FLUID FLOW

- Through porous/permeable rock

Chalk (N.S.) : $\phi \simeq 25-45\%$

$k \sim 0.01-5$ md

$d_p^{max} \sim 1\,\mu m$

Sandstone (N.S.) : $\phi \sim 10-25\%$

$k \sim 1-10,000$ md

$d_p^{max} \sim 200\,\mu m$

DARCY'S LAW

$[\mu m^2]$

Absolute Permeability [D]

Pressure [atm]

$$v = \left[\frac{k}{\mu}\right]\left(\frac{dp}{dx}\right)$$

"C"

Distance [cm]

Viscosity [cp]

"Darcy" velocity [cm/s]

$$1\ D = 1\ \mu m^2 = (10^{-6} m)^2$$
$$= 10^{-12}\ m^2$$

Assumptions:

(1) k is a "rock" property. (Not really so)

(2) μ is the "fluid" property

No rock-fluid interaction.

(3) Single fluid phase flowing

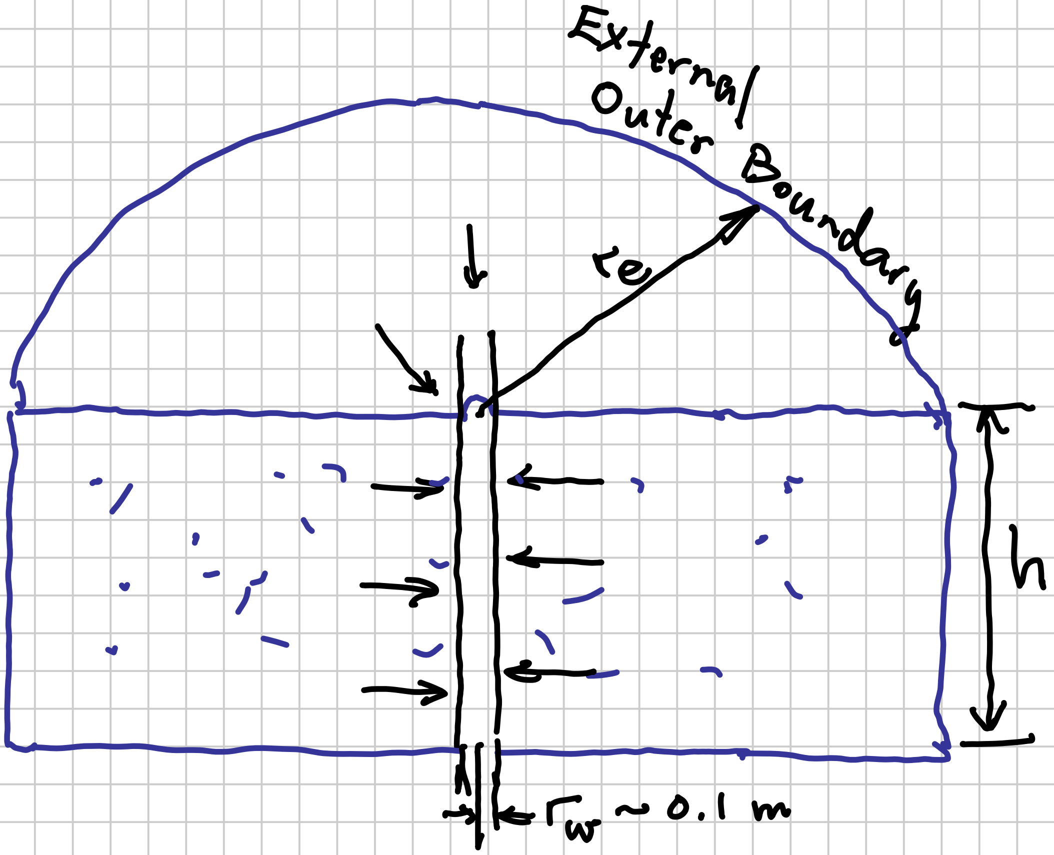
⋮

?

Apply Darcy's law to a producing well.

How?

(1) Define well geometry



Radial Cylindrical Geometry

How much (m^3) produces out of the reservoir per day. q = volumetric flow rate

$$q_o = f(k, \mu, p_e, p_w, B_o)$$

DARCY'S LAW - Development of Rate Equation

$$v = \frac{k}{\mu}\left(\frac{dp}{dr}\right)$$

B.C. (Boundary Conditions)

$p = p_w$ at $r = r_w$

$p = p_e$ at $r = r_e$

~ k & μ are constants w.r.t. p, r

Volumetric Rate at reservoir conditions

$*$ $$q_R = A_\perp \cdot v \Rightarrow v = \frac{\overset{\text{ss: const}}{q_R}}{A_\perp} = \frac{q_R}{2\pi h r}$$

$$A_\perp = 2\pi r h$$

$$q_R = 2\pi r h \cdot \frac{k}{\mu}\frac{dp}{dr} \quad ; \quad B \equiv \frac{V_R}{\underset{\uparrow}{V_{sc}}}$$

selling at surface cond.

Surface Volumetric Rate

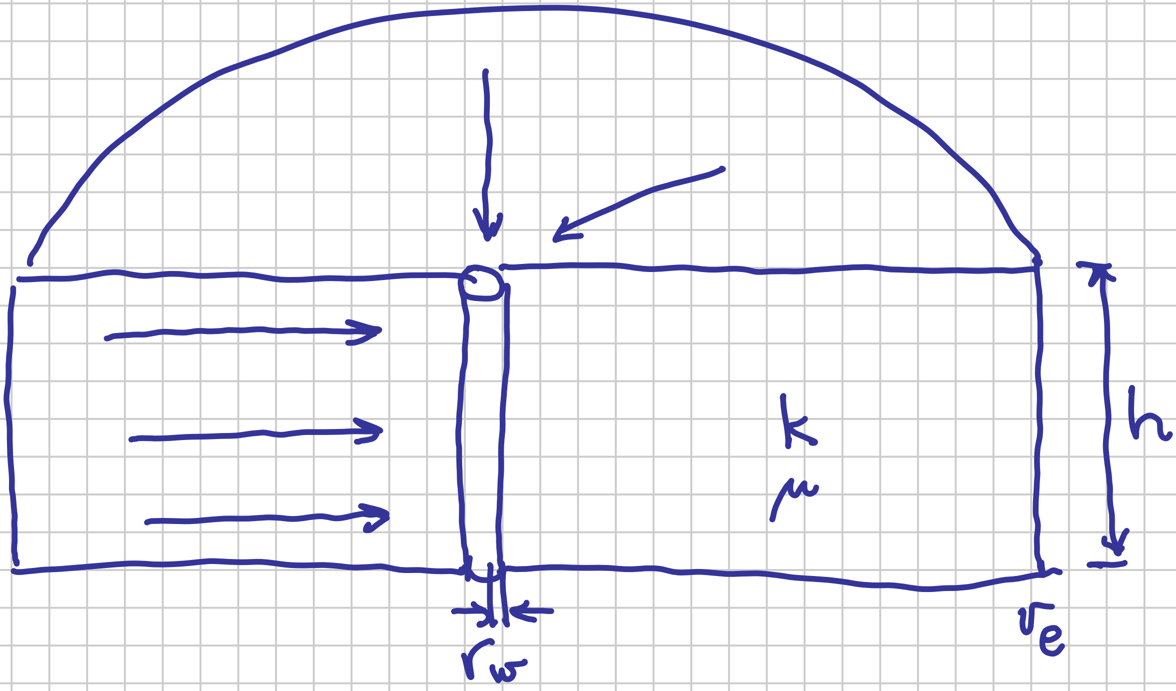
$$q = q_R \cdot \frac{1}{B}$$

$$q = \underbrace{\frac{2\pi k h}{\mu B}}_{\text{constant}} r \frac{dp}{dr}$$

e.g. $\frac{m^3}{s}$

$\frac{Sm^3}{d}$

~



~ q = constant independent of r

"Steady State"

"Pseudo Steady State"

$\underbrace{p_w \text{ and } p_e}_{\text{also } q}$ are also constant in time

Collect all constant terms

$$q = \frac{2\pi k h}{\mu B} \, r \frac{dp}{dr}$$

$$\int_{r_w}^{r_e} \frac{1}{r}\, dr = \underbrace{\frac{2\pi k h}{q \mu B}}_{\text{const}} \cdot \int_{p_w}^{p_e} dp$$

$$\ln \frac{r_e}{r_w} = \frac{2\pi k h}{q \mu B} (p_e - p_w)$$

(kh)
Permeability-Thickness
Product

Solve for q

$$\boxed{q = \underbrace{\frac{2\pi\, kh}{\mu B \cdot \ln \frac{r_e}{r_w}}}_{\text{Productivity Index}} (p_e - p_{wf})}$$

↑ p_{wf}
flowing

p_{ws}
"shut-in"
p_w

Summary Assumptions:

(1) $q(r) = \text{const.}$ PSS (SS)

(2) k, μ, B independent of p, r

(3) p_{wf} & p_e are const. in time

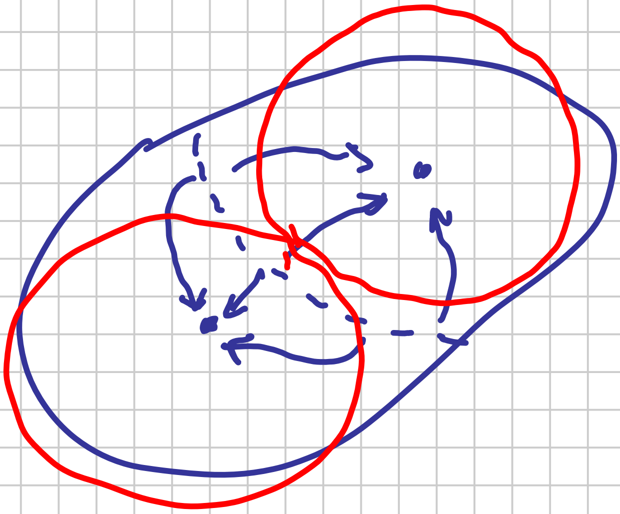
Term in Rate Eq.	Range	(OM)	[unit]
k	$0.001 - 10^4$	(7)	[md]
h	$1 - 1000$	(3)	[m]
μ	$0.01 - 10$ (10^4)	(6)	[cp]
P_e (initial)	$40 - 1000$	(1.5)	[bar]
P_{wf} (at abandonment)	$1 - 200$	(2^+)	[bar]
r_e	$100 - 10000$	(2)	[m]
r_w	0.1	(0)	[m]
$\ln \frac{r_e}{r_w}$	$\ln 10^3 - \ln 10^5$ $6.9 - 11.5$	(0.3)	
S	-6 to 100	(2)	

OM = Orders of Magnitude

$$= \log_{10}\left(\frac{\max}{\min}\right)$$

10^5 Pa = 1 bar

1 mPa·s = 1 cp



Quick Quiz

What is the relationship $p(r)$ and $v(r)$?

$$p \propto \ln r$$

$$v \propto \frac{1}{r}$$

Volumetric Average Reservoir Pressure

$\bar{p}_R \quad p_R \sim p_e$

Assumption:
$q = const \neq f(r)$

$$q = \frac{2\pi k h\,(\bar{p}_R - p_{wf})}{\mu B \left[\ln \frac{r_e}{r_w} - \frac{1}{2}\right]} \equiv \frac{2\pi k h (p_e - p_{wf})}{\mu B \cdot \ln \frac{r_e}{r_w}}$$

$$\bar{p}_R = \frac{1}{V_t}\int_{r_w}^{r_e} p \, dV$$

COMMON (MORE REALISTIC)

No-Flow Outer Boundary

BC $\left(\frac{dp}{dr}\right)_{r=r_e} = 0$

$$\Rightarrow \quad q = \frac{2\pi k h \left(\overset{\swarrow}{\bar{p}_R} - p_{wf}\right)}{\mu B\left[\ln\frac{r_e}{r_w} - \underset{\uparrow}{\frac{3}{4}}\right]}$$

SKIN: s

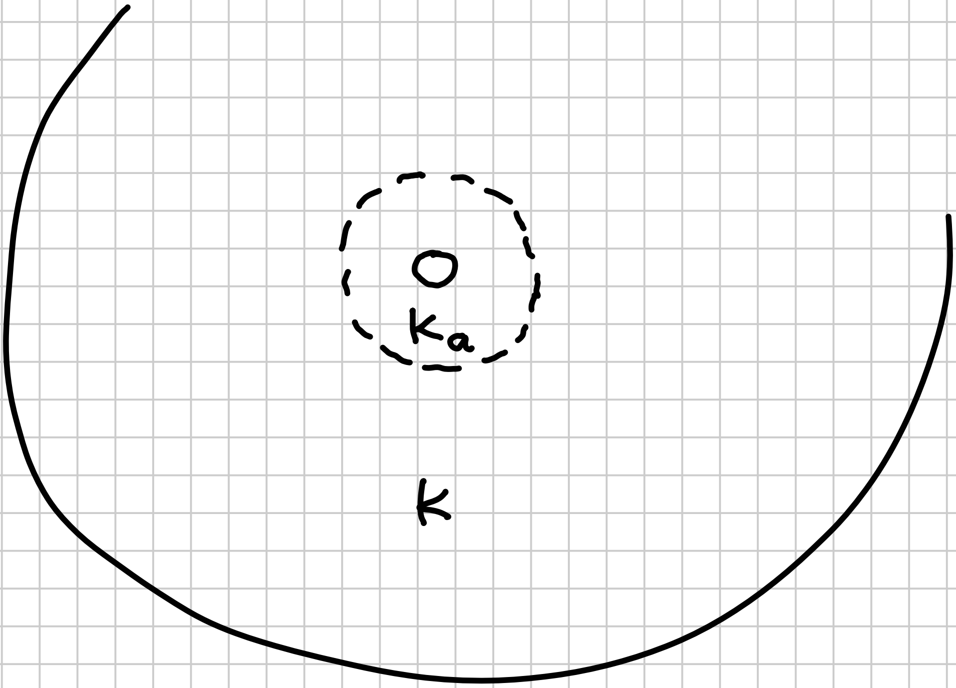
"Dimensionless" pressure drop "AT" (near) the well (r_w) due to a change in k 🟧 🟨 near the well, r_w + few cm

r_w + 10's m

Stimulation Skin	Damage Skin
$s < 0$	$s > 0$
[-6 to 0]	[0 to 100's]

$$q = \frac{2\pi kh(\bar{p}_R - p_{wf})}{\mu B\left[\underbrace{\ln\frac{r_e}{r_w} - \frac{3}{4}}_{\sim 8} + s\right]}$$

Skin "Composite Radial" Model



$$s = \left(\frac{k}{k_a} - 1\right)\cdot \ln\frac{r_a}{r_w}$$

Craft & Hawkins

GAS PVT BEHAVIOR

Ideal Gas Law:

$$pV = n\,\text{[redacted]}\,T$$

$\uparrow$

2.1 Pa (abs)

[p] Pressure: bara psia

(*) absolute (normal):

gauge $\equiv$ absolute p — atmospheric p

barg psig

1.1 Pa (gauge)

[V] Volume m^3 ft^3 S.C. Sm^3 scf

SI

$10^9\,m^3 = (km)^3$

$M\ Sm^3 = 10^6\ Sm^3$

$M_{\wedge}\ m^3 = 10^6\ m^3$

Field

$M = 10^3$

$MM = 10^6$

$(B)\ b = 10^9$

$(T)\ t = 10^{12}$

MMscf

Mscf (MCF)

$\left.\begin{array}{l}\text{Bcf}\\ \text{Tcf}\end{array}\right\}$ "scf"

b bbl

MM bbl

[T] Absolute Temperature [K, °R]

$$* \begin{cases} K = 273.15 + {}^\circ C \\ {}^\circ R = 459.67 + {}^\circ F \end{cases}$$

$${}^\circ R = 1.8\ K$$

$${}^\circ C = ({}^\circ F - 32)\frac{1}{1.8}$$

$${}^\circ F = 1.8\,{}^\circ C + 32$$

$$\frac{9}{5} = 1.8$$

[n] Moles

		mass unit		
SI:	mol	("g")	gmol	g-mol
	kmol	(kg)	kgmol	kg-mol

Field: lb-mol

$$23.48\ \text{lb-mol} = \qquad \text{g-mol}$$

2.2 lb / kg

$$23.48\ \cancel{\text{lb}}\text{-mol} \times \frac{\cancel{\text{kg}}}{2.2\ \cancel{\text{lb}}} \times \frac{1000\ \text{g}}{\cancel{\text{kg}}}$$

10 672 g-mol

Assume Propane (tank) $V = [l]$

$$M \equiv \frac{m}{n}$$

$$M_{C_3} = 44.09 \frac{g}{g\text{-mol}} = 44.09 \frac{lb}{lb\text{-mol}}$$

$$m = 10672 \text{ g-mol} \times 44.09 \frac{g}{g\text{-mol}}$$

$$= 470000 \text{ g} = 470 \text{ kg}$$

$$V = \frac{m}{\rho} = \frac{470 \text{ kg}}{507 \text{ kg/m}^3} = 0.93 \text{ m}^3$$

$$= 930 \text{ l}$$

"ρ_L" ~ 507 kg/m^3

↑ liquid volume

moles have a "mass" unit associated!

R = universal gas constant

$= 8.3143$	10.7315
p [Pa]	p [psia]
T [K]	T [°R]
n [g-mol]	n [lb-mol]
V [m^3]	V [ft^3]

"Real" Gas Law

$$pV = nRT\, Z$$

$$\uparrow$$

Z-factor

Gas Deviation Factor

- $Z = 1$: Ideal Gas
- No units
- Range 0.7 → 2
- Strong Function of
  - p
  - T
  - Composition (C_1 vs CO_2 vs Air)

van der Waals 1870s

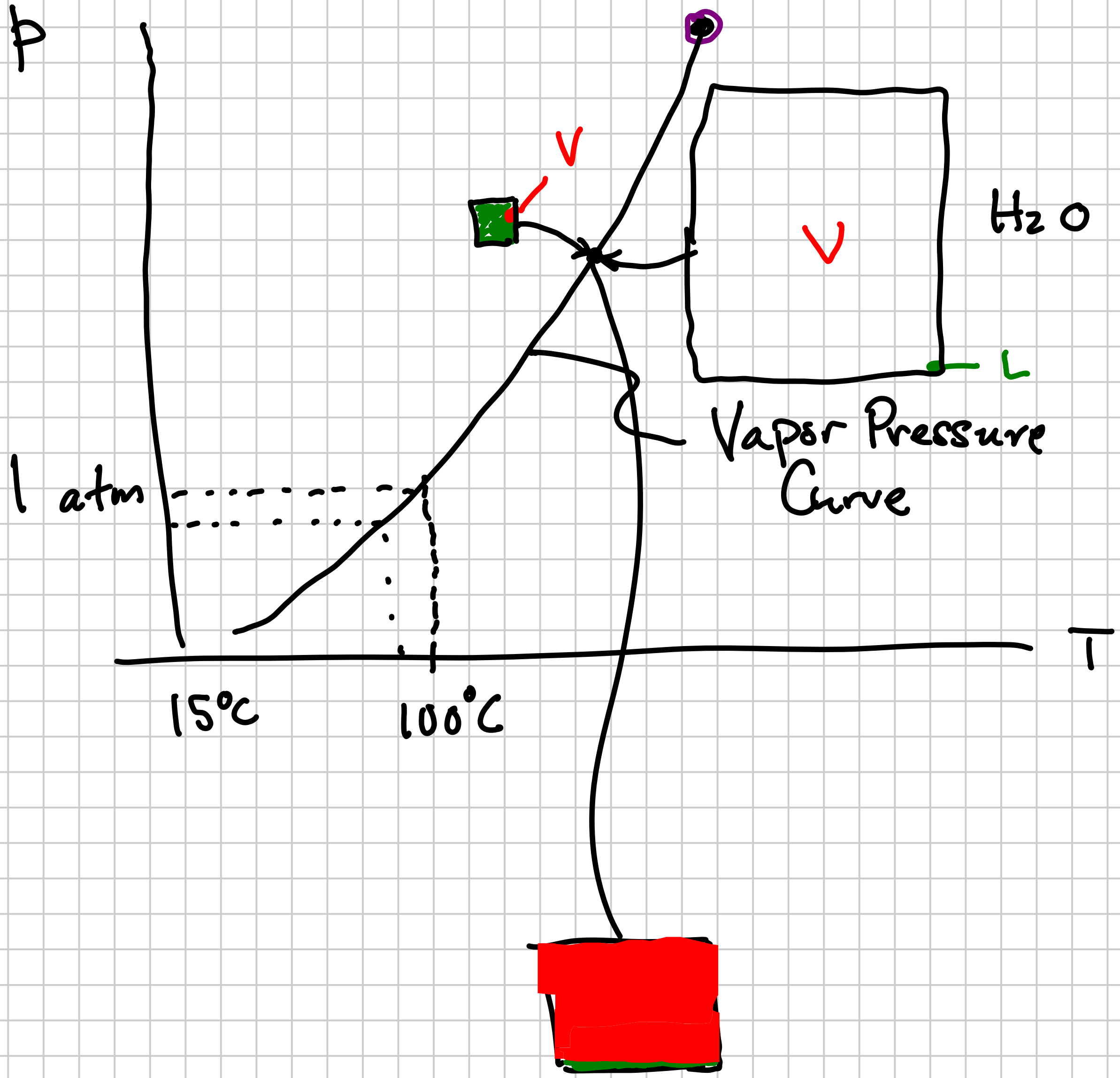
Law of Corresponding States

All gases behave the same IF we use a set of "transformed" variables (instead of P, T, V)

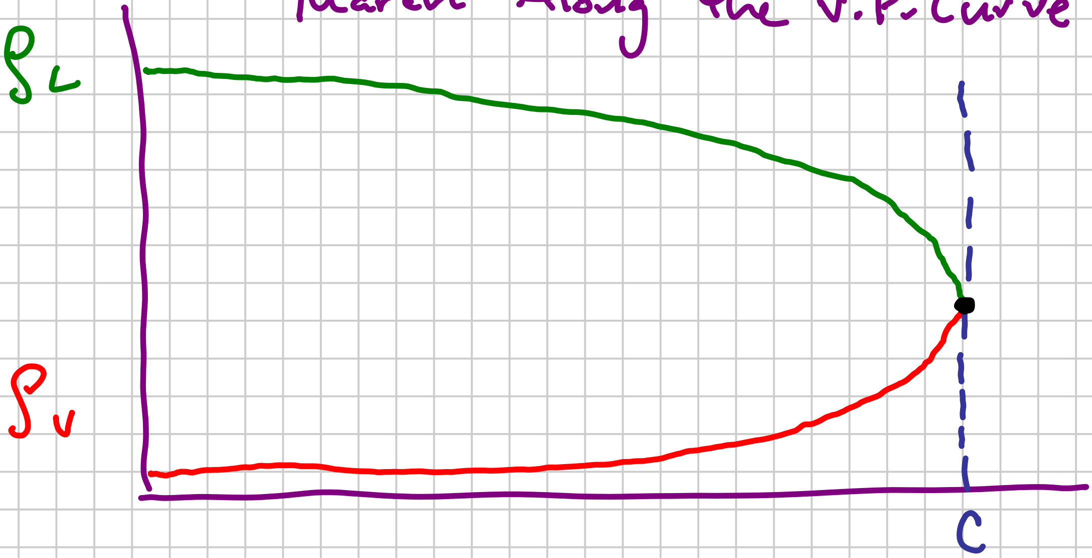
Transformation for Gases:

Reduced $p_r \equiv \frac{p}{p_c}$ ← Critical Pressure

$T_r \equiv \frac{T}{T_c}$ ← Critical Temp.



March Along the V.P. Curve



$Z(T_r, p_r)$ same for <u>any</u> gas

Katz

Standing

Mixtures: N_2, CO_2, H_2S, C_1, C_2... C_7...C_{20}

$$\text{“}T_c\text{”} = T_{pc} = \sum y_i \cdot T_{ci}$$

Pseudo Critical

mole fraction i

$$p_{pc} = \sum y_i \, p_{ci}$$

$$T_{pr} = \frac{T}{T_{pc}}$$

$$p_{pr} = \frac{p}{p_{pc}}$$

Pseudo
Reduced

$y_{C_1} = 0.8$ $p_{c,C_1} = 668$ psia

$y_{C_3} = 0.2$ $p_{c,C_3} = 616$ psia

$p_{pc} = 650$ psia

$$\boxed{Z(p_{pr}, T_{pr})}$$

GAS PHYSICAL PROPERTIES

ρ_g — Gravity Effects

- Initial S_w (depth)
- Gas-oil and/or Gas-Water segregation during flow

$$\Delta\rho = \rho_w - \rho_g \text{ or } \rho_o - \rho_g$$

B_g — Gas Rate Eq. $B_g(p)$

— Calculating B_{gi} IGIP / OGIP $\}$ from HCPV estimates

PTA

Pressure transient analysis

c_g

Well Test Analysis

$$\text{Diffusivity Constant} = \left(\frac{k}{\phi \mu c_t}\right)$$

$$c_t = \underset{\substack{\uparrow \\ \text{rock}}}{c_f} + S_w c_w + S_o c_o + S_g \underset{\uparrow}{c_g}$$

$$\mu_g = f(\rho_g)$$

$$\rho_g \equiv \frac{m_g}{V_g} = \frac{pM}{RTZ} = \frac{p \cdot M_{air} \cdot \gamma_g}{RTZ}$$

$$B_g \equiv \frac{V_g}{V_{gsc}} = \frac{p_{sc}}{T_{sc}} \cdot \frac{\text{[illegible]}\,T}{p}$$

assumes all moles n at (p, T)

remain as gas at (p_{sc}, T_{sc})

$$c_g \equiv -\frac{1}{V_g}\left(\frac{dV_g}{dp}\right)_T = \underset{\substack{\uparrow \\ \text{Dominant Term}}}{\frac{1}{p}} - \frac{1}{\text{[illegible]}}\underbrace{\left(\frac{d\,\text{[illegible]}}{dp}\right)_T}_{\text{tor-}}$$

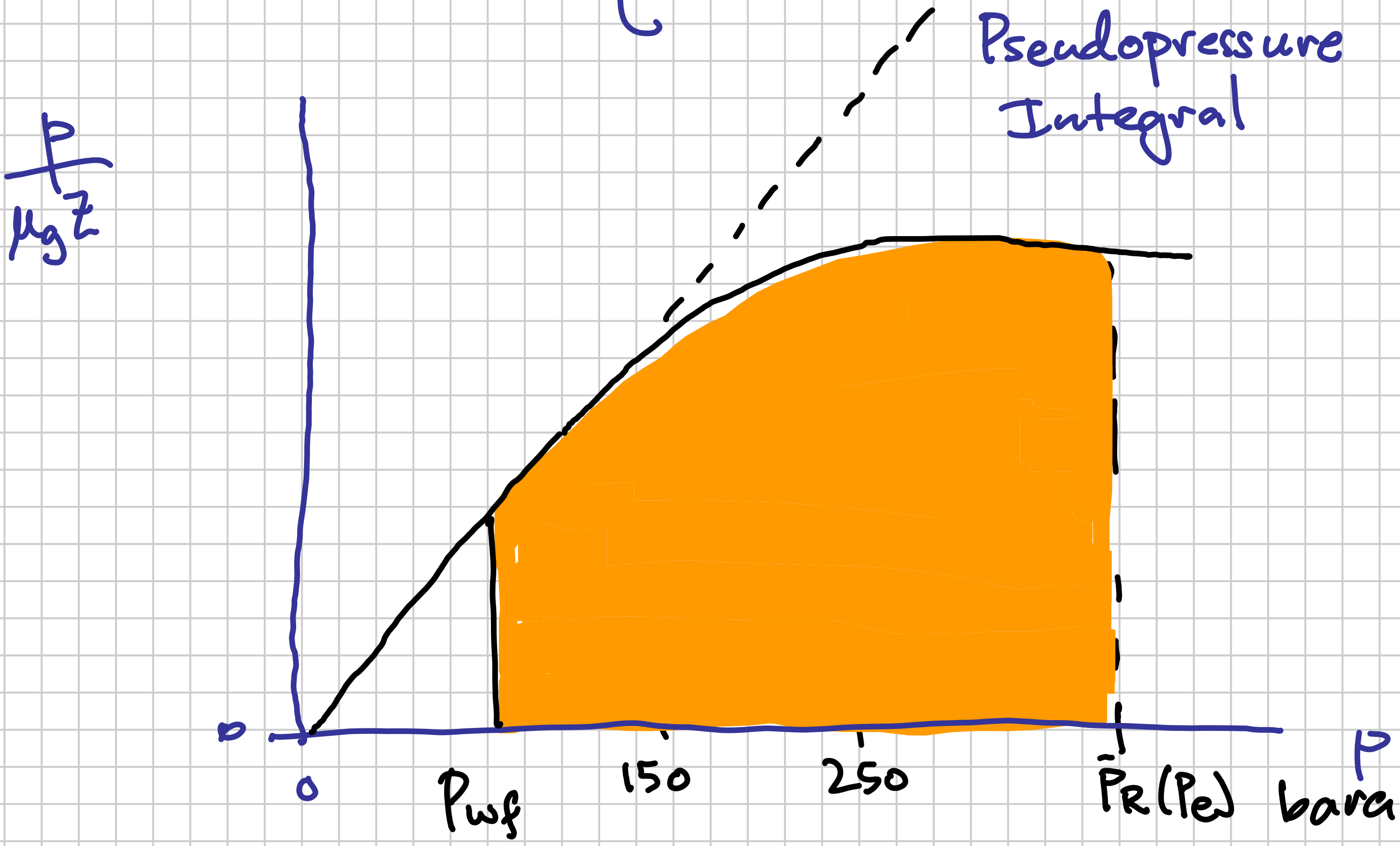
$c_g \simeq \frac{1}{p}$ ok up to ~ 100 bara

GAS RATE EQ

$$q_g = \frac{2\pi kh}{\ln\frac{r_e}{r_w} + s} \cdot \int_{p_{wf}}^{p_e} \frac{1}{\mu_g B_g}\,dp$$

$$\frac{1}{\mu_g B_g} = \frac{1}{\mu_g}\,\frac{T_{sc}\,p}{p_{sc}\,z\,T}$$

$$q_g = \left\{ \underbrace{\frac{\pi T_{sc}}{p_{sc} T}}_{\text{const. "C"}} \cdot \frac{kh}{\ln\frac{r_e}{r_w} + s} \right\} \boxed{2\int_{p_{wf}}^{p_e} \frac{p}{\mu_g z}\,dp}$$



$$p_p = m(p) \equiv 2\int_{\boxed{p_{sc}}}^{p} \frac{p}{\mu_g Z}\,dp$$

Pseudopressure

low ref.

Al-Husseiny,
Ramey,
Crawford

~1970

$$q_g = \underset{\uparrow}{C} \cdot \left[p_p(p_e) - p_p(p_{wf}) \right]$$

GAS MATERIAL BALANCE

Provides the relationship between average volumetric pressure $\bar{p}_R$ ($\bar{p}$ or p_R) and cumulative gas produced from the reservoir G_p. (remember: G = IGIP)

V_g at p_{sc}, T_{sc}

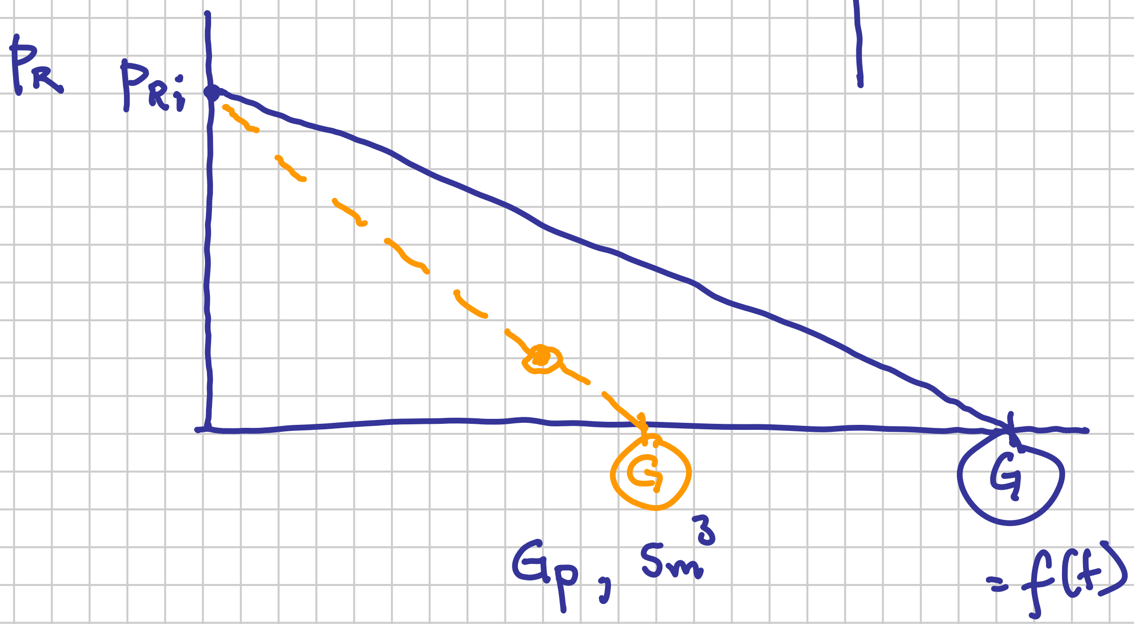
scf, Sm^3 $((Nm^3))$

std m^3

1st Order or Rule of Thumb:
(Ideal Gas Law)

$$\frac{p_R}{p_{Ri}} \cong 1 - \frac{G_p}{G}$$

~
- Ideal Gas Law
- HCPV = constant



Reasons for Deviation of the Simple MB

① Z-factor ; real gas behavior

② HCPV ≠ constant

- c_f : PV decreases as p_R ↓
- c_w : waterexpansion P_R ↓
  - Connate S_{wc}, S_{wi}
  - Aquifer *

③ Layer No-Crossflow (Differential Depletion)

- $\dfrac{(q_{gmax})_\ell}{G_\ell}$ "=" Voidage Ratio VR

$$(q_{gmax})_\ell = q_g \text{ @ low } P_{wf}$$
$$\sim \frac{(kh)_\ell \, (\bar{P}_\ell - P_{wf})}{\ln \frac{r_e}{r_w} + s_\ell}$$

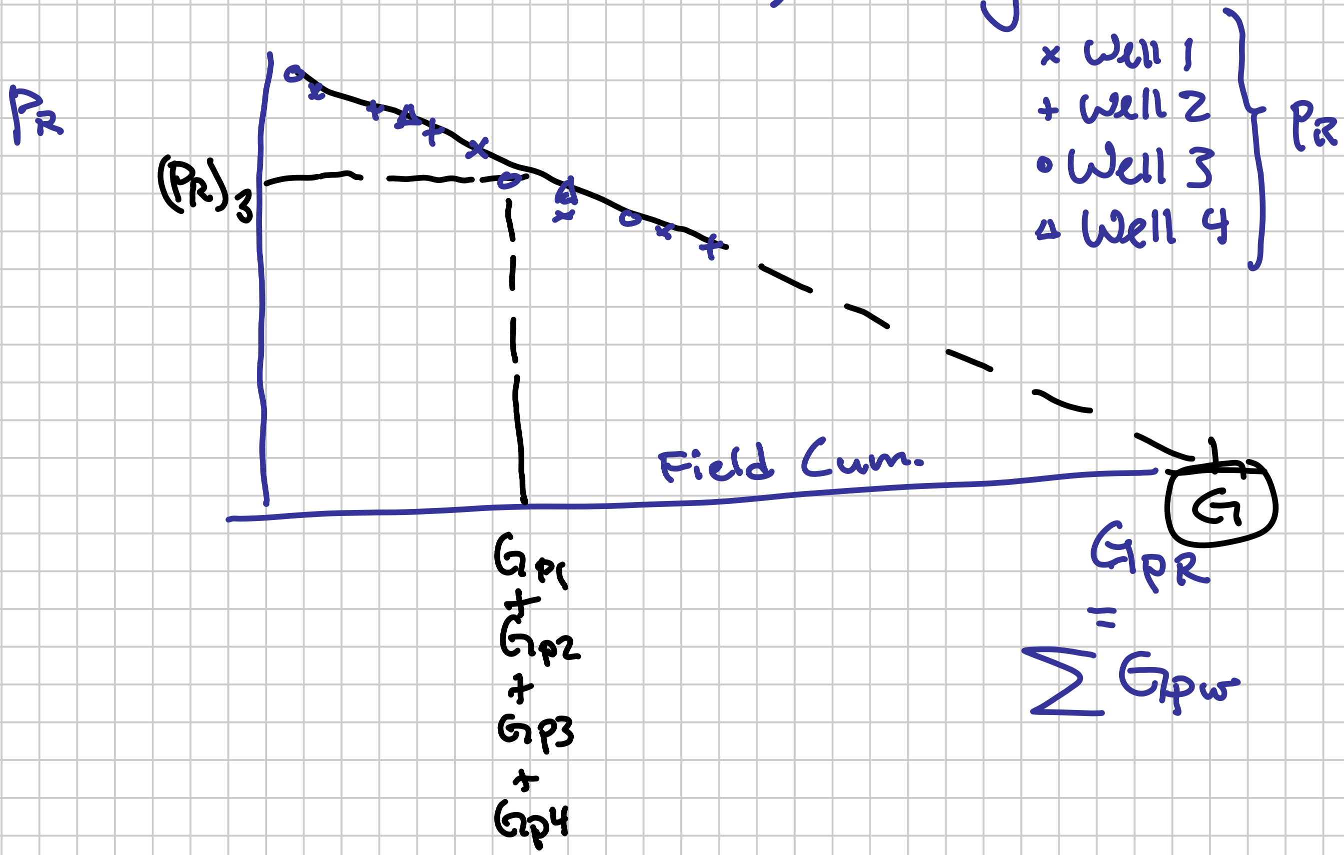
$\dfrac{(VR)_2}{(VR)_1} > 2\text{-}3$ ⇒ lead to significant differential depletion

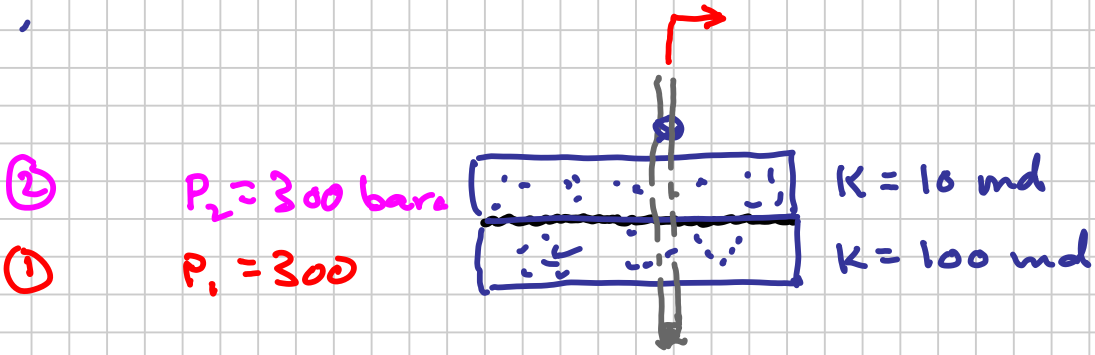
⇒ SI "P_R" are too low relative to the actual P_R

④ Well Interference (higher k reservoirs)

$$\frac{G_w}{G} \sim \underbrace{\frac{q_w}{\sum q_w}}_{\text{\% of the total rate from a given well}}$$

... Always use MB for the entire reservoir, not by well.





Common Situation
(NOT EXCEPTION)

$G = 200$ units

$G_1 = 100$

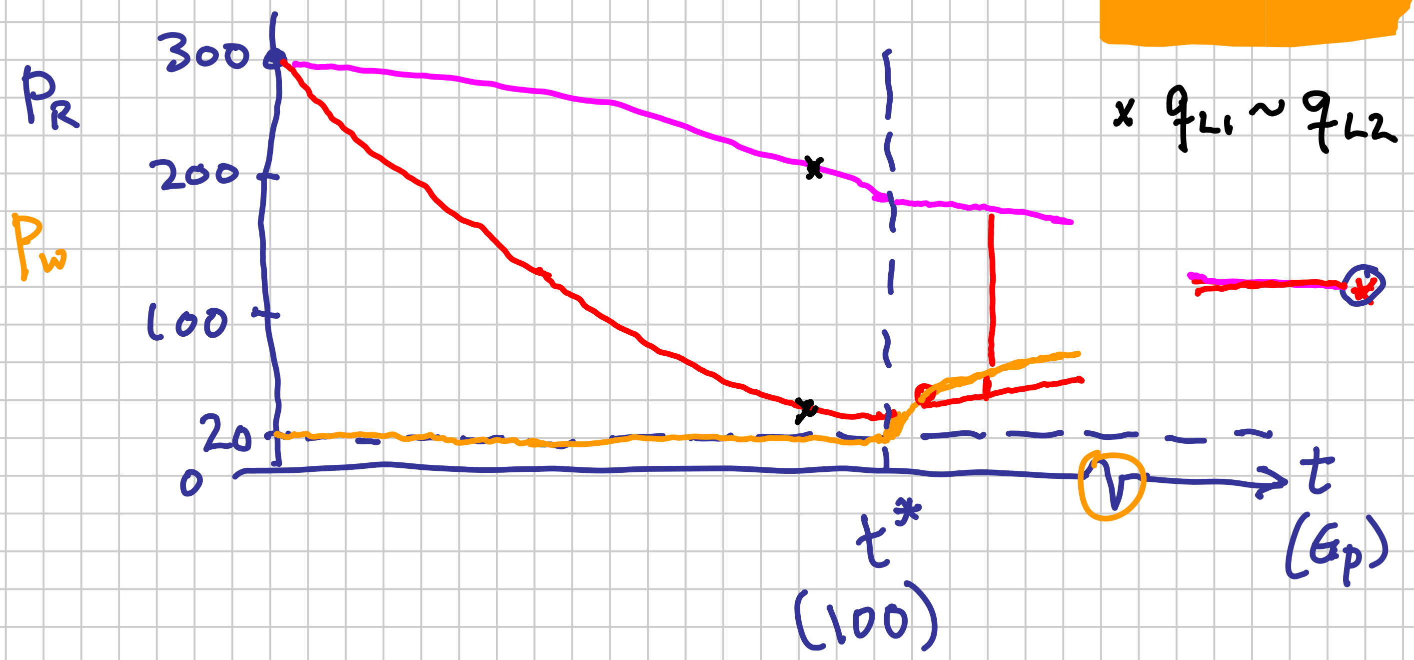
$G_2 = 100$

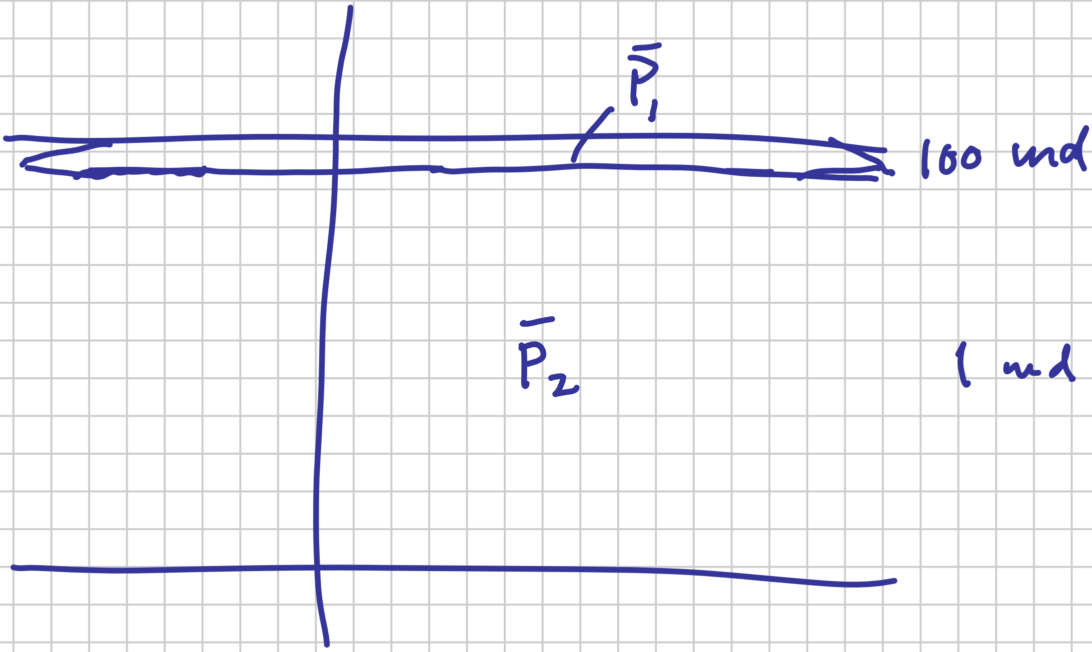
@ $P_{wf} = 20$ bara = constant

$G_p = 100$ units

$\bar{P}_R$

Plot P_{R1} & P_{R2}





Deviation of MB due to Real Gas, Z-factor

$$pV = nRTZ$$

~∘ V_R = HCPV = constant

$$V = \underset{\text{Initial}}{\frac{n_i R T_R Z_i}{P_i}} = \underset{\text{Current}}{\frac{(n_i - n_p)\, R\, T_R \cdot Z_R}{p_R}}$$

n_i = initial moles gas in R

n_p = cum. produced moles gas

Use surface gas volumes instead of moles:

$P_{sc} = 1$ atm

$T_{sc} = 15.56$ °C

$$P_{sc} V_{sc} = n R T_{sc}$$

e.g. $\frac{Sm^3}{kmol}$

$$\frac{V_{sc}}{n} = \underbrace{\frac{R T_{sc}}{P_{sc}}} = 23.64 \frac{Sm^3}{kmol}$$

$\frac{scf}{lbmol}$

$$= 379.14 \frac{scf}{lbmol}$$

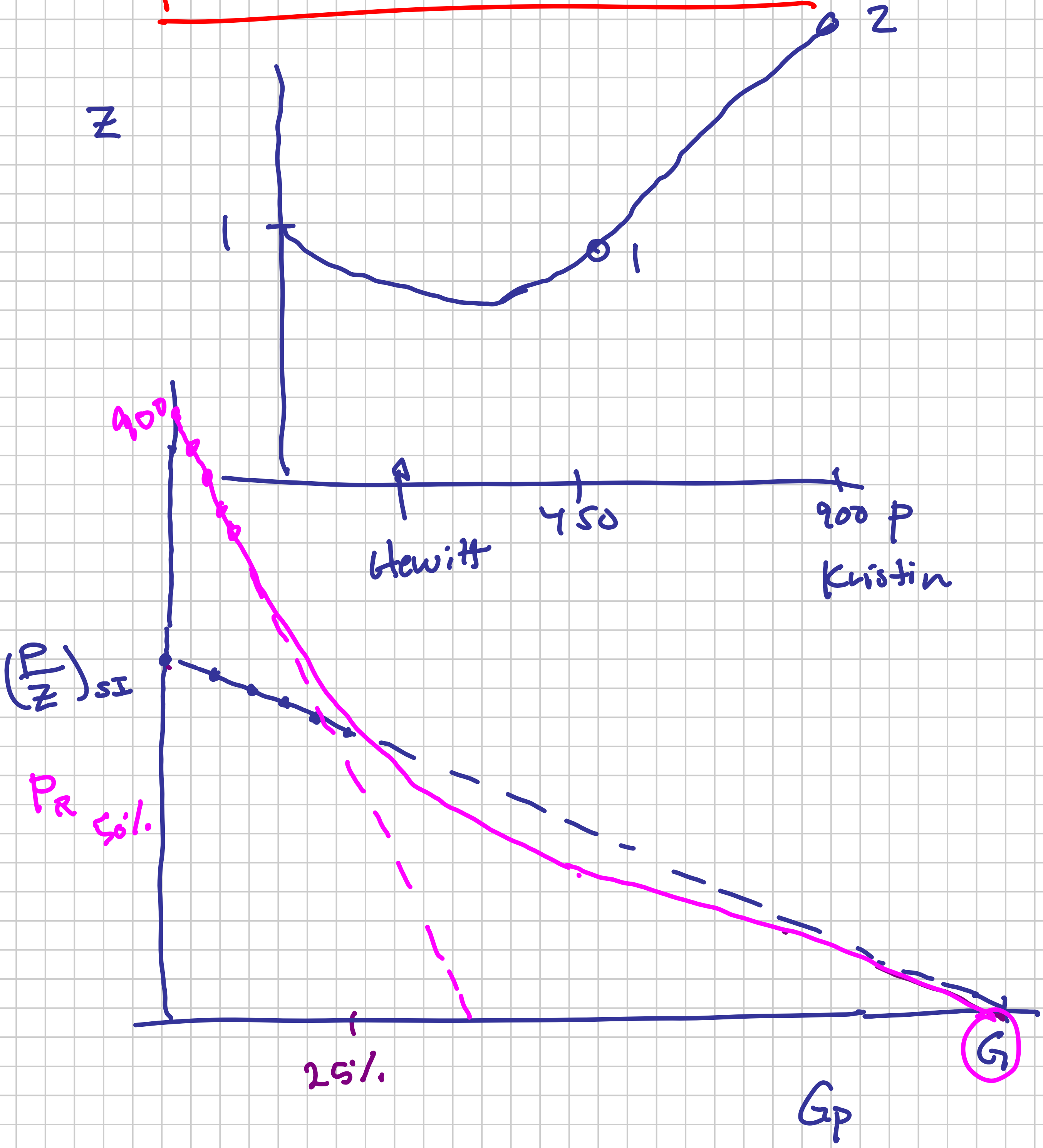
$$V = \frac{\cancel{n_i}\cancel{R}\cancel{T_R}Z_i}{P_i} = \frac{(n_i - n_p)\cancel{R}\cancel{T}_R \cdot Z_R}{p_R}$$

$$\times \frac{1}{\cancel{n_i}\cancel{R}} \qquad\qquad \times \frac{1}{n_i}\frac{1}{\cancel{R}}$$

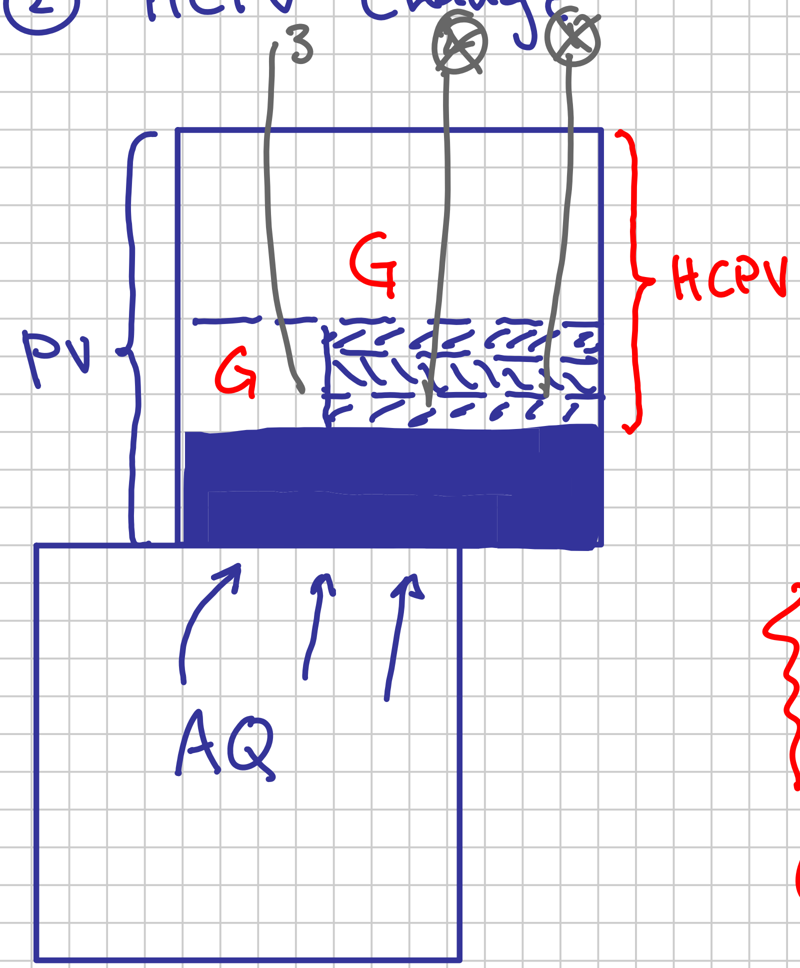
$$\frac{z_i}{P_i} = \left(\underset{1}{\frac{n_i}{n_i}} - \frac{n_p}{n_i}\right)\frac{z_R}{p_R}$$

$$\frac{n_p}{n_i} = \frac{G_p}{G}$$

$$\ldots \quad \boxed{\frac{p_R}{Z_R} = \frac{p_i}{Z_i}\left(1 - \frac{G_p}{G}\right)}$$



② HCPV Change



$p \downarrow$

✓ V_p PV ↓ $c_f = \frac{1}{V_p}\frac{dV_p}{dp}$

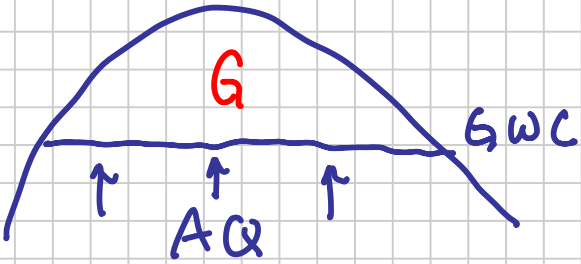
✓ V_w ↑ $c_w = -\frac{1}{V_w}\frac{dV_w}{dp}$

↓ $HCPV = V_p - V_w$

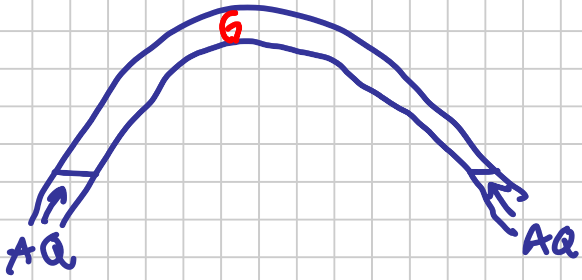
Same G_p *

Higher $\bar{p}_R$ (Good) → BAD (AQ)

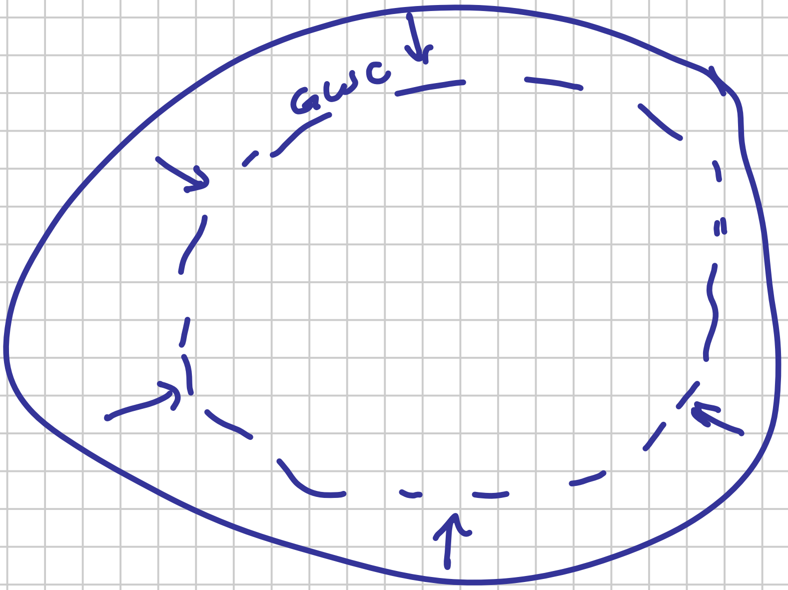
(G_p/G)



~ Linear AQ Flow



~ Radial AQ Flow



Calculating HCPV Change:

* $C_f = \frac{1}{V_p}\frac{dV_p}{dp}$ ~ constant ; may be f(p)

$$c_f \int_{p_i}^{p} dp = \int_{V_{pi}}^{V_p} \frac{1}{V_p}\, dV_p$$

$$c_f(p-p_i) = \ln \frac{V_p}{V_{pi}}$$

$$V_p = V_{pi} \cdot \exp\left[c_f(p-p_i)\right]$$

$$V_p \approx V_{pi}\left[1 - \underset{\substack{\uparrow \\ \sim 3-20\cdot 10^{-6}\ \text{psi}^{-1} \\ (4-10)}}{c_f}(p_i-p)\right]$$

* Connate Water Expansion

$$V_w = V_{wi} \cdot \exp\left[c_w(p_i-p)\right]$$

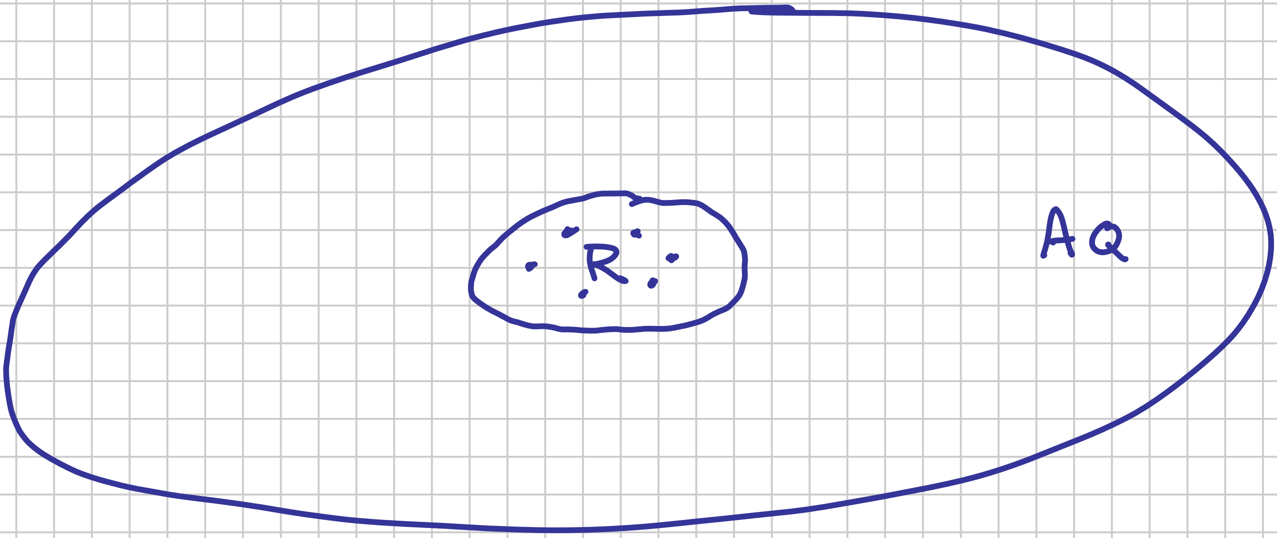
$$\approx V_{wi}\left[1 + c_w(p_i-p)\right]$$

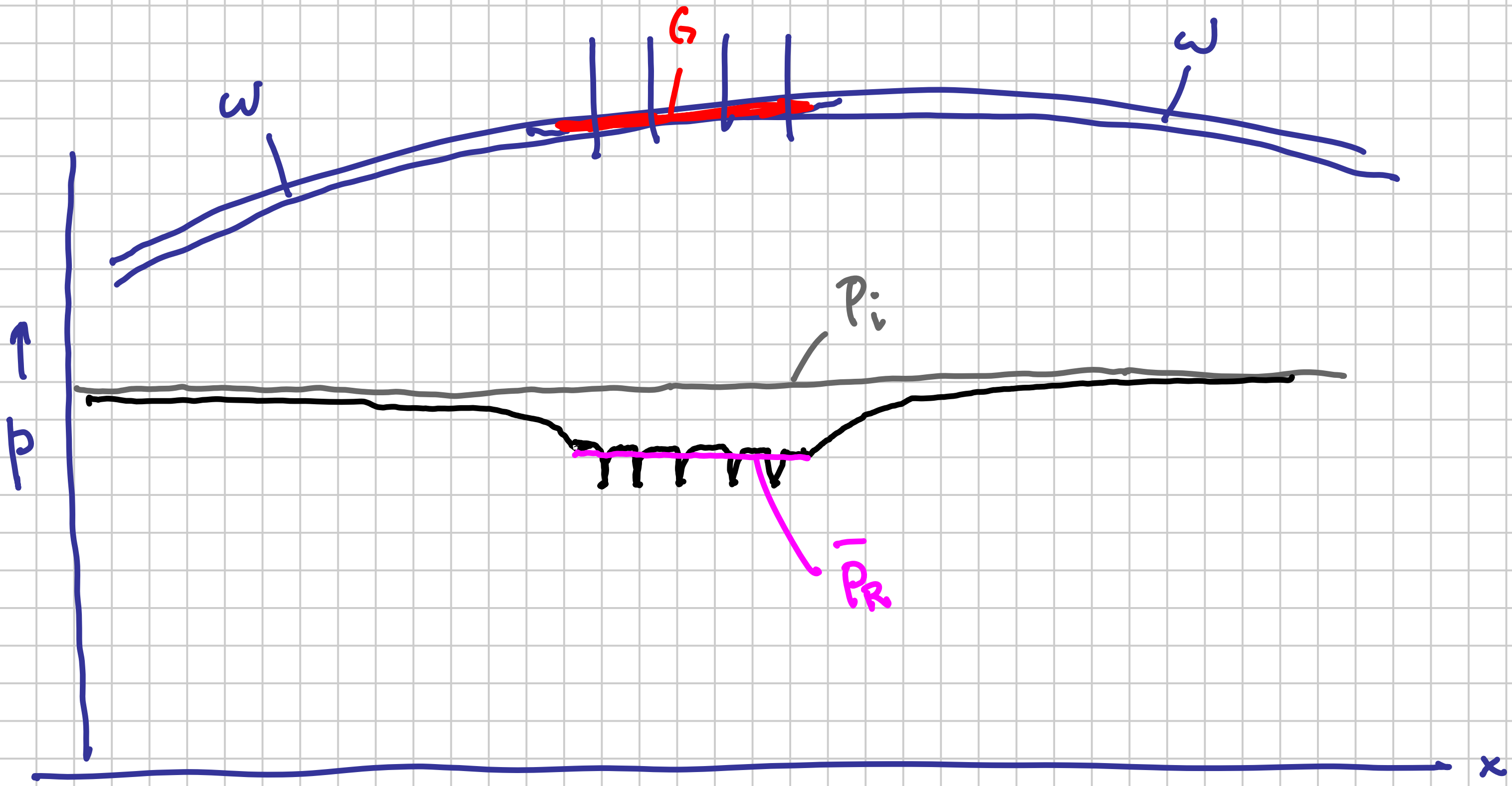
* Instantaneous volume changes in HCPV

AQUIFER-Induced HCPV Change

- May or may not be "instantaneous"
(~ 1-6 mo)

ΔV_w^* = ultimate (max) water expansion of the aquifer when the Reservoir pressure drops Δp, $t \to \infty$

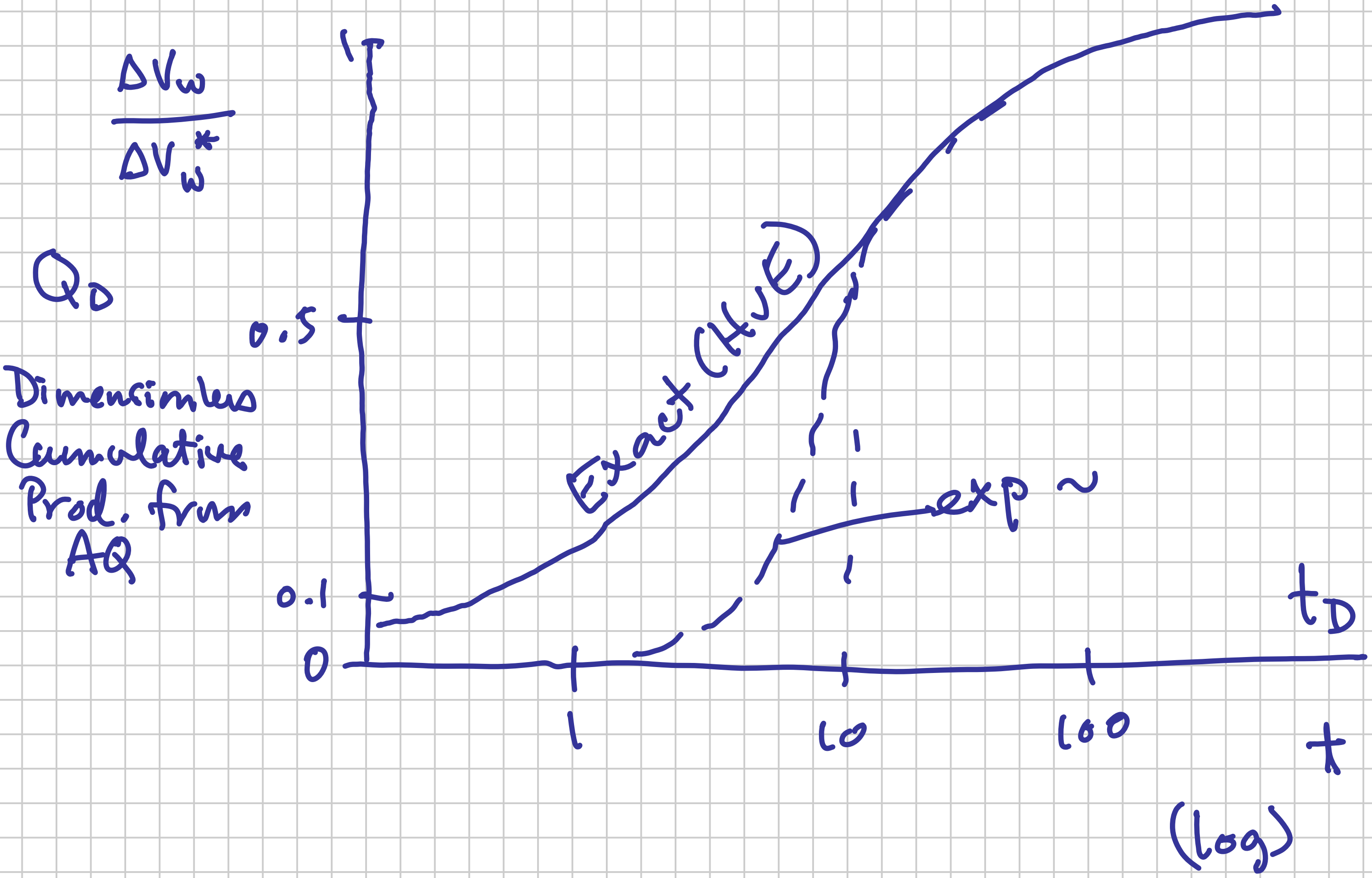




$$\Delta V_w^* = V_w - V_{wi}$$

$$\cong c_w \cdot \underset{\substack{\uparrow \\ \text{AQ} \\ \text{volume}}}{V_{wi}} \cdot (p_i - p)$$

Actual ΔV_w that encroaches from AQ to the R is ΔV_w at a given time after Δp occured.



$$t_D \sim \frac{k}{\phi \mu_w c_{tAQ} (r_{AQ}/r_R)^2} \cdot t$$

AQUIFER MODELS

(1) "POT" AQ

90-95%

AQ response is "instantaneous" (1-6 mo)

(2) PSEUDOSTEADY STATE (Fetkovich)

$Q_D \sim \exp(t_D) \Rightarrow$ Simple "Superposition"

(3) HURST-VAN EVERDINGEN

Includes all transient $Q_D(t_D)$ behavior

... complicated, messy, ... more accurate

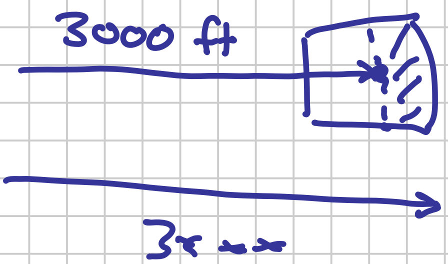
Today, model numerically

GAS MATERIAL BALANCE

AQUIFER

No Flow O.B. $r_e = 3000$ ft

Constant P O.B r_e = 3xxx ft



Injector with
PI ~ ∞
i.e. p_{wf} (PBH)
$\simeq \bar{P}_{G,20}$

Impact of Aquifer on Gas M.B.

HCPV decreasing

Given Gp ... G-Gp gas remaining

the $\bar{P}_R$ will be higher, the more/stronger aquifer – i.e. the more water that has encroached into the original HCPV.

⇒ Generally "good"

$$q_g = C \cdot \left[m(\bar{P}_R) - m(P_{wf}) \right]$$

... May be "not so good"

water encroaches finds its way to the (used-to-be) gas producing wells

DEATH.

Factors that decide how much water will encroach?

✓ * Initial reservoir pressure, P_{Ri}

* $\overset{\text{AQUIFER}}{\text{(Reservoir)}}$ $\underline{\underline{k}}$ — $f(t)$ 0 → 1

* $\boxed{V_{AQ}}$ ±

Maximum amount of encroachment

$$\boxed{W_e^*} = V_{AQ} \cdot C_{AQ} \cdot \Delta P_{AQ}$$

± ! → V_{AQ}

$(c_w + c_f)_{AQ}$ → C_{AQ}

8-10 · 10^{-6} 1/psi

$\boxed{P_{Ri}}$ → ΔP_{AQ}

$$C_{AQ} = \frac{1}{V_{AQ}} \frac{dV_{AQ}}{dp}$$

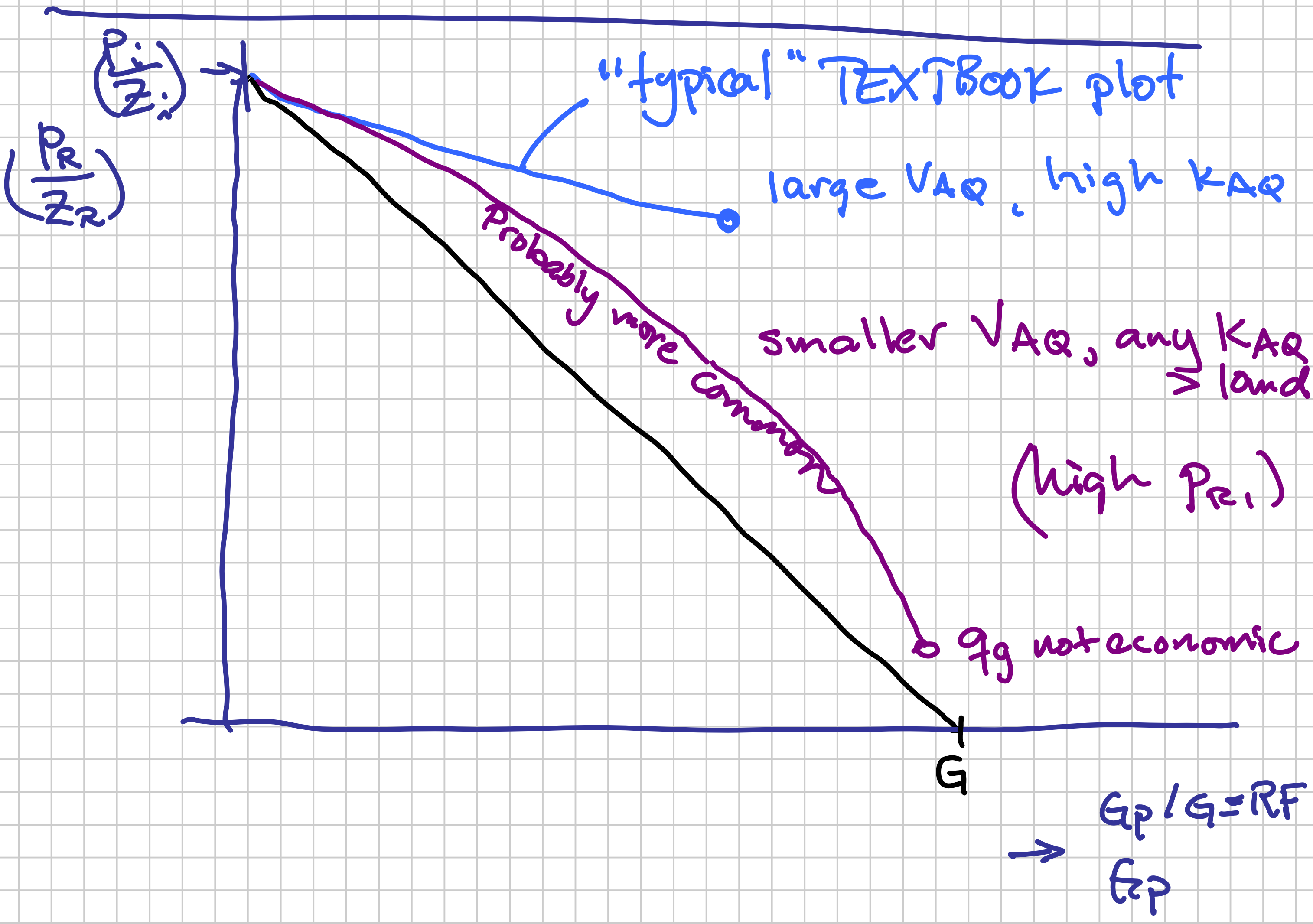
$k \lesssim 1$ md : AQUIFER is <u>NON</u> issue

$k > 500$ md : Instantaneous

(**) q_{gF} only an effect K ~> 500 md
& large aquifer
(extent)

~ Instantaneous influx,
very much water

To maximize gas recovery get the gas out as fast as you can.



"POT" AQUIFER MODEL : ΔW_e^* for Δp_{AQ} : < 6 mo.

$\bar{p}_R \approx \bar{p}_{AQ}$

$$\frac{p}{Z}\left(1 - c_e(p_i - p)\right) = \left(\frac{p}{Z}\right)_i \left(1 - \frac{G_p}{G}\right)$$

Moderate-to-High p_{Ri} "Small enough" V_{AQ}

"High enough" k

c_e = "effective" cumulative, total compressibility of the "system"

- Connate Water (in V_{PR})
- Reservoir rock (pore) compressibility
- V_{AQ}

~ constant ; can be $c_e(p)$

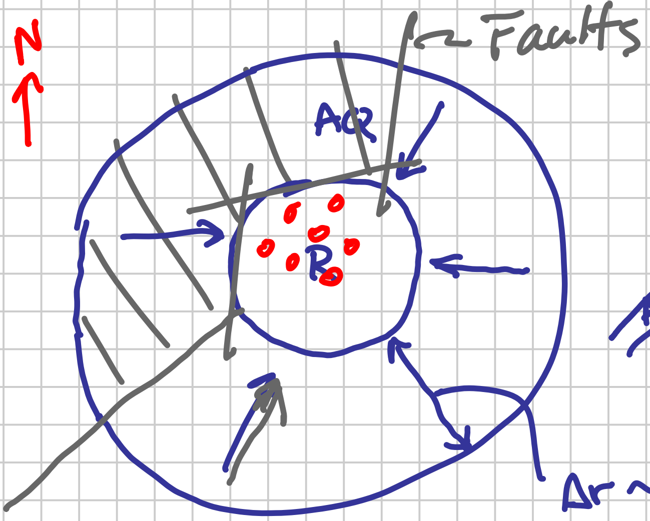
"Degree of Curvature"

$\Downarrow$

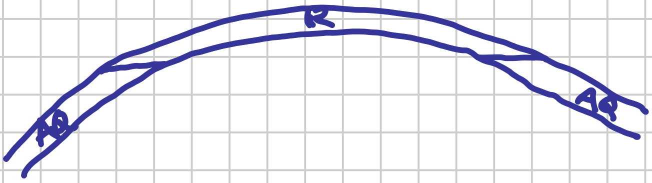
$$c_e = \frac{c_w S_{wc} + c_f + M(c_w + c_f)}{1 - S_{wc}}$$

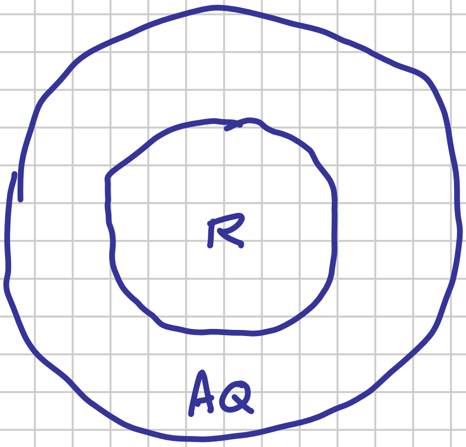
$$M = \frac{V_{AQ}}{V_{PR}}$$

R-AQ Geometries

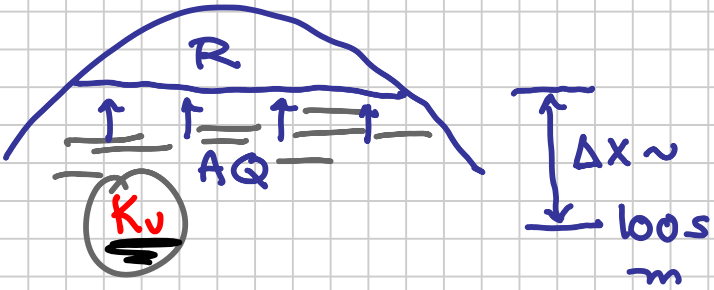


Radial





Linear



Bottom Water Drive

BUILDING MODEL REPRESENTATIONS OF AQUIFERS

1. Define V_{AQ}
2. Permeability "right"
3. Geometry R-AQ right
    - R-AQ boundaries
    - AQ flow distances (Δx, Δr)

4. (Time dependent influx)   10-15%

Sufficient <u>numerical</u> <u>gridding</u>.

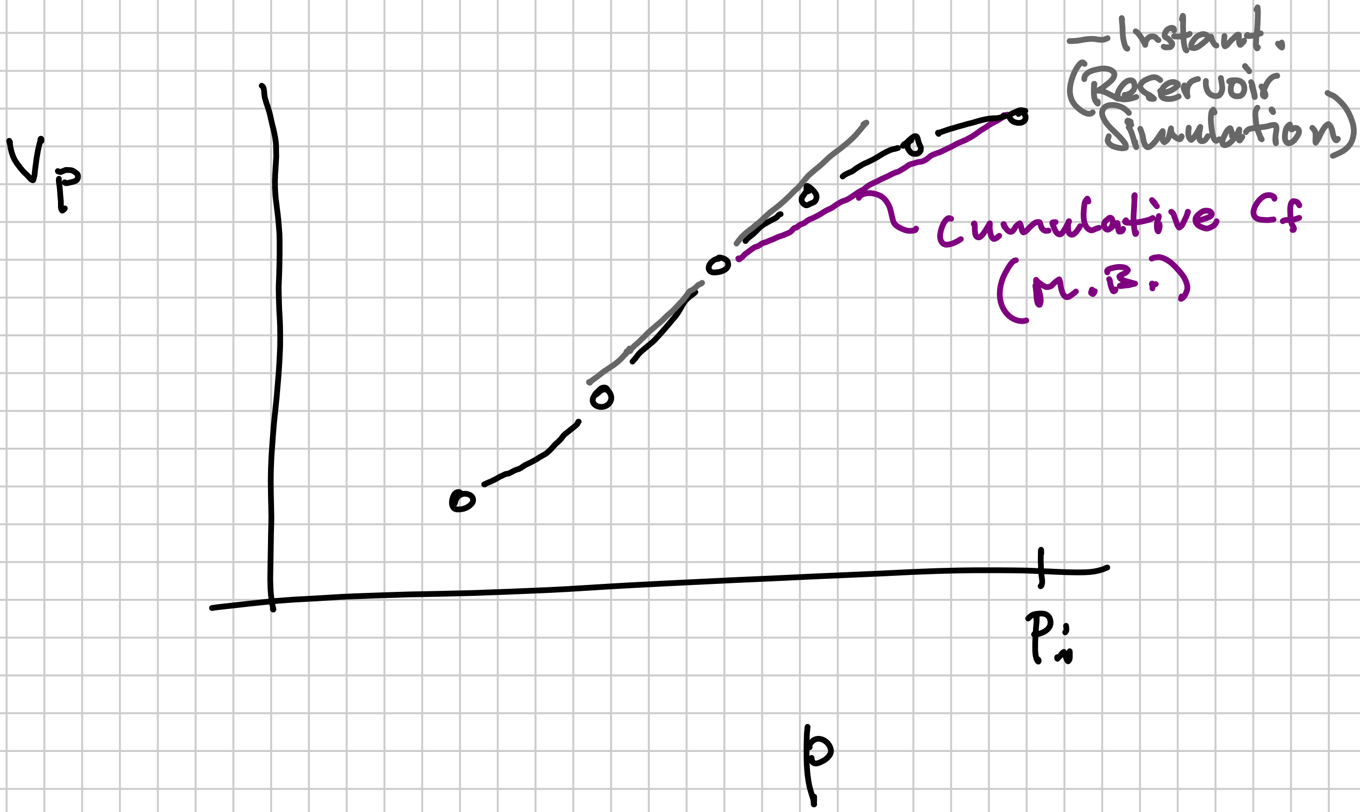
$\underline{\underline{f(t)}} \cdot \underbrace{\Delta W_e^*}_{\text{max influx}}$ (size of lego pieces).

5. Formation compressibility

$\sim \underbrace{4}_{\text{typical}} - \overline{25} \cdot 10^{-6}$ 1/psi

unusual $c_f(p)$

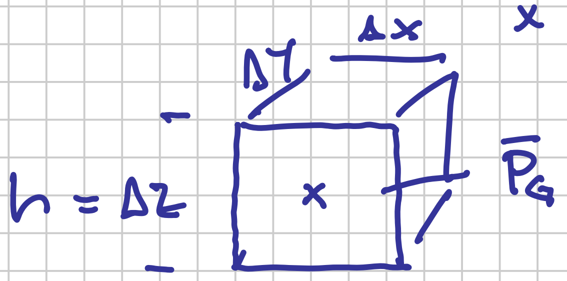
|| - instantaneous ?
|| - cumulative



Study the e-note Gas-Field-Development
→ Production-Forecasting.xls

Uses Pot Aq. Model.

Simulator Well Connection Factor (PI)



x Well Connection

$$q = \boxed{PI}\,\overset{\text{PBH}}{(\bar{P}_G - P_{wf})} \cdot \underbrace{\bar{\lambda}_G}_{\text{Mobility}}$$

$$\lambda = \frac{k_r}{\mu}$$

$$PI = \text{constant}$$

$$PI = \left\{ \frac{(k\Delta z)_G}{\ln\frac{r_o}{r_w} + s} \right\}$$

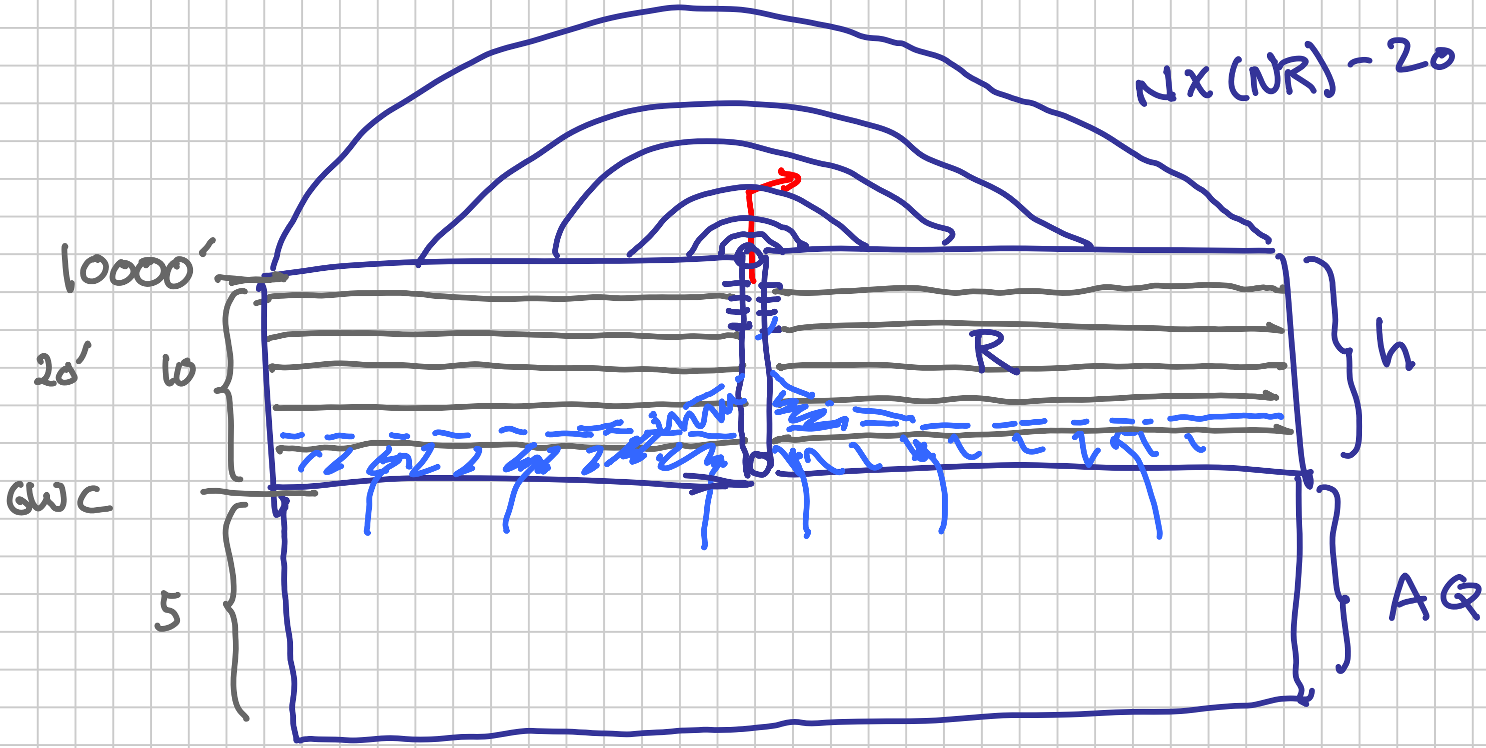
Peaceman Radius $$r_o \sim \left(\frac{\Delta x\ \Delta y}{\pi} \right)^{1/2}$$

$$\frac{p}{z}\left[1 - Ce(p_i - \bar{p})\right] = \frac{p_i}{z_i}\left(1 - \frac{G_p}{G}\right)$$

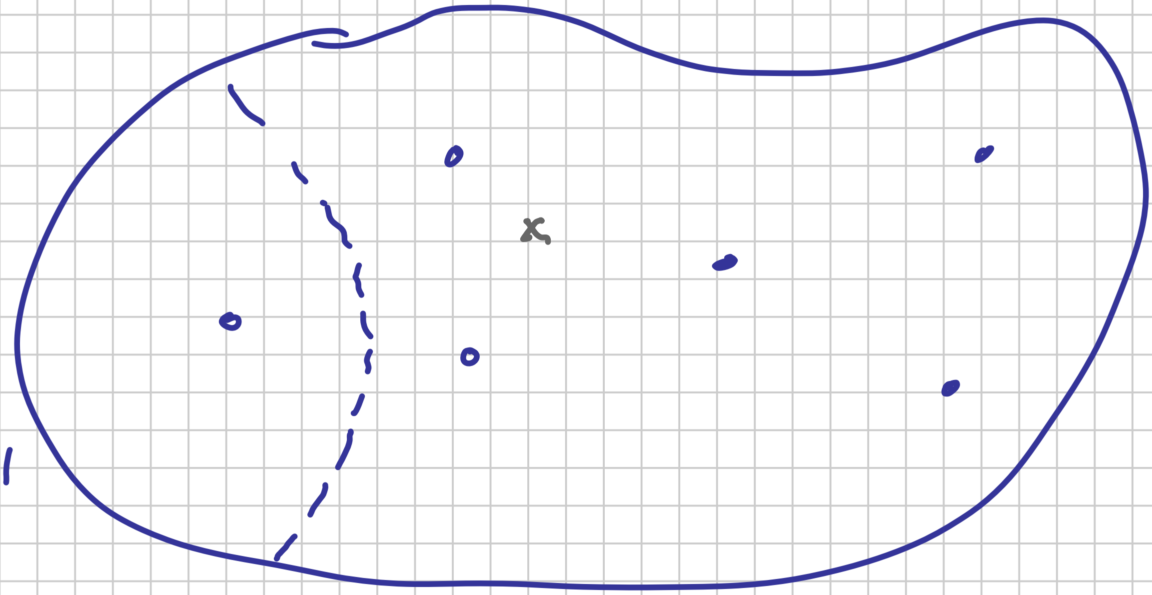
$$\left.\begin{array}{l} Z_i \\ Z(p) \end{array}\right\} \checkmark$$

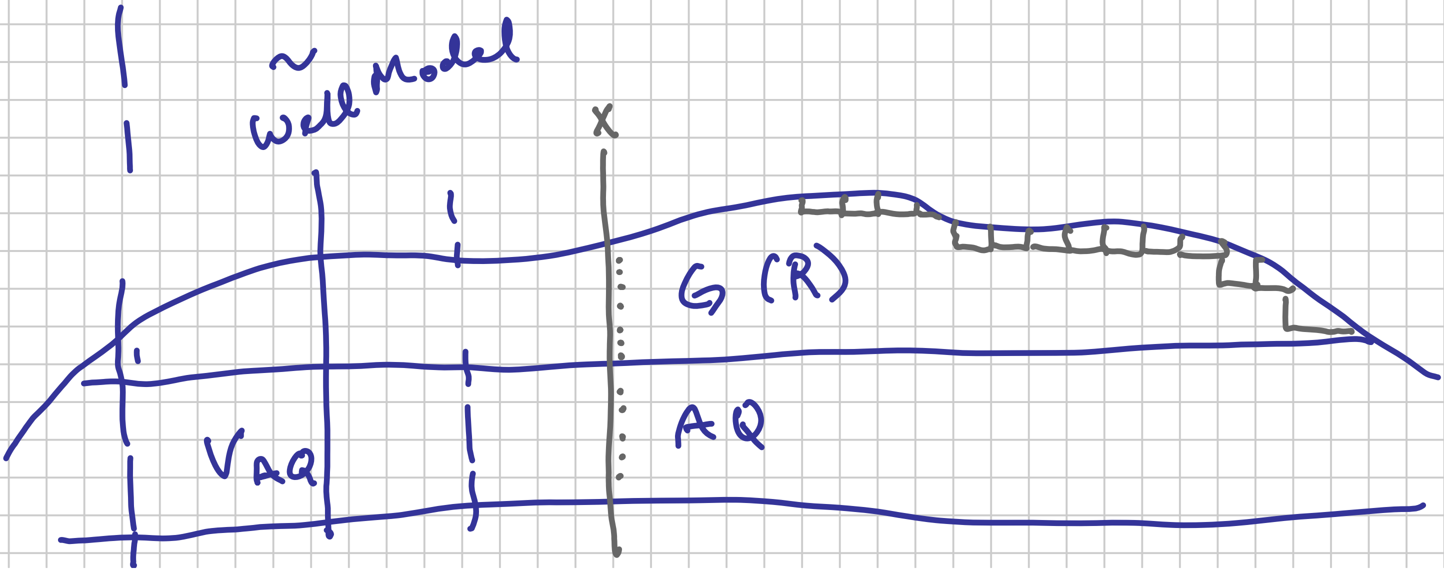
$$C_e: \quad \text{Excel}: \quad \underset{\substack{\uparrow \\ V_{AQ}/V_p}}{\overset{\checkmark}{M}}, \ \overset{\checkmark}{C_f}, \ \overset{\checkmark}{C_w}$$

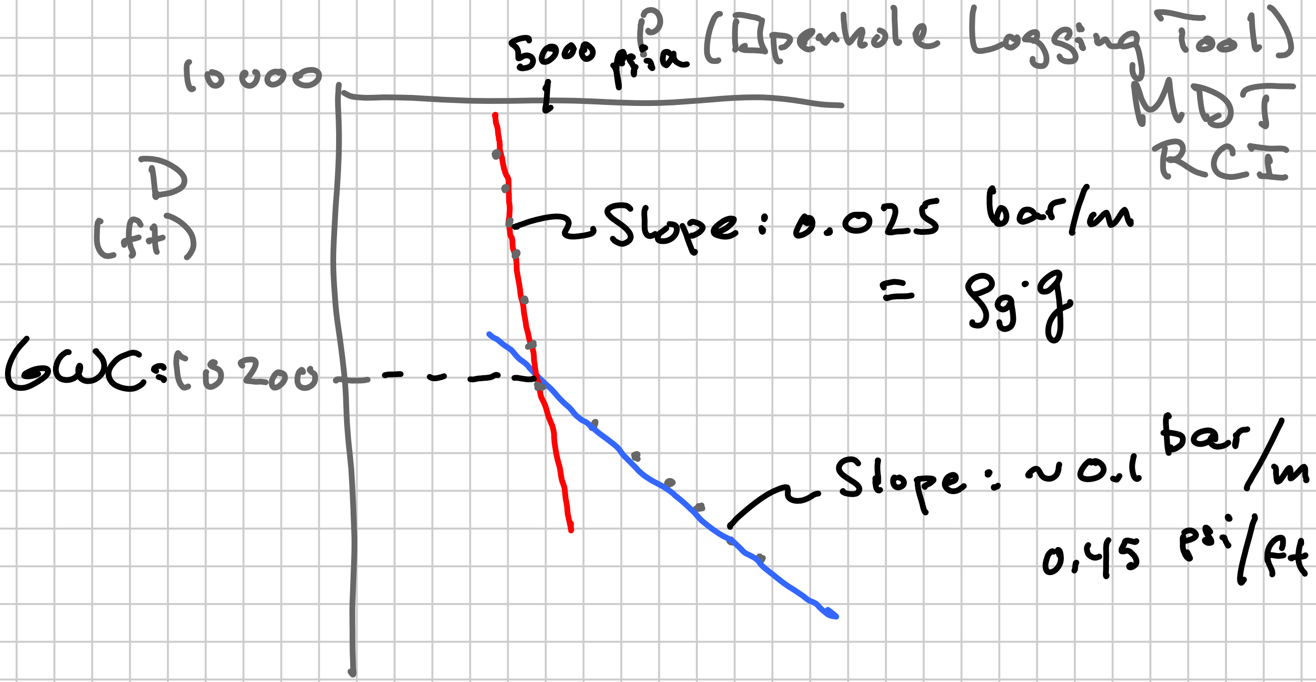
$$\text{Sensor}: \quad \underset{(V_p)}{V_{AQ}}, \ C_f, \ C_w$$



GWC = 10000 (top) + $\underset{\sim}{10}$ × 20' = 10200'







Slope: 0.025 bar/m $= \rho_g \cdot g$

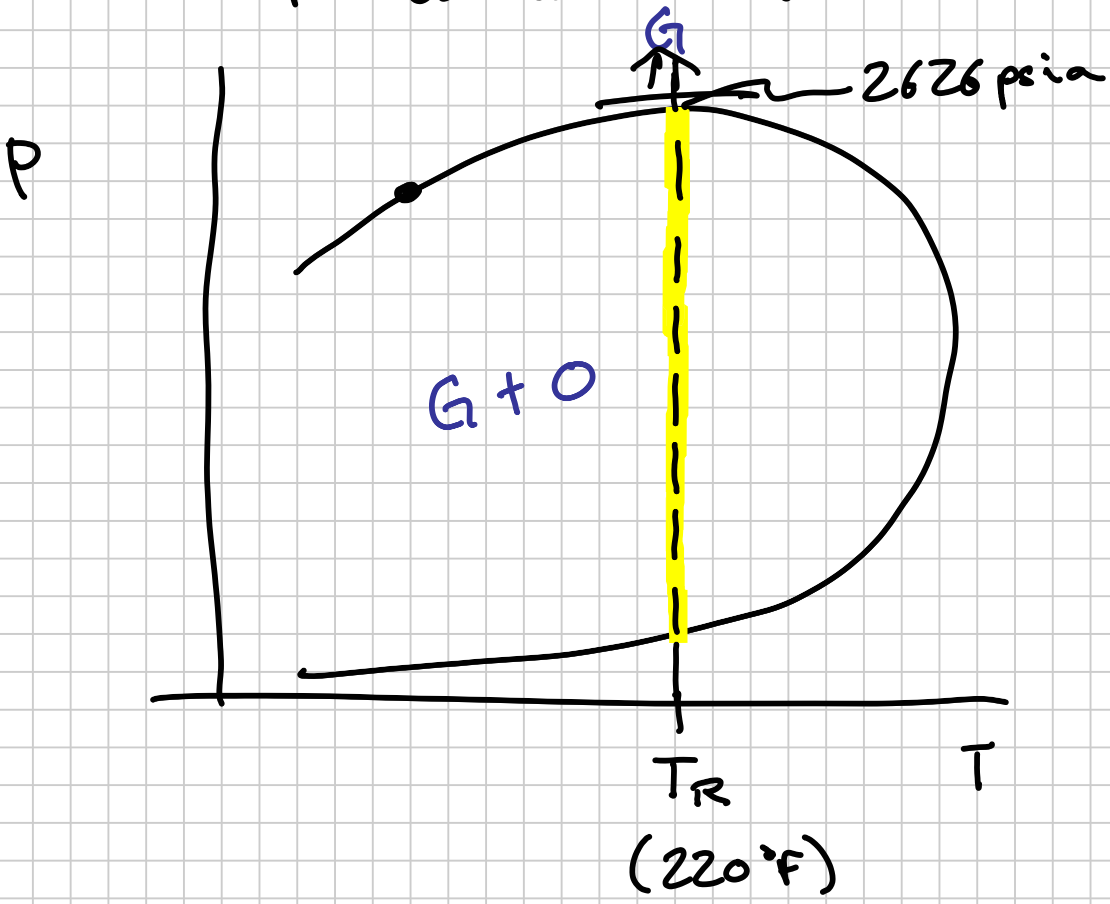
Slope: ~0.1 bar/m

0,45 psi/ft

Model Changes

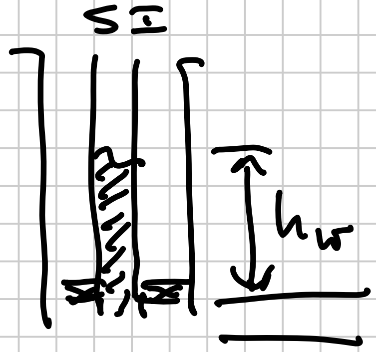
* Make single-phase gas

- Min PBH = 3000 psia Gas Condensate



Problem of Water Prod in Gas Wells

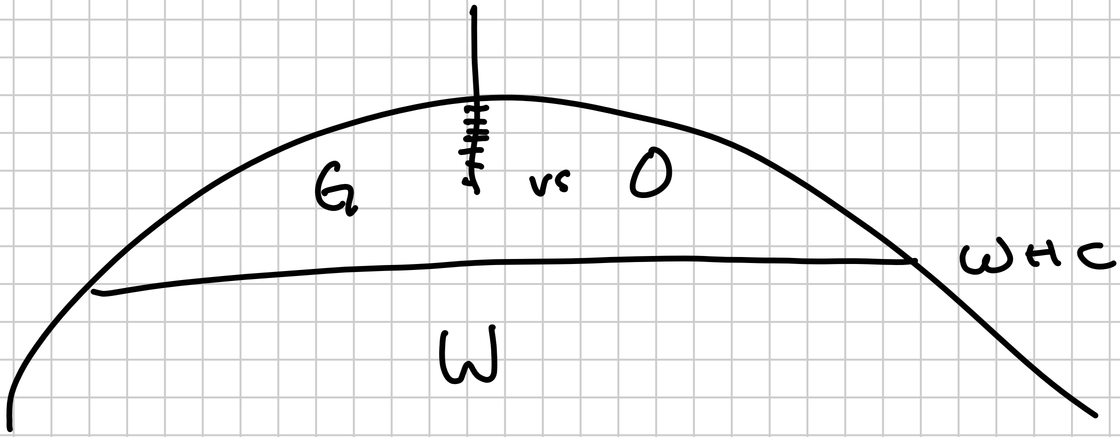
* Backpress $\rho_w \cdot h_w$



Unloading after SI

* Near-well "water damage"

⋮



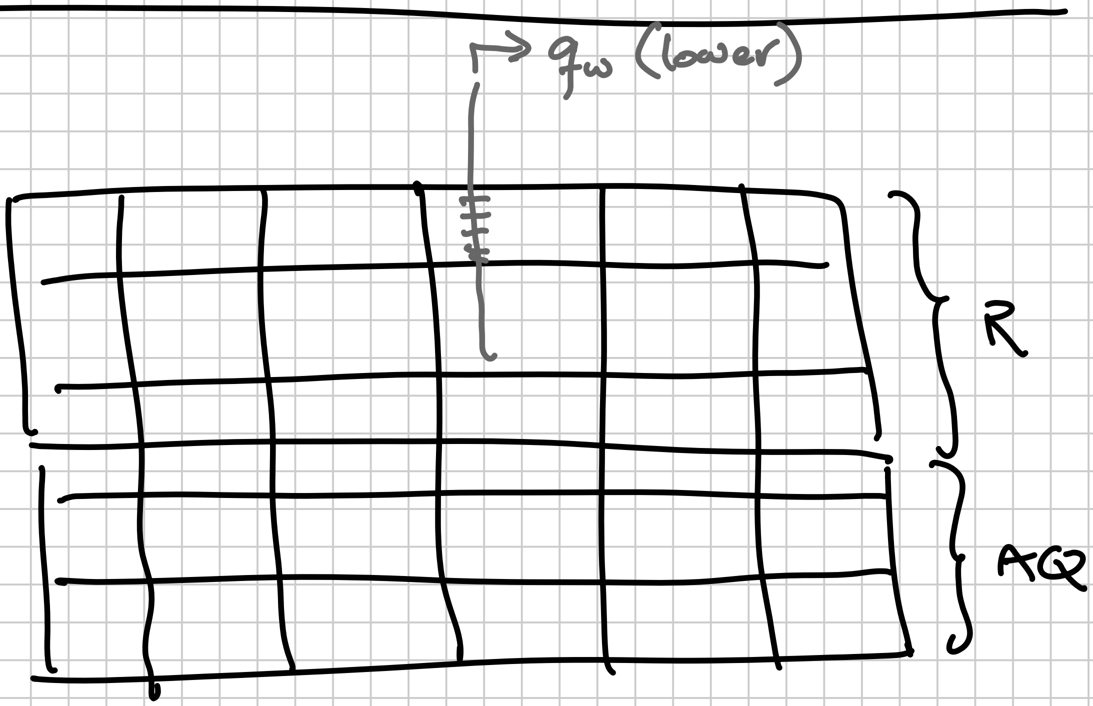
Coning:

Max — $(\rho_w - \rho_{HC})$ min

(μ_w / μ_{HC}) min

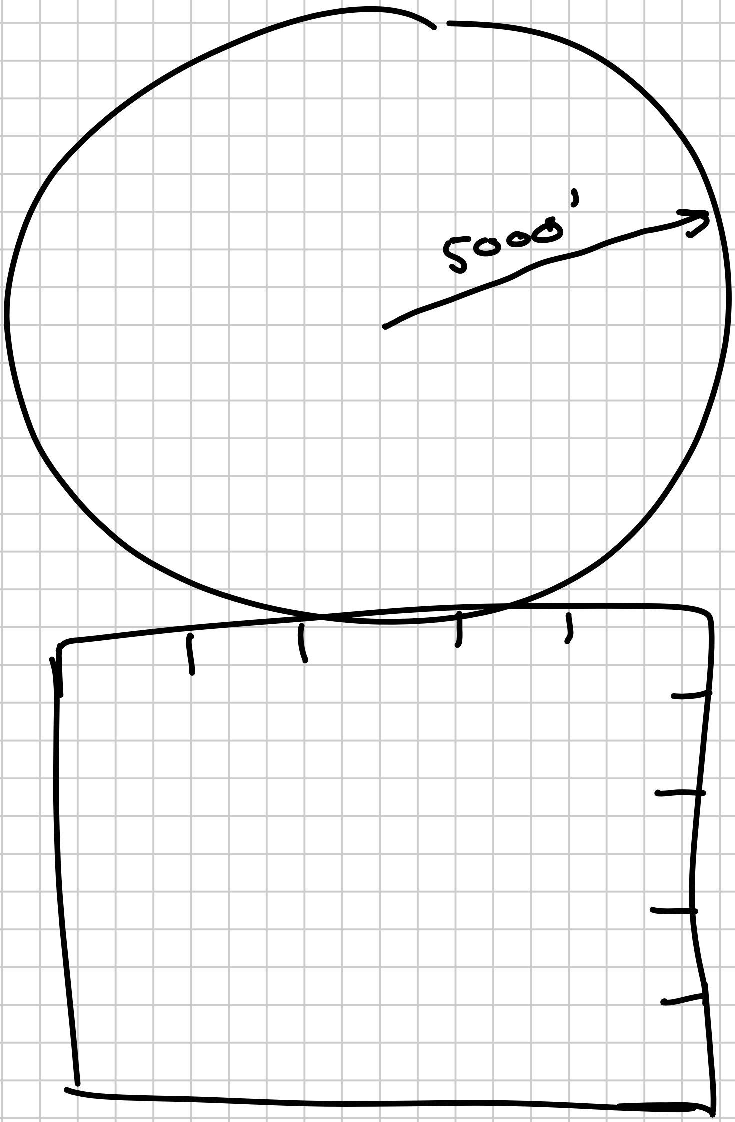
more coning in O-W

G HZ



$A = \pi r_e^2$

$\Delta x = \Delta y = \frac{\sqrt{A}}{5}$



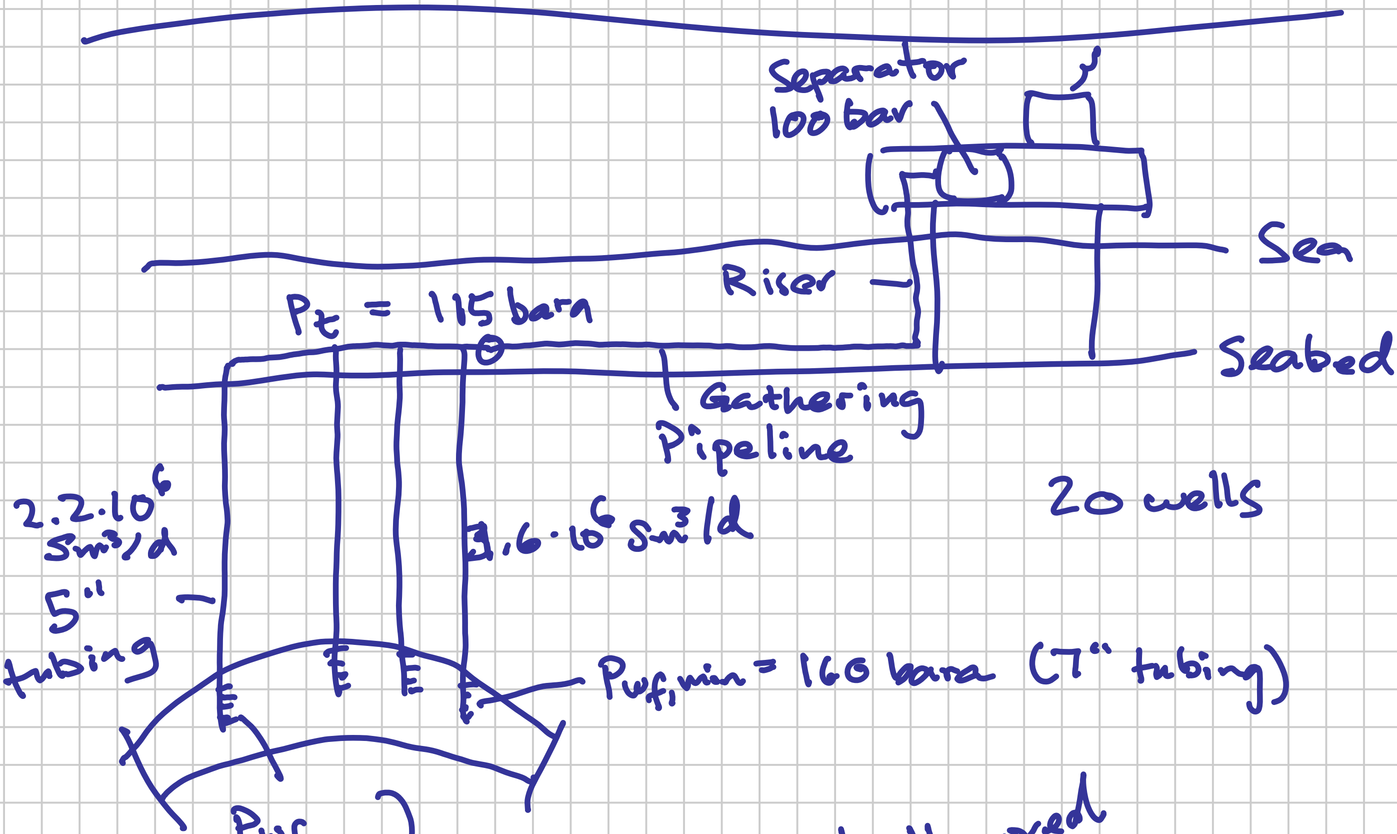
COUPLING RATE EQ. & MATERIAL BALANCE

⇒ RATE DECLINE EQ.

RATE-TIME FORECASTING

NUMBER OF WELLS REQUIRED

... Economics



Pwf
PBH
BHP
220 bara

BHP Constraint : Honored

"IPR"

Productivity Index

Rate Eq.

$$\boxed{q_g = C \cdot \left[m(\bar{P}_R) - m(P_{wf}) \right]}$$

f(G_p)
Material Balance

Pressure Constraint

$$G_p = \int_0^T q_g \, dt$$

Mat. Bal. $$\frac{p}{\enclose{circle}{z}}\left[1-c_e(p_i-p)\right] = \left(\frac{p}{z}\right)_i\left(1-\frac{G_p}{G}\right)$$

Post Aq. Model ↗

e-note:
Gas-Mat-Bal

Fetkovich-
Reese-
Whitson

$\bar{p}_R(G_p)$:

✓ - G

✓ - p_i (z_i) ← PVT

- c_e : ✓V_{Aq}, ✓c_w, ✓c_f

$$\underset{\uparrow \atop \enclose{circle}{K}}{\text{frac}}\left[: \underset{\uparrow}{c_e}(p_i-p)\right]$$

$$c_e = \frac{c_w S_{wc} + c_f + M(c_w+c_f)}{1-S_{wc}}$$

$K<1$
$f=0$

$K > 10-100$
$f \sim 1$

$$q_{g_F}(t) = \sum_{w=1}^{4} q_{gw}(t)$$

Operations/
Facility/
Equipment
↓

$$q_{gw} = f\left(C_w, \bar{p}_R(G_p), p_{wf}\right)$$

$$\frac{\overset{\downarrow}{k}\overset{\downarrow}{h}}{T_R\left[\ln\frac{r_e}{r_w} + \underset{\uparrow}{s}\right]} \nearrow C_w$$

$$\sum_{w=1}^{4}\int q_{gw}\,dt \uparrow \bar{p}_R(G_p)$$

$(P_t,\ q_g,\ P_{wf})$: TPR

Tubing Performance Rel.

e-note : Gas-Tubing

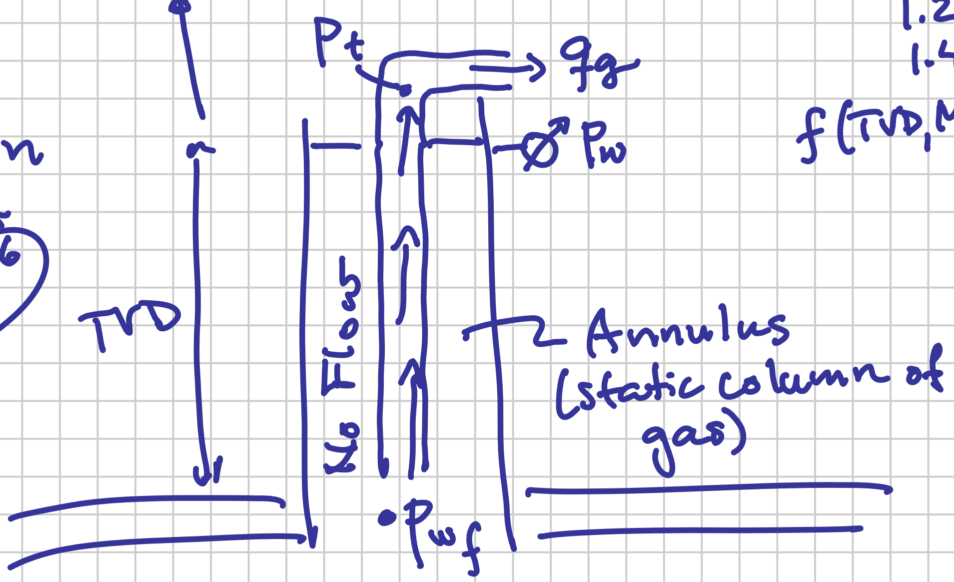
Fetkovich paper

VLP Vertical Lift Perf.

$$q_g = C_T\left(p_w^2 - p_t^2\right)^{0.5} \quad ; \quad P_{wf} = P_w \times \underbrace{\text{Const}}_{1.2\sim1.4}$$

$f(\text{TVD}, M_g)$

friction term $\propto \boxed{d_T^{2.6}}$



Coupling Rate Eq. (IPR) & Mat. Bal.

1. Current p_R known.

2. Calc. rate for a short time period, Δt, assuming p_R does not change.

   - Use constraint p_{wf} or constraint p_t } This gives the Maximum rate the well can prod., currently.

   Use Rate Eq (+ Tubing Eq.)

3. You decide how much of the max. rate to produce. $q_{sw,max}$

4. Calc. the incremental cum. production during Δt.

$$\Delta G_p = \underset{\underset{\leq q_{g\,max}\ (\text{at } p_{t,\text{constraint}})}{\uparrow}}{\bar{q}_g} \cdot \Delta t$$

5. Update total cum. produced,

$$G_p^n = G_p^{n-1} + \Delta G_p$$

6. Use Mat. Bal. to est. new $\bar{p}_R$ at G_p^n.

   Return to ①

How do I design the number of wells (as a function of time) needed to produce a specified, constant field gas rate for a specified period of time.

$P_{t,min}$ = specified, constant

Procedure.

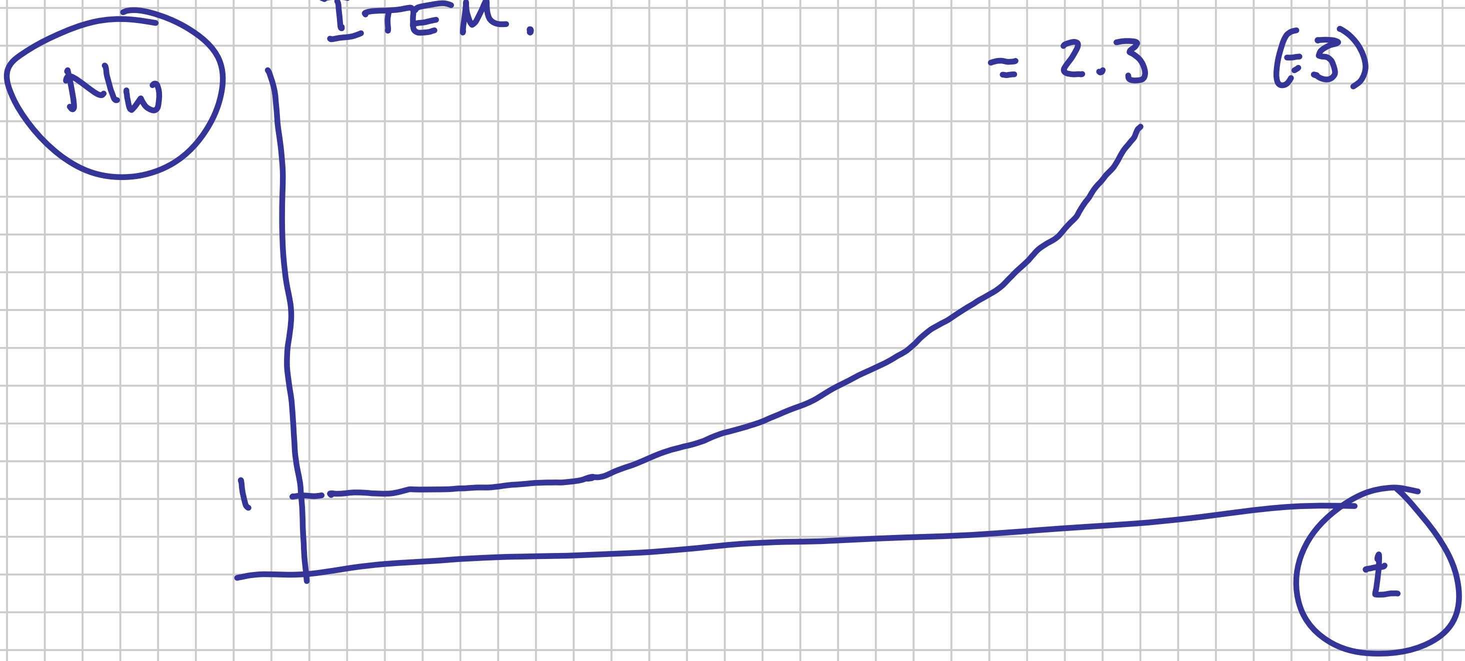
1. $q_{w,max}$ single well

$$\Delta t \left\{ \begin{array}{l} 2.\ \text{if } q_{w,max} > q_F ,\quad N_w = 1 ,\quad t = \Delta t \\ \quad \vdots \\ 3.\ \text{if } q_{w,max} < q_F ,\quad N_w \cdot q_{wmax} > q_F \end{array} \right.$$

$$N_w = \frac{q_F}{q_{wmax}}$$

$= 2.3 \quad (=3)$

KEY COST ITEM.



POT AQUIFER

$$\frac{p}{z}\left[1-c_e(p_i-p)\right]=\left(\frac{p}{z}\right)_i\left[1-\frac{\overset{\overset{\text{Solve}}{\downarrow}}{G_p}}{G}\right]$$

$$c_e=\frac{c_f+c_w S_{wc}+M(c_f+c_w)}{1-S_{wc}}=\text{const.}$$

PVT (i.e. z-factor) taken from Sensor files.

$[ft^3/scf]$

$$B_g=\underbrace{\left(\frac{p_{sc}}{T_{sc}}\right)\frac{(T_R)}{p}\cdot z} \Rightarrow \boxed{Z=B_g\cdot p\cdot\frac{\overset{\overset{{}^\circ R,\,K}{\downarrow}}{T_{sc}}}{p_{sc}}\cdot\frac{1}{\underset{\underset{{}^\circ R,\,K}{\nearrow}}{T_R}}}$$

Sensor : $\underline{p\geq p_{sat}\ (2626)}$

⋮

16 000

Sensor $\underline{B_g}$ $[~~rb~~/scf]$ $\times\frac{5.615\,ft^3}{bbl}$

$\underline{B_g\ [ft^3/scf]}$

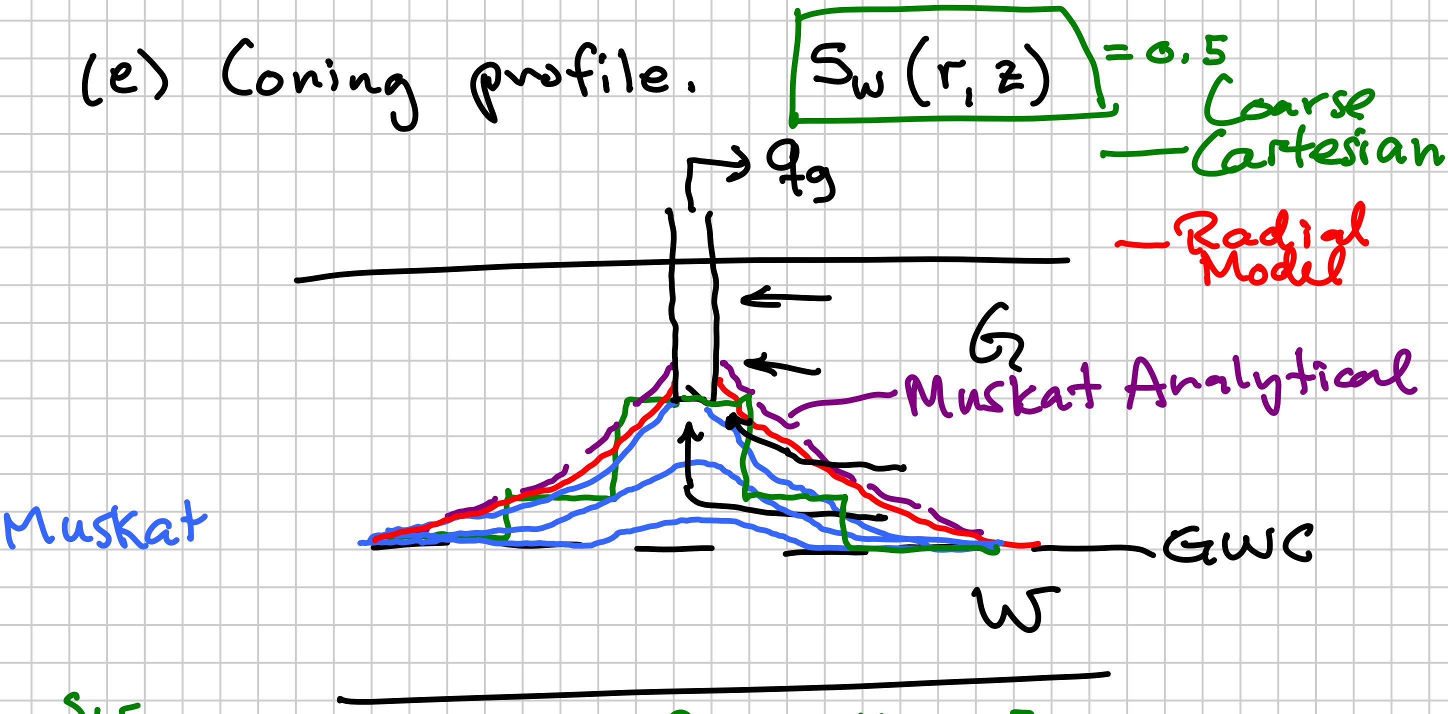
Calc. $\underline{Z}$

$$\frac{p}{z}\left[1-c_e(p_i-p)\right]=\left(\frac{p}{z}\right)_i\left[1-\frac{G_p}{G}\right]$$

$$\frac{G_p}{G}=1-\left\{\frac{(p/z)_i}{(p/z)\left[1-c_e\Delta p\right]}\right\}^{-1}$$

Problem 3.

(e) Coning profile.



Sw	1	2	3	4	5
6			0.22		
7	0.2	0.2	0.6		
8	0.21	0.3	1		
9	0.6	1	1		
10	1	1	1	1	1

PHASE BEHAVIOR REVIEW

Phases : Gas
Oil
(Water)

Behavior

- Volumetric $V(P,T)$
  (density; compressibility)
- Component
  - Equilibrium (Partitioning between phases)
  - Phase Amount
- Transport
  - Viscosity
  - Molecular Diffusion (‰)

PHASE BEHAVIOR

- Single-Component
  e.g. H_2O, CH_4, C_{10} ...
- Two-Component
  e.g. $C_1 - C_5$
- Multi-Component (Real Reservoir) Mixtures

CONCEPTS / TOPICS

- Saturated state : at least 2-phases in equilibrium
  (vs. undersaturated state) (1-phase)
  - Saturation Pressure
    - Bubble point
    - Dewpoint
      - Upper
      - Lower
    - Critical point
- Quantifying Amounts
  - Molar amounts n_v or n_g
    - Phase amounts, n, n_L or n_o

- F_v or F_g or $\beta \equiv \frac{n_v}{n}$

- Component amounts, n_i, n_{gi}, n_{oi}
  - Overall (total) mole frac., z_i
  - Vapor or Gas — " —, y_i
  - Liquid or Oil — " —, x_i

Total $$z_i \equiv \frac{n_i}{n}$$

Vapor $$y_i \equiv \frac{n_{vi}}{n_v}$$

Liquid $$x_i \equiv \frac{n_{Li}}{n_L}$$

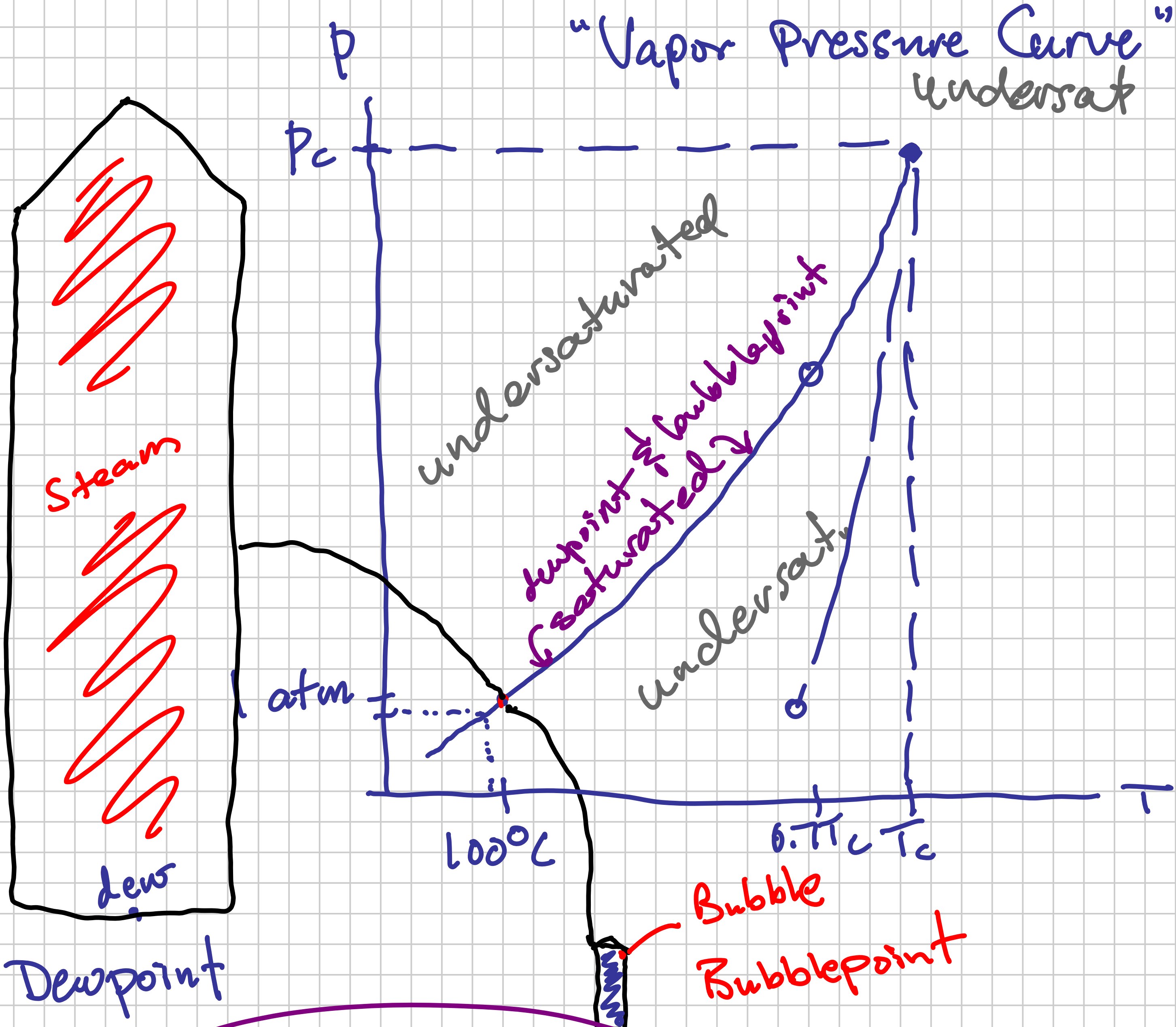
- Equilibrium Ratio, $K_i \equiv \frac{y_i}{x_i}$
  (K-value)

  Partioning Ratio

- Phase Densities
  (Viscosities)

Water Phase Behavior

100°C, 1 atm :
- Condition where we form a second phase.
- Saturated state.
- Point on the "Vapor Pressure Curve"



Liquid Phase = Vapor Phase
Chemical Pot   Chem Pot μ_i

Independent of phase amount

$$y_{H_2O} = \frac{(n_{H_2O})_v}{n_v} = 1$$

$$x_{H_2O} = \frac{(n_{H_2O})_L}{n_L} = 1$$

$\Big\}$ $K_{H_2O} = 1$ along the vapor pressure line.

Volumetric Behavior:

At 1 atm, 100°C $\Rightarrow$ $\rho_L \sim 1000 \ kg/m^3$

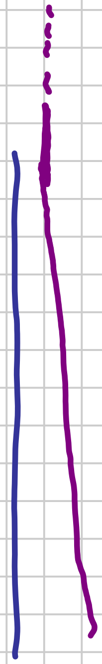
$\rho_v \sim 1 \ kg/m^3$

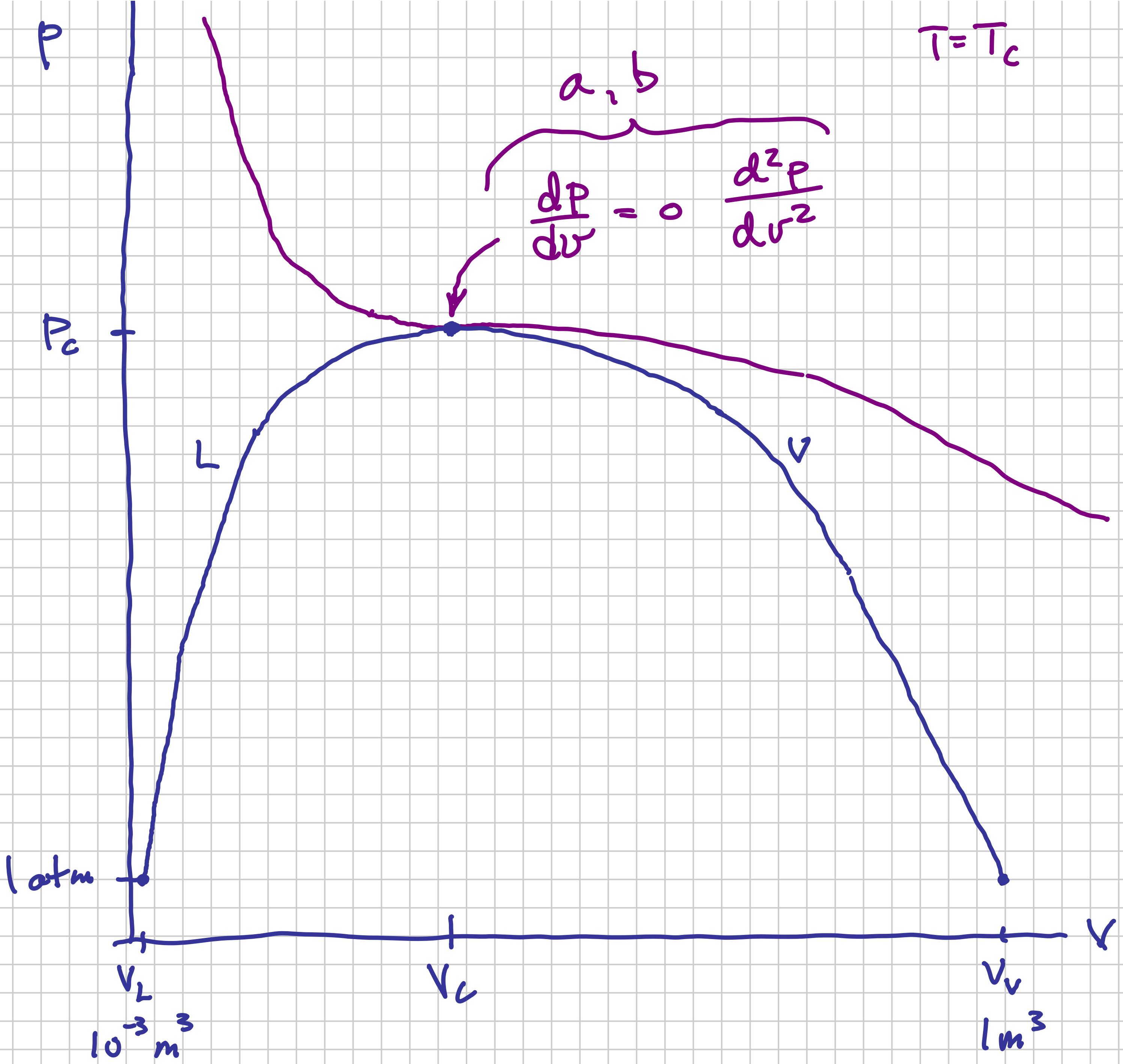
1 kg of H_2O

$V_L \sim 0.001 \ m^3$

$V_v \sim 1 \ m^3$

$$b = \frac{V_{min}}{n}$$





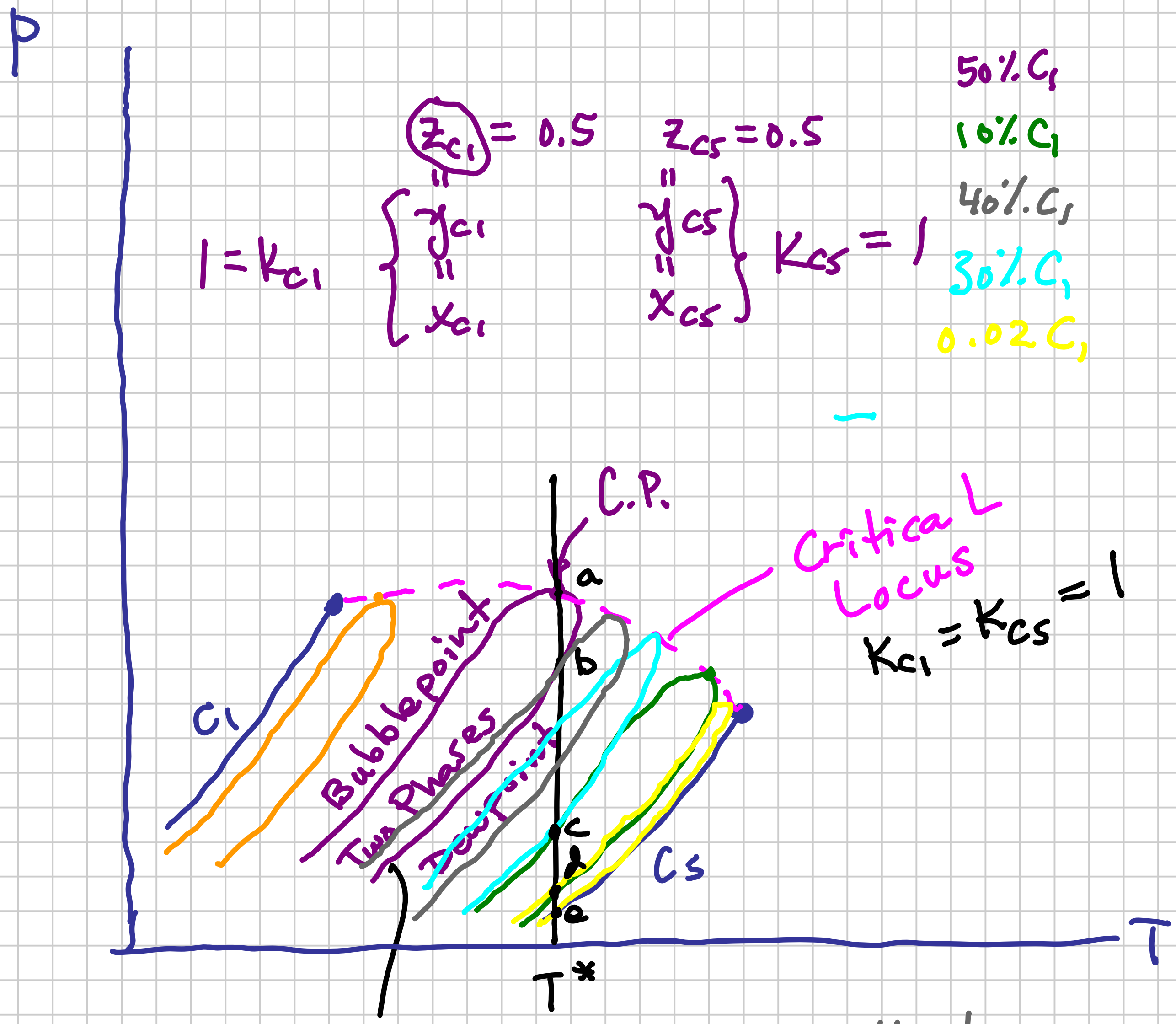
van der Waals Equation of State (EOS)
~1873

$$\boxed{p = \underbrace{\frac{RT}{v-b}}_{\text{Repulsive Forces}} - \underbrace{\frac{a}{v^2}}_{\text{Attractive Forces}}}$$

Liquids
Vapor
Saturated
Undersaturated
@ C.P.

$$v = \frac{V}{n}$$

Two-Component Systems



	K_{c_1}	K_{cs}
a	1.0	1.0
b	$1.25 = \frac{0.5}{0.4}$	$\frac{0.5}{0.6} = 0.83$
c	$3 = \frac{0.3}{0.1}$	$\frac{0.7}{0.9} = 0.78$
d	$5 = \frac{0.1}{0.02}$	$\frac{0.9}{0.98} = 0.92$
e		1.0

Hints:
At dewpoint
$y_i = z_i$
At bubblepoint
$x_i = z_i$

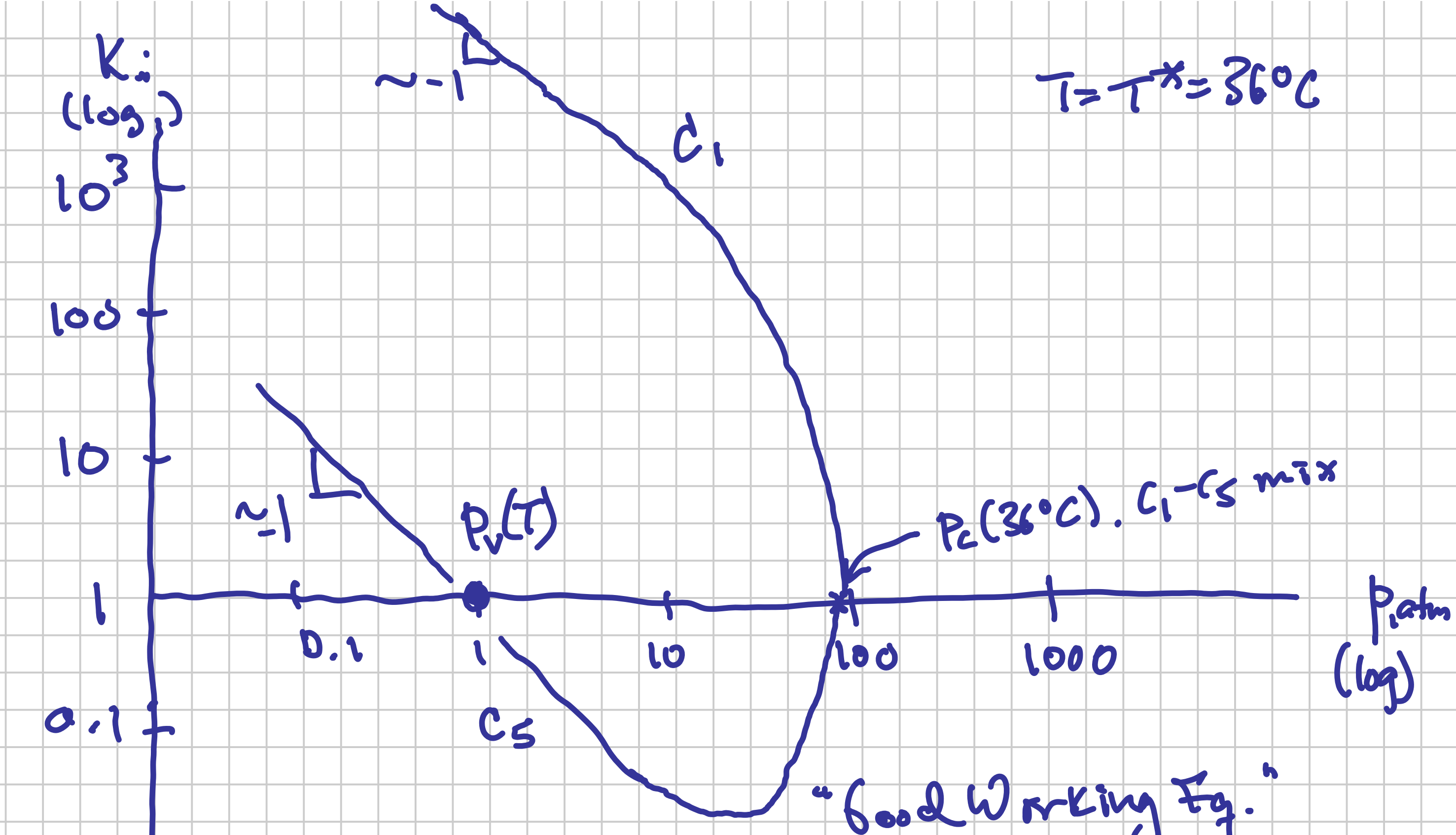
$K_i(p)$ at fixed T in a mixture

- Always be at least 1 $K_i > 1$ \
  ——— " ——— 1 $K_i < 1$ } guarantee molar i balance

- $K_i \simeq 1$ at $p = p_v(T)$ if V.P. exists @ T

- $$\frac{d \ln K_i}{d \ln p} \simeq -1 \qquad p < p_v(T)$$

- all $K_i \to 1$ at $p_c(T)$



"Good Working Eq."

Ch. 3: Modified Wilson K-value Eq.

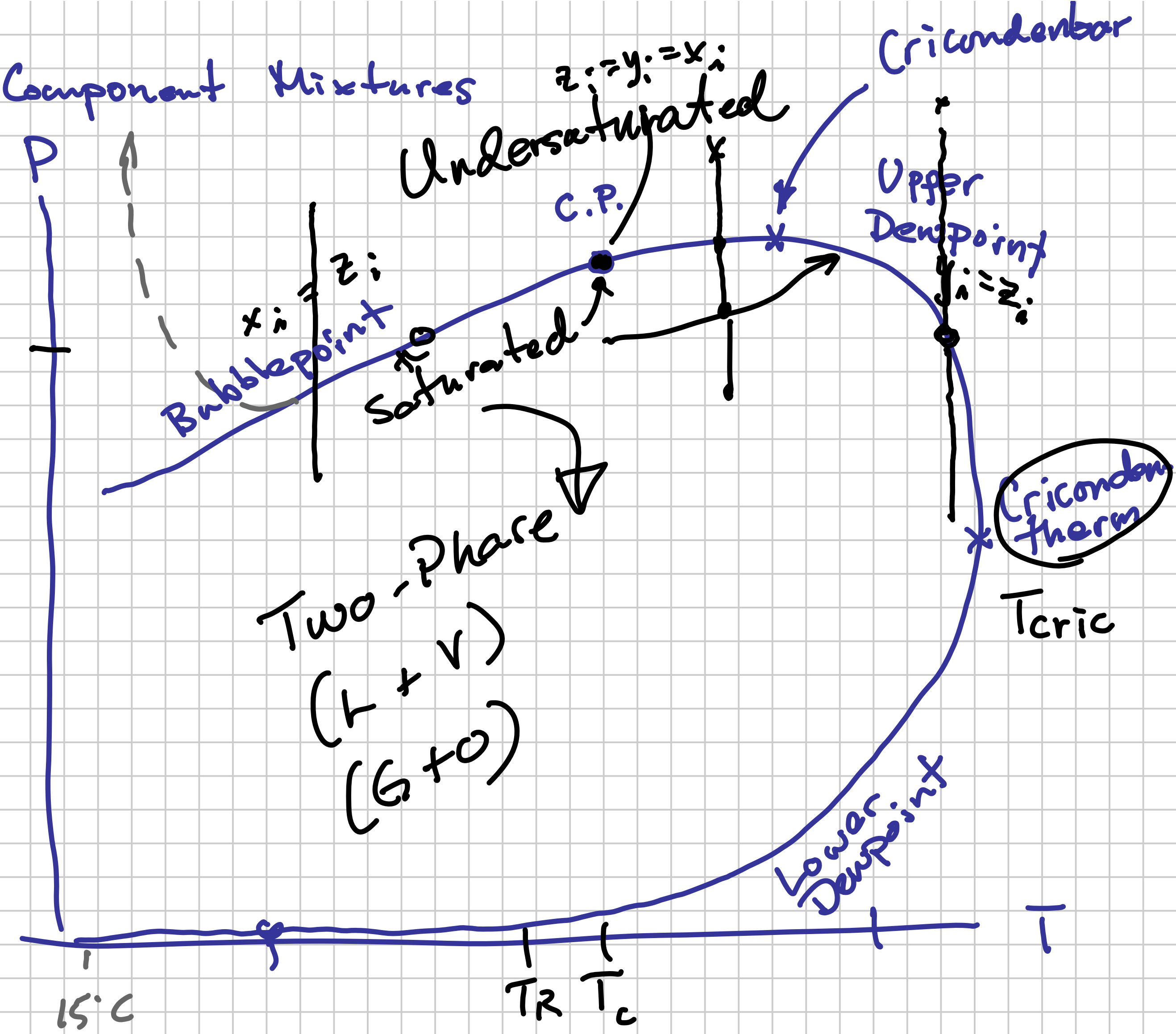
$$K_i\left(\underbrace{T_{ci},\ p_{ci},\ \omega_i}_{\text{Acentric Factor}},\ \underset{\text{Convergence Pressure ``}p_c\text{''}}{p_K},\ \overbrace{p,\ T}\right)$$

$T^* = 36°C$

(= T_b of C_5) $P_v = 1$ atm

→ Acentric Factor?

Multi-Component Mixtures



Reservoir Classification:

Oil : $T_R < T_c$

Gas : $T_R > T_c$

(a) Gas Condensate $T_c < T_R < T_{cric}$
(see liquid forming IN the reservoir)

(b) Wet Gas $T_R > T_{cric}$: get surface condensate

(c) Dry Gas $T_R > T_{cric}$: DON'T get surface cond.

Phase Diagrams

p-T Diagram

- Defines the boundary between a mixture being single-phase (undersaturated) or two phase. (saturated)

Line defining the boundary, (BP-CP-DP) line, is a saturated or two-phase state where the one phase is of infintesimal amount (ϵ).

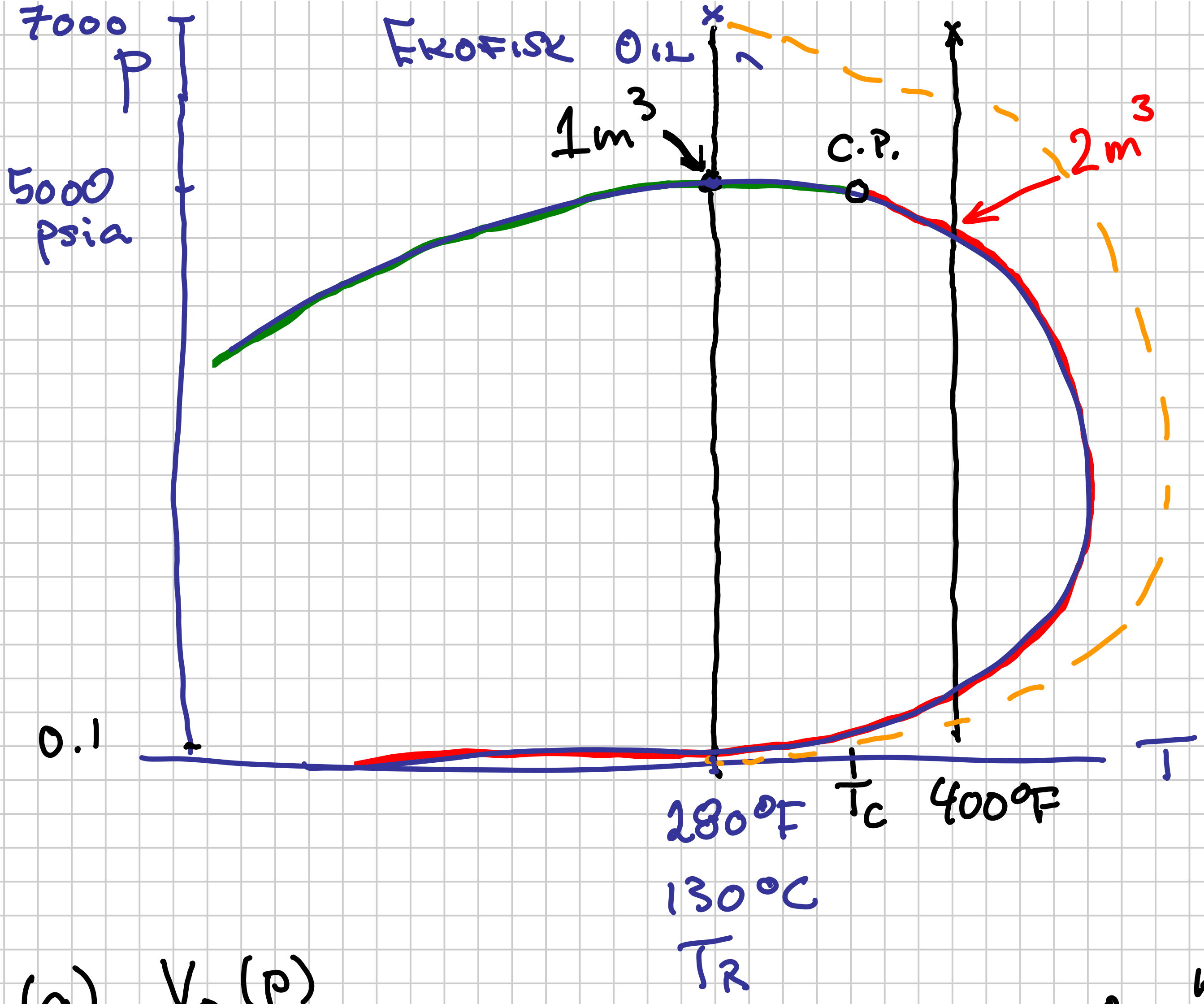
p-V Diagram (fixed temperature)

- $\rho(p)$ for single-phase (saturated)
(undersat.) conditions.

$$\rho_{(p)} \equiv \frac{m}{V} = \frac{\text{constant}}{V(p)}$$

- Changing relative phase volumes (V_o, V_g) vs p.
  - $V_o/(V_o+V_g)$ %

  or
  - V_o/V_{sat} $\left(\frac{V_o}{V_{dp}} \text{ or } \frac{V_o}{V_{bp}}\right)$ %



(a) $V_o(p)$

$V_t(p) = V_g(p) + V_o(p)$

7000 → 0.1 psia

$\rho_{ob} = 800 \frac{kg}{m^3}$

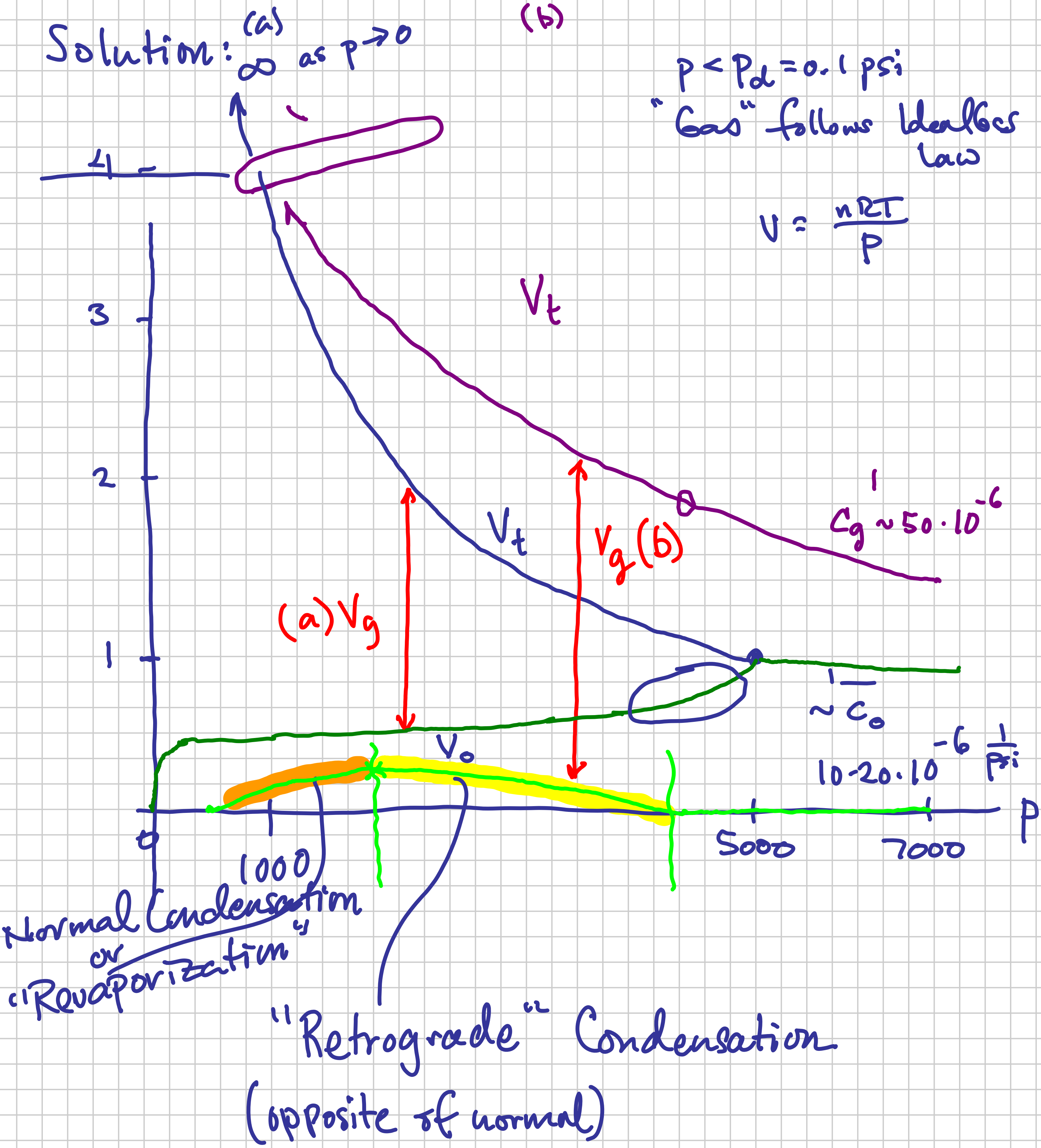
$M_o \sim 100$

✓ (b) Locate C.P.

(c) At $T > T_c$

Repeat (a)

Solution:



$p < p_d = 0.1$ psi
"Gas" follows Ideal Gas law

$$V = \frac{nRT}{p}$$

Mixture Compositions

("Chemical Makeup")

N_2
CO_2
H_2S
C_1
C_2
C_3
i-C_4
n-C_4
i-C_5
n-C_5
C_{6s}
C_7
C_8
⋮
C_{40+}

C_{7+}

⇒ "Liquified Natural Gas" LNG

Surface Gas

Stock-Tank Oil
Surface Oil ~ C_{6+}

Liquified Petroleum Gas

\$
C_2-C_3 → L "LPG"

C_{4+} in Surface Gas \$ → L "NGL"

Natural Gas Liquid

Quantifying Composition:

- Mass amount of each component is measured. m_i
- Molar amount $n_i = \dfrac{m_i}{M_i}$ is usually reported, as a mole fraction $= \dfrac{n_i}{\sum n_i}$

Total Overall Molar Composition z_i

Vapor (Gas) Phase —— " —— y_i

Liquid Phase —— " —— x_i

$$\sum_i z_i = 1 = \sum y_i = \sum x_i$$

Rule of Thumb

<u>mol-% C_{7+}</u>

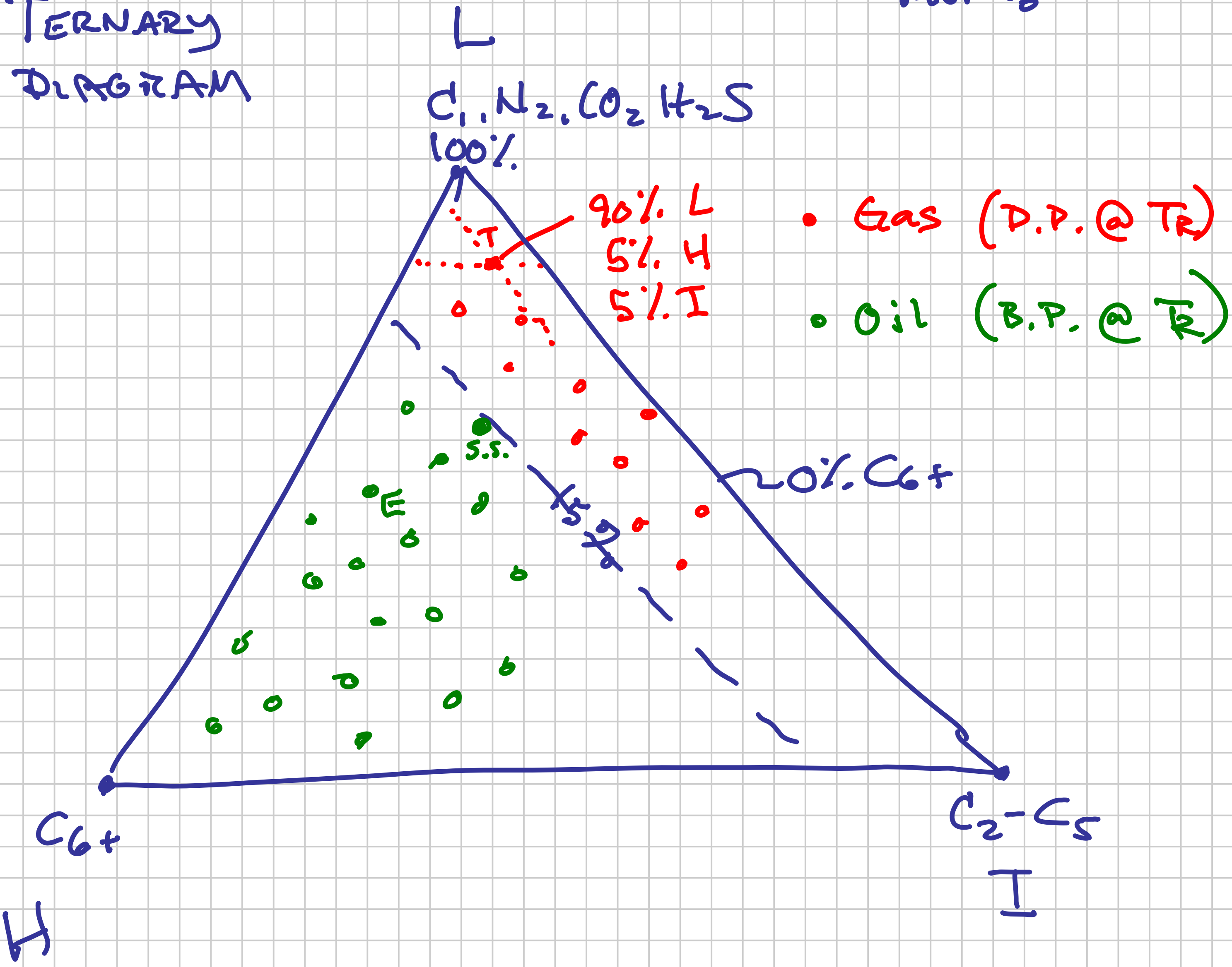
> 14 (12-15) : oil

< 14 : gas

> 1 : gas condensate, upper D.P. at T_R

< 1 : "wet" gas or "dry" gas $\Big\}$ $T_R > T_{cric.}$

TERNARY DIAGRAM



Gulf of Mexico

Unusual but Typical

Little C_2-C_6 content

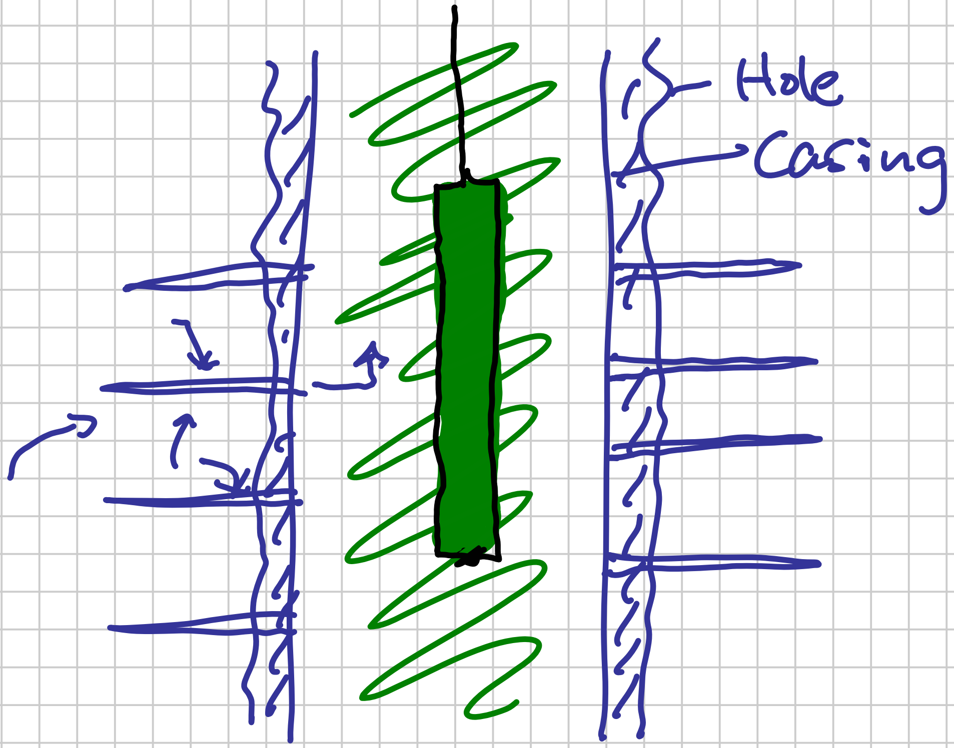
⇒ Unusually High Saturation Pressures (B.P. or D.P.)

How do we measure composition?

① Acquire samples.

ONLY FOR OILS!

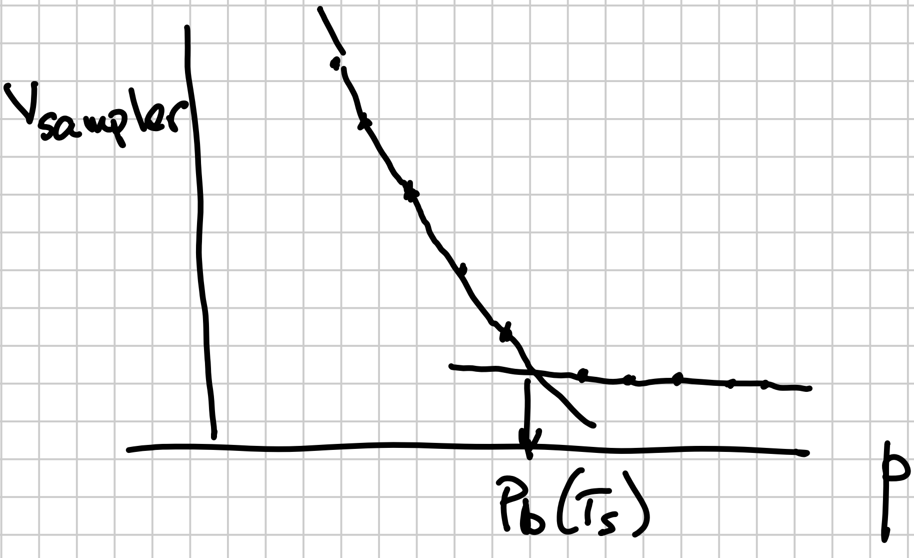
- (Cased-hole) Bottomhole Sample



* Collected during a shut-in

* —— " —— a low flow rate

(a) At the surface, the p_b is measured at ambient temperature.

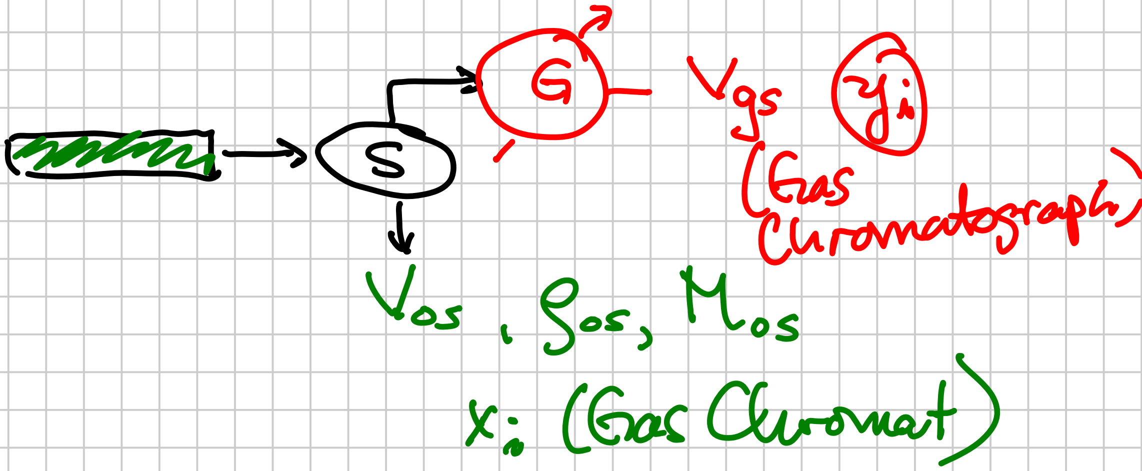


(b) Send the sample bottle (~ 1 L) to a laboratory.

- Measure same $p_b(T_s)$ (checking for leakage)

(c) Measure Composition:

- Bring sample single-phase ($p > p_b$)
- Remove a small sample @ const p to standard conditions (flash)



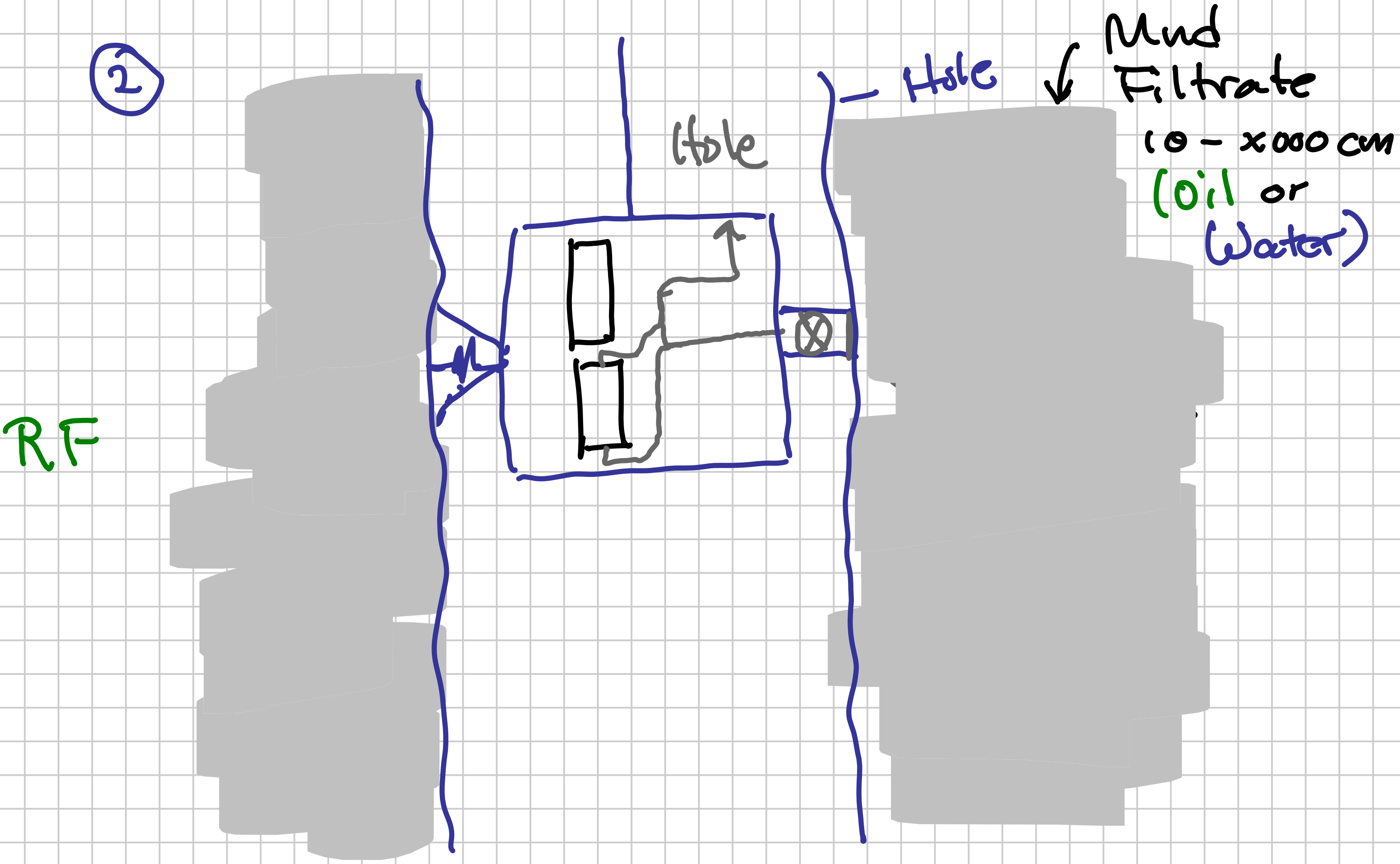
$$GOR = \frac{V_{gs}}{V_{os}}$$

- Mathematically recombine to total oil composition:

$$\boxed{z_i} = f_g \cdot y_i + \underbrace{(1-f_g)}_{f_o} \cdot x_i$$

$$f_g = \frac{GOR}{GOR + \left(\frac{\rho_{os}}{M_{os}}\right)\frac{RT_{sc}}{p_{sc}}}$$

②



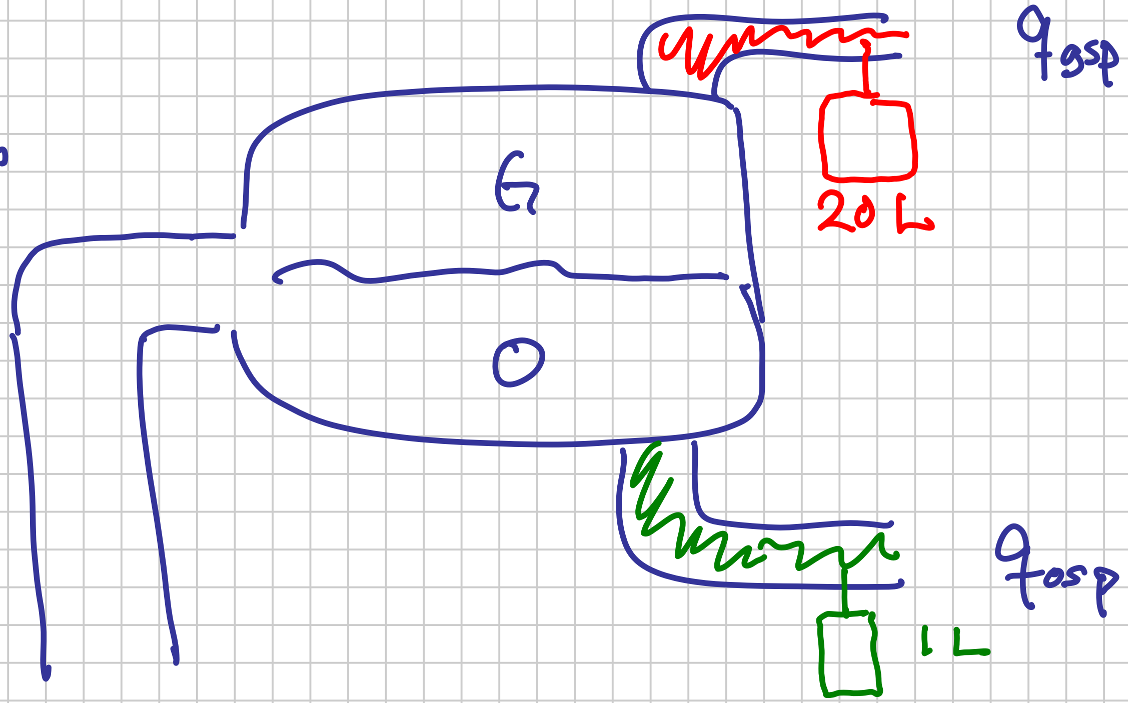
(Open-Hole) Bottomhole Sampler

- MDT
- RCI

} OILS & GASES

③ SURFACE SEPARATOR SAMPLES

During
Production
Test



$$R_{sp} \quad [Sm^3/sep\text{-}m^3] = \frac{q_{gsp}}{q_{osp}}$$

- Sep. Oil Flash - GC - recombination $\Rightarrow \begin{cases} x_{spi} \\ \rho_{osp} \\ M_{osp} \end{cases}$
- Sep. Gas GC $\Rightarrow y_{spi}$

$$\boxed{z_i} = f_g \cdot y_{spi} + (1-f_g) \cdot x_{spi}$$

$$f_g = \frac{R_{sp}}{R_{sp} + \left(\frac{\rho_{osp}}{M_{osp}}\right) \frac{RT_{sc}}{p_{sc}}}$$

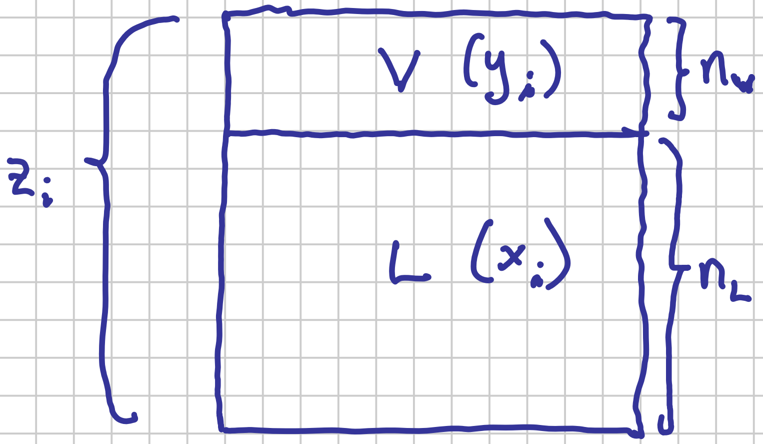
- GAS & OIL reservoirs

PHASE EQUILIBRIUM CALCULATIONS

Ch. 3 (K-values); Ch 4 — Rachford-Rice "Flash"

Given (Known):

2N
- z_i = overall (total) mixture molar composition
- $K_i \equiv y_i / x_i$ = K-values, equilibrium ratios



P, T are known or specified

. . . estimate of K_i

N = # components

Want (Don't Know):

β
- n_v, $n_L = n - n_v$
- y_i
- x_i

2N+1

Equations:

$$K_i = \frac{y_i}{x_i}$$

$$n_i = n_{vi} + n_{Li}$$

$$n = n_v + n_L$$

Definitions:

$$z_i = \frac{n_i}{n} \qquad \beta = \frac{n_v}{n} = \frac{n_v}{n_v + n_L}$$

$$y_i = \frac{n_{vi}}{n_v} \qquad \text{"}f_g\text{"} = \beta$$

$$x_i = \frac{n_{Li}}{n_L} \qquad \text{"}f_v\text{"} = \beta$$

$$\Rightarrow \quad z_i = \beta \cdot y_i + (1-\beta)\, x_i$$

$$\sum z_i = 1 = \sum y_i = \sum x_i$$

Muskat - McDowell, 1949 (small footnote)

$$\sum y_i - \sum x_i = 0$$

$$\sum (y_i - x_i) = 0 \qquad *$$

Rachford & Rice, 195x

$$z_i = \beta y_i + (1-\beta) x_i \qquad K_i = y_i / x_i$$

$$= \beta y_i + x_i - \beta x_i \qquad x_i = y_i / K_i$$

$$= \beta (y_i - x_i) + x_i$$

$$z_i = \beta \left(y_i - \frac{y_i}{K_i} \right) + \frac{y_i}{K_i}$$

$$= \beta\, y_i - \frac{\beta}{K_i}\, y_i + \frac{1}{K_i}\, y_i$$

$$= y_i \left[\beta - \frac{\beta}{K_i} + \frac{1}{K_i} \right]$$

$$z_i = y_i \left[\beta\left(1 - \frac{1}{K_i}\right) + \frac{1}{K_i} \right]$$

$$\boxed{y_i = \frac{\boxed{z_i}}{\beta\left(1 - \frac{1}{\boxed{K_i}}\right) + \frac{1}{\boxed{K_i}}}}$$

$$y_i - x_i = y_i - \frac{y_i}{K_i} = y_i\left(1 - \frac{1}{K_i}\right)$$

$$y_i - x_i = \frac{z_i\left(1 - \frac{1}{K_i}\right)}{\beta\left(1 - \frac{1}{K_i}\right) + \frac{1}{K_i}}$$

$$= \frac{z_i}{\beta + \frac{1}{K_i}\left(1 - \frac{1}{K_i}\right)^{-1}}$$

$$= \frac{z_i}{\beta + \frac{1}{K_i}\left(\frac{K_i - 1}{K_i}\right)^{-1}}$$

$$y_i - x_i = \frac{z_i}{\beta + \frac{1}{K_i - 1}}$$

$$\sum y_i - x_i = 0$$

$$\sum_{i=1}^{N} \frac{z_i}{\beta + \frac{1}{K_i - 1}} = 0$$

Solve for β, 1 EQ, 1 unknown

Muskat-McDowell

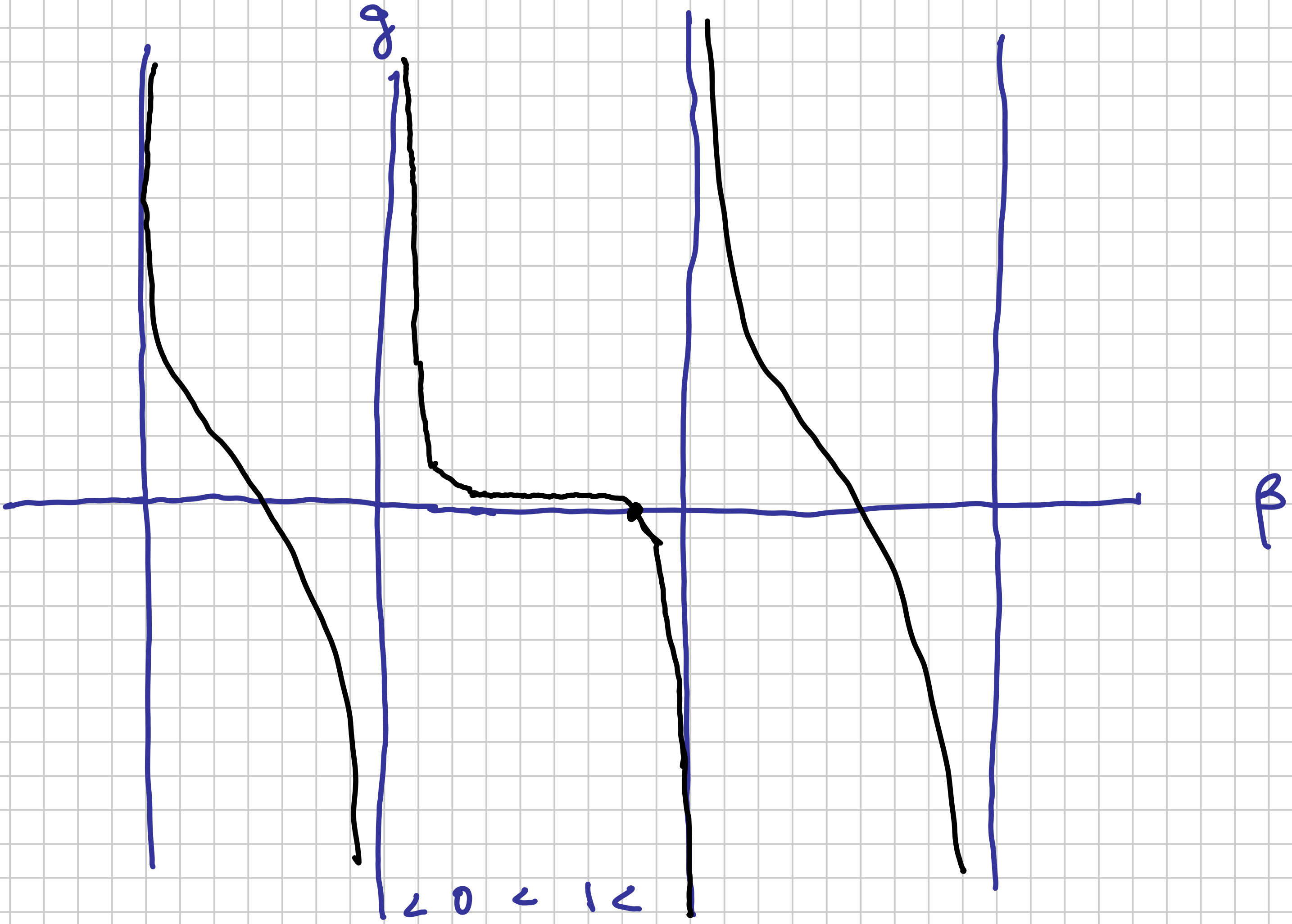
$c_i \equiv \frac{1}{K_i - 1}$ variable;

$$g(\beta) = \sum_{i=1}^{N} \left\{ \frac{z_i}{\beta + c_i} \right\} = 0$$

summation term $= 0$
$K_i = 1$

- Multiple Solutions $(N-1)$
  - Only one of these that is physically acceptable ($y_i > 0$, $x_i > 0$)
- Highly non-linear, can be

- Can be problems of machine round-off error



$$\beta_{min} = \frac{1}{1-K_{max}} \checkmark \qquad \beta_{max} = \frac{1}{1-K_{min}} \checkmark$$

- Monotonic function, $g(\beta)$
  - Newton-Raphson solution
- One K-value > 1

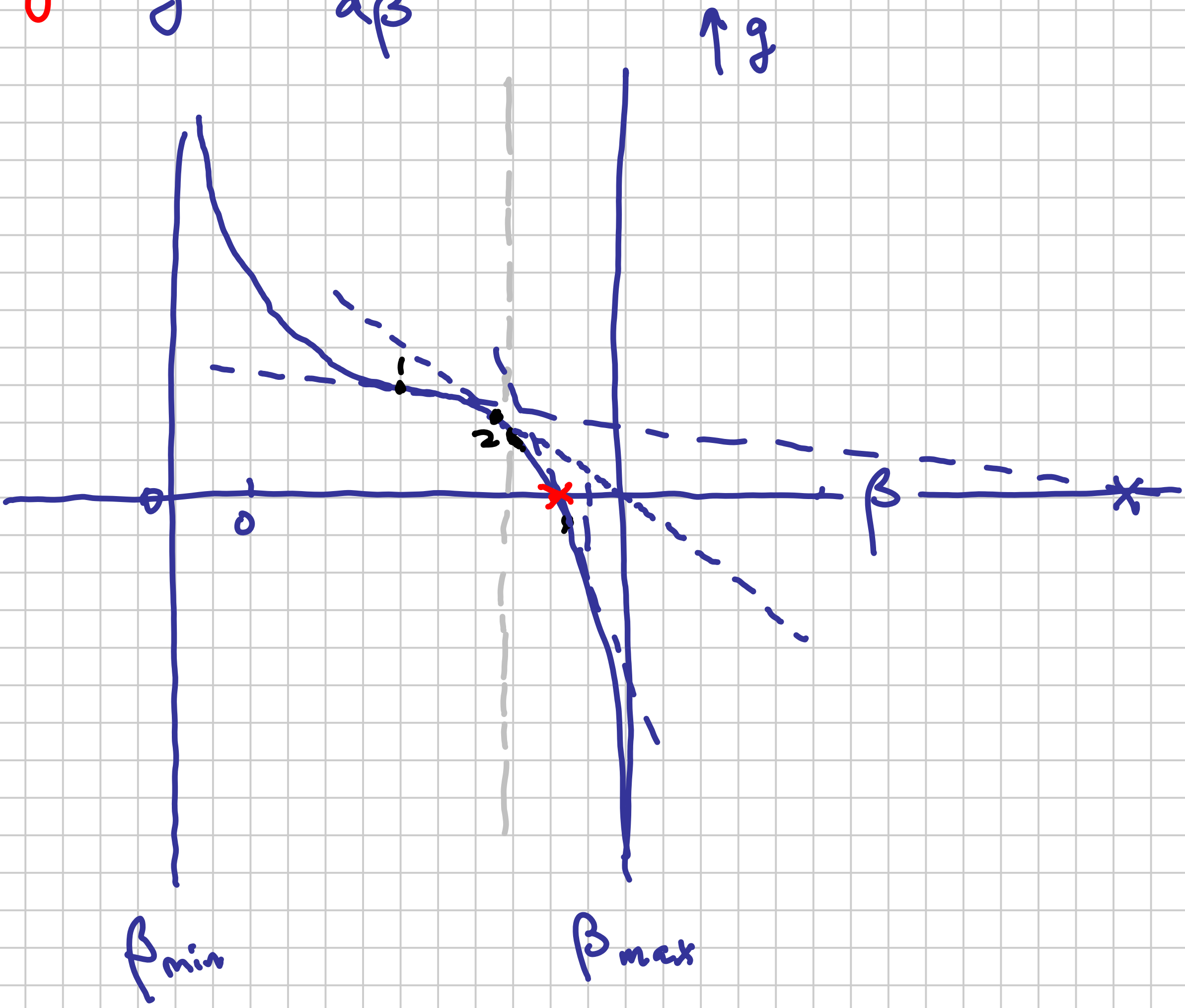
—— " —— < 1

NR: $$\beta^{n+1} = \beta^{n} - \frac{g(\beta^{n})}{g'(\beta^{n})}$$

Analytic

$\beta_{min} < \beta_{max}$
Interval Halving

$$g' = \frac{dg}{d\beta}$$



Solve for $\overset{\checkmark}{\beta} \Rightarrow n_v, n_L$

✓ $y_i =$

✓ $x_i =$

Special Case of Phase Equilibrium Calculation

(1) $\beta = 0 \quad (\epsilon)$: Bubblepoint

(2) $\beta = 1 \quad (1-\epsilon)$: Dewpoint

Appendix B: Example Calc. of P_b, P_d using modified Wilson Eq. $K_i(p, T, p_K; i)$

(1) **BP**: Know $x_i = z_i$

$$y_i = z_i K_i(\underset{\substack{\uparrow \\ \text{Change}}}{p}, T, p_K; i)$$

EQ $\boxed{1 = \sum y_i}$

(2) **DP**: Know $y_i = z_i$

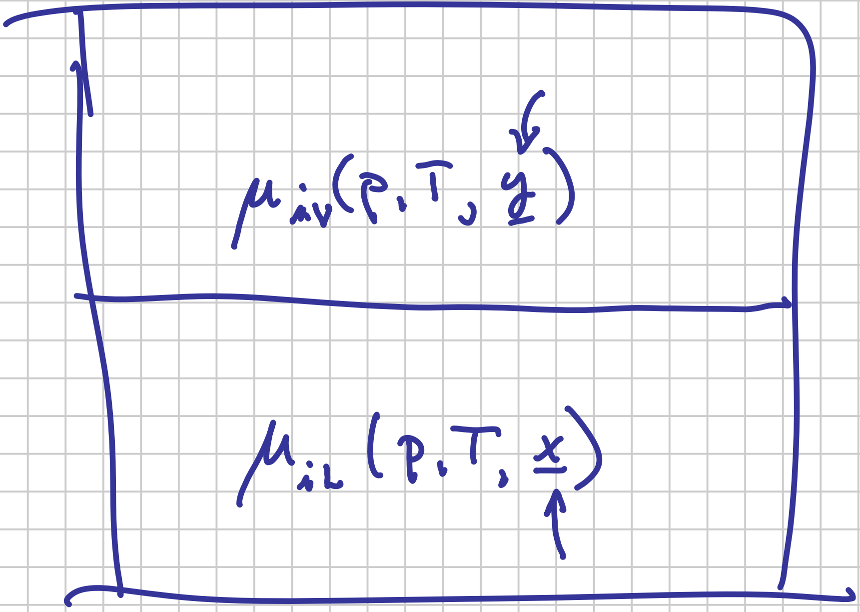
$$x_i = \frac{y_i}{K_i} = \frac{z_i}{K_i}$$

$$\boxed{1 = \sum x_i = \sum \frac{z_i}{K_i(\underset{\uparrow}{p}, \ldots)}}$$

Solve Phase Split Problem: RR

- Assumes we have K-values (estimate)

Check our solution with an Equation of State that tells us the GIBBS's chemical energy $\mu_i(p, \underset{V}{T}, \underline{z})$



$$\mu_{iV} = \mu_{iL} \quad \text{all } i$$

If not, UPDATE K_i estimates

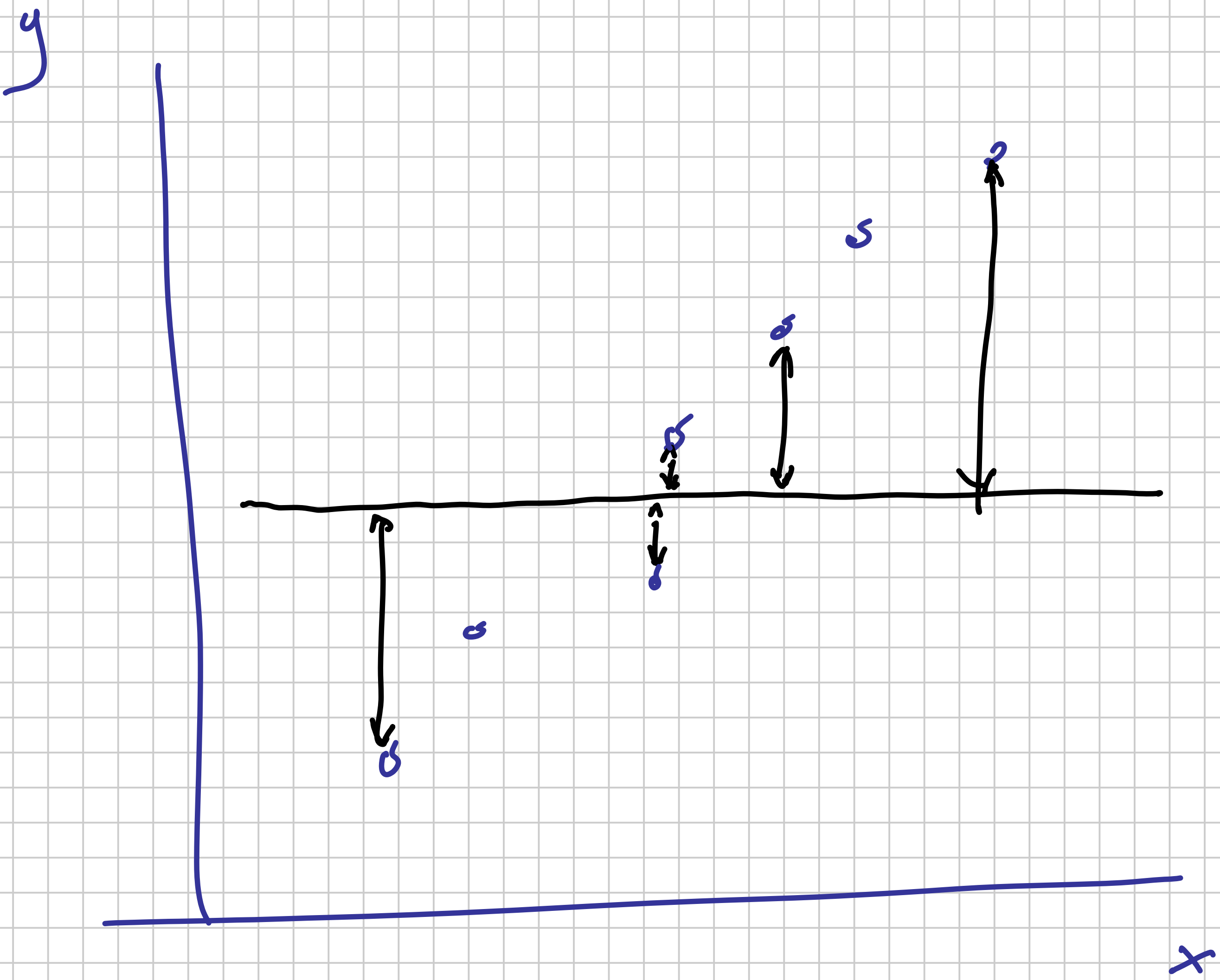
Redo RR, new $\underline{y}$ and $\underline{x}$

Go back to Mr. Gibbs.

Example Phase Equilibria Calculations

- Rachford-Rice Phase Split $(z_i, K_i)^*$
  - BP and DP ($\beta = 0$ and $\beta = 1$)
  - Excel Solver
- Build "our" own K-value correlation using the modified Wilson Eq.
  - Excel Solver

* We need a composition z_i and a set of K-values K_i : Sensor data set Problem 1.2 "an oil"



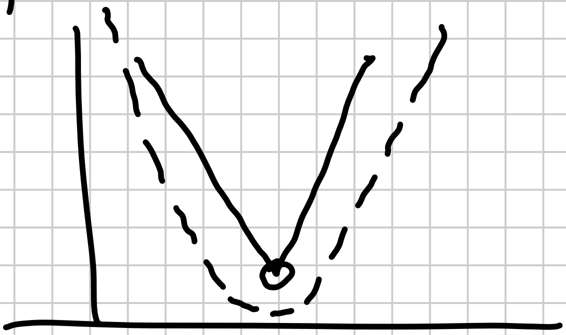
$y = ax + b$

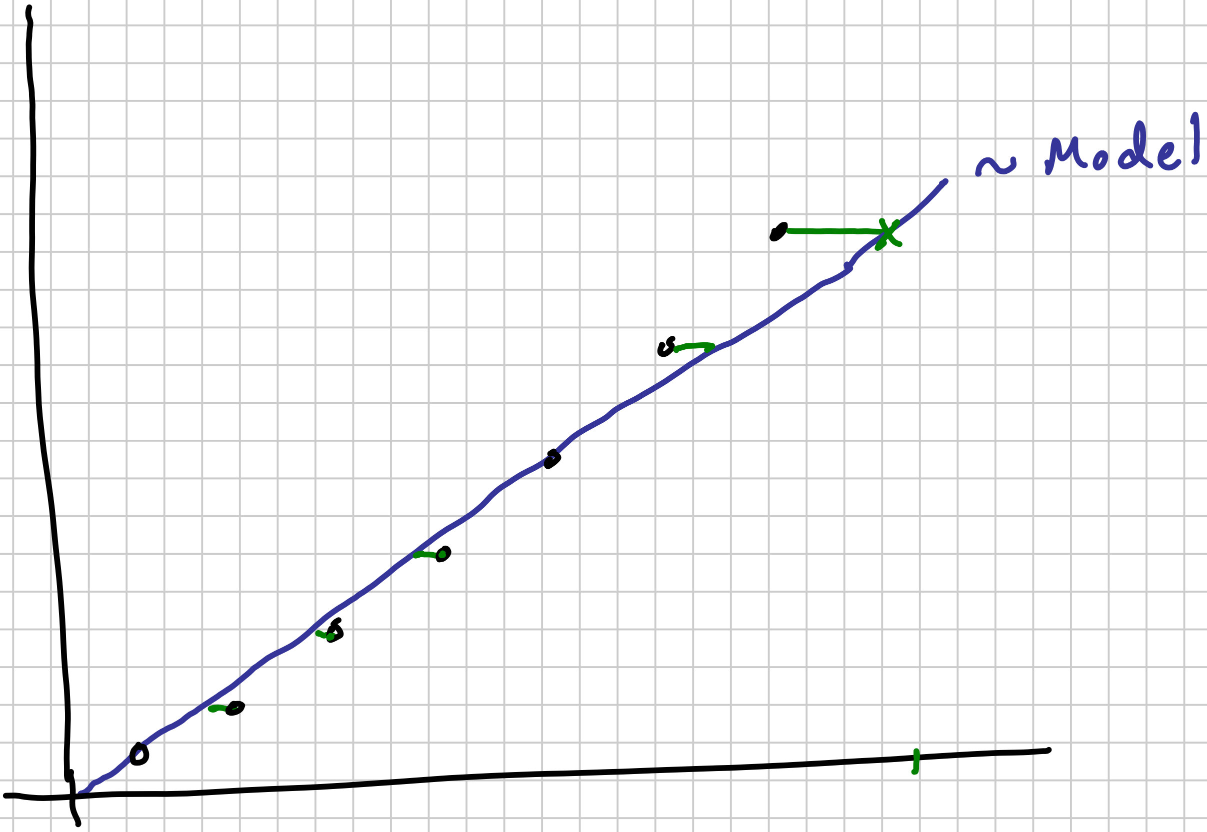
$\Sigma = 0$

S $\min S = \Sigma$ errors

errors $= |\Delta|$

$\Sigma \Delta^2$

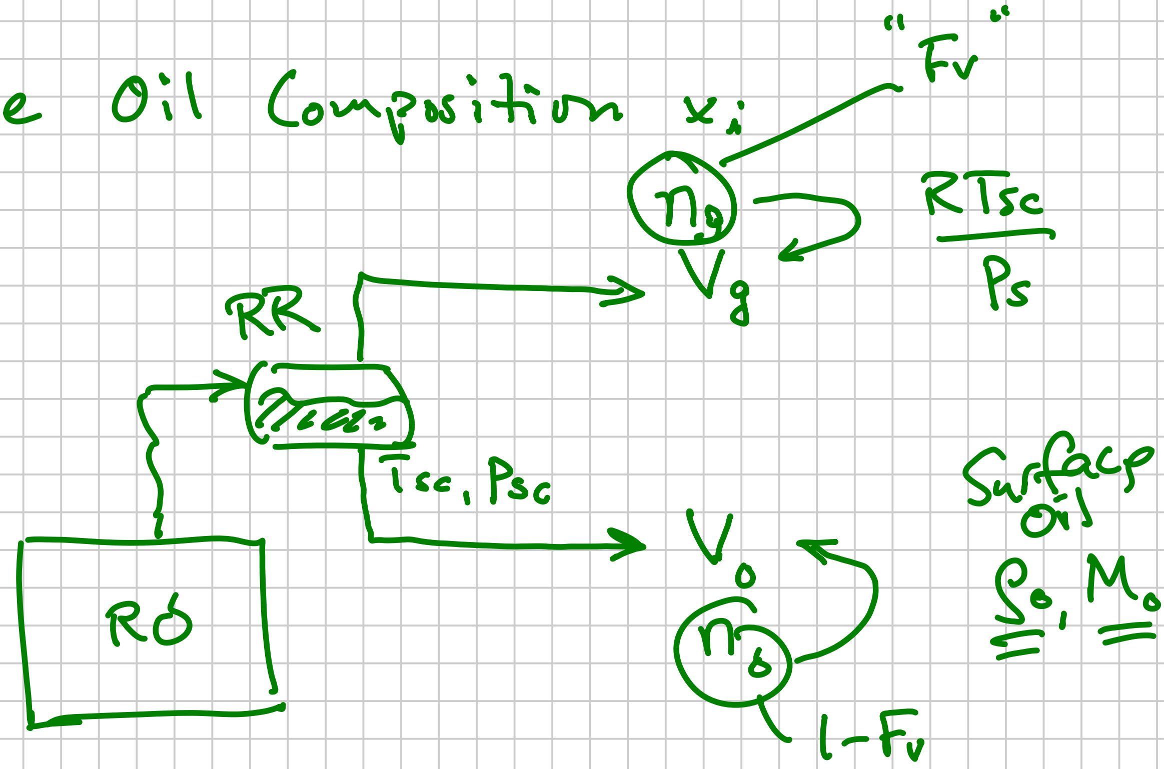




~ Model

Problem 4 –

Take Oil Composition x_i



Surface Separation Process

$$K_i \, (T_{sc}, P_{sc})$$

modified Wilson Eq.

Ideal Gas Eq. $(Z = 1)$

$pV = nRT$

$$\frac{V_g}{n_g} = \frac{R\,T_{sc}}{p_{sc}}$$

$\checkmark \quad = 23.64 \ \frac{Sm^3}{Kmol}$

$\checkmark \quad = 379 \ \frac{scf}{lbmol}$

$T_{sc} = 60°F$
$= 520°R$
$= 273 + 15.6 \ K$

$p_{sc} = 14.696$ psia
$= 1$ atm
$= 1.0135$ bar

$5.615 \ ft^3/bbl$

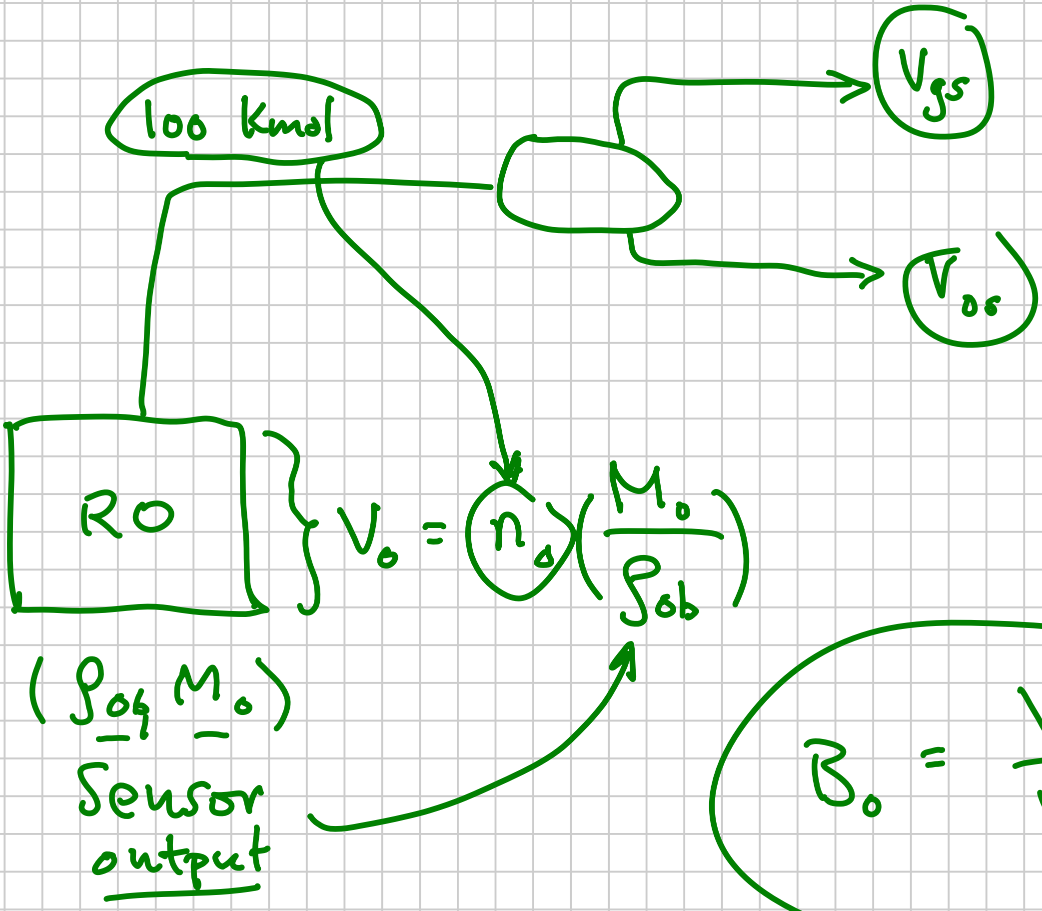
$$149 \ \frac{Sm^3}{Sm^3} = 149 \ \frac{scf}{\cancel{scf}} \times 5.615 \ \frac{\cancel{ft^3}}{bbl}$$

$$= 838 \ scf/STB$$

200 lb-mol of O_2

V at P_{sc}, T_{sc} ? cm^3

$M_{O_2} = 12$ lb/lb-mol



$$V_o = \boxed{n_o}\left(\frac{M_o}{\rho_{ob}}\right)$$

$(\underline{\rho_{og}}\ \underline{M_o})$

Sensor output

$$\frac{\rho_{os}}{\rho_w} = \gamma_o$$

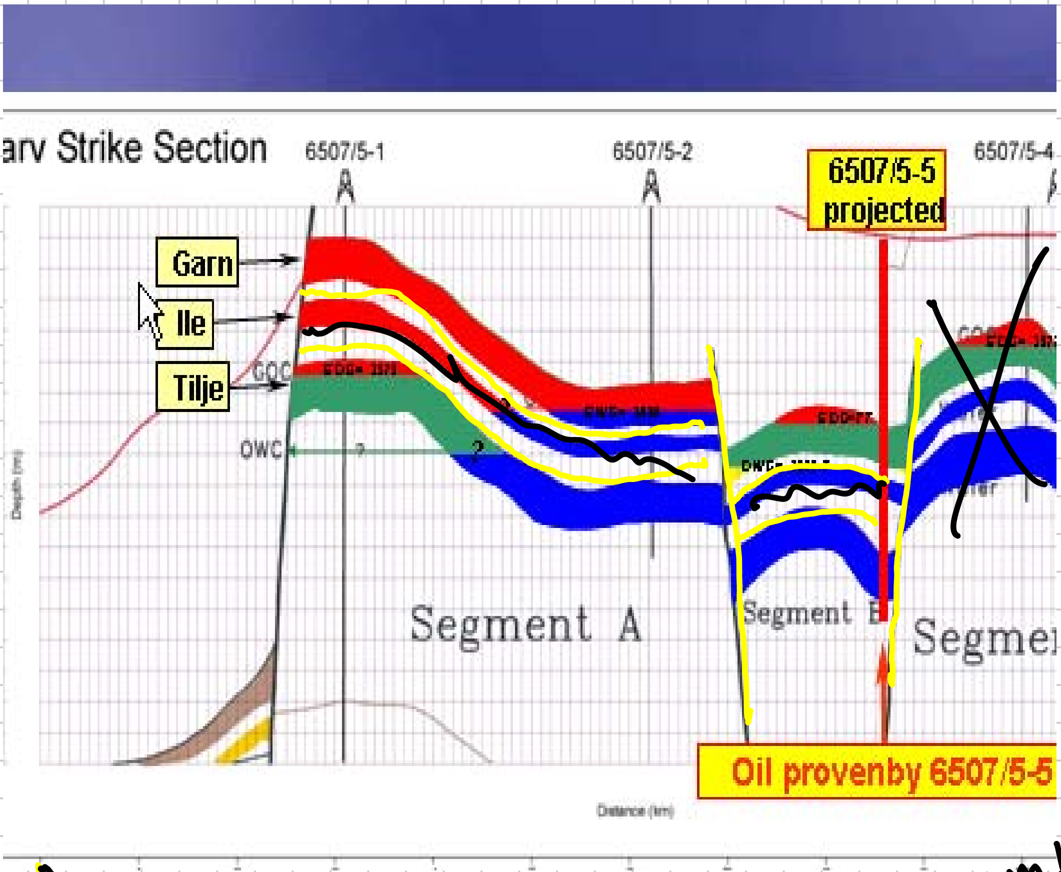
$$\boxed{B_o = \frac{V_o}{V_{os}}}$$

$$GOR \simeq \frac{F_v}{(1-F_v)} \cdot \text{const}$$

RG : $\boxed{F_v} \sim 1$

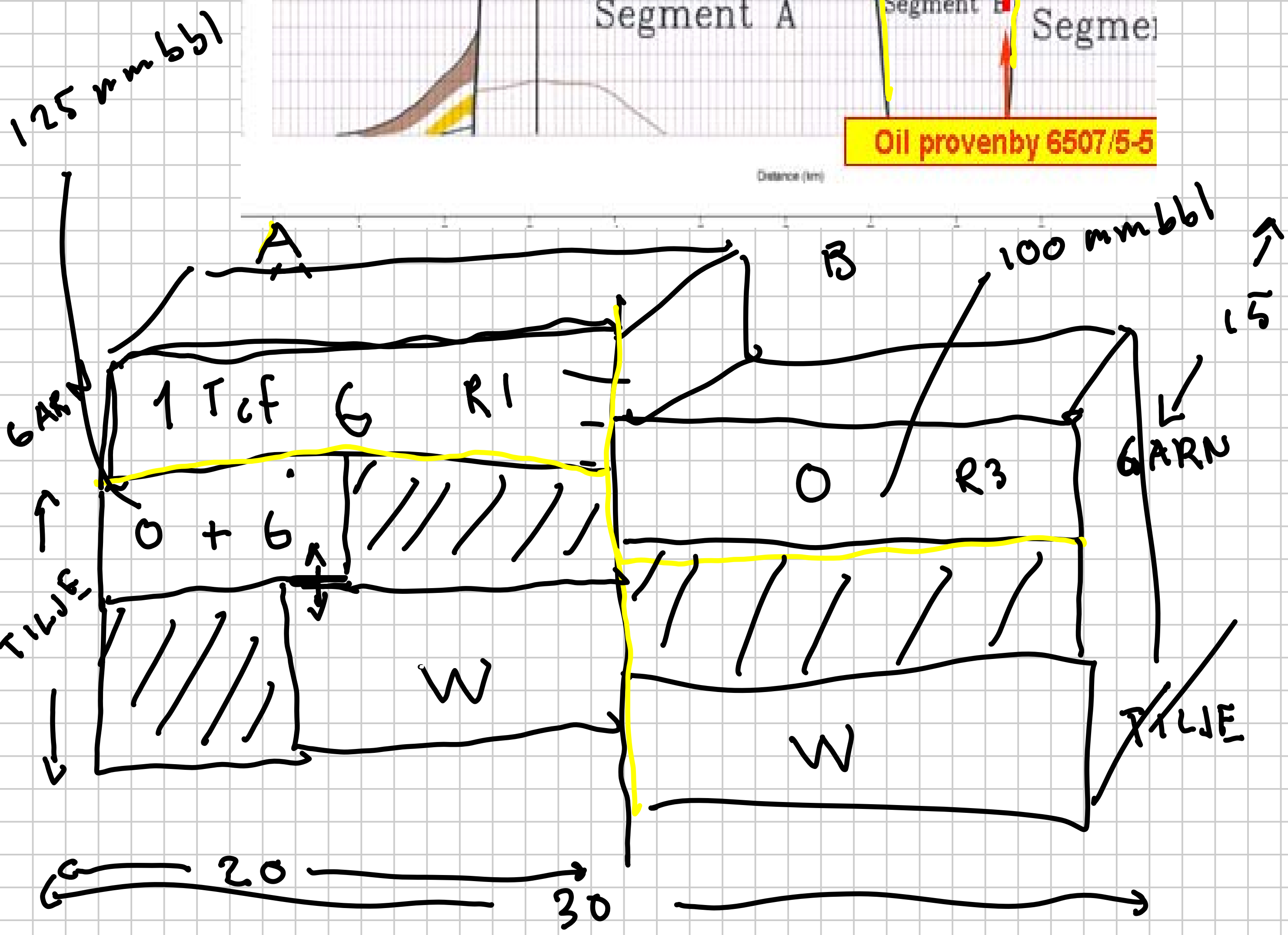
RR : Gas Feed

$F_v \sim 0.98,\ 0.99$



arv Strike Section

Oil provenby 6507/5-5



BLACK-OIL PVT FORMULATION (Ch. 7)

Simplified model describing the phase and volumetric behavior of reservoir fluids using two "components":

Surface Gas $\left(\overbrace{N_2, CO_2, H_2S, C_1, C_2}^{\text{all}}, \overset{\text{most}}{\text{“}C_3-C_5\text{”}}\right)$

Surface Oil $\left(\underset{\text{all}}{\sim\text{“}C_{6+}\text{”}}\right)$

Phase Behavior:

mass or volume

(1) How much of each reservoir phase (RG, RO) "$\underline{\underline{F_r}}$" at reservoir conditions

(2) Composition of each phase:

$$\left.\begin{array}{ll} RG & (\underline{\underline{y_i}}) \\ RO & (\underline{\underline{x_i}}) \end{array}\right\} \quad i = 1, 2$$

$i = 1$: Surf. Gas

$i = 2$: Surf Oil

Volumetric Behavior:

(1) Phase densities $\overbrace{\rho_{RO}, \rho_{RG}}^{EOS}$

(2) Volume of each phase

BO PVT Formulation

- We only use "volume" to quantify amounts ("volume fractions")
  - i.e. not mass, not moles

Note: Reservoir, Production, Pipeline engineering 95% of your work will be using BO PVT formulation.

Note: BO PVT "properties" are not "intuitive", not good scientific/eng. quantities ... STRANGE

How do we build a black-oil PVT model?

---

(1) Old way: (Ch. 3) using so-called "correlations" ... kinda stopped 1980's-1990's

(2) Measurements from laboratories ... also kinda stopped 1980's-1990's

Key Input

(3) From an Equation of State (EOS)

Compositional PVT model

– K_i, z_i, RR, P_{sat} ...

complicated stuff.

95% of time since mid 90's.

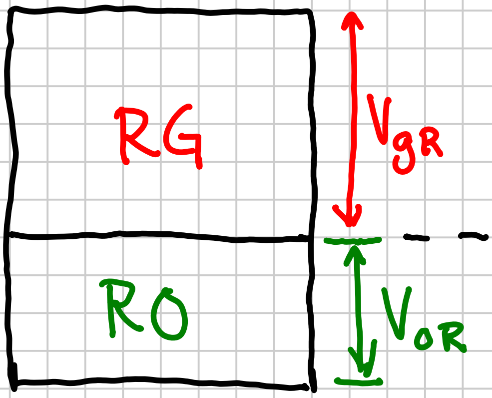
What are the BO PVT Properties?

RG BO PVT:

(1) $B_g \equiv \dfrac{V_{gR}}{V_{gs}}$

(2) $r_s \equiv \dfrac{V_{os}}{V_{gs}}$

$[R_v]$



T_R, P

RO BO PVT:

(1) $B_o \equiv \dfrac{V_{oR}}{V_{os}}$

Start with

End up with

(2) $R_s \equiv \dfrac{V_{gs}}{V_{os}}$

RG PVT Properties

r_s : solution oil-gas ratio

(solution condensate-gas ratio)

$10-300 \ \frac{STB}{MMscf}$ solution OGR

$60-1800 \ \frac{Sm^3}{10^6 \ Sm^3}$ solution CGR

$\overset{*}{\boxed{B_{gd}}}$: gas FVF (formation volume factor)

$\frac{1}{50} - \frac{1}{300} \ \frac{m^3}{Sm^3}$ "dry" gas FVF

↑
surface
or
sales
gas

At initial
pressures
$\gtrsim$ 50 bar

* Traditional Gas FVF B_g

$$B_{gw} = \frac{p_{sc}}{T_{sc}} \cdot \frac{Z T_R}{p}$$

| Assumes **all** RG stays as gas at surface

RO BO PVT:

B_o : Oil FVF

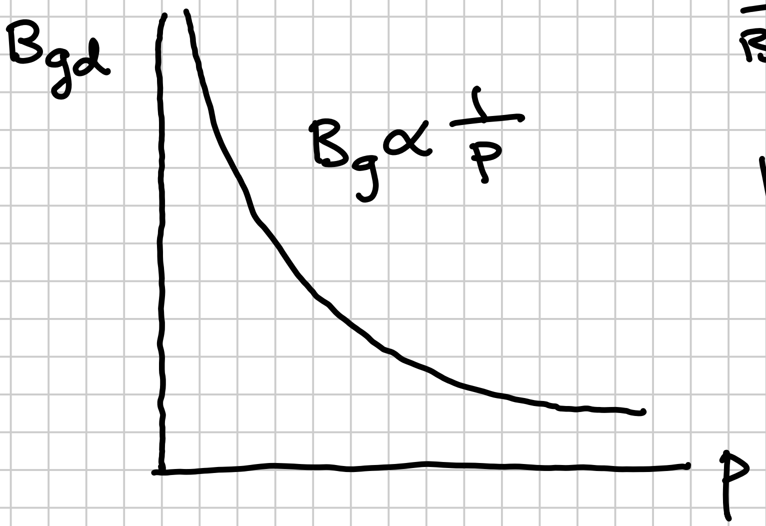
1.05 – 3.0 m^3/Sm^3 [RB/STB]

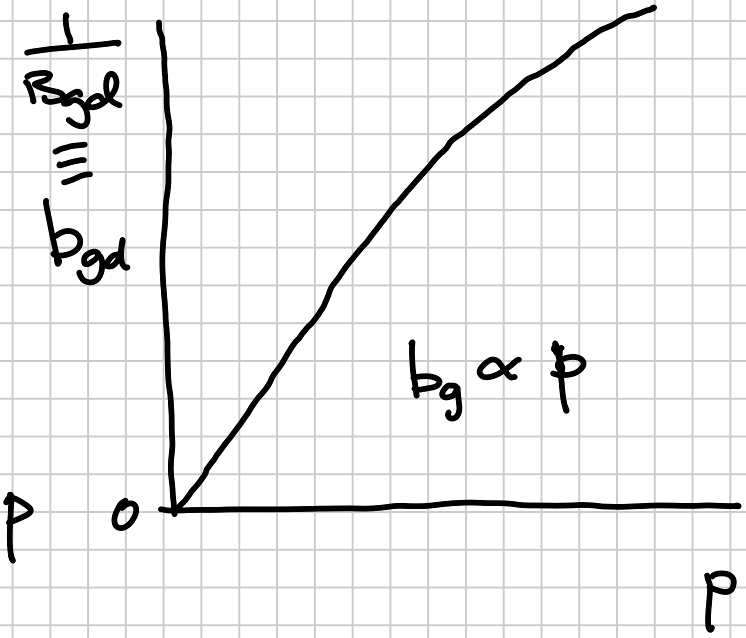
R_s : Solution Gas-Oil Ratio (GOR)

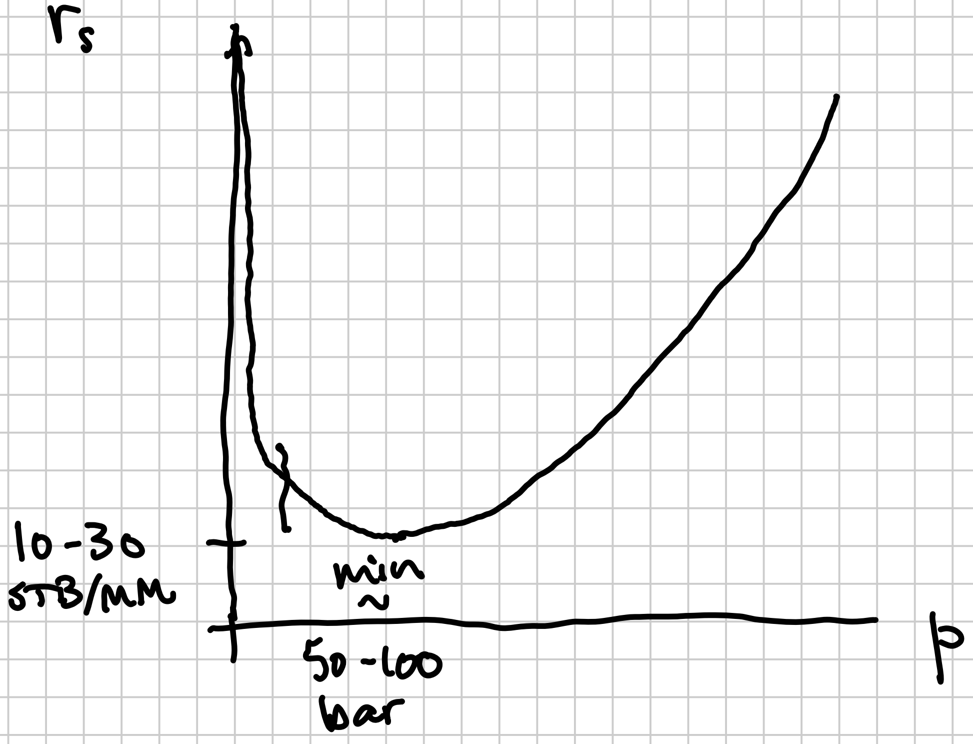
10 – 600 Sm^3/Sm^3

50 – 3500 scf/STB

Pressure Dependence of BO PVT Properties

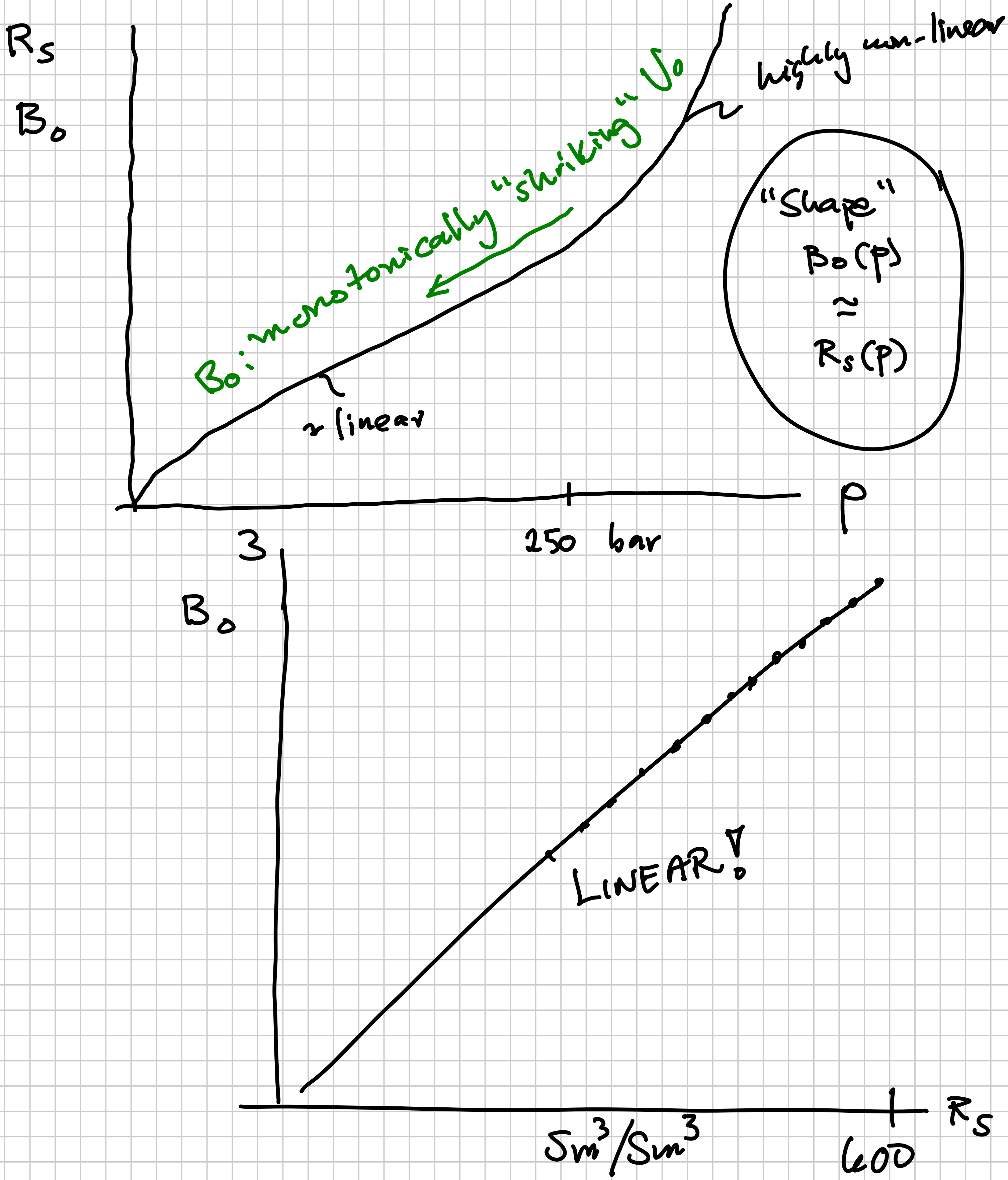




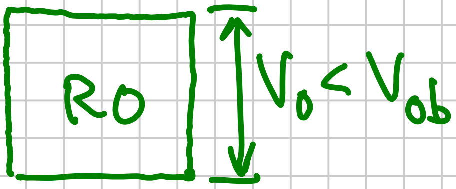


non-monotonic

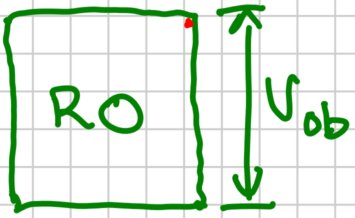
RO BOPUT (SATURATED OIL)



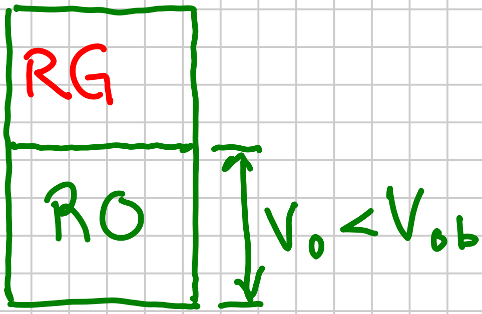
Undersaturated Condition



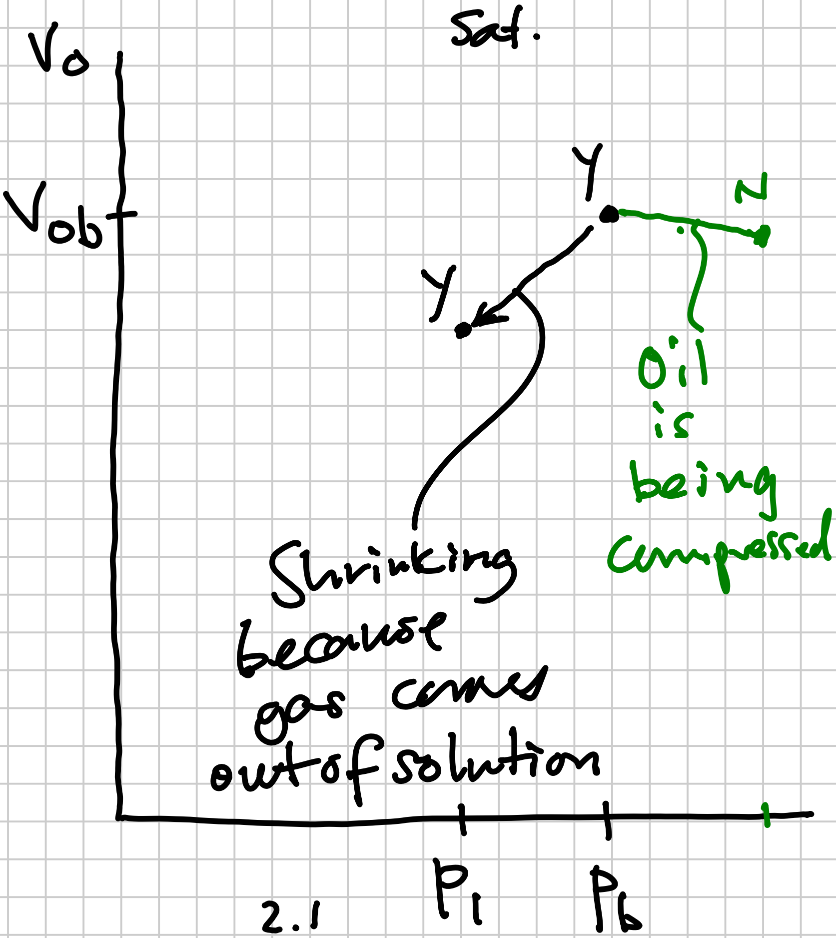
$P > P_b$

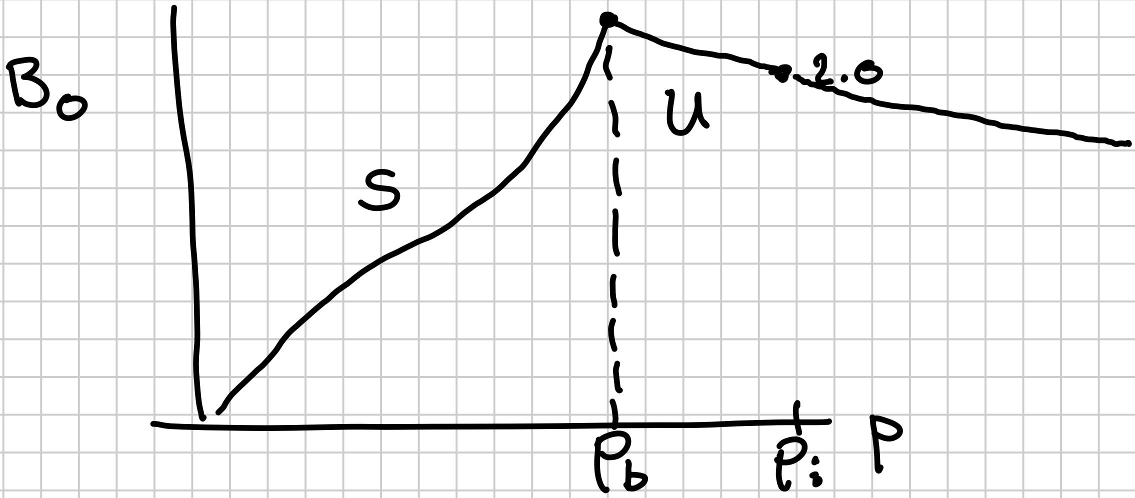


$P = P_b$



$P_1 < P_b$

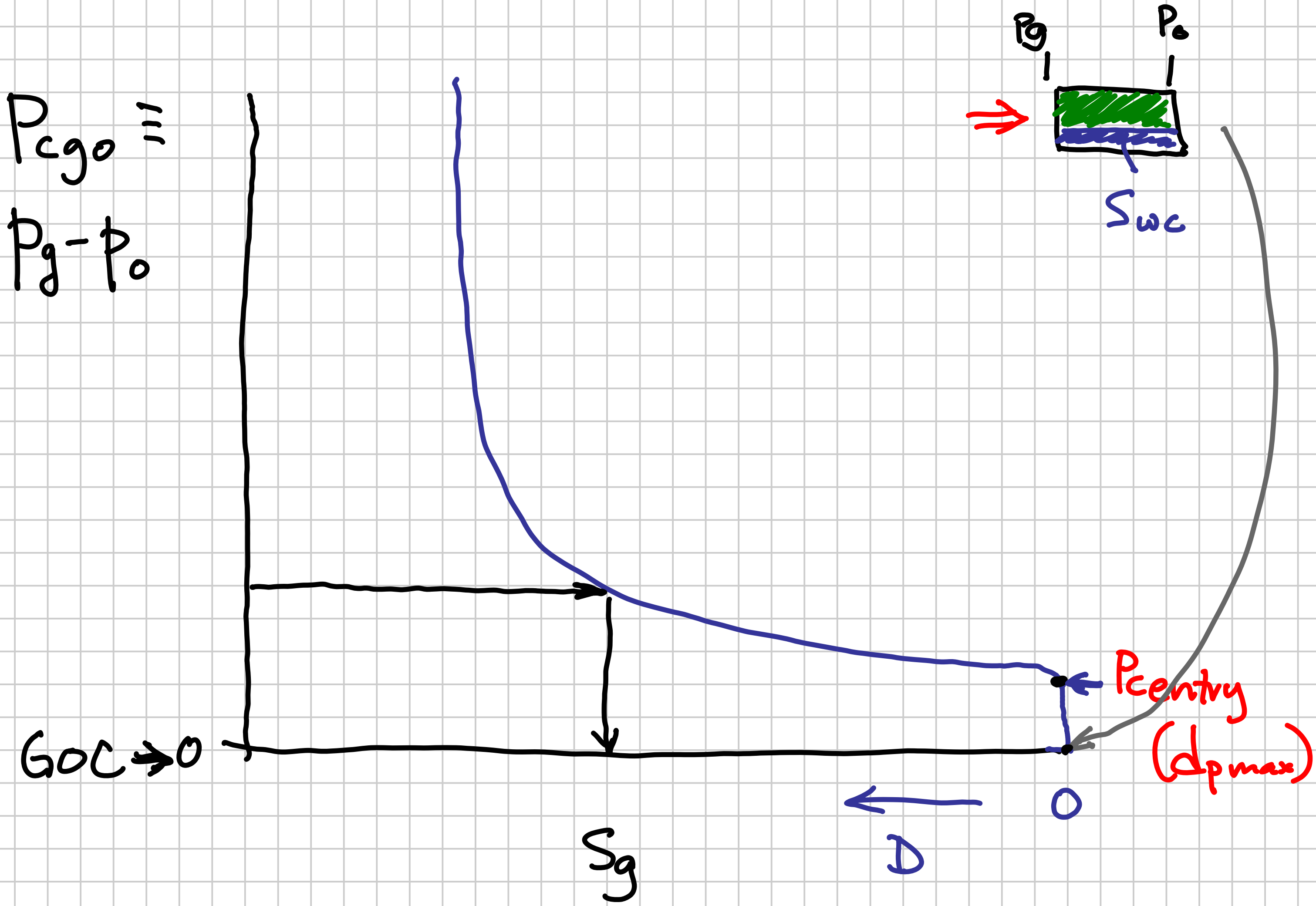


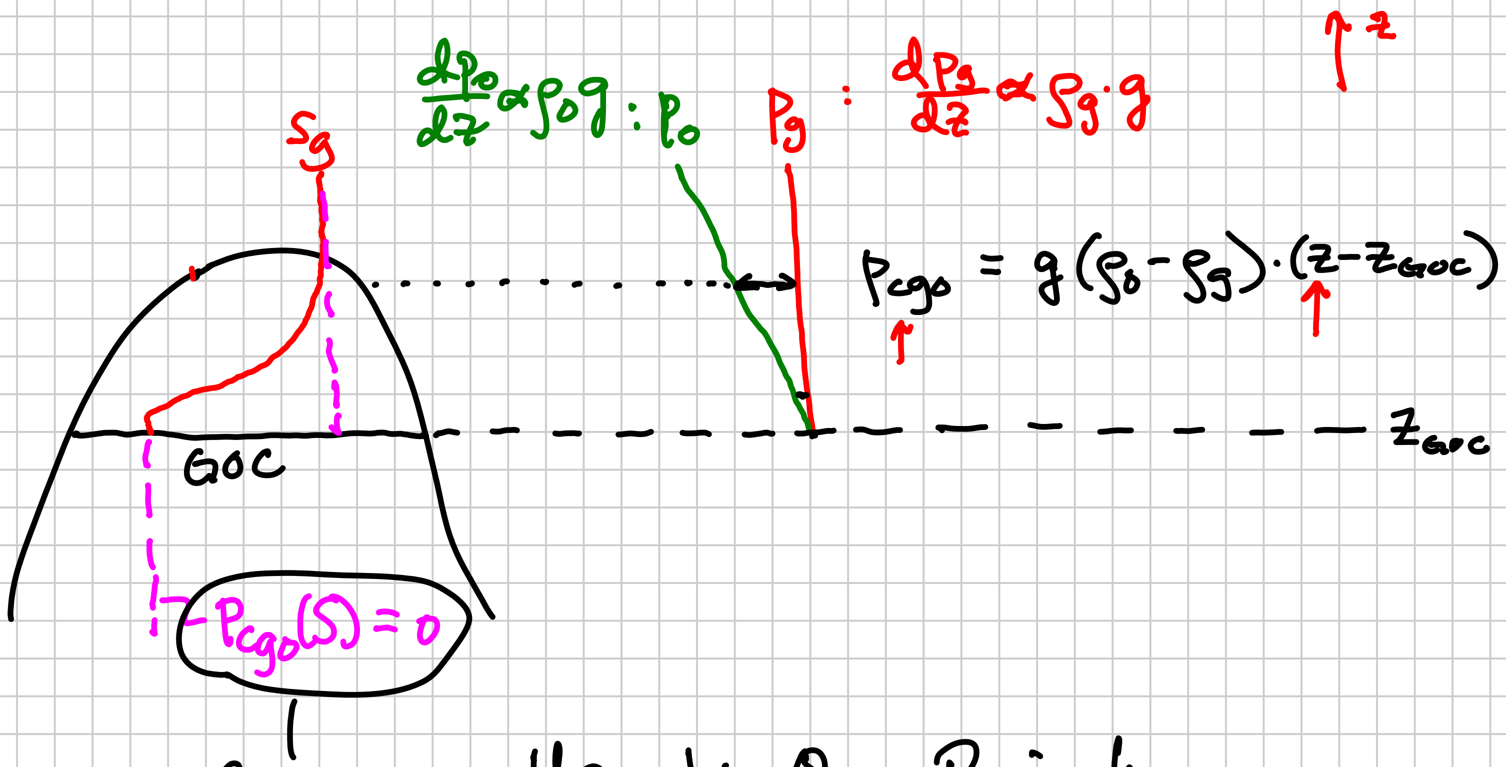


GOC defined as the depth where $P_{cgo} = 0$.

If no $\boxed{P_{cgo}(S)}$ used, then $S_o = 0$ above the GOC, $S_g = 0$ below GOC.

If $P_{cgo}(S_g)$ used in initialization, results in $S_o > 0$ above the GOC.





Common, Usual; Our Project

Black-Oil Model (Ch. 7)

$$\rho_o(p) = \frac{m_{gs} + m_{os}}{V_{oR}} = \frac{m_{\bar{o}} + m_{\bar{g}}}{V_{oR}}$$

$$\boxed{\rho_o = \frac{R_{s(p)} \cdot \rho_{\bar{g}} + \rho_{\bar{o}}}{B_{o(p)}}}$$

Constants

algebra

$$\rho_{\bar{g}} = \frac{m_{\bar{g}}}{V_{\bar{g}}} \; ; \; \rho_{\bar{o}} = \frac{m_{\bar{o}}}{V_{\bar{o}}}$$

$$R_s = \frac{V_{\bar{g}}}{V_{\bar{o}}}$$

$$\rho_{\bar{g}} = \frac{p_{sc} M_{\bar{g}}}{R T_{sc}} = \underbrace{\frac{p_{sc} M_{air}}{R T_{sc}}}_{\text{Known}} \cdot \gamma_{\bar{g}}$$

$$\boxed{\rho_g(p) = \frac{\rho_{\bar{g}} + \rho_{\bar{o}}\, r_s(p)}{B_{gd}(p)}}$$

Black-Oil Model

kg/m^3 $\rho_{\bar{g}}$
kg/m^3 $\rho_{\bar{o}}$

$\left.\begin{matrix}\rho_{\bar{g}} \\ \rho_{\bar{o}}\end{matrix}\right\}$ Are specified or given constants – i.e., $\underline{\text{not}}$ $f(p)$

Sm^3/Sm^3 $R_s(p)$
m^3/Sm^3 $B_o(p)$
Sm^3/Sm^3 $r_s(p)$
m^3/Sm^3 $B_{gd}(p)$

$\Rightarrow$ $\rho_o(p)$, $\rho_g(p)$

Eqs. have no conversion constants

Field Units:
$\rho_{\bar{o}}$ (API; SG, water = 1)
$\rho_{\bar{g}}$ (lb/ft^3; SG, air = 1)
R_s (scf/STB) Mscf/STB
B_o (RB/STB)
r_s (STB/scf) or ...
B_{gd} (ft^3/scf) or ...

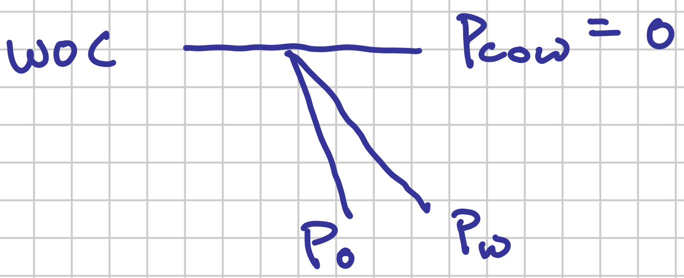
Importance of ρ_o, ρ_g

(1) Initialization $P_{cgo} = g(\rho_o - \rho_g)\,\Delta z$

$$p_o - p_w = P_{cow} = g\,\underline{(\rho_w - \rho_o)}\,(z_{woc} - z)$$

if small, ρ's are important

$S_o(z)$: Important!!



(2) v in high-k reservoirs:

controlled by $\Delta\rho_{wo}$ and/or $\Delta\rho_{og}$

Procedure for Generating Black-Oil PVT

(1) Correlations

(2) Lab Data

(3) EOS model

---

(1) Correlations:

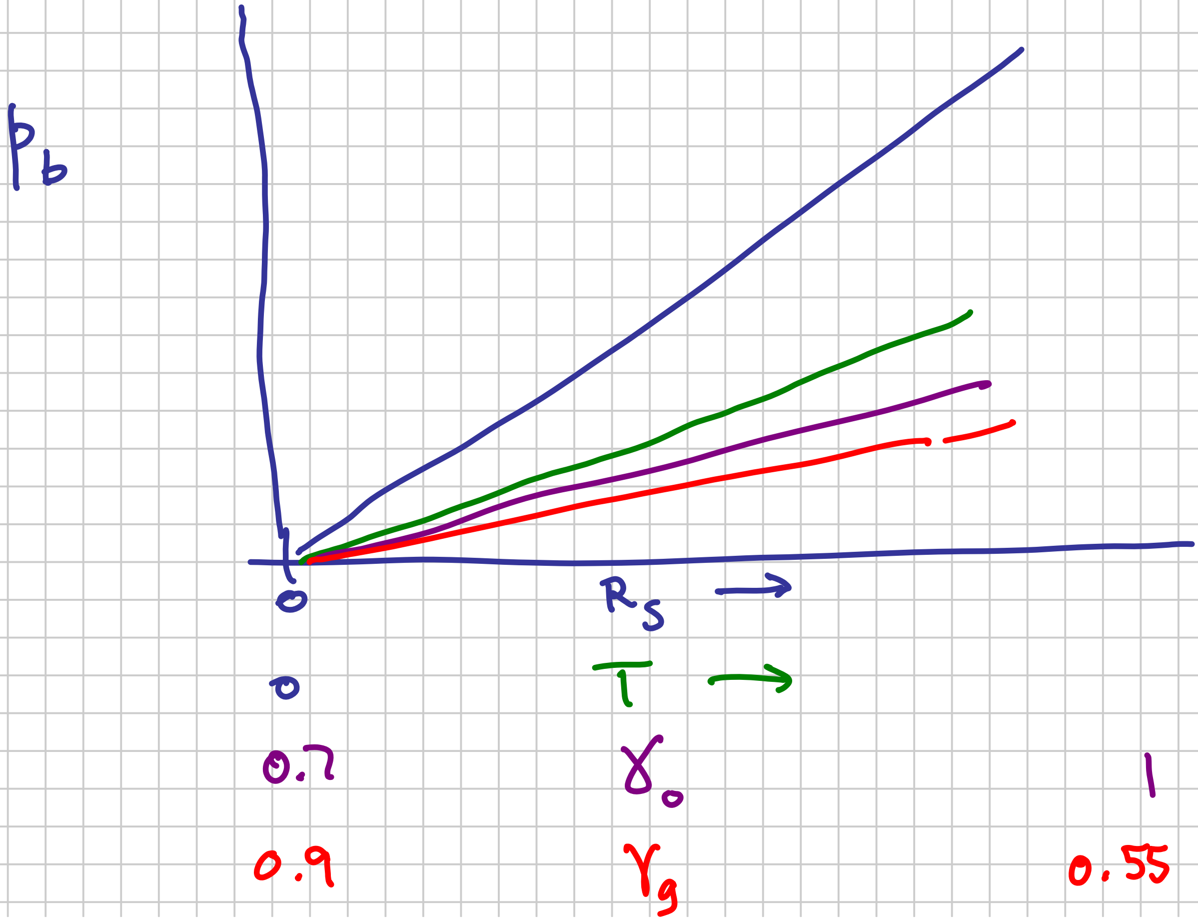
Oil (RO):

$P_b\left(R_s, T, \gamma_o, \gamma_g\right)$ ± 5-20% Bubblepoint

"Composition" of the RO

- Standing
- Lasater
- Glasø

⋮



Invert BP Correlation:

$$R_s(p, T, \gamma_o, \gamma_g)$$

for $p < p_{b\,(\text{initial})}$

- Oil FVF B_o ± 1-3% for $B_o < 1.7$

Saturated

$$B_o(R_s, T, \gamma_{\bar{o}}, \gamma_{\bar{g}})$$

NB!

~Linear $B_o(R_s)$

$p > p_b$: Undersat. B_o

Use $\boxed{c_o}$ to correct the B_{ob}

- Oil Viscosity ± 10-50%

Saturated

$$\mu_o(\mu_{oD}, R_s)$$

for $R_s = 0$ at T_R

OIL TYPE* is missing

$$\mu_{oD}(\gamma_{\bar{o}}, T_R)$$

± 10-50%

* Aromatic
Paraffinic

- Gas
  - Z-factor (Standing-Katz Chart)

    or an EOS fit to

    e.g. Yarborough

$$Z(p, T, \text{composition})$$

ρ_g c_g B_g

  - Viscosity

$\mu_g(\rho_g)$ Lee-Gonzalez (used by labs)

$\pm$ 5-10%

or

$\mu_g(\mu_g^\circ, p, T)$

0.02-0.03 cp

Gas Visc. of an Ideal Gas

$\mu_g^\circ(\gamma_g, T)$

Problem for Gas Condensates*:

<u>No</u> correlation

$$p_d(r_s, T, \gamma_g, \gamma_o)$$

$$\boxed{r_s(p, \dots)}$$

Very important for G.C. "Oil" recovery

RG "composition" $\left\langle \begin{matrix} \bar{g} \\ \bar{o} \end{matrix} \right.$

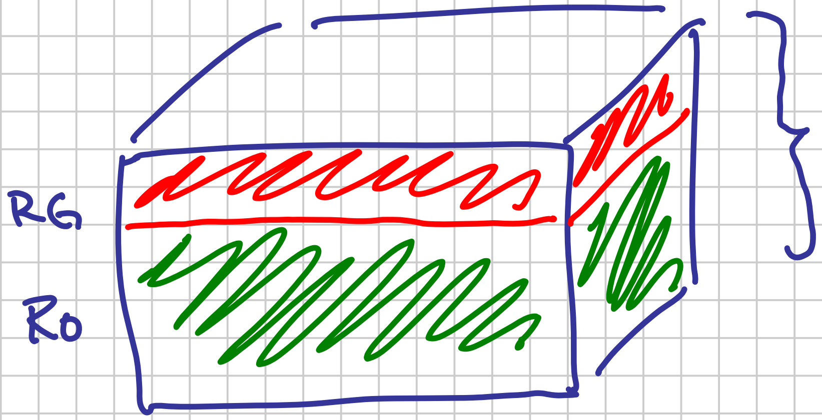
* Also a problem for <u>any</u> RG

$p \gtrsim 300$ bar

Initial

Solution

$$\mu_{oD} \quad \left.\begin{array}{l}\text{"dead" oil}\\ \text{surface oil}\\ \text{stock-tank oil}\\ \text{de-gassed oil}\end{array}\right\} \begin{array}{l} R_s=0\\ \text{no gas left}\\ \text{in solution}\\ @\ p_{sc},\ T_R\end{array}$$



$\Rightarrow$ Surface $\boxed{GOR}$ $= \dfrac{V_{\bar{g}}}{V_{\bar{o}}}$ $= \dfrac{V_{\bar{g}g} + V_{\bar{g}o}}{V_{\bar{o}g} + V_{\bar{o}o}}$

$$V_{gR}(RG) = 0.3 \cdot 10^9 \text{ ft}^3$$

$$V_{oR}(RO) = 0.7 \cdot 10^9 \text{ ft}^3$$

At P_{Ri}

RO: $B_o = 1.6$ RB/STB

$R_s = 800$ scf/STB $= 0.8$ MCF/STB

RG: $B_g = \dfrac{1}{250}$ ft^3/scf

$r_s = 100 \dfrac{\text{STB}}{\text{MMscf}} = 100 \cdot 10^{-6} \dfrac{\text{STB}}{\text{scf}}$

RO: $B_o = \dfrac{V_{oR}}{V_{\bar{o}o}}$ $\qquad R_s = \dfrac{V_{\bar{g}o}}{V_{\bar{o}o}}$

RG: $B_{gd} = \dfrac{V_{gR}}{V_{\bar{g}g}}$ $\qquad r_s = \dfrac{V_{\bar{o}g}}{V_{\bar{g}g}}$

... by algebra, by a calculator or both.

$$GOR_{RG+RO} = \underline{\underline{286 \quad \frac{scf}{scf}}} \quad \left[\frac{sm^3}{sm^3}\right] \Rightarrow \underline{\underline{1605 \; \frac{scf}{STB}}} \checkmark$$

$$939 \quad \frac{scf}{STB}$$

$$\left\{ 92 \quad \frac{Mscf}{STB} \qquad 92000 \; \frac{scf}{STB} \right\}$$

$$V_{RO} = 0.7 \cdot 10^9 \; ft^3 \qquad = 1.25 \cdot 10^8 \; RB$$

$$V_{RG} = 0.3 \cdot 10^9 \; ft^3$$

<u>RO:</u>

$$B_o \equiv \frac{V_{oR}}{V_{\bar{o}o}} = 1.6 \; \frac{RB}{STB} = 1.6 \; \frac{ft^3}{scf}$$

$$\overset{\checkmark}{V}_{\bar{o}o} = V_{oR} \cdot \frac{1}{B_o} = \frac{1.25 \cdot 10^8}{1.6} = 7.79 \cdot 10^7 \; STB$$

$$\overset{\checkmark}{R}_s \equiv \frac{V_{\bar{g}o}}{V_{\bar{o}o}} \qquad R_s = 800 \; \frac{scf}{STB}$$

$$\overset{\checkmark}{V}_{\bar{g}o} = R_s \cdot V_{\bar{o}o} = 800 \cdot 7.79 \cdot 10^7 \quad scf$$

$$= 6.23 \cdot 10^{10} \quad scf$$

$$0.0623 \quad Tcf$$

<u>RG:</u>

$$\overset{\checkmark}{B}_{gd} \equiv \frac{\overset{\checkmark}{V}_{gR}}{V_{\bar{g}g}} = \frac{1}{250} = 0.004 \; \frac{ft^3}{scf}$$

$$V_{\bar{g}g} = V_{gR} \cdot \frac{1}{B_{gd}} = 0.3 \cdot 10^9 \, ft^3 \times 250 \frac{scf}{ft^3}$$

$$= 7.50 \cdot 10^{10} \text{ scf}$$

$$0.075 \text{ Tcf}$$

$$\checkmark\, r_s \equiv \frac{V_{\bar{o}g}}{\checkmark V_{\bar{g}g}} = 100 \cdot 10^{-6} \frac{\text{STB}}{\text{scf}}$$

$$V_{\bar{o}g} = r_s \cdot V_{\bar{g}g} = 100 \cdot 10^{-6} \times 7.50 \cdot 10^{10}$$

$$= 7.5 \cdot 10^{6} \text{ STB}$$

We have ALL surface volumes:

$$GOR = \frac{V_{\bar{g}g} + V_{\bar{g}o}}{V_{\bar{o}g} + V_{\bar{o}o}}$$

$$= \frac{7.5 \cdot 10^{10} + 6.23 \cdot 10^{10}}{7.79 \cdot 10^{7} + 7.5 \cdot 10^{6}} \quad \frac{\text{scf}}{\text{STB}}$$

$$= 1607 \frac{\text{scf}}{\text{STB}}$$

$$= 286 \text{ Sm}^3/\text{Sm}^3$$

Surface Gas: 2 sources RG + RO

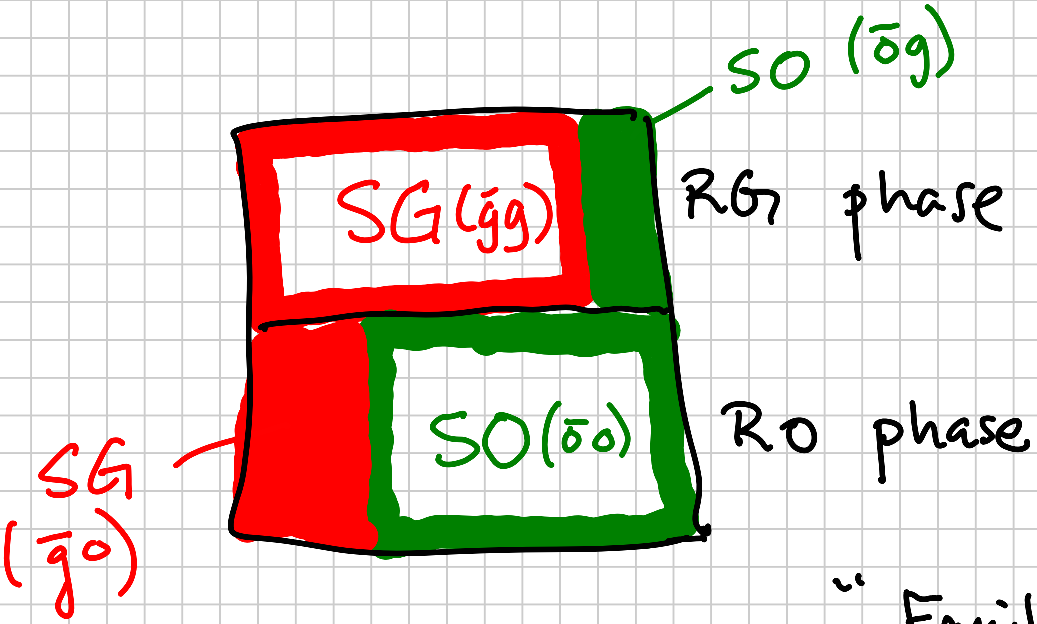
Surface Oil: 2 sources RG + RO

Black-Oil PVT Model

- 2 "pseudo" components SG SO

$\bar{g}$ $\bar{o}$

- 2 phase (in the reservoir) RG RO



@ P_R, T_R

"Equilibrium Ratio"

$$K_i = \frac{y_i}{x_i}$$

$$i \in \{SG, SO\}, \{\bar{g}, \bar{o}\}$$

$$\left.\begin{array}{l} K_{\bar{g}} > 1 \\ K_{\bar{o}} < 1 \end{array}\right\} \text{Eqs. 7.x}$$

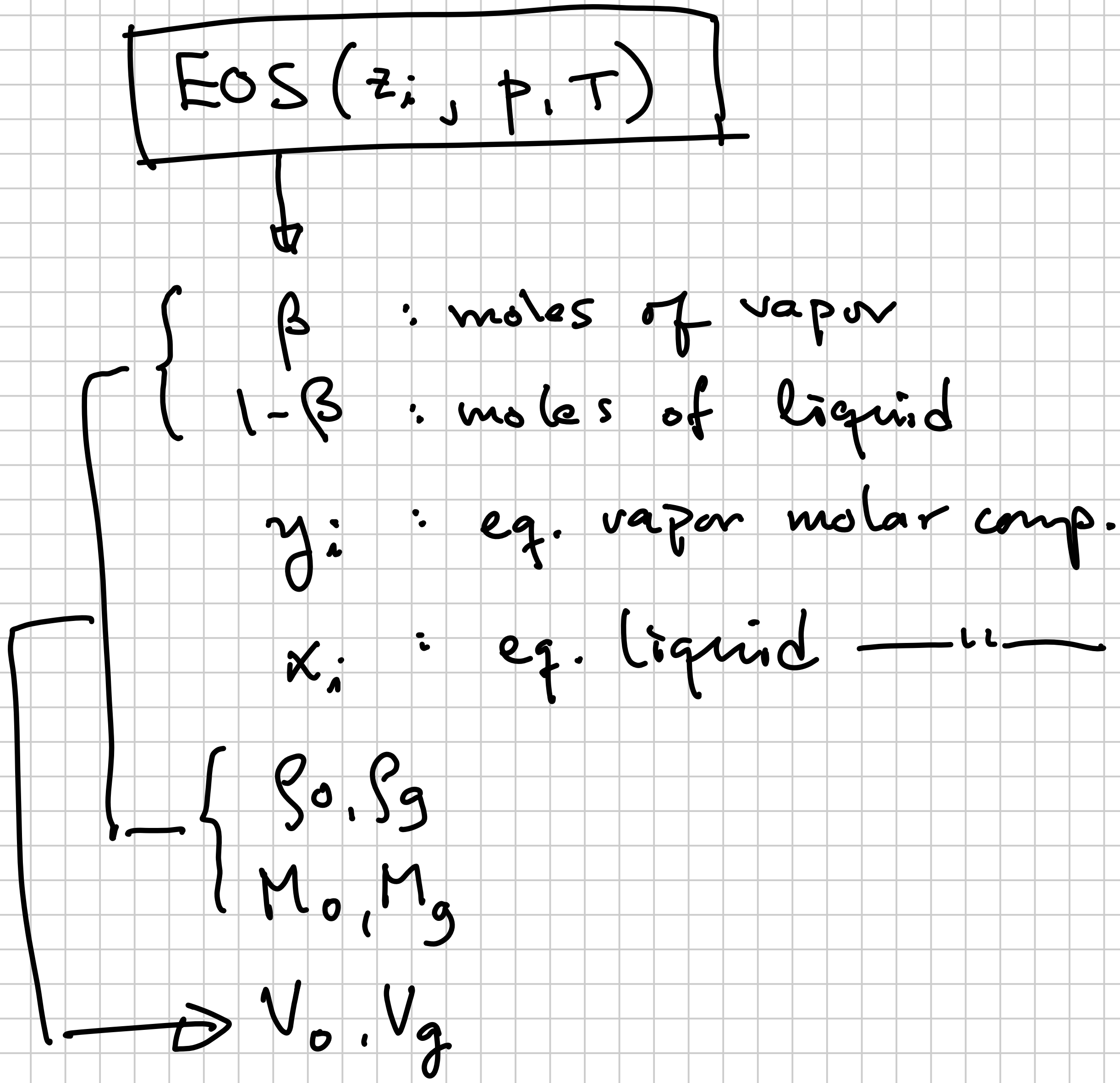
Conceptually:

R_s : 'gives' the RO "composition" x_i

r_s : 'gives' the RG "composition" y_i

Generating BO PVT Tables

(3) Using an EOS (Compositional) Model



$\boxed{EOS(z_i, p, T)}$

$\downarrow$

β : moles of vapor

$1-\beta$: moles of liquid

y_i : eq. vapor molar comp.

x_i : eq. liquid — " —

ρ_o, ρ_g

M_o, M_g

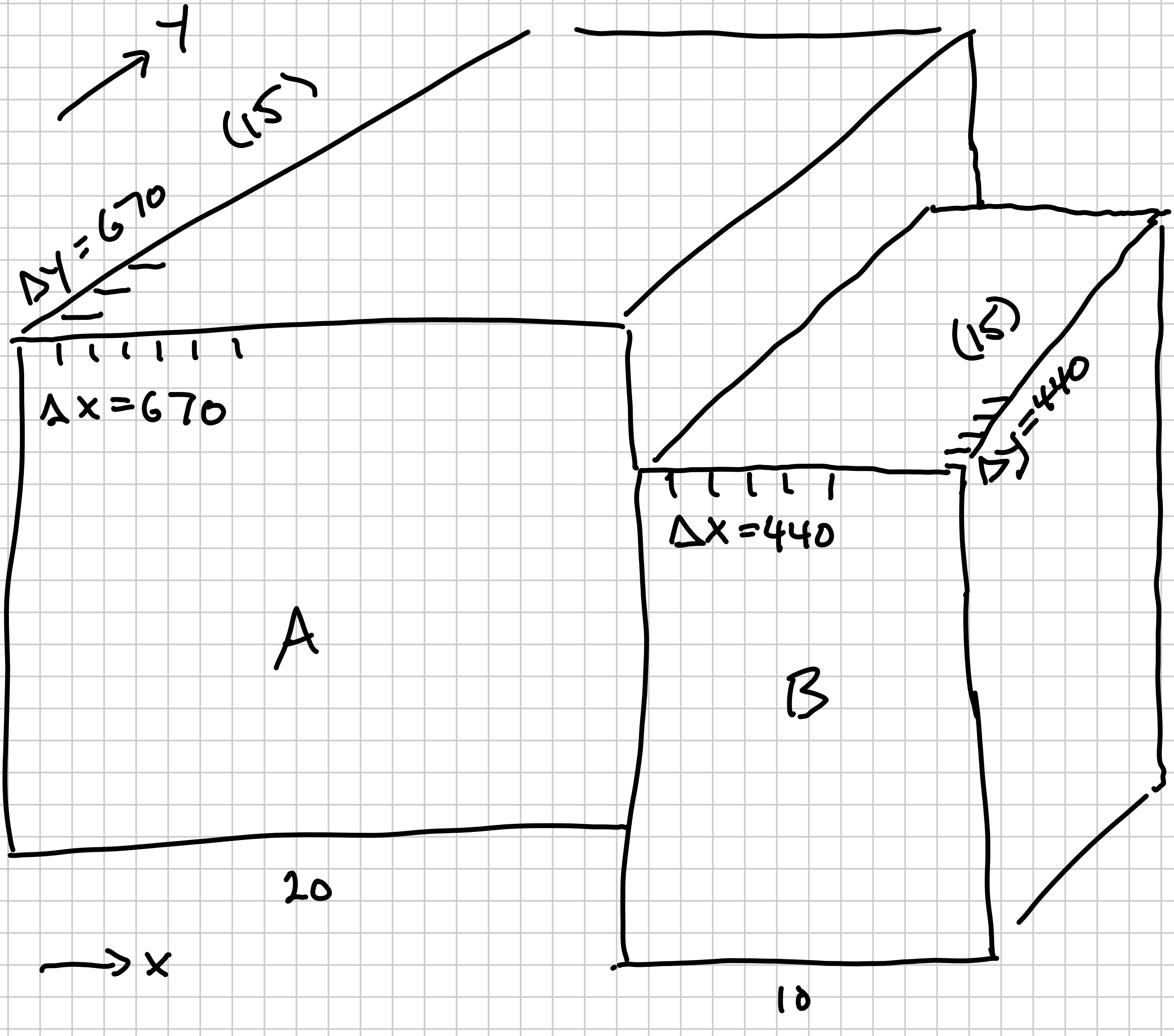
$\rightarrow V_o, V_g$

Economic Calculations:

Missing operational costs.

est. \$ 0.5 million / day (Check?)

<u>DX</u> <u>DY</u>



(1) AT CONSTANT RESERVOIR TEMPERATURE

* Simulate pressure depletion in a reservoir, starting at the initial saturation pressure.

Two Phases Initially

$$\left\{\begin{array}{l} RG : @\ p_d \\ RO : @\ p_b \\ RG+RO\ (\text{saturated } @\ GOC)\ @\ p_{GOC} = (p_d)_{RG} = (p_b)_{RO} \end{array}\right.$$

** Lower the pressure, bring to equilibrium with a new RG and a new RO.

o
o
o
repeat

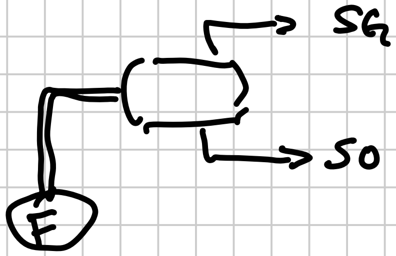
Each "stage" in this process, we collect two compositions, $y_i(p)$ and $x_i(p)$

RG's RO's

(2) Define a SURFACE PROCESS

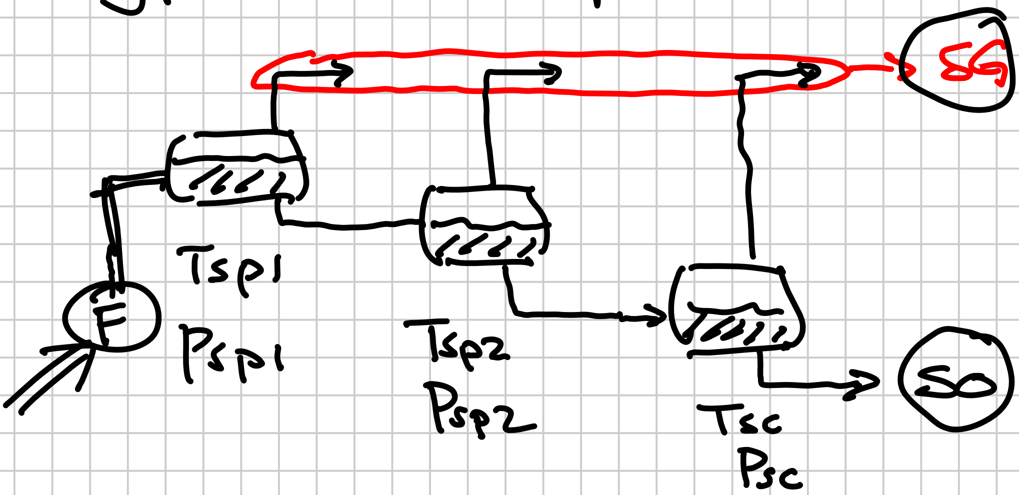
(a) Simplest

Single-stage separation at P_{sc}, T_{sc}

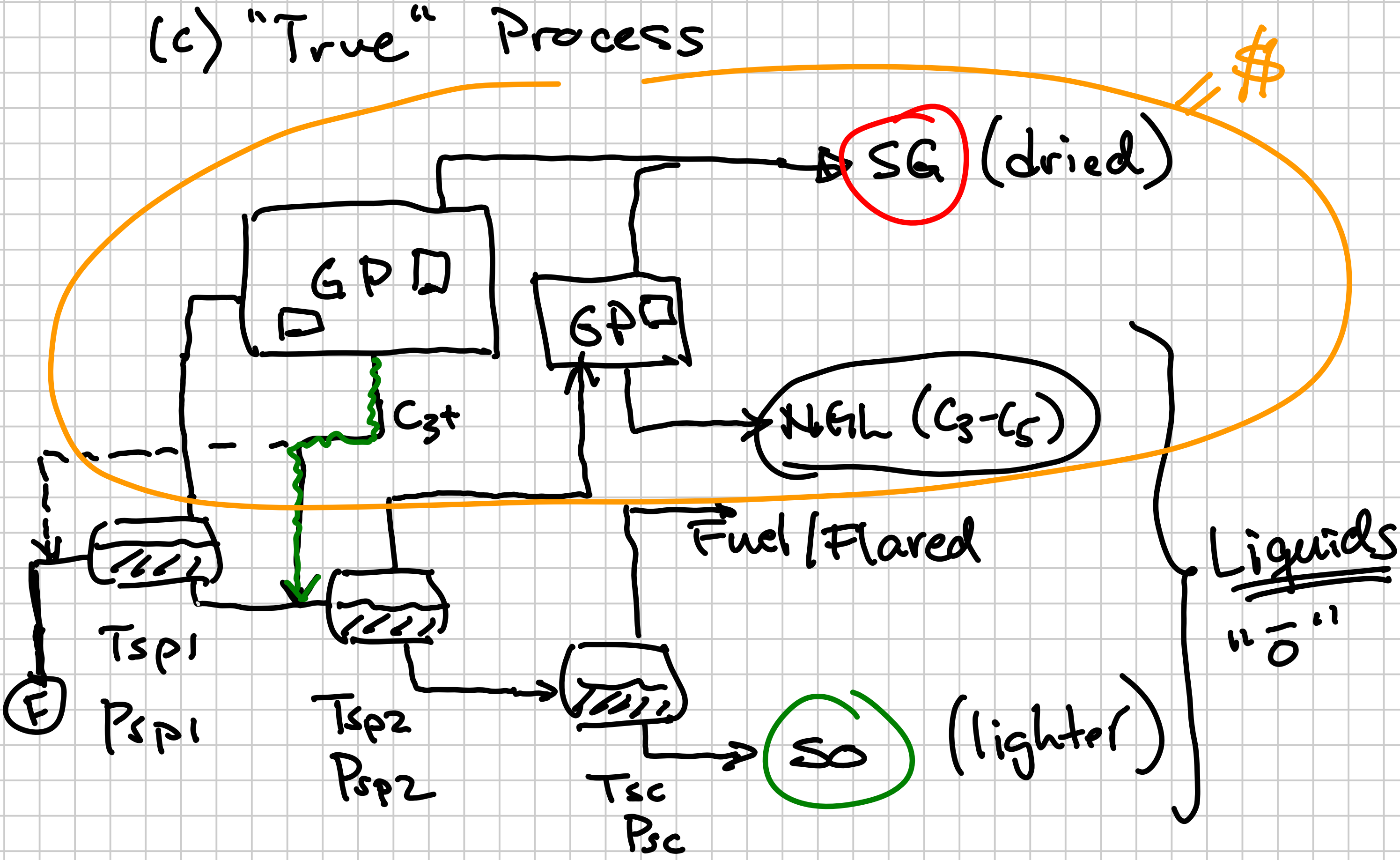


"F" Feed
(feeding the Process)

(b) "Typical" Mutistage Separator



(c) "True" Process



(3) Pass each RG composition $y_i(p_R, T_R)$
Depletion

as the "feed" to the Process.

$\Rightarrow$ SG + SO
$(\bar{g}g)$ $(\bar{o}g)$

B_{gd} , r_s @ p_R

" "

$\dfrac{V_{gR}}{V_{\bar{g}g}}$ $\dfrac{V_{\bar{o}g}}{V_{\bar{g}g}}$

Saturated Gas BO Properties

(4) Pass each RO composition $x_i(\underline{p_R}, T_R)$
Depletion

as the "feed" to the Process.

$\Longrightarrow$ SG + SO
$(\bar{g}o)$ $(\bar{o}o)$

B_o R_s at p_R's

" "

$\dfrac{V_{oR}}{V_{\bar{o}o}}$ $\dfrac{V_{\bar{g}o}}{V_{\bar{o}o}}$

(5) Calculate RG & RO Viscosities at each P_R.

$$\left.\begin{array}{l} \mu_g\,(y_i\,,P_R\,,T_R) \\ \mu_o\,(x_i\,,P_R\,,T_R) \end{array}\right\} \begin{array}{l}\text{Special} \\ \text{compositional} \\ \text{viscosity} \\ \text{correlation}\end{array}$$

DONE. ... almost ☺

⑥ "Choose" surface gas and surface oil densities:

$$\rho_{\bar{g}} = \text{constant}$$

$$\rho_{\bar{o}} = \text{constant}$$

$\rho_{\bar{o}g} \neq \rho_{\bar{o}o}$ at any p

$\rho_{\bar{g}g} \neq \rho_{\bar{g}o}$ — " —

10 pressures, 20 $\rho_{\bar{o}}$'s PICK 1

20 $\rho_{\bar{g}}$'s PICK 1

Which values to choose?

* Remember $\rho_{\bar{o}}$, $\rho_{\bar{g}}$ ONLY used to help calculate $\rho_o(p)$ and $\rho_g(p)$

$$\rho_o(p) = \frac{\rho_{\bar{o}} + R_s(p) \cdot \rho_{\bar{g}}}{B_o(p)}$$

$$\rho_g(p) = \frac{\rho_{\bar{g}} + r_s(p)\, \rho_{\bar{o}}}{B_{gd}(p)}$$

Choose $\rho_{\bar{o}}$ and $\rho_{\bar{g}}$ so that the Known (EOS or measured) $\rho_o(p)$ and $\rho_g(p)$ are "best" calculated.

- $\Delta\rho_{og} = (\rho_o - \rho_g)$
- $\Delta\rho_{wo} = (\rho_w - \rho_o)$

* high K with G and W injection

---

* low K with "heavier" oil S_w(depth)

$$G = \frac{\Delta p}{\Delta z} \quad \left[\frac{psi}{ft}\right] \qquad \left[\frac{Pa}{m}\right] \quad \left[\frac{bar}{m}\right]$$

Static Column is defined by

$$\frac{dp}{dz} = \rho g$$

Consistent units

$$\frac{Pa}{m} = \rho\left[\frac{kg}{m^3}\right]\cdot 9{,}81\left[\frac{m}{s^2}\right]$$

$$\boxed{\rho\ \left[\underline{\underline{lb_m}}\,/\,\underline{\underline{ft^3}}\right]}$$

Field Units:

$$\boxed{\frac{dp}{dz}\left[\frac{psi}{ft}\right]} = \rho\cdot\left(\frac{g}{g_c}\right) = \rho\left[\frac{lb_f/ft^2}{ft}\right]$$

$$= \rho\left[\frac{lb_f/ft^2}{ft}\right]\frac{ft^2}{144\,in^2}$$

$$= \rho/144\left[\frac{lb_f/in^2}{ft}\right]$$

$$= \frac{\rho}{144}\left[\frac{psi}{ft}\right]$$

$$g = 32.\ ft/s^2$$

$$g_c = 32\left(\frac{lb_f\cdot s^2}{lb_m\cdot ft}\right)^{-1}$$

$$psi \equiv \frac{lb_f}{in^2}$$

SI: kg/m^3 $\quad$ g/cm^3

Example: $\boxed{\rho = 50 \, \frac{lbm}{ft^3}} \longrightarrow = 801 \, \frac{kg}{m^3}$

$$G = \frac{dp}{dz} = \rho\left(\frac{g}{g_c}\right)$$

$$= 50 \, \underbrace{\frac{1}{144}} \cdot \left(\frac{32}{32}\right)$$

$$= 0.347 \, \frac{psi}{ft}$$

Sensor Output G× psi/ft

$$G = \rho g$$

$$= 801(9.81)$$

$$= 7849 \, \frac{Pa}{m}$$

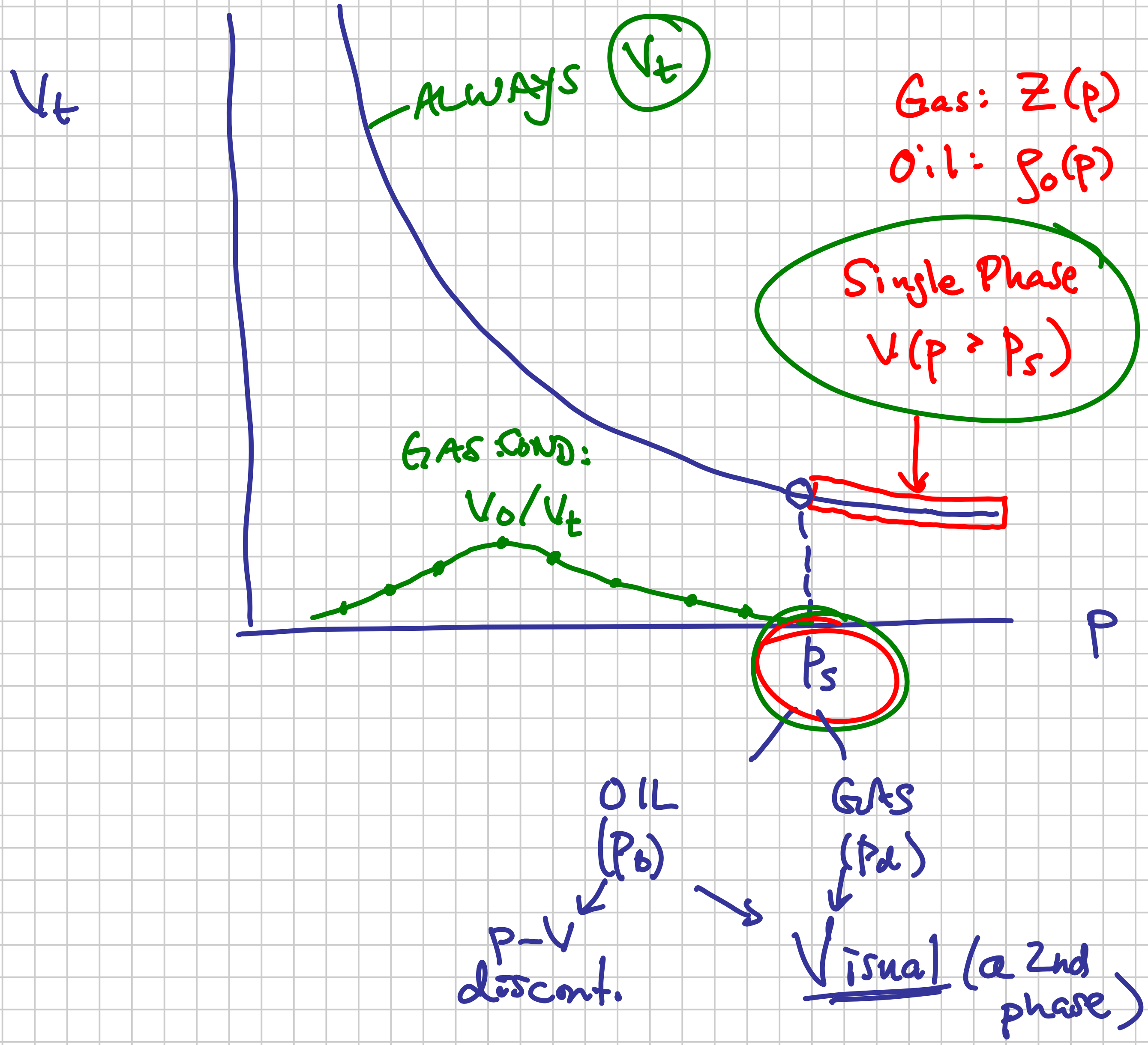
$$= 0.0785 \, \frac{\cancel{bar}}{\cancel{m}}$$

$$\times \frac{14.50377 \, psi}{\cancel{bar}}$$

$$\times \frac{\cancel{m}}{3.28 \, ft}$$

$$= 0.347 \, \frac{psi}{ft}$$

CCE:


V_t
ALWAYS V_t
Gas: Z(p)
Oil: ρ_o(p)
Single Phase
V(p > p_s)
GAS COND:
V_o/V_t
P
p_s
OIL
(p_b)
P-V
discont.
GAS
(p_d)
Visual (a 2nd phase)

DLE Test:

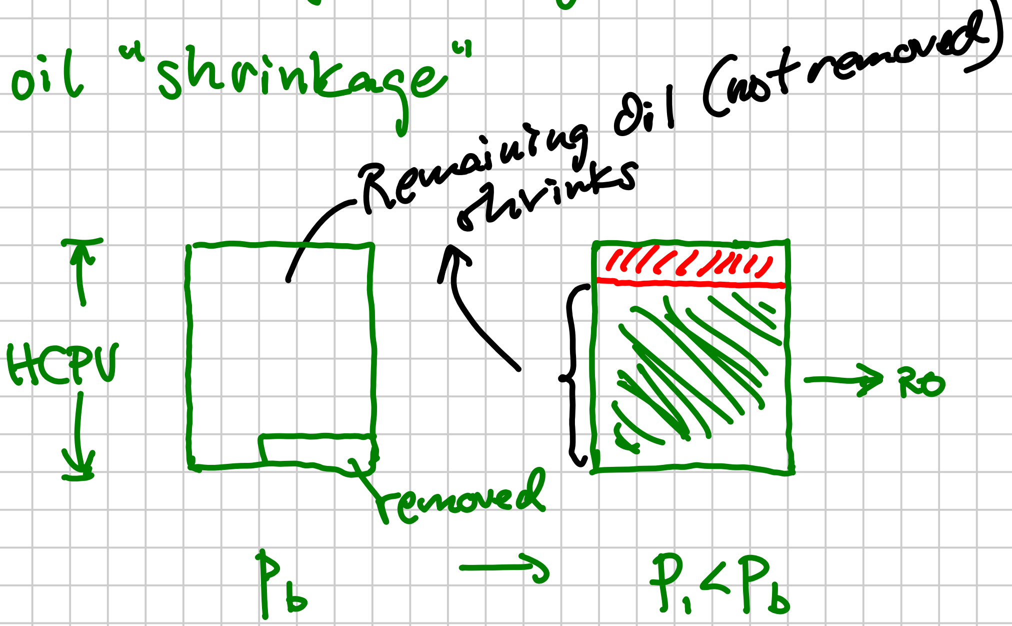
Gives a good description of phase INTRINSIC properties vs pressure

$$\left\{ \begin{matrix} y_i \checkmark & \\ \boxed{x_i} & \\ \rho_g \checkmark & Z_g \checkmark \\ \rho_o \checkmark & V_o/V_{ob} \checkmark \end{matrix} \right\}$$

Back-calculate from Mat. Bal. if needed.

OK by DLE

Also it gives a good estimate of oil "shrinkage"



$$\text{"Shrinkage"} = \left[\frac{V_o(?)}{V_{ob}}\right]_{DLE} \sim \left[\frac{V_o}{V_{ob}}\right]_{\text{Oil Not Produced (remaining)}}$$

$\underline{\underline{S_o(p)}}$ changes
- Take Out Oil
- Shrinkage $(V_o/V_{ob})_{DLE}$ $= 1 - S_w - \underset{\uparrow}{S_g}$

$S_{oi} = 1 - S_w$

$K_{rg}(S_o)$ $\uparrow$ Not Good $q_{RG} \uparrow$

$K_{ro}(S_o)$ $\downarrow$ $q_{RO} \downarrow$

$$R_{sd,k} = \frac{\overbrace{\overbrace{\left(\sum_{i=1}^{n} \Delta V_{gs}\right)}^{\text{Total Gas Removed}} - \overbrace{\left(\sum_{i=1}^{k} \Delta V_{gs}\right)}^{\text{Cum. Gas Removed to } p_k}}^{\text{Gas Remaining in Solution @ } p_k}}{\underbrace{V_{o,n}\,(T_{sc})}_{\text{"residual oil" @ } T_{sc},\, p_{sc}}} \qquad \frac{\text{scf}}{\text{bbl}}$$

NB! Not the same as a "surface processed" solution gas-oil ratio!

$$(R_{sd,b}) > R_{sb}$$

↑

Original BP Oil through a "surface sep. process."

100 bar, 80 °C

20 bar, 30 °C

1 bar, 15.5 °C

$$B_{od,k} \equiv \frac{\boxed{V_o}(p_k, T_R) \leftarrow \text{Shrinking oil}}{V_o(\underset{\uparrow \, p_{sc}}{p_n}, T_{sc}) \leftarrow \text{residual oil}}$$

$$B_{td} = \frac{V_o + \left(\sum \Delta V_{gs}\right) B_g}{\boxed{V_{o\,residual}}}$$

$$\boxed{B_{td} = B_{od} + (R_{sd,b} - R_{sd}) B_g}$$

@ p

$$B_g = \frac{\boxed{\Delta V_g}(p, T_R)}{\boxed{\Delta V_{gs}}(p_{sc}, T_{sc})} = \frac{p_{sc}}{T_{sc}} \cdot \frac{\boxed{z_s} T_R}{p}$$

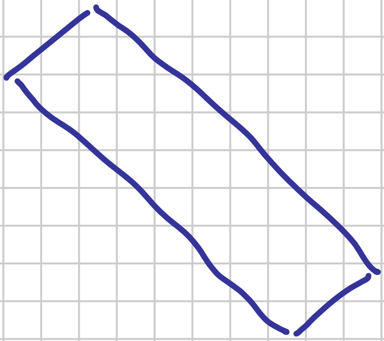
} assumes all RG becomes SG

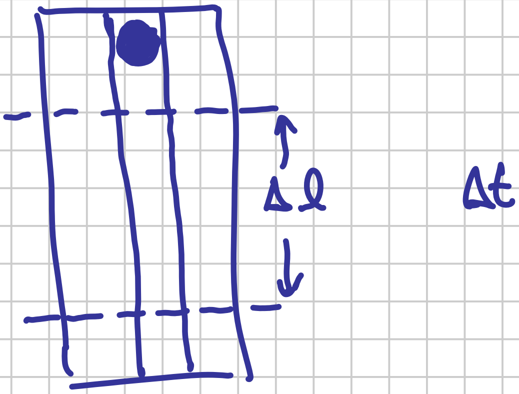
○ measured quantities

○ Calculated

$$\boxed{\rho_{o,k}} = \frac{\boxed{m_{o\,residual}} + \left(\sum_{i=n}^{k+1} \boxed{\Delta m_g}\right)}{\boxed{V_{o,k}}}$$

μ_o Rolling ball viscometer





$$\{\Delta t, \Delta l, \rho_o\} \Rightarrow \mu_o$$

Oil Shrinkage: $\dfrac{V_o}{V_{ob}} = \dfrac{B_{od}}{B_{od,b}}$

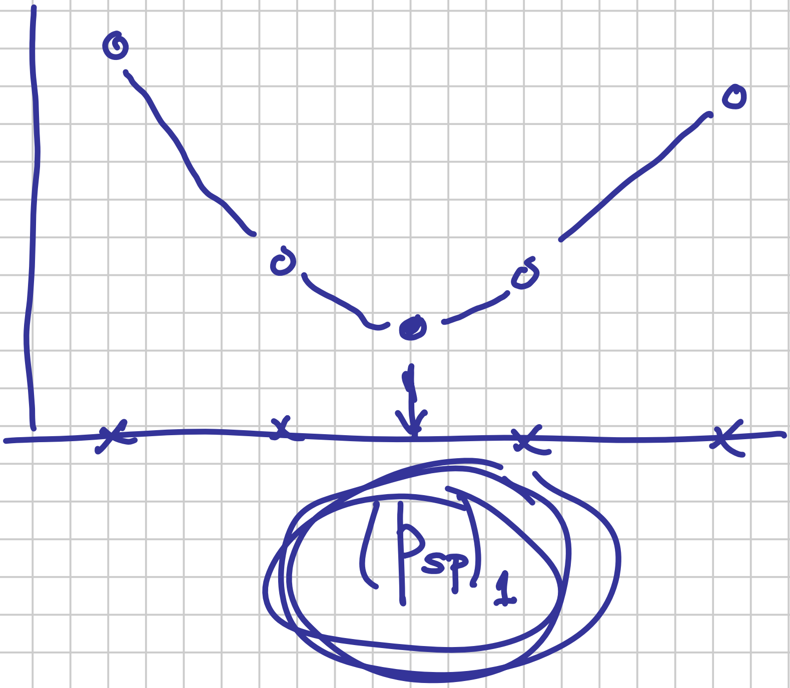
Surface Separation Process

Tries to maximize the total liquid product(s) : surface oil

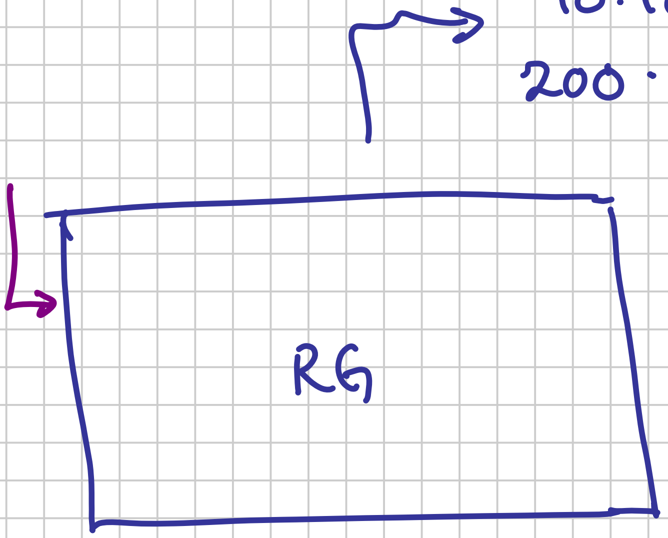
* Primary $(p_{sp}, T_{sp})_1$

* Processing Gases $\underset{L}{\Downarrow} C_{5+}\ C_4$ — R.E. don't understand

$$B_{ob} = \frac{\boxed{V_{ob}}}{\underset{\underset{\text{max}}{\uparrow}}{V_{\bar{o}}}}$$



Poor boy Process Optimization



$10 \cdot 10^{12}$ scf ($6 \cdot 10^{12}$ scf)

$200 \cdot 10^{6}$ STB ($50 \cdot 10^{6}$ STB)

30%

C_1

N_2

CO_2

$120 \cdot 10^{6}$ STB

10 mol% C_{6+}

Intensive Property

Independent of amount

ρ M x_i y_i

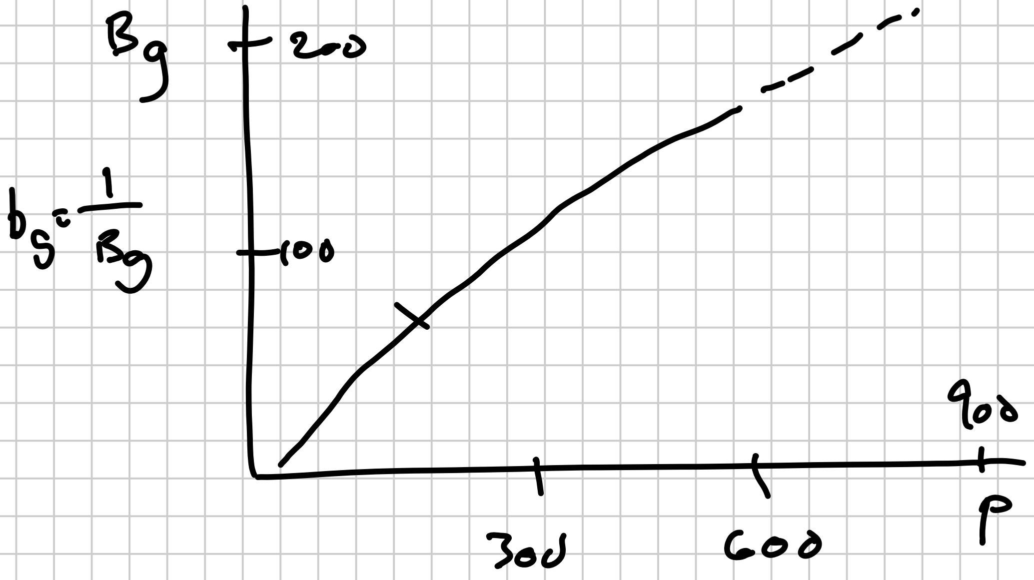
μ K_i

Extensive

Depends on the amount.

V m n

E



Never plot B_g vs p.

p.s. Why.

Curtis might

Fig. 1 – Inverse gas FVF (form...) as a function of pressure. b_g is $\propto$ with pressure, deviation fer that is caused by the Z(p).
b_g is order of magnitude 100–200 Sm³/Rm³.

$5 - 8 \cdot 10^9$ usd

50-60% RF oil

70 + RF gas

Gas, Water Inj
WAG Inj

$1,5 \cdot 10^9$ USD depletion w/ 3 wells

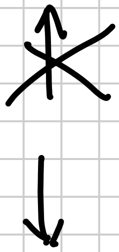
@ 10 years

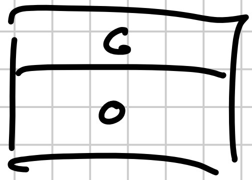
Opt. Depletion
$4 - 5 \cdot 10^9$ USD 2-wells
10

Depletion RF $= f(\bar{P}_R,$ abandonment$)$

Inj RF $\downarrow$ $= f($ No. Wells, Well Placement, $\underbrace{k(x,y,z)}$, . . . $)$

P_{Ri}:





Model: $300\cdot10^6$ STB
$1.2\cdot10^{12}$ scf

$\Rightarrow$ $P_{Ri} = 4000$ psia — PVT:

r_s	B_{gd}
R_s	B_o

✓ (1) RG : $r_{si} = 300\cdot10^6 / 1.2\cdot10^{12} \Rightarrow P_d$ ×

✓ (2) RO : $R_{si} = 1.2\cdot10^{12} / 300\cdot10^6 \Rightarrow P_b$ ×

✓ (3) RG + RO — $r_{si} = r_s(4000)$ B_{gi}
$R_{si} = R_s(4000)$ B_{oi}

HCPV : ✓ $IGIP = IGIP_{RG} + IGIP_{RO}$
✓ $IOIP = IOIP_{RG} + IOIP_{RO}$

$P_{ri} =$ $T_R =$ T_{pc} P_{pc}

Z

T_{pr} P_{pr}

$$\rho_g \longleftarrow \textcircled{Z}$$

$$\rho_g = \frac{pM}{ZRT}$$

Print the Data Book

Wed. 12-14 Q & A