

Volumetric Gas Flow Rate

$$q_{fg}(t)$$

Production Forecast

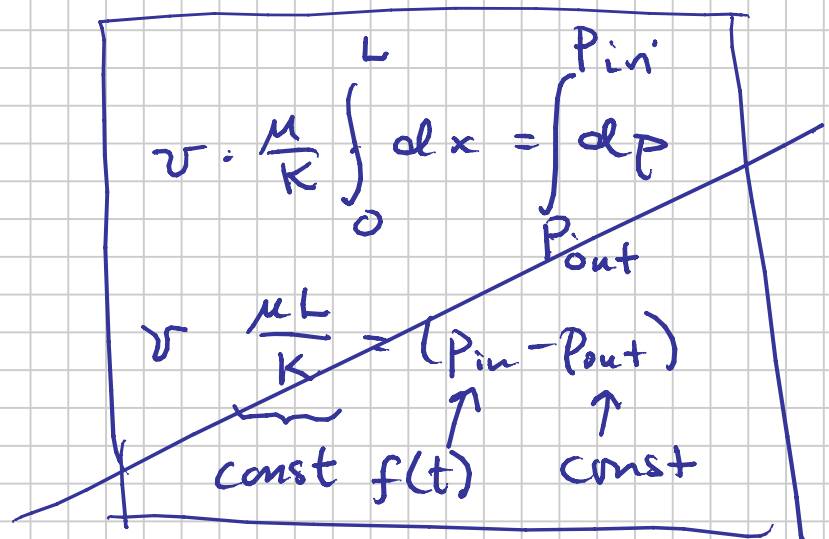
Darcy's Law

EQ: 
$$v = -\frac{k}{\mu} \left( \frac{dp}{dx} \right)$$

Ideal Gas Law

$$pV = nRT$$

const  $\left\{ \begin{array}{l} k: \text{permeability (D or md)} \\ \mu: \text{viscosity (cp} \equiv \text{mPa}\cdot\text{s)} \end{array} \right.$



Local Flow rate  $q$  is (at  $p$ )

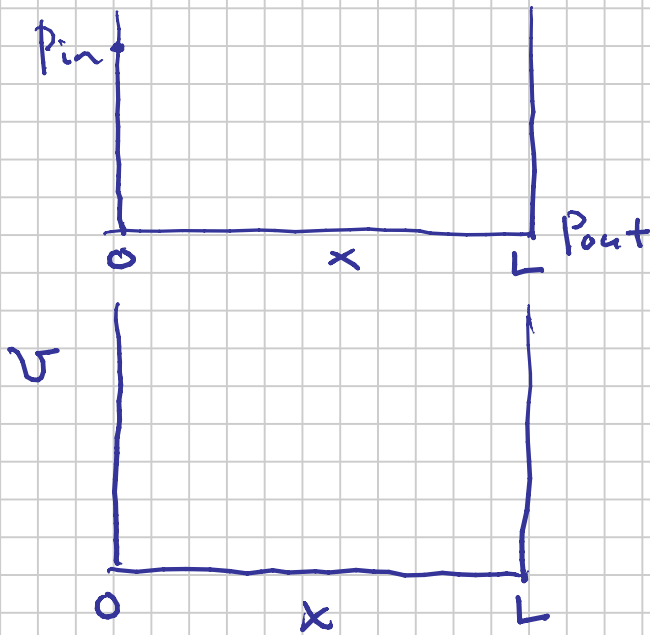
$$q = v \cdot A_{\perp} \quad \text{m}^3/\text{s}$$

$$= v \cdot \pi r^2 \quad \text{bbt/D}$$

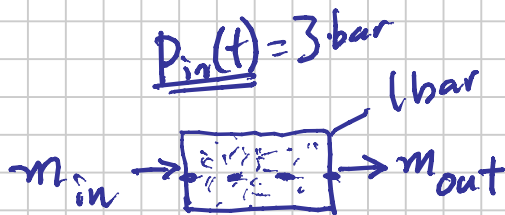
Convert to Surface Flow rate

q<sub>g</sub> at ambient conditions

1 atm 20°C

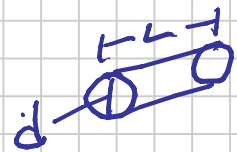


$$v = \left(\frac{k}{\mu}\right) \cdot \underbrace{\frac{dp}{dx}}_{\approx \frac{p_{in} - p_{out}}{L}}$$



$m_{in} = m_{out}$  : Steady State Flow Condition

$$\dot{m}_{in} = v_{in} \cdot A_{\perp} \cdot \rho(p_{in})$$



$$\dot{m}_{out} = v_{out} \cdot A_{\perp} \cdot \rho(p_{out})$$

$$\dot{m}(x) = v(x) A_{\perp} \cdot \rho_g(p(x)) = \text{constant}$$

mass flow rate

$$\left[ \frac{k}{\mu} \frac{dp}{dx} \right] \cdot \frac{\pi r^2}{A_{\perp}} \cdot \frac{pM}{RT} = \text{constant} \left( \frac{kg}{s} \right)$$

$\frac{m_g}{V_g} = \rho_g(p)$  : Equation of State P-V-T

$$pV = nRT$$

$n$  = moles, kmol  
 $P$  = pressure, Pa  
 $V$  = volume,  $m^3$   
 $T$  = temperature, K  
 $R$  =

$$\frac{n}{V} = \frac{P}{RT} \quad \frac{\text{kmol}}{m^3}$$

$$\rho_g = \frac{m}{V} = \frac{P}{RT} \cdot \left(\frac{m}{n}\right) \quad \frac{\text{kg}}{m^3} = \frac{PM}{RT}$$

$\uparrow$   
 $M$  [molecular weight]  
 molar mass

$$M_{C_1} = 16.04 \frac{\text{kg}}{\text{kmol}}$$

$$\frac{k}{\mu} \frac{dp}{dx} \cdot \frac{\pi r^2}{A_c} \cdot \frac{PM}{RT} = \text{constant} \left(\frac{\text{kg}}{s}\right)$$

$$\dot{m} = q_m = \text{const} = \underbrace{\frac{k \pi r^2}{\mu} \left(\frac{M}{RT}\right)}_{\text{const}} \cdot P \frac{dp}{dx}$$

$$q_m \cdot \int_0^L dx = \frac{\pi r^2 k}{\mu} \left(\frac{M}{RT}\right) \int_{P_{out}}^{P_{in}} P dp$$

$$q_m = \left(\frac{\pi r^2 k}{L}\right) \left(\frac{M}{\mu RT}\right) (P_{in}^2 - P_{out}^2)$$

Geometry Rock  
Property

Fluid  
Property  
&  
State

Nightmare: UNITS

$p_R$  drops as a function of  
cumulative mass (moles)  
of gas removed.

$q(t)$



# Darcy's Law & Units

d'Arey

$$v = \frac{k}{\underbrace{\mu}_c} \frac{dp}{dx}$$

## cgs Units

$$v \text{ [cm / s]}$$

$$p \text{ [atm]}$$

$$x \text{ [cm]}$$

$$\mu \text{ [cp]}$$

↑ centi Poise

$$k \text{ [D]}$$

↑ "Darcy"

## Pure SI Units

$$v \text{ [m / s]} \leftarrow \text{Correct!}$$

$$p \text{ [Pa]}$$

$$x \text{ [m]}$$

$$\mu \text{ [Pa}\cdot\text{s]}$$

$$k \text{ [m}^2\text{]}$$

Can be shown:

$$\left\{ \begin{array}{l} 1 \text{ D} \approx 10^{-12} \text{ m}^2 = 1 \mu\text{m}^2 \\ 1 \text{ md} = 10^{-3} \text{ D} \\ 1000 \text{ md} = 1 \text{ D} \end{array} \right. \begin{array}{l} 1 (\mu\text{m})^2 \\ 1 (10^{-6} \text{ m})^2 \\ 10^{-12} \text{ m}^2 \end{array}$$

Markus Hays Lab Exp Aug. 17, 2008

Core 1

$L = 5 \text{ cm}$  (2")

$d = 3.8 \text{ cm}$  (1.5")

Core 2

Gage — barg bara bar

$P_{in}$ [atm]	$P_{out}$ [atm]	$q$ [L/min]
2.01	1.87	3.0
0.98	0.90	1.5
0.36	0.325	0.75

Calculate  $k$  assuming a simplified Darcy equation.

$$v = \frac{k}{\mu} \frac{\Delta P}{\Delta x} \approx \frac{k}{\mu} \frac{\Delta P}{L}$$

$M_{N_2}$  @ Low P  $\sim 0.012 \text{ cp}$

Convert L/min  $\rightarrow \text{cm}^3/\text{s}$

$$\left( \frac{q}{\text{min}} \right) \cdot \frac{1000 \text{ cm}^3}{\text{L}} \cdot \frac{\text{min}}{60 \text{ s}}$$

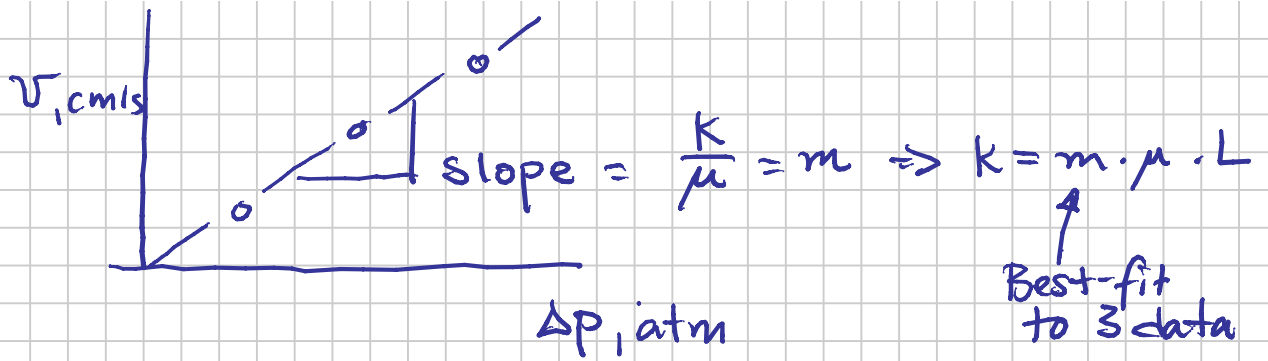
$\underbrace{\hspace{10em}}_1 \quad \underbrace{\hspace{10em}}_1$

$q \rightarrow v = q/A$

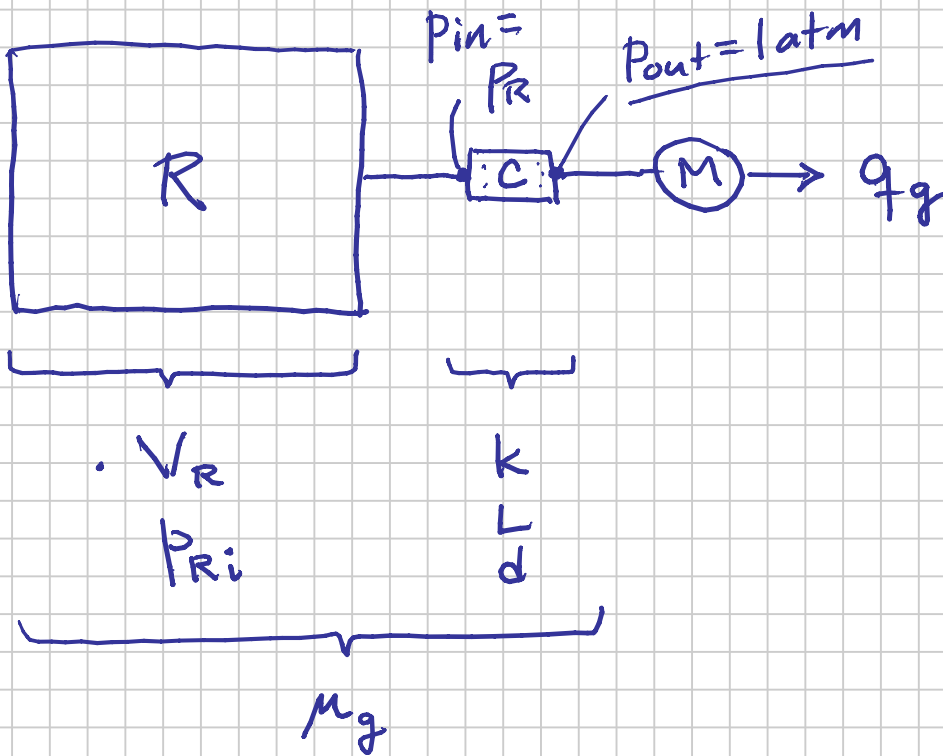
$$k \approx v \frac{L}{\Delta P} \mu$$

$$v = \frac{k}{\mu L} \cdot \Delta P$$

$k = m \cdot \mu \cdot L$

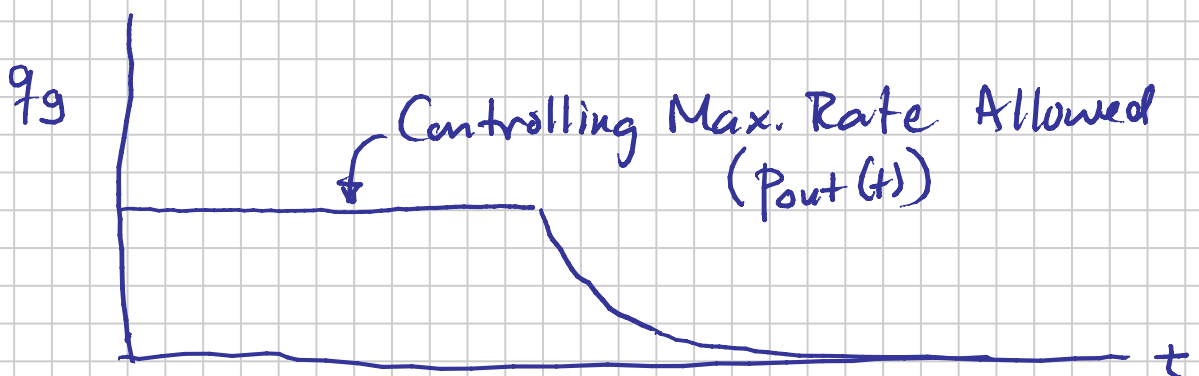
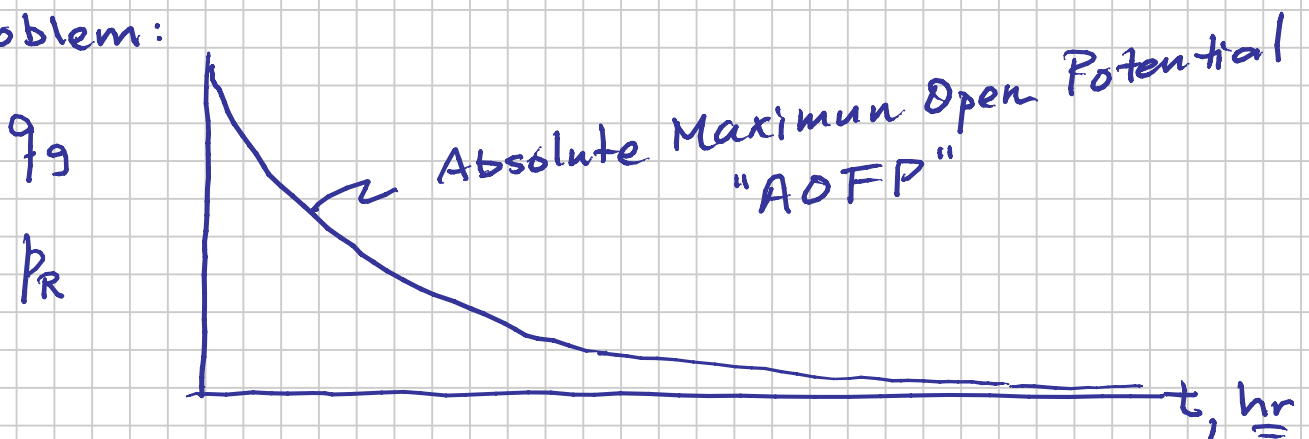


## Reservoir Flow Analogue



R: Reservoir  
 C: Core ("Rock")  
 M: Meter

Problem:



# Gas Rate (Field & Lab)

Volumetric Rates at "Standard Conditions" (SC)

$$\underline{\text{SPE}}: \left. \begin{aligned} P_{sc} &= 1 \text{ atm} = 14.696 \text{ psia} \\ &= 1.0135 \text{ bara} \end{aligned} \right\}$$

$$T_{sc} = 60^\circ\text{F}$$

$$15.56^\circ\text{C}$$

$$14.50377 \frac{\text{psi.}}{\text{bar}}$$

Relationship between S.C. gas volume and moles of gas "n"

$$\text{Ideal Gas Law: } p_{sc} V_{sc} = n R T_{sc}$$

$$\Rightarrow \left( \frac{V_{gsc}}{n} \right) = \frac{R T_{sc}}{P_{sc}} \quad \leftarrow \text{abs. (K or } ^\circ\text{R)}$$

$$\text{SI Units } \left\{ \begin{array}{l} \text{m}^3, \text{ kmol}, \text{ K}, \text{ bara} \\ \text{Kg-mol} \end{array} \right\}$$

$$\underline{\text{Sm}^3} \quad \text{m}^3 \text{ of gas at s.c.}$$

std m<sup>3</sup> (SPE)

$$R = 0.083143$$

$$T_{sc} = 15.56 + 273.15$$

$$P_{sc} = 1.0135 \text{ bara}$$

ANY GAS:

$$\frac{V_{gsc}}{n} \frac{[\text{Sm}^3]}{\text{Kg-mol}} = \frac{R T_{sc}}{P_{sc}} = \underline{\underline{23.68}} \frac{\text{Sm}^3}{\text{Kg-mol}}$$
$$= 379. \frac{\text{scf}}{\text{lb-mol}}$$

TABLE A-2—UNIVERSAL GAS CONSTANT FOR DIFFERENT UNITS

Pressure Unit	Volume Unit	Temperature Unit	Mass (mole) Unit	Gas Constant $R$
psia	ft <sup>3</sup>	°R	lbm	10.7315
psia	cm <sup>3</sup>	°R	lbm	303,880
psia	cm <sup>3</sup>	°R	g	669.94
bar	ft <sup>3</sup>	°R	lbm	0.73991
atm	ft <sup>3</sup>	°R	lbm	0.73023
atm	cm <sup>3</sup>	°R	g	45.586
Pa	m <sup>3</sup>	K	kg	8314.3
Pa	m <sup>3</sup>	K	g	8.3143
kPa	m <sup>3</sup>	K	kg	8.3143
kPa	cm <sup>3</sup>	K	g	8314.3
bar	m <sup>3</sup>	K	kg	0.083143
bar	cm <sup>3</sup>	K	g	83.143
atm	m <sup>3</sup>	K	kg	0.082055
atm	cm <sup>3</sup>	K	g	82.055

Reservoir "Tank"

$$P_R V_R = n_R (R T_R)$$

Assume  $V_R = \text{const. w.r.t. time}$

$$R T_R = \text{--- " ---}$$

$$P_R(t) = \underbrace{\left( \frac{R T_R}{V_R} \right)}_{\text{const}} \cdot \underbrace{n_R(t)}_{\text{changing}}$$

$$n_R(t) = n_{Ri} - \int_0^t q_g \cdot \left( \frac{R T_{sc}}{P_{sc}} \right)^{-1} dt$$

"Material Balance"

e.g. 23.68

$$* \quad n_R(t) = n_{Ri} - \left( \frac{R T_{sc}}{P_{sc}} \right)^{-1} \int_0^t q_g dt$$

$$q_g = \left( \frac{RT_{sc}}{P_{sc}} \right) \dot{n}_g$$

↑  
molar gas rate

Steady State Flow Assumption

$$\dot{n}_{gin} = \dot{n}_{gout} = \dot{n}_g(x)$$

$$\dot{n}_g = \underbrace{v_g \cdot A}_{m^3/s} \cdot \left( \frac{\rho_g}{M_g} \right) c(x)$$

$$v_g = \frac{k}{\mu_g} \frac{dp}{dx} \quad \text{Darcy}$$

Gas Law

$$\frac{\rho_g}{M_g} = \left( \frac{n}{V} \right) c = \left( \frac{P}{RT} \right) c = \left( \frac{1}{RT} \right) \cdot \underset{\uparrow}{P(x)}$$

$$q_g = \left( \frac{RT_{sc}}{P_{sc}} \right) \cdot \frac{k}{\mu_g} \frac{dp}{dx} \cdot A \cdot \frac{1}{RT} P$$

$P_{in}$

0

$P_{out}$

L

$$q_g = \left( \frac{1}{2} \frac{T_{sc}}{P_{sc} T_R} \right) \frac{A}{L} \left( \frac{1}{\mu_g} \right) k (P_{in}^2 - P_{out}^2)$$

Constant	G	PVT	Rock	Driving Potential
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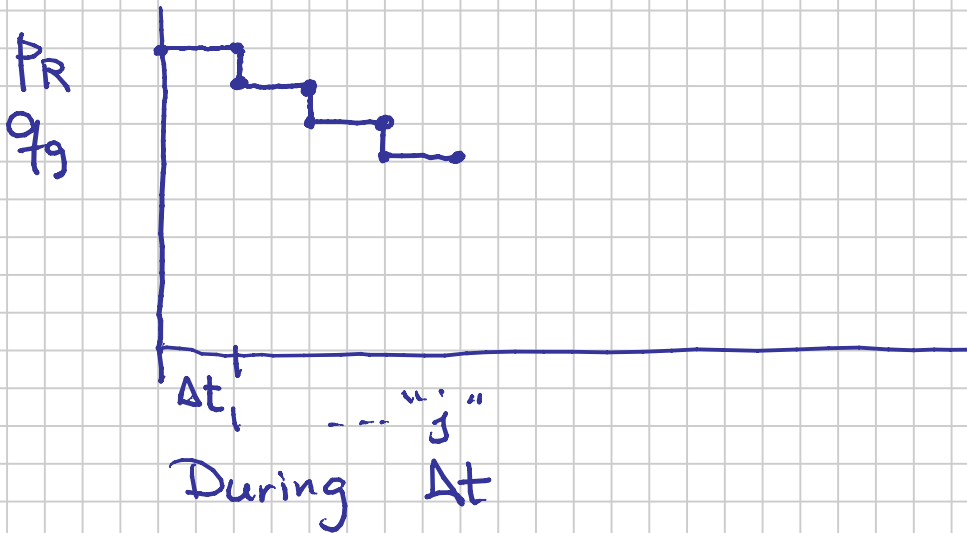
↑  
geometry

$$q_g = \underset{\uparrow}{C} (P_{in}^2 - P_{out}^2)$$

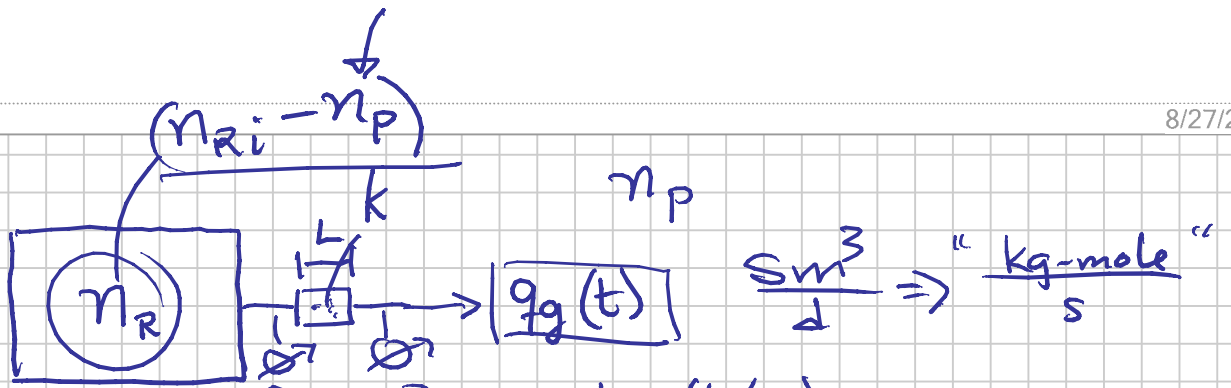
$$P_R = P_{in}$$

Solution Strategy:

Time-lag Solution (Direct @ each Time Step) T.S.



- (1) We know current  $P_{R,j}$
- (2) Calculate  $q_g = C (P_{R,j}^2 - P_{out}^2)$   
↑ ↑  
const
- (3) Calculate remaining moles gas in R at end T.S.
- (4) Calculate the new  $P_R$  for next T.S.  $j+1$
- (5) Go to (1)



constant = "10 L"  $P_R$   $P_{sc} = 1 \text{ atm (1 bar)}$   
10 bara  
 15.56 °C

(1)  $p \check{V} = n \check{R} \check{T}$   
 Ideal Gas Law

(2)  $v = \frac{k}{\mu} \frac{\Delta p}{\Delta x}$   
 L

$\Rightarrow q = C (P_R^2 - P_{sc}^2)$  (2)

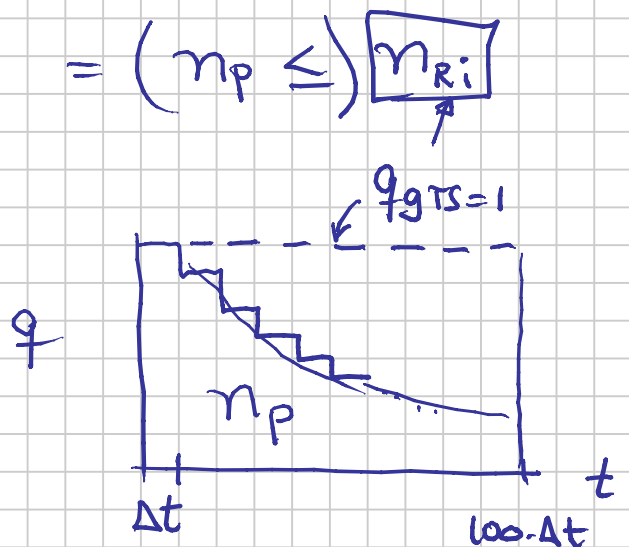
Problem:

\*  $\Delta t$  too big  $\Rightarrow$  garbage

Limit

100 TS is "default"

$\left\{ \frac{1}{\left( \frac{RT_{sc}}{P_{sc}} \right)} (q_g)_{TS=1} \right\} \times \Delta t \times 100 = (n_p \leq) n_{Ri}$   
 kg-mole/s                      s total





Recommended Estimate  $\Delta t \sim \frac{n_{Ri}}{\left\{ \sum_{i=1}^{n_{Ri}} (Q_{s,i}) \right\} \cdot 100}$

Ultimate Recovery =  $\frac{n_p}{n_{Ri}} = \frac{n_{Ri} - n_R}{n_{Ri}}$   
 $= 1 - \frac{n_R}{n_{Ri}}$

Ideal Gas Law

$$= 1 - \frac{P_R}{P_{Ri}}$$

$$= 1 - \frac{1}{10}$$

$$= 90\%$$

$$x \frac{\text{cm}^3}{\text{s}} \cdot \frac{1}{\text{Pa}^2} \cdot \frac{35.31 \text{ scf}}{\text{cm}^3} \cdot \frac{3600 \cdot 24 \text{ s}}{D} \cdot \left( \frac{10^5 \text{ Pa}}{\text{bar}}, \frac{\text{bar}^2}{14.5 \text{ psia}} \right) \sim \frac{\text{ft}^3}{\text{m}^3}$$

$\Rightarrow$

$$2.24 \frac{\text{lb}}{\text{kg}}$$

# "Gas Law" $p-V-T$ (PVT)

of "Real" Gases ( $N_2, CO_2, H_2S, C_1, C_2, C_3, C_4, \dots$ )  
 at "Real" Conditions (States), "high"  $p, T$

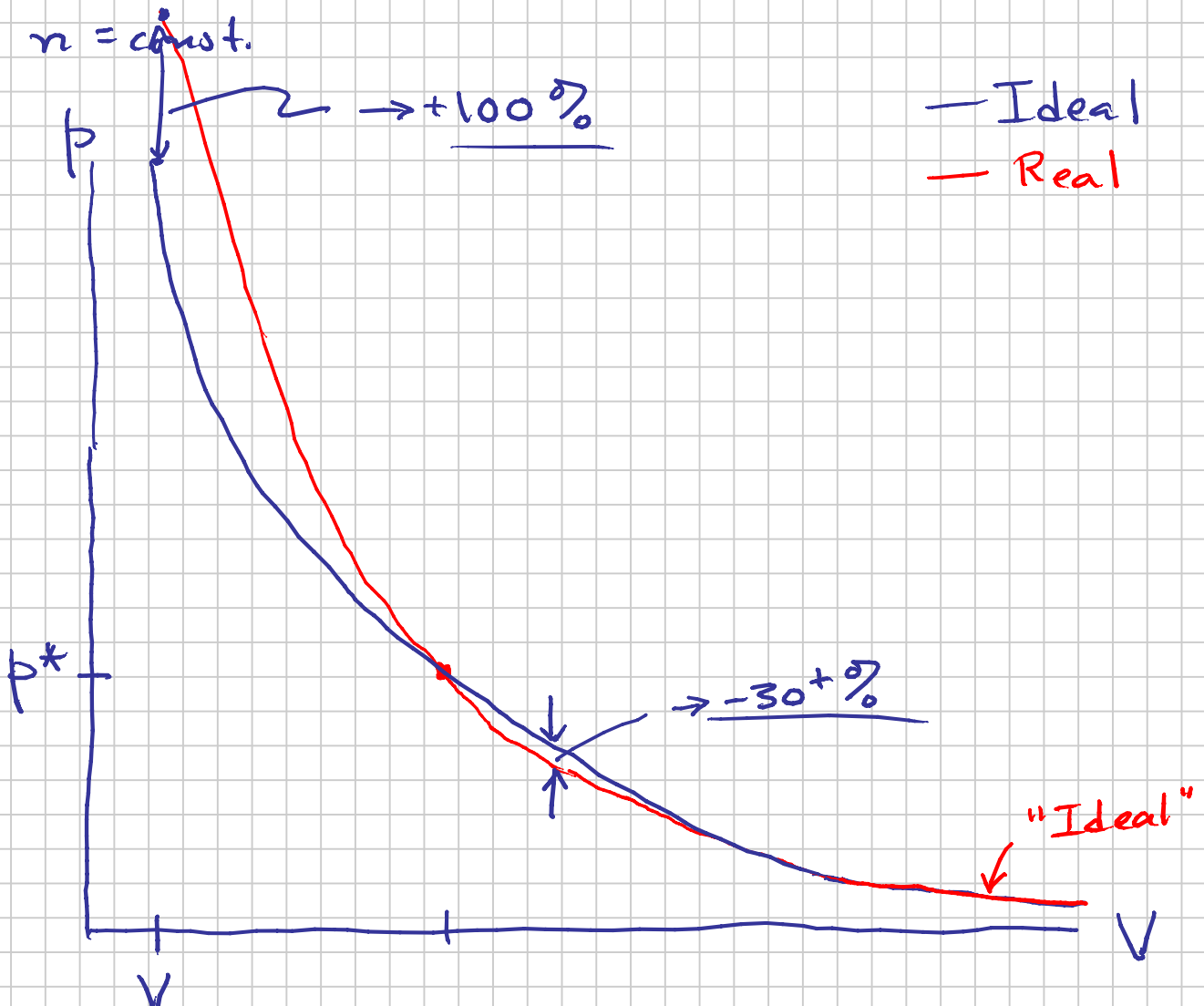
## Ideal Gas Law

$$pV = nRT \quad \leftarrow \text{absolute (K, R)}$$

↑  
 absolute  
 ↑  
 Be Careful  
 (units)

$T = \text{const.}$   
 $n = \text{const.}$

$$p \propto \frac{1}{V}$$



Ideal Gas Law **CANNOT** be used in engineering at  $p \gtrsim 10$  bar

Introduce a "Correction Factor"  
"Deviation Factor"  
...

$$Z \equiv \frac{V_{\text{actual}}(p, T, n)}{V_{\text{ideal}}(p, T, n)}$$

"Z-factor"

$$V_{\text{actual}} = V = V_{\text{ideal}} \cdot Z$$

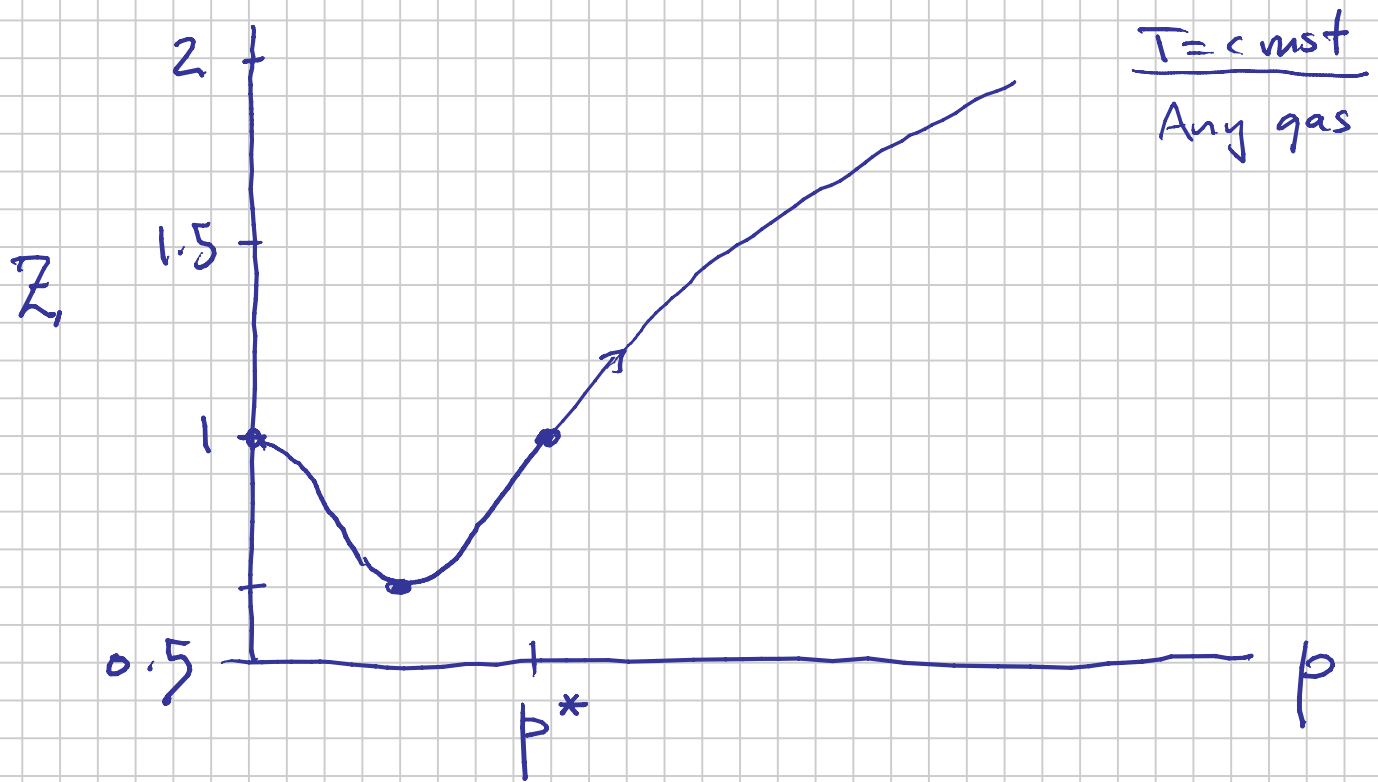
↑  
 $\frac{nRT}{p}$

$$\Rightarrow V = \frac{nRT}{p} \cdot Z$$

$$\boxed{pV = nRT \cdot Z}$$

"Real Gas Law"

## Class Exercise



History:

(1) 1870s van der Waals

⇒ Law of Corresponding States

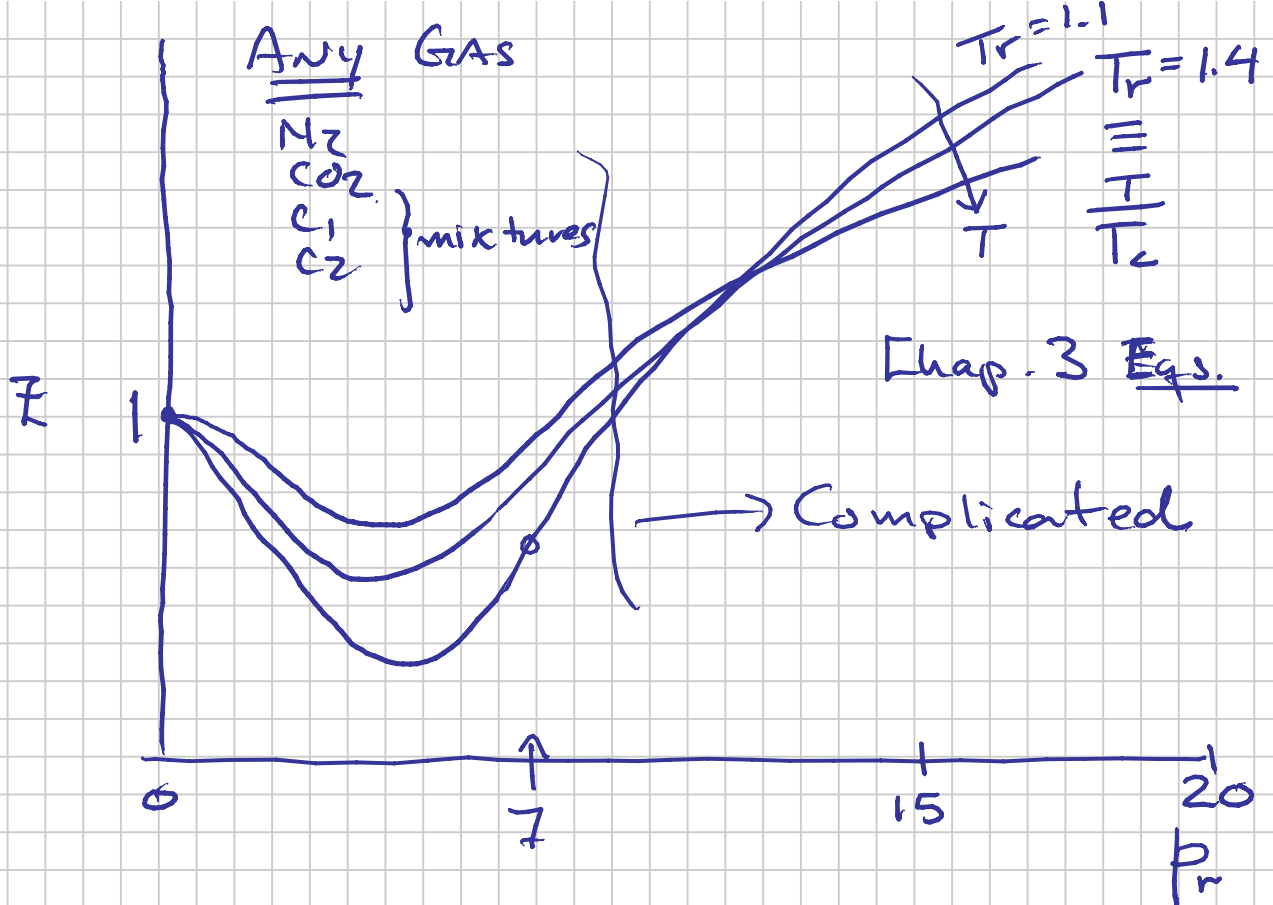
Any compound or mixture will behave "the same" in terms of  $p$ - $V$ - $T$  if you recast in terms of "reduced" (new)

variables  $p_r$ - $V_r$ - $T_r$   
"Single"

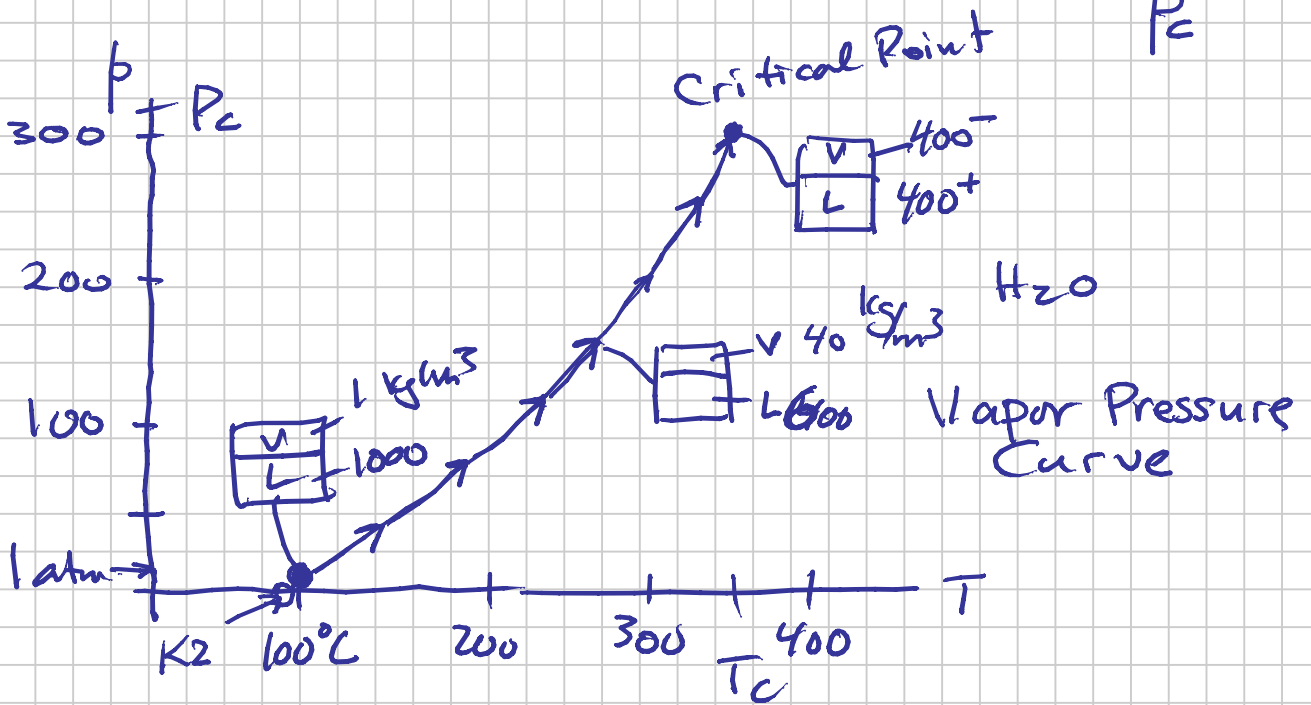
(2) Donald Katz U. Michigan

↳ PhD student Marshall B. Standing  
"Muz"

⇒ Publication ~ 1941



Review:  $(p_c, T_c)$  "Critical Point"



Gas Specific Gravity  $\gamma_g \equiv \frac{\rho_{g,sc}}{\rho_{air,sc}} = \frac{M_g}{M_{air}}$

est.  $T_c, P_c$   $Z_{sc} = 1$   $28.97$

Methane:  $M=16 \Rightarrow \gamma_g = 0.55$  (air=1)

Reservoir Gases:  $\gamma_g \sim 0.55 \rightarrow 1.5$  (2)

Calculating  $Z$

(1) Fetch  $P, T$

(2) Get

$P_c, T_c$   
Mixtures

$$P_{pc} \approx \sum_{i=1}^n \gamma_i P_{ci}$$

Gas mole fraction

$$\approx (P_c)_i$$

$$T_{pc} = \sum_{i=1}^n \gamma_i T_{ci}$$

$$\approx (T_c)_i$$

(3)  $T_{pr} \equiv \frac{T}{T_{pc}}$

$$P_{pr} \equiv \frac{P}{P_{pc}}$$

(4) SK Chart  $\Rightarrow Z_f$  at  $(P, T)$

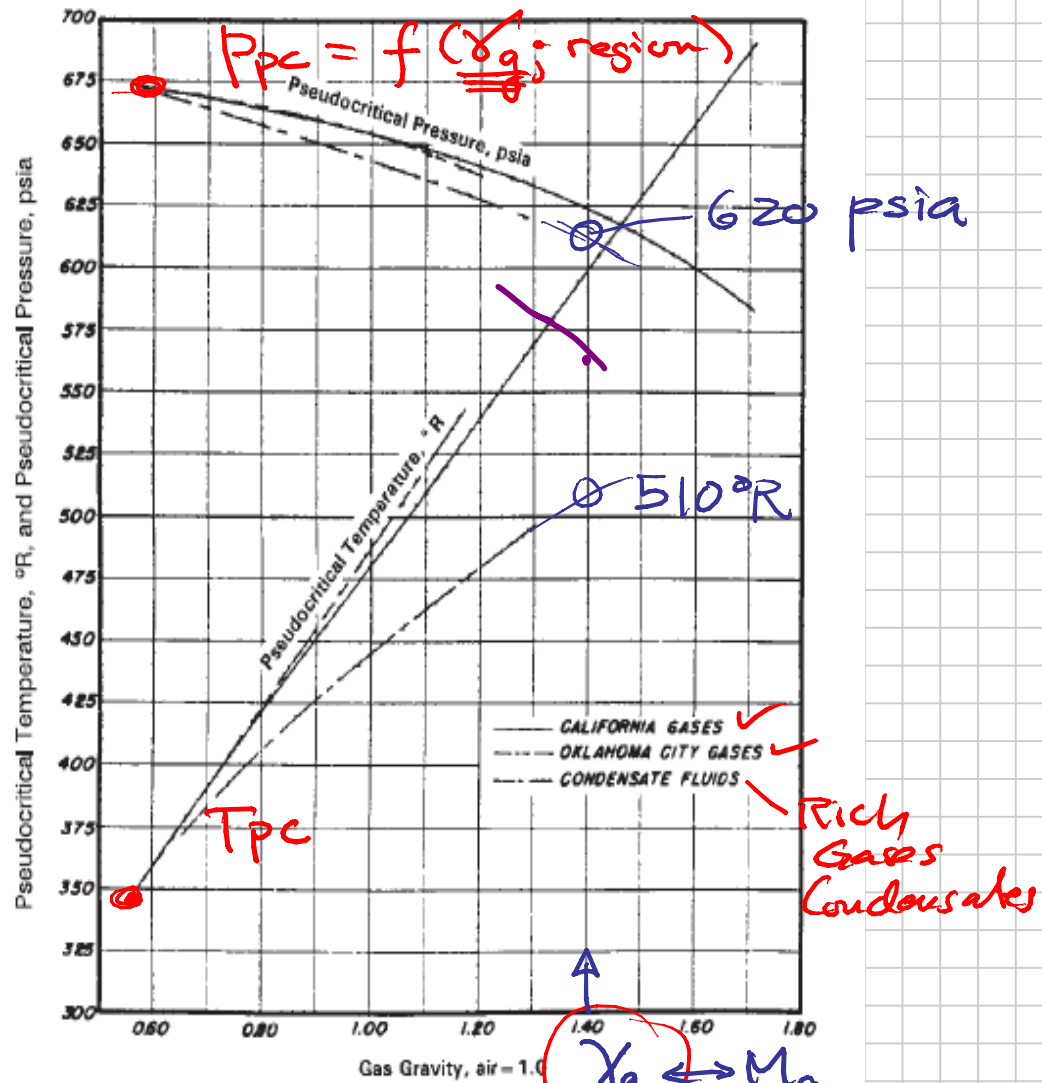


Fig. 3.7—Gas pseudocritical properties as functions of specific gravity.

Kristin:  $P_{Ri} = 900 \text{ bara} \quad \checkmark$

$$T_R = 170 \text{ }^\circ\text{C} \quad \checkmark$$

$$\gamma_g = 1.4$$

$$P_{PC} = 620 \text{ psia}$$

$$T_{PC} = 510 \text{ }^\circ\text{R}$$

$$T_{Pr} = \frac{T(K)}{T_c(K)} = \frac{170 + 273}{(510 / 1.8)}$$

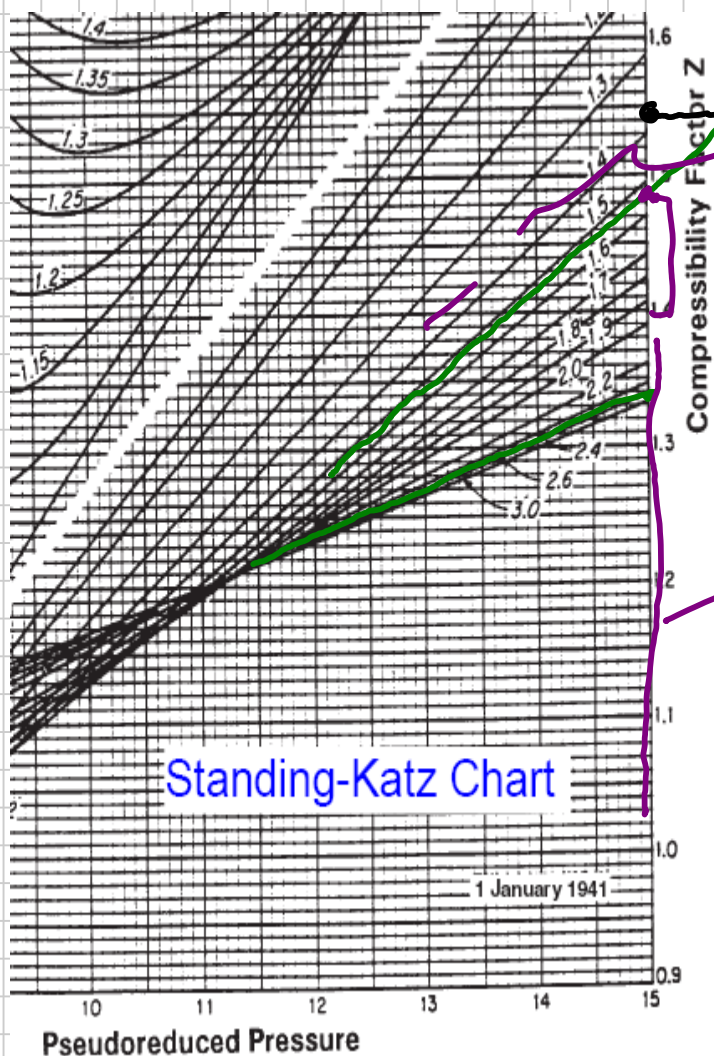
$$= 1.56 \quad \checkmark$$

$$P_{Pr} = \frac{900 \text{ bara}}{620 \text{ psia} \cdot \frac{1 \text{ bara}}{14.5 \text{ psia}}}$$

$$P_{Pr} = 21.0$$

(off the chart)

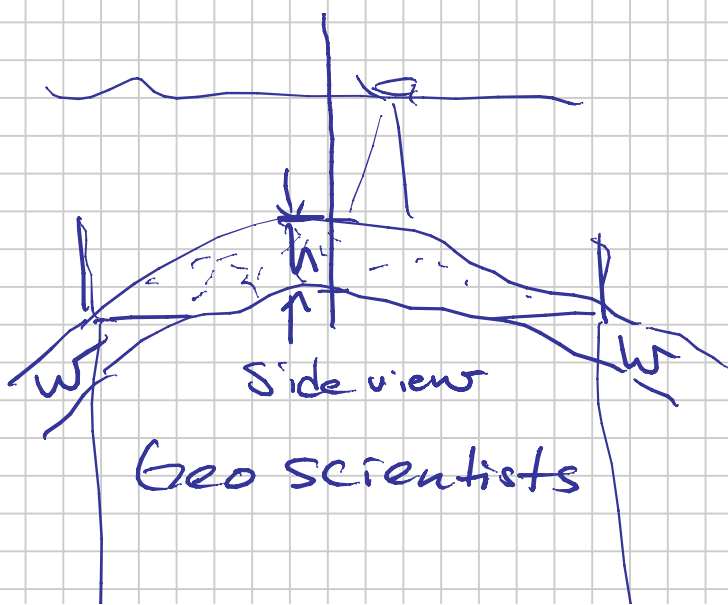
~~21.0~~  $\approx 16.55$



$$0.38 + 1.48 = \boxed{1.86}$$

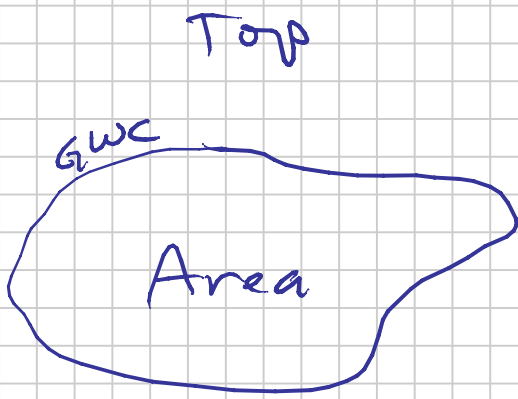
$$Z \sim 1.86$$
 extrap ✓

What & how do we use  $Z(p)$



Log  $\Rightarrow$  GAS Hydrocarbon estimate  
 Pure Katz Volume  $\bar{A} \times \bar{h}$   
 $\times \bar{\phi}_{\text{logs}} \times (1 - \bar{S}_w)_{\text{cores Gas}}$





$$V_{GR} = \bar{A} \bar{h} \phi (1 - \bar{S}_w)$$

We don't sell the Germans gas "down there"

$\sim 10^+$  \$/Mscf

$V_{GR} \rightarrow$  Sellable (s.c.) Gas Volume

$$= 100 \cdot 10^9 \text{ m}^3$$

@ Kristin conditions

$$P_{GR} V_{GR} = n R T_R Z_R$$

$$n_{GR} = \frac{P V}{R T Z} \quad \text{kg-mole}$$

$$= \frac{900 (1 \cdot 10^9)}{(0.08314)(170+273) (2)} \quad \pm 0.03 \quad 3\%$$

$$= 1.22 \cdot 10^{10} \text{ kg-mole} \times 23.68 \frac{\text{Sm}^3}{\text{kg-mole}}$$

$$= 2.9 \cdot 10^{11} \frac{\text{Sm}^3}{290 \cdot 10^9 \text{ Sm}^3} \times 35.31 \frac{\text{scf}}{\text{Sm}^3}$$

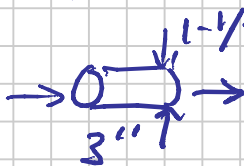
$$= \frac{10 \cdot 10^{12} \text{ scf} \times \text{USD} \frac{10}{1000 \text{ scf}}}{100 \cdot 10^9 \text{ USD}} \sim \text{Price}$$

Introduce the Real Gas Law  $pV = nRTZ$

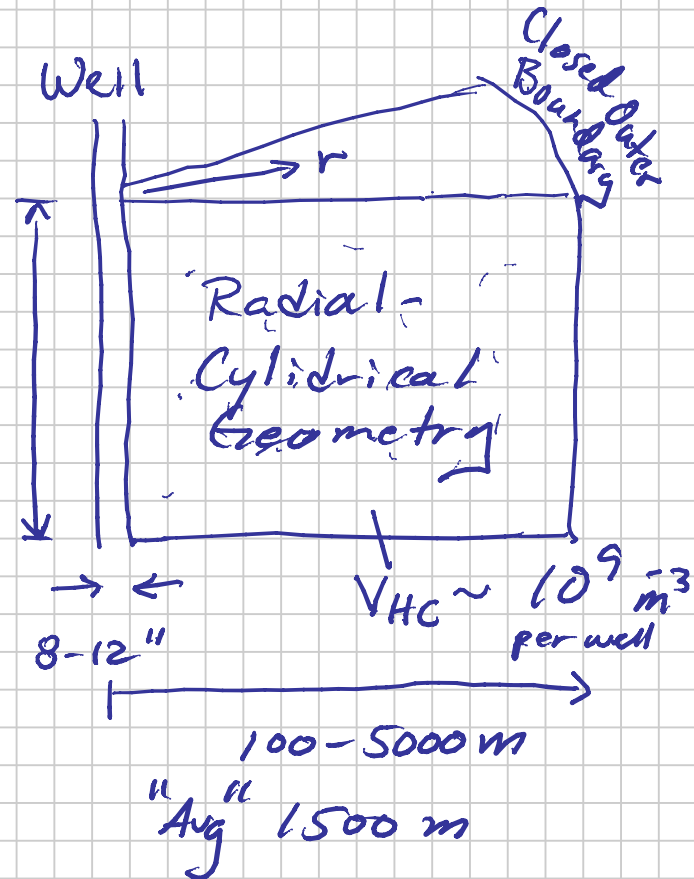
into Darcy Eq & Gas "Material Balance" (MB)  $Z(p,T)$

"Rate Eq"

Realistic Flow Geometry



1-1000m ~ h



45 · 10<sup>12</sup> scf Troll <sup>50 wells</sup>

900 · 10<sup>12</sup> scf NORTH FIELD (Qatar) Tcf

250 wells

\$0.5m

\$5B

By Well IGRIP ~ 50 · 10<sup>6</sup> scf → 1-2 · 10<sup>12</sup> scf

Initial Gas In Place

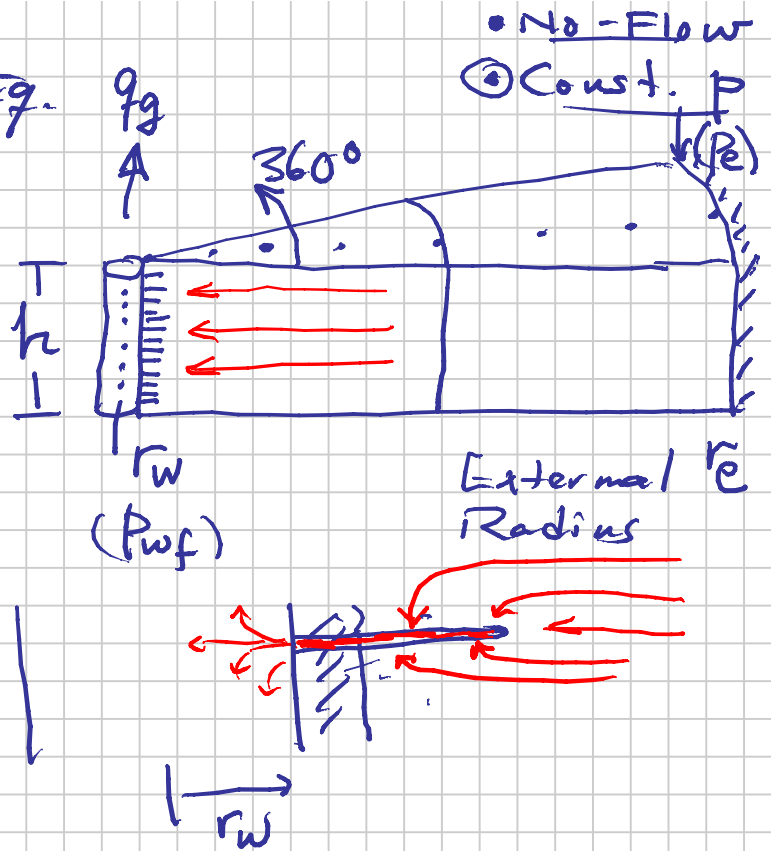
↑  
S.C.I

Develop the Gas Rate Eq.

⊙  $p @ r_e = \text{const}$

⊙ Steady State Flow:

$\dot{m}_g(r) = \text{constant}$



$q_g$  = surface gas volumetric rate at S.C. latm, 60°F

$$v = \frac{k}{\mu} \frac{dp}{dr}$$

$$\rho = \frac{M}{RT} \cdot \left( \frac{p}{Z} \right)$$

Mass Flow Rate at Surface

Mass Flow Rate at any r

Given:

$$q_g \cdot (\rho_g)_{sc}$$

$$[Sm^3/s] [kg/Sm^3]$$

$$= \underbrace{v_g \cdot A_L}_{q_{gR}} \cdot \rho_g$$

$$[m^3/s] [kg/m^3]$$

$$q_g \cdot \frac{M p_{sc}}{RT_{sc}}$$

Given

$$= \underbrace{\frac{k}{\mu_g} \frac{dp}{dr}}_{=v_g} \cdot 2\pi r h \cdot \frac{M}{RT_R} \cdot \frac{p}{Z}$$

Collect all "constants" on RHS

$$q_g = \underbrace{\frac{T_{sc}}{T_R P_{sc}} \cdot 2\pi kh \cdot r}_{\text{constant}} \frac{p}{\mu z} \frac{dp}{dr}$$

Given  $\uparrow$   $q_g$

$$\int_{r_w}^{r_e} \frac{1}{r} dr = \frac{2\pi kh}{\underbrace{(T_R P_{sc} / T_{sc})}_{\text{const.}}} \int_{P_w}^{P_e} \underbrace{\frac{p}{\mu z}}_{f(p)} dp$$

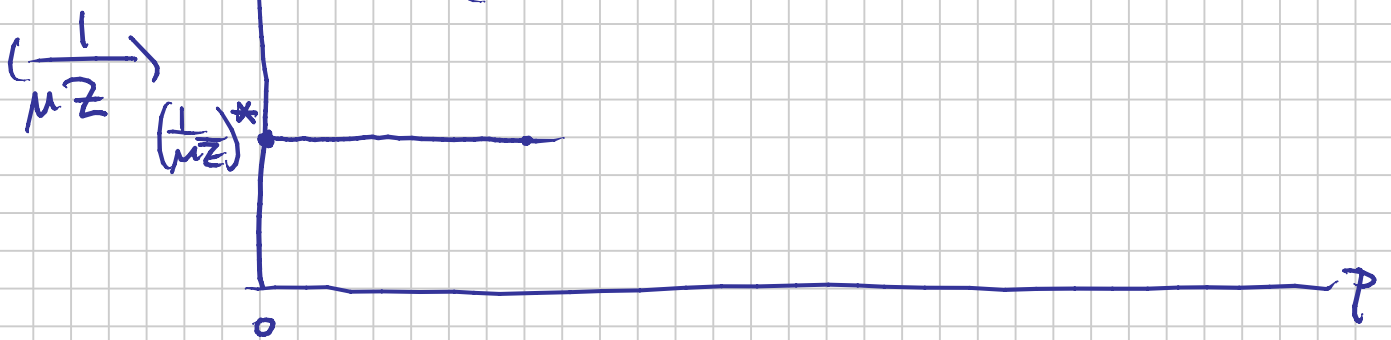
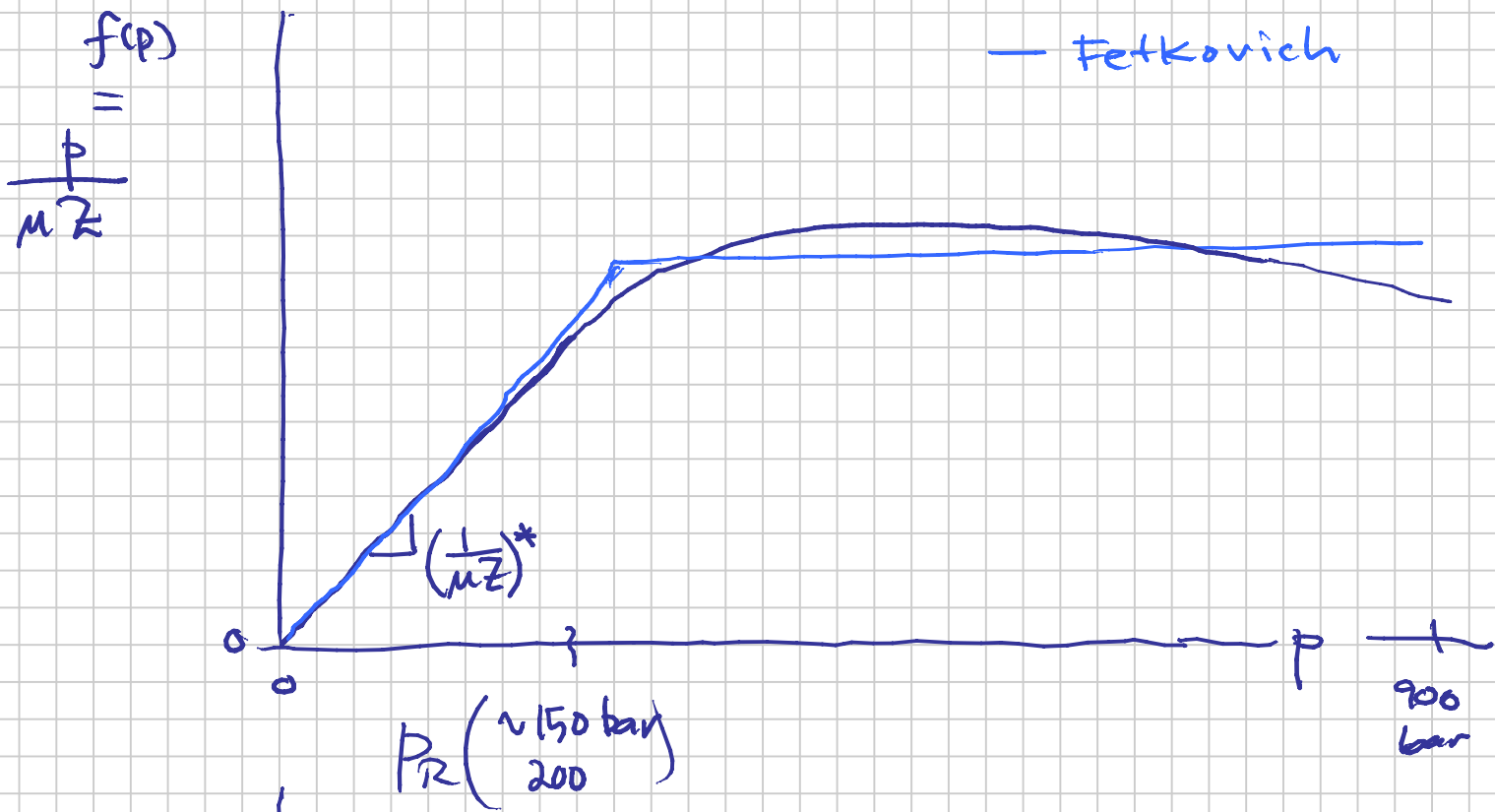
$\ln r_e / r_w$

$\sim 6-10$   
"G"

$$q_g = \underbrace{\frac{2\pi kh}{\left(\frac{T_R P_{sc}}{T_{sc}}\right) \left[\ln(r_e/r_w) + \underline{\underline{"G"}}\right]}}_C \int_{P_{wf}}^{P_e} \underbrace{\frac{p}{\mu z}}_{f(p)} dp$$

$\uparrow$   
flowing

"G"  $\sim 0$  non-perfect flow effects  
in perforations



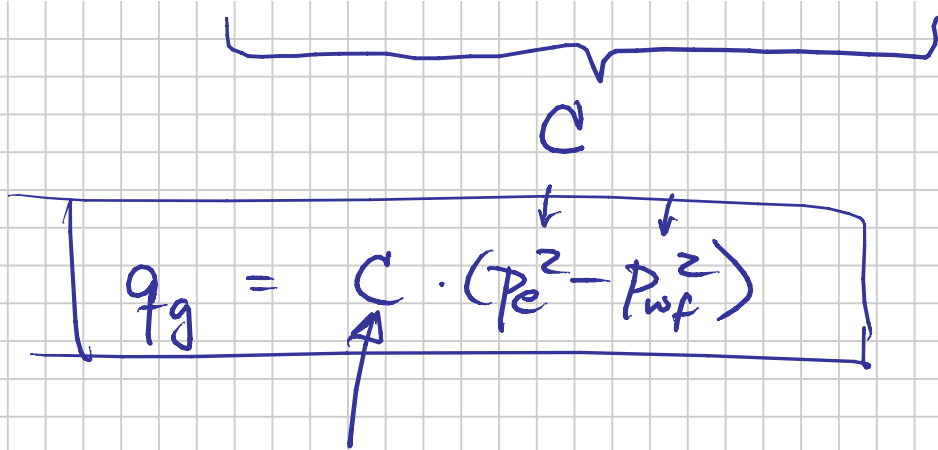
$$\left(\frac{1}{\mu z}\right)^* \sim \frac{1}{\mu_g^0} \text{ Gas Viscosity @ } p \sim 0 \text{ (1 atm)} \sim 0.015 \text{ cp}$$

{ Low- $P \leq 200$  bara }

$$\int_{P_{wf}}^{P_e} \frac{p}{\mu z} dp = \frac{1}{2 \mu_g^0} (P_e^2 - P_{wf}^2)$$

$$q_g = \frac{2\pi k h}{\left[ \left( \frac{T_R P_{sc}}{T_{sc}} \right) \rho_n \frac{r_e}{r_w} + \left( \right) \right]} \cdot \frac{1}{2 \mu_g^0} \cdot (P_e^2 - P_{wf}^2)$$

Usually small



Ormen  
Large

(1) kh

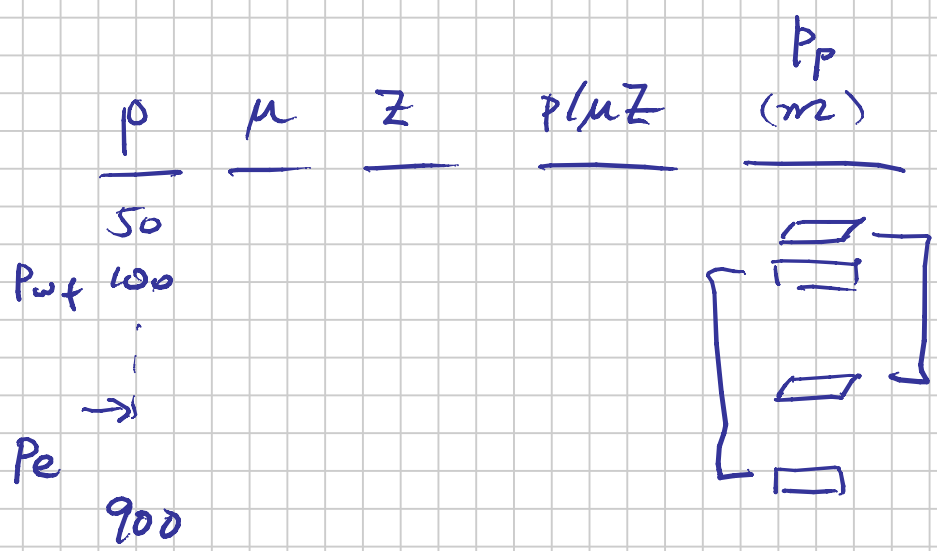
(2) Geometry terms

1960s : Al-Hussainy, Ramey, Crawford

Defined: Gas Pseudopressure

$$m \equiv 2 \int_0^P \frac{p}{\mu z} \cdot dp$$

( $p_p$ )  
> 1990



$$\int_{P_{wf}}^{P_e} \frac{p}{\mu z} dp = \int_0^{P_e} \frac{p}{\mu z} dp - \int_0^{P_{wf}} \frac{p}{\mu z} dp$$

$$= (P_{re} - P_{pwf})$$

Final General Gas Rate Eq

$$q_g = \frac{\pi k h (\bar{P}_R - P_{pwf})}{\left( \frac{T R P_{sc}}{T_{sc}} \right) \left[ \ln \frac{r_e}{r_w} - 0.75 \right]}$$

$\frac{m^2 \cdot m}{\pi k h}$       Pa/s  
 $\frac{k}{R}$  Pa       $\sim 8$        $\uparrow$  No-Flow @  $r_e$   
 $\bar{P}_R$  @  $0.47 r_e$

Use  $\bar{P}_R$  = vol. avg. p of drainage volume  
instead of  $P_e$

$$\ln \frac{0.6 r_e}{r_w} = \ln \frac{r_e}{r_w} - 0.5$$



# GAS RATE EQ


$$10^6 q_g = \frac{(\bar{k}h)^{\text{same}}}{\left(\frac{T_R B_{sc}}{T_{sc}}\right)^{\text{same}}} \left[ \underbrace{\ln \frac{r_e}{r_w}}_{\sim 8} - \frac{3}{4} + \underbrace{s}_{\text{No-Flow O.B.}} \right] \cdot 2 \int_{P_{wf}}^{P_R} \frac{p}{\mu z} dp$$

$P_R \checkmark$  Vol. Avg. Res Press 300 ✓  
 $P_{wf} 250 \checkmark$   
 $(P_{PR} - P_{pwf})$

"Constant" for a given well with a given  $\bar{k}h$  and a given well geometry

$$P_p \equiv 2 \cdot \int_0^P \frac{p}{\mu z} dp$$

## "s" ("Skin")

Non-idealities  $\Rightarrow$  deviation from our simple geometric model 

### • Damage

$$s_d = \left( \frac{\bar{k}}{k_d} - 1 \right) \cdot \ln \frac{r_d}{r_w}$$

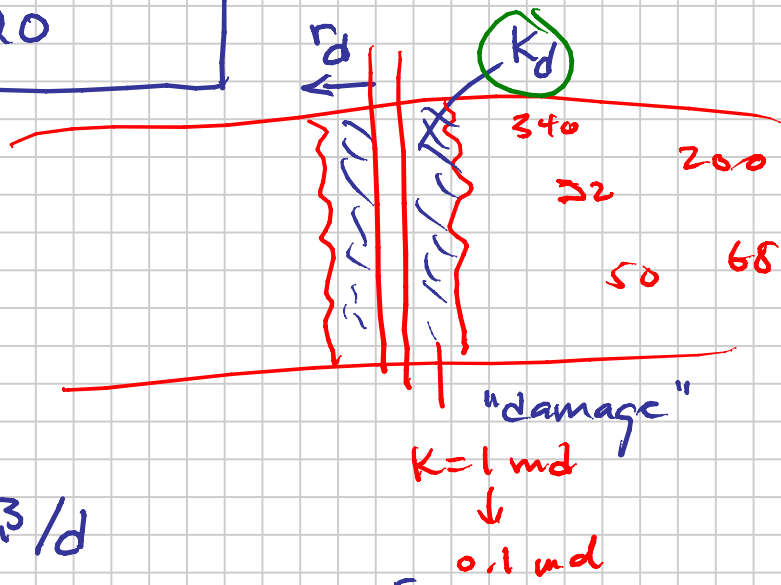
$\bar{k} = 100 \text{ md}$   
 $k_d = 1 \text{ md}$   
 $r_d = 2 \text{ ft } (0.7 \text{ m})$   
 $r_w = 8 \text{ in } (0.2 \text{ m})$

and  $k(r, z, \theta) = \text{constant}$

$\bar{k} = 100 \text{ md}$   $\updownarrow$  2-5 orders of magnitude

Exception: low  $k(r \sim r_w)$

$$S_d = +120$$



No damage

$$q_g = 10^6 \text{ Sm}^3/\text{d}$$

for some  $p_{wf}$

$$p_R = 300 \text{ bar}$$

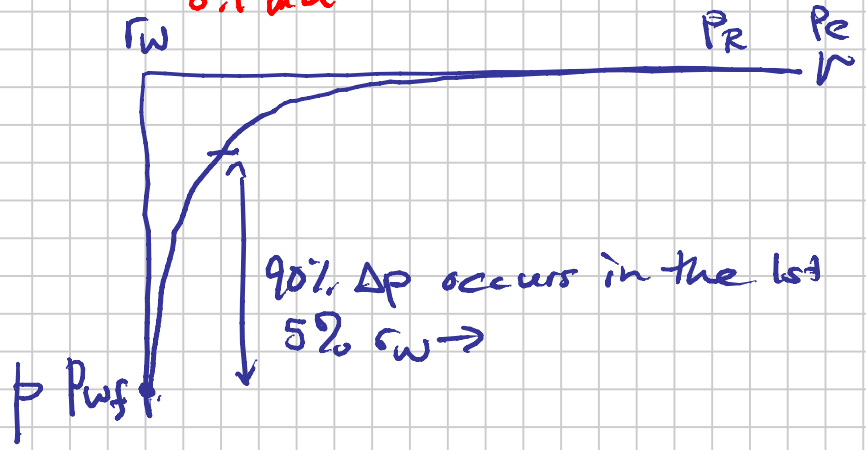
$$p_{wf} = 250 \text{ bar}$$

SAME

$$q_g = ?$$

$$S_d = +120$$

$$\left( \ln \frac{r_e}{r_w} - \frac{3}{4} \right) = 8$$



90%  $\Delta p$  occurs in the last 5%  $r_w \rightarrow$

$$p \propto \ln r$$

$$\frac{q_{gs}}{q_{gs=0}} = \frac{(\ln \frac{r_e}{r_w} + 0)}{(\ln \frac{r_e}{r_w} + S)}$$

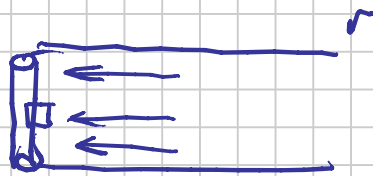
$$= \frac{8}{8+120}$$

$$(q_g)_{s=120} = (q_g)_{s=0} \cdot \frac{8}{128} = 62500 \frac{\text{Sm}^3}{\text{d}}$$

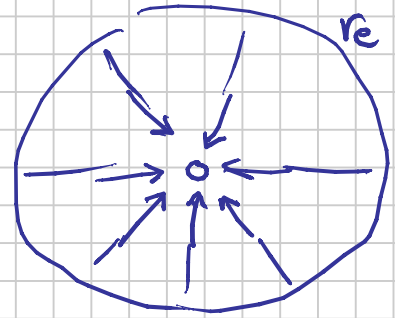
$$10^6$$

• Flow Geometric Skin

Ideal:



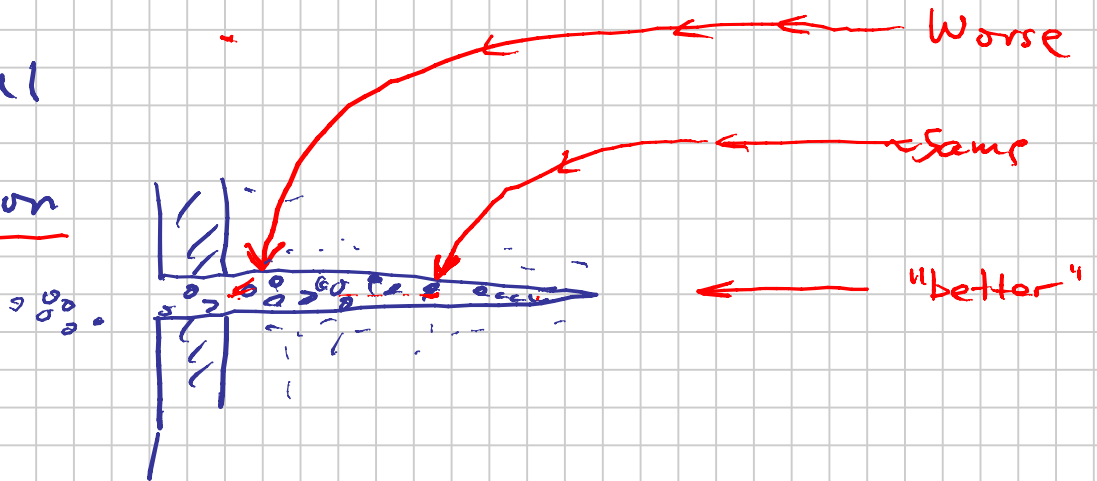
Side



Top

Non-Ideal

• Perforation

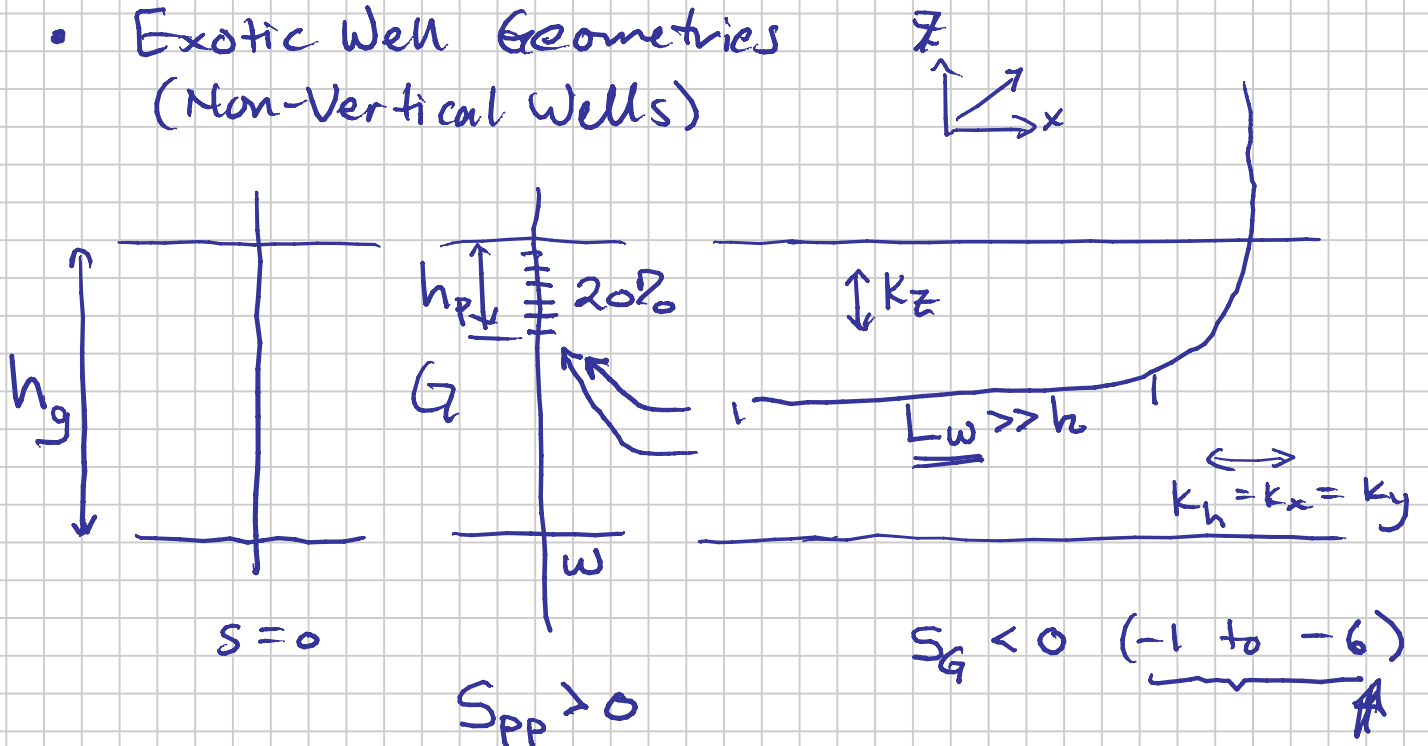


Casing  
+ 0.5 ish

Sperf ~ 0

- 0.5 ish

• Exotic Well Geometries  
(Non-Vertical Wells)



$$S_{pp} = f\left(\underbrace{\frac{h_p}{h}}_{0.1-1}, \underbrace{\sqrt{\frac{K_v}{K_h}}}_{\frac{1}{2}-\frac{1}{10}}\right)$$

+1 to +50

Generalized & Realistic Gas Material Balance (MB)

$$MB: \bar{P}_R \sim G_p \leftrightarrow n_p$$

↑  
Cumulate Surface Gas  
Volume Produced

Real Gas Laws:

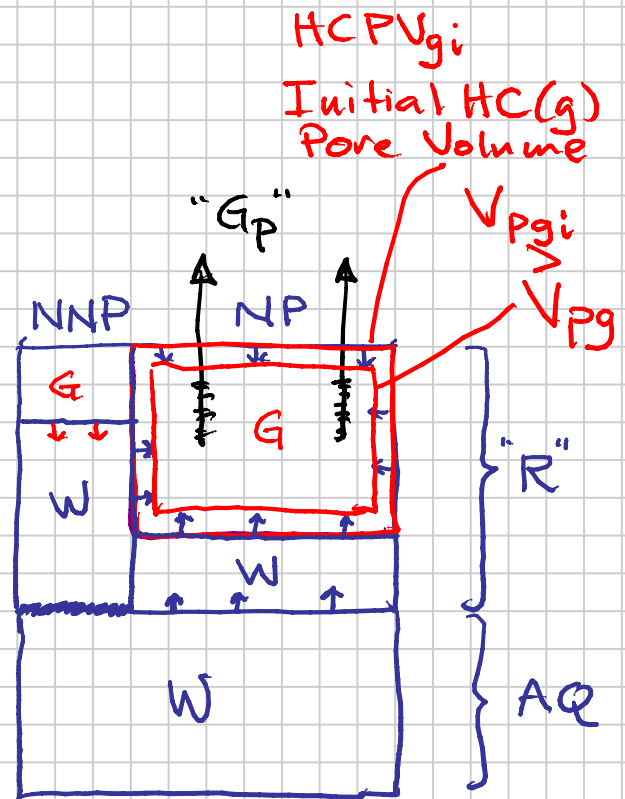
$$pV = nRTZ$$

"Simplified" Gas MB

$$V_{pgi} = \text{constant}$$

"R": Rock containing pores containing gas

"AQ" Rock "connected" (flow) to R but containing only water (Aquifer)



R: (1) NP - Net Pay ("good k"; significant flow of gas)

W: Connate Water  
5% - 50%  $k_w \sim 0$

(2) NNIP Non-Net Pay  $k_g \sim 0$

$S_w \sim 50-100\%$

often interspersed between NP rock

- Shale
- "Dirty" (low-k) sandstone
- Carbonate

$$V_{pgi} = \text{constant}$$

$$pV = nRT_R Z$$

$$V_{pgi} = n_{Ri} R T_R (Z_{Ri} / p_{Ri})$$

$$V_{pg} = n_R R T_R (Z_R / p_R) =$$

$$n_R = n_{Ri} - n_p$$

$$n_{Ri} (Z_{Ri} / p_{Ri}) = (n_{Ri} - n_p) (Z_R / p_R)$$

$$\frac{p_R}{Z_R} = \left( \frac{p_{Ri}}{Z_{Ri}} \right) \cdot \left( 1 - \frac{n_p}{n_{Ri}} \right)$$

Cum. Produced Surface Gas Volume  $G_p = n_p \cdot \frac{RT_{sc}}{p_{sc}}$

Initial " in Place  $G = n_{Ri} \cdot \frac{RT_{sc}}{p_{sc}}$

$$\frac{P_R}{Z_R} = \left( \frac{P_{Ri}}{Z_{Ri}} \right) \cdot \left( 1 - \frac{G_p}{G_i} \right)$$

Kristin Field

$$P_{Ri} = 900 \text{ bara} \quad Z_{Ri} = 1.7$$

$$P_R = 450 \text{ bara} \quad Z_R = 1.15$$

Recovery Factor  $RF_g$  ( $G_p/G_i$ )

at 450 bara =  $P_R$  ?

$$\frac{G_p}{G_i} = 1 - \frac{(P/Z)}{(P/Z)_i} = 1 - \frac{(450/1.15)}{(900/1.7)}$$

$$= 0.26 \quad \underline{\underline{25\%}}$$

vs "intuitive" (ideal gas) 50% RF

Include the change in  $V_{pg} (P_R)$  due to

- (1) AQ water expanding
- (2) Connate water NP expanding
- (3) ——— " ——— NMP ———
- (4) Pore Volume reduction

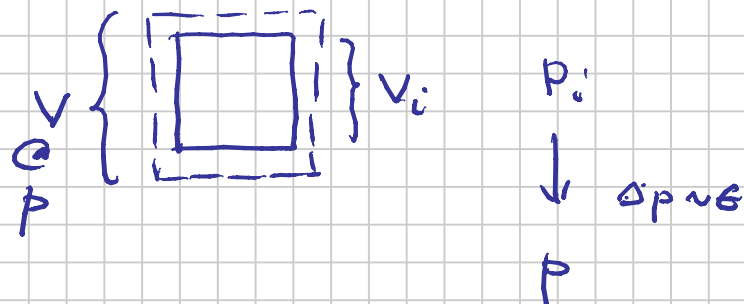
Two Physical Properties

- (1)  $c_w \rightarrow$  water expansion
- (2)  $c_f, c_r \Rightarrow$  pore volume change

Isothermal Compressibility

$$c \equiv -\frac{1}{V_i} \frac{dV}{dp}$$

Instantaneous  
compressibility



$V_i \rightarrow V$   
 $P_i \rightarrow P$ 
} requires integrating this equation

e.g. assume  $c = \text{const.}$

$\Rightarrow$

Use a concept of "cumulative compressibility"

$$\bar{c} = -\frac{1}{V_i} \frac{\Delta V}{\Delta p}$$

any  $\Delta p$  change

$$V_{pgi} \rightarrow V_{pg}$$

Volume of R gas  $\Rightarrow$  Gas Law

$$V_{pgi} - V_{pg} = \left\{ \begin{array}{l} \Delta V_{pg} + \Delta V_{pg} \\ \uparrow \quad \quad \uparrow \\ \text{rock} \quad \text{water} \\ \text{comp.} \quad \text{entering} \end{array} \right\}$$

$$\left\{ \begin{array}{l} c_w \quad V_w \\ c_f \quad V_p \end{array} \right\} (NP + NNP + Ad)$$

Composite all of these effects into a single

Total Cumulative Effective Compressibility

$$\boxed{\bar{c}_t} \text{ est.}$$

Real Gas Law:

$$pV = nRTz$$

$$V = V_{pg}$$

$$n = n_R = n_{Ri} - n_p$$

$$\bar{c}_t = \frac{1}{V_{pgi}} \cdot \frac{\Delta V_{pg}}{\Delta p}$$

Est.

Solve for  $V_{pg}$

$$\Delta V_{gp} = -V_{pgi} \cdot \bar{c}_t \Delta p$$

$$P_R \cdot V_{pgi} (1 - \bar{c}_t \Delta p) = (n_{Ri} - n_p) RTz_R$$

$$V_{pgi} = \frac{n_{Ri} RTz_{Ri}}{P_{Ri}}$$

$$V_{pgi} - V_{pg} =$$

$$\dots V_{pg} = V_{pgi} (1 - \bar{c}_t \Delta p)$$



$$\Delta p = P_{ri} - P_R$$

$$\frac{P_R}{Z_R} (1 - \bar{C}_t \Delta p) = \left( \frac{P_{ri}}{Z_{ri}} \right) \left( 1 - \frac{n_P}{n_{Ri}} \right)$$

POT AQUIFER MODEL

↑  
G<sub>P</sub>/G

$$\frac{P_R}{Z_R} (1 - \bar{C}_t \Delta p) = \left( \frac{P_{ri}}{Z_{ri}} \right) \left( 1 - \frac{G_P}{G} \right)$$

↑  
Recovery  
Factor

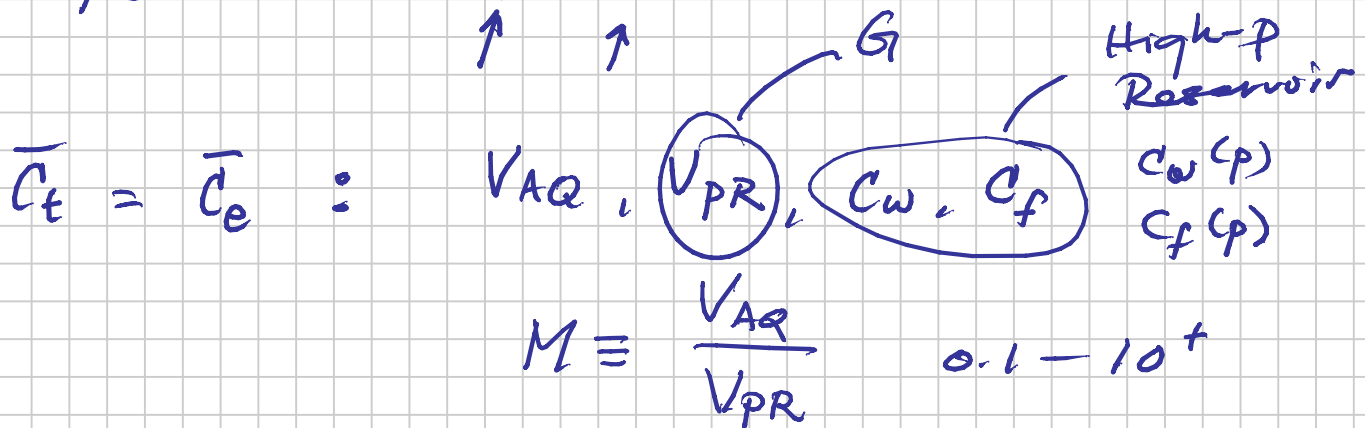
Traditional  
Simplified MB

$$\Delta C_t = 0$$

$$V_{Pg} = V_{Pgi}$$

SPE 22921

Fetkovich et al.

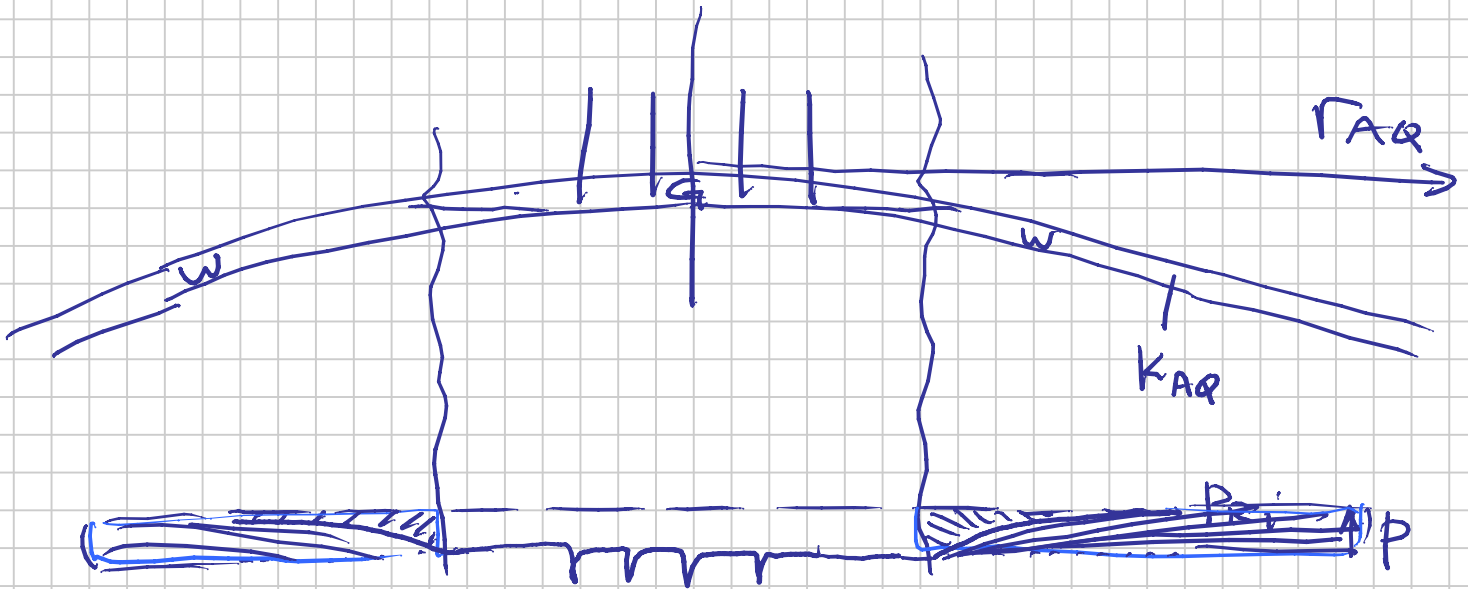


# MOST GENERAL GAS M.B.

- Takes into account

$$\bar{p}_{AQ} \neq \bar{p}_R$$

(>)

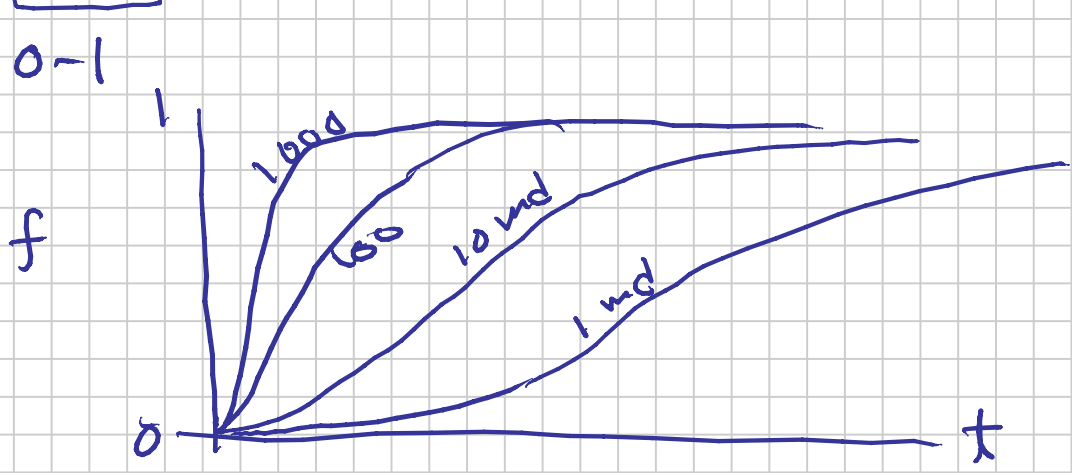


## Max Water Volume Encroachment

$$f(t) V_{AQ} \cdot C_{AQ} \cdot (P_{Ri} - P_R)$$

Superposition  
von Erweiterungen  
Hurst

Fetkovich  
Simplified

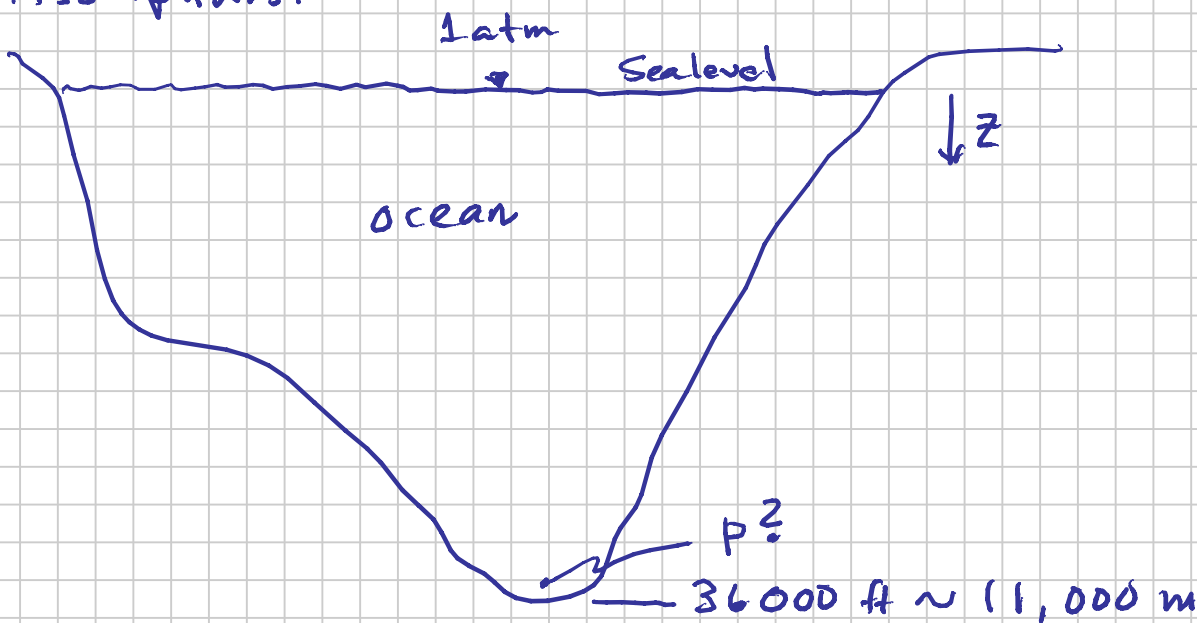


...

TODAY : QUIZ

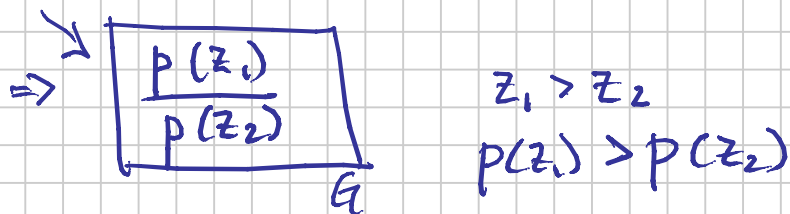
Simple PVT Models to estimate the pressure-us-depth relationship of {Gas, Oil, Water} in a "static column" of fluid

Assumptions:



•  $T(z) = \text{constant}$

• Static :  $\frac{dp}{dz} = \rho g$



Water :  $c_w \equiv -\frac{1}{V} \left( \frac{dV}{dP} \right)_T = \text{constant}$   
 Oil :  $c_o = \text{const}$

( $\rho_w^0$ ) at 1 atm  
 e.g. 1025 kg/m<sup>3</sup>  
 Seawater

Gas :  $pV = nRTZ$

$$\Rightarrow \rho_g = \frac{m}{V} = \frac{nM}{V} = \frac{pM}{RTz} \approx \frac{pM}{RT\bar{z}}$$

$\bar{z}(p)$  complex

Assumption:  $p(z_2) \rightarrow p(z_1)$   
 $\uparrow$  depth  $z$        $\uparrow$  depth  
 $\underbrace{\hspace{10em}}$   
 $\bar{z}$  at  $\frac{p_1 + p_2}{2}$

Quiz:

(1)  $p$  at deepest ocean depth\*

$$\frac{dp}{dz} = \rho g$$

(2)  $\frac{p(z_1)}{p(z_2)}$  for gas

\*  $c_w$ ? Ch. 9,  $\sim 2.5 \cdot 10^{-6} \frac{\text{vol}}{\text{vol}} \frac{1}{\text{psi}} \times \frac{14.5 \text{ psi}}{\text{bar}}$   
 $= 36 \cdot 10^{-6} \frac{1}{\text{bar}}$

Ignore  $d$

$$\rho_w = \rho_w^0 = 1025 \text{ kg/m}^3$$

$$\frac{dp}{dz} = \rho g$$

$$p_2 - p_1 = \rho g (z_2 - z_1)$$

$$p_1 = 1 \text{ atm} \sim 1 \text{ bar}$$

$$\rho = 1025 \text{ kg/m}^3$$

$$g = 9.8 \text{ m/s}^2$$

$$z_1 = 0$$

$$z_2 = 11000 \text{ m}$$

$$\begin{aligned}
 p_2 &= p_1 + \underbrace{(1025)(9.8)(11000)}_{\text{Pa}} \\
 &= 10^5 + \text{Pa} \\
 &= 10^5 + 1.1 \cdot 10^8 \\
 &= 1.1 \dots \cdot 10^8 \\
 &= 1100 \text{ bar}
 \end{aligned}$$

$$\boxed{\rho = \rho_0 \cdot \exp(c \Delta p)}$$

$$c = -\frac{1}{v} \frac{dv}{dp}$$

$$c \int_{p_1}^{p_2} dp = - \int_{v_1}^{v_2} \frac{1}{v} dv$$

$$c(p_2 - p_1) = \ln v_1 / v_2$$

$$v_2 = v_1 e^{-c \Delta p}$$

$$\rho = \frac{m}{v}$$

$$\rho_2 = \rho_1 e^{c \Delta p}$$

$$1025 \cdot \exp\left(\frac{36 \cdot 10^{-6}}{\text{bar}} \cdot 1100 \text{ bar}\right)$$

$$= 1025 \cdot 1.$$

$$= 1066 \text{ kg/m}^3$$

$$p_{\text{bottom}} = 1100^+ \text{ bar}$$

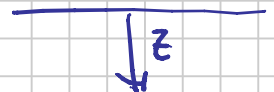
Gas:

$$\frac{dp}{dz} = \rho g$$

$$\rho = \frac{pM}{RTz} \approx \underbrace{\left(\frac{M}{RTz}\right)}_{\text{const.}} \cdot p$$

$$\int \frac{1}{p} dp = (\quad) \int dz$$

$$\ln p_2/p_1 = \left(\frac{M}{RTz}\right) (z_2 - z_1)$$



$$\frac{P_2}{P_1} = \exp \left\{ \left( \frac{Mg}{RT\bar{z}} \right) (z_2 - z_1) \right\} \quad \checkmark$$

True Vertical Depth TVD

$z_1 > z_2$

Bottomhole  $\rightarrow$   $\frac{P_1}{P_2} = \exp \left\{ \underbrace{\left( \frac{Mg}{RT\bar{z}} \right)}_{>0} \underbrace{(z_1 - z_2)}_{>0} \right\}$

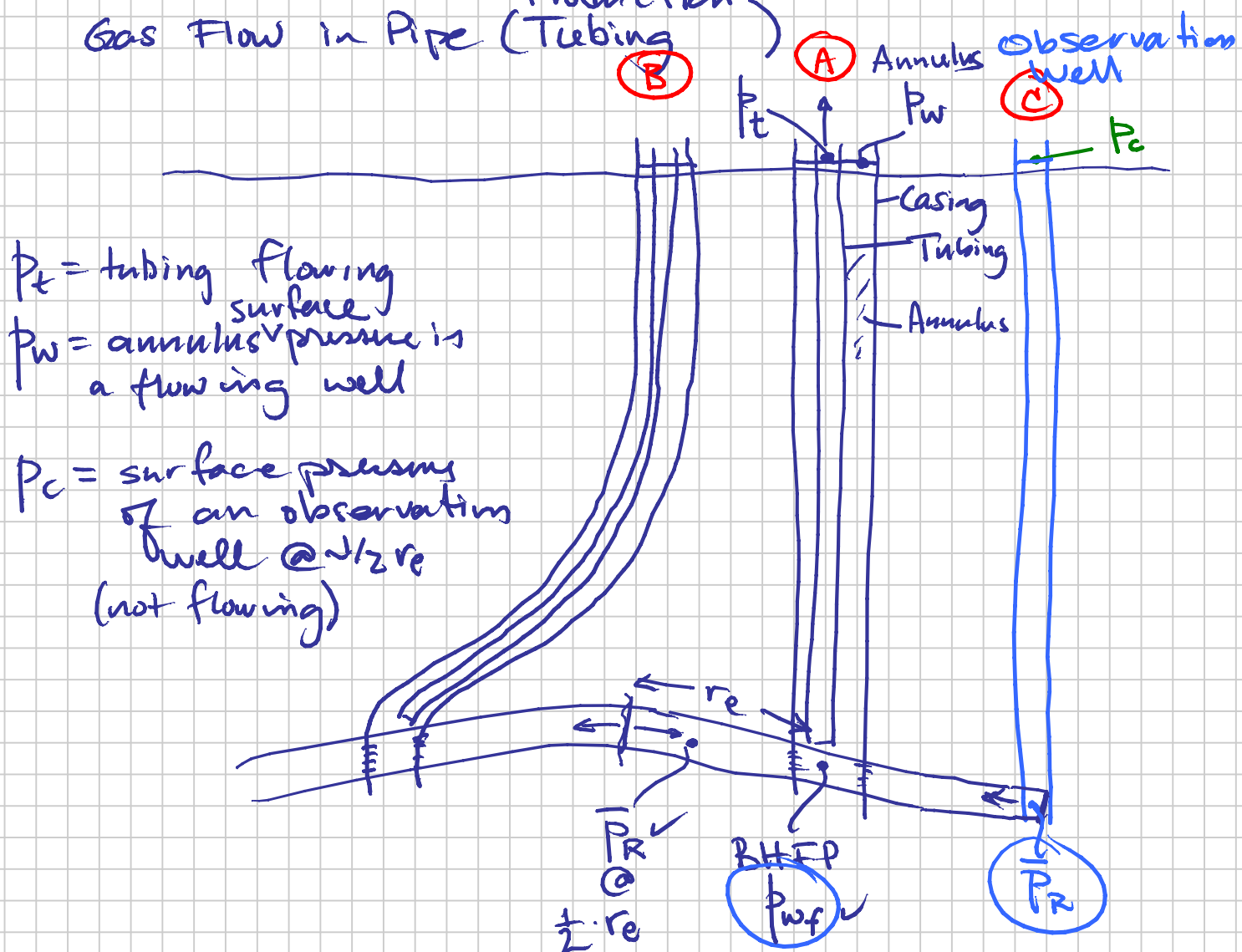
Surface  $\rightarrow$

$\underbrace{\hspace{10em}}_{>0}$

constant

1.1 - 1.25

Gas Flow in Pipe (Tubing) Production



$$\exp\left\{\frac{Mg}{RTz}\right\} = 1.1$$

Observed  
Surface  
Pressure  
bara

(A)

220

(B)

170

(C)

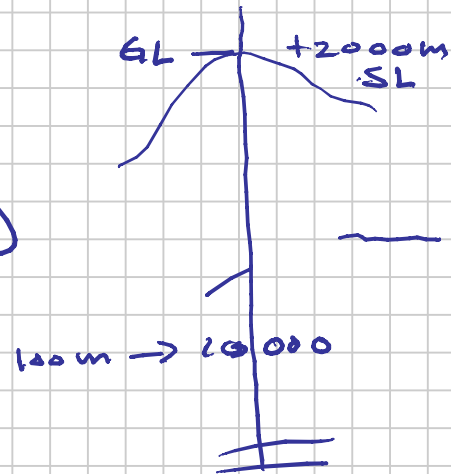
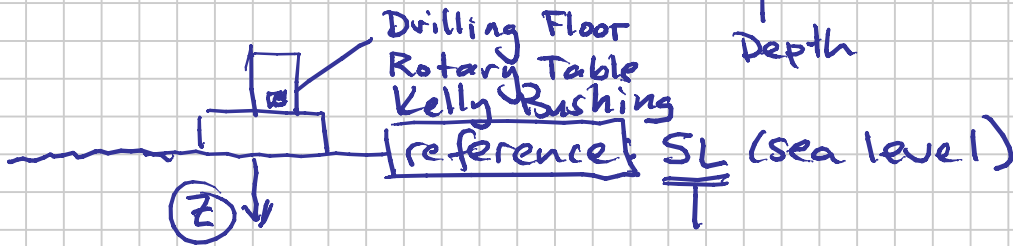
380

Calculate:  $(p_{wf})_A$        $(p_{wf})_B$        $P_R$

Can I calculate  $(p_t)_A$        $(p_t)_B$  ?

What do I need to calculate ?

Summarize Static Column  $p(z)$  Issue



$$p(z) \propto \rho$$

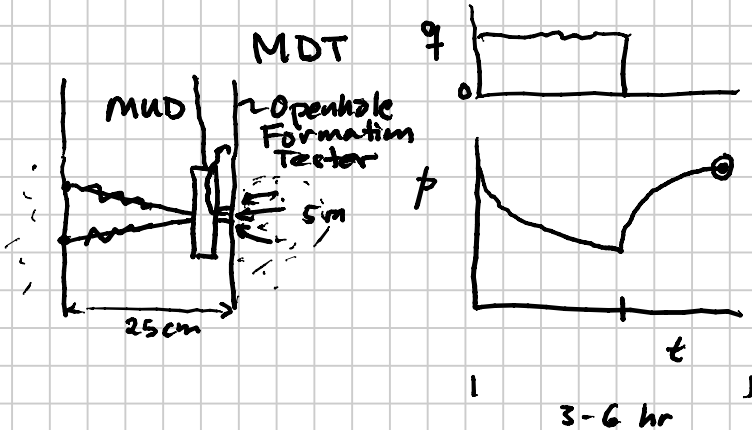
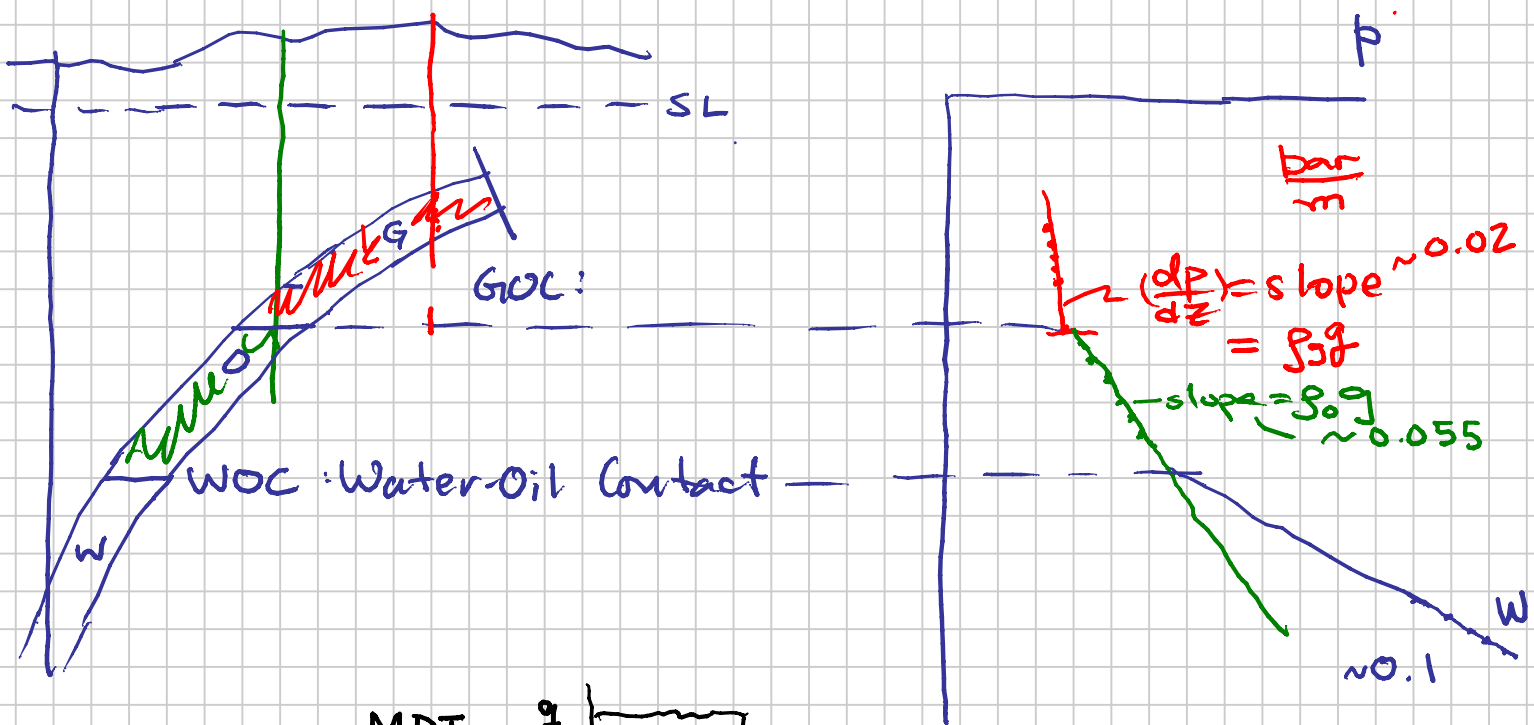
1st Approx:  $\rho_p = \text{constant}$

Within the Reservoir  $p(z, \rho, w)$

$$\frac{dp}{dz} = \bar{\rho} g$$

Water Gradient (Slope) 0.1 bar/m

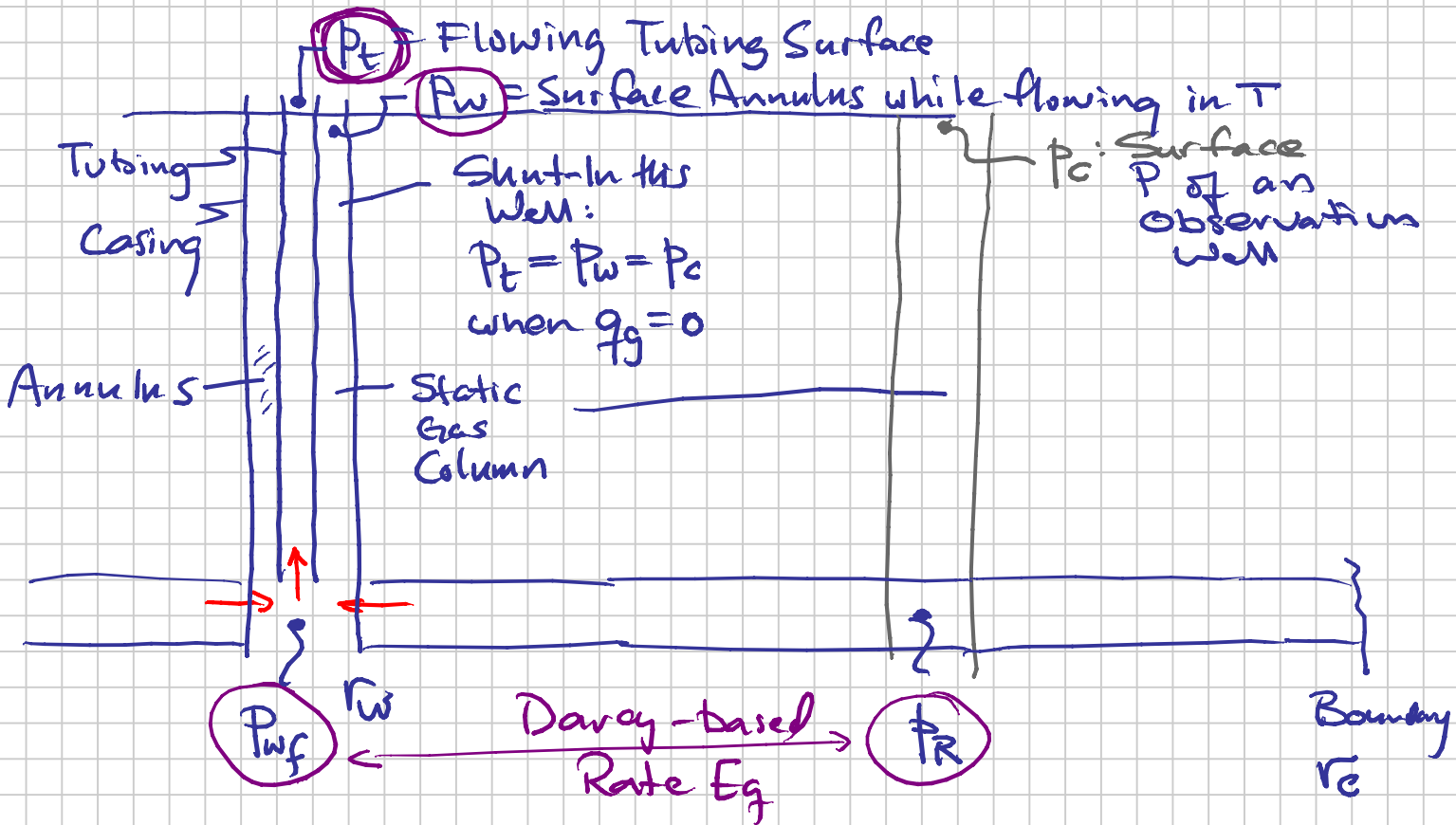
Gas Grad. 0.015-0.035  $\frac{\text{bar}}{\text{m}}$





# Gas Reservoirs

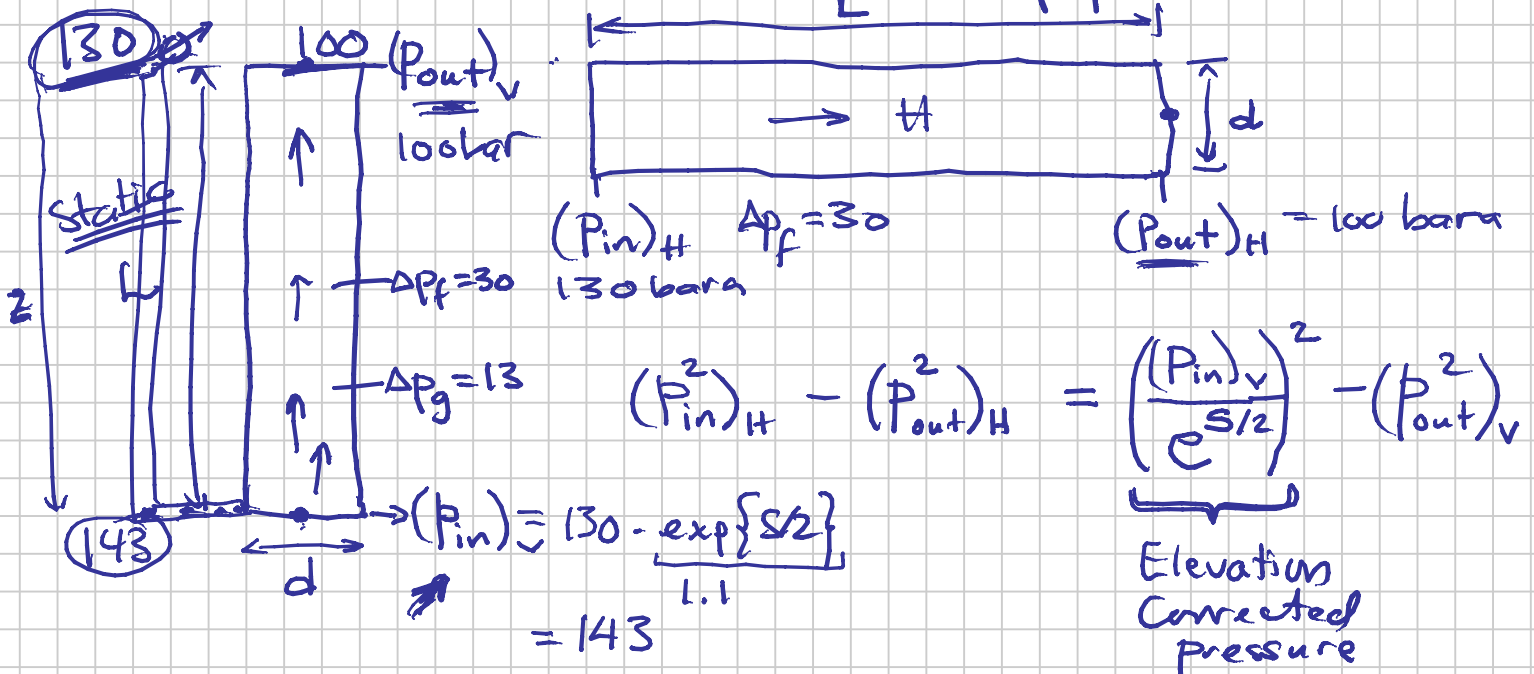
## Gas Flow through (Tubing) Pipe



$P_t, P_w, P_c$  all SURFACE elevation pressures

$$\frac{P_{wf}}{P_w} = \frac{P_R}{P_c} = \exp \left\{ \frac{Mg}{RTz} \right\}$$

## Friction Pressure Losses in pipe:

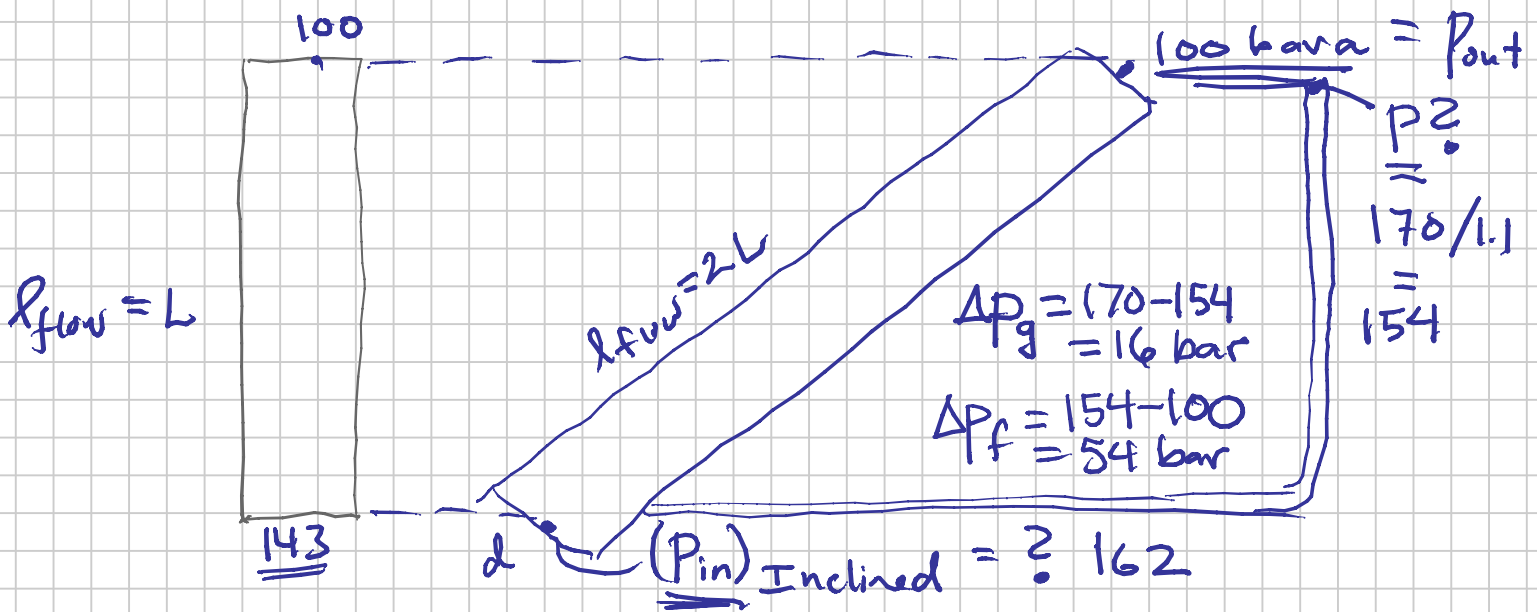


$\rho \approx \rho$

$$S \equiv \frac{2 \cdot Mg}{RTz} L \quad \text{TVD}$$

$$\rightarrow \frac{P_1}{P_2} = \exp \left\{ \frac{Mg}{RTz} \Delta z \right\}$$

$$P_1 > P_2 \quad \Delta z > 0$$



$$(\Delta P_f^2) \propto l_{flow}$$

$$\left( \frac{(P_{in})_I}{1.1} \right)^2 - P_{out}^2 = 2 \cdot \Delta P_f^2 = 2 \cdot (130^2 - 100^2)$$

$l_{flow} = 13800$

$$\Rightarrow (P_{in})_I = \left[ (13800 + 100^2) \right]^{1/2} \cdot 1.1 = 170 \text{ bara}$$

# Summary:

$$\dot{m} = a \cdot (\bar{P}_{in}^2 - \bar{P}_{out}^2)^{1/2}$$

Volumetric surface gas rate  $q_g = \dot{m} \cdot (\rho_g)_{sc}^{-1}$   
 $\frac{kg}{s} \cdot \frac{1}{\frac{kg}{m^3}}$

$$q_g = C (\bar{P}_{in}^2 - \bar{P}_{out}^2)^{1/2}$$

$\bar{P}$  = equivalent pressure at a common depth reference "datum" @  $\bar{z}$

$$\bar{P}(\bar{z}) = P(z) \cdot \exp \left\{ \frac{Mg}{RT\bar{z}_1} \cdot (z - \bar{z}) \right\}$$

↑  
Gas Z-factor!

## Tubing Flow Constant $C_T$ or "T"

↑ Fetkovich paper

Units, Mair, "2"

where  $S = 0.0375 GH/T_a Z_a$  (This S should not be confused with skin)

and  $e =$  natural log

$G =$  gas gravity ( $\gamma_g$ )

$H =$  vertical depth, ft TVD

$T_a =$  average temperature,  $^{\circ}R$   $\bar{T}$

$Z_a =$  gas deviation factor at average pressure

$$P_c^2 = P_R^2 / e^S$$

$$P_w^2 = P_{wf}^2 / e^S$$

$$Q = Mscfd$$

$$M = Mair \cdot \gamma_g$$

$$P_c / P_R = 1/e^{S/2}; \quad \bar{P}^2 = P_c^2 \cdot \exp\{S\}$$

$$q_g \quad Mscf/D = 10^3 \text{ scf/D}$$

## Tubing Friction Curves

The basic equation relating wellhead static column pressure  $p_w$  and the wellhead flowing tubing pressure  $P_t$  as given in the IOCC Manual<sup>(4)</sup> is

$$P_{wf}^2 = e^s P_t^2 + \left( \frac{F_r Q T_a Z_a}{31.62} \right)^2 (e^s - 1) \quad \dots(21)$$

where

$$F_r = \frac{0.10797}{D^{2.612}}$$

with D in inches.

Dividing both sides by  $e^s$ , we obtain

$$\frac{P_{wf}^2}{e^s} = P_t^2 + \left( \frac{F_r Q T_a Z_a}{31.62} \right)^2 \frac{(e^s - 1)}{e^s} \quad \dots(22)$$

with  $P_{wf}^2/e^s = p_w^2$  we can rearrange and obtain

$$Q = \left[ \frac{31.62 e^{s/2}}{\sqrt{(e^s - 1) F_r T_a Z_a}} \right] (P_w^2 - P_t^2)^{0.5} \quad \dots(23)$$

The general form is simply

$$Q = T (P_w^2 - P_t^2)^{0.5} \quad \dots(24)$$

$$q_g = C_T (P_w^2 - P_t^2)^{0.5}$$

↑  
 $\tilde{P}_{wf}$   
 corrected  
 to  
 surface  
 datum

$$C_T = \frac{31.62 e^{s/2}}{\sqrt{e^s - 1} T_a Z_a} \cdot \left\{ \frac{1}{F_r} \right\} = \text{const.} \cdot d_T^{2.6}$$

Given well tubing  $\approx$  constant

↑  
 $\left\{ \frac{1}{F_r} \right\} \approx d_T^{2.6}$

Main engineering parameter controlling

$\Delta P_f$

## Gas Rate Eq.

$$\text{Darcy: } q_g = C_R (P_R^2 - P_{wf}^2)$$

low-p approx.

$$C_R = \frac{(kh)}{(\quad) \left[ \ln \frac{r_e}{r_w} - \frac{3}{4} + s \right]}$$

## Tubing

$$q_g = C_T (P_w^2 - P_t^2)^{0.5}$$

↑  
Control!

Bring all pressures to surface datum

$$P_w, P_t \quad \checkmark$$

$$P_c = P_R / e^{s/2}$$

$$P_R^2 = P_c^2 \cdot e^s$$

$$P_w = P_{wf} / e^{s/2}$$

$$P_{wf}^2 = P_w^2 \cdot e^s$$

$$q_g = \{ C_R \cdot e^s \} (P_c^2 - P_w^2)$$

$$q_g = C'_R (P_c^2 - P_w^2)$$

$$q_g = C_T (P_w^2 - P_t^2)^{1/2}$$

} Combine

"R" ΔP

$$P_c^2 - P_w^2 = \frac{1}{C_R} qg$$

"T" ΔP

$$P_w^2 - P_t^2 = \frac{1}{C_T} qg^2$$

$$(P_c^2 - P_t^2) = \frac{1}{C_T} qg^2 + \left(\frac{1}{C_R}\right) qg$$

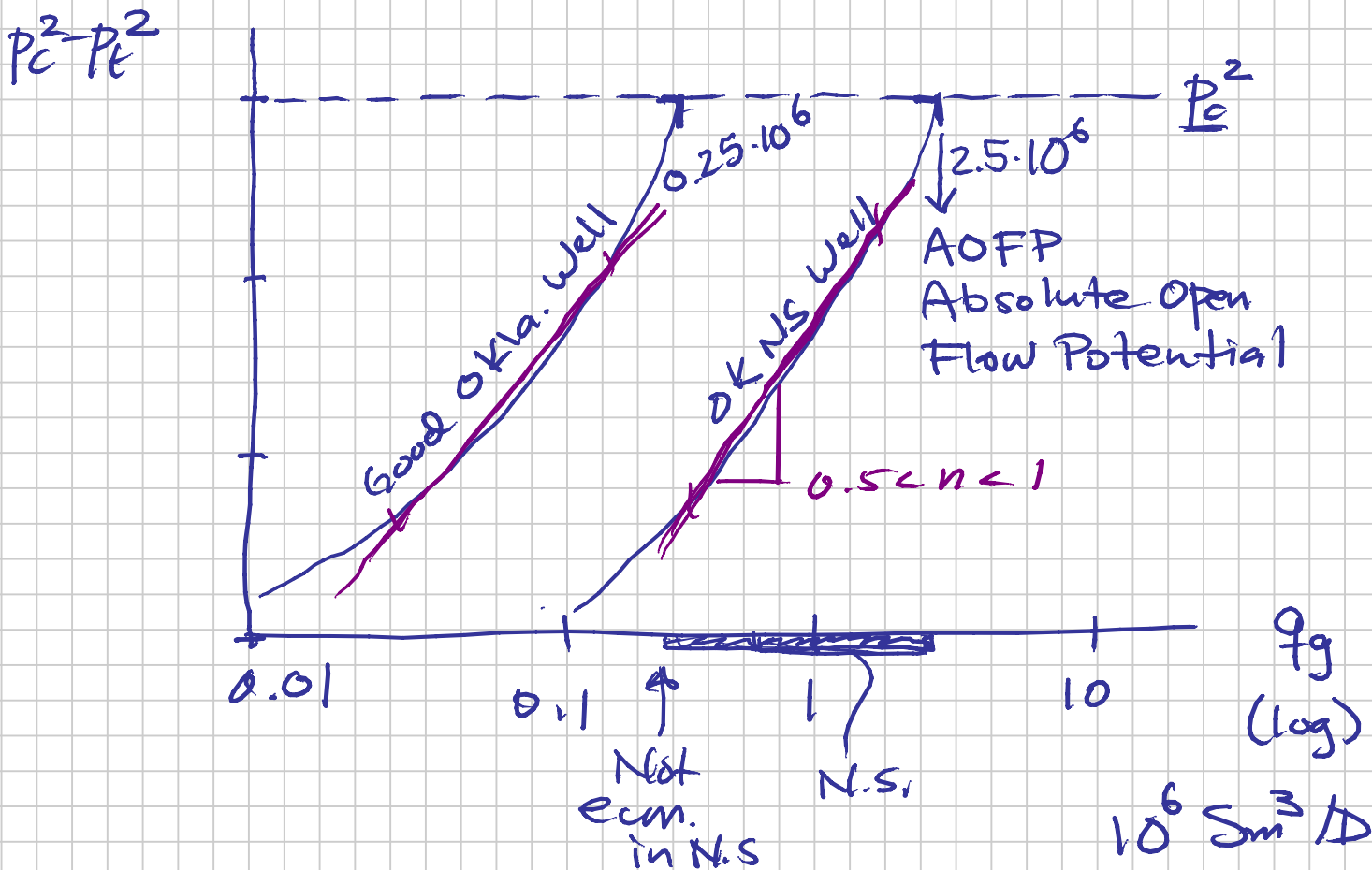
Quadratic Eq q

$$0 = ax^2 + bx + c$$

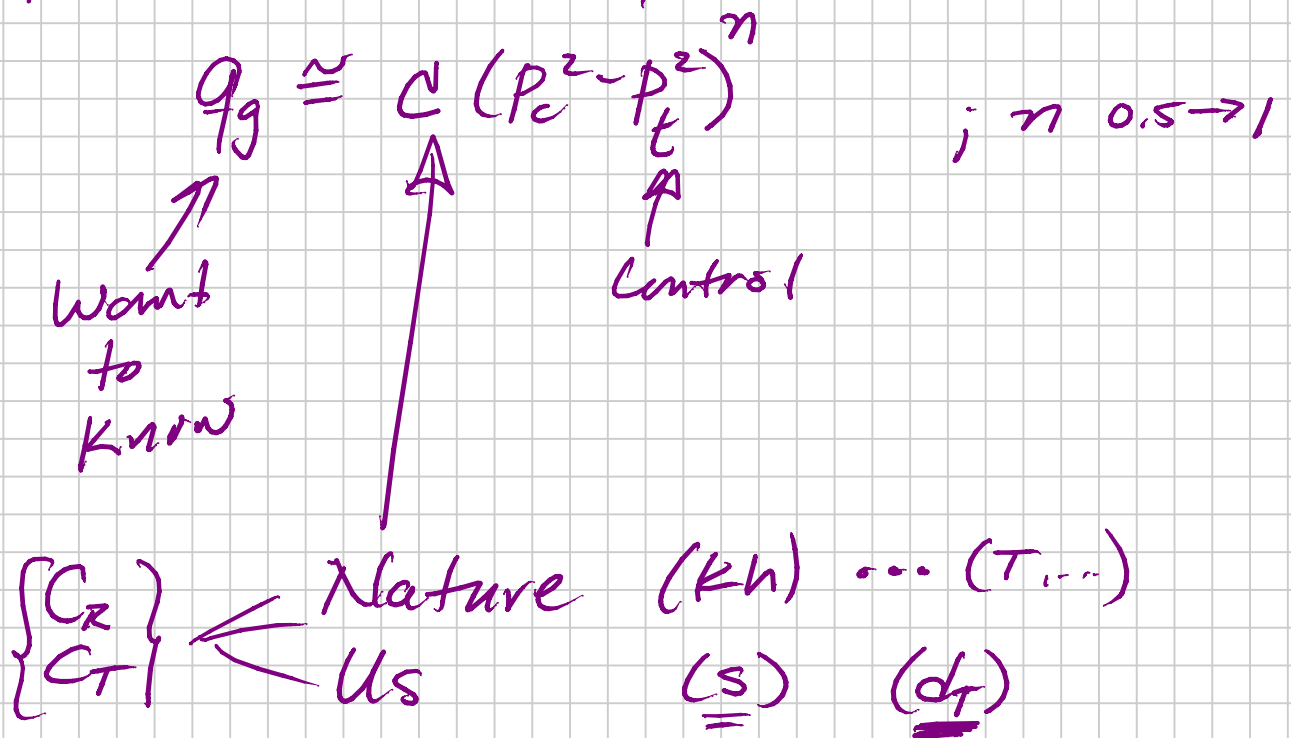
$$qg = \frac{-C_R^{-1} + \left[ C_R^{-2} - 4\left(\frac{1}{C_T}\right) \Delta P^2 \right]^{1/2}}{2\left(\frac{1}{C_T}\right)}$$

$$(P_c^2 - P_t^2) = \Delta P^2$$

Wellhead (Tubinghead) Backpressure Eq.



# Approximate "Model" Eq.



$$\beta \propto \left(\frac{1}{k}\right)$$

Forchheimer Rock Property

190x

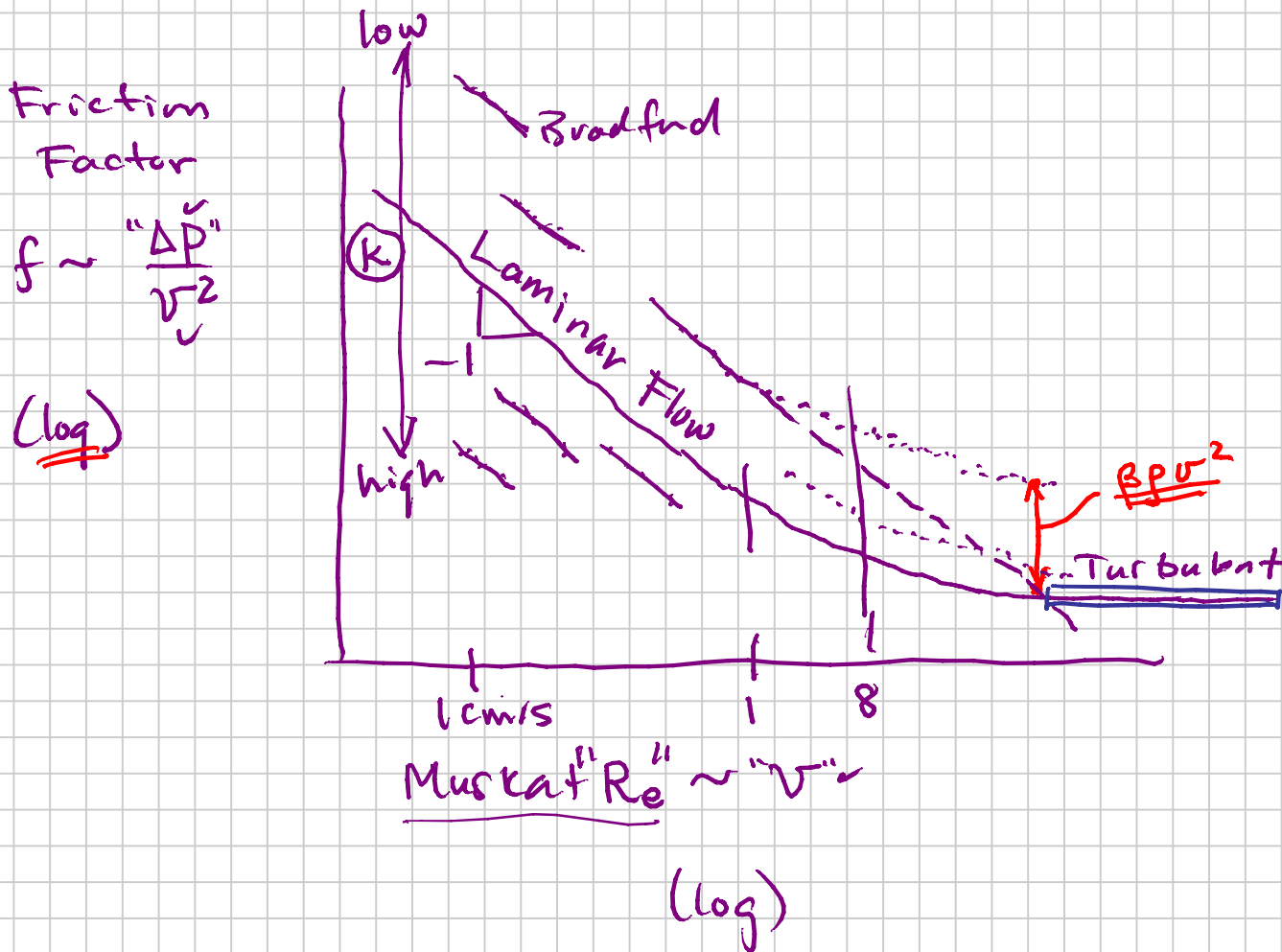
$$\frac{dp}{dx} = \underbrace{\left(\frac{\mu}{k}\right) v}_{\text{Darcy}} + \underbrace{\beta \rho v^2}_{*}$$

Darcy

$$\frac{dp}{dx} = \left(\frac{\mu}{k}\right) v$$

Morris Muskat

1930s



Gas Flow in Pipe:

$$q_g = C_p (P_{in}^2 - P_{out}^2)^{0.5}$$

⇒ f = constant  
"totally" turbulent flow.

$$f_g = \frac{\Delta P}{\rho v^2}$$

vs q



$$q \propto v$$

$$\frac{\Delta p^2}{v^2} \text{ vs } v$$

$$q_{fg} = \left[ \frac{kh}{T_R \left[ \ln \frac{r_e}{r_w} - \frac{3}{4} + s \right]} \right] \left[ \frac{P}{\mu Z} \frac{dp}{p_{wf}} \right]$$

$\sim 8$   
 " " "

$p_{PR} - p_{pwf} = \underline{\underline{\Delta p_p}}$

Forchheimer "Modification"

Constant  $\alpha \beta$

Total  $s_t = s^* + (D q_{fg}) \Rightarrow \left\{ \begin{array}{l} \text{Rate-dependent} \\ \text{Skin} \end{array} \right\}$

Constant:

- Damage
- Flow Geometry

$$\frac{D q_{fg}}{\mu} \sim 1 \rightarrow 100$$

⊗ Needs to be quantified.

Not to be used  $\rightarrow \beta = a k^{b v^{-1}}$  X

$\Rightarrow$  Tend to estimate a D value

$\sim 10$  times too small

### NOMENCLATURE

- $f$  = FRICTION FACTOR  
 $d$  = DIAMETER OF AVERAGE GRAIN  
 $\Delta p$  = PRESSURE DROP  
 $L$  = LENGTH OF CORE  
 $\gamma$  = FLUID DENSITY  
 $V$  = VELOCITY  
 $Q$  = RATE OF FLOW  
 $A$  = GROSS-SECTIONAL AREA  
 $\mu$  = ABSOLUTE VISCOSITY

### SYMBOLS

- $\square$  OIL } PRESENT  
 $\circ$  WATER } DATA  
 $\triangle$  AIR }  
 $\square$  OIL, W.F. CLOUD  
 $\cdot$  GAS, U.S. BUREAU OF MINES

SAMPLE NO.	SAND	PERCENT
<u>CONSOLIDATED</u>		
1	BRADFORD	12.5
2	BRADFORD	12.3
3	3rd VENANGO	16.9
4	CERAMIC A	31.0
5	ROBINSON	20.8
6	CERAMIC B	37.8
7	WOODBINE	19.7
8	WILCOX	19.9
9	3rd VENANGO	11.9
10	ROBINSON	19.5
11	ROBINSON	19.4
12	3rd VENANGO	23.3
13	WILCOX	16.3
14	WARREN	19.2
15	3rd VENANGO	21.4
16	ROBINSON	20.6
17	CERAMIC C	33.2
18	3rd VENANGO	21.4
19	WOODBINE	23.9
20	"	26.9
21	"	27.7
22	"	22.1
23	"	29.8
<u>UNCONSOLIDATED</u>		
24	FLINT	36.5
25	OTTAWA	30.3
26	20-30 OTTAWA	24.5
27	LEAD SHOT	24.2

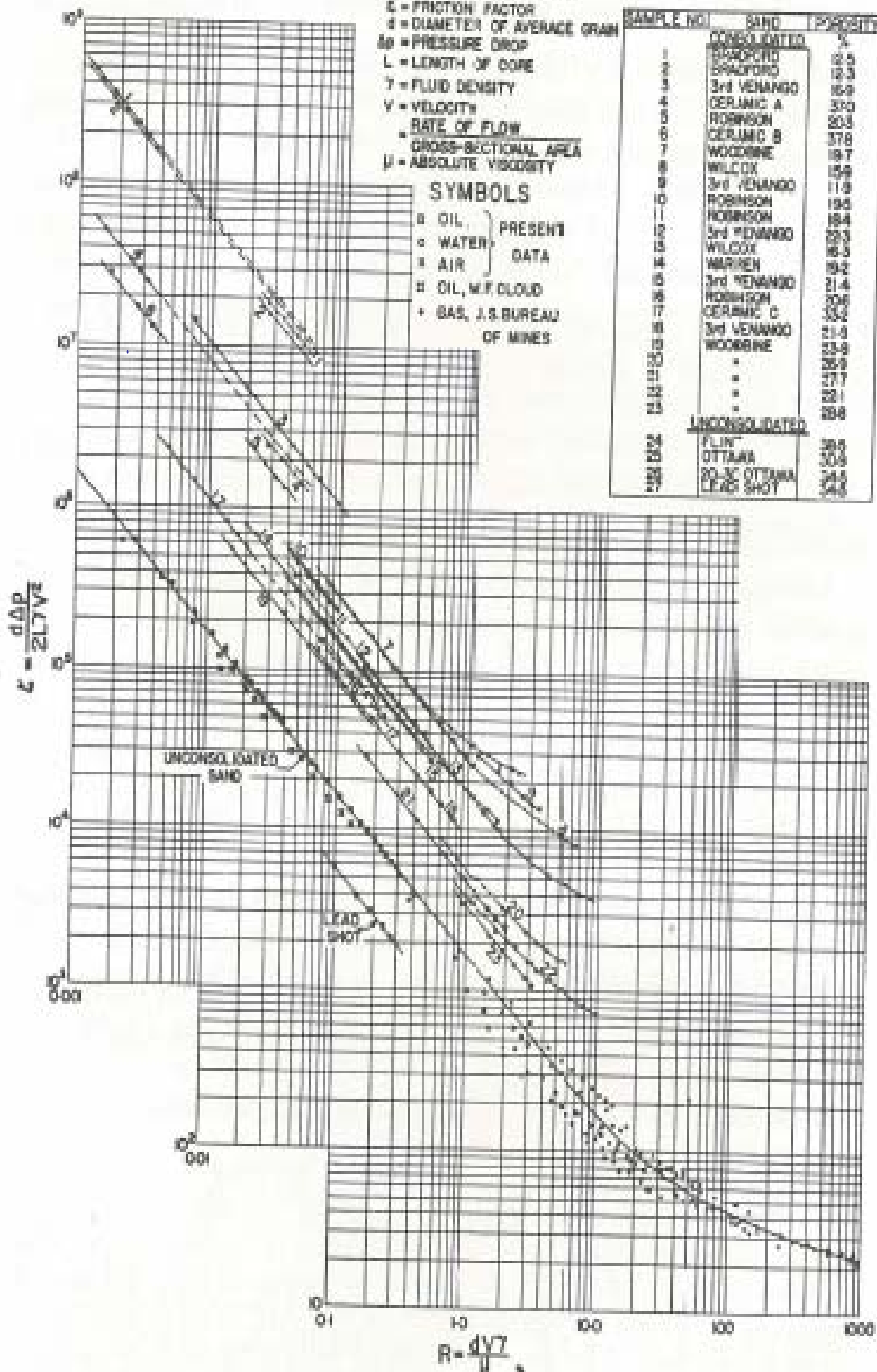


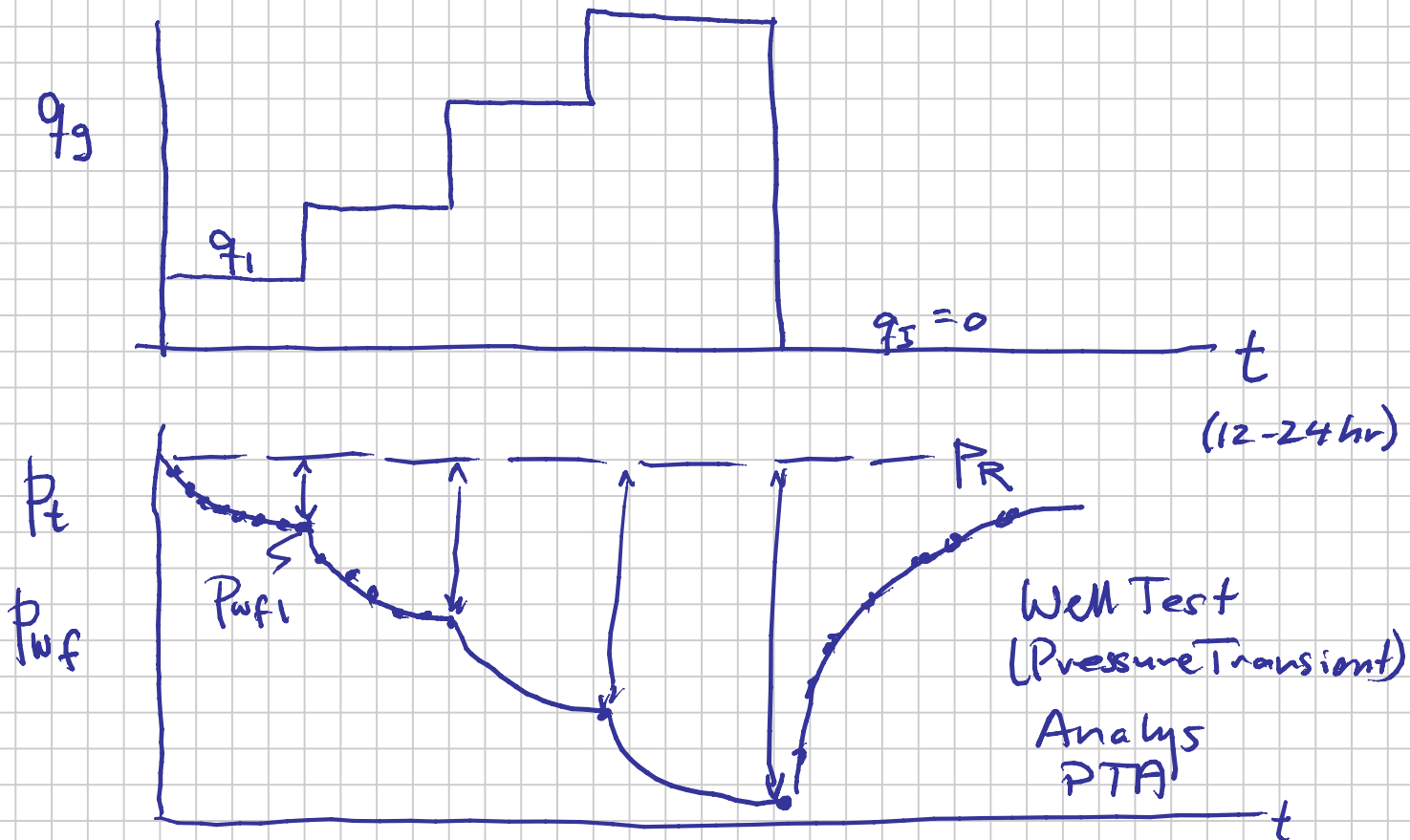
FIG. 8.—Friction-factor chart for the flow of fluids through sands. (After Fancher, Lewis, and Barnes, Bull. Pa. State Expt. Sta.)

# Need for MULTI-RATE TESTING OF (ALL) GAS WELLS

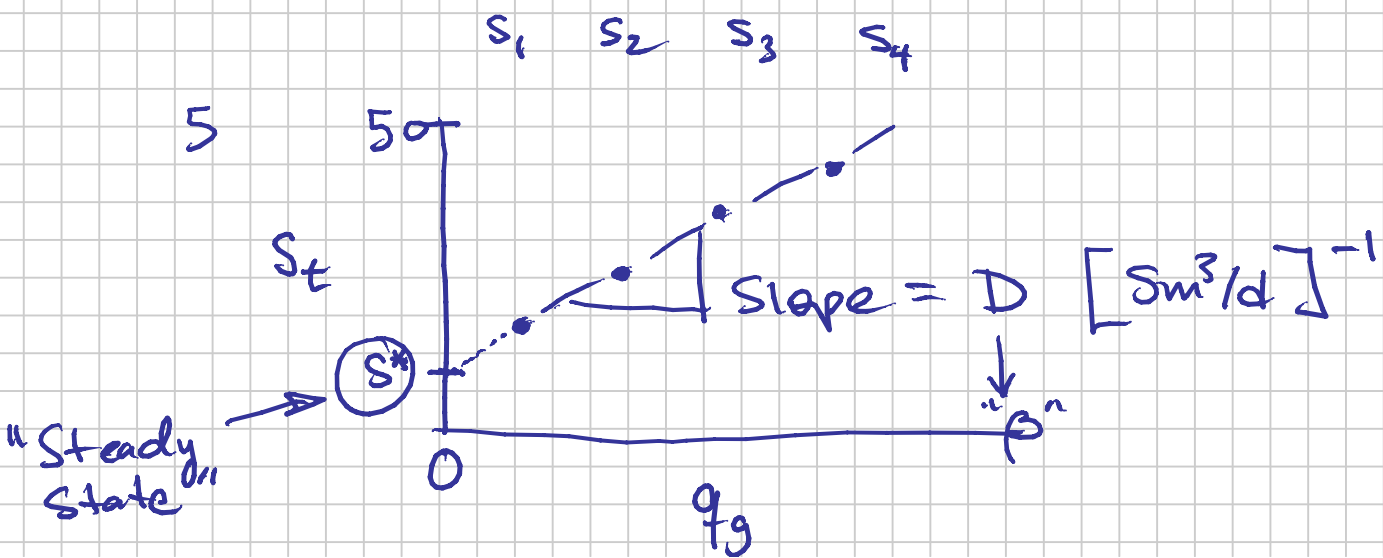
⇒ Fetkovich

Multi-Rate Test Methods:

## ① Isochronal



know:  $(kh)$



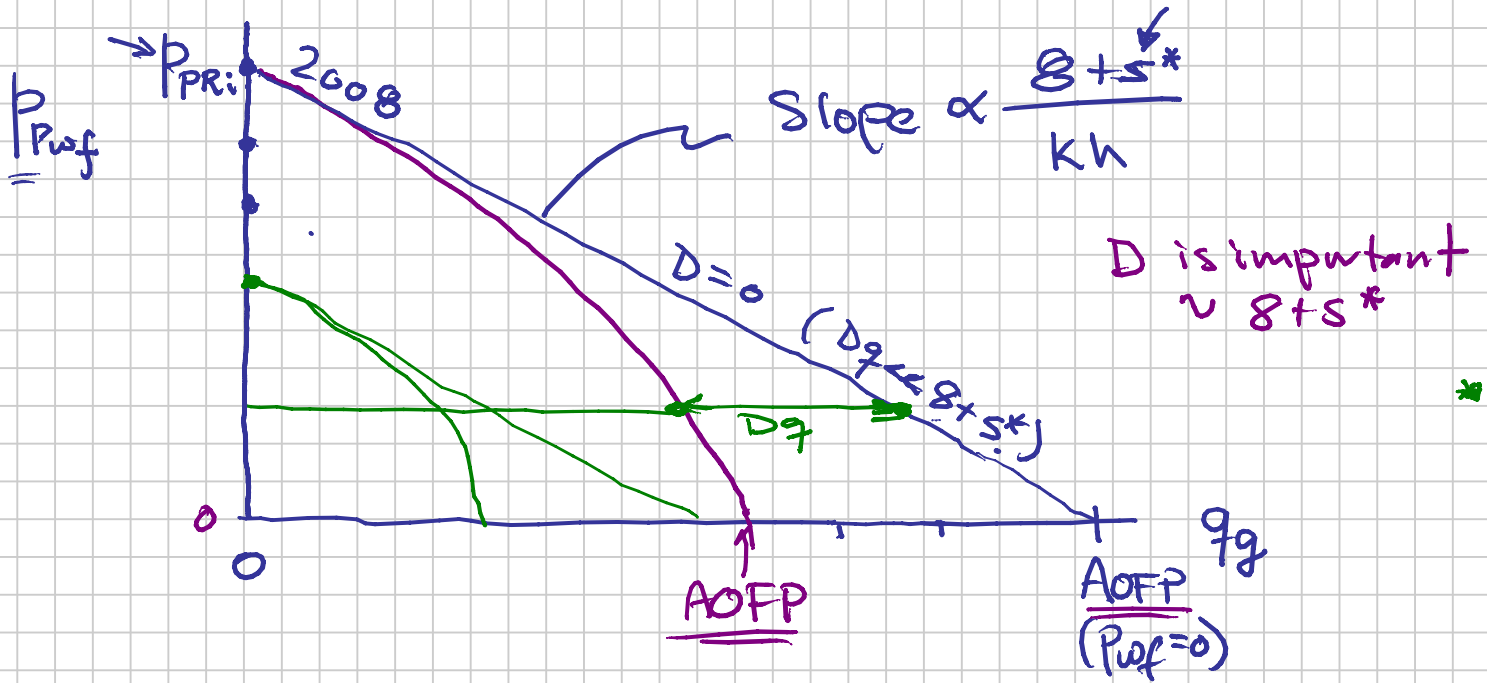
# Damage + Flow Geometry

## Resolve Rate equation

$$A q_g^2 + B q_g + \Delta p_p = 0$$

## Graphical Representation

### ① Inflow Performance Relation (IPR)



### ② Backpressure Plot: Fetkovich

# Gas Rate Eq.

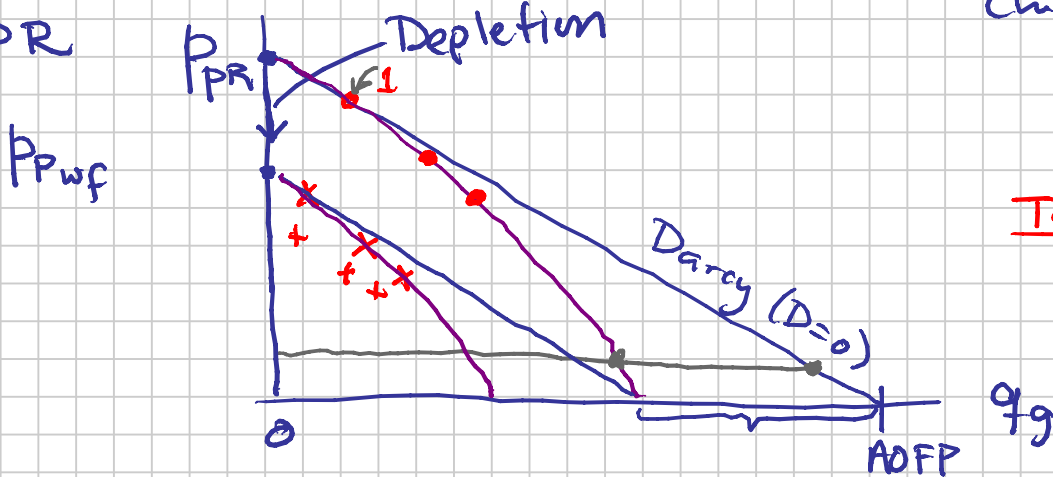
Based on Forchheimer "Law"

Graphical Presentation

$$q_g = \frac{KH (P_{PR} - P_{pwf})}{T_R \left[ \ln \frac{r_e}{r_w} + s^* + Dq_g \right]}$$

$\uparrow$   $\uparrow$   
 Const Rate Dependent

(1) IPR



Test Data

● 2008

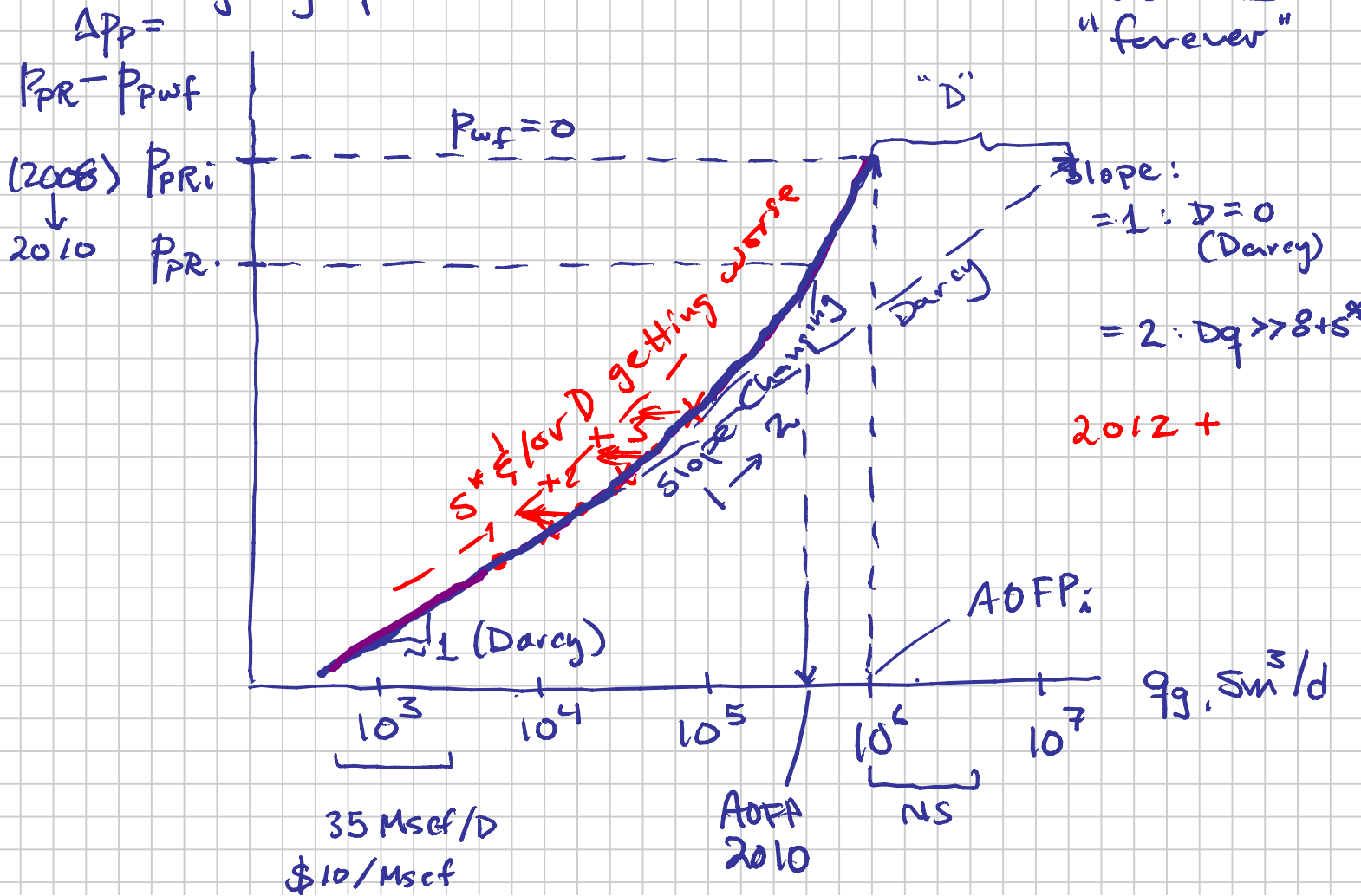
× 2010

Prosper

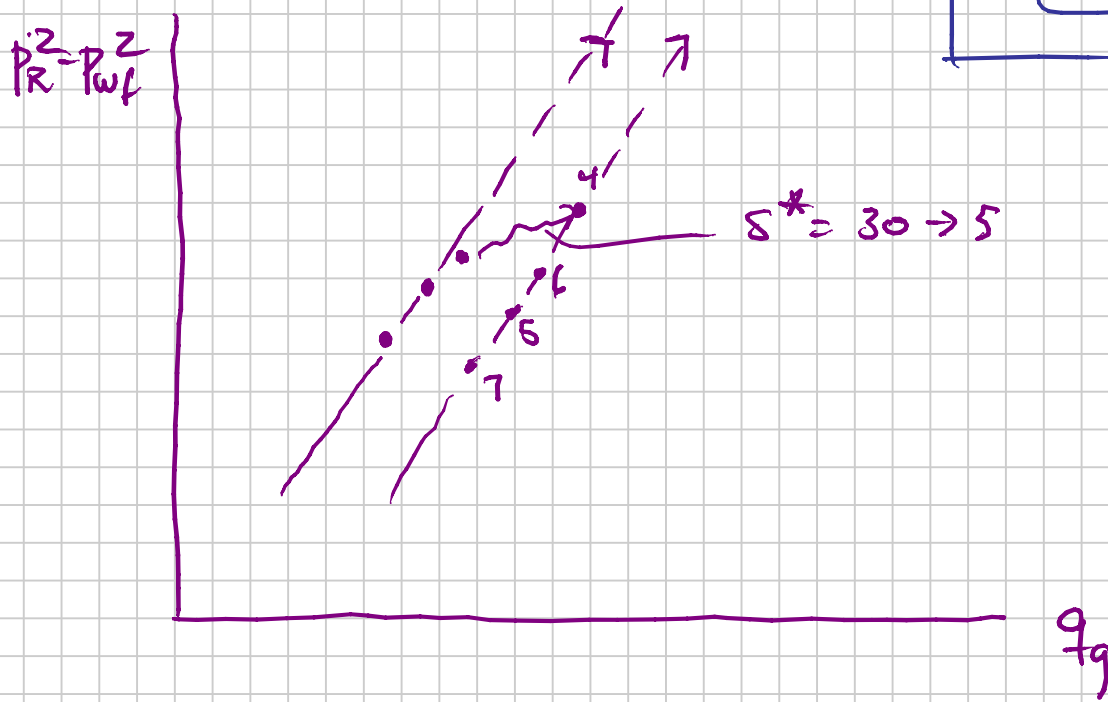
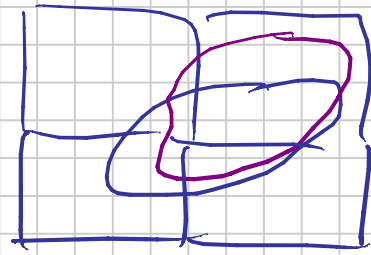
(2) Backpressure Plot (Diagnostic)

log-log plot

$\bar{K}h, s^*, D$   
 Constants  
 "forever"



Fetters



Combine R + T =>

$$q_g (p_R, p_t)$$

↑  
Surface  
Flowing  
Tubing

R:  $\bar{p}_R \rightarrow p_{wf}$  : "Darcy"  $\Rightarrow$  Forchheimer

T:  $p_{wf} \rightarrow p_t$

$p^2$  is an ok  $\sim$  of  $p_p$  ( $p_R < 200^+$  bar)

$$(P_R^2 - P_w^2) = A' q_g + B' q_g^2 \quad \text{"R"}$$

$$\begin{aligned} &\uparrow & \uparrow \\ &\text{Darcy} & \text{"D"} \\ &\frac{q_g}{kh, S^*} & \end{aligned}$$

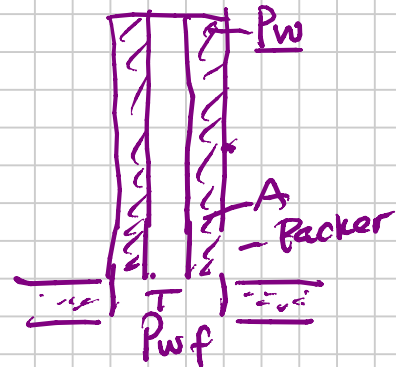
$$P_c^2 - P_w^2 = A_R q_g + B_R q_g^2 \quad \text{"R"}$$

↑  
Surface Pressure of an Observing Well

↑  
Flowing Annulus Pressure w/NO Packer

$$\begin{aligned} A_R &= A'_R e^{s/2} \\ B_R &= B'_R e^{s/2} \end{aligned}$$

$$\frac{P_R}{P_c} = e^{s/2} \sim 1. x = \frac{P_w}{P_c}$$



$$P_c^2 - P_w^2 = A_R q_g + B_R q_g^2 \quad \text{"R"}$$

$$P_w^2 - P_t^2 = B_T q_g^2 \quad \text{"T"}$$

Add

$$(P_c^2 - P_t^2) = A_R q_g + (B_R + B_T) q_g^2$$

$$\begin{aligned} q_g &= C_T (P_w^2 - P_t^2)^{1/2} \\ B_T &= \frac{1}{C_T^2} \end{aligned}$$

$$(P_c^2 - P_t^2) = \bar{A}_R q_g + B q_g^2$$

$$B_T \propto (d_T)^{2.6}$$

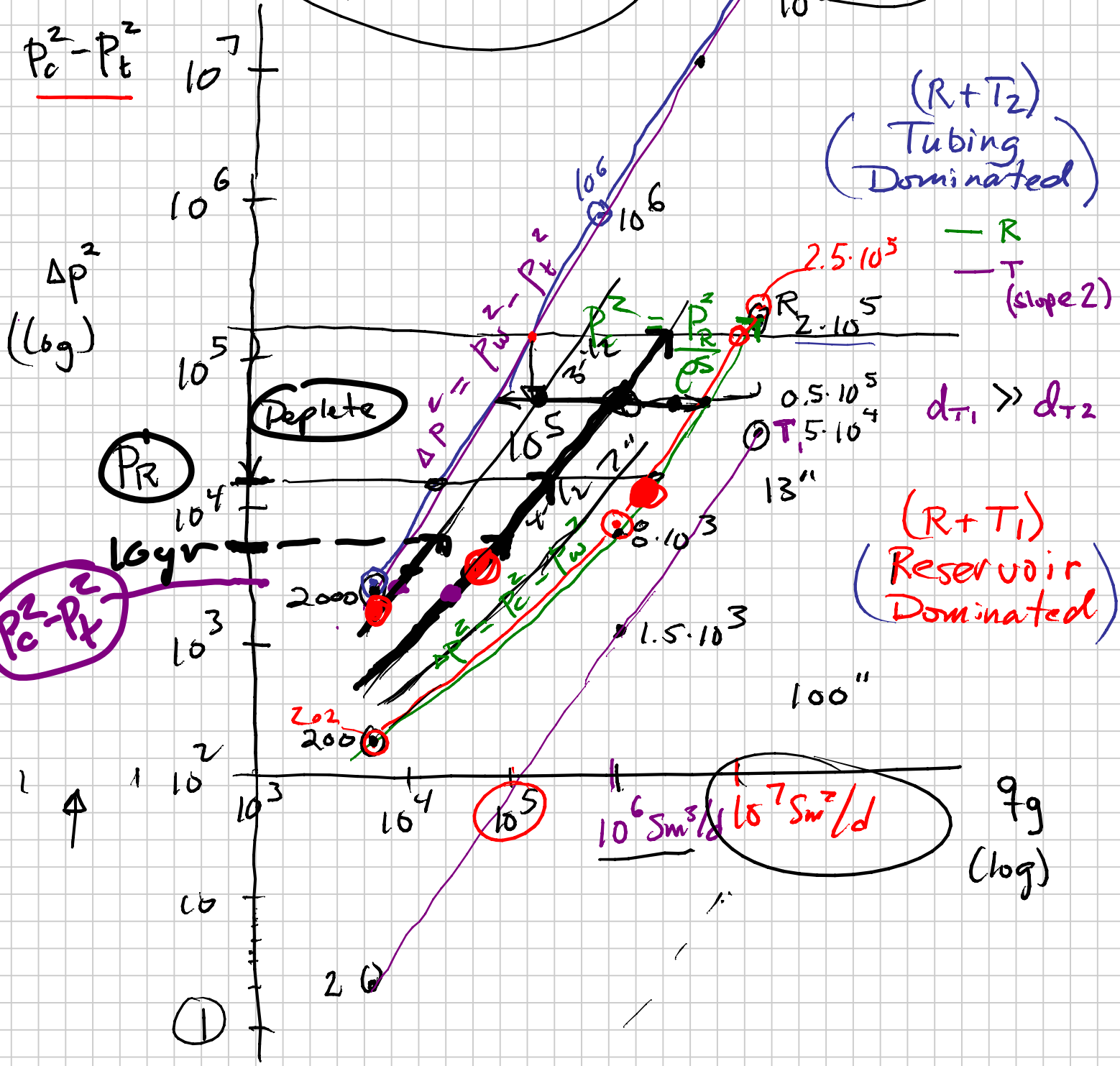
↑  
We Central

$\frac{kh}{s^*}$

Like to have max.  $q_g$  for a given  $p_t$

Price( $d_T > 7''$ )

IKEA (Straw)  $2\frac{3}{8}''$



(R+T<sub>2</sub>)  
Tubing  
Dominated

— R  
— T (slope 2)

$d_{T1} > d_{T2}$

(R+T<sub>1</sub>)  
Reservoir  
Dominated

$10^6 \text{ Sm}^3/\text{d}$   $10^7 \text{ Sm}^3/\text{d}$

$q_g$   
(log)

$P_c^2 - P_t^2$

$\Delta p^2$   
(log)

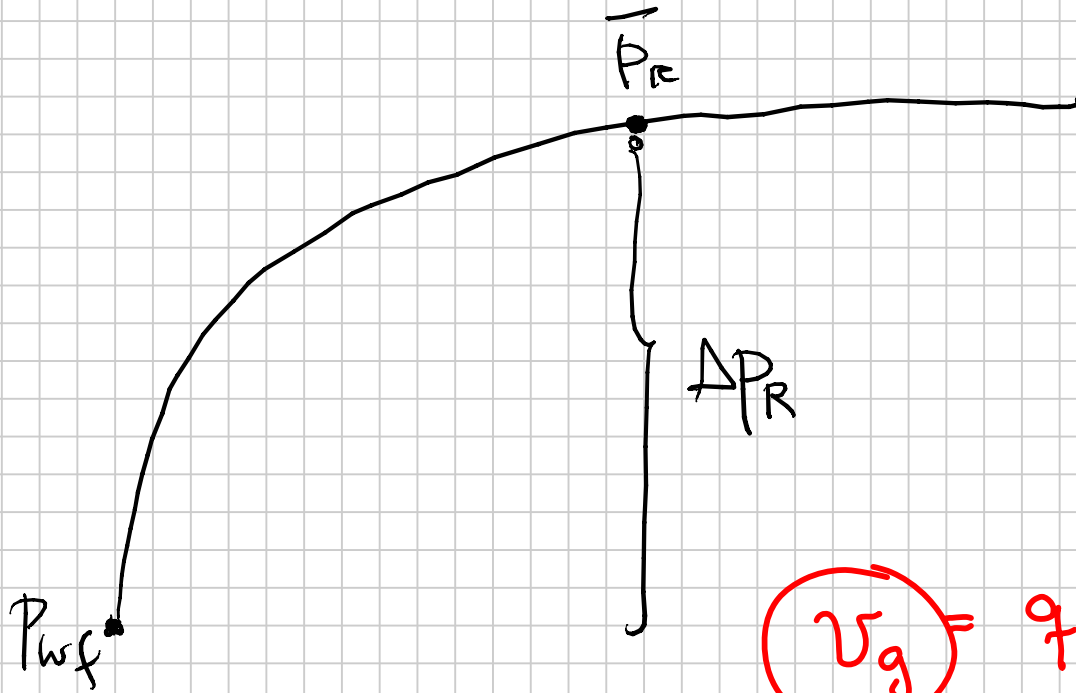
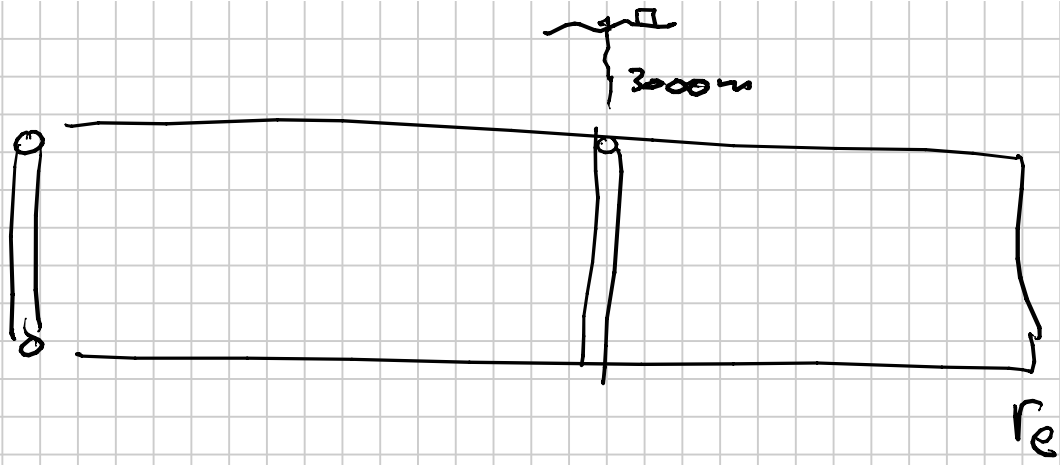
$P_R$

$P_c^2 - P_t^2$

↑

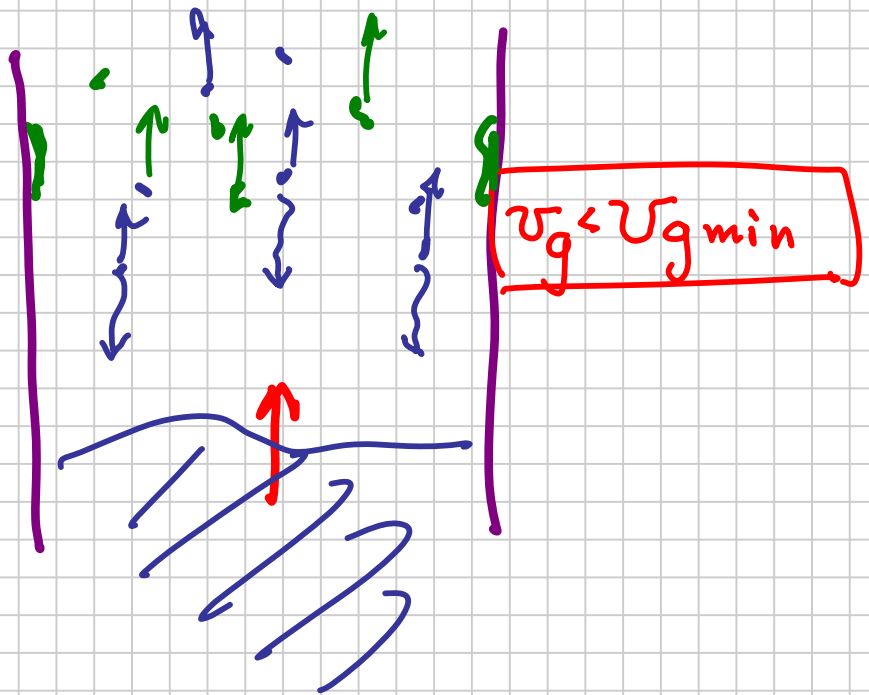
①





$$v_g = 979 / \pi \left( \frac{d_p}{2} \right)^2$$

Liquid  
Lifting

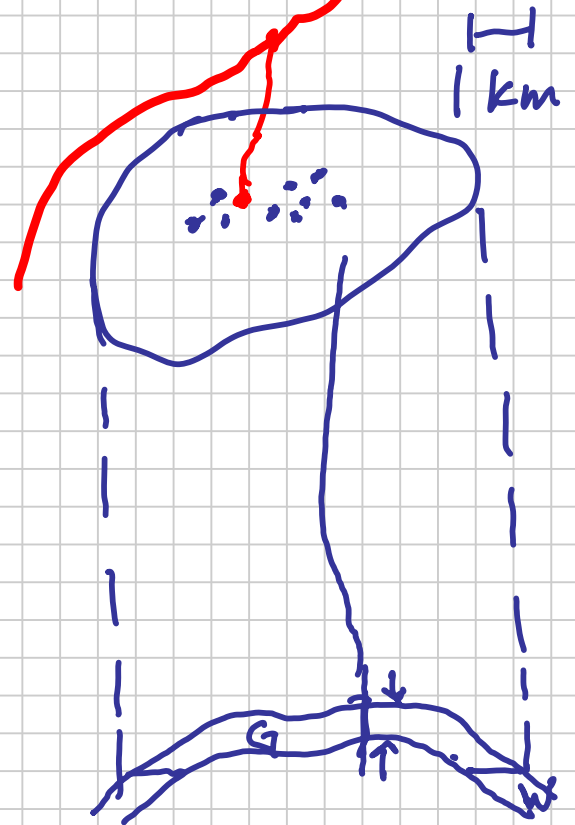


① Estimate Initial Gas in Place (IGIP),  $G$  [ $\text{Sm}^3$ ]

•  $\bar{A}, \bar{h}_g, \bar{\phi}, \bar{S}_w$

Hydrocarbon PV  
HCPV

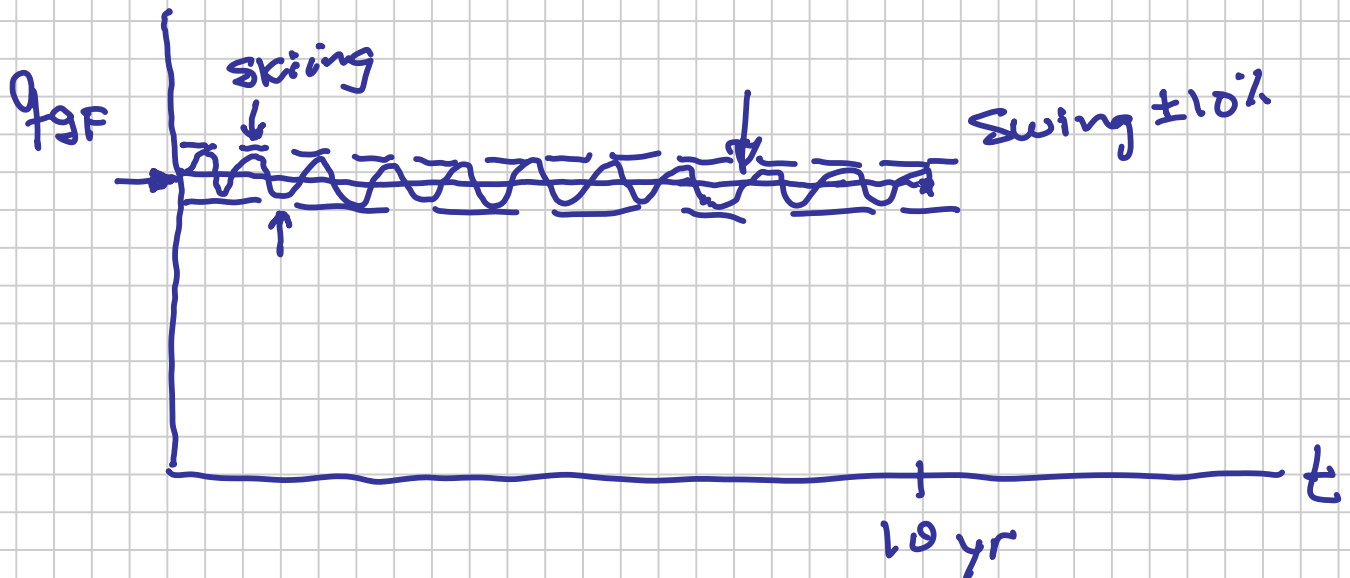
$V_{PR} = (1 - \bar{S}_w)$   
PV HC



② Purchaser specifies

$q_{GF} \sim \% G / \text{year}$

(3-6) x Contract Period  $< 50\% G$   
(Plateau)  
(7-15 yr)

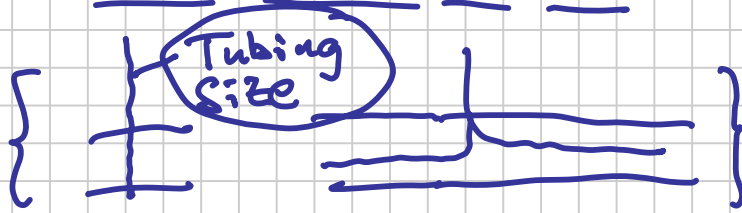


How many wells do we need to satisfy the GEMAS for 10 yr?

$N_w$

Major Cost Variable

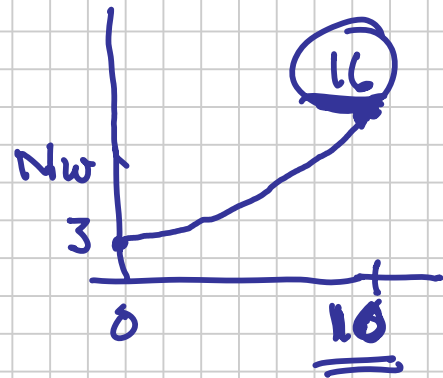
What kind of wells



How do we do this?

①  $q_{gF}$  defined ( $\pm$ )

②  $q_{gF} = \underbrace{q_{gw}(t)}_{= \text{constant}} - \underbrace{N_w(t)}_{= ?}$



$q_{gw}(t) = f(A, B, \Delta p^2)$

$\Delta p^2 = p_c^2 - p_t^2$

Well rate expected when delivering against "back pressure"

$p_c(t) = \frac{P_R(t)}{L e^{srz}}$

$p_{tmin} = \text{constant}$

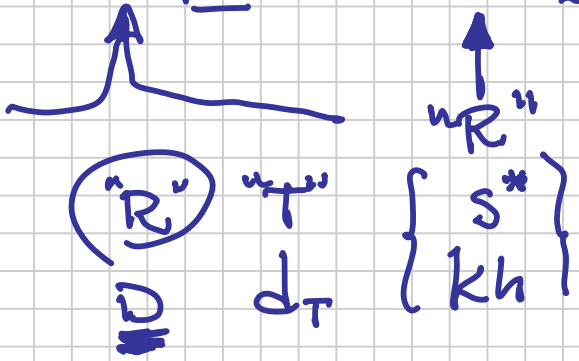
Constraint on how low this can be

- Pipeline pressure (no compression)
- Compressor

( $B_R + B_T$ ) Simple eqs.

inlet

$$B q_{gw}^2 + A q_{gw} - \underline{\underline{\Delta p^2}} = 0$$



$$A q_{gw} = \Delta p^2 \quad \text{Darcy Law}$$



$$q_{gw} = \frac{kh (P_R^2 - P_w^2)}{\underbrace{\ln \frac{r_e}{r_w} - \frac{3}{4}}_{7-9} + S^*}$$

$r_e$

$k(x,y,z) \sim \text{const.}$



$$A_w \approx \pi r_e^2$$

$$A_w \approx \frac{A_R}{N_w}$$

# PROBLEM DESCRIPTION

- $Q = 10 \text{ Tcf} \leftarrow \bar{\phi} \bar{S}_w \bar{h}$   
 $\pm 2 \text{ Tcf}$
- $1 \text{ Tcf} = 10^{12} \text{ scf}$   
 $35.31 \text{ scf / Sm}^3$
- after 1 discovery exploration well

- Germans want to buy 4%  $Q$  / year, 12 yr  
 (you can optionally solve for 10% swing)

## Initial Well Test Results

$$q_g = 1.5 \cdot 10^6 \text{ Sm}^3/\text{d}$$

$$p_{Ri} = 250 \text{ bara} \quad (T_R = 80^\circ\text{C})$$

$$\text{Depth } 2500 \text{ m BSL} \quad \sim 8000'$$

$$\Delta p_R = 30 \text{ bar}$$

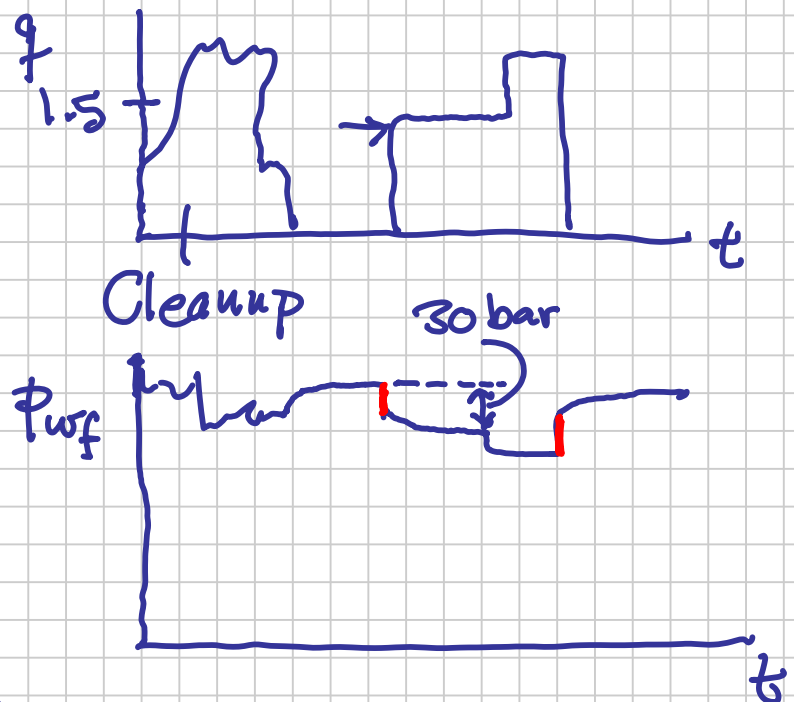
$$\Delta p_{S^*} = 15 \text{ bar}$$

$$\Delta p_{\text{TD}} \sim 0$$

$$\left\{ \begin{array}{l} \Delta p_T \sim 100 \text{ bar} \\ @ 1.5 \cdot 10^6 \text{ Sm}^3/\text{d} \\ \text{rate} \end{array} \right.$$

$$\text{Test Tubing } d_T = 3.5'' \text{ ID}$$

$$h(x,y) = \bar{h} = \text{constant}$$



$$p_{tmin} = 50 \text{ bara}$$

PVT: e-note on gas-prf

$$\bar{\gamma}_g = 0.7$$

Tubing:    5-1/2"    7"    9"    OD  
                  ~5"    ~6.5"    8.5"    ID    ?

$$e^{S/2} = 1.2$$

---

Solve:

- Assume Production Wells  $S^* = 0$ .

- 7" OD tubing as "base" case
- $N_w$  at 12\_yr.