

Final Topics (Next Three Weeks)

① Gas Rate Equation (Reservoir / Tubing)

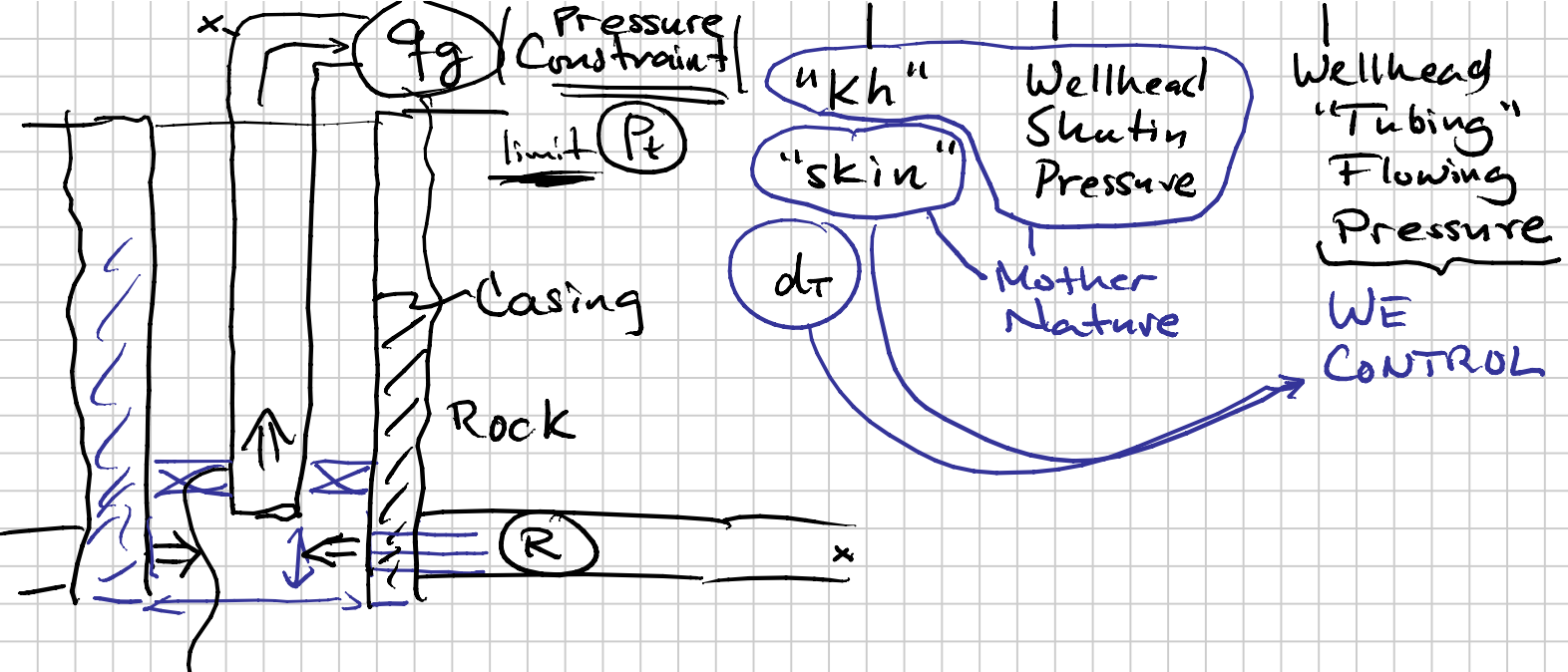
- (R) {
  - "Inflow Performance Relation" (IPR)
  - Backpressure Equation
- Darcy's Law or Forcheimer Eq.

- (T) • Tubing Flow
  - "Vertical Lift Curve" (VLC)

• Composite (R+T)

- "Wellhead Backpressure Equation" 0.5-1

$$\underline{(q_g)_{well}} = C_{WH} (P_c^2 - P_t^2)^{n_{WH}}$$



Tubing (O.D.  $2\frac{3}{8}" \rightarrow 7" \rightarrow 9\frac{5}{8}" \rightarrow 13"$ )  
 $d_t$  We control

Baku  
 "Old Technology"  
 - Casing  
 - Hanging Tubing

RESERVOIR RATE EQUATION

Darcy's Law:

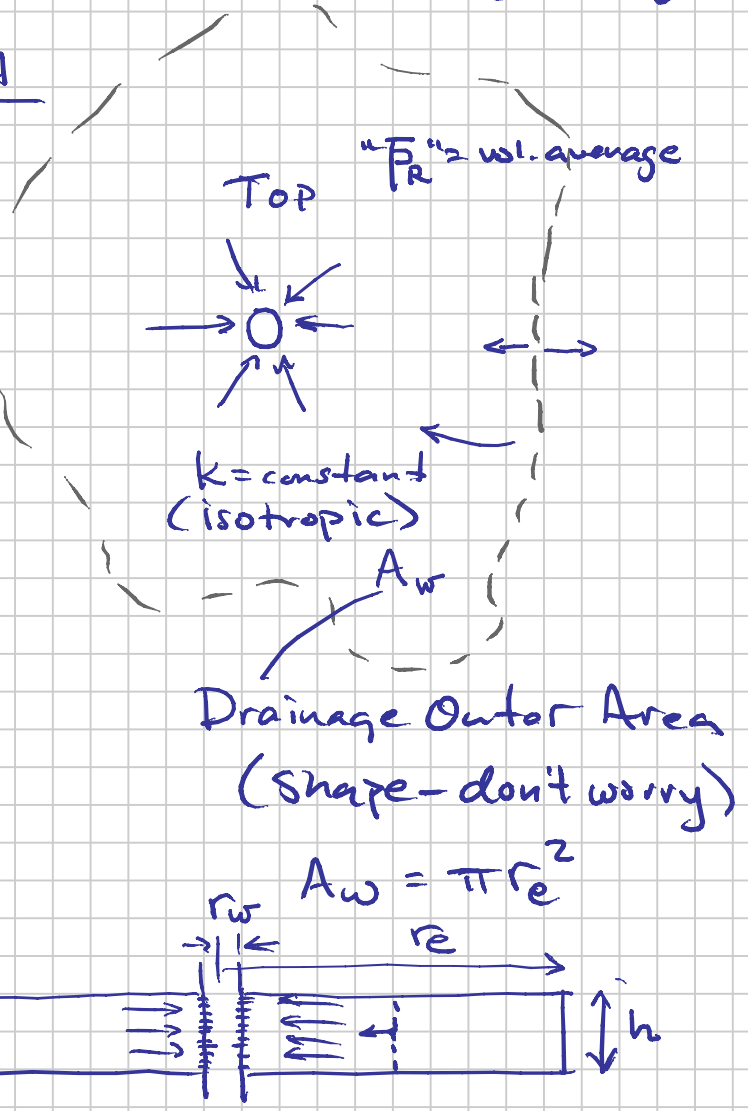
$$v = \frac{k}{\mu} \cdot \frac{dp}{dr}$$

↑  
 Darcy Velocity  
 ≠  
 Pore Velocity

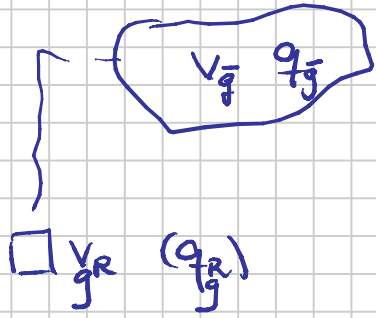
volumetric rate

$$q_R \left[ \frac{m^3}{s} \right] = v \cdot \underbrace{2\pi r h}_{A_z}$$

• p at r ("reservoir" rate)



Conversion from  $q_R$  to surface rate  $q_g$  (gas) in  $\frac{\text{Sm}^3}{\text{s}}$



$$B_g \equiv \frac{V_{gR}}{V_g} = \frac{q_{gR}}{q_g}$$

$$q_g = q_{gR} \cdot B_g^{-1}; \quad B_g = \frac{p_{sc}}{p} \cdot \frac{Z_{sc}}{Z}$$

$$= v \cdot 2\pi r h B_g^{-1}$$

↑  
Darcy's Law

$$q_g = \frac{k}{\mu_g} \cdot \frac{2\pi h}{B_g} \cdot r \frac{dp}{dr}$$

Constant Rate Assumption  
(Steady-State)

p-dependant

r-dependant

$\mu_g$   
 $B_g$

$r$

" $q_g$ "  $\frac{\text{moles}}{\text{day}}$   
= constant at ALL  $r$

$$\int_{r_w}^{r_e} \frac{1}{r} \cdot dr = \left\{ \frac{2\pi k h}{q_g} \right\} \int_{p_{wf}}^{p_e} \frac{1}{\mu_g B_g} dp$$

$$\ln \frac{r_e}{r_w} = \frac{2\pi k h}{q_g} \int$$

$p(r_w) \equiv p_{wf}$   
= wellbore flowing pressure  
BHFP

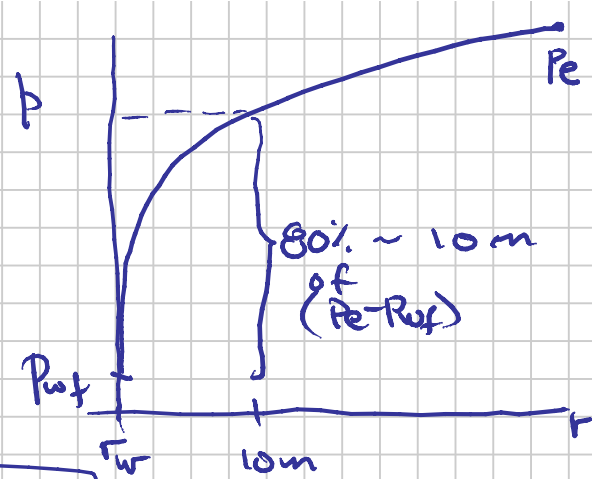
$$q_g = \frac{2\pi k h}{\ln \frac{r_e}{r_w}} \cdot \int_{p_{wf}}^{p_e} \frac{1}{\mu_g B_g} dp$$

↑  
Can't measure

$\Rightarrow p = f(\ln r)$ ; plot  $p$  vs  $\ln r \Rightarrow$  linear

$$\frac{\ln r^*/r_w}{\ln r_e/r_w} = 0.8$$

Solve for  $r^*$



You (or me) or someone else

can show that  $\bar{p}_R @ p(r=0.472 r_e)$

$$q_g = \frac{2\pi kh}{\ln \frac{0.472 r_e}{r_w}} \int_{p_{wf}}^{\bar{p}_R} \frac{1}{M_g B_g} dp$$

Reservoir  
Steady state  
Rate  
Equation

Some folks write  $\ln \frac{0.472 r_e}{r_w} = \ln \frac{r_e}{r_w} - \frac{3}{4}$

### Pressure Integral

SPE

$$p_p (m(p)) \equiv \int_0^p \left( \frac{1}{M_g B_g} \right) dp = \text{"pseudopressure"}$$

"gas mobility term"  $\lambda_g$

Sometimes:  $B_g = \frac{P_{sc}}{T_{sc}} \frac{z_{TR}}{P}$

$$p_p \equiv \left( \frac{1}{2} \right) \int_0^p \frac{P}{M_g z} dp$$

$$\frac{1}{2} \cdot \left( \frac{T_{sc}}{P_{sc} T_R} \right)$$

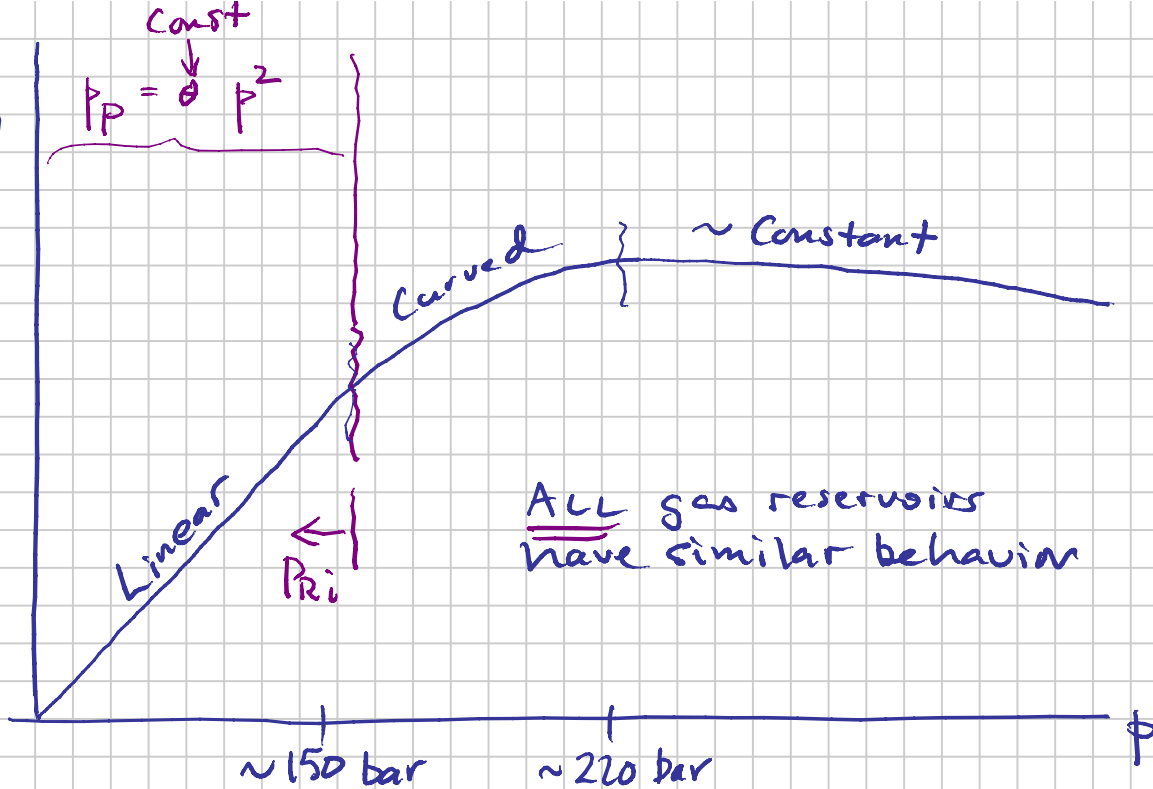
Original Form  
(Al-Husseiny, Ramey,  
Crawford)



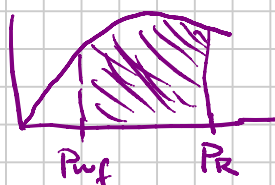
$$\lambda_g \equiv \frac{1}{\mu_g B_g}$$

(OR)

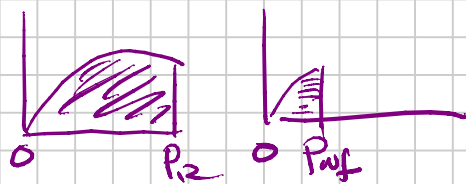
$$\frac{p}{\mu z}$$



Calculus memory lane



$$\left[ \int_{p_{wf}}^{p_R} \right] = \int_0^{p_R} - \int_0^{p_{wf}} = (p_{pR} - p_{p_{wf}})$$



$$q_g = \frac{2\pi kh (p_{pR} - p_{p_{wf}})}{\ln(r_e/r_w) - \frac{3}{4}}$$

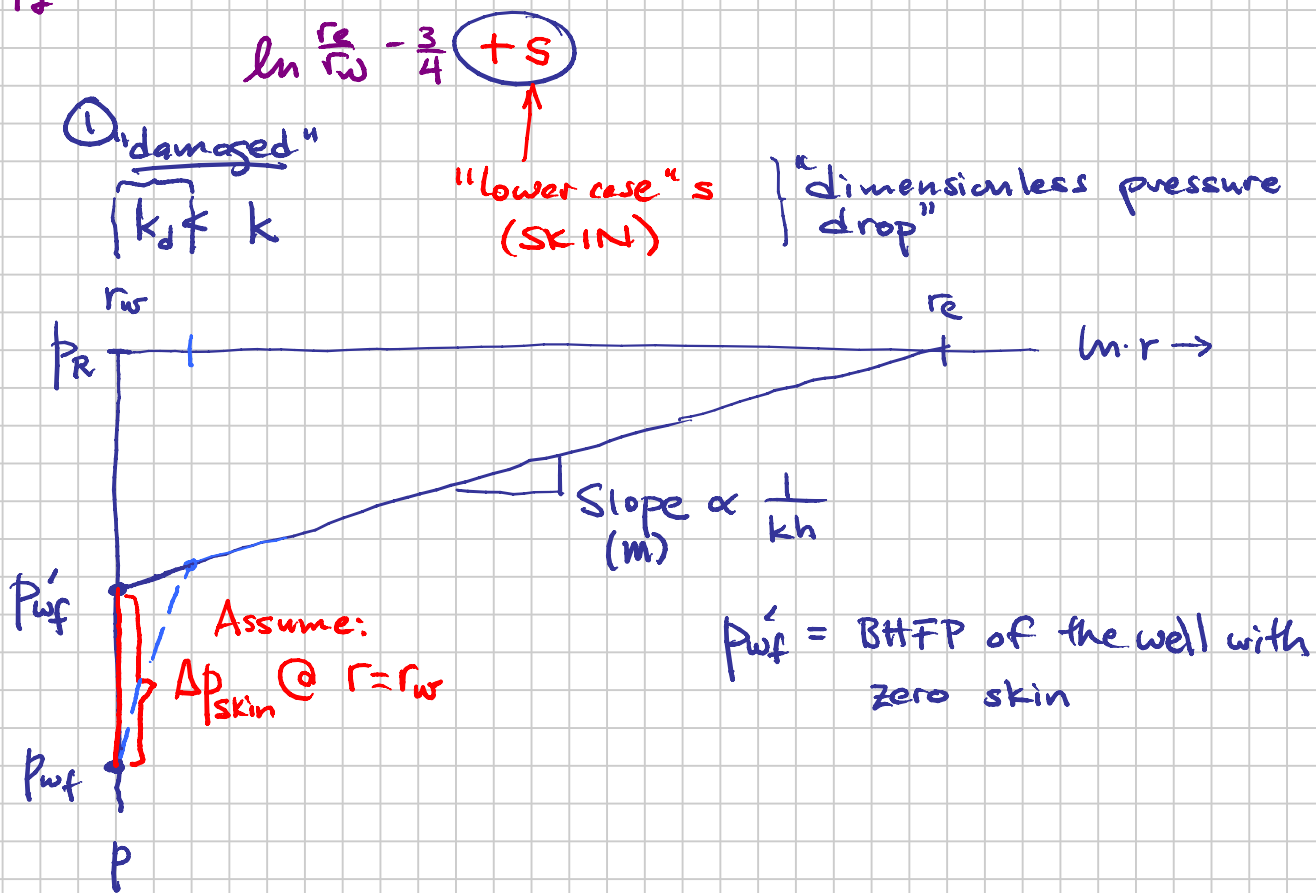
$$p_R < 2500 \text{ psia (130 bar)} \Rightarrow p_P = p^2$$

$$q_g \approx \frac{2\pi(\lambda) kh (p_R^2 - p_{wf}^2)}{\ln \frac{r_e}{r_w} - \frac{3}{4} + S}$$

low-pressure approximation

# Non-Ideal Flow Conditions (Concept: SKIN)

$$q_g = \frac{2\pi kh (P_{PR} - P_{pwf})}{\ln \frac{r_e}{r_w} - \frac{3}{4} + S}$$



$$\Delta p_t = \Delta p_R + \Delta p_{skin}$$

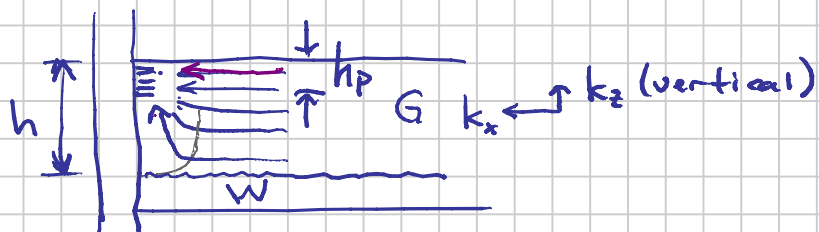
Solve rate eq.

$$\Delta p_{PR} = P_{PR} - P_{pwf}' = \frac{q_g}{2\pi kh} \cdot \underbrace{\ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4}}_{\text{dimensionless}}$$

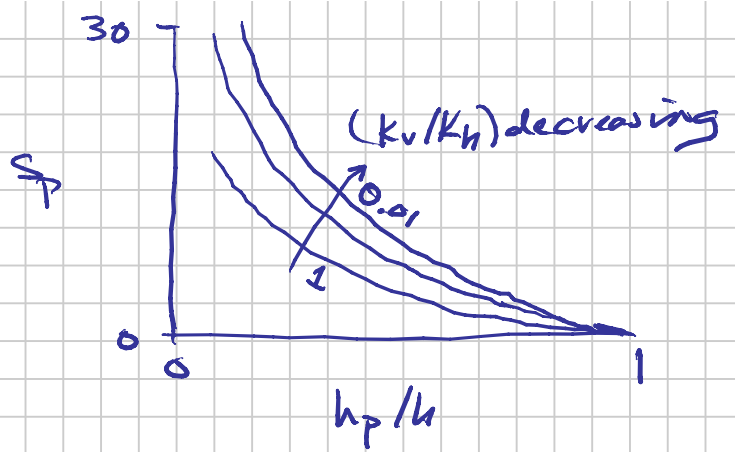
$$\Delta p_{P_{skin}} = P_{pwf}' - P_{pwf} = \underbrace{\left( \frac{q_g}{2\pi kh} \right)}_{\text{slope}} \cdot \underbrace{s}_{\text{dimensionless } \Delta p_{P_{skin}}}$$

## ② Partial Penetration

$$|S_p| > 0$$



$$\uparrow S_p \left( \underbrace{h_p/h}_\downarrow, \underbrace{k_v/k_x}_\downarrow \right)$$



③ ..... ⑫ ..... ⑮  
Skin Effects

"High Velocity Flow"  
"non-Darcy Flow"  
"Turbulent"

Darcy's Law isn't good enough  
"Laminar"  
Like  
Pore  
Flow

Forcheimmer

$$\frac{dp}{dr} = \underbrace{\frac{\mu}{k} v}_{\text{Darcy}} + \underbrace{\beta \cdot \rho \cdot v^2}_{\text{PUT}}$$

Rock (like k)  
↓ NEW

$\left. \begin{matrix} (kh) \\ s \\ D \\ R \end{matrix} \right\} q_g(P_{wf})$   
"IPR"

you ...

$$q_g = \frac{2\pi kh (P_{PR} - P_{wf})}{\ln \frac{R}{r_w} - \frac{3}{4} + \left[ \underset{\substack{\uparrow \\ \text{Damage, } P_D, \dots}}{s} + \{ D q_g \} \right]}$$

FINAL GENERAL  
RESERVOIR  
RATE  
Eq.

Total Skin  $s_t$

$$D = \frac{\beta}{z}$$

OR

• Correlations

$$\beta \propto \frac{1}{k}$$

• Well Tests (Special)

Rate-Dependent  
Skin

$$s_t = s + D q_g$$

$$A q_g^2 + B q_g - (P_{PR} - P_{wf}) = 0$$

Quadratic  
Equation

$$A = \frac{D}{2\pi kh}$$

$q_g (A_p)$

$$B = \frac{\ln \frac{R}{r_w} - \frac{3}{4} + s}{2\pi kh}$$

$$\left(\ln \frac{r_e}{r_w} + s\right) q_g + D q_g^2 - 2\pi kh \Delta p_p$$

$$\underbrace{\frac{D}{2\pi kh}}_{A \Rightarrow D} \cdot q_g^2 + \underbrace{\frac{\ln r_e/r_w + s}{2\pi kh}}_{B \Rightarrow s} \cdot q_g - \Delta p_p = 0$$

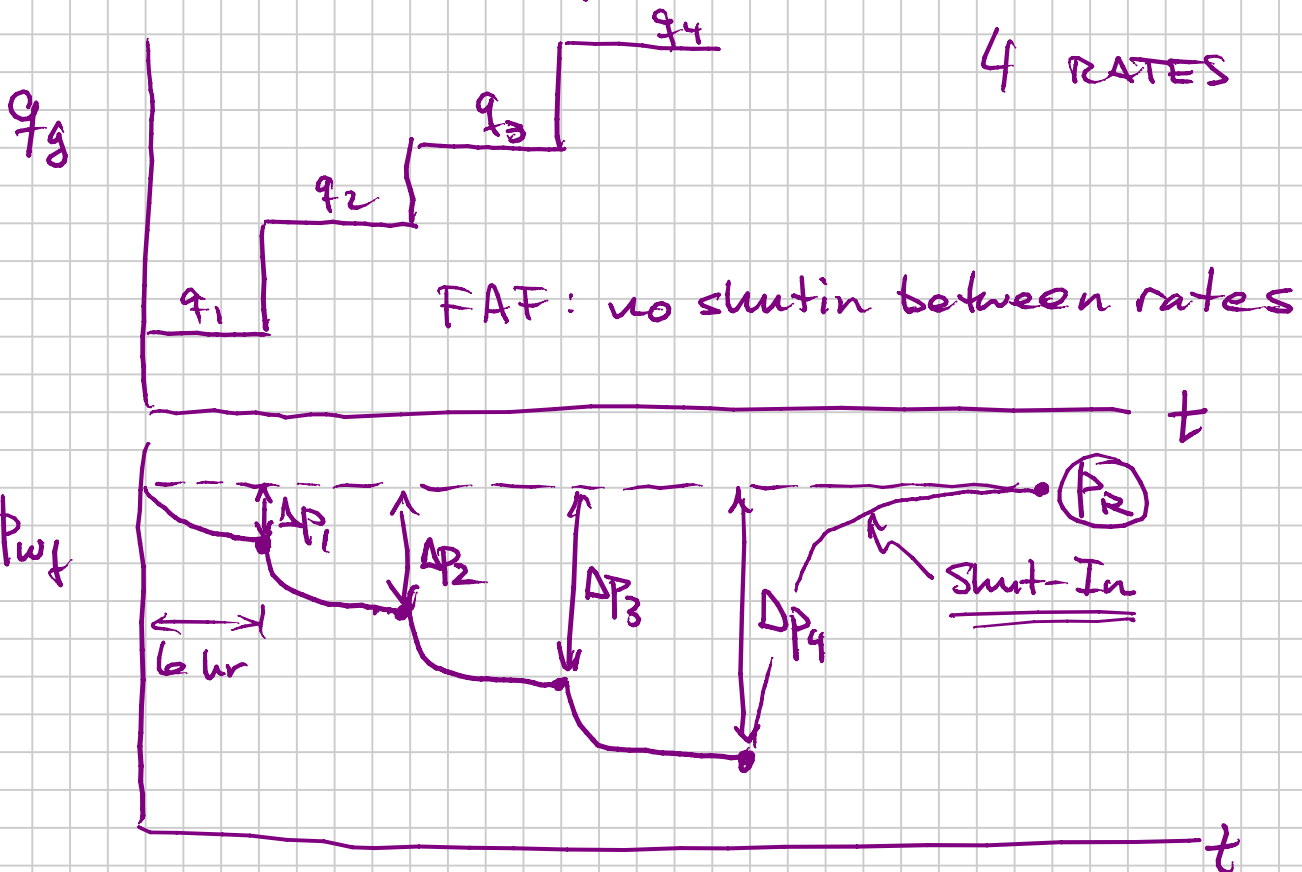
Normally you have an estimate of  $(kh)$   $\left\{ \begin{array}{l} \text{Core (Log)} \\ \text{PTA (Well Testing)} \end{array} \right.$

Special Well Tests  $\Rightarrow \beta, D$

• SI between each rate

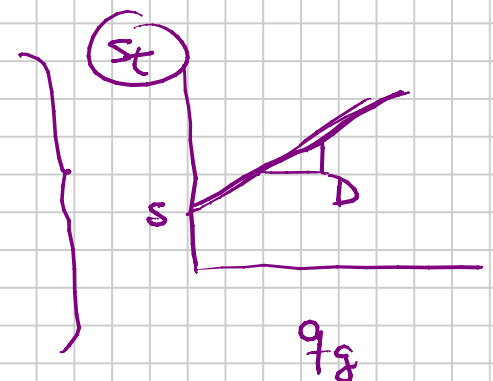
Multi-Rate Flow-After-Flow or Isochronal (FAF)

•  $\Delta p$  each rate



Best-Fit  $\Rightarrow A, B$

$\left( \begin{array}{l} kh \\ s \end{array} \right), D(\beta)$



# Multi-Rate Test

From Smit-In:  $P_R = \boxed{\quad}$

2007.11.07

Rate #	$q_g$	$P_{wf}^*$	$\Rightarrow \int \Rightarrow$	$P_{pwf}$	$P_{PR} - P_{pwf}$
1					
2					
3					
4					

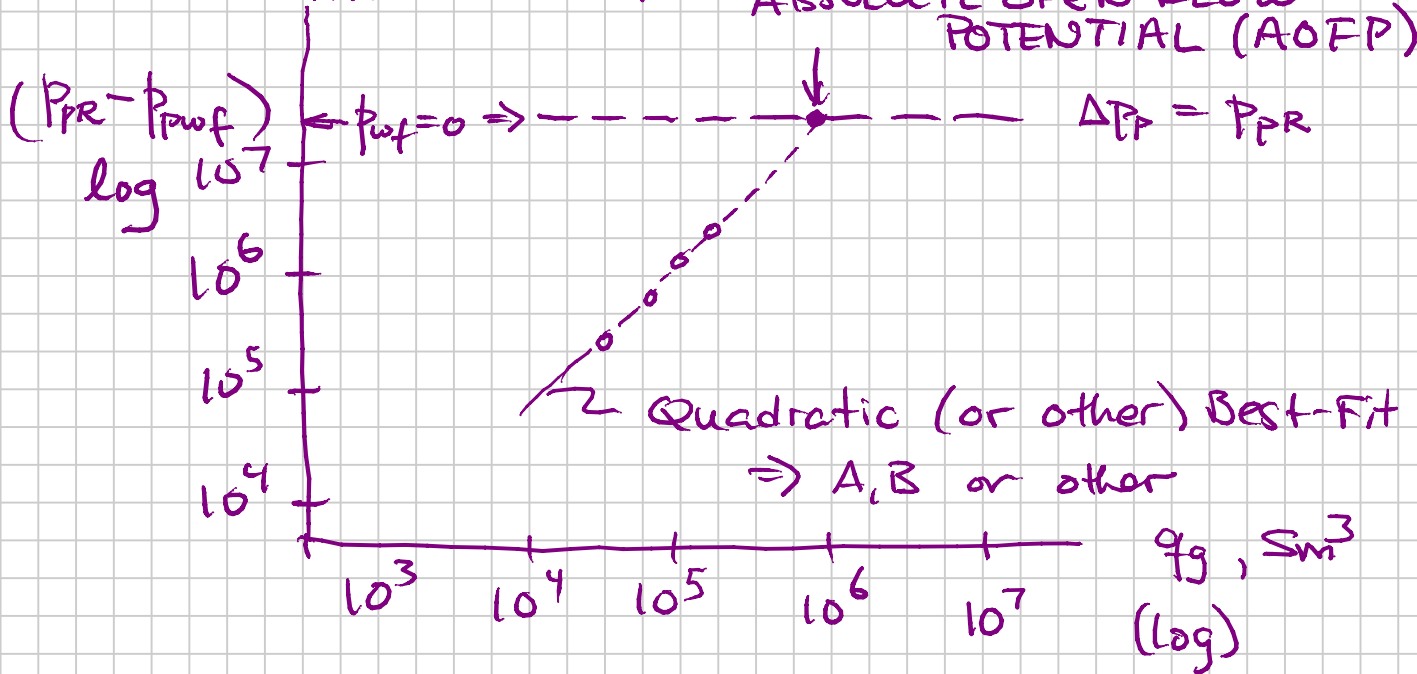
     Measured

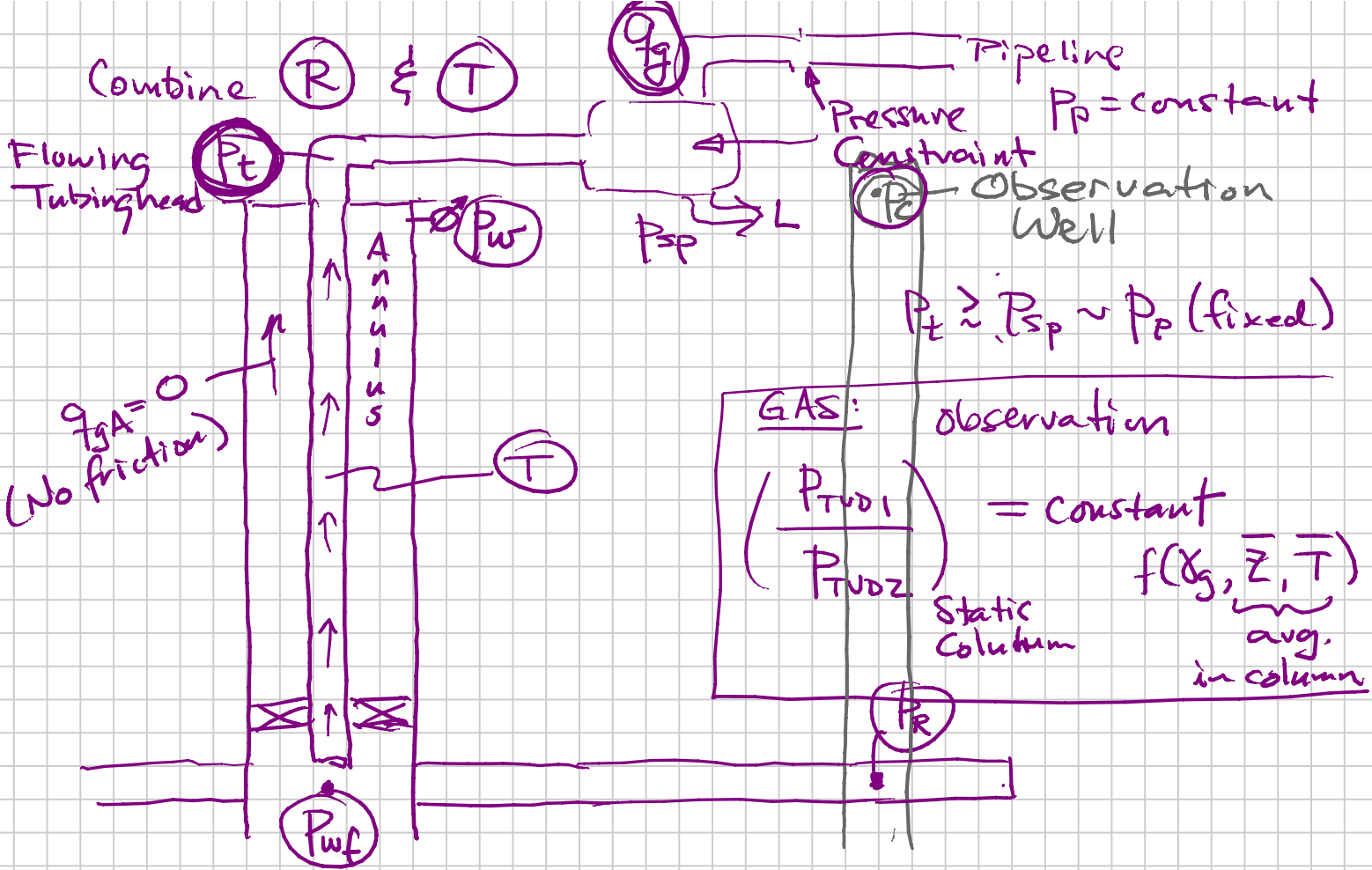
\*<sup>(a)</sup> Wireline-hung or permanent BHP pressure gauge

(b) Surface pressures  $\rightarrow$  Converted  $\rightarrow$  BHP's Accurately

Plot (tradition)  
"BACKPRESSURE PLOT"

Theoretical  
ABSOLUTE OPEN FLOW  
POTENTIAL (AOFP)





Tubing Flows:

$$\frac{dp}{dz} = \rho g$$

$$\rho = \frac{PM}{ZRT} \sim (\cdot) P$$

$$q_{Tg} = C_T (P_w^2 - P_t^2)^{0.5}$$

↑                    ↑  
Constant

$C_T (d_t^{2.7}, L_t, \rho_g, TVD)$

↑                    ↑                    ↑                    ↑  
 Tubing Diameter    Length of Tubing    Gas Gravity    True Vertical Depth

Bring  $\bar{R}$  pressures to a surface datum:

$$\frac{(p)_R}{(p)_s} = e^{S/2} = \text{constant} \sim 1.2 - 1.4$$

$S(\gamma_g, \bar{z}, \bar{T}, \text{TVD}) = \text{constant for all wells in a common } \bar{R}$

$$\left(\frac{p_R}{p_c}\right) = \left(\frac{p_{wf}}{p_w}\right) = e^{S/2} = \text{constant} \quad (1-32)$$

1.16  
1.42

$$p_R = p_c e^{S/2}$$

$$A q^2 + B q = (p_R^2 - p_{wf}^2)$$

$$A q^2 + B q = \underset{\uparrow}{p_c^2} \cdot e^S - \underset{\uparrow}{p_w^2} e^S$$

$$A' q^2 + B' q = p_c^2 - p_w^2 \quad \textcircled{R}$$

$$A' = A/e^S, \quad B' = B/e^S \quad +$$

$$\left(\frac{1}{C_T}\right)^2 q^2 = p_w^2 - p_c^2 \quad \textcircled{T}$$

$$\underbrace{\left(A' + \frac{1}{C_T^2}\right) q^2 + B' q}_{A''} = \underbrace{p_c^2 - p_w^2}_{\text{Surface-datum pressures}}$$

" $p_R$ "

1st  
or 2nd  
account

# Wellhead Backpressure Eq. (Fetkovich)

At a given time:

$$\downarrow P_R(Q_p)(t)$$

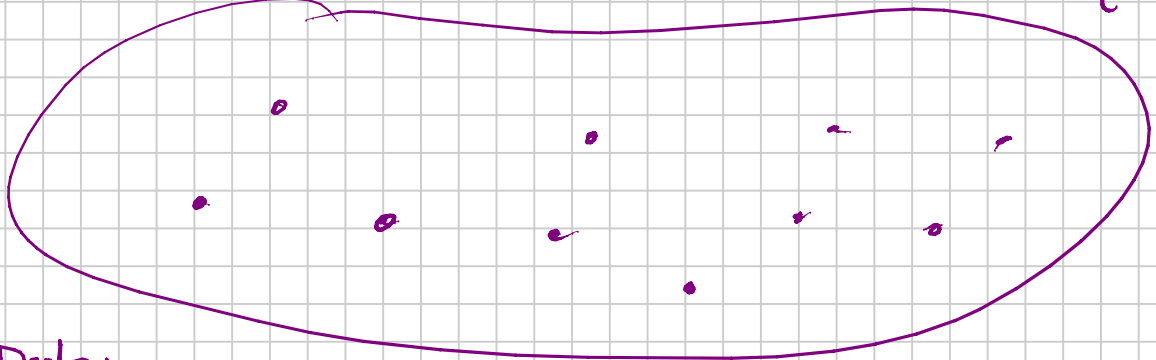
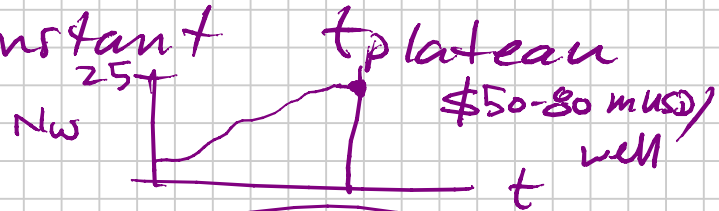
Material Balance

- Say let's set  $P_R = \text{minimum (constraint)}$

$$\Rightarrow q_{g, \text{max, well}} \checkmark$$

$$\Rightarrow q_{g, \text{Field}} = \text{constant}$$

$$N_{w, \text{wells}}(t)$$

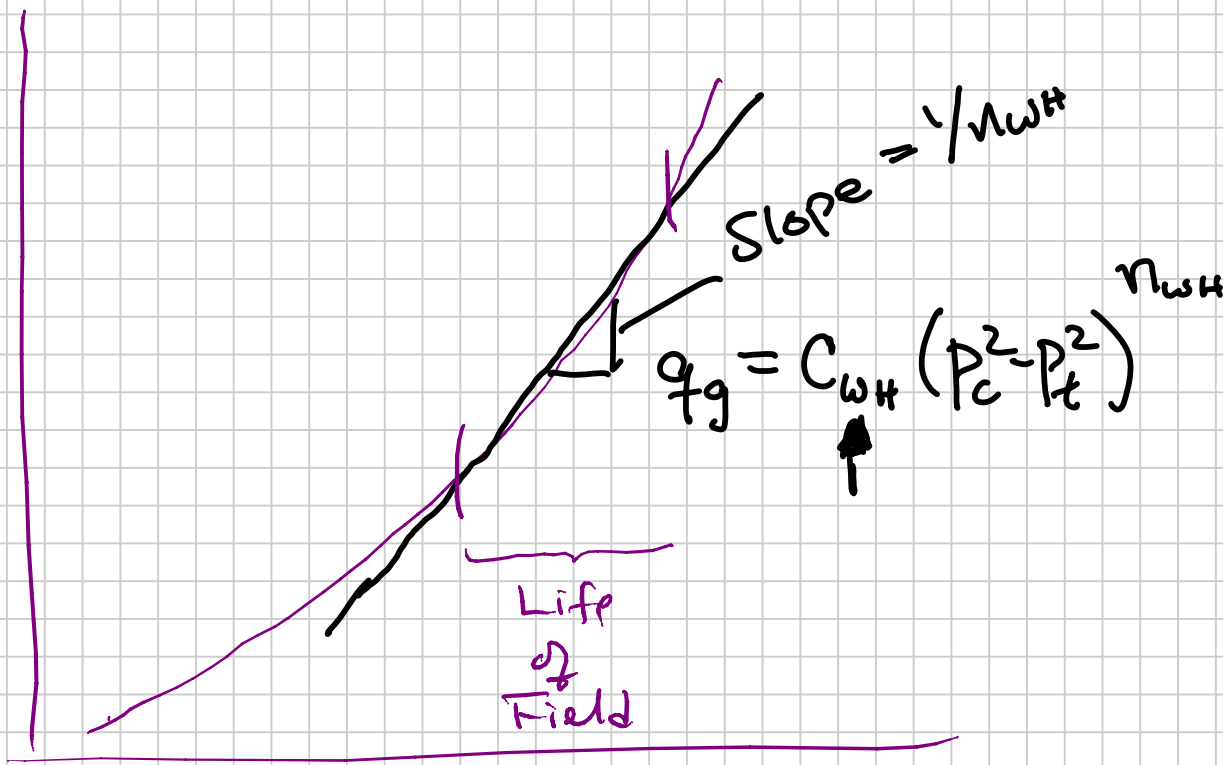


Key Data:

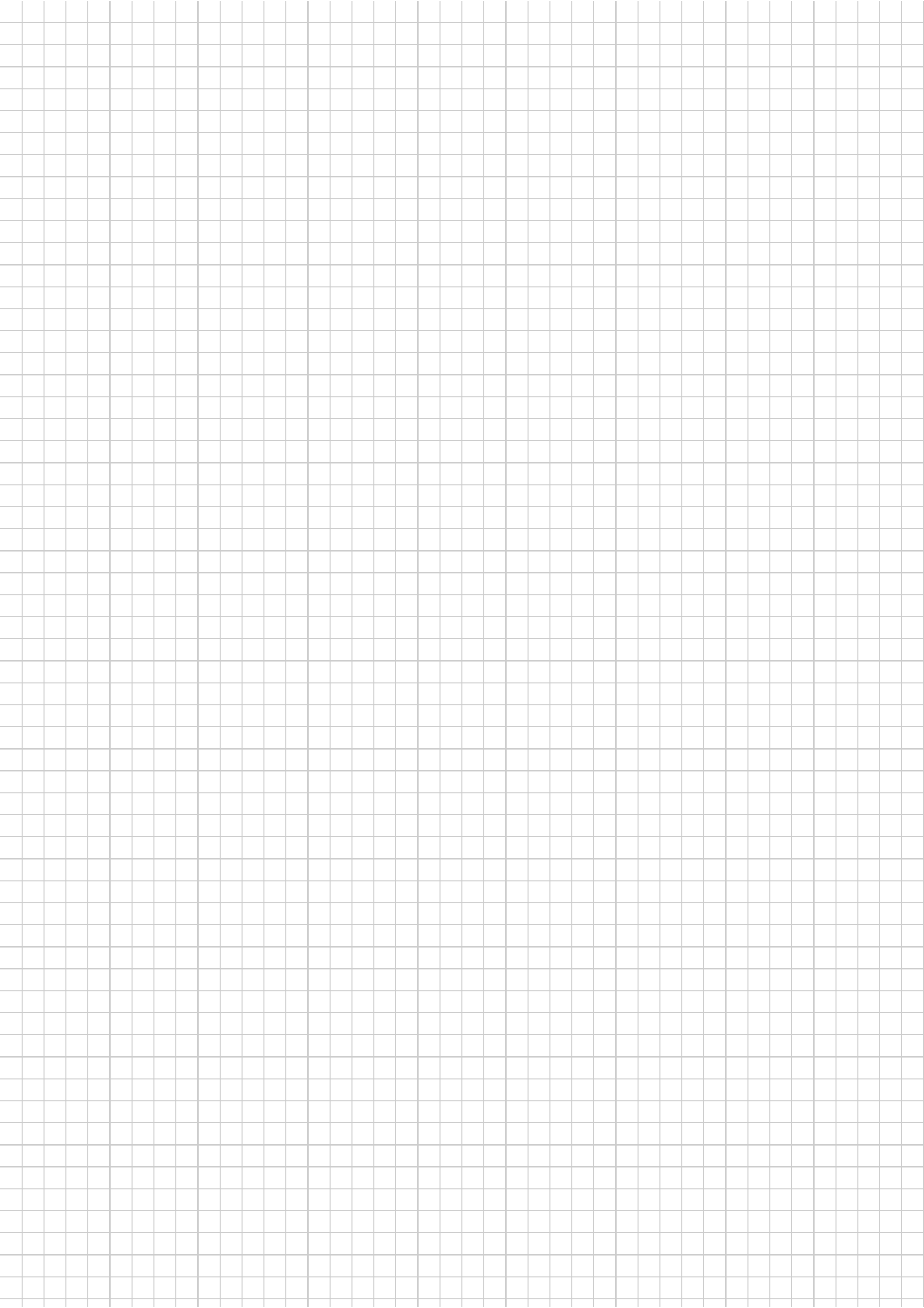
- IGIP (tank)
- All wells are identical  $\checkmark$
- $q_{g, \text{Field}}$  / year



$P_c^2 - P_t^2$   
(log)



$q_g$   
(log)



# GAS MATERIAL BALANCE

Note Title

11/13/2007

## GAS PVTEQ. - REAL GAS LAW

$$pV = nRTZ$$

Assumptions:

$$V = HCPV = \text{constant} \otimes$$

$$Z = f(p, T)$$

$$T = T_R = \text{const.}$$

$$n_R = n_i - n_p$$

Introduce: Surface Gas Volumes

$$V_{gsc} = \frac{RT_{sc}}{p_{sc}} \cdot n$$

379 scf/lbmol  
23.64 Sm<sup>3</sup>/kgmol

$$(G) \quad IGIP = n_i \cdot 379$$

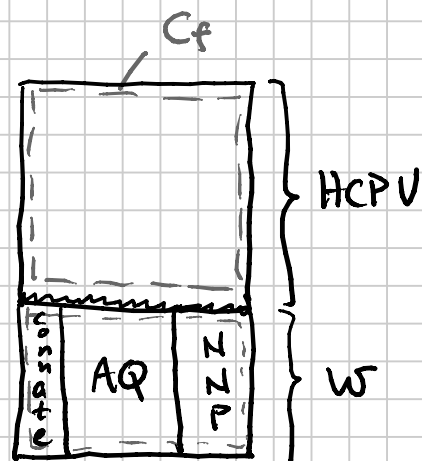
$$(G_p) \quad \text{Cum. Gas Prod.} = n_p \cdot 379$$

⇒ Straight-Line Gas. M.B

$$\left( \frac{p}{Z} \right) = \left( \frac{p}{Z} \right)_i \left( 1 - \frac{G_p}{G} \right)$$

↑  
RF

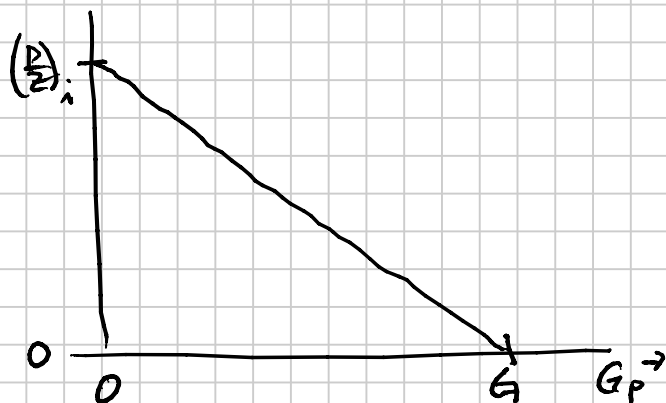
$$\frac{G_p}{G} = 1 - \frac{(p/Z)}{(p/Z)_i}$$



"Pot Aquifer"  
Small AQ  
High k

"s.c." 1 atm (14.7 psia)  
60°F (15.56°C)

"POT AQ MODEL" -  
 $\Delta P_g = \Delta P_w$   
↑ instantaneously  
By Production



# Pot Aquifer Model

$$\frac{p}{z} \left[ 1 - \bar{c}_e (p_i - p) \right] = \left( \frac{p}{z} \right)_i \left( 1 - \frac{G_p}{G} \right)$$

$$\bar{c}_e = \frac{c_f + c_w S_{wc} + \underbrace{M}_{(AQ+NHP)} (c_f + c_w)}{1 - S_{wc}}$$

$$S_{wc} \sim 0.25$$

$$c_f \sim c_w \sim 5 \cdot 10^{-6} \text{ psi}^{-1}$$

$$M \equiv \frac{V_{AR}}{V_{PR}} \sim 1-5 \text{ (Pot Aq.)}$$

Containing HC

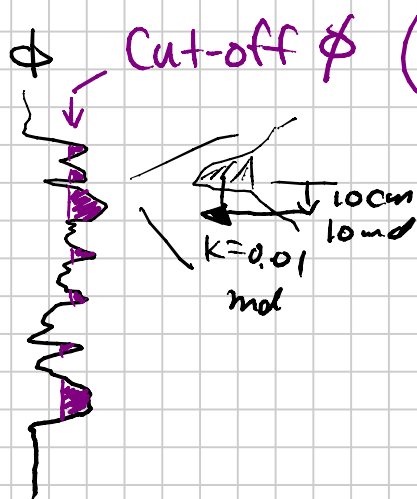
$c_e =$

$$\frac{G_p}{G} = 1 - \frac{(p/z) [1 - \bar{c}_e (p_i - p)]}{(p/z)_i}$$

NMP: Non-Net Pay — Water-bearing "dirty" sand  
 $\Delta z \sim 10-100 \text{ cm}$

- (1) Interbedded shales
- (2) "Low- $\phi$ " (low- $k$ ) 'sand'

Additional "interbedded" water

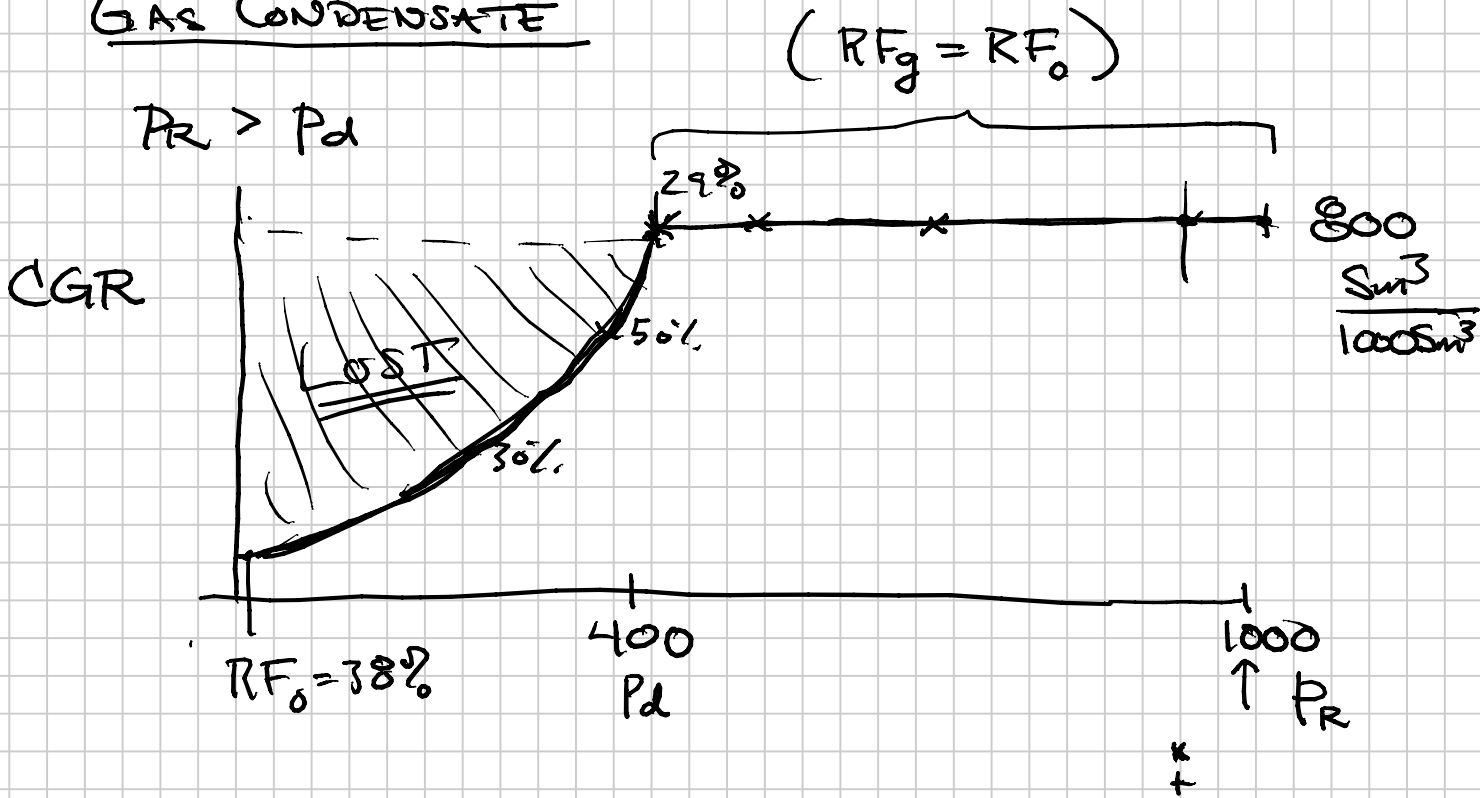


Cut-off  $\phi$  (DON'T USE...)

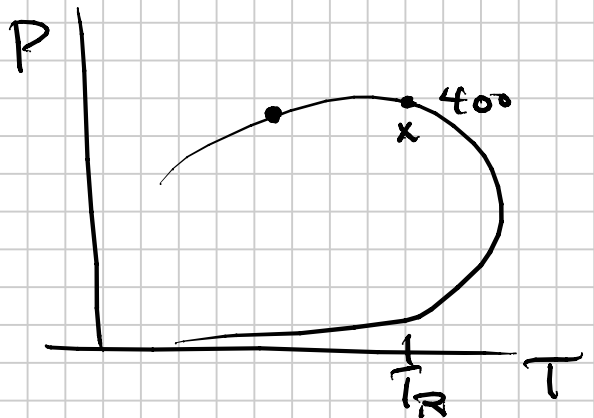
particularly for Gas Reservoirs

$$\underline{M} = \{ M_{AQ} + M_{NNP} \}$$

GAS CONDENSATE



~~PVT~~

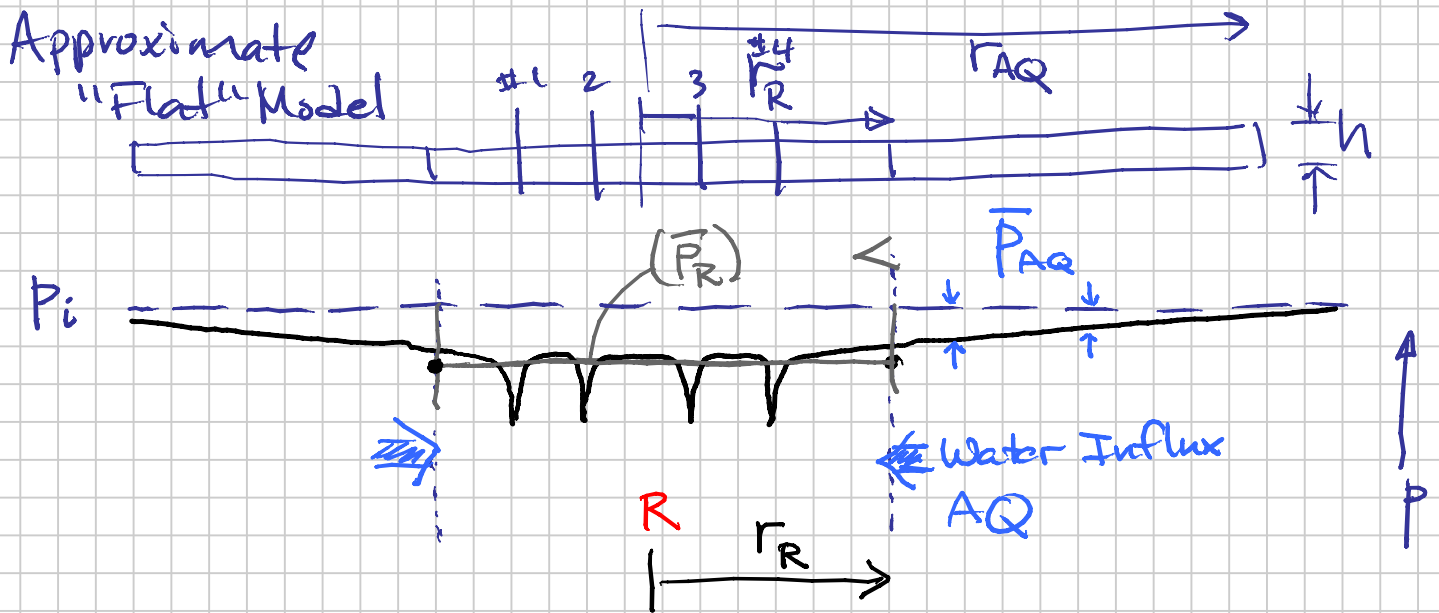
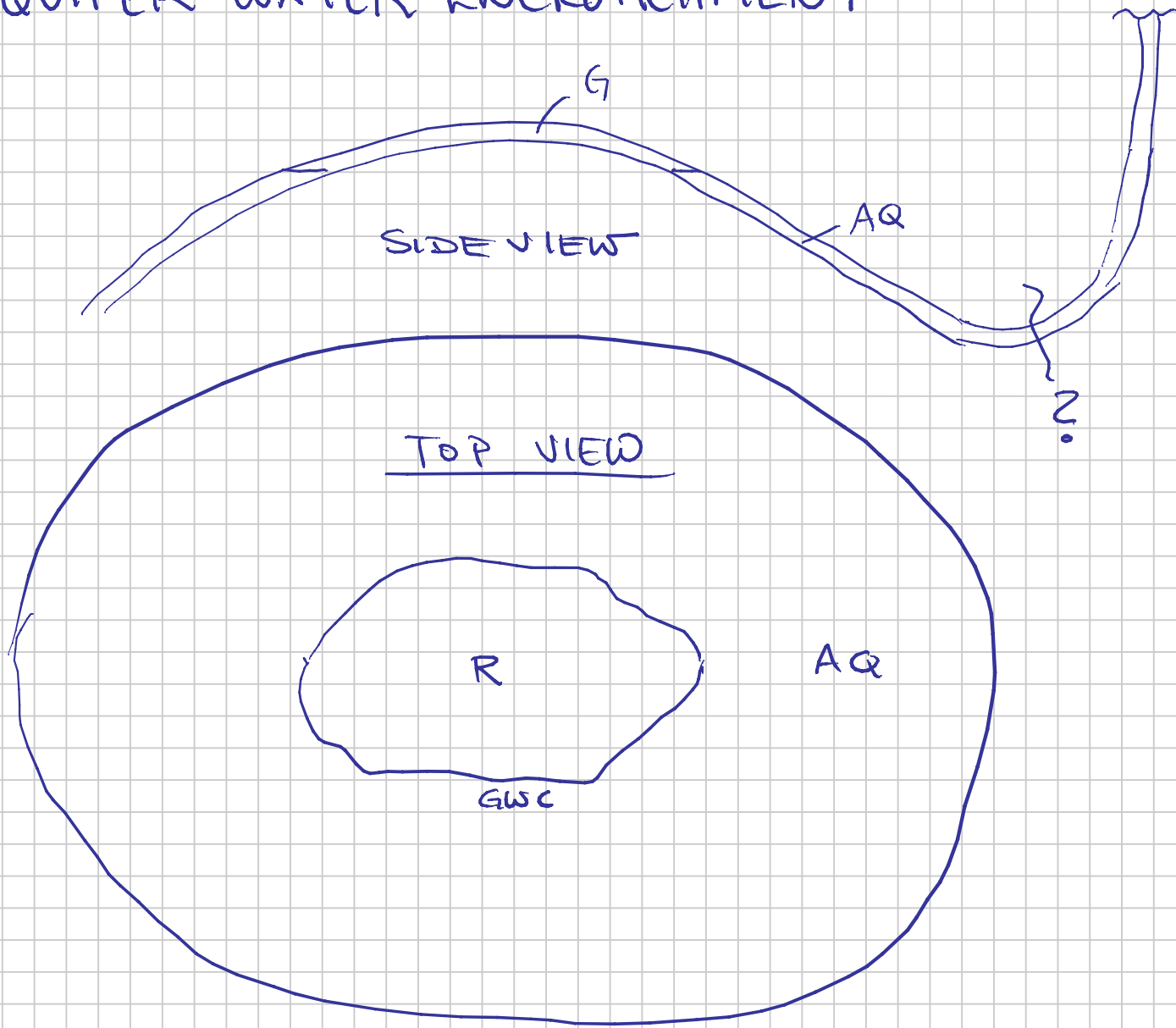


Limited pressure

bbl RB STB

5.615  $ft^3$

# AQUIFER WATER ENCROACHMENT



$$q_g = \frac{2\pi kh (P_r - P_{wf})}{\ln r/r_w}$$

Mathematical Solution  $(q_w)_{AQ \rightarrow R}$  ?

- $p_R(r) = \text{const}$   $r < r_R$  @  $t$

( •  $p_{g-w}$  @  $r_R$  changes in time )

(according to the rate of gas production)

Flow in the aquifer  $r_R < r < r_{AQ}$   
single-phase water flow

$p_R(t)$  boundary condition

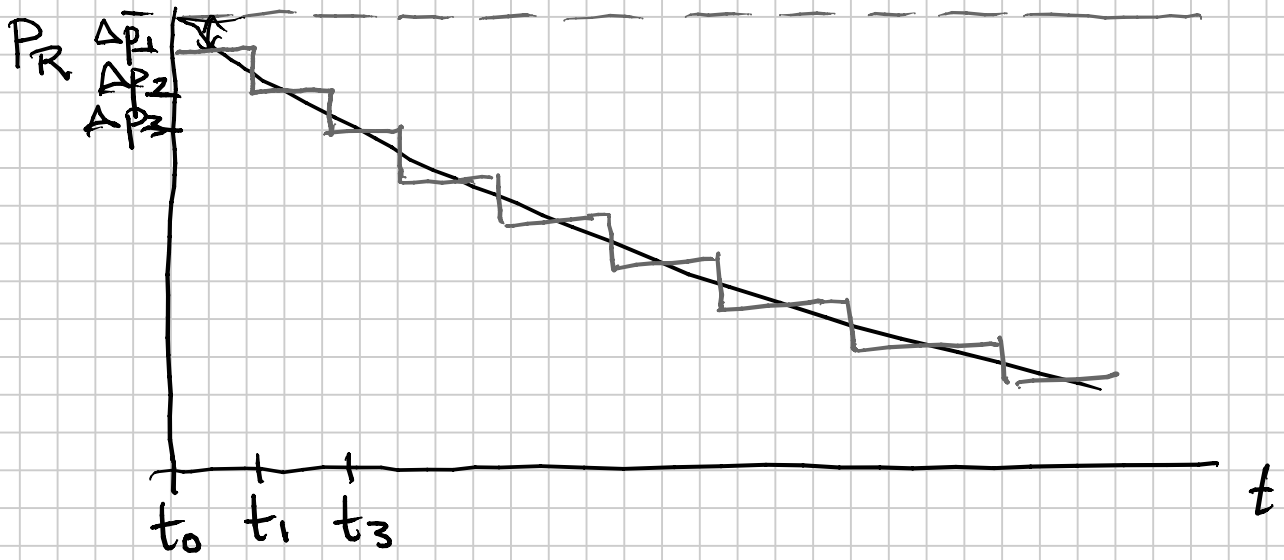
Solution is to use superposition  
to calculate  $q_w(t)$

- van Everdingen-Hurst general solution

- Fetkovich PSS Approximation

$$q_w = q_{wi} \cdot e^{-Dt}$$

---



$$q_w(t) = \sum \Delta q_w (\Delta P_{\Delta t})$$

$\Delta p_1$  starts at  $t_0 \rightarrow t$

$\Delta p_2$  starts at  $t_1 \rightarrow t$

⋮

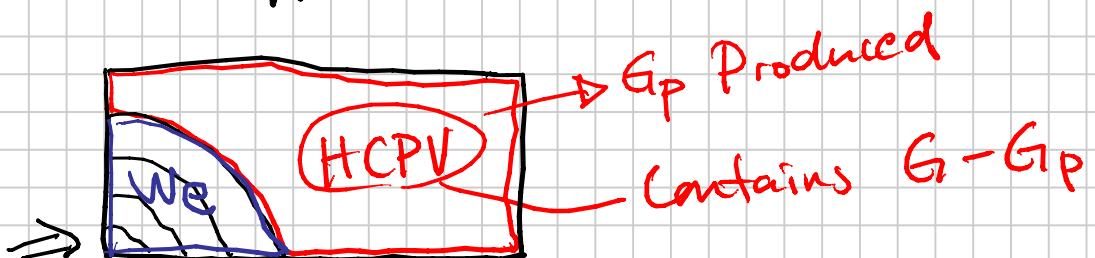
Cumulative Water Encroachment

$$W_e = \int_0^t q_w(t)$$



Filling the Gas R PV

HCPV decreases

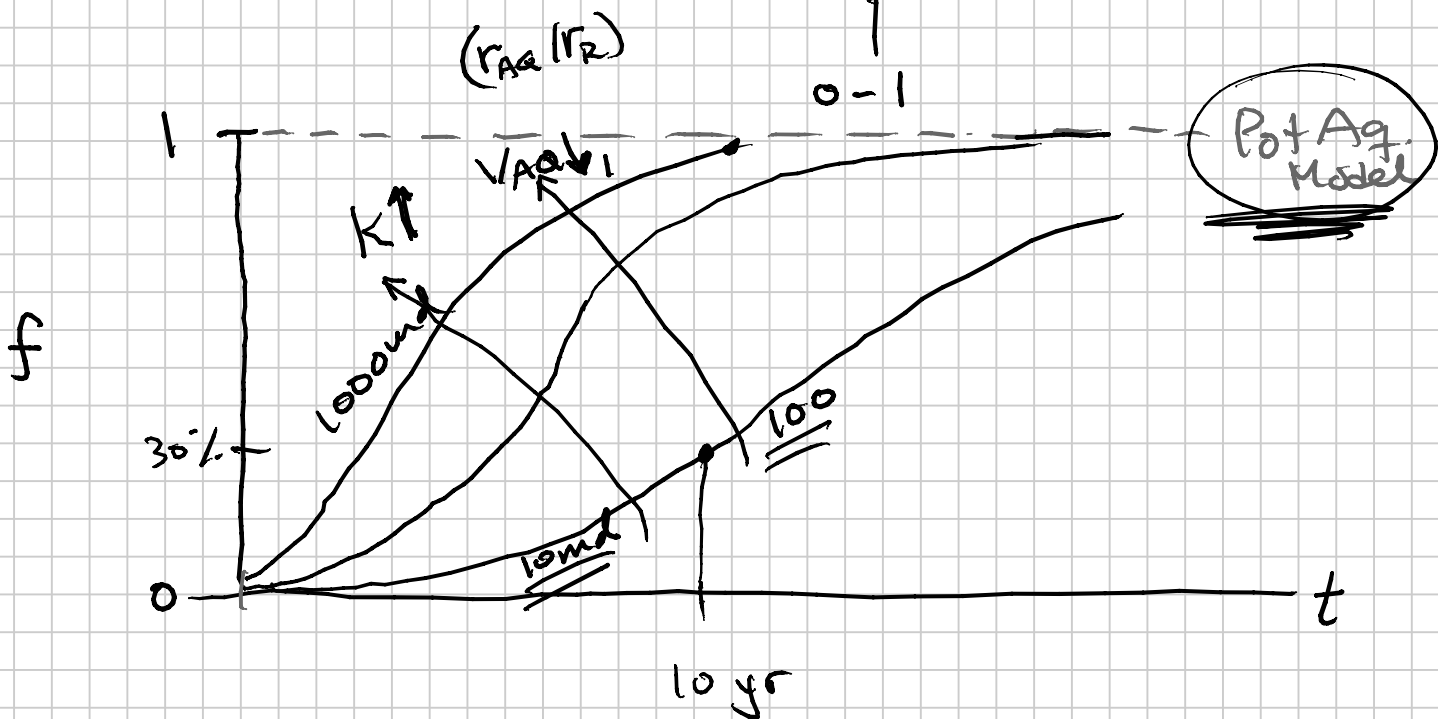




Pot Aq. Model gives the maximum encroachment

$$W_{\max} = V_{Aq} \cdot (c_w + c_f) \underbrace{(P_i - P_R)}_{\text{total } \Delta p}$$

$$W_e(t) = W_{\max} \cdot \underbrace{f(t)}_{0-1}$$



$k \lesssim 1 \text{ md}$  : little-no aquifer support in a normal 10-20 yr depletion life

$(P_{Ri})$   $(HP)$  reservoirs (> 500 bar)

even "small" water volumes can become important

