
1. $p_b (R_s)$
2. $R_s (p)$
3. $B_{ob} (R_s)$

Bubble point correlation for saturated oil

p_b increases with R_s

$$p_b = f(R_s, T, \gamma_{\bar{o}} \ \gamma_{\bar{g}})$$

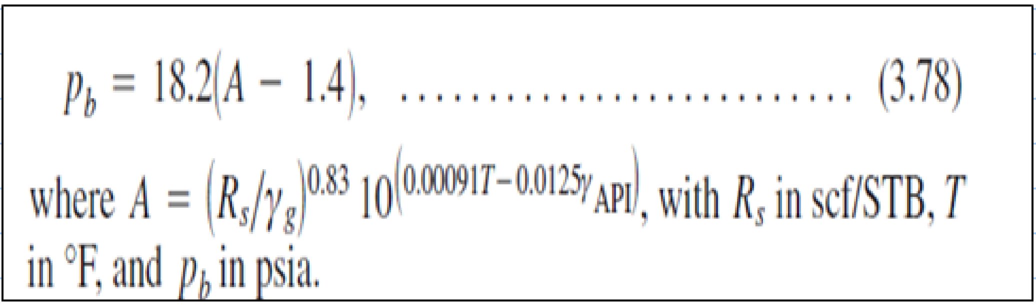




In this lecture we will list correlations developed by M.B. Standing for Californian oil field

$$P_b = 18.2\,(A - 1.4)$$

$$A = \left(\frac{R_s}{\gamma_g}\right)^{0.83} \cdot 10^{\left(0.00091\,T - 0.0125\,\gamma_{API}\right)}$$



$$p_b = 18.2(A - 1.4), \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.78)$$

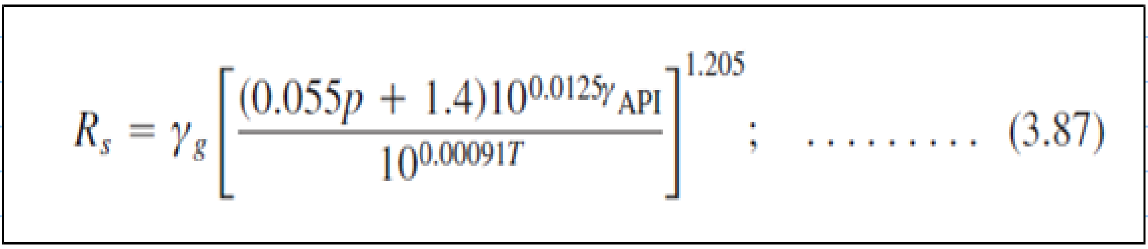
where $A = (R_s/\gamma_g)^{0.83}\,10^{(0.00091T - 0.0125\gamma_{API})}$, with R_s in scf/STB, T in °F, and p_b in psia.

standing correlation – Californian oils

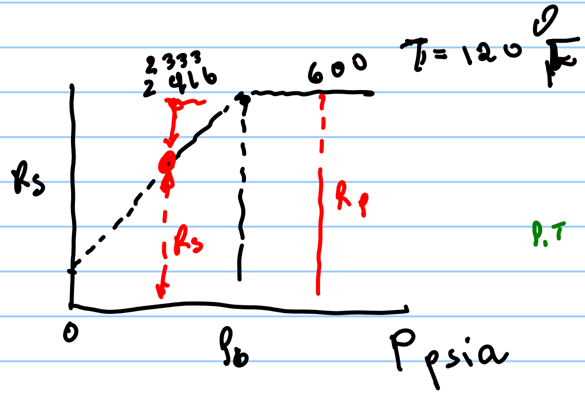
glasø corrations – North sea oils

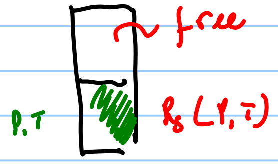
solution gas correlation

$$R_s = f(P, T, \gamma_o\ \gamma_g)$$



$$R_s = \gamma_g\left[\frac{(0.055p + 1.4)10^{0.0125\gamma_{API}}}{10^{0.00091T}}\right]^{1.205}; \quad \ldots\ldots\ldots \quad (3.87)$$





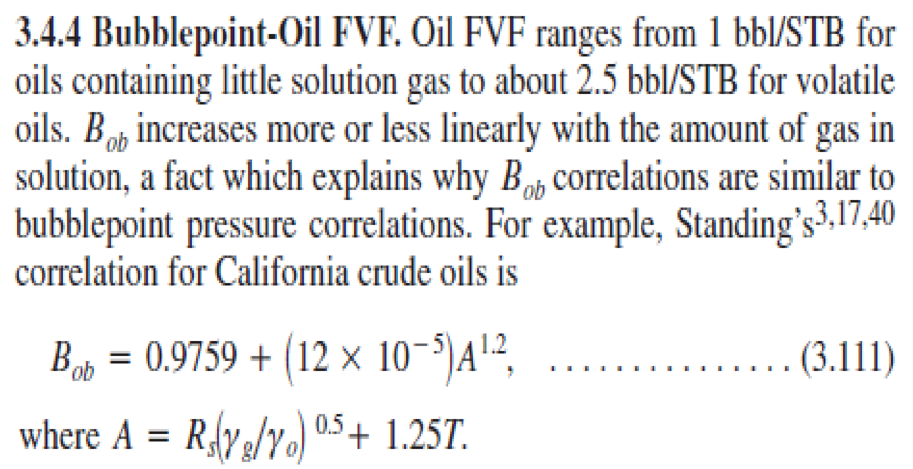
FVF correlation

B_{obp}

$$B_{ob} = f(R_s \quad T \quad \gamma_o \quad \gamma_g)$$

$$B_{ob} = 0.9759 + 12\left(10^{-5}\right) A^{1.2} \qquad (3.111)$$

$$A = R_s\left(\frac{\gamma_g}{\gamma_o}\right)^{0.5} + 1.25\,T$$

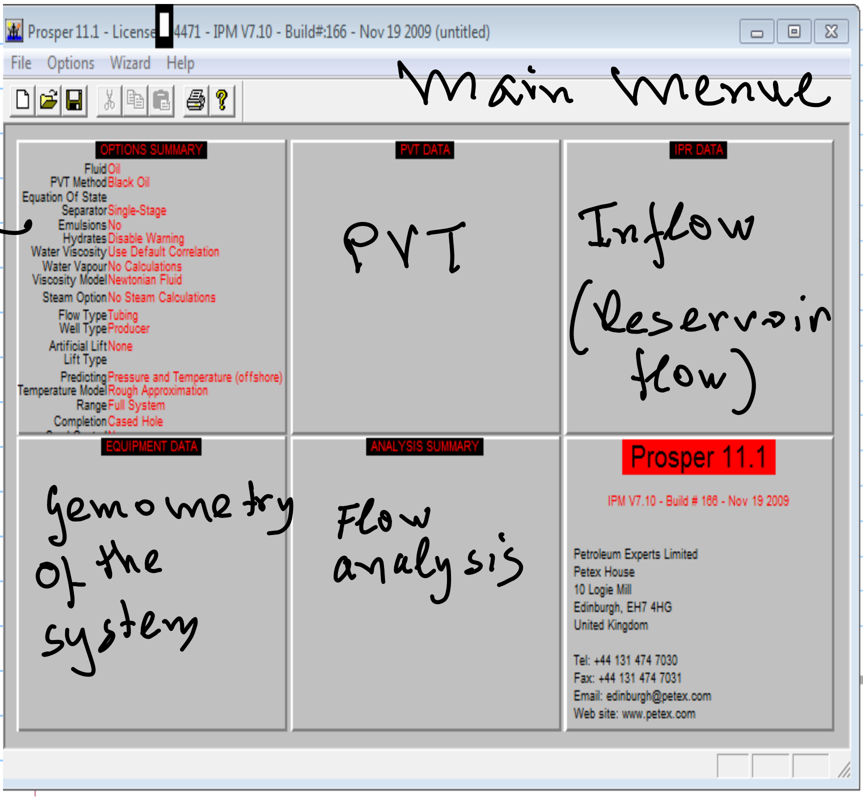


3.4.4 Bubblepoint-Oil FVF. Oil FVF ranges from 1 bbl/STB for oils containing little solution gas to about 2.5 bbl/STB for volatile oils. B_{ob} increases more or less linearly with the amount of gas in solution, a fact which explains why B_{ob} correlations are similar to bubblepoint pressure correlations. For example, Standing's[3,17,40] correlation for California crude oils is

$$B_{ob} = 0.9759 + \left(12 \times 10^{-5}\right)A^{1.2}, \quad \dots\dots\dots\dots\dots (3.111)$$

where $A = R_s\left(\gamma_g/\gamma_o\right)^{0.5} + 1.25T$.

We will demonstrate the use of these three correlation to develop Black oil tables for production or reservoir applications using the Production program PROSPER by Petroleum Experts system



Prosper 11.1 - License[redacted]4471 - IPM V7.10 - Build#:166 - Nov 19 2009 (untitled)

Main menue

OPTIONS SUMMARY

Fluid Oil
PVT Method Black Oil
Equation Of State
Separator Single-Stage
Emulsions No
Hydrates Disable Warning
Water Viscosity Use Default Correlation
Water Vapour No Calculations
Viscosity Model Newtonian Fluid
Steam Option No Steam Calculations
Flow Type Tubing
Well Type Producer
Artificial Lift None
Lift Type
Predicting Pressure and Temperature (offshore)
Temperature Model Rough Approximation
Range Full System
Completion Cased Hole

PVT DATA

PVT

IPR DATA

Inflow (Reservoir flow)

EQUIPMENT DATA

gemometry of the system

ANALYSIS SUMMARY

Flow analysis

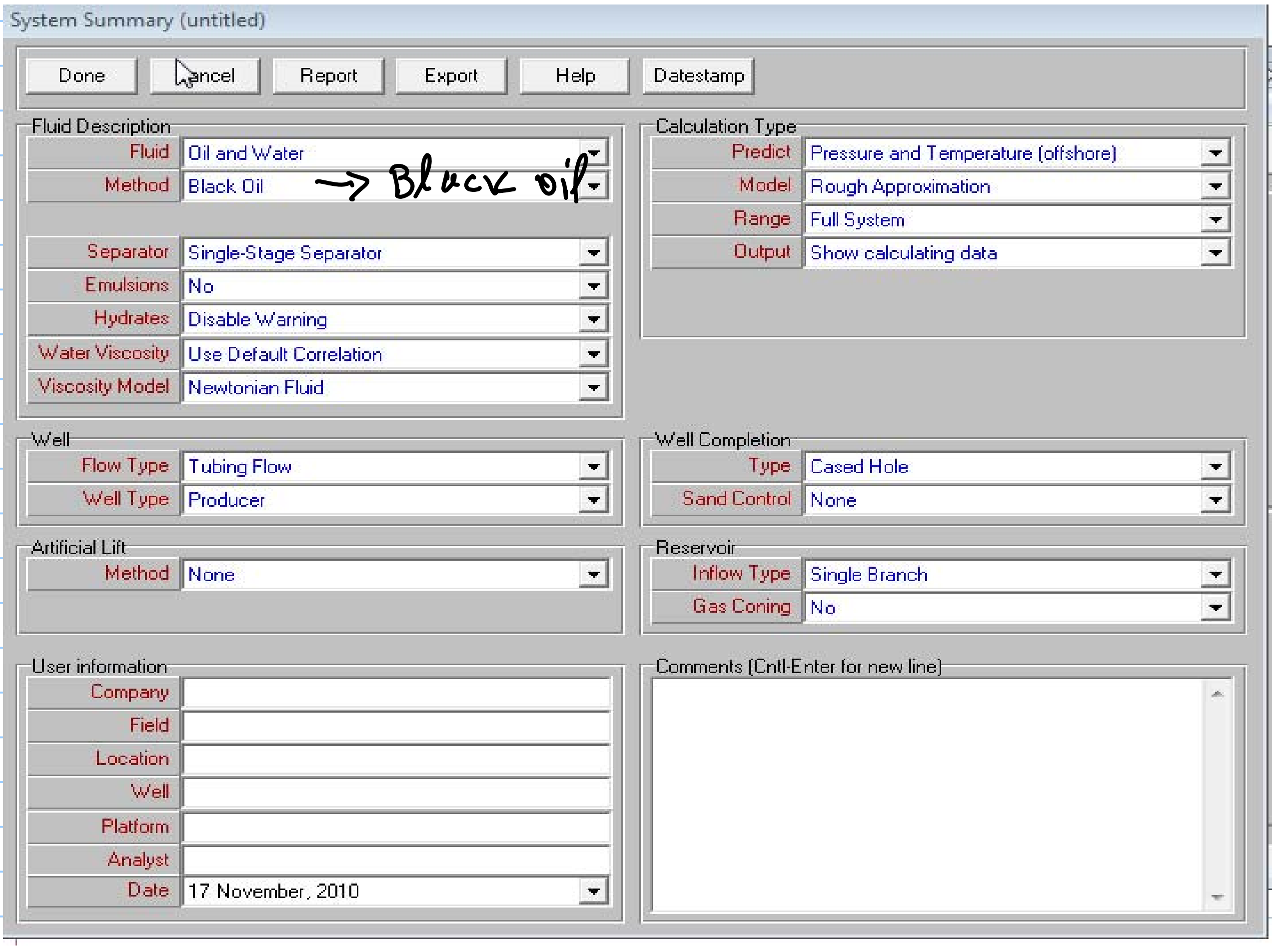
Prosper 11.1

IPM V7.10 - Build # 166 - Nov 19 2009

Petroleum Experts Limited
Petex House
10 Logie Mill
Edinburgh, EH7 4HG
United Kingdom

Tel: +44 131 474 7030
Fax: +44 131 474 7031
Email: edinburgh@petex.com
Web site: www.petex.com

system window



System Summary (untitled)

Fluid Description
Fluid: Oil and Water
Method: Black Oil → Black oil
Separator: Single-Stage Separator
Emulsions: No
Hydrates: Disable Warning
Water Viscosity: Use Default Correlation
Viscosity Model: Newtonian Fluid

Calculation Type
Predict: Pressure and Temperature (offshore)
Model: Rough Approximation
Range: Full System
Output: Show calculating data

Well
Flow Type: Tubing Flow
Well Type: Producer

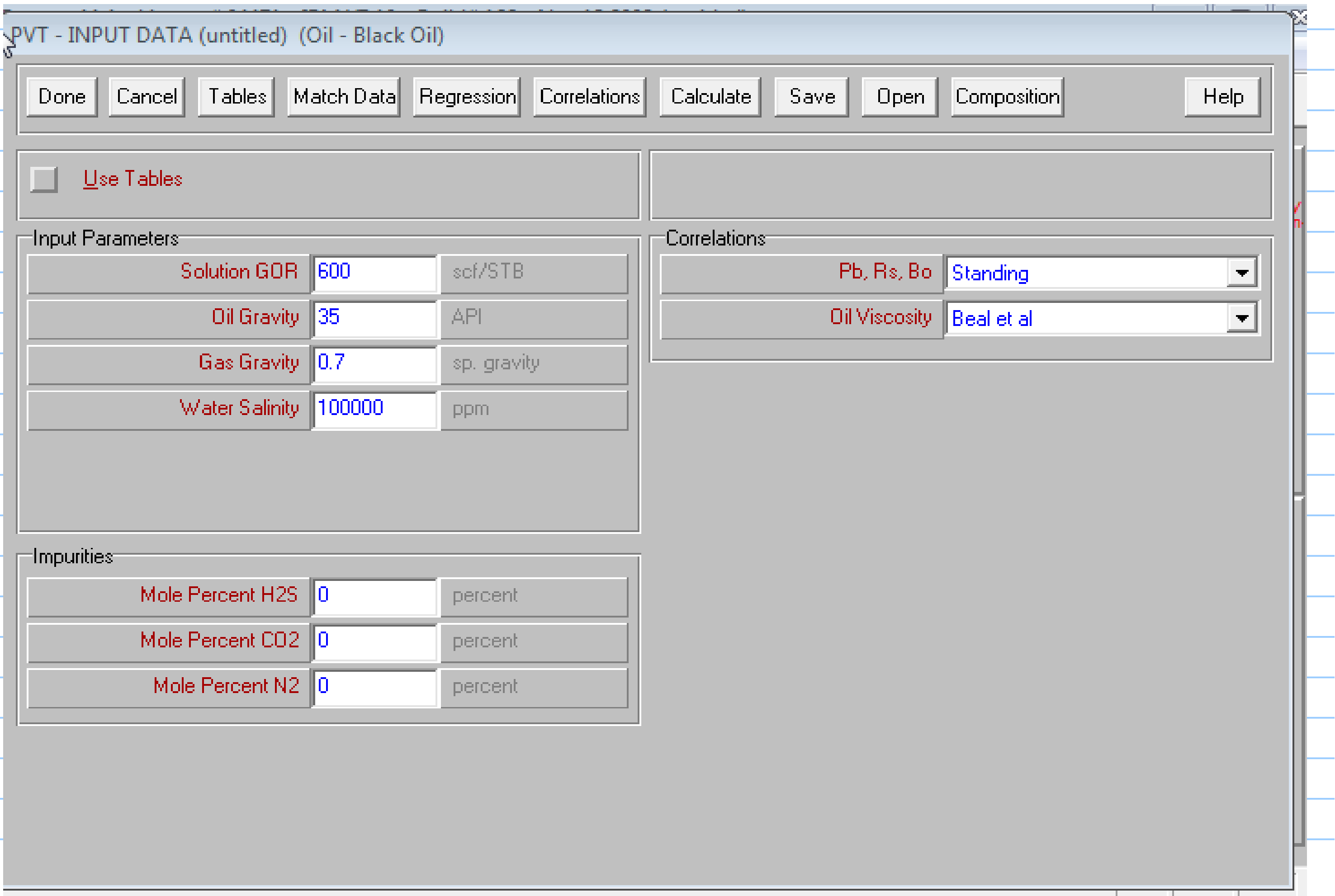
Well Completion
Type: Cased Hole
Sand Control: None

Artificial Lift
Method: None

Reservoir
Inflow Type: Single Branch
Gas Coning: No

User information
Company:
Field:
Location:
Well:
Platform:
Analyst:
Date: 17 November, 2010

Comments (Cntl-Enter for new line)



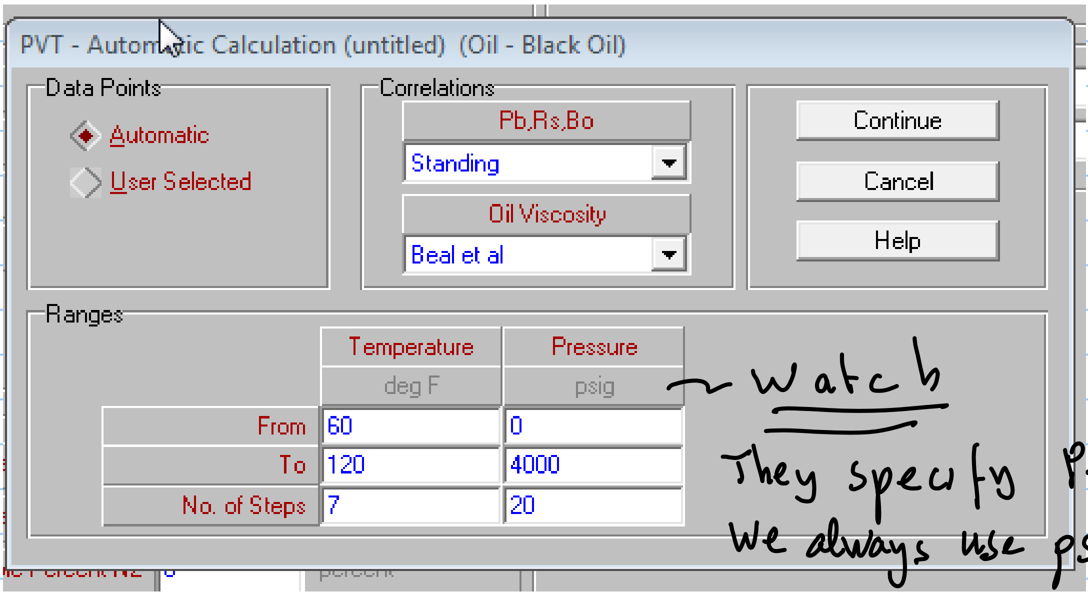
PVT - INPUT DATA (untitled) (Oil - Black Oil)

☐ Use Tables

Input Parameters
Solution GOR: 600 scf/STB
Oil Gravity: 35 API
Gas Gravity: 0.7 sp. gravity
Water Salinity: 100000 ppm

Correlations
Pb, Rs, Bo: Standing
Oil Viscosity: Beal et al

Impurities
Mole Percent H2S: 0 percent
Mole Percent CO2: 0 percent
Mole Percent N2: 0 percent



PVT - Automatic Calculation (untitled) (Oil - Black Oil)

Data Points
● Automatic
○ User Selected

Correlations
Pb,Rs,Bo: Standing
Oil Viscosity: Beal et al

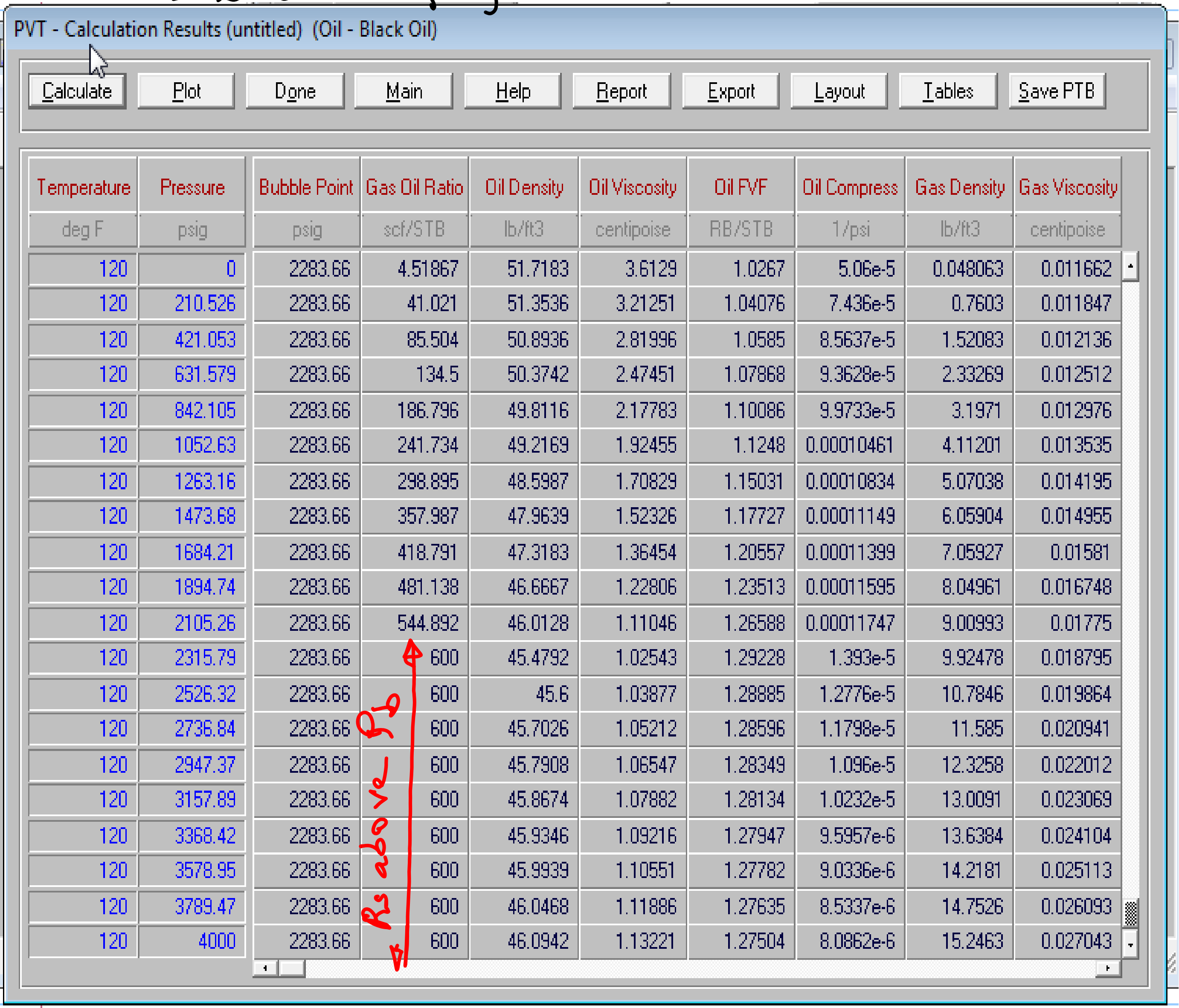
Ranges

	Temperature deg F	Pressure psig
From	60	0
To	120	4000
No. of Steps	7	20

→ watch

They specify psig. We always use psia.

The program calculates tables for temp = 60°F → 120 F°. We included the one for T = 120°F. For the Temp the P_b=2283.66 psig



PVT - Calculation Results (untitled) (Oil - Black Oil)

Temperature deg F	Pressure psig	Bubble Point psig	Gas Oil Ratio scf/STB	Oil Density lb/ft3	Oil Viscosity centipoise	Oil FVF RB/STB	Oil Compress 1/psi	Gas Density lb/ft3	Gas Viscosity centipoise
120	0	2283.66	4.51867	51.7183	3.6129	1.0267	5.06e-5	0.048063	0.011662
120	210.526	2283.66	41.021	51.3536	3.21251	1.04076	7.436e-5	0.7603	0.011847
120	421.053	2283.66	85.504	50.8936	2.81996	1.0585	8.5637e-5	1.52083	0.012136
120	631.579	2283.66	134.5	50.3742	2.47451	1.07868	9.3628e-5	2.33269	0.012512
120	842.105	2283.66	186.796	49.8116	2.17783	1.10086	9.9733e-5	3.1971	0.012976
120	1052.63	2283.66	241.734	49.2169	1.92455	1.1248	0.00010461	4.11201	0.013535
120	1263.16	2283.66	298.895	48.5987	1.70829	1.15031	0.00010834	5.07038	0.014195
120	1473.68	2283.66	357.987	47.9639	1.52326	1.17727	0.00011149	6.05904	0.014955
120	1684.21	2283.66	418.791	47.3183	1.36454	1.20557	0.00011399	7.05927	0.01581
120	1894.74	2283.66	481.138	46.6667	1.22806	1.23513	0.00011595	8.04961	0.016748
120	2105.26	2283.66	544.892	46.0128	1.11046	1.26588	0.00011747	9.00993	0.01775
120	2315.79	2283.66	600	45.4792	1.02543	1.29228	1.393e-5	9.92478	0.018795
120	2526.32	2283.66	600	45.6	1.03877	1.28885	1.2776e-5	10.7846	0.019864
120	2736.84	2283.66	600	45.7026	1.05212	1.28596	1.1798e-5	11.585	0.020941
120	2947.37	2283.66	600	45.7908	1.06547	1.28349	1.096e-5	12.3258	0.022012
120	3157.89	2283.66	600	45.8674	1.07882	1.28134	1.0232e-5	13.0091	0.023069
120	3368.42	2283.66	600	45.9346	1.09216	1.27947	9.5957e-6	13.6384	0.024104
120	3578.95	2283.66	600	45.9939	1.10551	1.27782	9.0336e-6	14.2181	0.025113
120	3789.47	2283.66	600	46.0468	1.11886	1.27635	8.5337e-6	14.7526	0.026093
120	4000	2283.66	600	46.0942	1.13221	1.27504	8.0862e-6	15.2463	0.027043

R_s above P_b

we have not discussed correlations for viscosity μ_o and compressibility, c_o

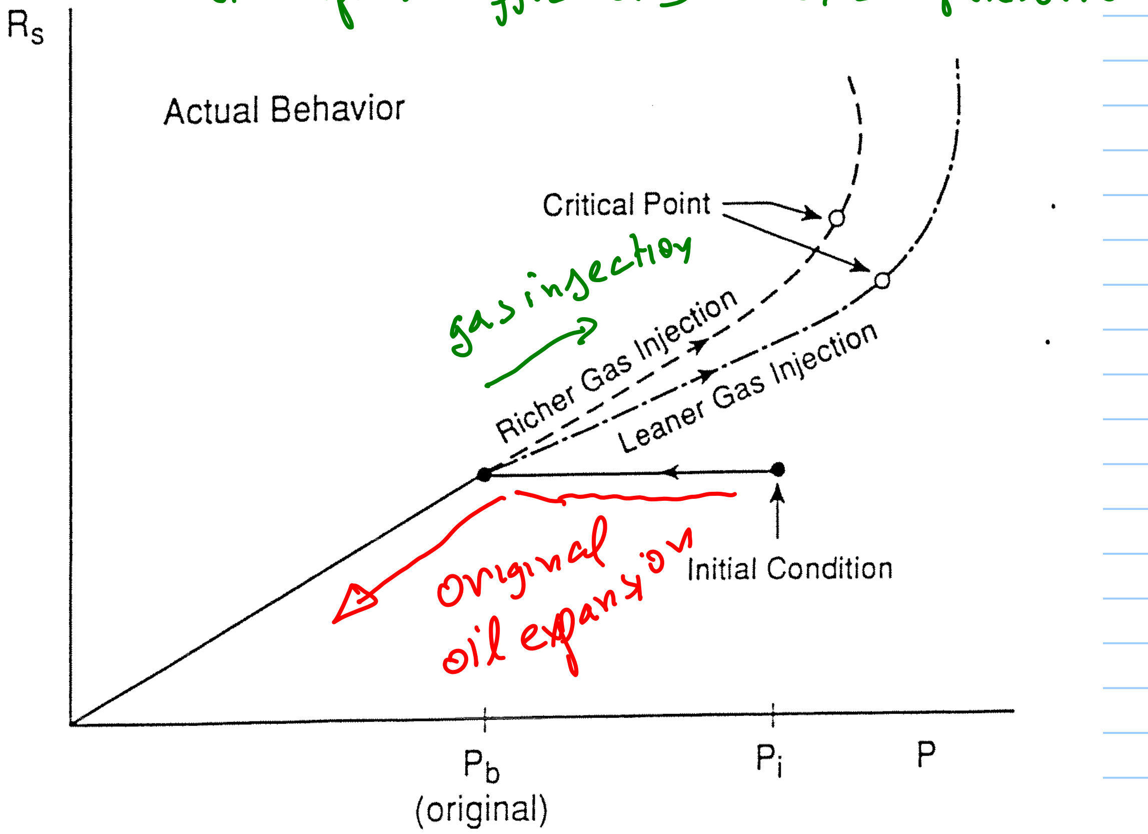
we have to recognize the limitation of the correlations. They are not unique and not universal, they have limitation on the range of validity and are field specific

problems with correlation

- No general → API_o
- can not be extrapolated

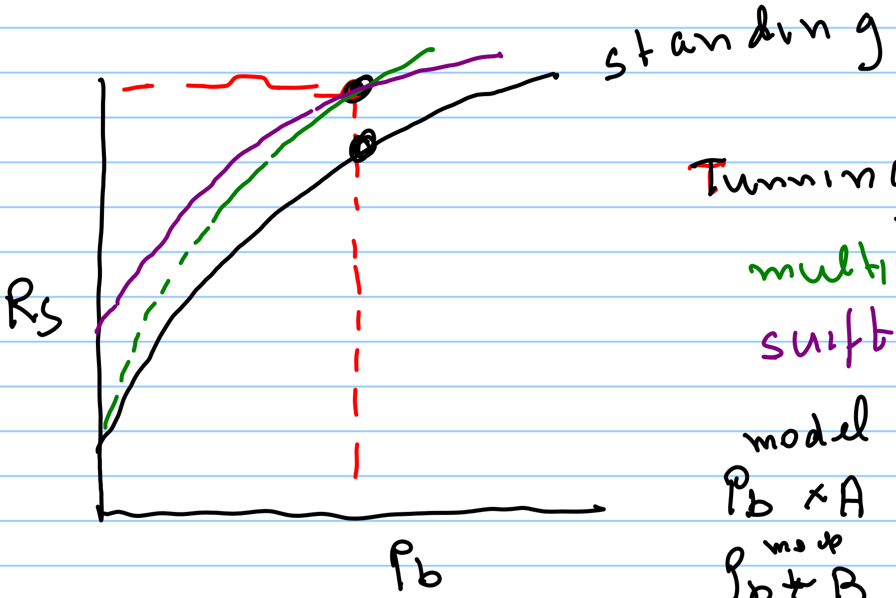
Normally assume $r_s = 0$

Example: Difficulties in extrapolation



Problems with tunning correlations to available data. Example tuning standing Rs correlation to a single point data, Rs at the Pb

Prosper uses to factors: multiplier and shift $X^{tunned} = (X^{model} \times A) + B$



$P_b^{model} \times A$

$P_b^{model} + B$

$P^{model} \times A + B = \text{data}$

How prosper tune the correlations to one data point ?

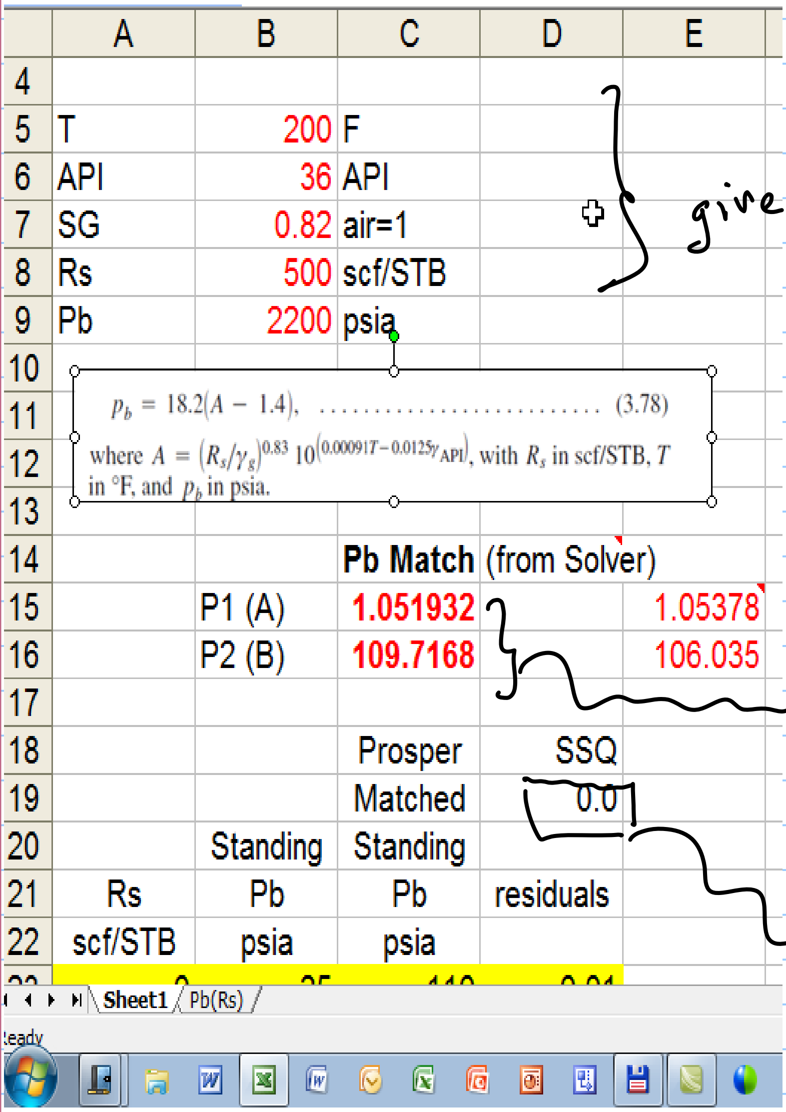
Define a new function (Prosper)

$$\phi = \left(P_b^{model} - P_b^{data}\right)^2 + (A-1)^2 + \left(\frac{B}{P_b^{data}}\right)^2 \rightarrow \text{minimize}$$

A = multiplier

B = shifting

$P_{actual} = P_{model} \times A + B$



	A	B	C	D	E
4					
5	T	200	F		
6	API	36	API		
7	SG	0.82	air=1		
8	Rs	500	scf/STB		
9	Pb	2200	psia		
14			**Pb Match**	(from Solver)	
15		P1 (A)	**1.051932**		1.05378
16		P2 (B)	**109.7168**		106.035
17					
18			Prosper	SSQ	
19			Matched	0.0	
20		Standing	Standing		
21	Rs	Pb	Pb	residuals	
22	scf/STB	psia	psia		

$p_b = 18.2(A - 1.4)$, (3.78)

where $A = (R_s/\gamma_g)^{0.83}\, 10^{(0.00091T - 0.0125\gamma_{API})}$, with R_s in scf/STB, T in °F, and p_b in psia.

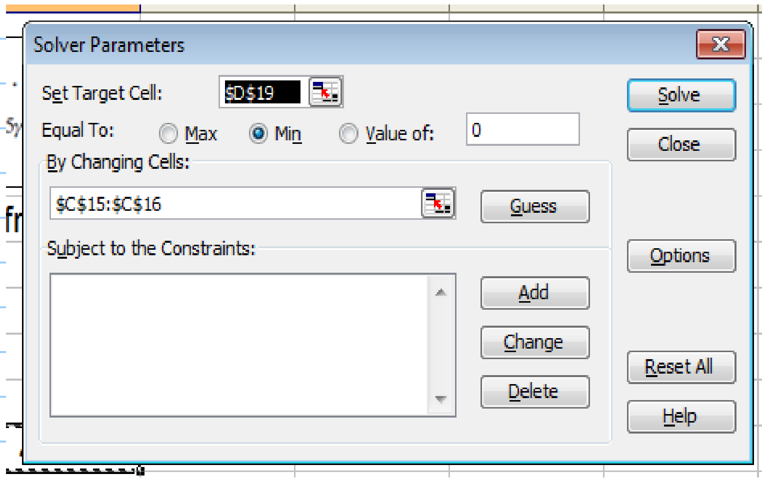
given data

tunning parameters. First assume

A = 1.0

B = 0.0

Solver cell to minimize the Ø



Solver Parameters

Set Target Cell: $D$19

Equal To: ○ Max ● Min ○ Value of: 0

By Changing Cells: $C$15:$C$16

Subject to the Constraints:

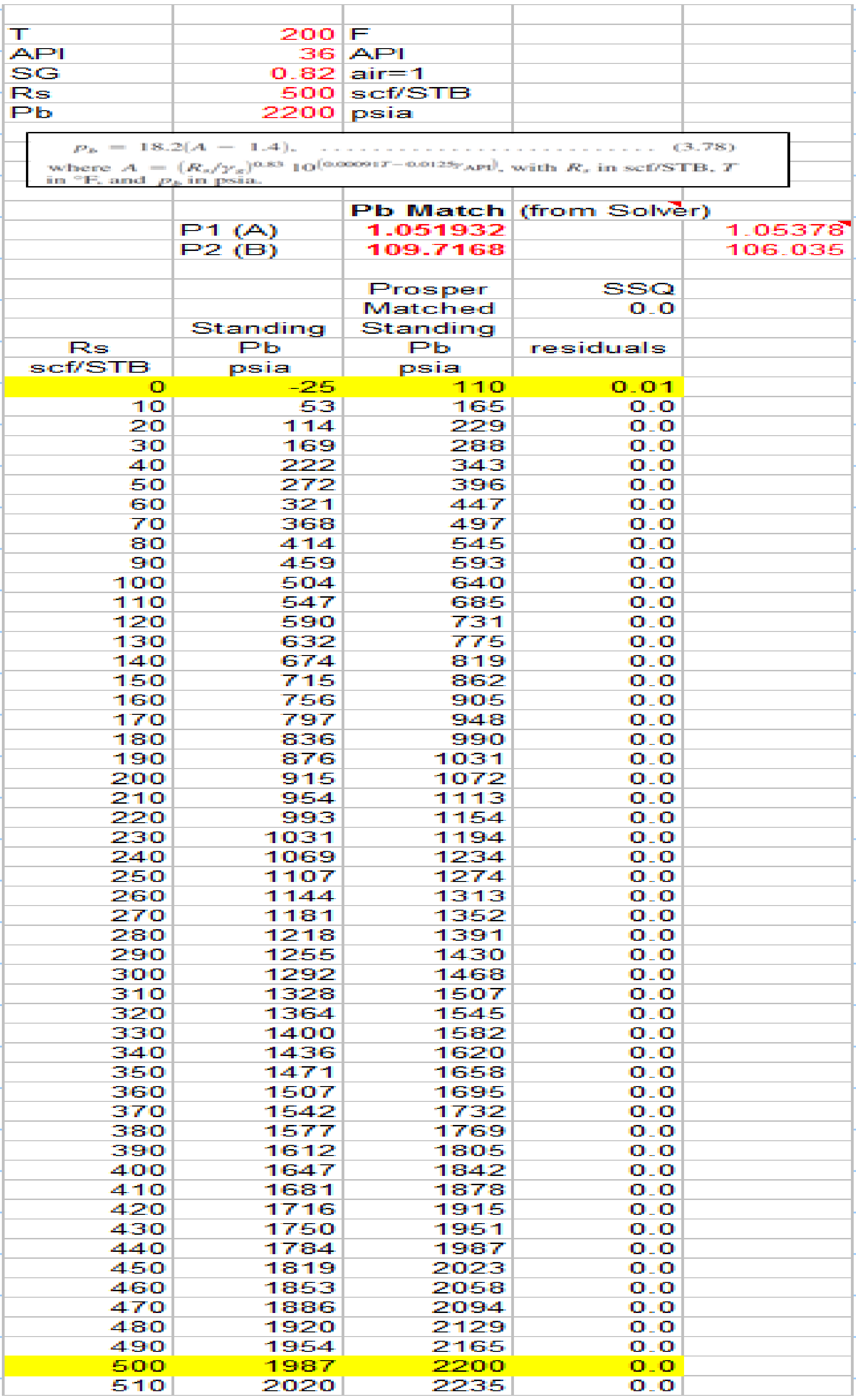
solver window

The target cell D19 has the function

$$\left(P_b^{m} - P_b^{Data}\right)^2 + (1 - A)^2 + \left(\frac{P_b^{dat}}{B}\right)^2 =$$

$= 0$

Below is the entire table after the tunning



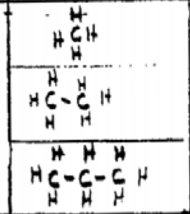
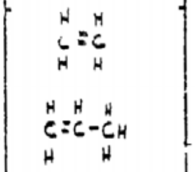
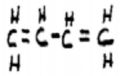
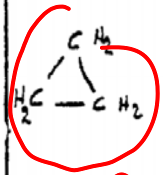
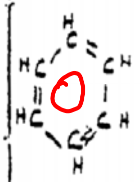
T	200	F		
API	36	API		
SG	0.82	air=1		
Rs	500	scf/STB		
Pb	2200	psia		

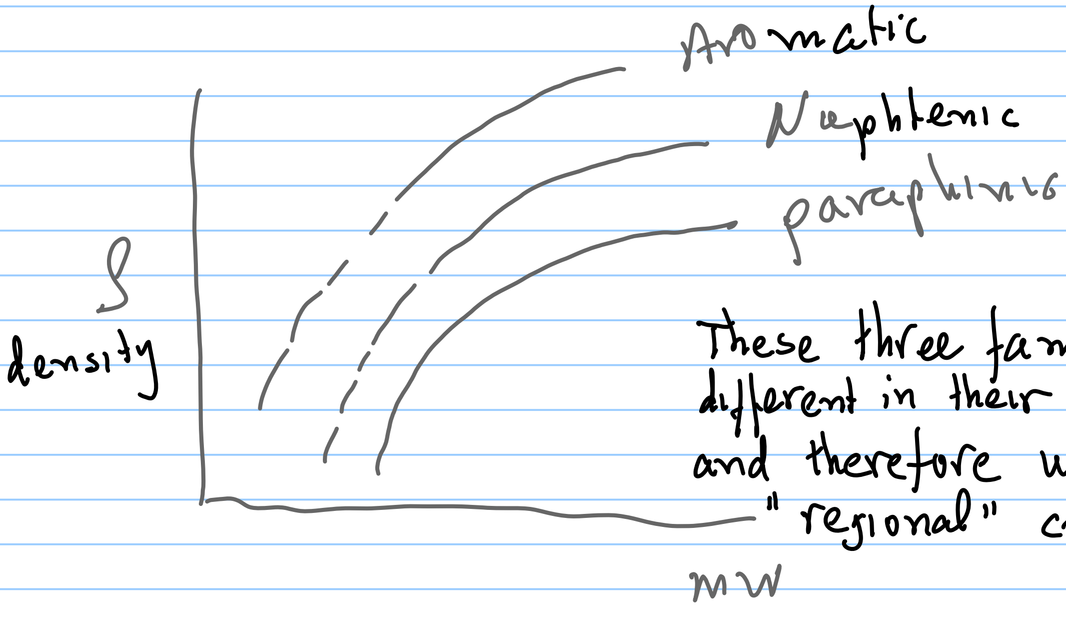
$$p_b = 18.2(A - 1.4), \quad (3.78)$$

where $A = (R_s/\gamma_g)^{0.83}\, 10^{(0.00091T - 0.0125\gamma_{API})}$, with R_s in scf/STB, T in °F, and p_b in psia.

		Pb Match	(from Solver)	
	P1 (A)	1.051932		1.05378
	P2 (B)	109.7168		106.035
		Prosper	SSQ	
		Matched	0.0	
	Standing	Standing		
Rs	Pb	Pb	residuals	
scf/STB	psia	psia		
0	-25	110	0.01	
10	53	165	0.0	
20	114	229	0.0	
30	169	288	0.0	
40	222	343	0.0	
50	272	396	0.0	
60	321	447	0.0	
70	368	497	0.0	
80	414	545	0.0	
90	459	593	0.0	
100	504	640	0.0	
110	547	685	0.0	
120	590	731	0.0	
130	632	775	0.0	
140	674	819	0.0	
150	715	862	0.0	
160	756	905	0.0	
170	797	948	0.0	
180	836	990	0.0	
190	876	1031	0.0	
200	915	1072	0.0	
210	954	1113	0.0	
220	993	1154	0.0	
230	1031	1194	0.0	
240	1069	1234	0.0	
250	1107	1274	0.0	
260	1144	1313	0.0	
270	1181	1352	0.0	
280	1218	1391	0.0	
290	1255	1430	0.0	
300	1292	1468	0.0	
310	1328	1507	0.0	
320	1364	1545	0.0	
330	1400	1582	0.0	
340	1436	1620	0.0	
350	1471	1658	0.0	
360	1507	1695	0.0	
370	1542	1732	0.0	
380	1577	1769	0.0	
390	1612	1805	0.0	
400	1647	1842	0.0	
410	1681	1878	0.0	
420	1716	1915	0.0	
430	1750	1951	0.0	
440	1784	1987	0.0	
450	1819	2023	0.0	
460	1853	2058	0.0	
470	1886	2094	0.0	
480	1920	2129	0.0	
490	1954	2165	0.0	
500	1987	2200	0.0	
510	2020	2235	0.0	

Additional Issue that makes it difficult to have universal correlation is the type of crude oil (chemistry)

Series of H.C	Paraffins	Olefines	Diolefines	Acetylene	Naphthene	Aromatic
Saturated/under saturated HC	Saturated HC	undersaturated	undersaturated	undersaturated	saturated (closed ring chains)	under saturated
General formula	C_nH_{2n+2}	C_nH_{2n}	C_nH_{2n-2}	C_nH_{2n-2}	C_nH_{2n}	C_nH_{2n-6}
Name of the lowest compunds in the series. (Geneva Terminology)	methane Ethane Propane	Ethylene Propylene butylen	butadiene	ethine (acetylen) propine butine	cyclopropan	Benzen methyl benzen (toluene)
Suffix/preffix = according to the Geneva system	.ane	.ene	.adiene	.ine	cyclo.	. Benzen
Structural Formula of the simplest compounds in the series.				$HC\equiv CH$		
Chemical stability (high/low)	high	low	low	low	high	high
Occurens in crude oil and Natural gas.	yes	yes.	very little	very little	yes	yes



These three families are different in their properties and therefore we see "regional" correlations

TPG 4145 SUMMARY / Q's & A's

Note Title 2010-11-23

GAS RATE EQUATIONS

GAS MATERIAL BALANCE EQS

RATE DECLINE EQS.

* * *

PVT

BASIC PHASE & VOLUMETRIC BEHAVIOR

COMPONENTS, PARTITIONING

FLASH CALCULATION

SATURATION PRESSURE CALC

PVT LABORATORY TESTS

BLACK-OIL PVT

* * *

GAS RATE EQUATIONS

- RESERVOIR (Darcy's or Forcheimmer)

$$v = \frac{k}{\mu}\frac{dp}{dx} \Rightarrow \frac{dp}{dx} = \frac{\mu}{k} v \; ; \qquad \text{Gases:} \quad \frac{dp}{dx} = \frac{\mu}{k} v + \beta \rho v^2$$

low-p: $p_R \lesssim 150$ bara

$$\Rightarrow \quad q_g = \frac{7.7\, kh \left(p_R^2 - p_{wf}^2\right)}{T_R (\mu_g Z_g)_R \left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s + D q_g\right]}$$

$$\left[\frac{Sm^3}{d}\right]$$

k[md] h[m] T_R[K] μ_g[cp] p[bara]

Darcy : $q_g = C_R\left(p_R^2 - p_{wf}^2\right)$; D = 0

Proxy (Forcheimmer) : $q_g = C_R\left(p_R^2 - p_{wf}^2\right)^n$

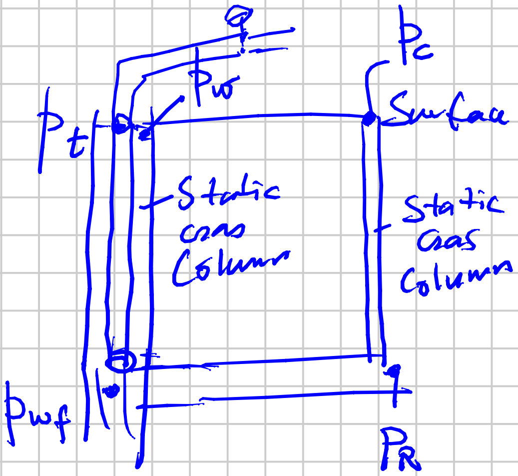
n = 1 : Darcy

$n = \frac{1}{2}$: Dq >> $\left(\ln\frac{r_e}{r_w} + s\right)$

$0.5 \le n \le 1$

TUBING RATE EQ

$$\boxed{q_g = C_T\left(p_w^2 - p_t^2\right)^{0.5}}$$ ✓



$$\frac{p_{@D}}{p_{@s}} \approx \text{constant } (1.x)$$

$$= \exp\left[S/2\right]$$

$S(D, \gamma_g)$ Static Gas Column term

$$\frac{p_R}{p_c} = \exp\left[S/2\right] = \frac{p_{wf}}{p_w}$$

Fetkovich:

$$\underset{\uparrow}{B_R} q_g^2 + \underset{\uparrow}{A_R} q_g = \left(p_c^2 - p_w^2\right)$$

"D", kh kh, s

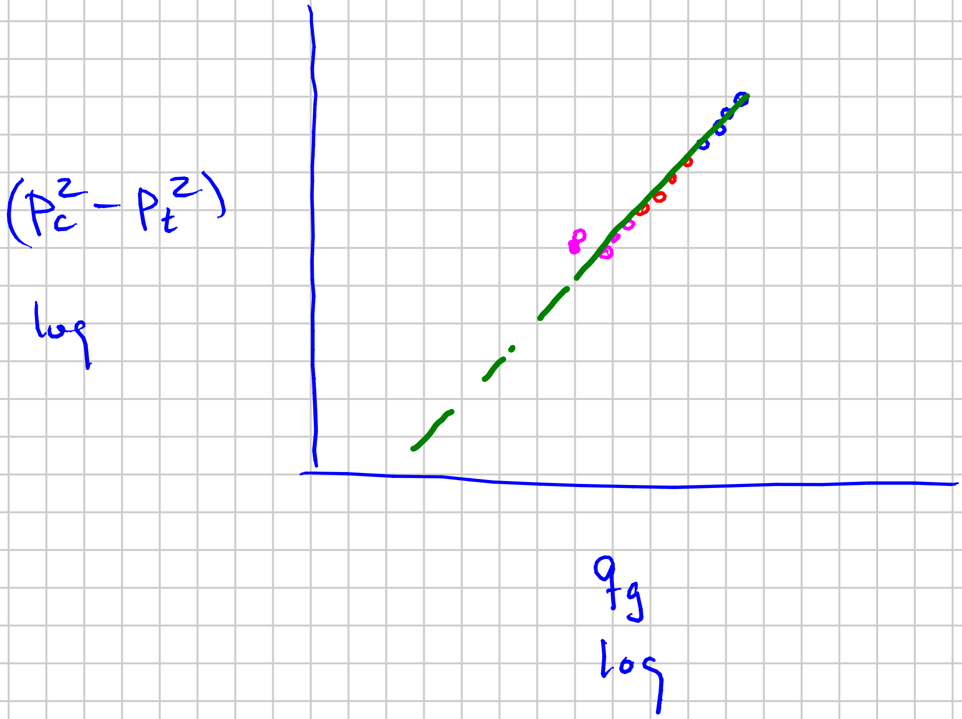
$$\frac{1}{C_T^2}\, q_{tg}^2 = (p_w^2 - p_t^2)$$

$$\boxed{\left(B_R + \frac{1}{C_T^2}\right) q_{tg}^2 + A_R q_{tg} = (p_c^2 - p_t^2)}$$

measureable
- inexpensively
- continuously

Healthy Gas wells

$$\left.\begin{matrix} B_R \\ C_T \\ A_R \end{matrix}\right\} = \text{cnst}$$



GAS MATERIAL BALANCE EQS

$$\frac{p_R}{Z_R} = \frac{p_{Ri}}{Z_i}\left(1 - \frac{G_p}{G}\right)$$

Assumption:
: $HCPV \cong$ constant

$\uparrow$

Gas Recovery Factor

G_p = Cum. Gas Prod

G = IGIP

$$Z_g\left(p_R,\ T_R,\ y_i\,(\gamma_g)\right) :$$

$$\{ y_i \to \gamma_g \} \Rightarrow T_{pc} \quad p_{pc}$$

$\uparrow$

$$\frac{T_R\,[K]}{T_{pc}\,[K]} = T_{pr}$$

Standing-Katz Chart

$Z_g(T_{pr},\ p_{pr})$

$$\frac{p_R}{p_{pc}} = p_{pr}$$

HCPV to reduce : Water expansion
PV compaction

$$\frac{p_R}{Z_R}\left[1 + c_e\,(p_{Ri} - p_R)\right] = \frac{p_{Ri}}{Z_{Ri}}\left(1 - \frac{G_p}{G}\right)$$

$\uparrow$

~ constant

c_e {Water Volume ; c_w ; c_f ; S_{wc}}

Mat. Bal. Eq. $p_R = f(G_p)$

RATE DECLINE EQS.

Arps - empirical

Fetkovich - Defined terms in Arps eqs. in terms of physical quantities from rate eqs. & mat. eq.

$$q = q_i \cdot \frac{1}{\left[1 + b \underset{\uparrow}{D} t\right]^{1/b}} \qquad \text{Hyperbolic}$$

NOT the same D as in Forcheimmer eq.!

$(\epsilon)\quad 0 \leq b < 1$

$$b = 0 : \quad q = q_i \exp\left[-Dt\right]$$

Apply for $\underline{p_{wf} = \text{cnst}}$ $\underset{\uparrow}{(t > 0)}$

from start of decline!

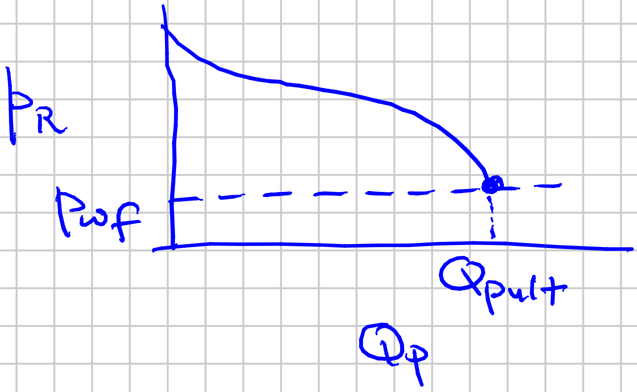
$\swarrow$ p_R @ $t = 0$

$$q_{gi} = \frac{7.7\,kh\,(p_R^2 - p_{wf}^2)}{T\mu z\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s + D\,q_{gi}\right]}$$

$$= q_g \text{ at } t = 0$$

$$D = \frac{1}{1-b} \cdot \frac{q_i}{Q_{p\,ultimate}}$$

$$\int_0^\infty q\,dt \;:\; p_R \to p_{wf}$$



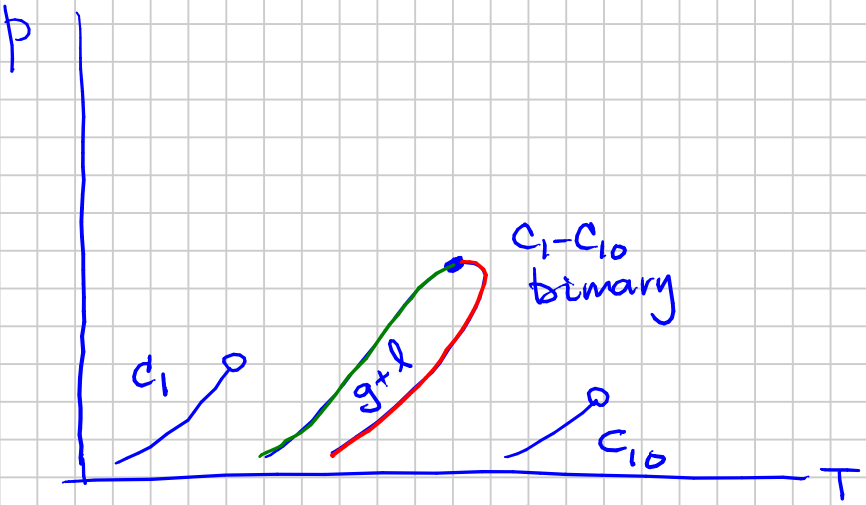
b : related to the "shape" of the mat. bal. relation

$0 < b < 0.5$; $b = 0.5$ for

Darcy low-p rate eq.

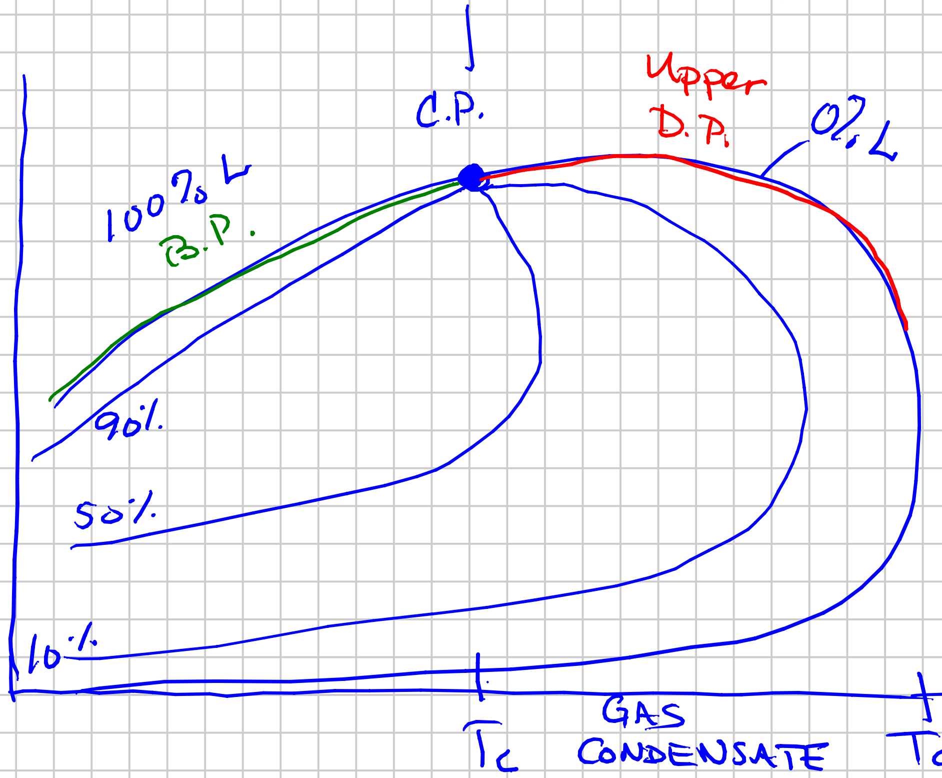
$$q_g = C_R(p_R^2 - p_{wf}^2)$$

BASIC PHASE & VOLUMETRIC BEHAVIOR



– Vapor pressure curve: gas-liquid equilibrium

$$K_i = \frac{p_{v_i}(T)}{p}$$



$p \lesssim 50$ bara

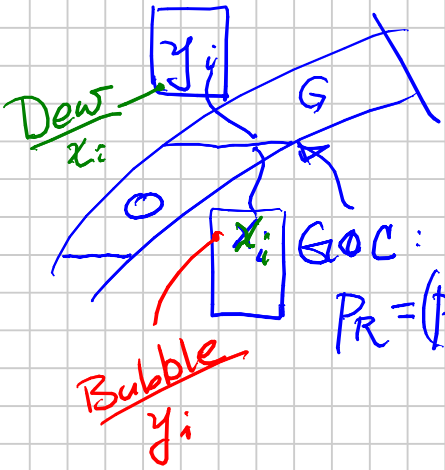
SURFACE PROCESSING

$\{z_i\}$

$T_R < T_c$: Oil Reservoir

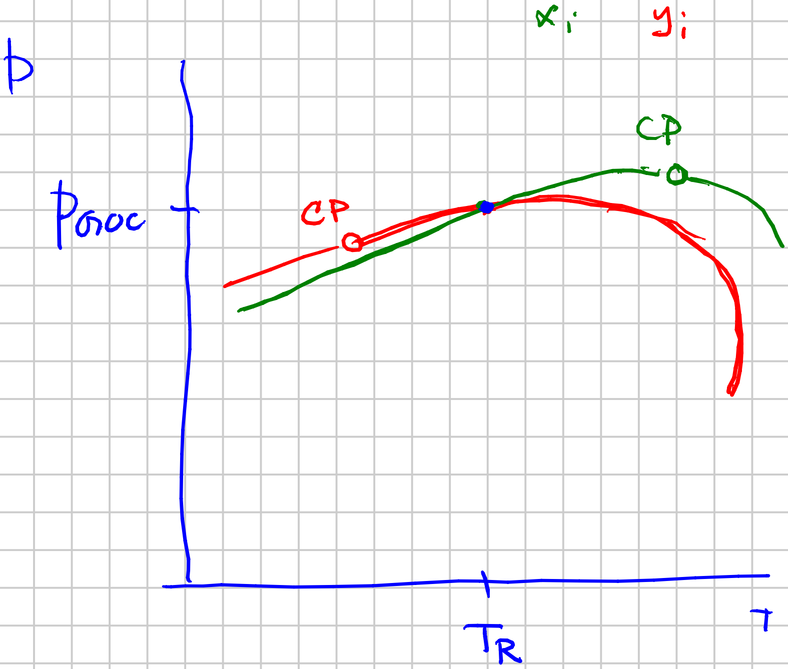
$T_R > T_c$: Gas Reservoir

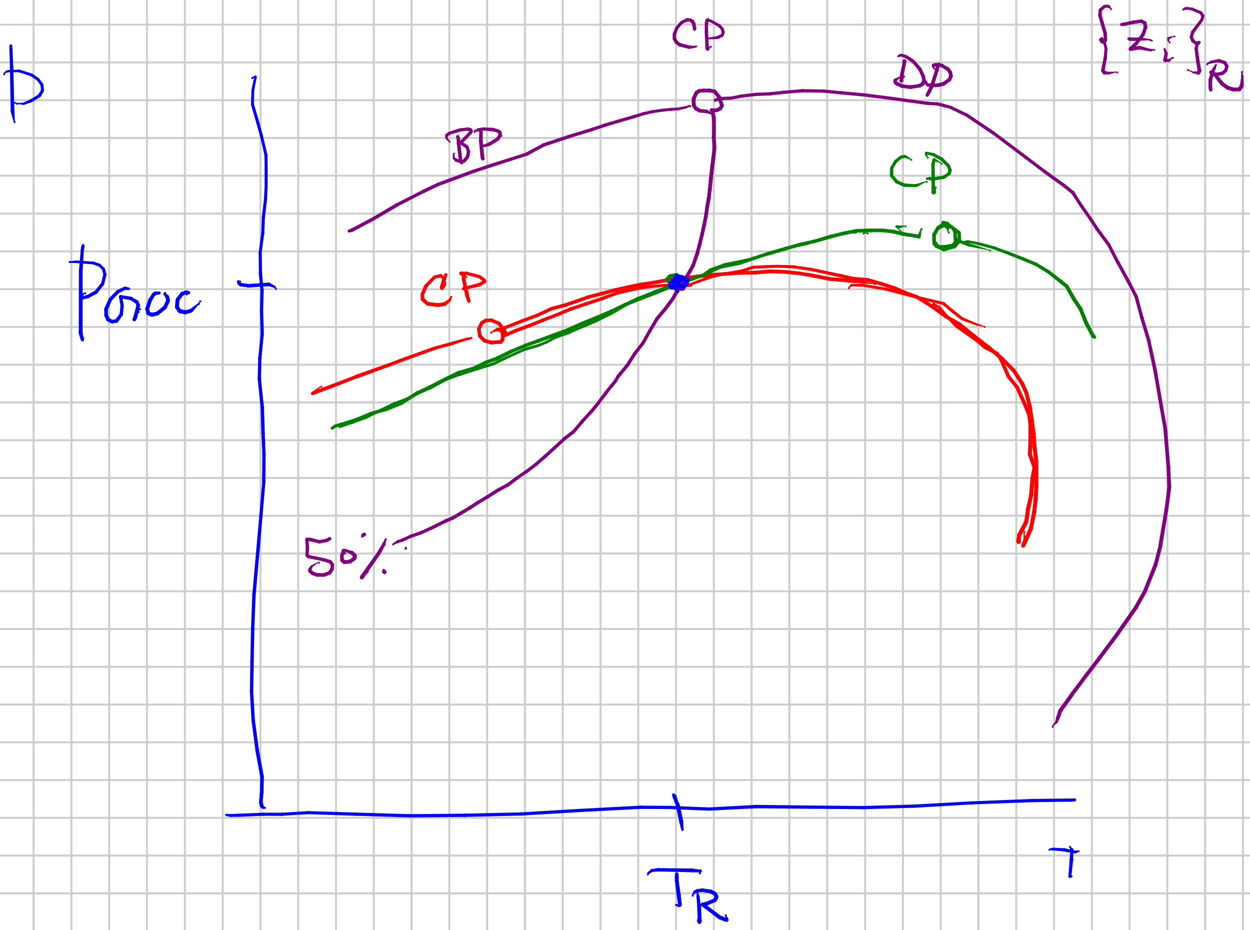
$$\frac{V_g}{V_S+V_o} = \frac{1}{2}$$



$\left.\right\} (z_i)_R$

GOC : $P_R = (p_b)_{GOC} = (p_d)_{GOC}$





Flash Calc. @ (P_{GOC}, T_R)

using
$$\left.\begin{aligned} z_i &= \boxed{y_i} \\ &= \boxed{x_i} \\ &= (z_i)_R \end{aligned}\right\} K_i\ ?$$

Gas: $Z_g(p,T)$

$$V_g = \left(\frac{nRT}{p}\right) Z(p,T)$$

$\uparrow$ Highly non-linear

Oils: $\rho_o(p, T, x_i)$

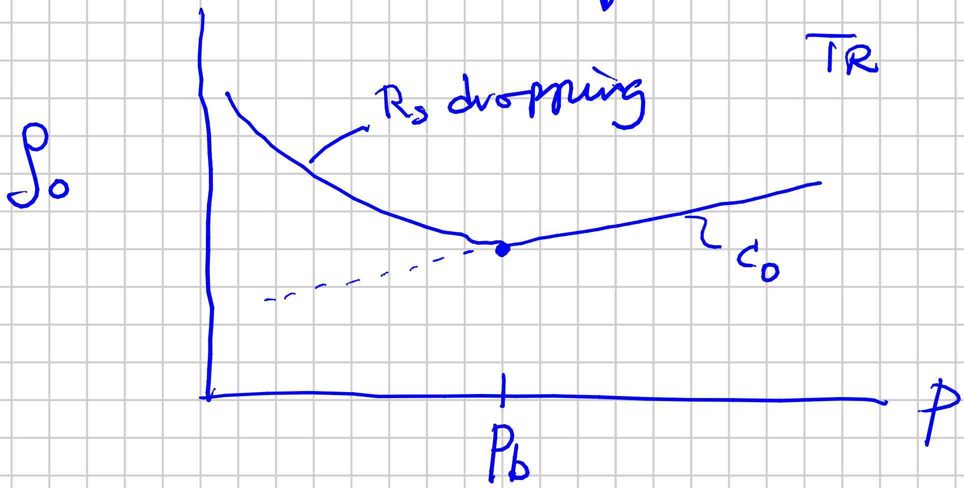
$\uparrow$

Surface (Solution) Gas amount $\Big\}$ R_s $\quad x_{C4-}$

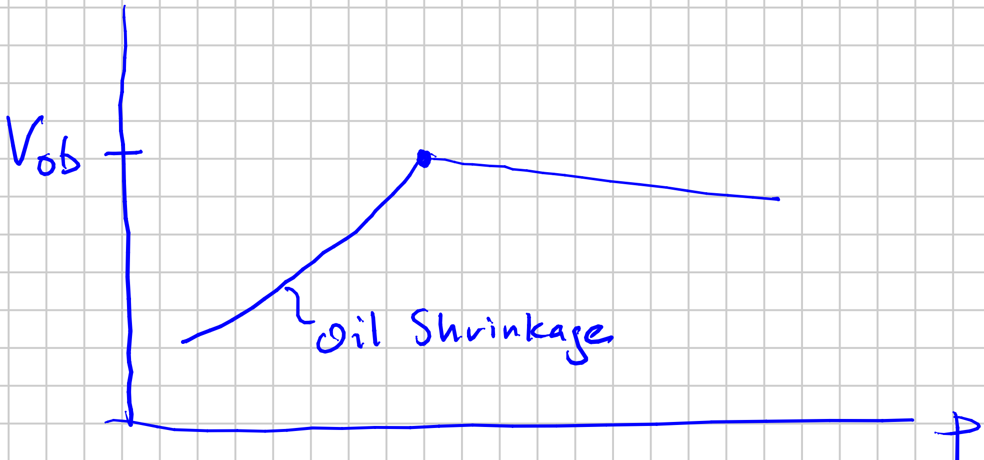
Surface Oil Gravity (density)

$\Big(c_o(R_s, \rho_{\bar{o}})$ $\quad \rho_{\bar{o}} \sim \rho_{C5+}$

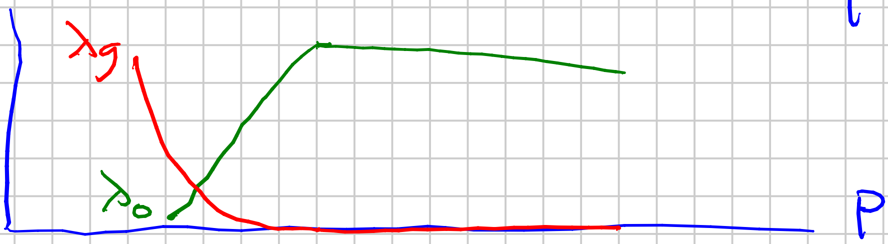
$\downarrow$



$S_o \sim V_o$
$= 1 - S_g$



$\frac{k_r}{\mu} = \lambda$

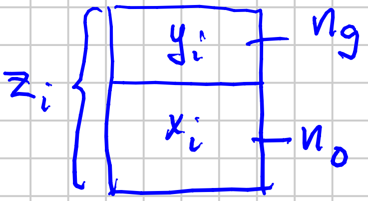


COMPONENTS, PARTITIONING

FLASH CALCULATION

SATURATION PRESSURE CALC

: Find p where $f_g \nearrow 1$, $\searrow 0$



$$\boxed{f_g = \frac{n_g}{n_g+n_o}}$$

$$\boxed{K_i = \frac{y_i}{x_i}}$$

$$\boxed{z_i = f_g \cdot y_i + (1-f_o)\, x_i}$$

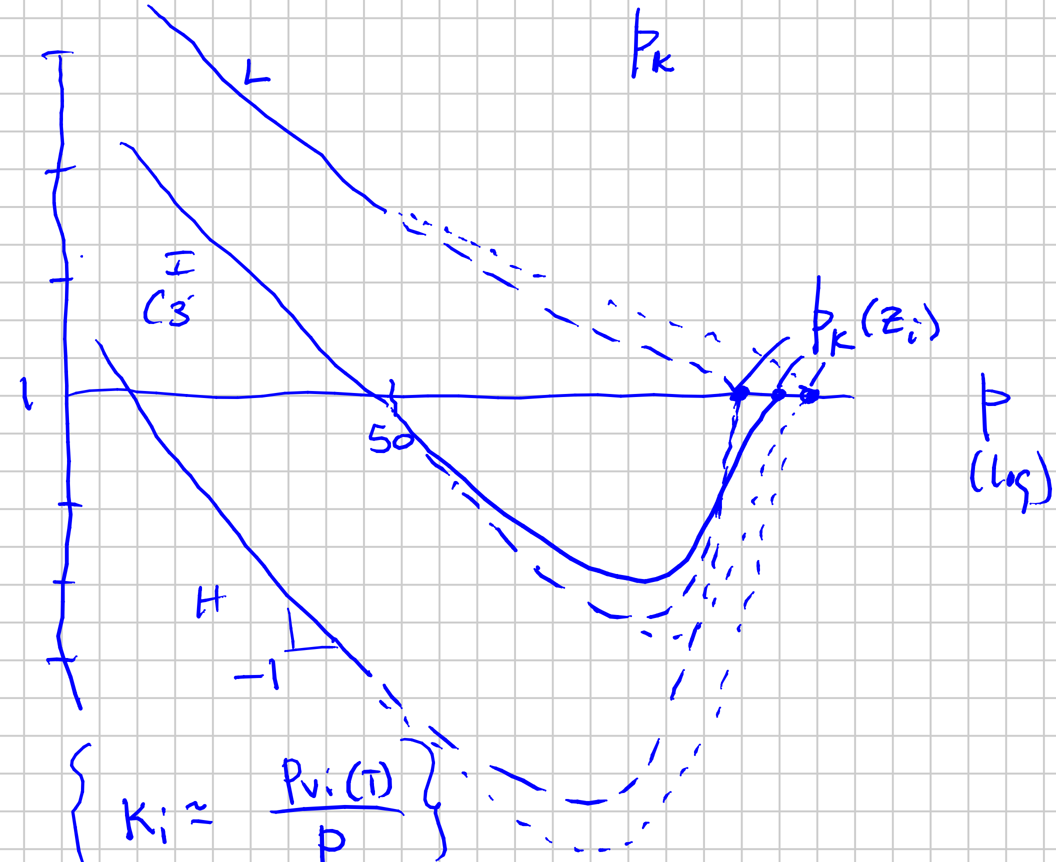
Given: p, T, z_i

Solve: f_g, y_i, x_i

$$\left\{\begin{matrix} \sum y_i = 1 \\ \sum x_i = 1 \end{matrix}\right\} \sum (y_i - x_i) = 0$$

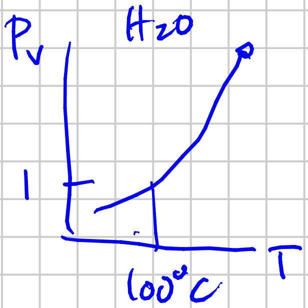
Assume or know $K_i(p, T, z_i)$

p_K



$$\left\{ K_i \simeq \frac{p_{vi}(T)}{p} \right\}$$

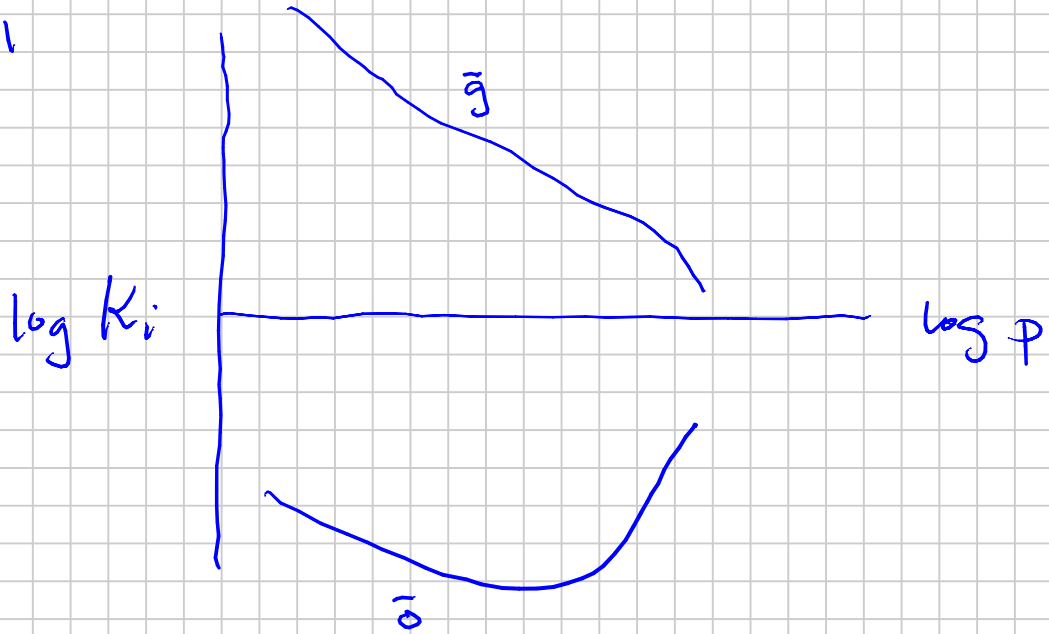
Wilson



Black Oil

$r_s(p) \to y_{\bar{o}}$

$R_s(p) \searrow x_{\bar{g}} \quad \sim$



$$r_s \Rightarrow \frac{n_{\bar{g}g}}{n_g}$$

$$R_s \Rightarrow \frac{n_{\bar{o}o}}{n_o}$$

PVT LABORATORY TESTS

CCE : $V_t(p)$: Oils

$\{V_o(p)\}$

$\Rightarrow \quad p_s \{p_b \text{ or } p_d\}$ @ $T_R \quad \Rightarrow$ Gas or Oil

$\Rightarrow \quad Z_g(p > p_d)$ and $\rho_o(p > p_b)$

Depletion Tests:

Oils: Differential Liberation

Gas Condensates: Constant Volume Depletion

Amount &
Phase Property Changes vs $p < p_s$

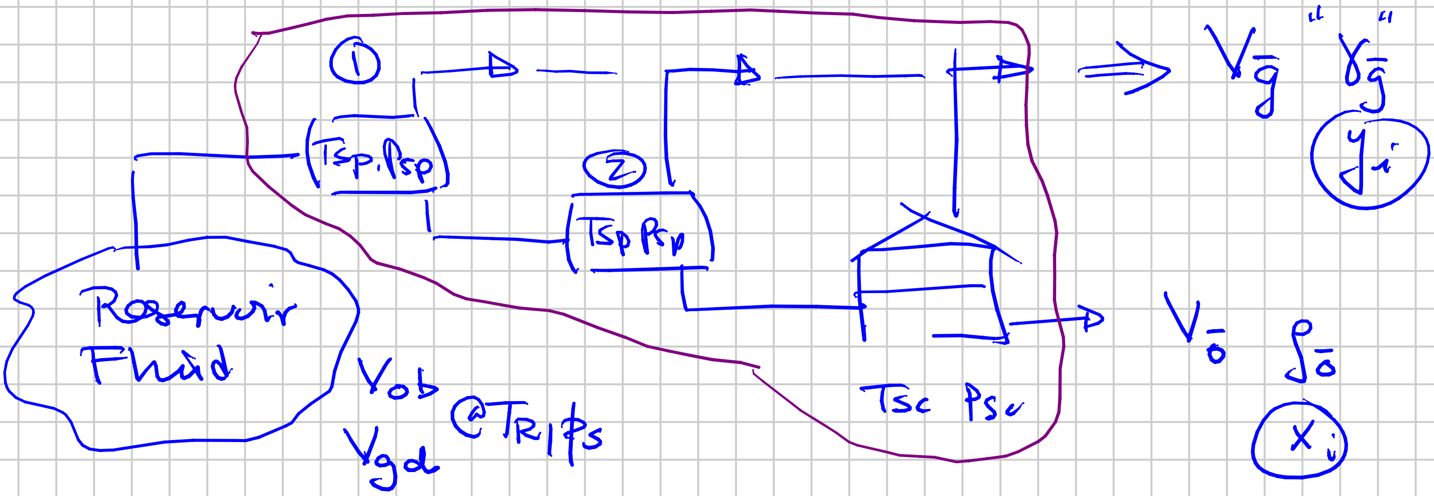
Amount

$\{n_g\ m_g\ V_g\}\ \rho_g \qquad \left\{\begin{matrix}\mu_g \\ \mu_o\end{matrix}\right\} \qquad y_i$

$\{n_o\ m_o\ V_o\}\ \rho_o \qquad\qquad\qquad x_i\ (R_s)$

$\boxed{\underline{p}}$

SEPARATOR TEST:



OIL: $R_s = \dfrac{V_{\bar{g}}}{V_{\bar{o}}}$ | GAS: $r_s = \dfrac{V_{\bar{o}}}{V_{\bar{g}}}$

$B_{ob} = \dfrac{V_{ob}}{V_{\bar{o}}}$ | $B_{gd} = \dfrac{V_{gd}}{V_{\bar{g}}}$

BLACK-OIL PVT

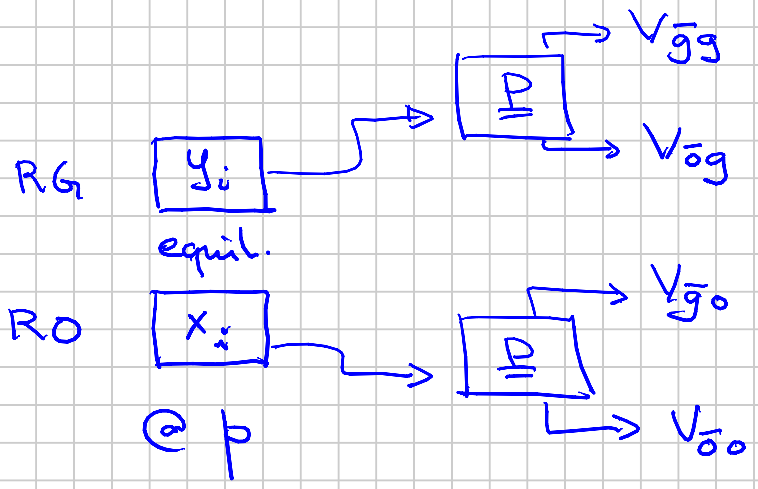
Two "Components"

$\bar{g}$ surface gas ~ C_{5-}

$\bar{o}$ surface oil ~ C_{6+}

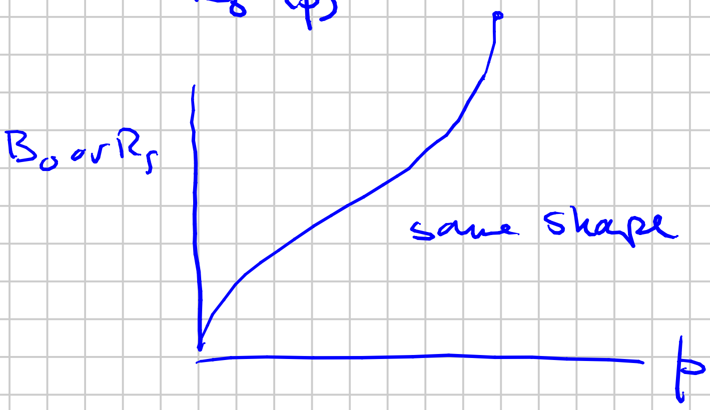
r_s (p)

B_{gd} (p)



R_s (p)

B_o (p)



RG $$\rho_g(p) = \frac{\rho_{\bar{g}} + \rho_{\bar{o}}\, r_s(p)}{B_{gd}(p)}$$

RO $$\rho_o(p) = \frac{\rho_{\bar{o}} + \rho_{\bar{g}}\, R_s(p)}{B_o(p)}$$

$\rho_{\bar{g}}$

$\rho_{\bar{o}}$ constants

low-p $K_i \cong \frac{p_{vi}(T)}{p}$

$T_R = 200\,°F$ (660 °R) $\leftarrow \sim (T_b)_{C_7}$

→ $p_{sp} = 40$ bar

C_7 $p_v(T_b) \equiv 1$ atm $= 1$ bara

$K_{i,est} \approx \frac{1\text{ bar}}{40\text{ bar}} = 0.025$

Quiz 3

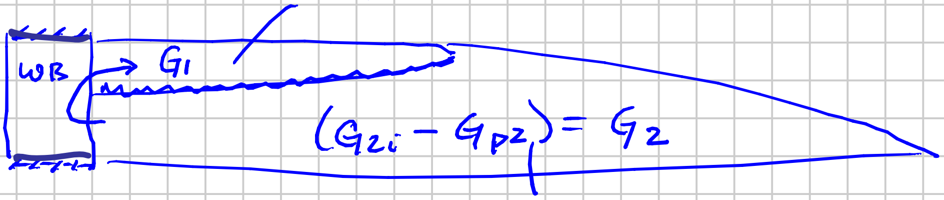
Q.5

$G_{1i} = 10^6\ Sm^3$

$C_{R1} = 31\ \frac{Sm^3/d}{bar^2}$

$G_1 = G_{1i} - G_{p1}$

$p_{c1} = 20$ bara



$(G_{2i} - G_{p2}) = G_2$

$p_{c2} = 28$ bara

$C_{R2} = 11\ \frac{Sm^3/d}{bar^2}$

Max Volume 2→1: $\overbrace{G_2 - G_2^{\infty}}$

$(\underline{p_{c2}} \to \underline{p_c^{\infty}})$

$G_{2i} = 3 \cdot 10^6\ Sm^3$

$t \to \infty$

$p_{c1}^{\infty} = p_{c2}^{\infty} = p_c^{\infty}$

$$p_c^{\infty} = \left(\frac{HCPV_1 \cdot p_{c1}^{-1} + HCPV_2\ p_{c2}^{-1}}{HCPV_1 + HCPV_2} \right)^{-1} \Leftarrow$$

$$\boxed{G = G_1 + G_2}$$

$$\boxed{G = G_1^{\infty} + G_2^{\infty}}$$

$$\boxed{G_1 = \frac{HCPV_1}{P_{c1}} \qquad G_2 = \frac{HCPV_2}{P_{c2}} \qquad G = \frac{HCPV_1 + HCPV_2}{P_c^{\infty}}}$$

$t = \infty$

$$G = \frac{HCPV}{B_g}$$

$$B_g = \frac{P_{sc}}{T_{sc}} \frac{z T_R}{p_R}$$

$$= \left\{ \frac{P_{sc}\,(1)\,T_R}{T_{sc}} \right\} \frac{1}{p_R} = \left\{ \frac{P_{sc}\,(1)\,T_R}{T_{sc}} \exp(S/2) \right\} \frac{1}{P_c}$$

constant for 1, 2

$$G_i P_c = P_{ci} \left(1 - \frac{G_p}{G_i}\right) \cdot G_i \Rightarrow P_{ci}(G_i - G_p) = G_i P_c$$

$$G_i - G_p = \boxed{\underset{\substack{\uparrow \\ \text{remaining} \\ @\, P_c}}{G} = G_i \frac{P_c}{P_{ci}}}$$

Max $2 \to 1$ $\Delta t_{S1} = \infty$

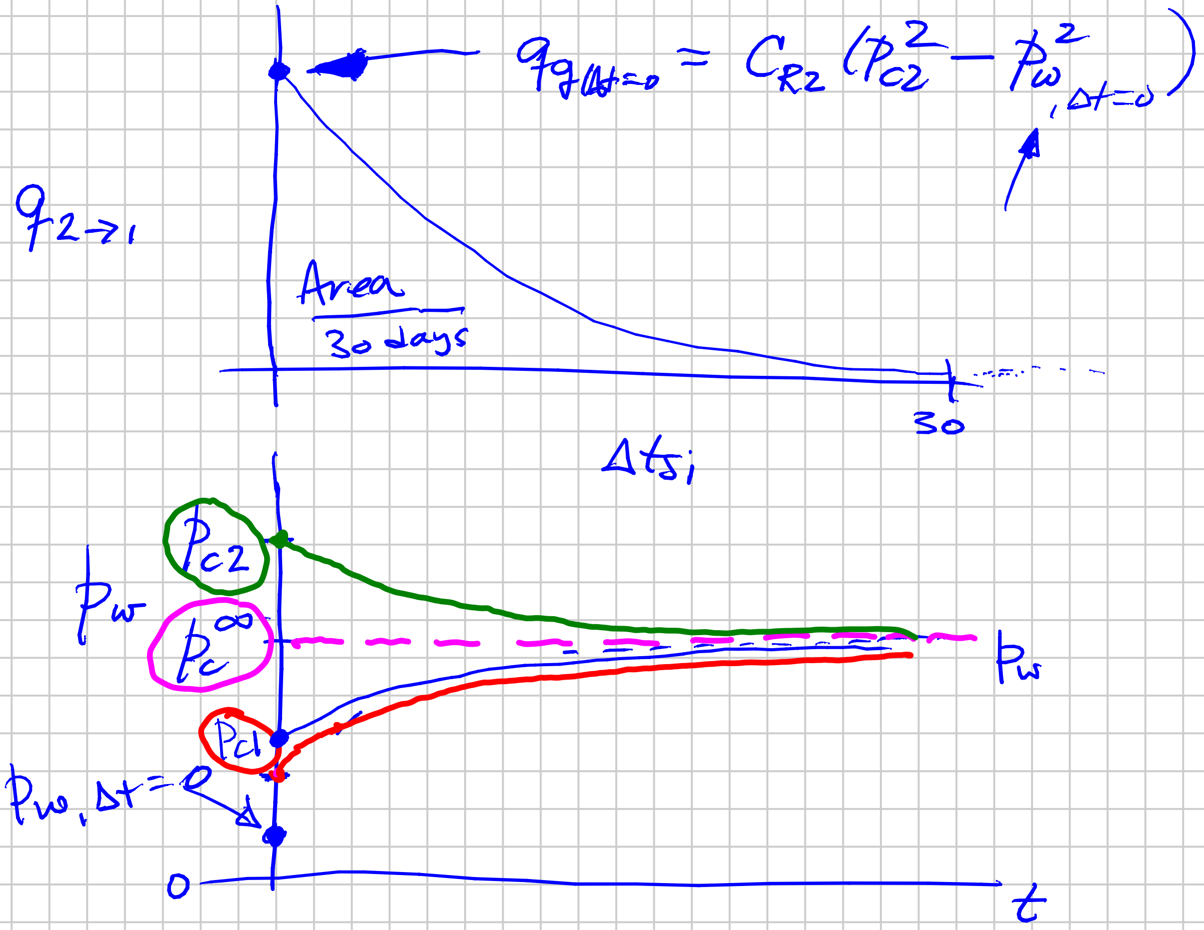
$$P_{c2} \to P_c^{\infty}$$

$$\Delta G_2 = \left[\left(G_1 \cdot \frac{P_{c2}}{P_{ci}} \right) - \left(G_i \cdot \frac{P_c^{\infty}}{P_{ci}} \right) \right]$$

$$= G_i \left(\frac{P_{c2}}{P_{ci}} - \frac{P_c^{\infty}}{P_{ci}} \right) = \frac{\overset{\checkmark}{G_i}}{\underset{\checkmark}{P_{ci}}} \left(\overset{\checkmark}{P_{c2}} - \overset{\downarrow}{P_c^{\infty}} \right)$$

Sm^3

$$\bar{q}_{max,2} = \boxed{\frac{\Delta G_2}{30\ days}} = -\bar{q}_{max,1}$$



At $\Delta t = 0$

$$q_{g2} = -q_{g1}$$

$$C_{R2}(p_{c2}^2 - p_{w,0}^2) = C_{R1}(p_{c1}^2 - p_{w,0}^2)$$

$\Rightarrow$ Solve $p_{w,0} = \Delta t$

Black Oil PVT

- ~~Traditional~~ (1980) $r_s = 0$
- Modified (1980)   :   Traditional (2010)

Using $r_s = 0$ OK

- If you get the ✓ "same" oil and gas recoveries and well performance using $r_s = 0$ or $r_s(p)$ $\parallel \neq$

$N_{pult} = 100 \cdot 10^6$ STB   15% IOIP

$G_{pult} = 80 \cdot 10^6$ Mscf   80% IGIP   {Out of Solution}

$(\ \bar{r}_s = 10 \text{ STB}/10^3 \text{ Mscf})$

$$(\Delta N_p)_{Cond.} = 80 \cdot 10^6 \text{ Mscf} \times 10 \frac{\text{STB}}{10^3 \text{ Msf}}$$

$$= 800 \times 10^3 \text{ STB}$$

$$\sim 0.8 \cdot 10^6 \text{ STB}$$

(~1% extra oil)

2006 Prob 2e

Sep. Test : $(y_{7+})_{sp}$ 0.24 mol-%

$(x_{7+})_{sp}$ 33.12 mol-%

$\Big\}(K_{7+})_{sp} = 0.0072$

@ 60 barg
50 °C

low-p : $K_i \cong \dfrac{p_{vi}(T_{sp})}{p}$

$$0.0072 = \frac{p_{vC_{7+}}(T_{sp})}{p_{sp}}$$

$$p_{vC_{7+}}(T_{sp}) = 0.0072(60)$$

$$= 0.43 \text{ bara}$$

1 atm $K_{C_{7+}}(T_{sp}) = \dfrac{p_{vC_{7+}}(T_{sp})}{1 \text{ bara}} = 0.43$