FLUIDS & FLOW

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Arif
Olu (mide) } Stud. Assis.

Sissel - PhD stud. / assis.

W : 12 - 14 L (2 hr)

Th : 15 - 16 P Q&A

F : 8 - 10 L (1 hr)
P (1 hr)

P: Problem Development

~15

Economic Revenues from Oil and Gas

Surface

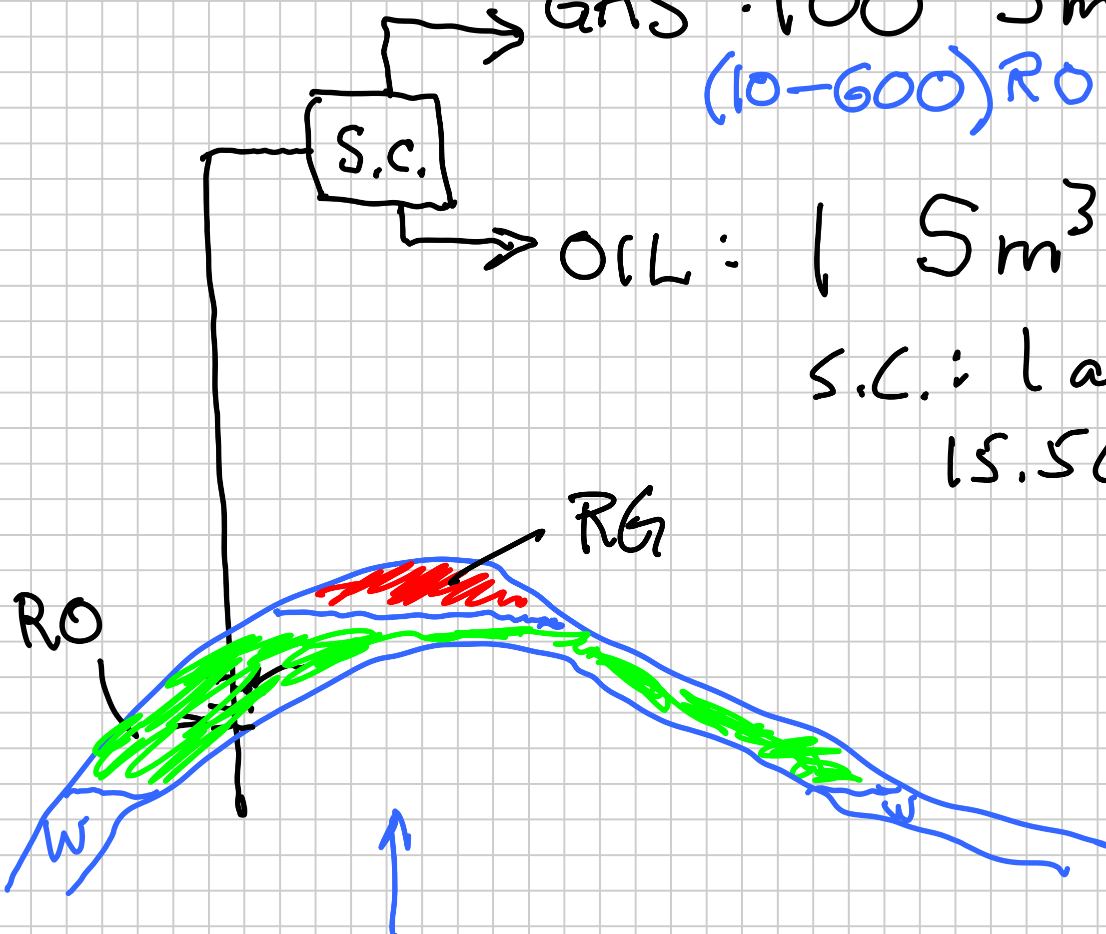
GAS : 100 Sm^3/d

(10–600) RO

OIL : 1 Sm^3/d

s.c. : 1 atm

15.56 °C



"barrel"

Prices: \$70 / bbl

NOK 6.5 / USD

\$8 / M scf

M: 1000

scf: standard ft^3

$6.28 \frac{bbl}{m^3}$

$35.31 \frac{ft^3}{m^3}$

\$/d are being produced?

\$/yr vs \$75,000

Solution:

$1\ m^3$ oil $= 6.28$ bbl

$\times 70$ \$/bbl

$=$ \$420

$100\ m^3$ gas $= 3531\ ft^3$ (scf)

$\simeq 3.531$ Mscf

$\times$ \$8/Mscf

$=$ \$28

Total = \$450/d

= \$160,000/yr

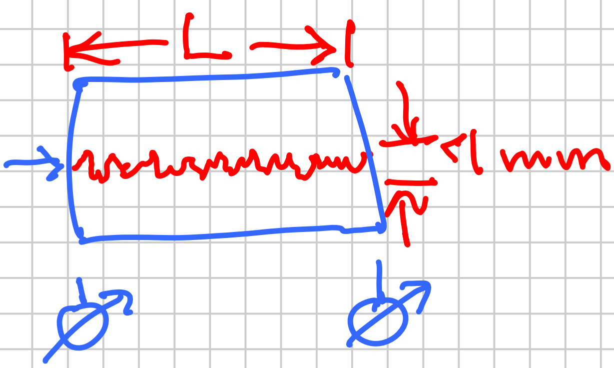
= NOK 1 000 000

How to calculate oil and gas rates from a well?

(1) Darcy's Law

$$v = \frac{k}{\mu}\cdot\frac{\Delta p}{\Delta L}$$

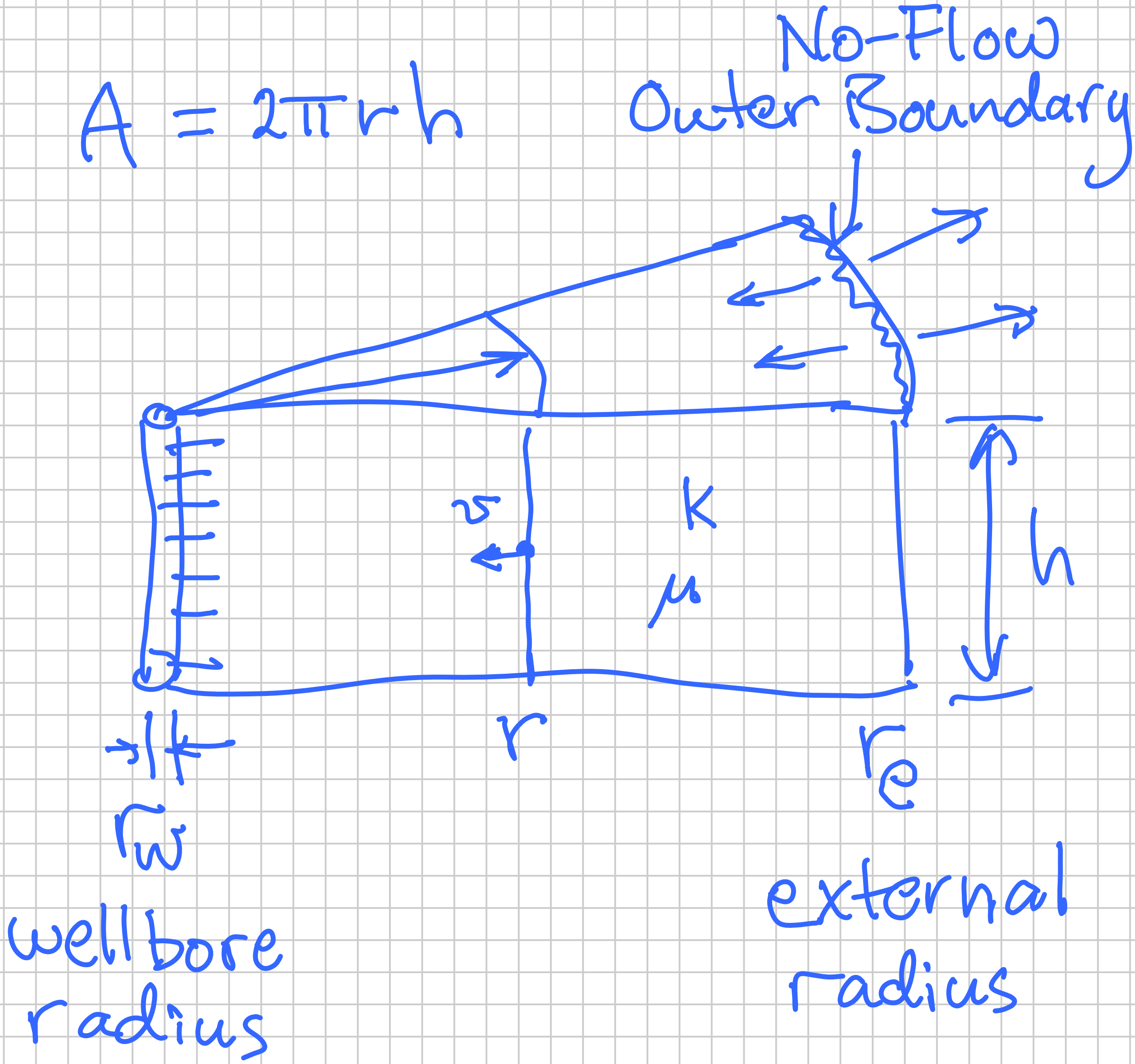
$$q\left(\frac{Sm^3}{d}\right) = \underbrace{v \cdot A}_{\text{rate at reservoir conditions}} \cdot SF$$



$$P_{in} - P_{out} = \Delta p$$

$$SF = \frac{1}{B} = \frac{\text{volume surface oil}}{\text{volume reserv. oil}}$$

$A = 2\pi r h$



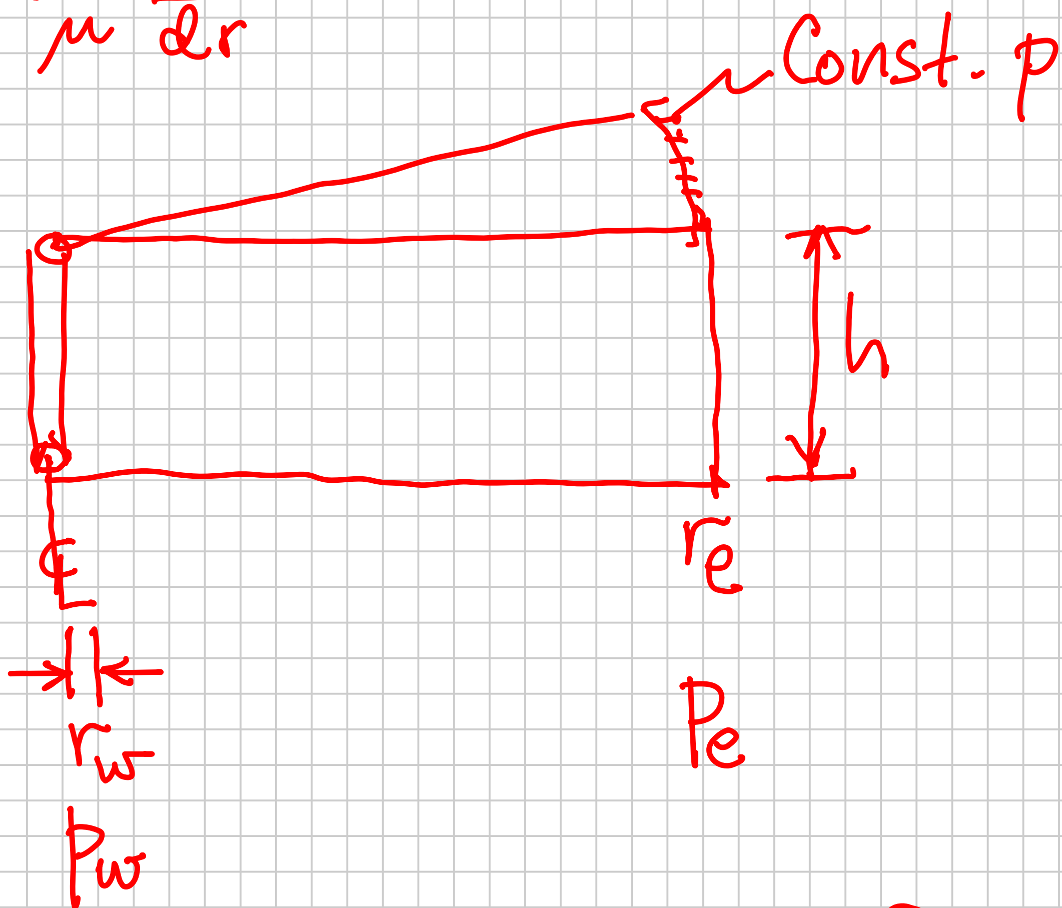
p_w

p_e

$$q_o = f(k, \mu, r_e, r_w, p_e, p_w, h)$$

Darcy's Law

$$v = \frac{k}{\mu}\frac{dp}{dr}$$



area ⊥ to flow

$$q_{fR} = v \cdot A$$ @ reservoir conditions

$$A = 2\pi r h$$

k, μ are constants

Shrinkage: $\frac{1}{B_o} = 0.3 - 0.98$

$$q_o = \underbrace{q_{oR}}_{\text{Darcy}} \cdot \frac{1}{B_o}$$

Assume Constant

Solution:

$$v = \frac{k}{\mu}\frac{dp}{dr}$$

$r = r_w \,,\; p = p_w$

$r = r_e \,,\; p = p_e$

Separate variables p & r, collect constants,

$$q = 2\pi r h v \quad = \quad \text{constant}$$

$$q = 2\pi r h \frac{k}{\mu}\frac{dp}{dr}$$

$$\int_{r_w}^{r_e} \frac{1}{r}\, dr = \frac{2\pi k h}{\mu q}\int_{p_w}^{p_e} dp$$

$$\ln \frac{r_e}{r_w} = \frac{2\pi k h}{\mu q} \cdot (p_e - p_w)$$

$$q = \frac{2\pi k h}{\mu \cdot \ln r_e/r_w} (p_e - p_w)$$

Reservoir rate,

convert using $\frac{1}{B_o}$

Surface Oil

$$q_o = q_{oR} \cdot \frac{1}{B_o}$$

$$\boxed{q_o = \frac{2\pi k h (p_e - p_w) \, b_o}{\mu_o \cdot \ln r_e/r_w}}$$

Rate Equation

$$b_o = \frac{1}{B_o}$$

Parameter (units)	Range	Orders of Magnitude
k (md)	$0.01 - 10^4$	[6]
h (m)	$1 - 10^3$	[3]
μ_o (mPa.s)	$0.1 - 100$ (00)	[4]
B_o (m^3/Sm^3)	$1 - 3$	[0.3]
$\ln r_e/r_w$ *	$7 - 9$	[0.1]
p_e (bar)	$100^{(-)} - 1000$	Δp [3]
p_w (bar) *	10	

* We control.

Units:

$1000\ md = 1\ D$

$1\ D \cong 10^{-12}\ m^2$

$10^3\ mPa.s = Pa{\cdot}s$

$10^5\ Pa = 1\ bar^*$

$60 \times 60 \times 24\ s = 1\ d$

d'Arcy

atm

cm

cp

$$v = \frac{k}{\mu}\frac{dp}{dx}$$

$$\left(\frac{cm}{s}\right) \quad \frac{(D)}{(cp)} \cdot \frac{(atm)}{(cm)}$$

absolute

* bara

barg

gauge

difference is local atmospheric pressure

~ 1 atm

Volumes & rates

m^3

Sm^3 std m^3 m^3 at standard conditions

1 atm

15.56 °C

ft^3

scf

Gas standard ft^3

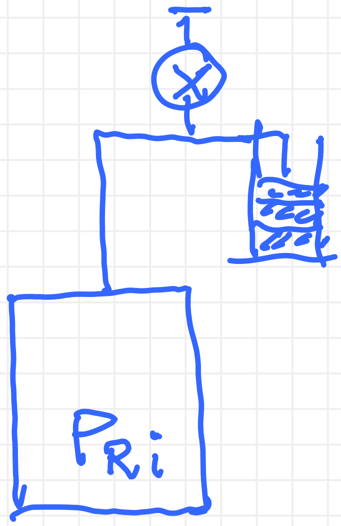
bbl

STB Oil & Water stock-tank bbl (standard)

RB "reservoir" bbl

$$\ln \frac{r}{r_w} = \frac{2\pi k h b}{\mu q} \cdot (p - p_w)$$

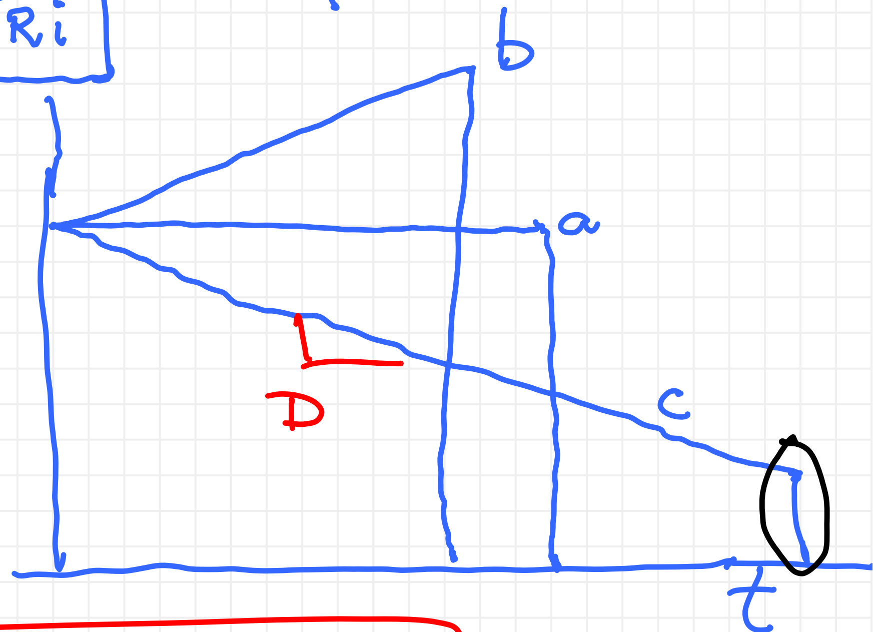
$$p(r) = p_w + \frac{\mu_o q_o}{2\pi k h b_o} \cdot \ln \frac{r}{r_w}$$



1 atm

$$q = C(P_R - P_{sc})$$

log $q_o(t)$



$$\boxed{q_o = q_{oi} \cdot e^{-D \cdot t}}$$

kh
P_{Ri}
μ_o
b_o
P_w

?
10IP
Recovery
Factor
"C_o"

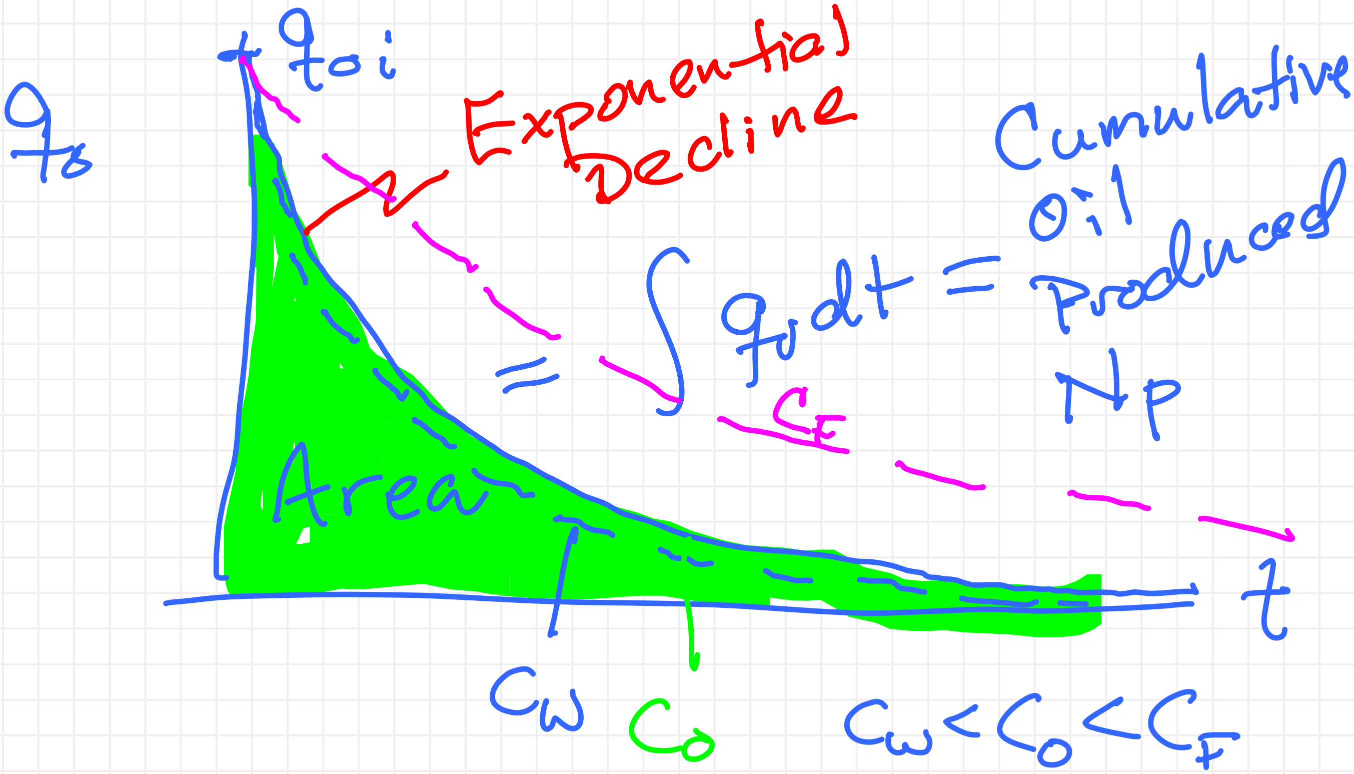
Prob. 1

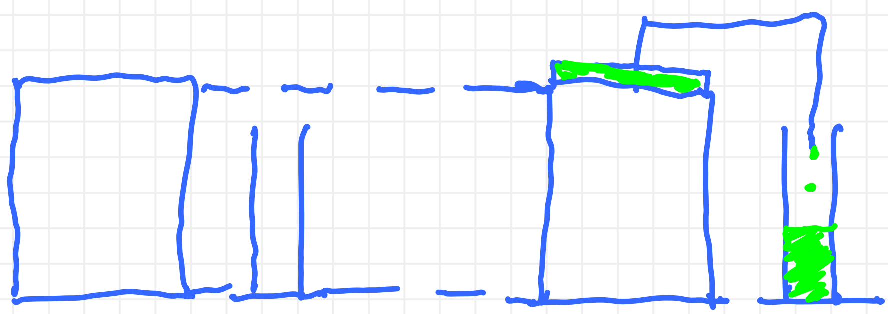
(F)

$$q_o = \alpha \frac{kh(P_e - P_w)}{\mu_o B_o \cdot \ln r_e/r_w}$$

const?

$$q_o\left(\frac{Sm^3}{d}\right) = \alpha \frac{k(md)\, h(m)\, \Delta p(bar)}{\mu(cp) \cdot B_o \cdot \ln r_e/r_w}$$





10 L
900 bara

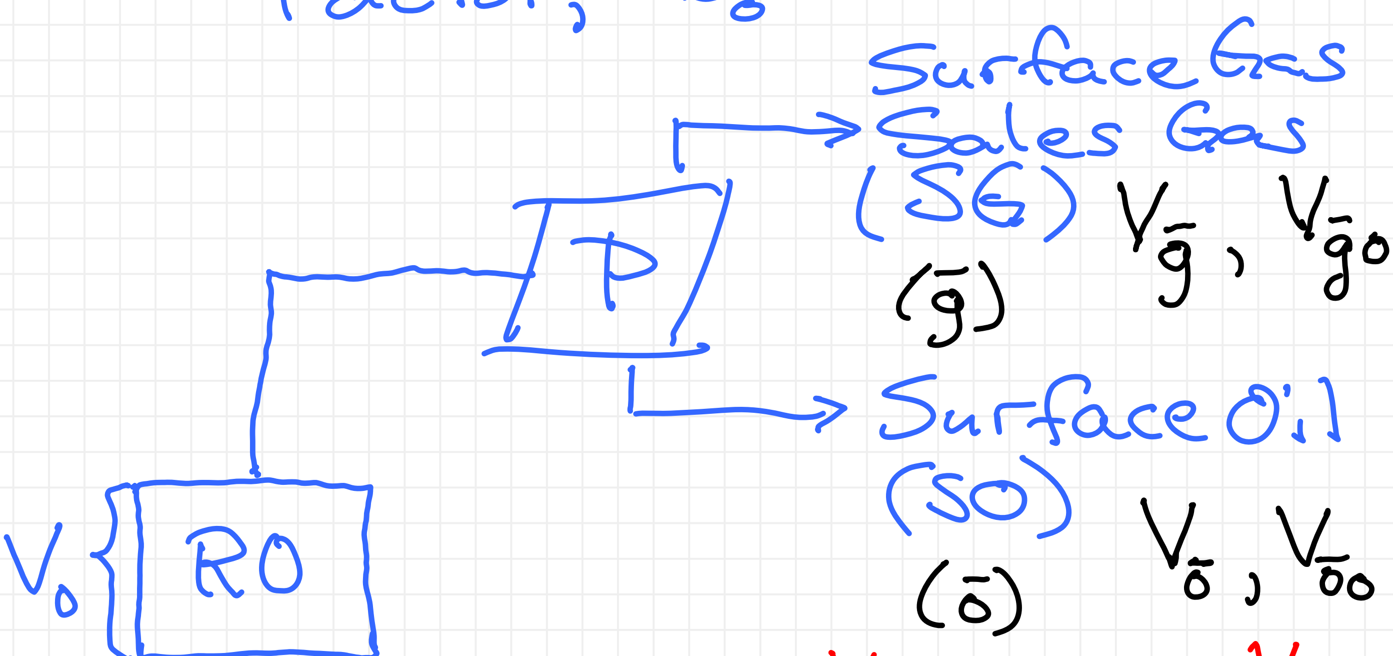
10 L
1 bara

$$\boxed{c = \frac{1}{V}\frac{dV}{dp}} \leftarrow f(\text{fluid})$$

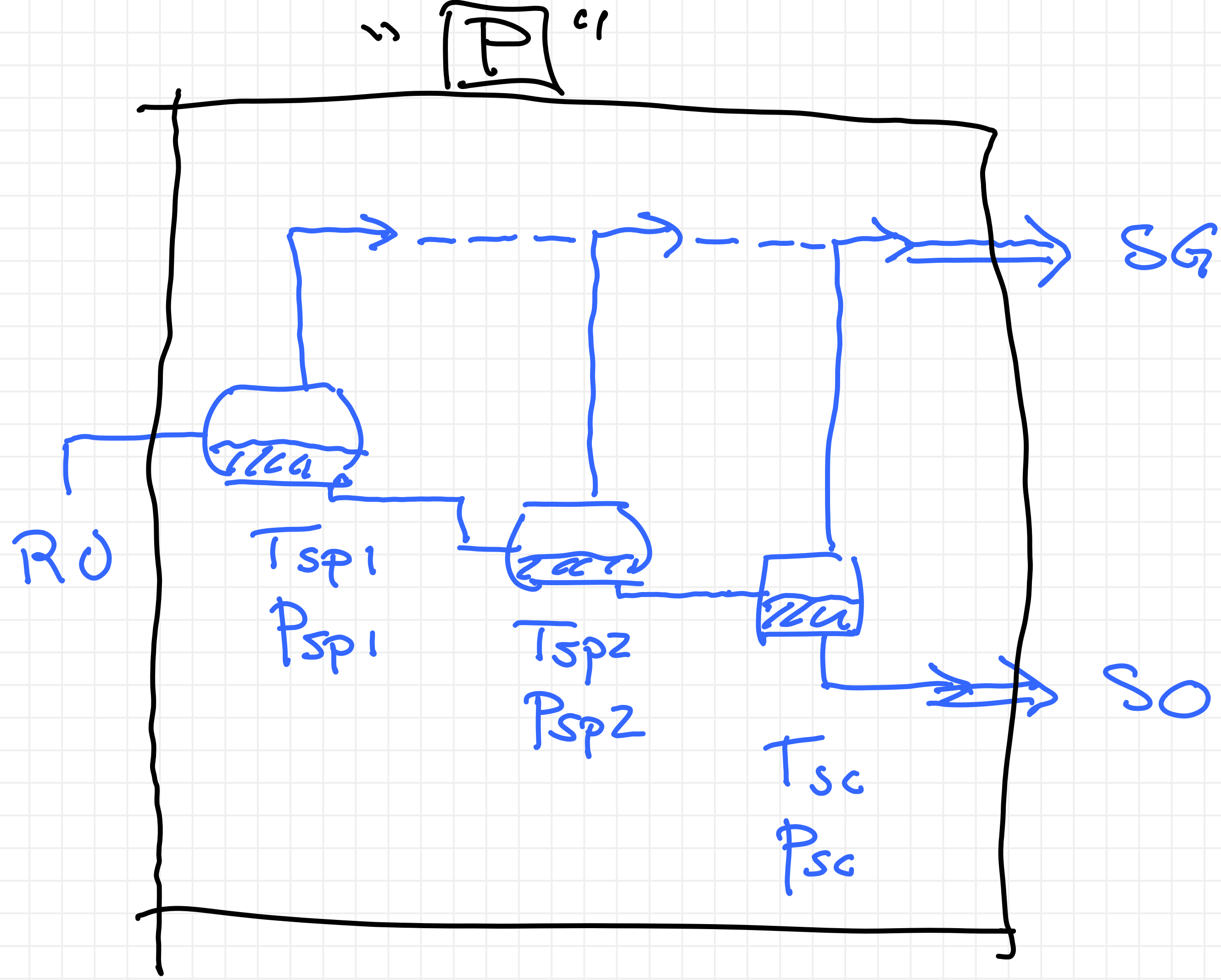
RESERVOIR OIL (RO)

PHYSICAL PROPERTIES

- Oil Density, ρ_o
- Oil Compressibility, c_o
- Solution Gas-Oil Ratio, R_s
- Oil Formation Volume Factor, B_o



$$R_s \equiv \frac{V_{\bar{g}o}}{V_{\bar{o}o}}, \quad B_o \equiv \frac{V_o}{V_{\bar{o}o}}$$



Oil Density

$$\rho_o \equiv \frac{m_o}{V_o} = \frac{m_{\bar{o}o} + m_{\bar{g}o}}{V_o}$$

$$\rho_{\bar{o}o} = \frac{m_{\bar{o}o}}{V_{\bar{o}o}} \qquad \rho_{\bar{g}o} = \frac{m_{\bar{g}o}}{V_{\bar{g}o}}$$

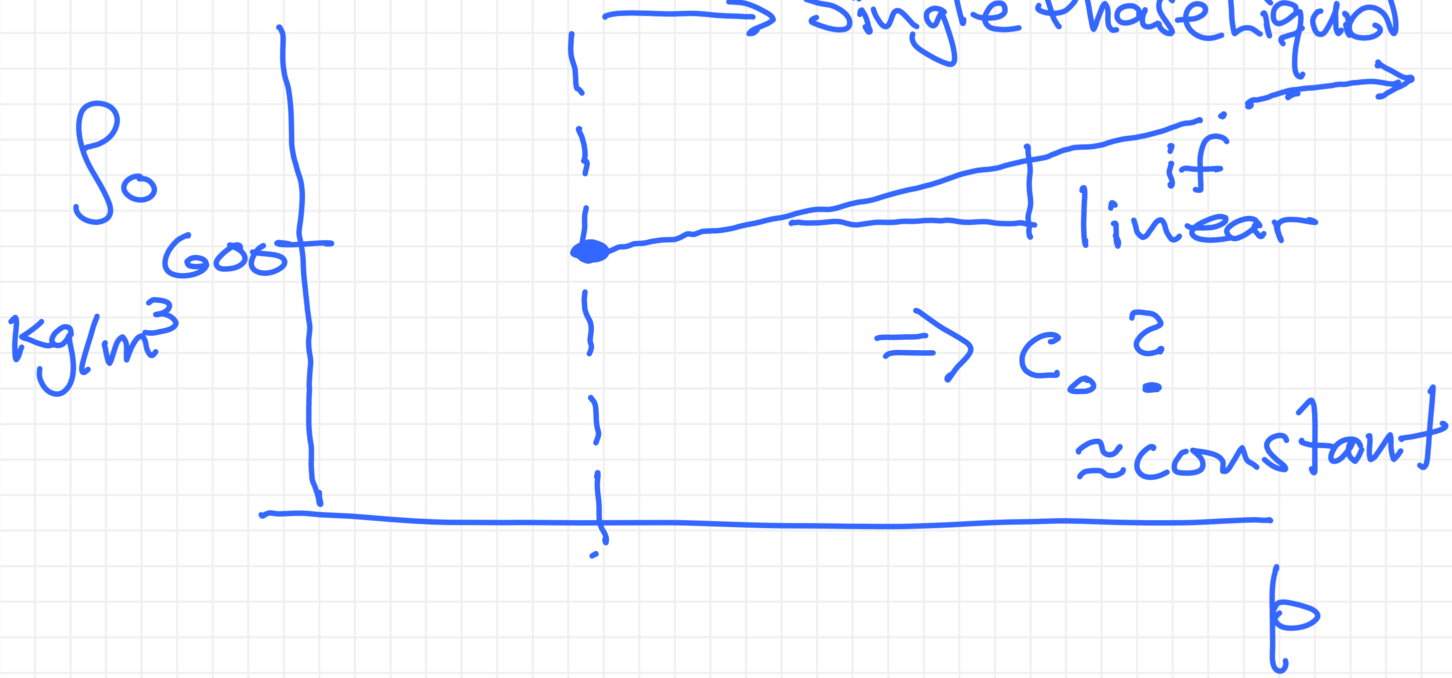
As engineers we usually "know" (have a table) of

p	B_o	R_s
⋮	⋮	⋮

and $\rho_{\bar{o}o}$, $\rho_{\bar{g}o}$

DERIVE by Wed.

$$\rho_o = f(B_o, R_s, \rho_{\bar{o}o}, \rho_{\bar{g}o})$$



$$c_0 = -\frac{1}{V_0}\left(\frac{dV_0}{dp}\right)_{T,m_0}$$

$$V_0 = m_0 / \rho_0$$

$$= \underbrace{m_0}_{\text{const}} \cdot \frac{1}{\rho_0} \Rightarrow \rho_0 = m_0 \frac{1}{V_0}$$

$$\frac{d\rho_0}{dp} = m_0 \cdot (-1) V_0^{-2}$$

$$\boxed{c_0 = \frac{1}{\rho_0}\left(\frac{d\rho_0}{dp}\right)}$$ Derive

Constant c_0

$\Rightarrow$ B_0 is linear in p

because

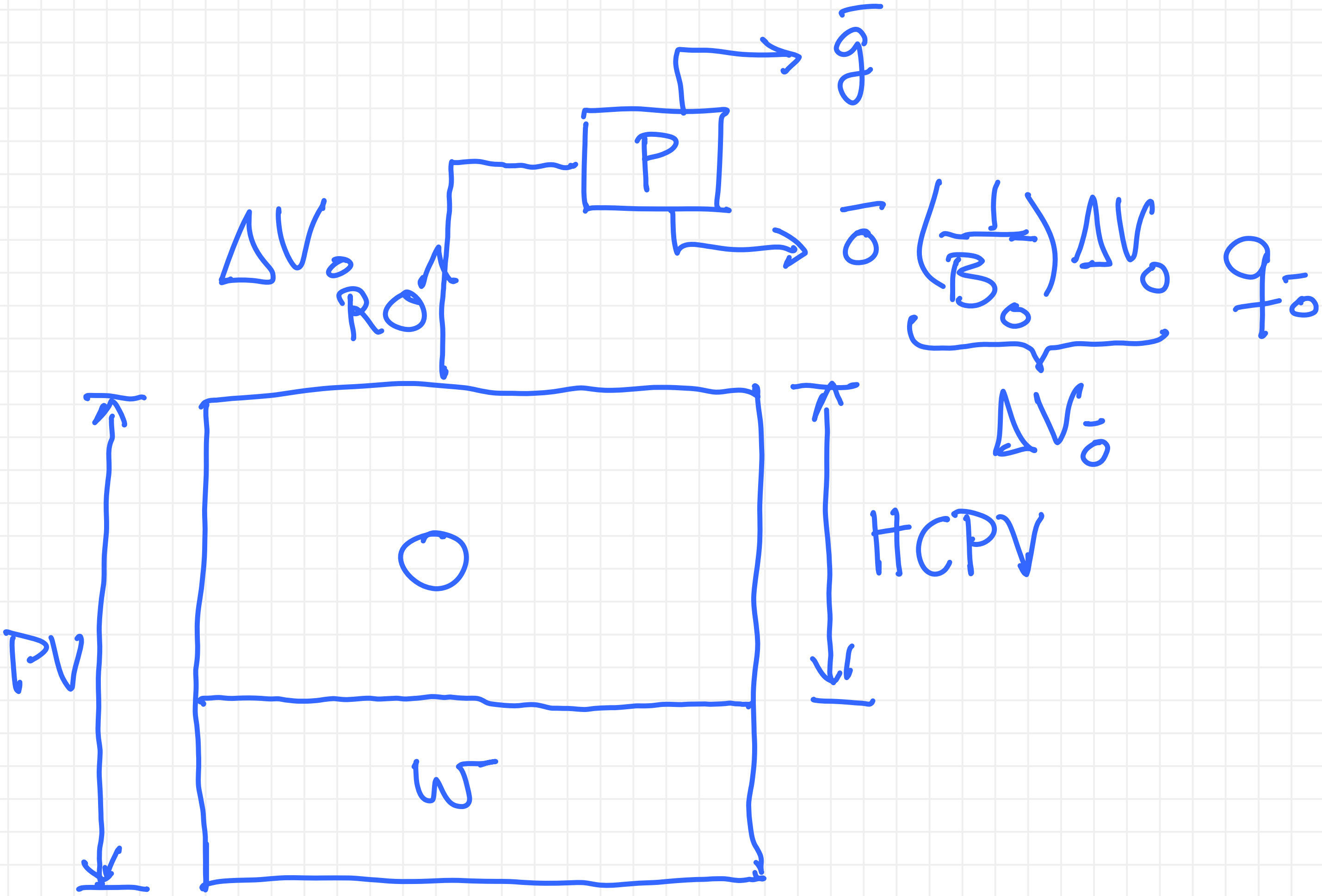
$$\boxed{c_0 = \frac{1}{B_0}\left(\frac{dB_0}{dp}\right)}$$ Derive

PROBLEM 2.

Pressure depletion of an oil reservoir.

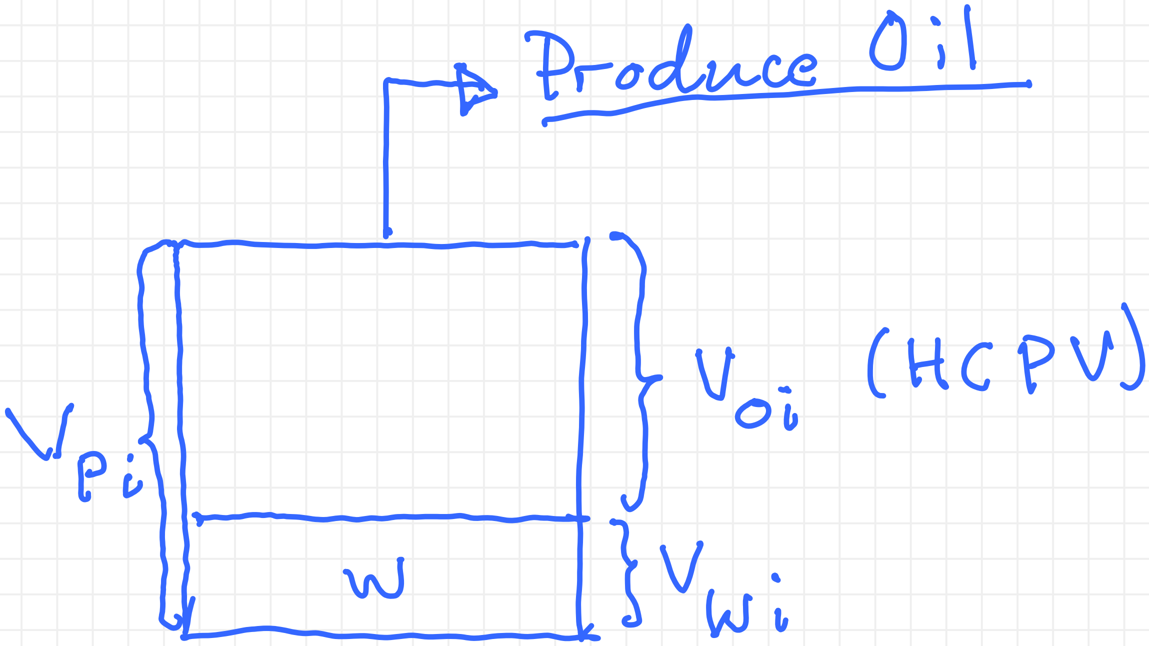
- Rate Eq. from Prob. 1
- Assume constant compressibilities
  - oil
  - water
  - pore

optional



P_{Ri}

$\left.\begin{array}{l} c_o \\ c_w \\ c_f \text{ (pore)} \end{array}\right\}$ compressibilities



$$c = -\frac{1}{V}\left(\frac{dV}{dp}\right)_{T,m}$$

P_{Ri} $\quad V_{pi} = V_{wi} + V_{oi}$

P_R $\quad V_p = V_w + V_o$ $\quad$ *

$\rightarrow$ c = constant,
Know V_i
Solve for V at p

$$c\int_{p_i}^{p} dp = -\int_{V_i}^{V} \frac{1}{V}\, dV$$

$$c(p-p_i) = -\ln\frac{V}{V_i} = \ln\frac{V_i}{V}$$

$$V = V_i \exp\left[-c\,(p-p_i)\right]$$

$$\boxed{V = V_i \cdot \exp\left[c(p_i-p)\right]}$$

* Water & Oil expand as p drops

* Pore volume contracts as p drops

$$V_o = V_{oi} \cdot \exp\left[c_o(p_i-p)\right]$$

$$V_w = V_{wi} \cdot \exp\left[c_w(p_i - p)\right]$$

$$V_p = V_{pi} \cdot \exp\left[-c_f(p_i - p)\right]$$

Need:

p_{Ri}, V_{pi}, V_{wi} (or V_{oi})

c_f c_w c_o

B_{oi}

Convert ΔV_o produced (to honor $V_p = V_w + V_o$) to surface $\Delta V_{\bar{o}}$, use B_o at current pressure.

$$c_o = -\frac{1}{B_o}\left(\frac{dB_o}{dp}\right)$$

$$\boxed{B_o = B_{oi} \exp\left[c_o(p_i - p)\right]}$$

$$\Delta V_o = (V_w + V_o) - V_p$$

$$\Delta V_{\bar{o}} = \frac{\Delta V_o}{B_o}$$

$$q_{\bar{o}} = \text{Rate Eq.}$$

$$\Delta t = \frac{\Delta V_{\bar{o}}}{q_{\bar{o}}}$$

Rate Eq. $q_o - p_R$

Material Balance (Volumetric)

$N_p - p_R$

Single Phase

Oil

$q_o(t)$

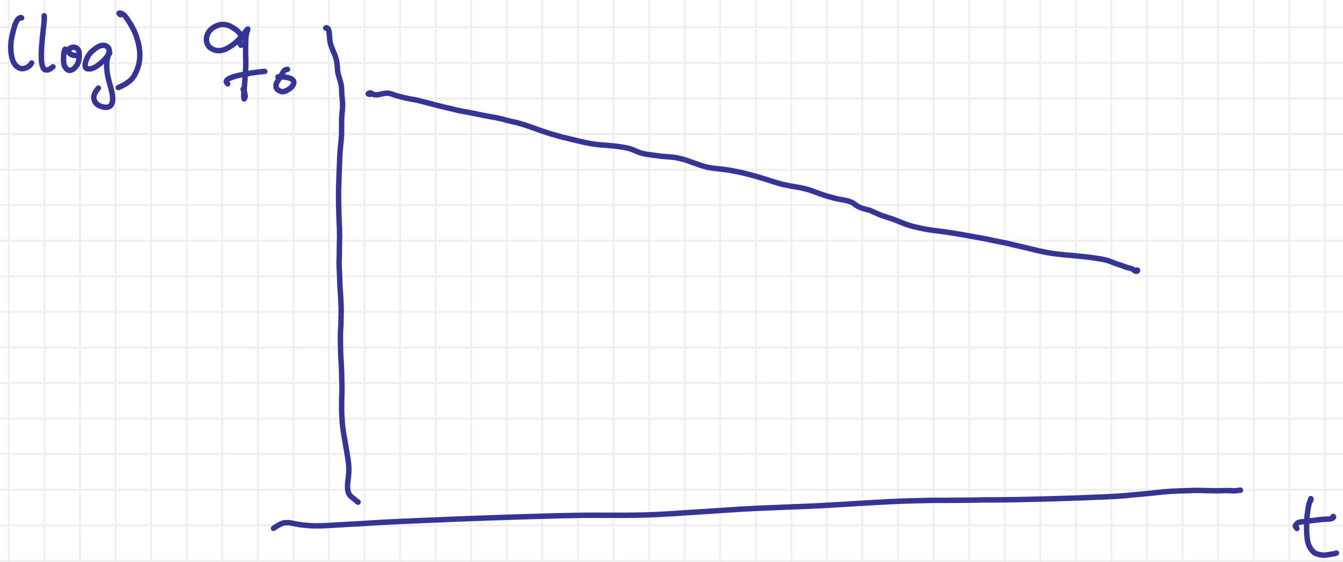
$N_p(t) = \int q \, dt$

$\bar{p}_R(t)$

$$\frac{dN_p}{dt} = q$$

Found from Solution

Problem 2

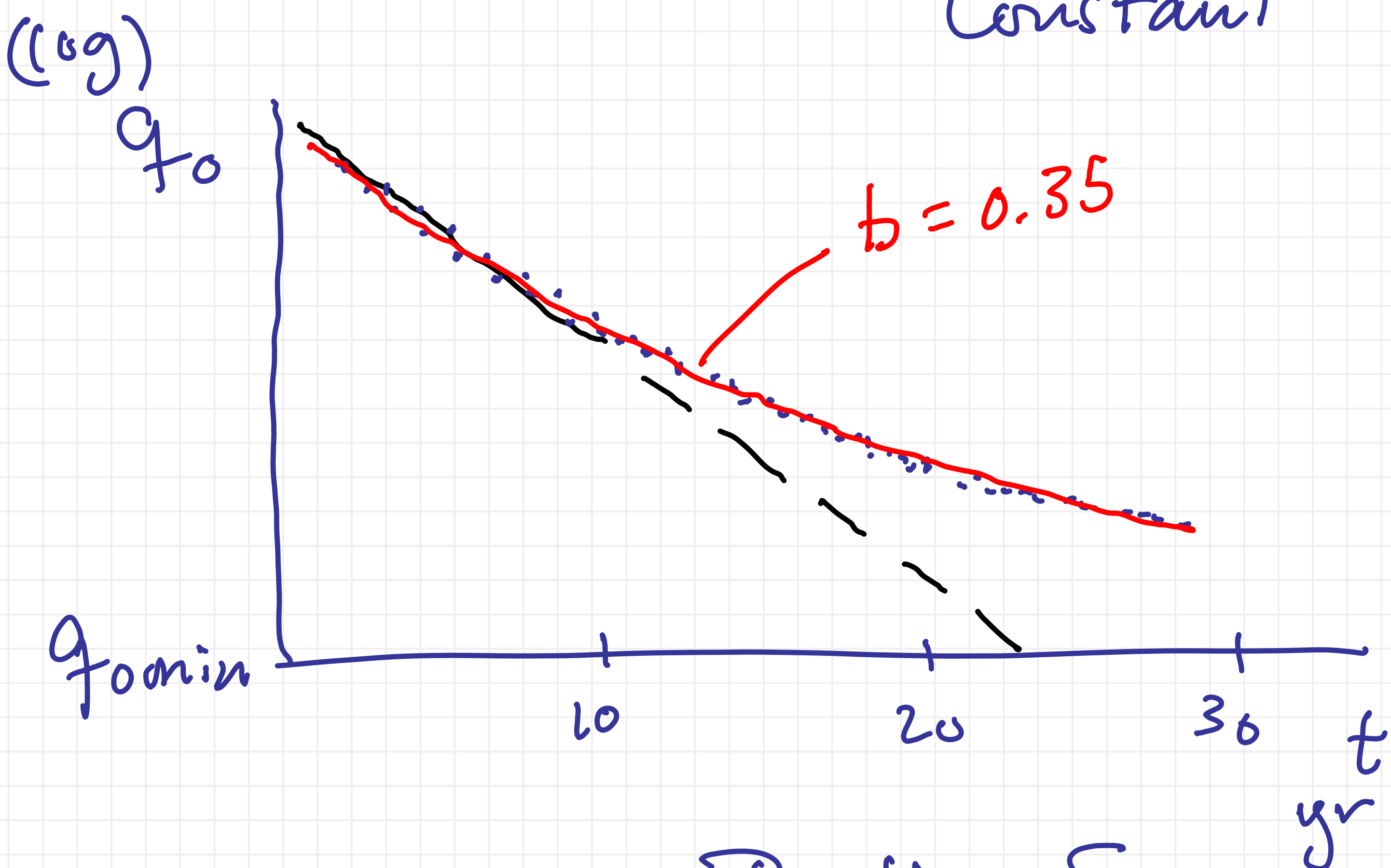


Exponential Decline

$$q_o = q_{oi} \cdot \exp[-Dt]$$

Decline Constant

Arps



General Arps Decline Eq.

Hyperbolic Decline

$$\boxed{q = \frac{q_i}{[1 + bDt]^{1/b}}}$$

$0 < b < 0.5$ Single Layer Production

$b = 0$: exponential
$\in$

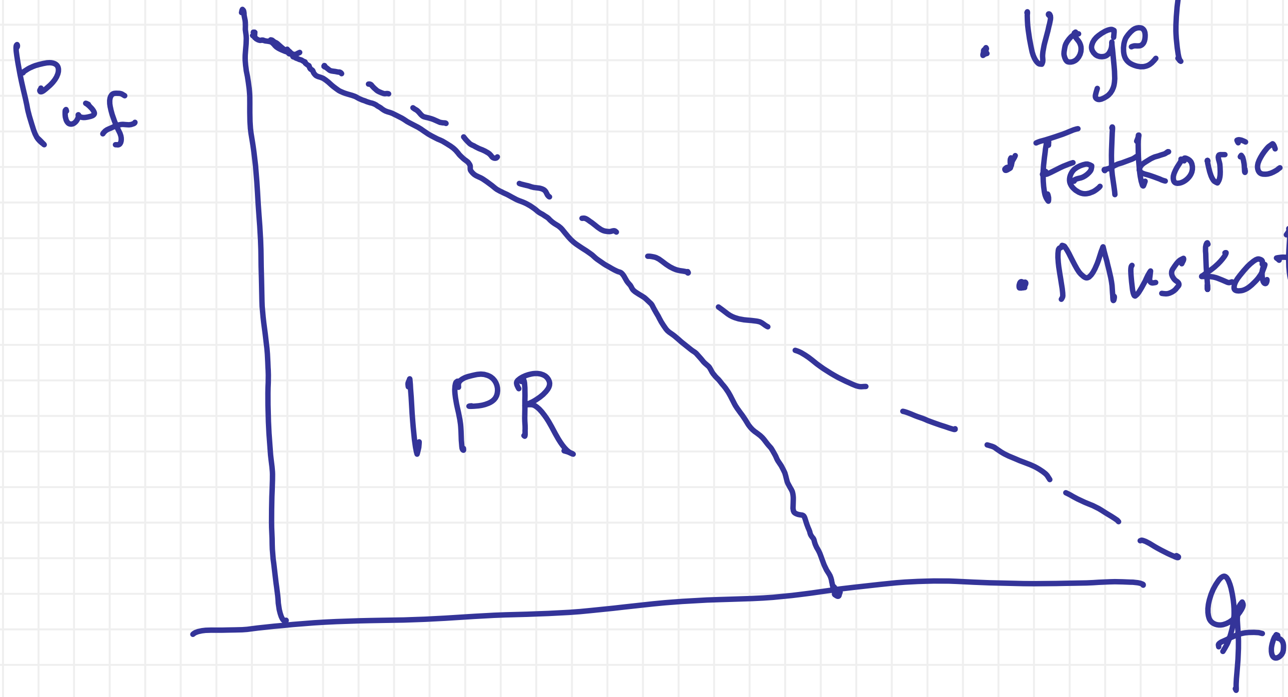
$b > 0$: • gas
(Depletion) • solution-gas drive SGD
($p_R < p_b$)
• water drive

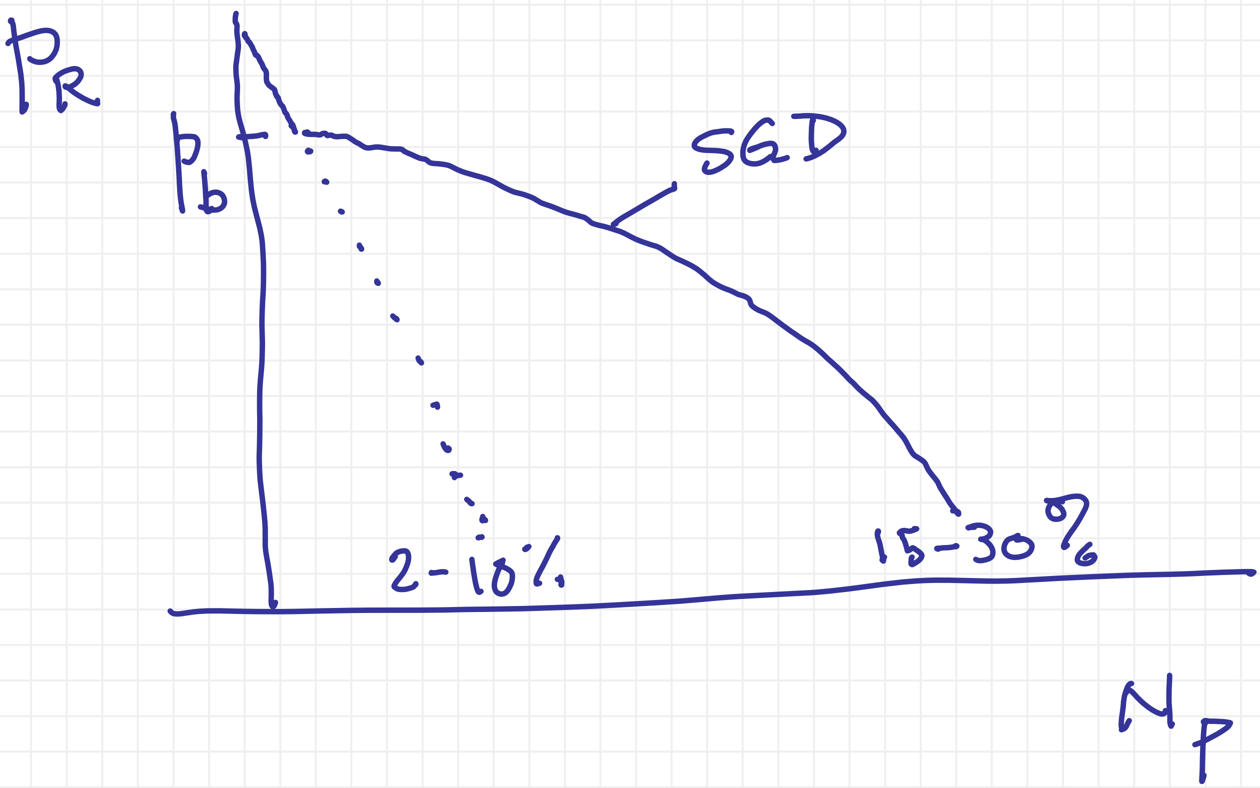
$\underline{b>0}$

$$\left.\begin{array}{l} \text{• Rate Eq. not linear} \\ \quad q(p_{wf}) \\ \quad \underline{\text{and/or}} \\ \text{• } N_p(p_R) \text{ not linear} \end{array}\right\} \text{cause } b>0$$

SGD

- Vogel
- Fetkovich
- Muskat





$b > 0 \Rightarrow$ Recovery Factor Increasing

b	$(N_{p\,ult}/N) = RF_{ult}$
0	10%
⋮	↓
0.5	35%

N = Initial Oil in Place (IOIP)

Problem 2 "Hints"

N = IOIP = Initial (Surface) Oil in Place

STOIIP = Stock-Tank Oil Initially in Place

$$[Sm^3]\ N = \frac{V_{oi}\ [m^3]}{B_{oi}\ [m^3/Sm^3]}$$

G = IGIP = Initial (Surface) Gas in Place

$$= N \cdot R_{si}$$

Recovery Factor RF

$$\text{(Surface) Oil} = RF_{\bar{o}} = \frac{N_p}{N}$$

$$GOR \equiv \frac{q_{\bar{g}}}{q_{\bar{o}}}$$

Problem 2 ($p_R > p_b$)

$$GOR = R_{si} = \text{constant}$$

$$\therefore \; q_{\bar{g}} = q_{\bar{o}} \cdot R_{si}$$

G_p = cumulative (surface) gas produced

$$= \int q_{\bar{g}} \, dt = \sum q_{\bar{o}} \cdot R_{si} \cdot \Delta t$$

$$= R_{si} \cdot \underbrace{\sum \Delta N_p}_{N_p}$$

$$= R_{si} \cdot N_p$$

$$RF_g = \frac{G_p}{G} = \frac{R_{si} \cdot N_p}{R_{si} \cdot N}$$

$$= \frac{N_p}{N} = RF_o$$

Solver in Excel to Solve the Non-Linear Regression Problem

$$\min \sum_{i=1}^{N} \underbrace{\left(\frac{q_{oArps} - q_{oProb.2}}{q_{oi}} \right)^2}_{\text{Residual}}$$

minimize sum (of squares)

SSQ

by changing

q_{oi}, b, D

with constraints

$q_{oi} > 0$

$D > 0$

$\epsilon < b < 0.5$

CHW

$$D = \left(\frac{1}{1-b}\right)\frac{q_i}{(N_p/N)_{\underbrace{ult}_{q=0}}}$$

Check if this is right, using results from Problems 2 & 3.

Problems 2 & 3 <u>DUE</u> Friday Sept. <u>16</u>

Gas Properties

Ideal Gas Law

Absolute Units (K or °R)

"PVT"

$$pV = nRT$$

8.314

Absolute Pressure

moles

{mol, gmol} — (g)

{kmol, kg-mol} — (kg)

lb-mol — (lb)

R depends on the units being used

{psia, ft^3, lb-mol, °R}

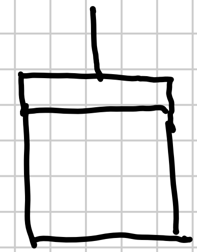
R = 10.7315

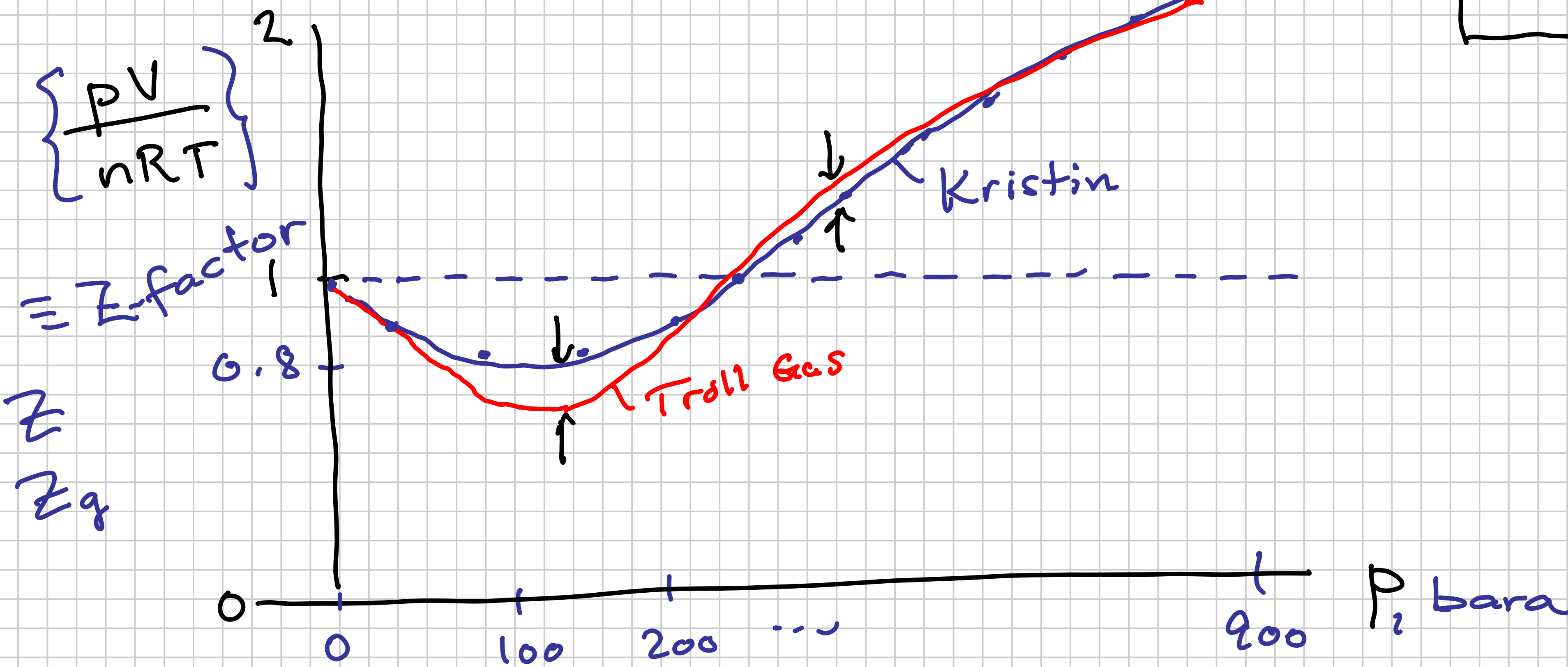
{Pa, m^3, gmol, K}

R = 8.314 ?

"Real" Gas Law
(non-ideal)

$$\frac{pV}{nRT} = 1 \quad \text{Ideal Gas}$$





$Z(p, T, \text{composition})$

Range $0.7 - 2^{+}$

Real Gas Law

$$pV = nRT \cdot Z_{(p,T,n_i)} \qquad i = C_1, C_2, N_2, CO_2, \ldots$$

Gas Properties: ρ_g, G_g, c_g, B_g

Surface Molar Gas Volume

$$\frac{V_{\bar{g}}}{n} = \frac{V_{gsc}}{n} = \frac{T_{sc} R}{p_{sc}}.$$

$T_{sc} = 15.56\ °C \quad (60\ °F)$

$p_{sc} = 1\ \text{atm} = 1.0135\ \text{bara} = 14.696\ \text{psia}$

$\{ \text{kg-mol}, m^3, K, \text{bara} \}$ $R = 0.08314$

$$\frac{V_{sc}}{n} = \frac{(15.56 + 273.15)(0.08314)}{1.0135}$$

$$= 23.68 \ \ Sm^3/kmol$$

$$= 379.15 \ \ scf/lb\text{-mol}$$

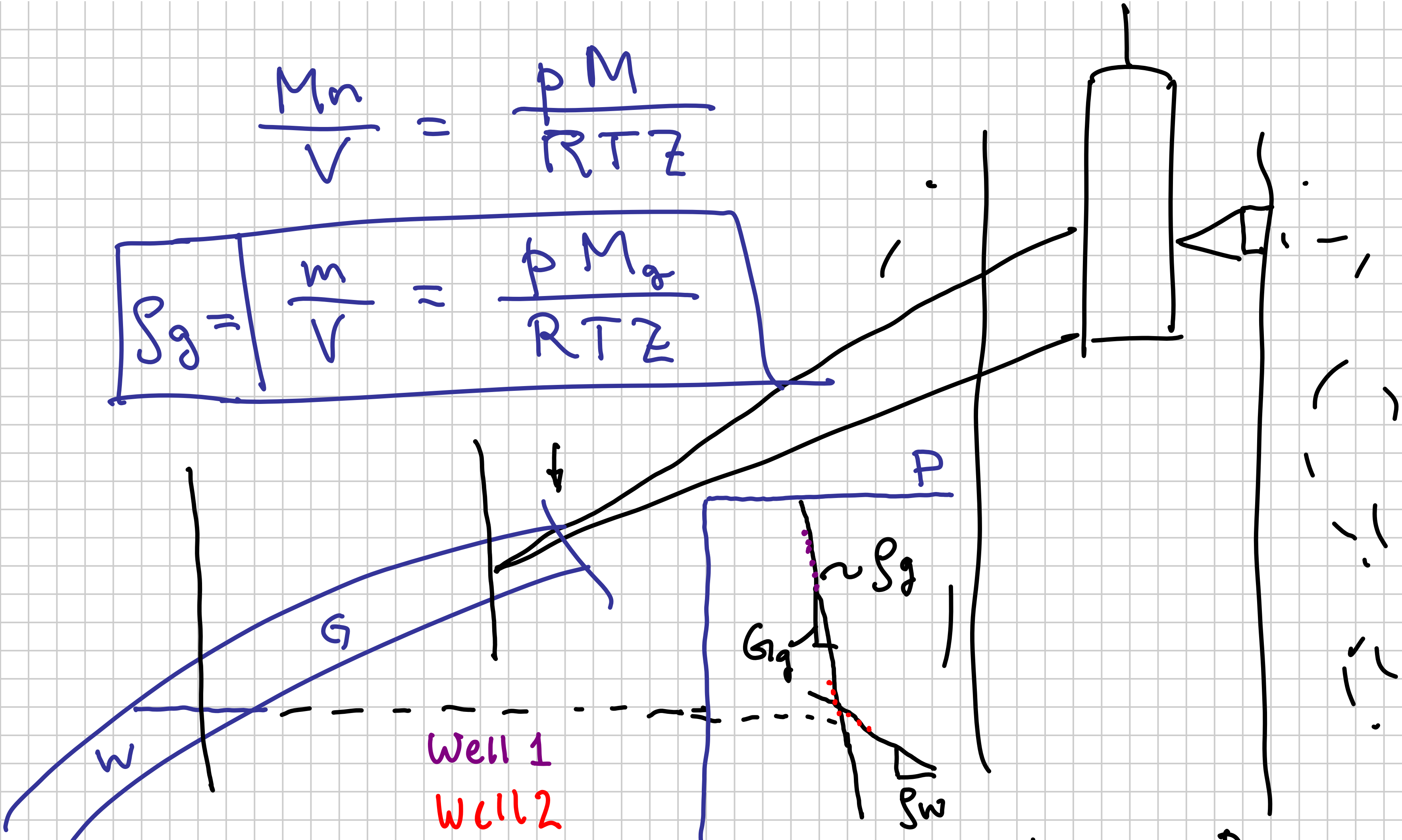
$$\rho_g = \frac{m_g}{V_g}$$

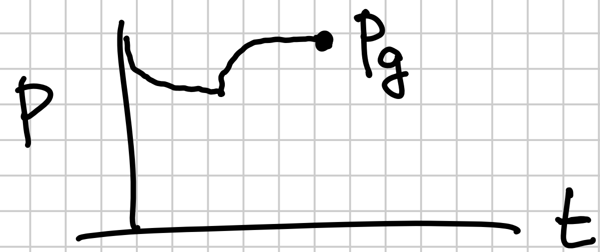
$$pV = nRTZ \Rightarrow M\frac{n}{V} = \frac{p}{RTZ} M$$

Need M (molecular weight or molar mass) $\equiv \frac{m}{n}$

$$\frac{M_n}{V} = \frac{pM}{RTZ}$$

$$\boxed{\rho_g = \frac{m}{V} = \frac{pM_g}{RTZ}}$$





Depth P.
Gradient, $G_g = \rho_g \cdot g$

$$G_w = \rho_w \cdot g$$

$$= 1000 \, \frac{kg}{m^3} \cdot 9.8 \, \frac{m}{s^2}$$

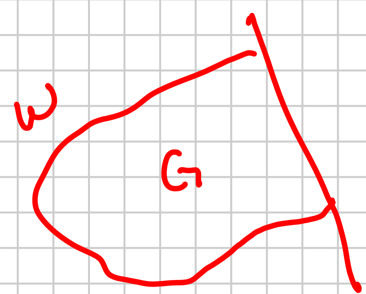
$$= 9800 \ \text{Pa/m}$$

$$= 0.098 \ \text{bar/m}$$

$$\sim 0.1 \ \text{bar/m} = 1 \ \text{bar} / 10 \ \text{m}$$

$$\rho_g \sim 100 - 400 \ \text{kg/m}^3$$

$$G_g \sim 0.01 - 0.04 \ \text{bar/m}$$



Find <u>HCPV</u>

Gas FVF $B_g \equiv \frac{V_g}{V_{\bar{g}}} = \frac{V_g(p_R, T_R)}{V_g(p_{sc}, T_{sc})}$

sell

$$B_g = \frac{V_g}{V_{gsc}}$$

$$\frac{P_{Ri} V_g = \cancel{n} \cancel{R} T_R z_{gi}}{P_{sc} V_{gsc} = \cancel{n} \cancel{R} T_{sc} \cdot 1}$$

$$B_{gi} = \frac{V_g}{V_{gsc}} = \frac{P_{sc}}{T_{sc}} \cdot \frac{T_R \cdot z_{gi}}{P_{Ri}}$$

Kristin: $B_{gi} = \frac{1.0135}{(15.56+273)} \cdot \frac{(130+273)(2)}{900}$

$$= 0.003145 \quad m^3/Sm^3$$

Expansion Factor $\quad b_{gi} \equiv \frac{1}{B_{gi}} = 317 \quad Sm^3/m^3$

$$c_g = -\frac{1}{V_g}\left(\frac{dV_g}{dp}\right)_{\underline{\underline{T,n}}}$$

$$pV = nRTZ$$

$$V = \underbrace{(nRT)}_{\text{const.}} \cdot \frac{Z}{p} = (nRT) \cdot Z \cdot p^{-1}$$

Chain Rule: $\quad \frac{dV}{dp} = (nRT)\left\{Z \cdot (-1)\, p^{-2} + \frac{dZ}{dp} \cdot p^{-1}\right\}$

$$\boxed{c_g = -\frac{1}{V}\frac{dV}{dp} = \underbrace{\frac{1}{p}}_{\substack{\text{1st Order} \\ \text{(most important)}}} - \underbrace{\frac{1}{Z}\left(\frac{dZ}{dp}\right)_{T,n}}}$$

$$c_g \simeq \frac{1}{p}$$

OK for Troll

$p_i \sim 180$ bar

__NOT__ OK for Kristin

$p_i = 900$ bara

Applications of Gas PVT Properties (Z-factor)

$$Z = \frac{pV}{nRT} \Rightarrow pV = nRT \cdot Z$$

$Z = 1$ Ideal Gas (low-p)

Z (p, T, gas composition)

$0.7 < Z_g(p_R, T_R) < 2^+$ } error vs "1" if neglect the Z-factor

Accuracy of Z – laboratory measurement OR calculation (Standing-Katz) ~ 1-2% (<3%).

Used to Calculate ρ_g, G_g, c_g, B_g

Rate Equation :

$$v = \frac{k}{\mu}\left(\frac{dp}{dr}\right)$$

$$\mu_g \propto \rho_g = \frac{pM}{ZRT} = \left(\frac{M}{RT}\right)\left(\frac{p}{Z}\right)$$

$\mu_g(p)$

$$v = q/(2\pi r h)$$

Reservoir in-situ conditions

$$q = q_g \cdot B_g \Rightarrow v = q_g B_g/(2\pi r h)$$

Surface Gas Rate

$f(p)$

$$B_g = \left(\frac{p_{sc}}{T_{sc}} T_R\right)\left(\frac{Z}{p}\right)$$

Gas : Include pressure dependence of B_g and μ_g.

$$v = \frac{q_g B_g}{2\pi r h} = \frac{k}{\mu_g}\frac{dp}{dr}$$

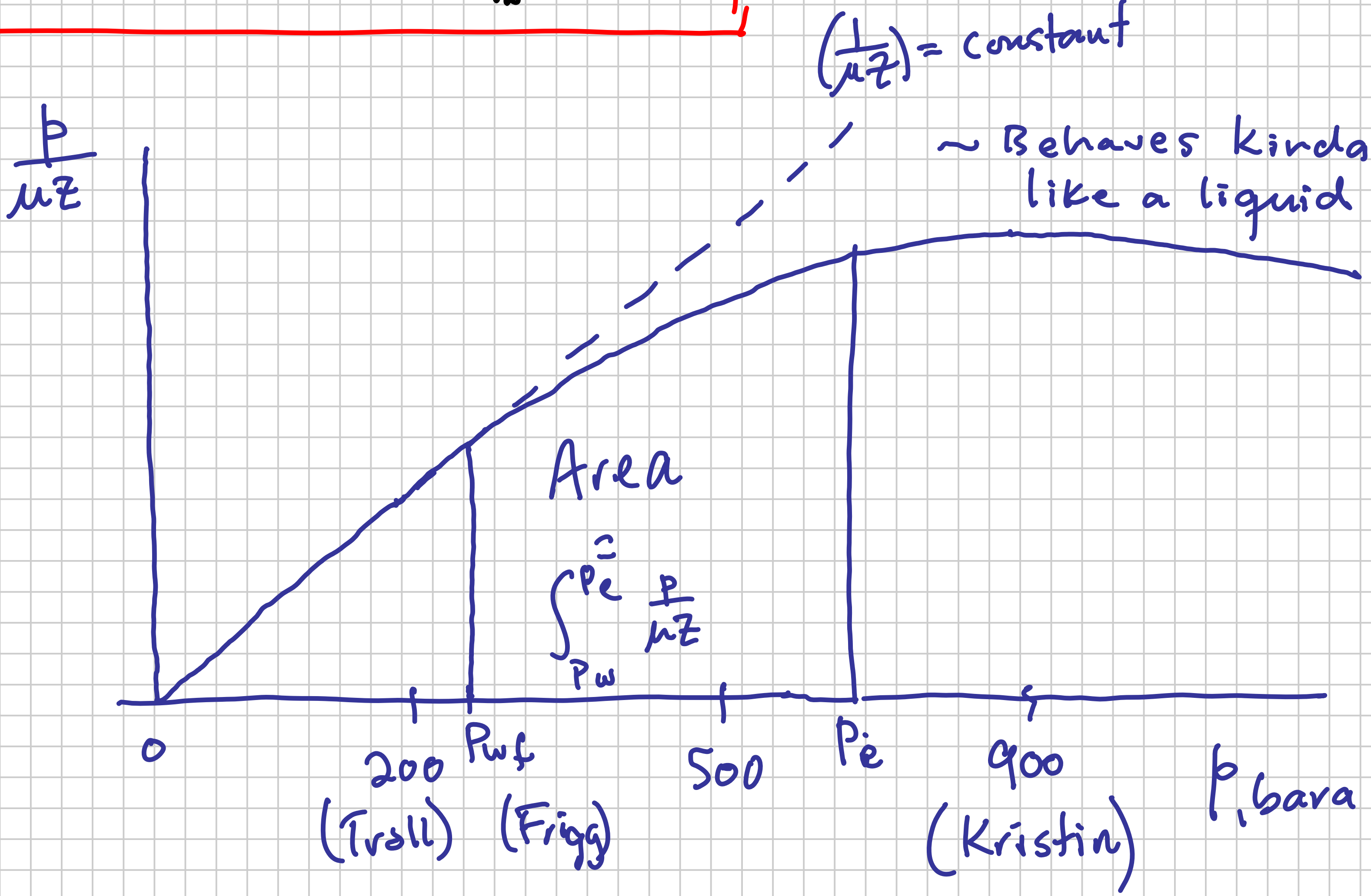
~ "volumetric average reservoir pressure"

$$q_g \cdot \int_{r_w}^{r_e} \frac{1}{r}\, dr = 2\pi k h \cdot \int_{p_w}^{p_e} \frac{1}{\mu_g B_g}\, dp \quad ; \quad B_g = \frac{p_{sc} T_R}{T_{sc}} \cdot \left(\frac{z}{p}\right)$$

$$q_g \ln\frac{r_e}{r_w} = \frac{2\pi k h \cdot T_{sc}}{p_{sc}\, T_R} \cdot \int_{p_w}^{p_e} \frac{p}{\mu_g Z_g}\, dp$$

$$\boxed{q_g = \frac{2\pi k h}{\ln r_e/r_w}\left(\frac{T_{sc}}{p_{sc} T_R}\right) \cdot \int_{p_w}^{p_e} \frac{p}{\mu Z}\, dp}$$

$$\boxed{q_g = \frac{2\pi k h}{\ln r_e/r_w}\left(\frac{T_{sc}}{p_{sc}T_R}\right)\cdot\int_{p_w}^{p_e}\frac{p}{\mu Z}\,dp}$$



Al-Hussainy, Ramey, Crawford

Gas Pseudopressure m or $m(p)$ p_p

$$\boxed{p_p \equiv 2 \cdot \int_{p_{ref}}^{p} \frac{p}{\mu Z}\, dp}$$

(usually p_{sc})

Calculate once, tabulate

$$2\int_{p_{wf}}^{p_e} = 2\int_{p_{ref}}^{p_e} - 2\int_{p_{ref}}^{p_w}$$

$$= p_{pe} - p_{pw}$$

$$\boxed{q_g = \underbrace{\frac{\pi k h}{\ln r_e/r_w}\left(\frac{T_{sc}}{T p_{sc}}\right)}\cdot\left[p_{pe} - p_{pw}\right]}$$

$p < 200$ bara

same constant

Pressure-Squared Approximation

...

$$q_g = \underbrace{\frac{\pi k h}{\ln(r_e/r_w)}\left(\frac{T_{sc}}{T p_{sc}}\right)\left(\frac{1}{\mu_{gi} z_{gi}}\right)}_{\text{constant}}\left[p_e^2 - p_w^2\right]$$

Gas Volumetric Material Balance (MB)

Assumptions: (1) $V_p \cong$ constant $(c_f \sim 0)$

(2) $V_w \cong$ constant $(c_w \sim 0)$

(3) No oil "around"

(4) Single "layer" (reservoir volume)

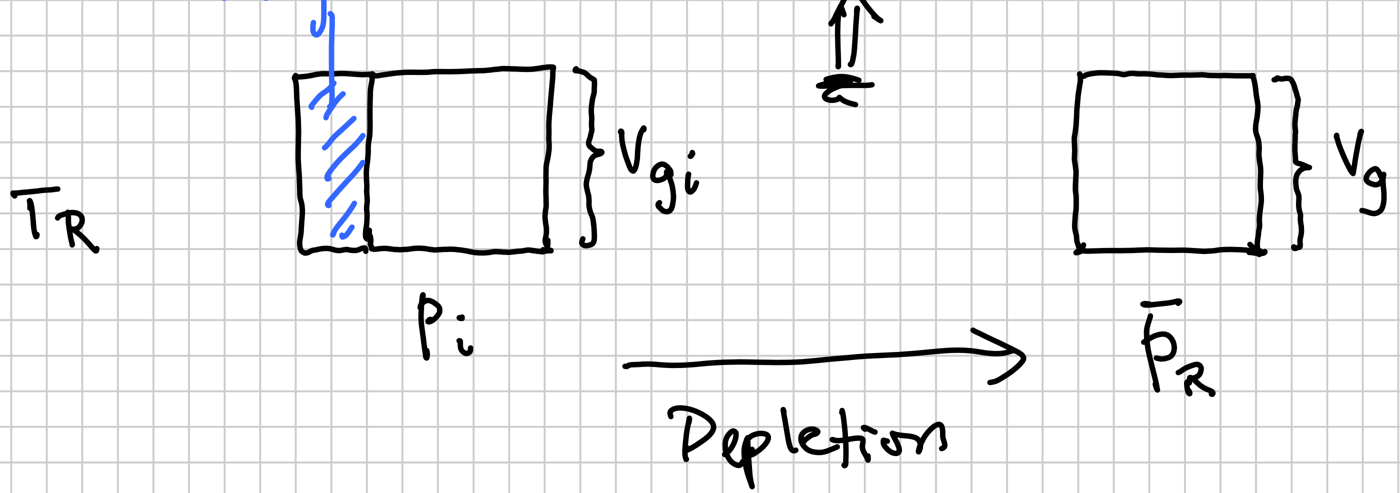
often violated.

$\Downarrow$

Straight-Line Gas MB

Produce Gas, G_p = cum. surface gas volume produced

Forget Water



Real Gas Law

✓ ✓ ✓ ✓ ✓

$$p_{Ri} V_{gi} = n_i R T_R Z_i$$

?   ✓   ? @ p_R

$$p_R V_{gi} = n R T_R Z_R$$

$$= n_i - n_p$$

✓   ✓

Measure G_p [Sm^3]

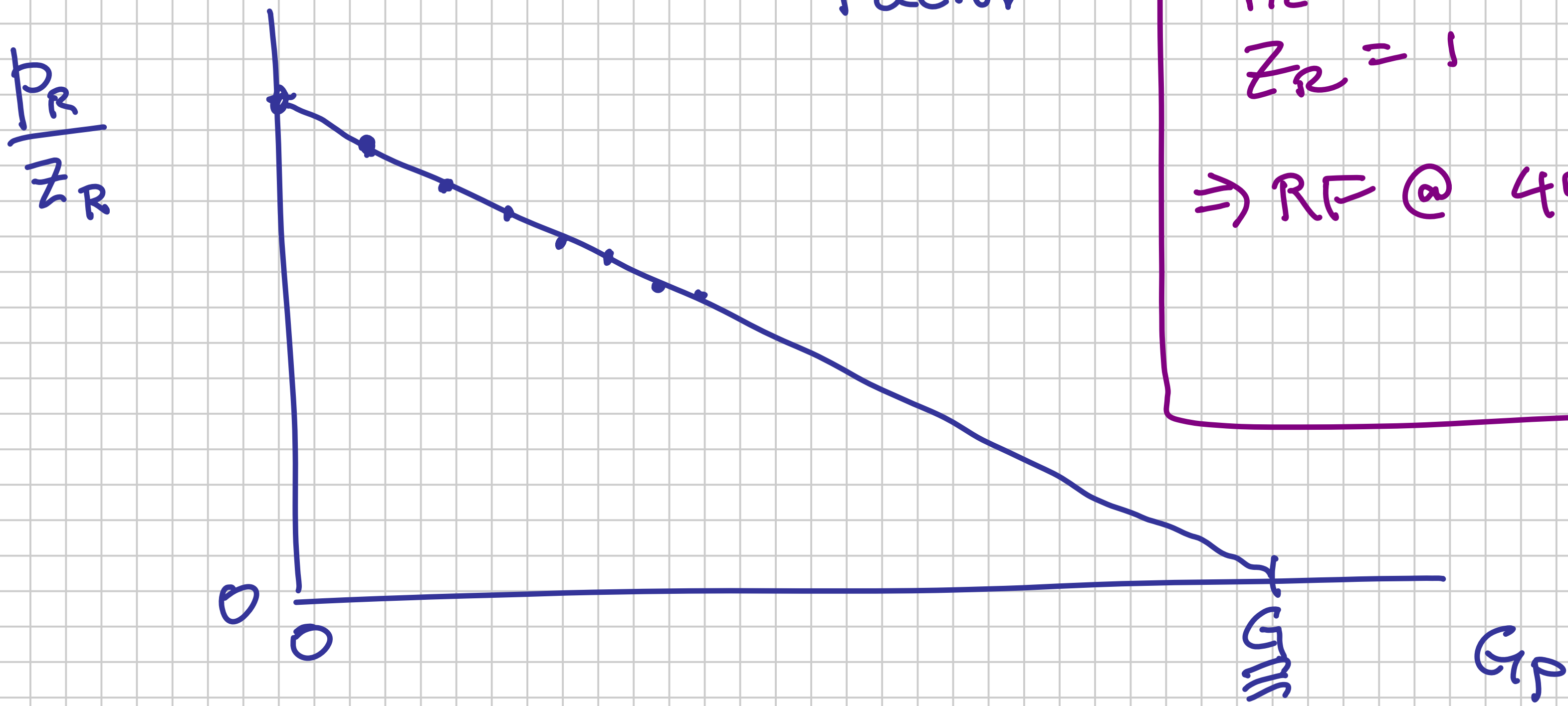
$$n_p \text{ [kg-mol]} = G_p \cdot \frac{1}{23.64}$$

$$\frac{V_g}{n} = \frac{T_{sc} R}{P_{sc}} \left[\frac{Sm^3}{\text{kg-mol}}\right]$$

$$n_i = G \cdot \frac{1}{23.64}$$

Initial Surface Gas in Place (IGIP)

$$\boxed{\frac{P_R}{Z_R} = \left(\frac{P_{Ri}}{Z_{Ri}}\right)\left(1 - \underbrace{\frac{G_p}{G}}_{\text{Recovery Factor}}\right)}$$



Kristin

$Z_i = 2$

$P_i = 900$ bara

$P_R = 450$

$Z_R = 1$

$\Rightarrow$ RF @ 450

Procedure for Calculating Z-factor

Recall: $Z(p, T, \text{gas composition})$

GASES Quantify AMOUNTS (mass, mole, volume)

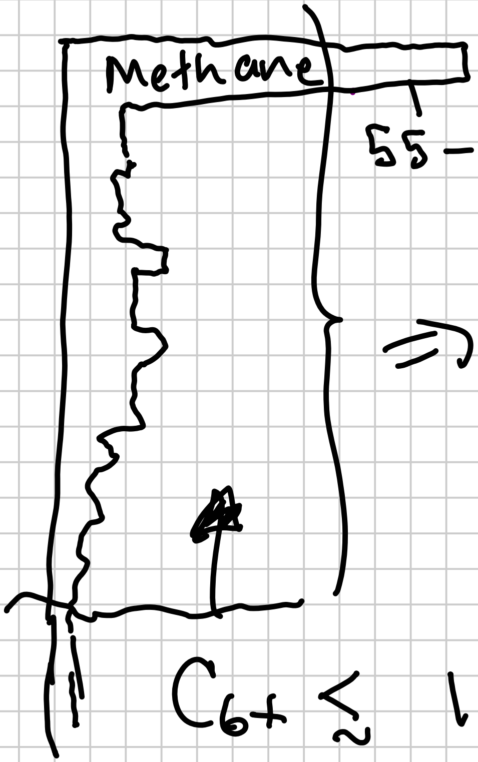
mass: Physically Measured

mole: most commonly USED

Molecular Weight (Molar Mass) M

non-HC: N_2, CO_2, H_2S — N.S. < 10 mol-%; H_2S — "0"

HCs: C_1, C_2, C_3, iC_4, nC_4, iC_5, nC_5, C_6, C_{7+}



55–95 mol-% $\Rightarrow$ Gas Pipeline

$C_{6+} \lesssim 1 - 13$ mol-% $\Rightarrow$ Tankers

Conversion Molar-to-Mass:

Gas Molar Composition y_i

AIR:

$y_{N_2} = 78$ mol.%

wt.% N_2 in Air: ?

$M_{N_2} = 28.02$

$M_{air} = 28.97$

$y_{N_2} = 0.78$ ...

$w_{N_2} = \dfrac{m_{N_2}}{m_{air}}$

(Mass) Weight Fraction

Solve:

Arbitrary: 1 kmol Air

$n_{air} = 1$ kmol

$n_{N_2} =$

$m_{air} =$

$m_{N_2} =$

⋮

Solution:

$$m_{air} = n_{air} \cdot M_{air}$$

$$m_{N2} = \underbrace{(y_{N2} \cdot n_{air})}_{n_{N2}} \cdot M_{N2}$$

$$\omega_{N2} = \frac{y_{N2} \cdot \cancel{n_{air}} \cdot M_{N2}}{\cancel{n_{air}} \cdot M_{air}} = y_{N2} \cdot \frac{M_{N2}}{M_{air}}$$

$$\sim 75.4 \text{ wt-\%}$$

Calculation of Z-factor

$Z(p, T, y)$

van der Waals ~ 1870's

"Law of Corresponding States"

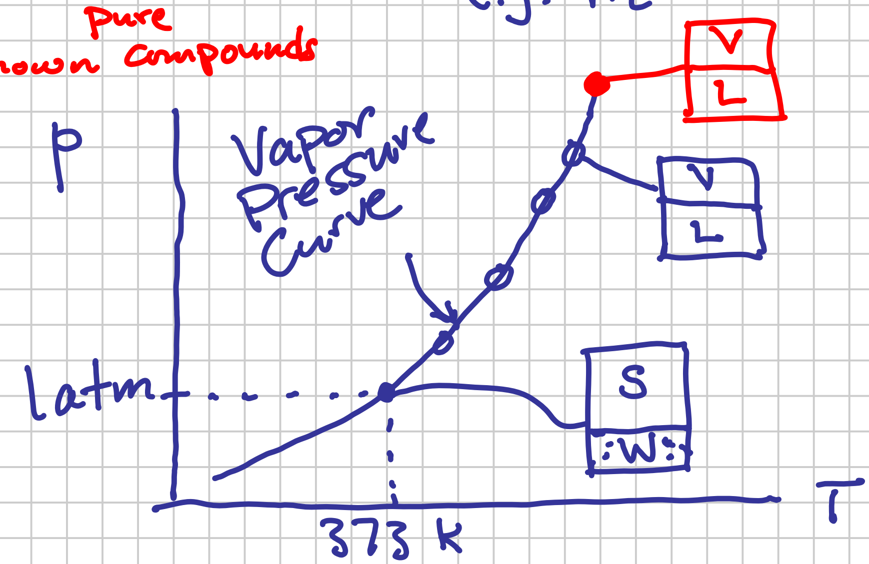
$$\boxed{Z(p_r, T_r)}$$

Reduced (Dimensionless) Pressure

$$p_r \equiv \frac{p}{p_c} \leftarrow \text{known}$$

$$T_r \equiv \frac{T}{T_c} \leftarrow \text{known for all pure compounds}$$

e.g. H_2O



gas composition

Mixtures:

$$p_{pc} = \bar{p}_c = \sum y_i \, p_{ci}$$

$$T_{pc} = \bar{T}_c = \sum y_i \, T_{ci}$$

Gas Mix : 90 mol-% C_1
10 mol-% C_2

$$T_{pr} \equiv \frac{T}{T_{pc}}$$

$$p_{pr} = \frac{p}{p_{pc}}$$

$T_R = 110°C$
$p_{Ri} = 450$ bara

$z_i(T_{pr}, p_{pr})$

$$T_{pc} = 0.9(190.6\ K) + 0.1(305.4\ K) = 202.1\ K$$

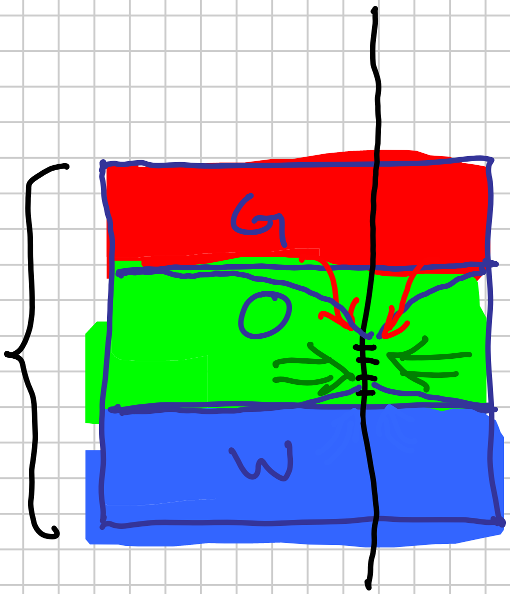
$$p_{pc} = 0.9(46.04 \text{ bara}) + 0.1(48.80 \text{ bara}) = 46.32 \text{ bara}$$

$$T_{pr} = \frac{(110+273)}{202.1} = 1.89$$

$$P_{pr} = \frac{450}{46.32} = 9.7$$

$$Z_i \simeq 1.12$$

V_p $\quad$ V_{pi}



	Initial		Current	Produced
	V_{gi}	? ✓	V_g	ΔV_g ↑
	V_{oi}	$\xrightarrow{c_o}$	V_o	ΔV_o
	V_{wi}	$\xrightarrow{c_w}$	V_w	ΔV_w Forget
	V_{pi}	$\xrightarrow{c_f}$	V_p	

$$\left.\begin{aligned} c_w &\sim \text{const} \\ c_o &\sim \text{const} \\ c_f &\sim \text{const} \end{aligned}\right\} \text{reasonable 1st approx.}$$

$$c_g \sim \frac{1}{p} \neq \text{const.}$$

$$V_p = V_g + V_o + V_w \quad ✓$$

$$p_R V_g = n_g R T_R \cdot Z_R$$

$$\boxed{V_g = R T_R \cdot (n_{gi} - \Delta n_{gp}) \left(\frac{Z_R}{p_R}\right)}$$

Free Gas moles produced

Input:

$$\boxed{\frac{\Delta n_{gp}}{n_{gi}} = \frac{G_{pf}}{G_f}} = \frac{\text{cumulative free gas } (Sm^3) \text{ produced}}{\text{initial free gas } (Sm^3) \text{ in place}}$$

* RF_{gf} = recovery factor of free gas

Specified manually in your solution (arbitrarily) $= f(p_R)$

* Strong influence on RF_o

$$q_o = C \cdot (\bar{P}_R - P_{wf})$$

$$C = \frac{kh}{\mu_o B_o \left[\underbrace{\ln(r_e/r_w)}_{\sim 7-10} + S_p \right]}$$

Partial Penetration
(Perforation)

s = "skin"

S_p = PP skin   5 - 50

$$\boxed{V_g = RT_R \cdot (n_{gi} - \Delta n_{gp})\left(\frac{Z_R}{P_R}\right)}$$

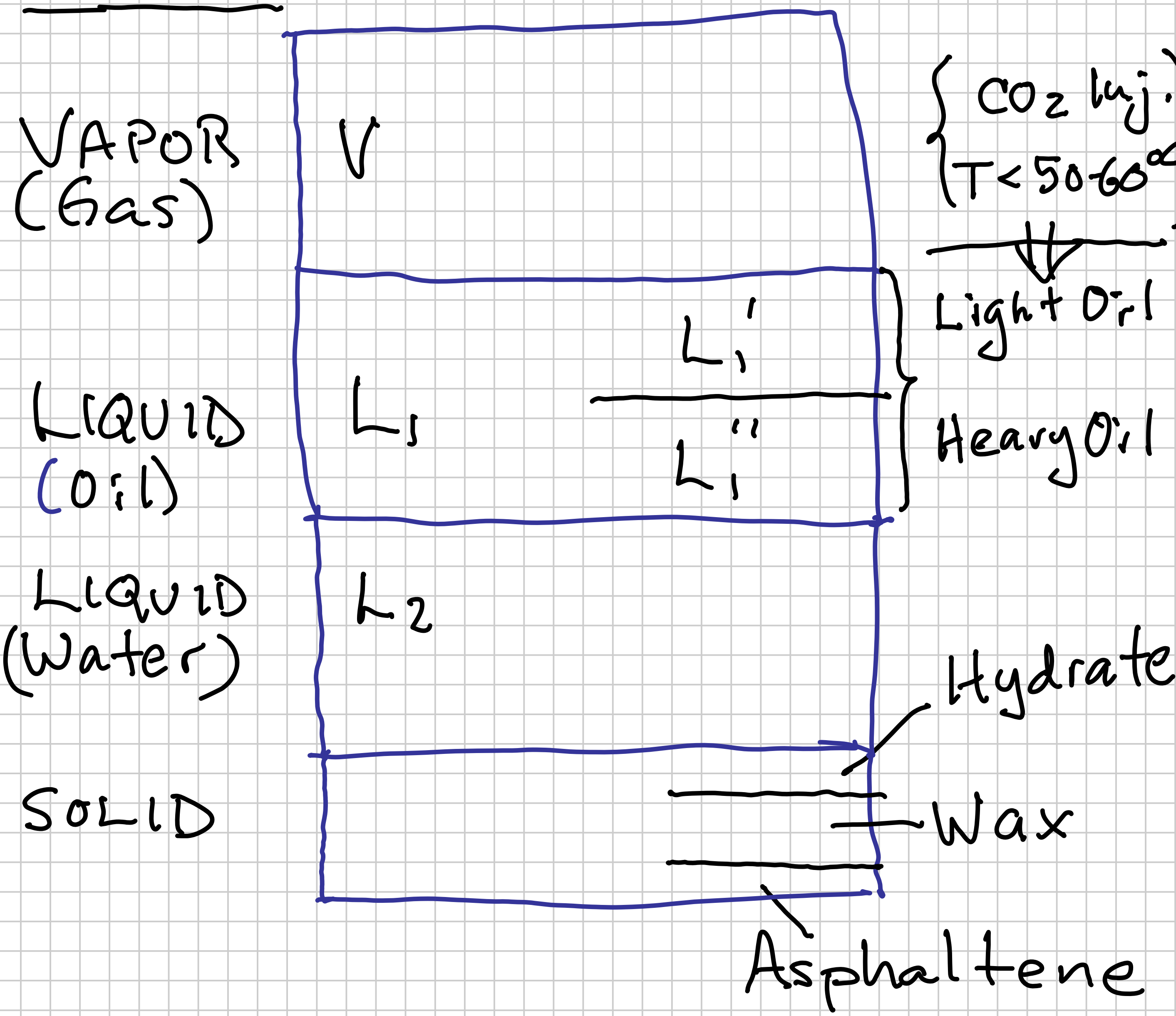
$$V_g = RT_R \, n_{gi} \left(1 - \frac{\Delta n_{gp}}{n_{gi}}\right)\left(\frac{Z_R}{P_R}\right)$$

$$n_{gi} = \frac{V_{gi} \, P_{Ri}}{RT_R Z_i}$$

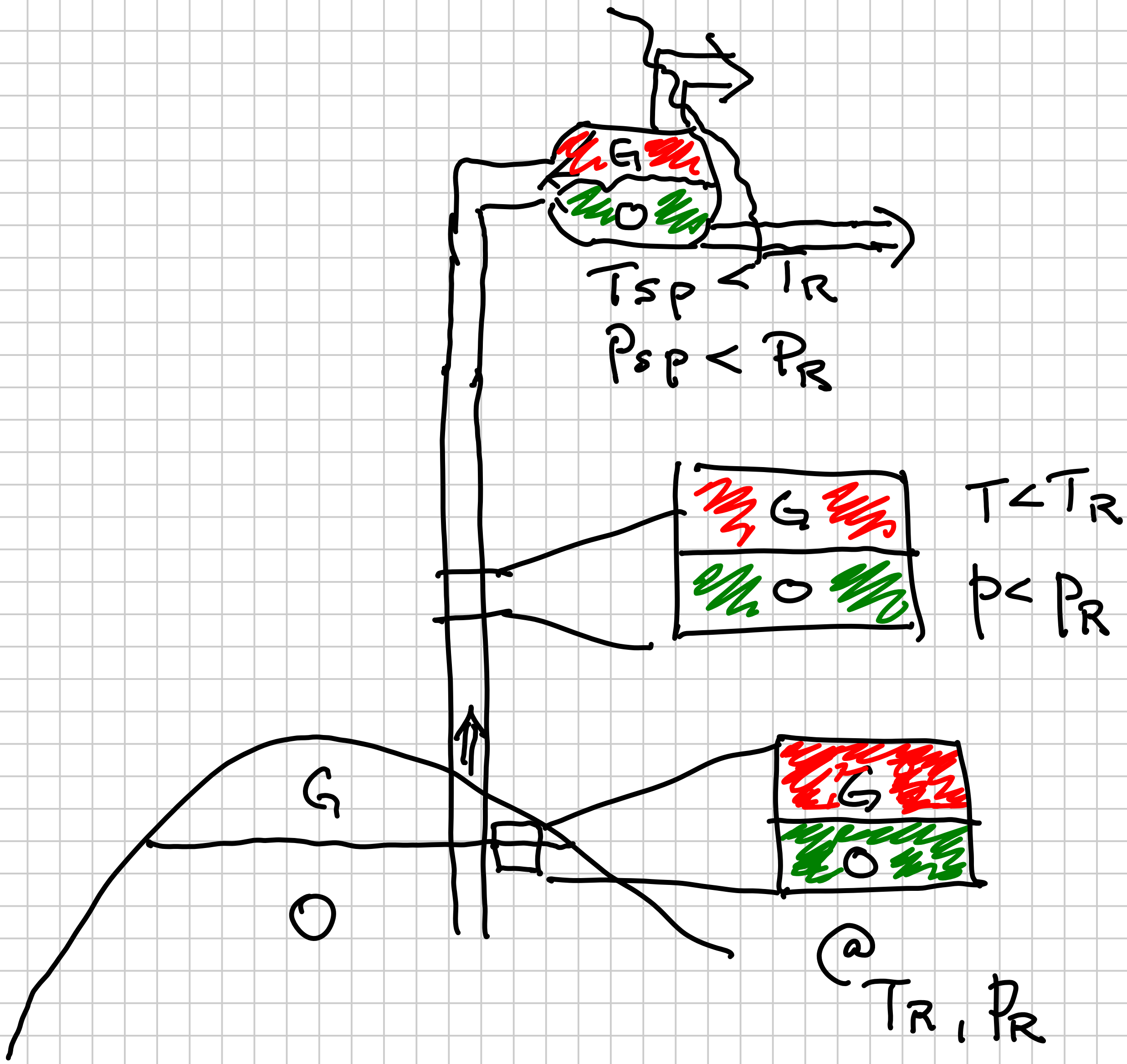
PHASE BEHAVIOR

VOLUMETRIC   EQUILIBRIA

PHASES



TWO-PHASE GAS-OIL PHASE EQUILIBRIUM



* INSTANTANEOUS EQUILIBRIUM
  (very good assumption)

QUANTIFICATION
(OF PHASE EQUILIBRIA)

Know:

Data:
- z_i — Total molar amount for each comp i
- P, T

A Model (equations)
Equation of State (EOS)
(Component Energies)

Definitions:

Fractions:
$$z_i \equiv \frac{n_i}{n}, \quad n = \sum_j n_j$$

(V) (L) Component Mole

$$\left.\begin{aligned} y_i &\equiv \frac{n_{vi}}{n_v}, \quad n_v = \sum_j n_{vj} \\ x_i &\equiv \frac{n_{Li}}{n_L}, \quad n_L = \sum_j n_{Lj} \end{aligned}\right.$$

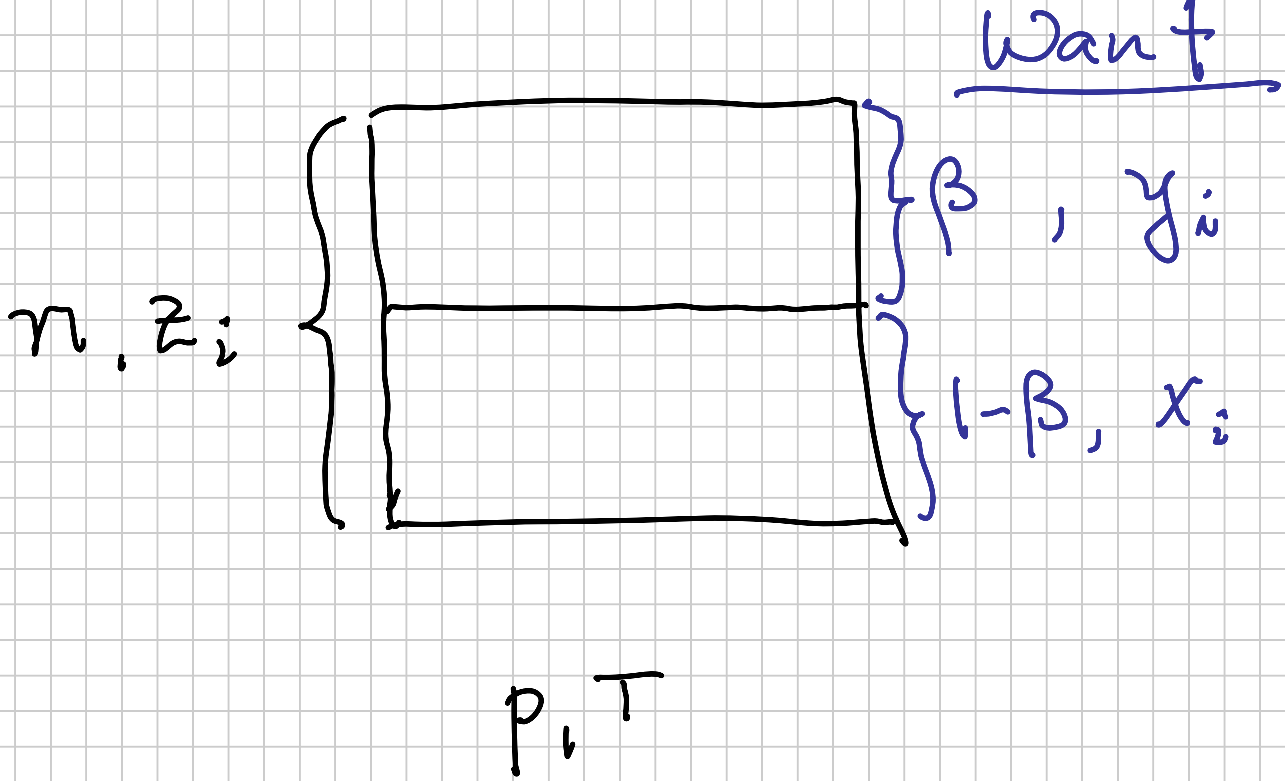
$$n = n_v + n_L$$

$$\beta = f_v = f_g \equiv \frac{n_v}{n} \quad \Big\} \text{ Vapor Phase Mole Fraction}$$

$$\left.\begin{aligned} \sum y_i &= 1 \\ \sum x_i &= 1 \\ \sum z_i &= 1 \end{aligned}\right\}$$

Quiz:

$$\sum y_i - x_i = \,?$$



Satisfy:

(1) $\underline{\mu_{Li}} = \underline{\mu_{vi}}$; $\mu =$ chem. energy

$= \mu_i$

(2) Minimum Total Energy

$$\mu = \sum n_{vi} \cdot \mu_{vi} + n_{Li}\mu_{Li}$$

$$\mu = \sum \mu_i \cdot (n_{vi} + n_{Li})$$

Min $\mu = \sum n_i \mu_i$

Where do we get μ_i

$\Rightarrow$ from EOS

$$(z_i, p, T) \longrightarrow \boxed{EOS} \longrightarrow \mu_i$$

Gibbs
vdW
Michelsen

Graphical Phase Diagrams

- p-T : pressure-temperature
- p-V : pressure-volume
- p-x : pressure-composition

Saturation Pressure

Oils : Bubblepoint

Gases : Dewpoint

?

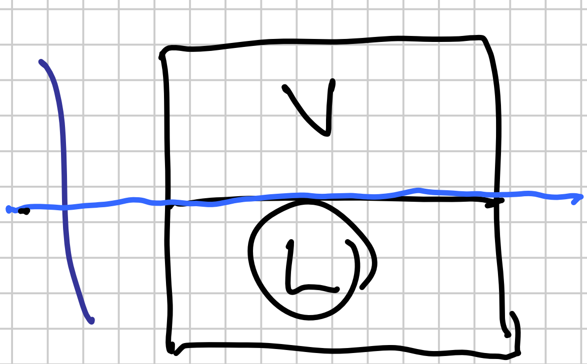
Saturated Phase

vs

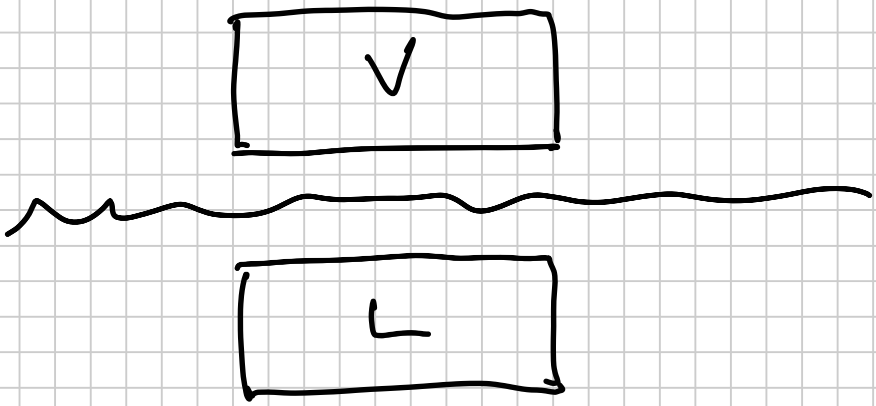
Undersaturated Phase

Answer:

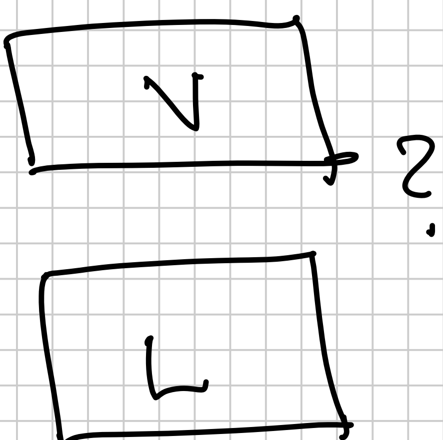
A saturated phase is a phase that is in equilibrium with another phase.



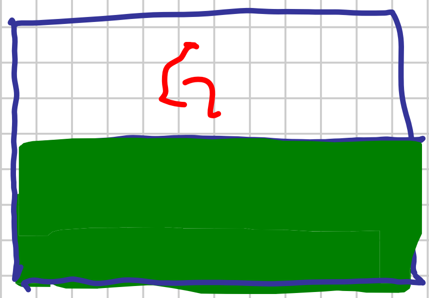
In Equilibrium



Test if two phases are saturated with each other?

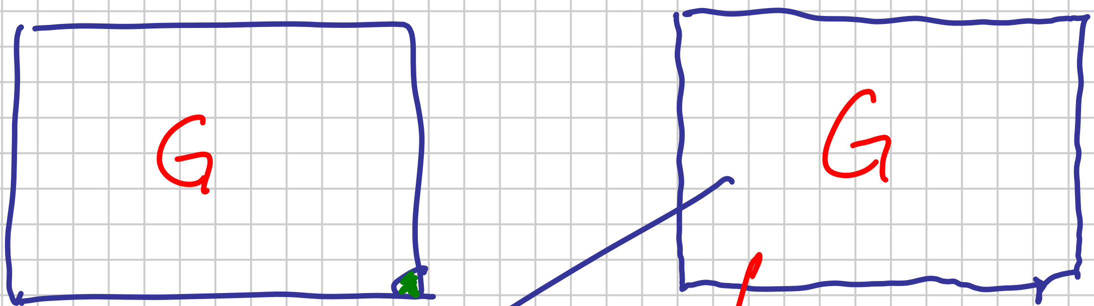


Put the two phases together. If nothing happens, they are saturated with each other.



If two phases present together

- Assume they are in equilibrium
- Each phase is "SATURATED" (with the other phase)



1 Is this gas saturated?

If yes, with what?

2 Is this gas undersaturated?

Yes – to many liquids (e.g. C_{10} C_{20})

Saturation Pressure

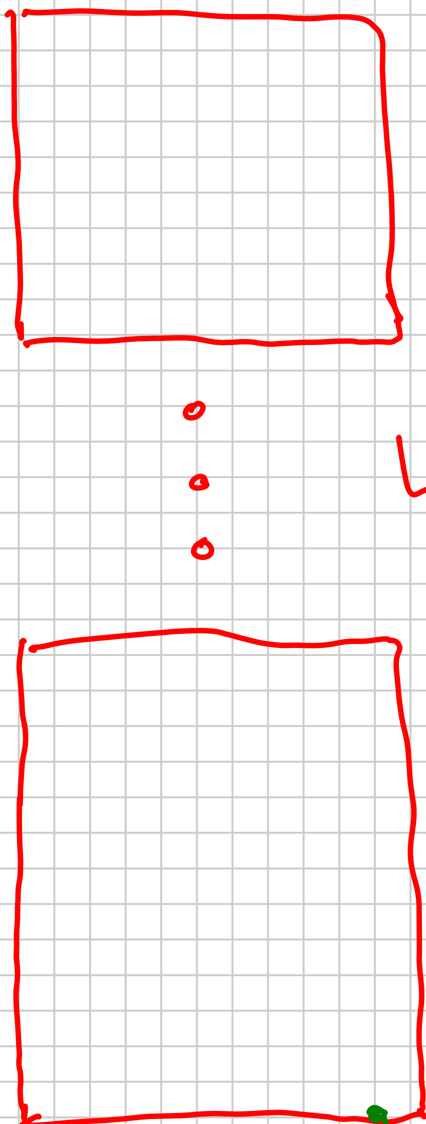
(1) Bubblepoint (p_b)

(2) Dewpoint (p_d)

KRISTIN

GAS FIELD

$P_{Ri} \sim 900$ bara



Const. Temp. T_R

Lower Pressure

P_d

1st Drop of Liquid

"Condensate"

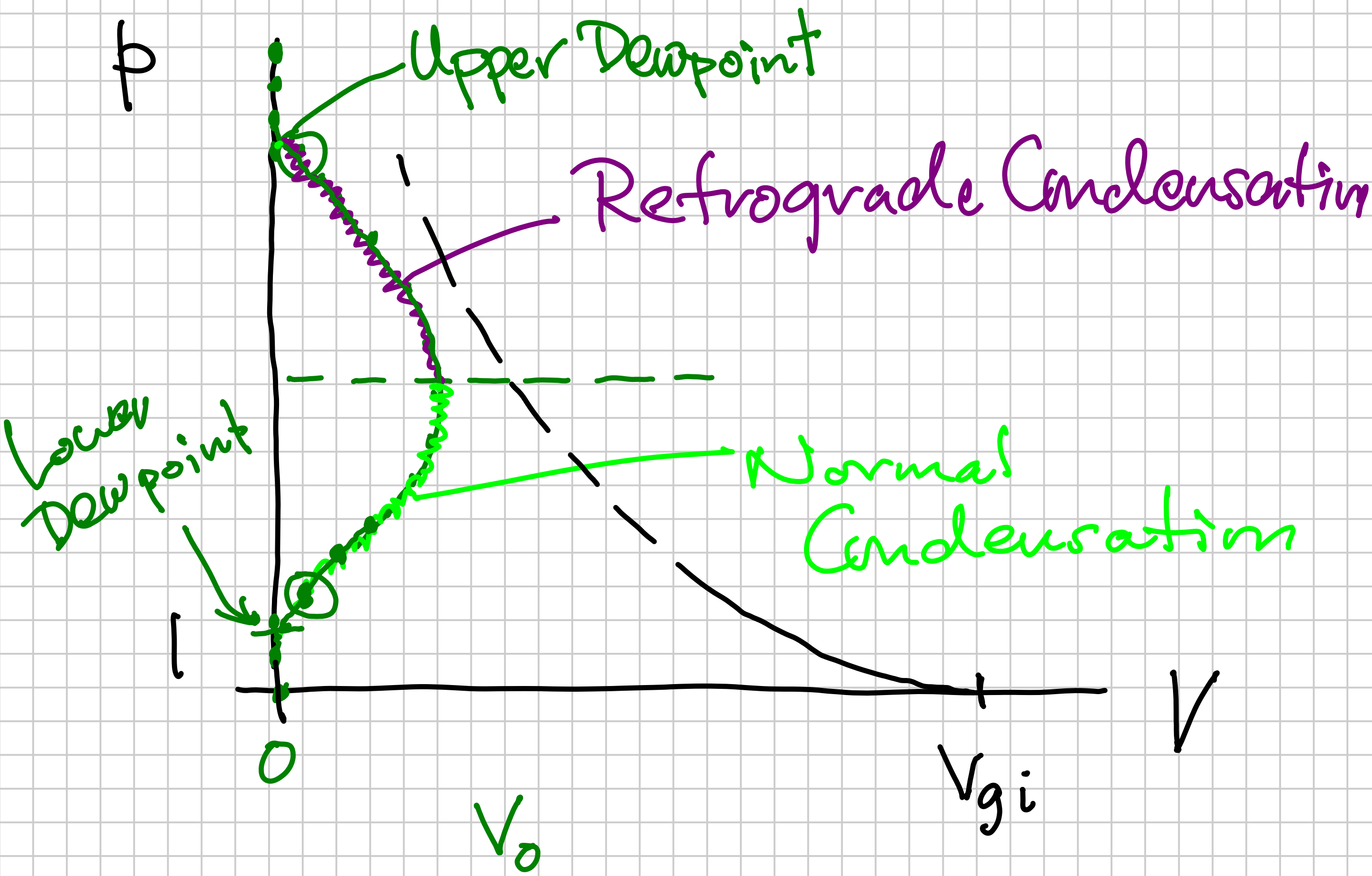
"Retrograde Condensate"

Gases: Two Dewpoints

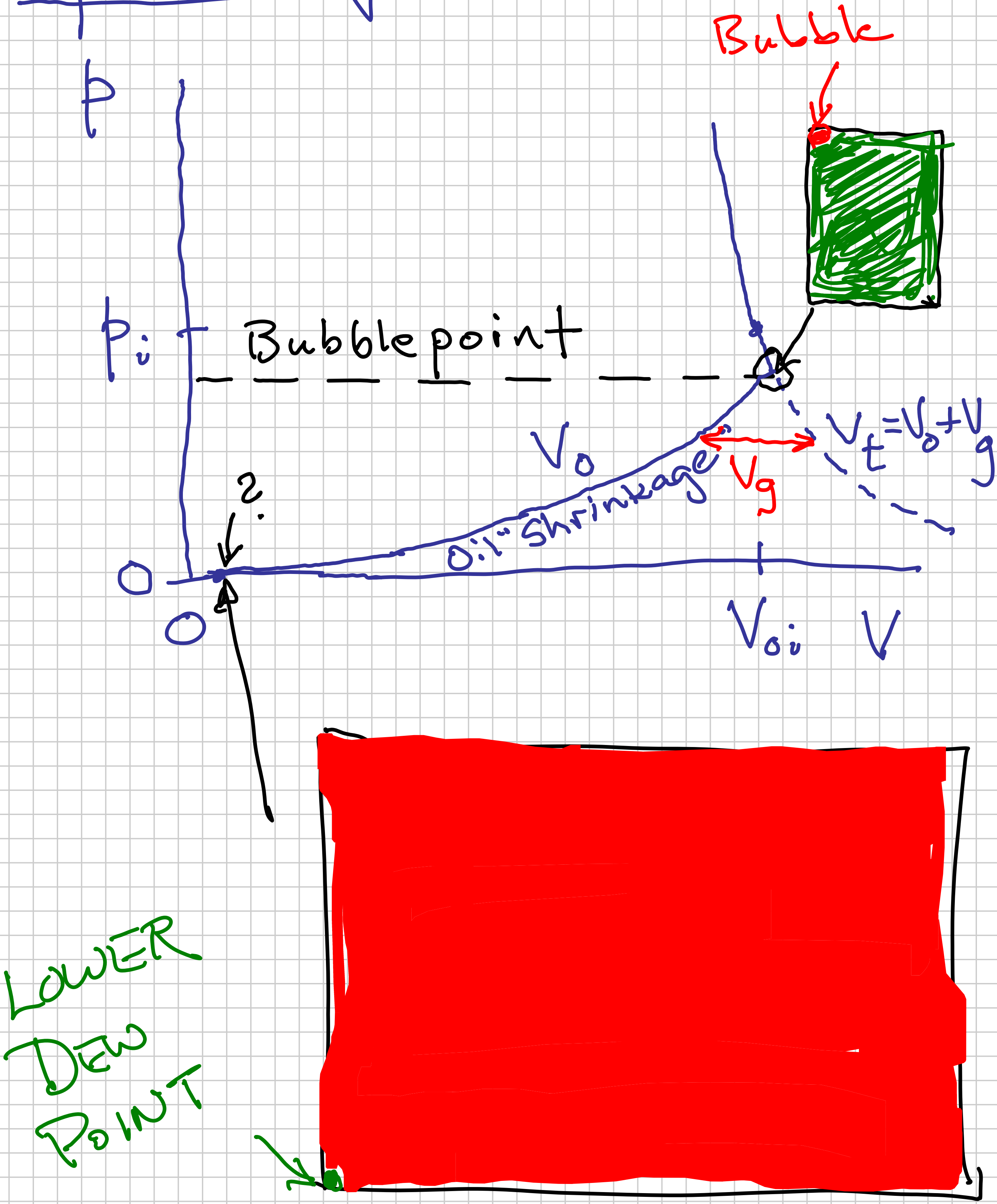
- Lower
- Upper ("retrograde DP")

p-T & p-V Diagrams

at T=const

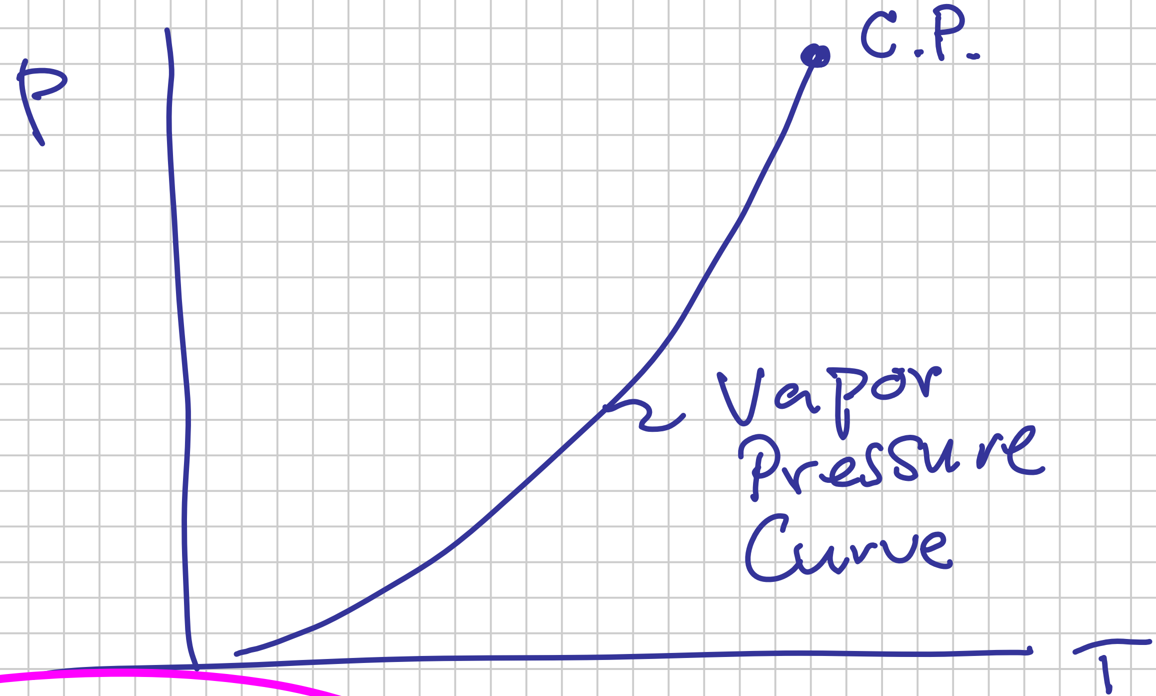


p-V Diagram for an OIL

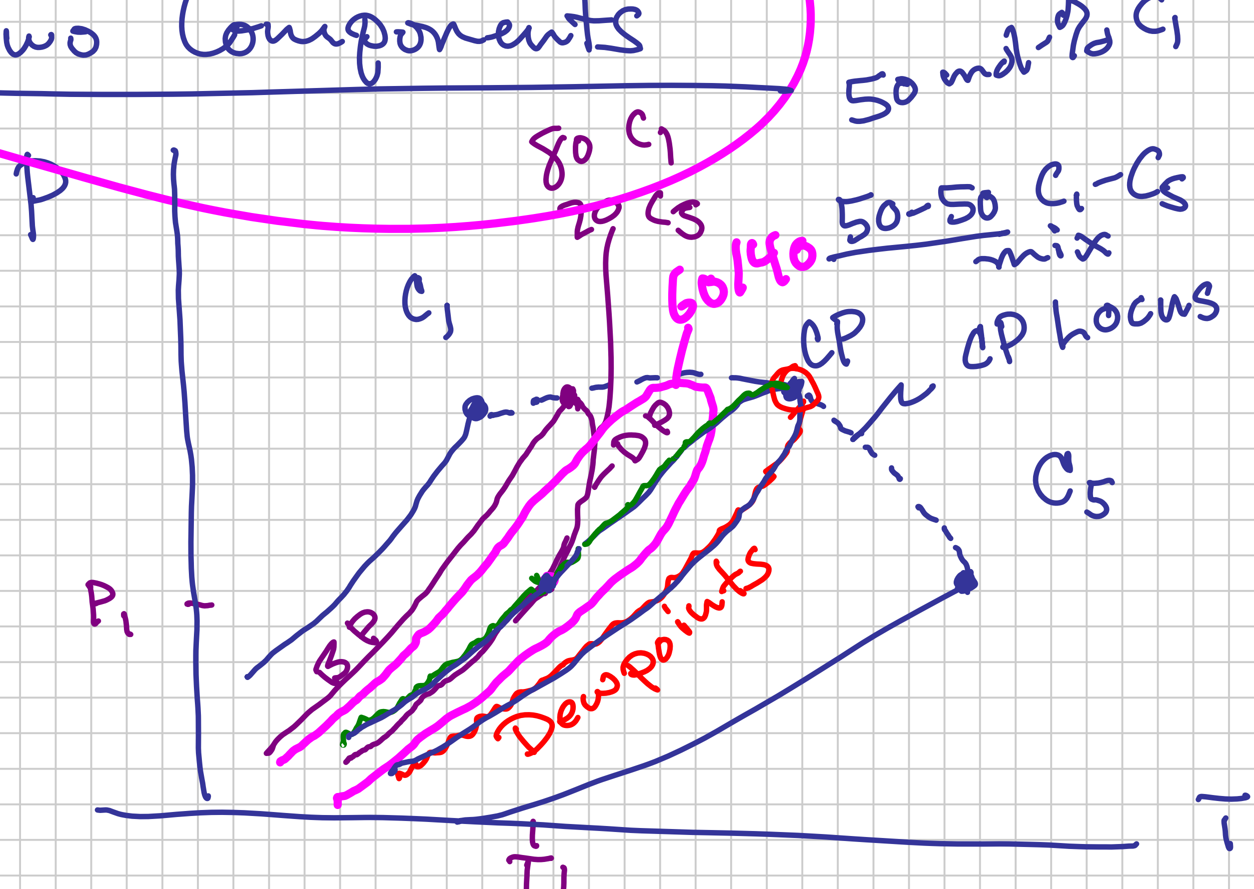


p-T Phase Diagram

Single Component



Two Components



CP

G

O

OIL

$\epsilon\,(10^{-30})$

$y_{C_1} > 50$

$y_{C_5} < 50$

50 mol-% C_1 x_{C_1}

50 mol-% x_{C_5}

$y_{C_1} = x_{C_1} = 50$ P_1, T_1

$y_{C_5} = x_{C_5} = 50$ P_1, T_1

GAS

$y_{C_1} = 80$

$y_{C_5} = 20$

$\epsilon\,(10^{-30})$

Dew drop $\begin{cases} x_{C_1} = < 80\,\% \\ x_{C_5} = > 20\,\% \end{cases}$

FACT

Binary Mixtures (2-components)

At a given (p, T) $\dfrac{y_i}{x_i} = \text{const.}$

independent of z_i (overall composition)

Question: At P_1, T_1

- What is y_{C_1}, y_{C_5} of gas bubble?

- What is x_{C_1}, x_{C_5} of the dew drop?

Bubble. Dew Drop

$$\frac{y_{C_1}}{x_{C_1}} = \frac{y_{C_1}}{x_{C_1}}$$

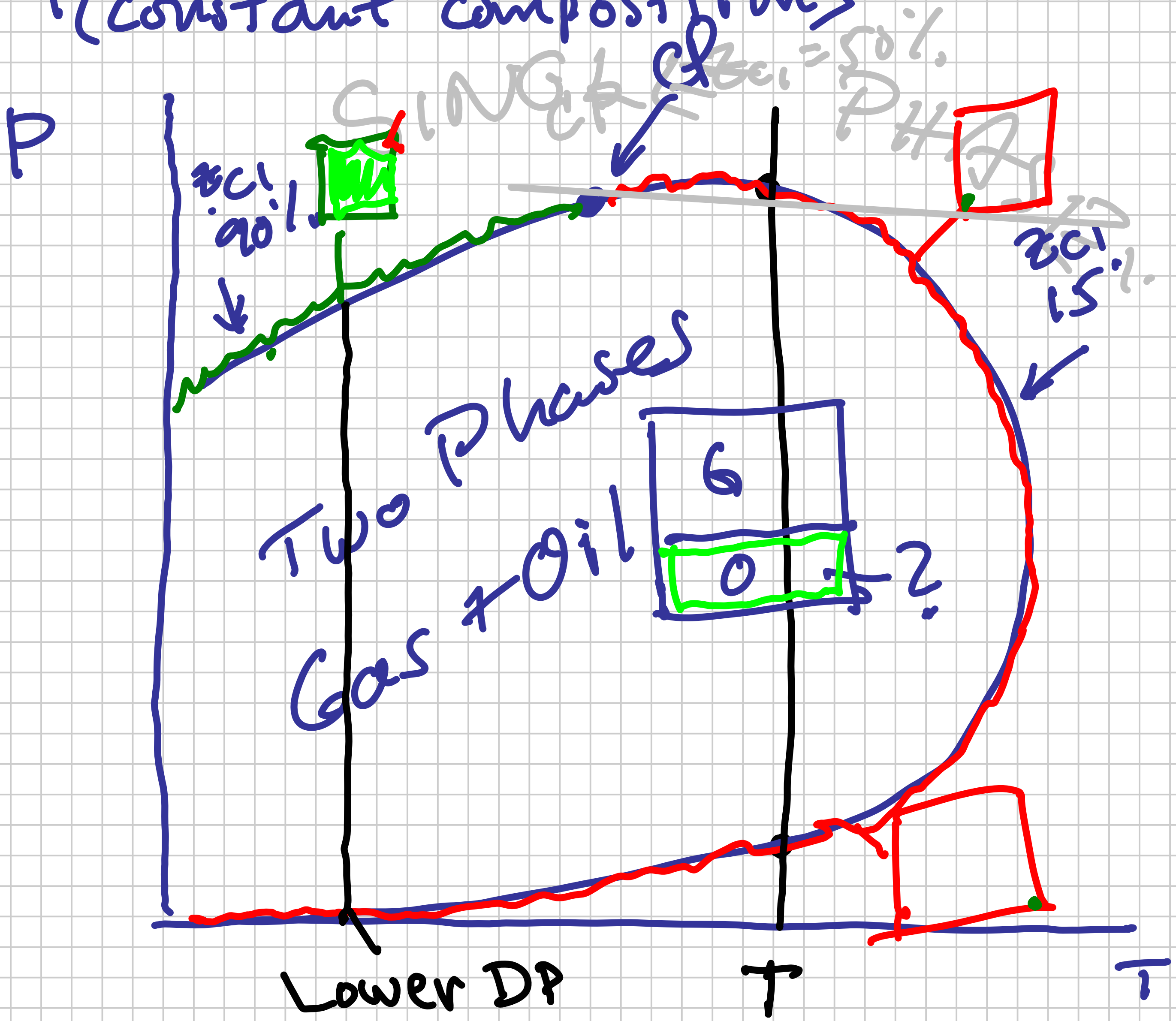
$$\boxed{\frac{y_{C_1}}{50} = \frac{80}{x_{C_1}}} \quad (1)$$

$z_{C_1} = 50$ $z_{C_1} = 80$

$$\boxed{y_{c_1} + y_{c_5} = 100} \quad (2)$$

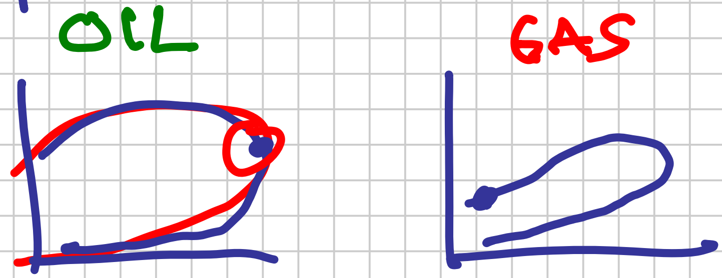
$$\Rightarrow y_{c_1} = 80 \qquad x_{c_1} = 50$$

P-T of a real HC mixture (constant composition)



C.P. depends on z_i

Shape

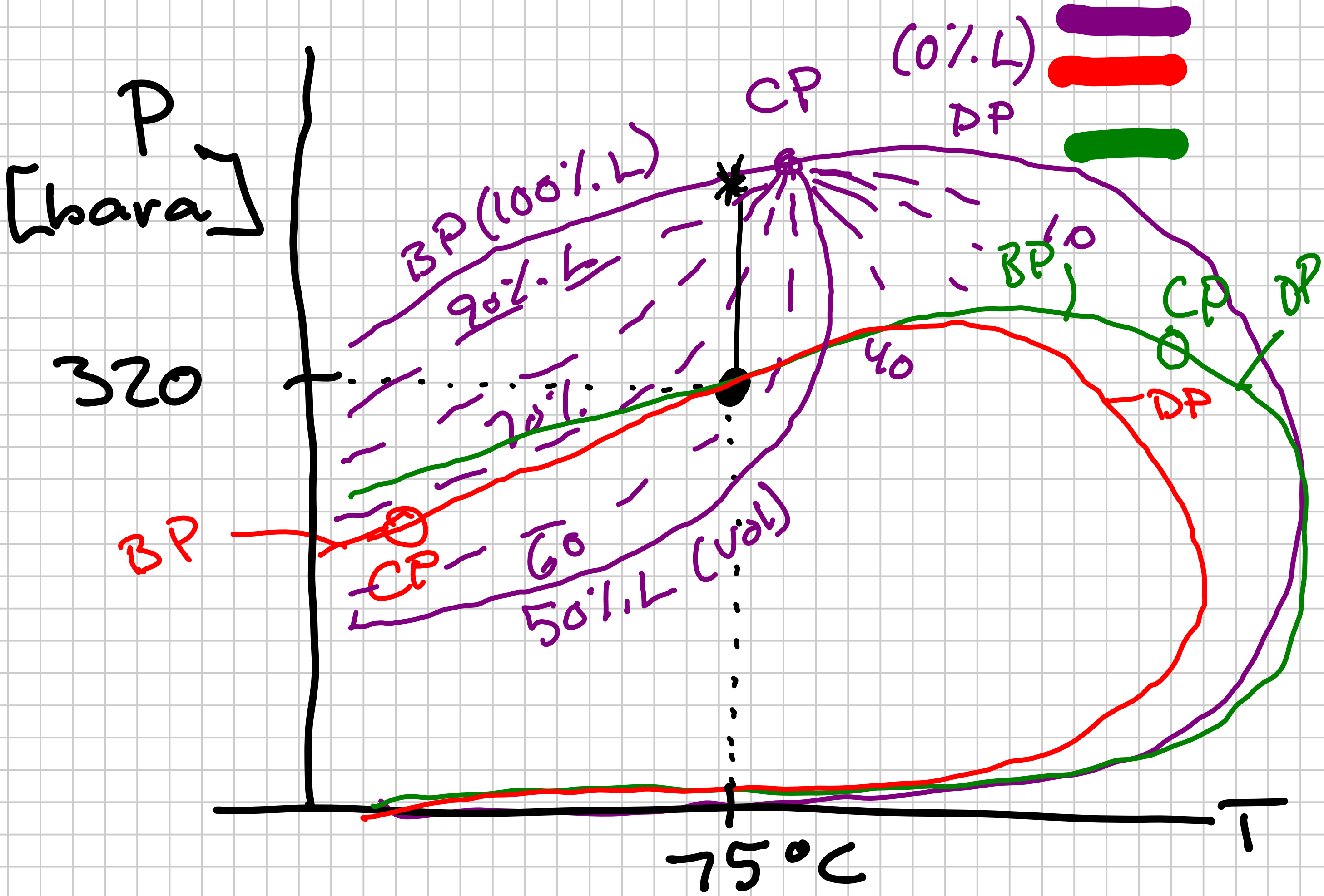


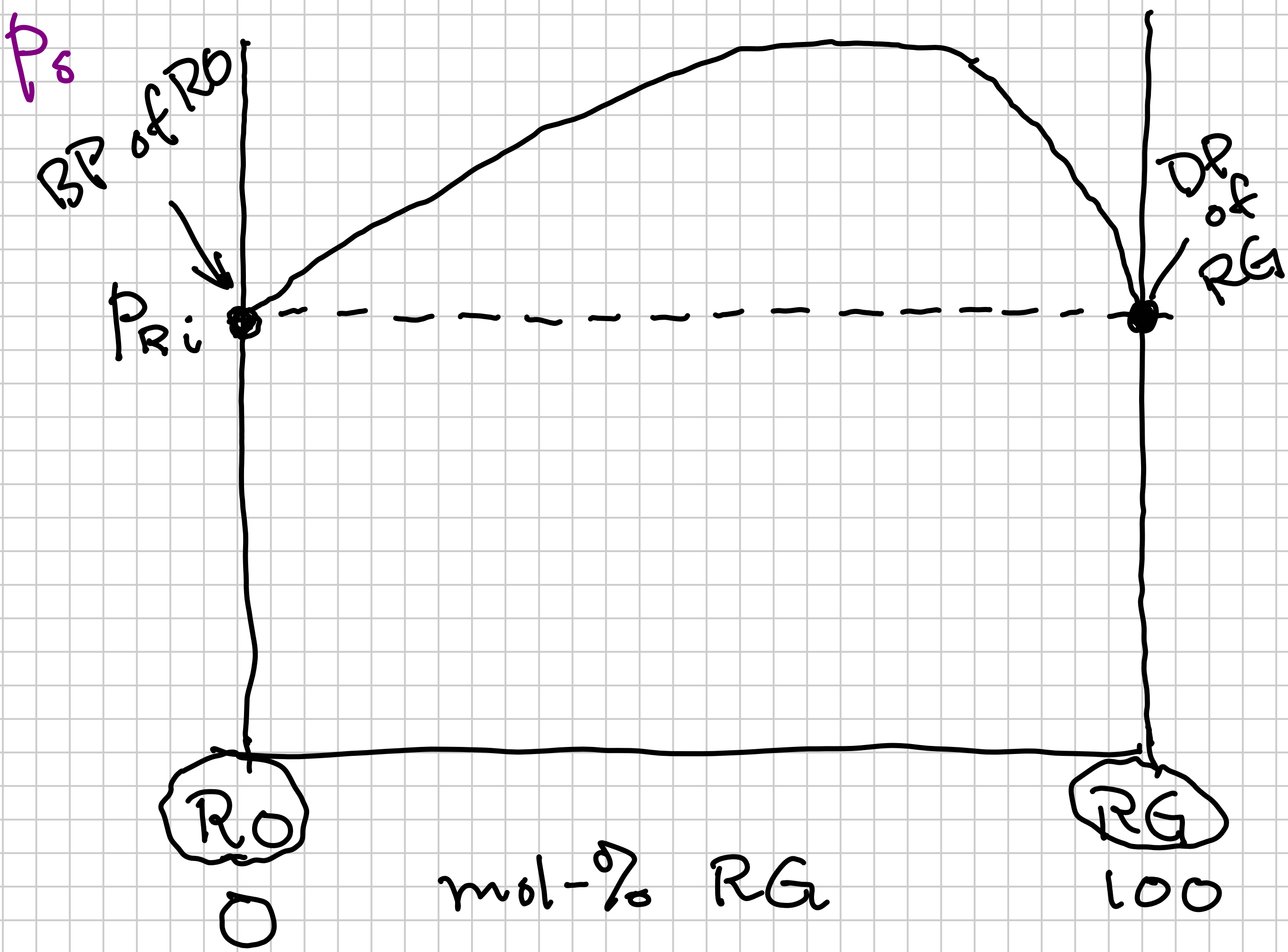
depends on z_i

- $f(C_1)$
- $f(C_{7+}$ amount, type $)$

OSEBERG FIELD

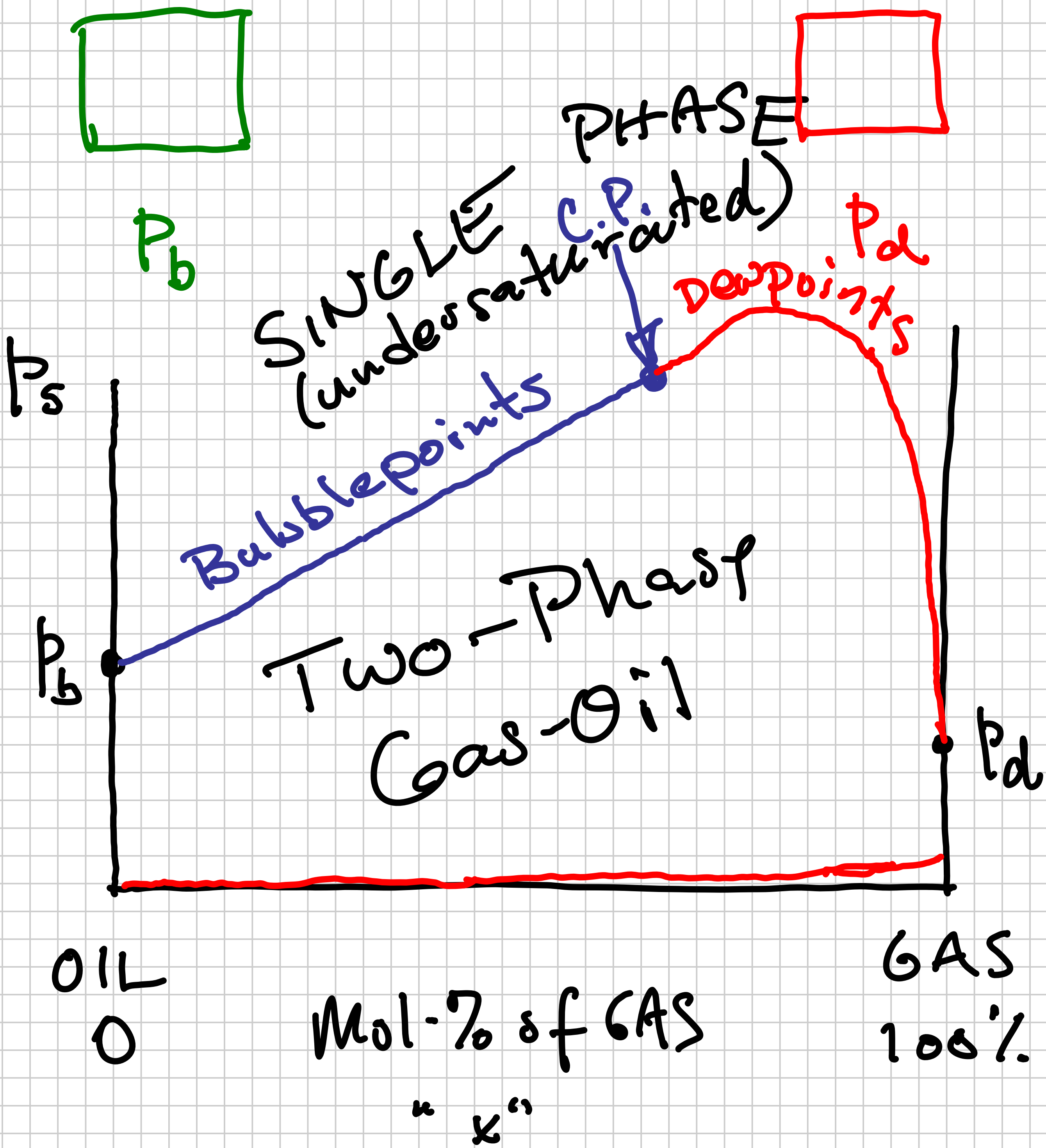
$P_{Ri} = 320$ bara, $T_R = 75°C$


P [bara]
320
75°C
T
(0% L)
CP
DP
BP (100% L)
90% L
70%
BP
CP
60
50% L (vol)
40
BP
CP
DP
DP


P_s
BP of RO
P_{Ri}
DP of RG
RO
RG
0
mol-% RG
100

Pressure-Composition $p-x$

Const T_R



After "oil gone"

q_g : use anything that makes engineering sense

"oil gone"

Calc. G_{pf}/G_f

$V_g = V_p - V_w$ ✓ ✓

$\dfrac{G_{pf}}{G_f} = \left(1 - \dfrac{n_{grem}}{n_{gi}}\right)$

Gas Law:

$\underbrace{V_g,\ P_R,\ T_R,\ Z_R,}_{n_{grem}}$

$\Delta V_g = \underbrace{\Delta n_{gp}} \cdot \dfrac{R\, T_R\, Z_R}{P_R}$

$n_{grem}\Big|_{P_R - \Delta P_R} - n_{grem}\Big|_{P_R}$

$$q_g = \frac{2\pi K h (T_{sc}/p_{sc})}{T_R \cdot \ln r_e/r_w} (\phi_{pR} - \phi_{pwf})$$

$$\phi_p = 2\int_{p_{sc}}^{p} \frac{p}{\mu Z}\, dp$$

$$\phi_p(p) = a_1 p + a_2 p^2 + a_3 p^3$$

Need q_g to convert

$$\frac{\Delta G_{pf}}{G_p} \Rightarrow \Delta t$$

PHASE EQUILIBRIA

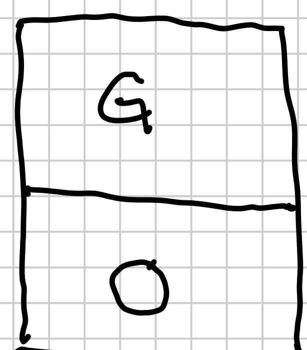
K_i: "equilibrium ratio"

"K-value"

Variable that "we" define to help make phase eq. calculations more efficient.

* Also helps to "understand" how a component i partitions into the phases.

$$K_i \equiv \frac{y_i}{x_i}$$



$K_i > 1$: component i has a relative preferance to be in the Gas phase (e.g. C_1, H_2, ...

$K_i < 1$: comp. i has a rel. preferance to be in the Liquid phase (C_6, C_7, ... ; H_2O)

$$K_i(\overbrace{\underbrace{p, T}_{\text{always}}}^{p_v(T)\text{ Key for low-p}}, \underbrace{\text{overall composition}}_{z_i})$$

sometimes
- NOT low-p ($\lesssim 100$ bar)
- YES higher-p ($\gtrsim 300$ bar)

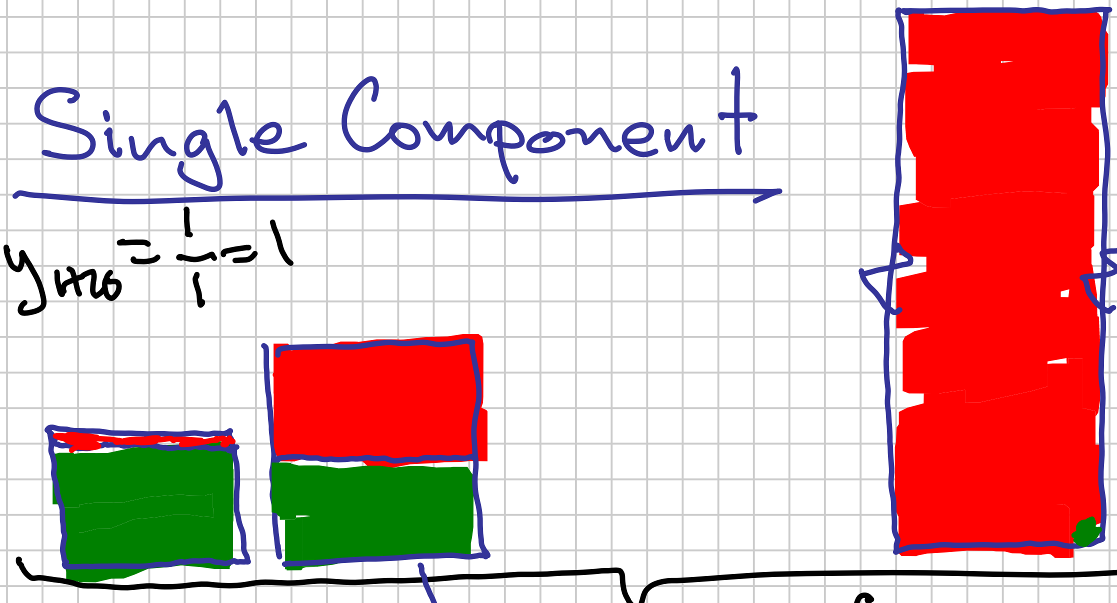
$$y_i \equiv \frac{n_{vi}}{n_v}$$

$$x_i \equiv \frac{n_{Li}}{n_L}$$

Low-p K-value Behavior

Single Component

$y_{H_2O} = \frac{1}{1} = 1$

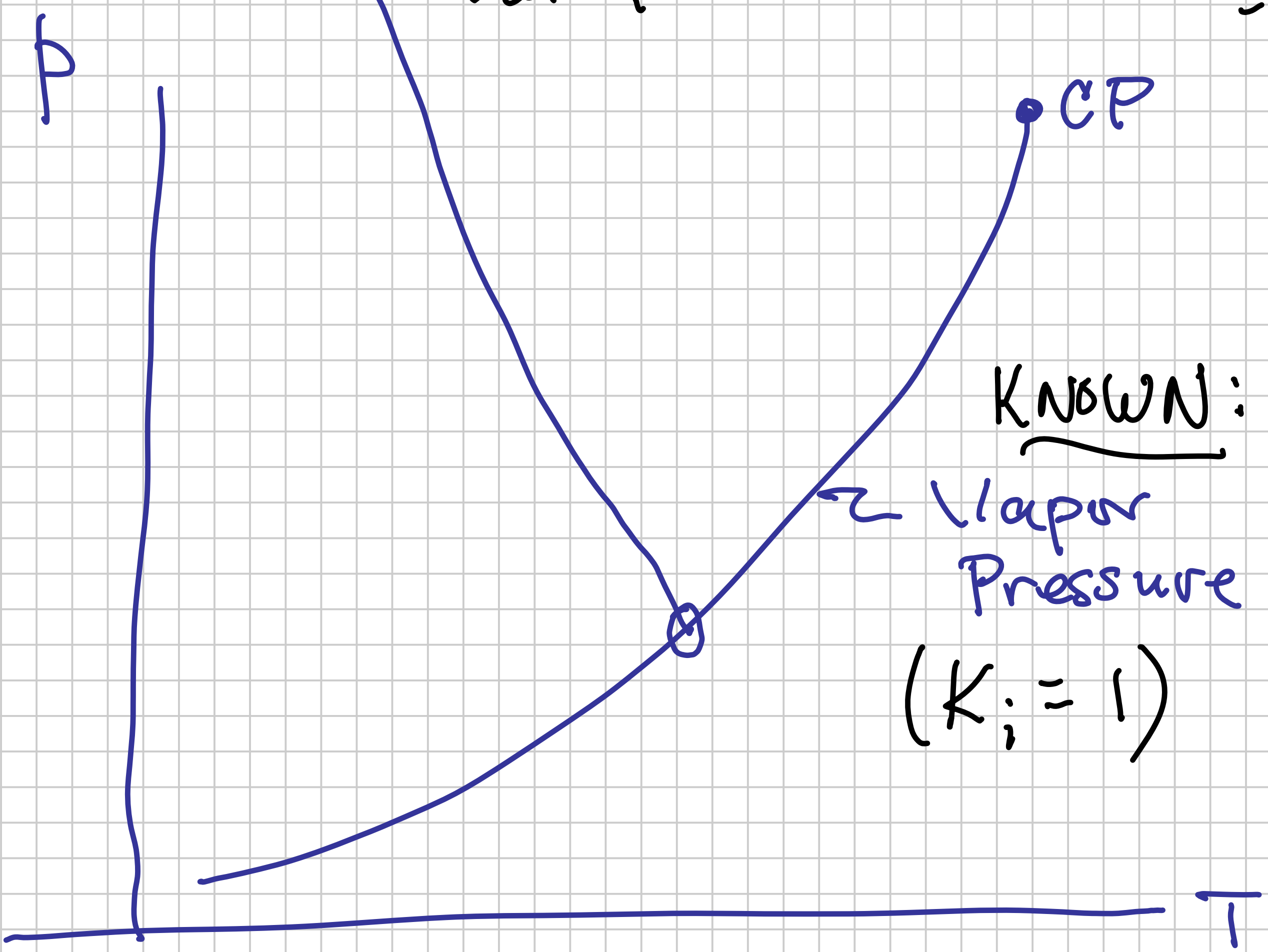


$K_i = 1$

$y_{H_2O} = \frac{99}{99} = 1$

$x_{H_2O} = \frac{1}{1} = 1$

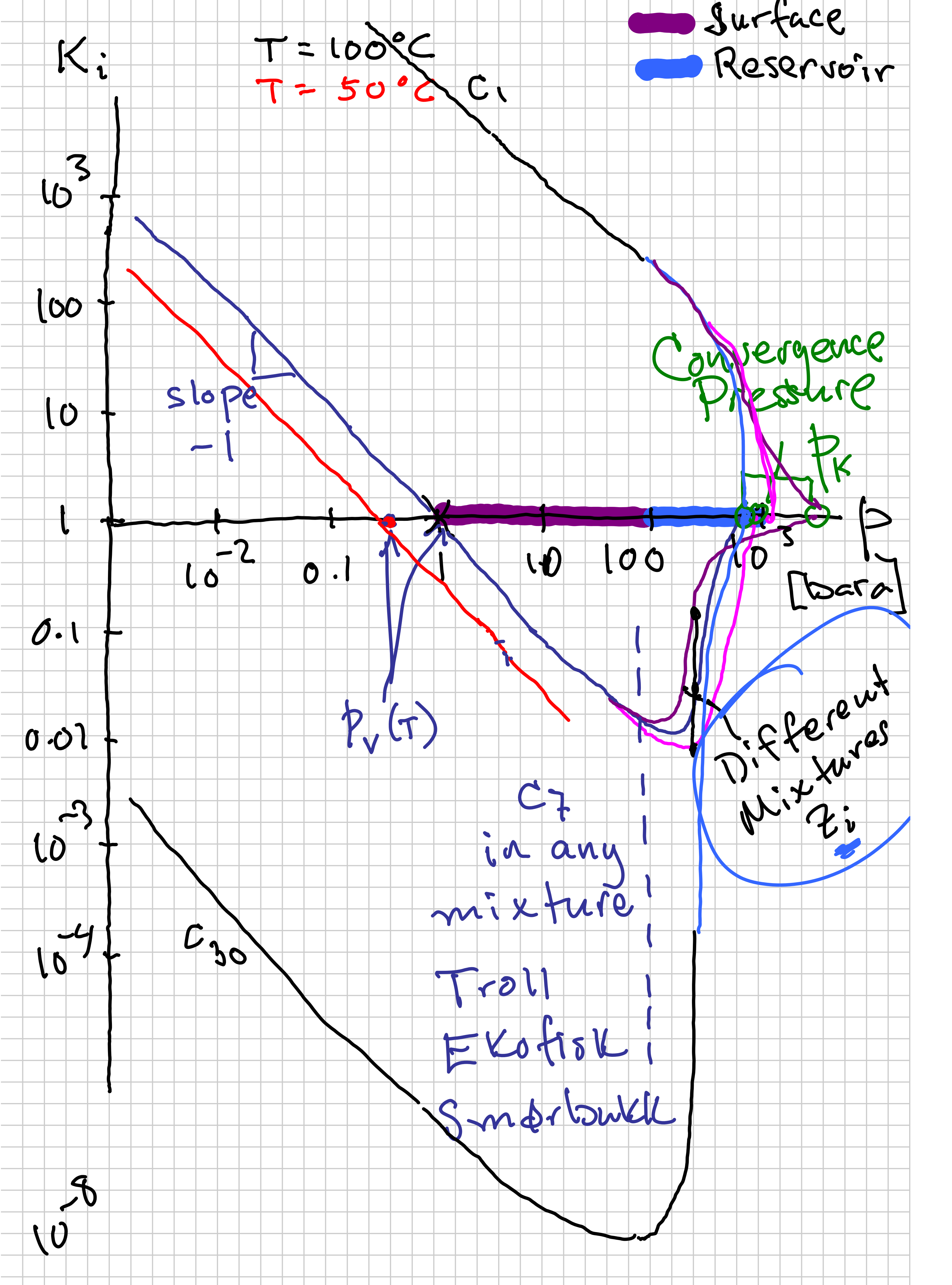
1 atm, 100°C



At low-p K_i not (much) dependent on z_i.

If a comp has $K_i = 1$ at $p_v(T)$ it will also have $K_i = 1$ at $p = p_v$, T in a mixture with many other components.

What is the pressure dependence of K_i at low-p?



K_i
T = 100°C
T = 50°C
C_1
Surface
Reservoir
10^3
100
10
1
0.1
0.01
10^{-3}
10^{-4}
10^{-8}
slope -1
10^{-2}
0.1
1
10
100
10^3
P [bara]
Convergence Pressure
P_K
$P_v(T)$
C_7 in any mixture
Troll
Ekofisk
Smørbukk
C_{30}
Different Mixtures z_i

Equation in "the book"

Wilson K-value equation.

$$K_i\,(\omega_i\,,\ p_{ri}\,,\ T_{ri})$$

$$p_{ri} = \frac{P}{P_{ci}}$$

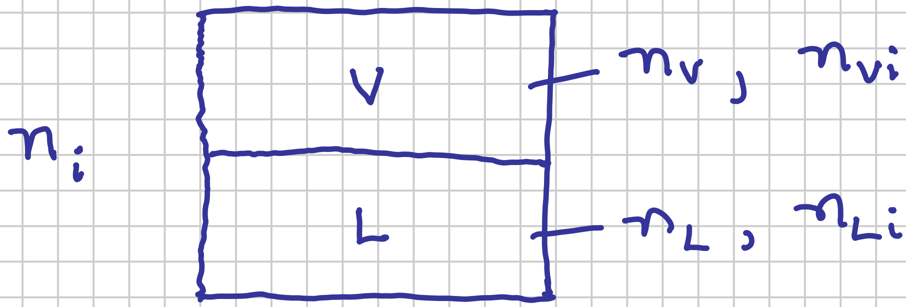
$$T_{ri} = \frac{T}{T_{ci}}$$

ω_i = "acentric factor"

$\sim$ slope of the $p_v(T)$
near the C.P.
$0 - 1\ (\dots 2)$

MODIFIED Wilson Eq.
(convergence press. p_K)

EQUILIBRIUM "FLASH"
(Isothermal)



$p = \text{const.}$
$T = \text{const.}$

(1) How many phases exist.

(2) How much (many moles) of each phase, if 2 (or more) phases exist.

n_V, n_L $\qquad n_V + n_L = n \qquad (+1)$

(3) How many moles of each component i in each phase.

n_{Vi} $\qquad n_{Li}$

Recall some definitions:

$$z_i = \frac{n_i}{n} \quad ; \quad n = \sum_{i=1}^{N_c} n_i$$

N_c = no. of components

Vapor: $y_i = \dfrac{n_{vi}}{n_v}$; $n_v = \sum_i n_{vi}$

Liquid: $x_i = \dfrac{n_{Li}}{n_L}$; $n_L = \sum n_{Li}$

$2 \times N_c$ unknowns

SOLUTION:

Need: z_i (or n_i)

$2 \times N_c$

From (p, T) also (z_i)

estimate K_i

Muskat (1949) in a footnote.

Rachford & Rice (195x)

$$\boxed{\sum_{i=1}^{N_c} (y_i - x_i) = 0}$$

$$\sum y_i - \sum x_i = 0$$

$$1 - 1 = 0$$

Component Material Balance

$$\frac{n_i}{n} = \frac{n_{vi}}{n} + \frac{n_{Li}}{n}$$

$$z_i = \frac{n_v \cdot y_i}{n} + \frac{n_L \cdot x_i}{n}$$

$$z_i = \underbrace{\frac{n_v}{n}}_{f_v} \cdot y_i + \underbrace{\frac{n_L}{n}}_{1-f_v} \cdot x_i$$

Introduce a variable

$$f_v \equiv \frac{n_v}{n}$$ vapor phase mole fraction

Phase Molar Balance:

$$\frac{n}{n} = \frac{n_v}{n} + \frac{n_L}{n}$$

$$1 = \left(\frac{n_v}{n}\right) + \frac{n_L}{n}$$

$$\frac{n_L}{n} = 1 - f_v$$

$$\Rightarrow \boxed{z_i = f_v\, y_i + (1-f_v)\, x_i} \quad \text{II}$$

Component Material Balance

$$\text{III} \quad \left.\sum (y_i - x_i) = 0 \right\} \Rightarrow \quad z_i, K_i$$

Know: z_i and $K_i = \frac{y_i}{x_i}$

$$z_i = f_v \cdot y_i + (1-f_v)\frac{y_i}{K_i}$$

$$= y_i\left[f_v + (1-f_v)\frac{1}{K_i}\right]$$

$$= y_i\left[f_v\left(1-\frac{1}{K_i}\right) + \frac{1}{K_i}\right]$$

$$y_i = \frac{z_i}{f_v\left(1-\frac{1}{K_i}\right) + \frac{1}{K_i}} \cdot \frac{K_i}{K_i}$$

$$\boxed{y_i = \frac{z_i K_i}{f_v(K_i-1)+1}}$$

$$x_i = \frac{y_i}{K_i} \qquad K_i = \frac{y_i}{x_i}$$

$$\boxed{x_i = \frac{z_i}{f_v(K_i-1)+1}}$$

$$\boxed{\sum_{i=1}^{N_c} \frac{z_i(K_i-1)}{f_v(K_i-1)+1} = 0} \quad [1]$$

Muskat:

$$\boxed{g(f_v) = \sum \frac{z_i}{f_v + c_i} = 0}$$

$$c_i \equiv \frac{1}{K_i - 1}$$ variable transform.

sum. term $= 0$ if $K_i = 1$

i	z_i	K_i
1	0.6	10
2	0.2	1
3	0.2	0.1

How does $g(f_v)$ "look"

$$-\infty < f_v < \infty$$

How to Solve?

e.g. in Excel

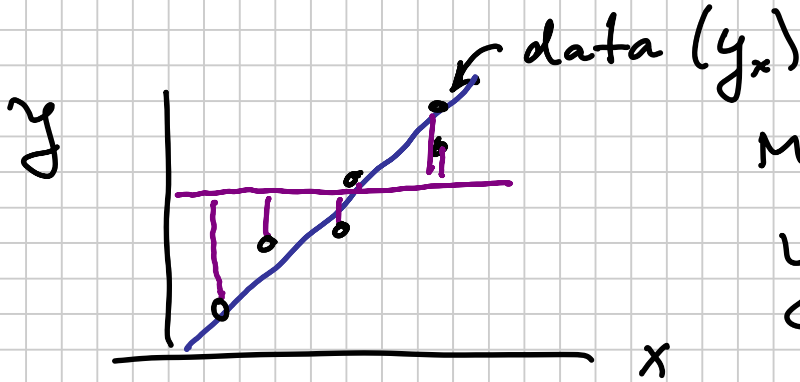
To use Solver to minimize (...)

you should create a single-cell "function"

$\boxed{} \leftarrow \sum (y_m - y_x)^{2}$

$\boxed{a}$

$\boxed{b}$



Model: (e.g.)

$y_m = ax + b$

?

$\sum (y_i - x_i)^2 = 0$ No.

$\left[\sum (y_i - x_i)\right]^2 = 0$

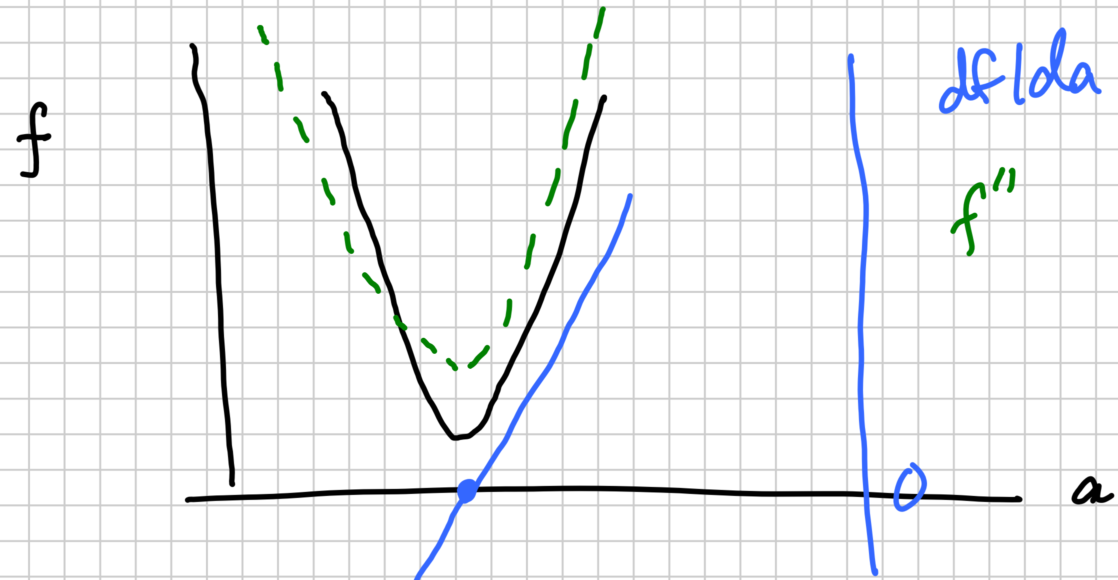
$\sum (y_i - x_i) = 0$

Problems found, $f_r \gg 1$ (maybe $f_r \ll 0$) - non physical

What is Solver doing?

Target Cell contains function $f(\underbrace{a, b, c \ldots}_{\text{parameters}})$

- e.g. minimize



min. $f(a)$ is found when $\frac{df}{da} = 0$

$$\frac{d^2f}{da^2} > 0$$

$$f \to \underbrace{f' = 0}_{\text{Newton-Raphson method}}$$

(needs f'')

Rachford-Rice Problem

$$\sum \frac{z_i}{c_i + f_v} = 0$$

$$c_i = \frac{1}{K_i - 1}$$

What if choose $f_v = -\dfrac{1}{K_1 - 1}$

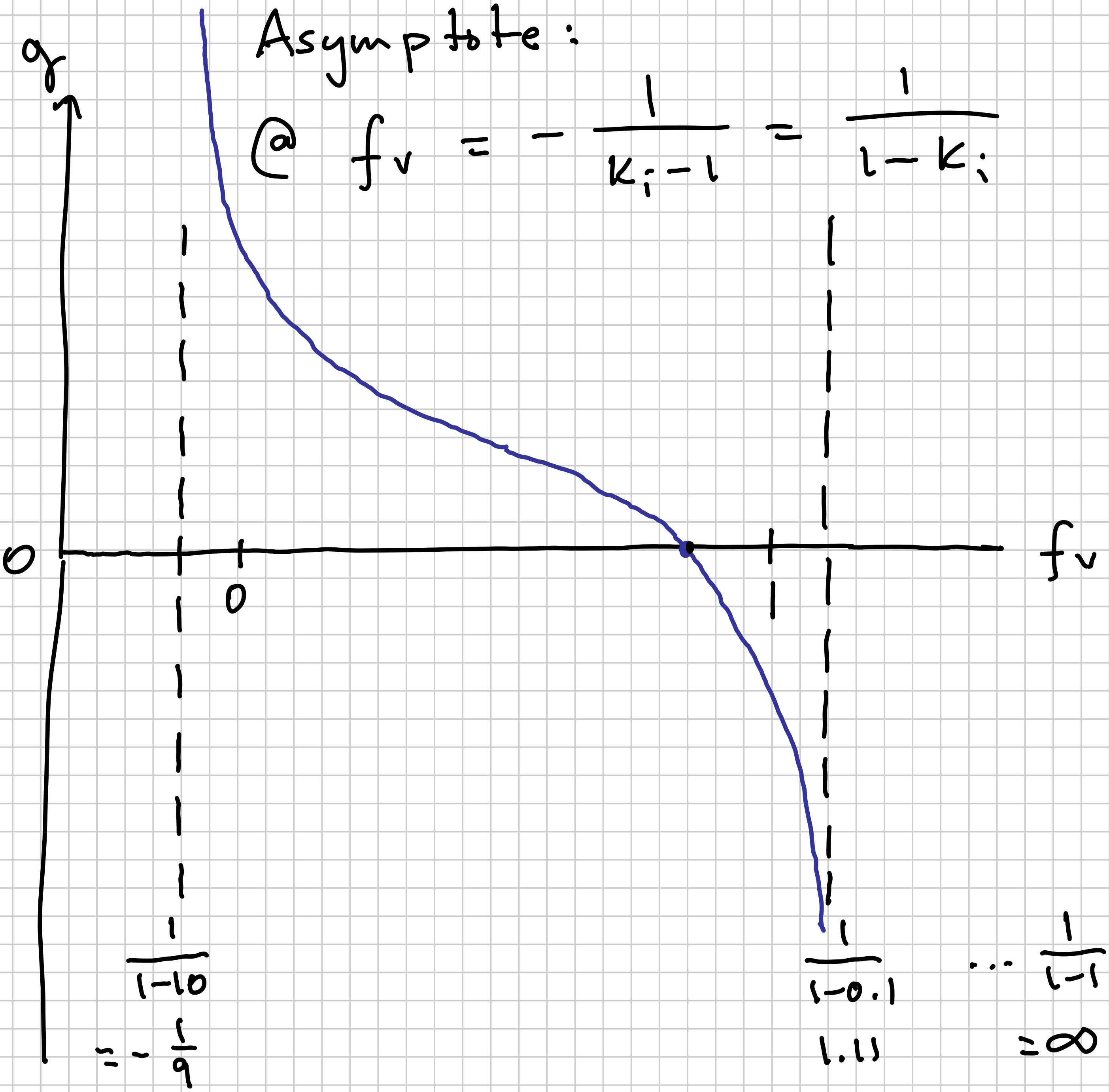
$$\sum = \frac{z_1}{\left\{\left(\frac{1}{K_i - 1}\right) - \left(\frac{1}{K_1 - 1}\right)\right\}} + \cdots$$

$$\underbrace{\left\{\left(\frac{1}{K_i - 1}\right) - \left(\frac{1}{K_1 - 1}\right)\right\}}_{0}$$

$\therefore \sum \to \infty$

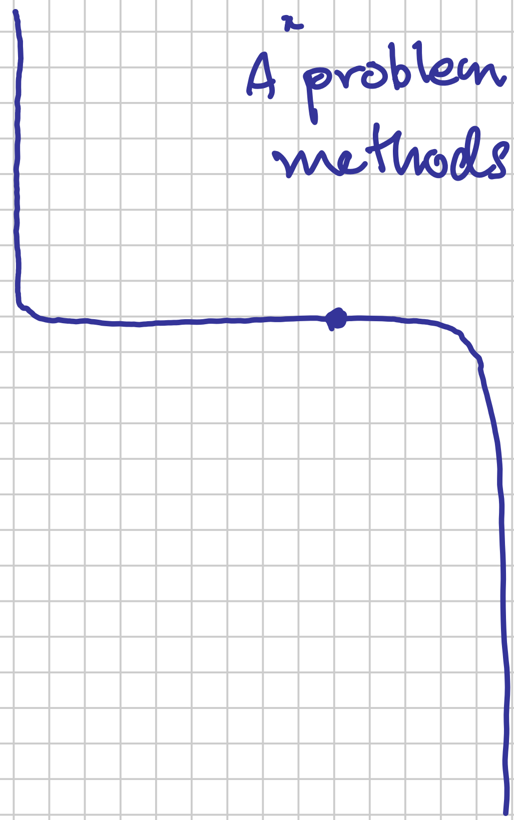
Asymptote:

$@\ f_v = -\dfrac{1}{K_i - 1} = \dfrac{1}{1 - K_i}$



$z_i \sim \epsilon$ $\quad K_i \begin{cases} \nearrow \text{large} \\ \searrow \epsilon \end{cases}$

$g(f_v)$



A "problem" for NR methods.

Numerical Roundoff

General RR Characteristics:

(1) Asymptotes at $-\dfrac{1}{K_i - 1}$ or $\dfrac{1}{1-K_i}$

solution giving $g(f_v) = 0$ between each asymp.

(2) The only PHYSICAL SOLUTION $(x_i \geq 0,\ y_i \geq 0)$

is $\dfrac{1}{1-K_{max}} < f_v < \dfrac{1}{1-K_{min}}$

negative $\qquad$ positive

$K_{max} > 1$

$K_{min} < 1$

BOUNDS $\begin{cases} f_{min} = \dfrac{1}{1-K_{max}} \\ f_{max} = \dfrac{1}{1-K_{min}} \end{cases}$

$c_i = \dfrac{1}{K_i - 1}$

(3) $f_{min} < 0 \quad < 1 < f_{max}$

PHYSICAL Solution $0 \leq f_v \leq 1$
is always within the bounds
of search

(4) If $f_v < 0$ or $f_v > 1$
a single phase exists.

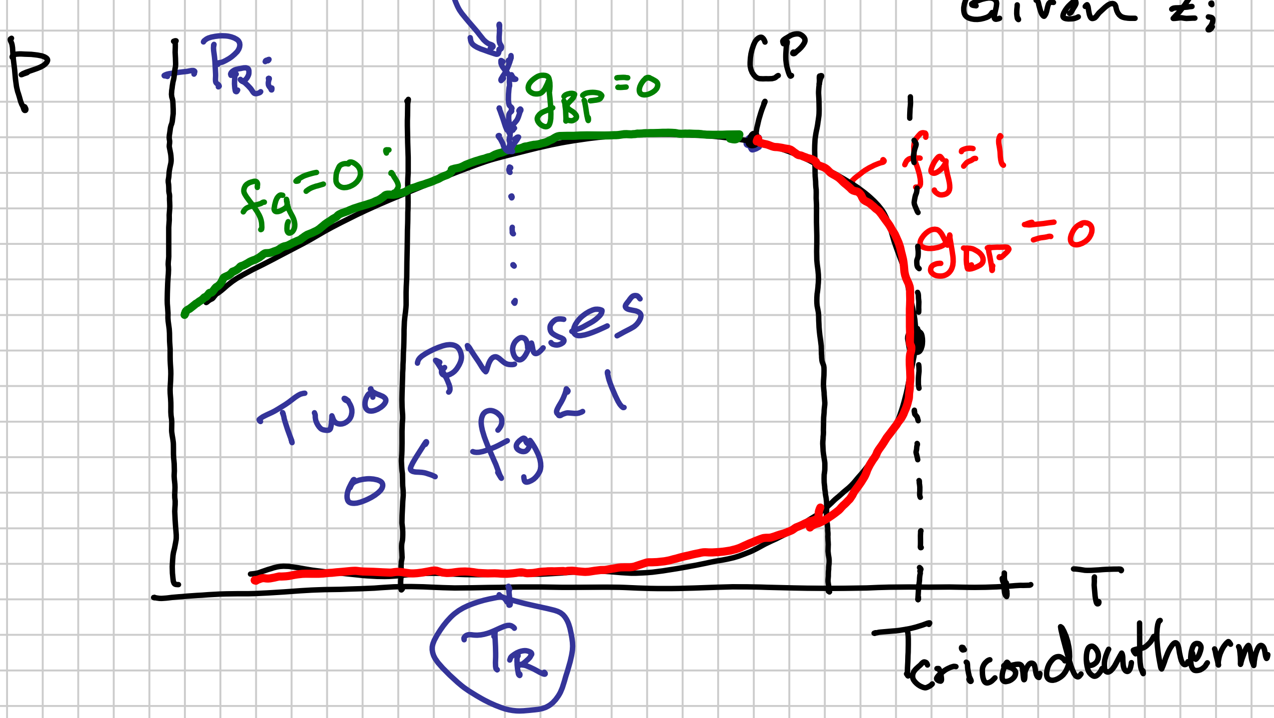
(5) If $0 \leq f_v \leq 1$, calculate
the x_i and y_i, done.

SATURATION PRESSURE CALCULATION

- DEWPOINT
- BUBBLEPOINT

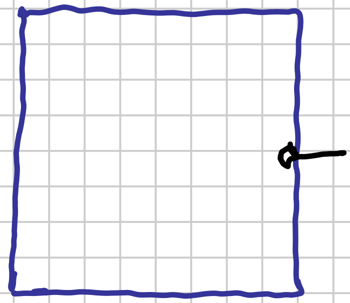
Following Flash Calculation where the result says the mixture z_i is single phase.

Given z_i



- Is it "oil" or "gas"?
- p_s?

What is the characteristic of a phase at its saturation pressure?



A small (ϵ) amount of a second phase appears.

$$f_v$$

$$z_i = f_g \cdot y_i + (1-f_g)\,x_i$$

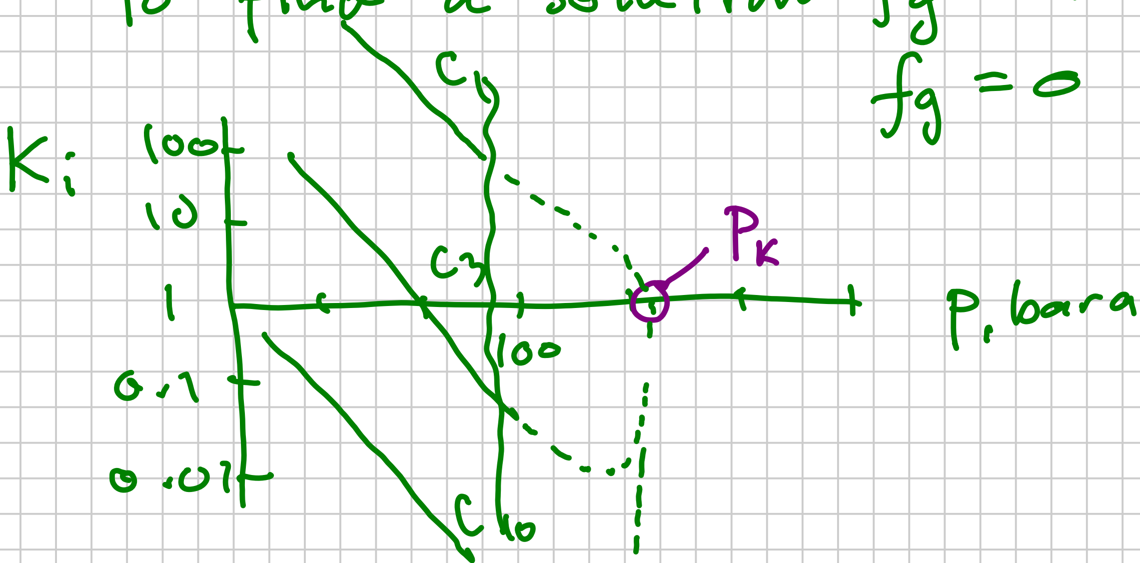
Dewpoint: $f_g = \dfrac{n_g}{n_g + n_o} = \dfrac{n_g}{n_g + \epsilon} = 1$

Bubblepoint: $f_g = 0 \;\; (\epsilon)$

If you have a flash solution how might I calculate sat. pressure?

$K_i(p)$, trial and error with p to find a solution $f_g = 1$ " DP

$f_g = 0$ " BP



ALTERNATIVE:

(1) Bubblepoint

Know $x_i = z_i$

$$\frac{y}{x} = K$$

$$y_i = K_i \cdot x_i$$
$$= K_i \cdot z_i$$

$$1 = \sum y_i = \sum K_i z_i$$

z_i is given

$K_i(p)$ < simplified Wilson ($p < 100$ bar) / modified Wilson ($p \leq p_K$)

$$\boxed{g_{BP}(p) = 1 - \sum z_i \cdot K_i(p) = 0}$$

when $p = p_b$

(2) DEWPOINT:

Know $y_i = z_i$

$$x_i = y_i / K_i$$

$$K_i = \frac{y_i}{x_i}$$

$$1 = \sum x_i = \sum y_i / K_i(p) = \sum \frac{z_i}{K_i(p)}$$

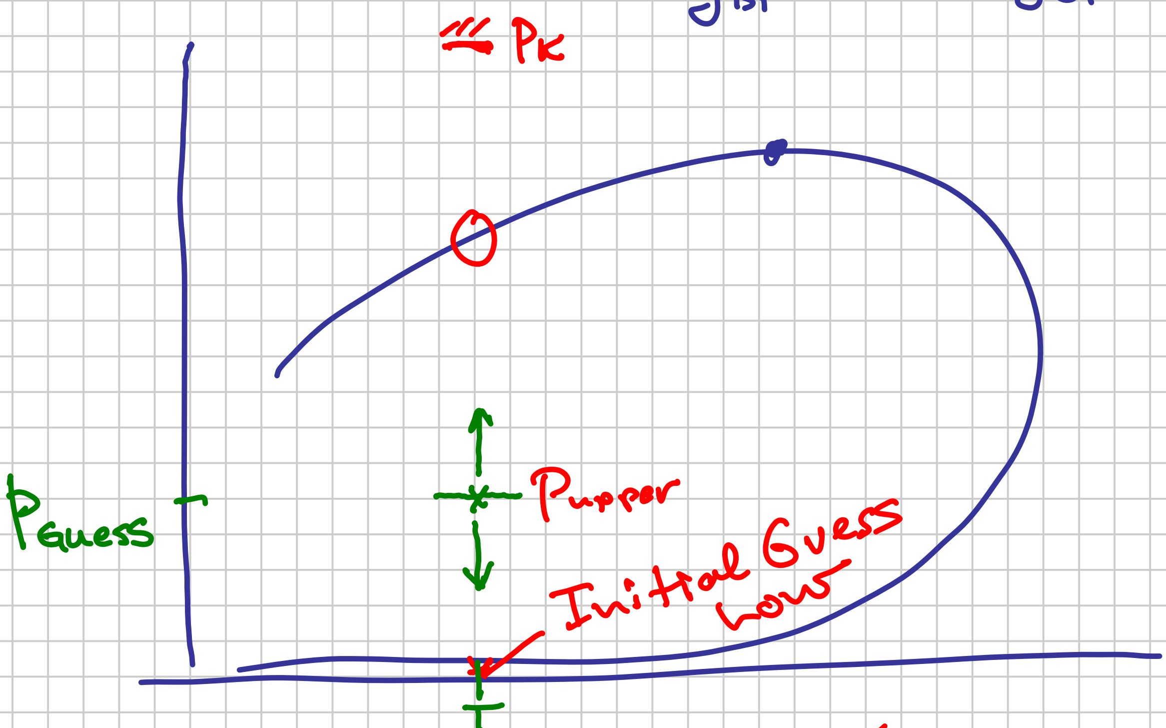
$$\boxed{g_{DP} = 0 = 1 - \sum \frac{z_i}{K_i(p)}}$$

Given z_i, T

Solve for (1) Lower DP $\quad g_{DP} = 0$

(2) Upper BP or DP

$g_{BP} = 0 \qquad g_{DP} = 0$

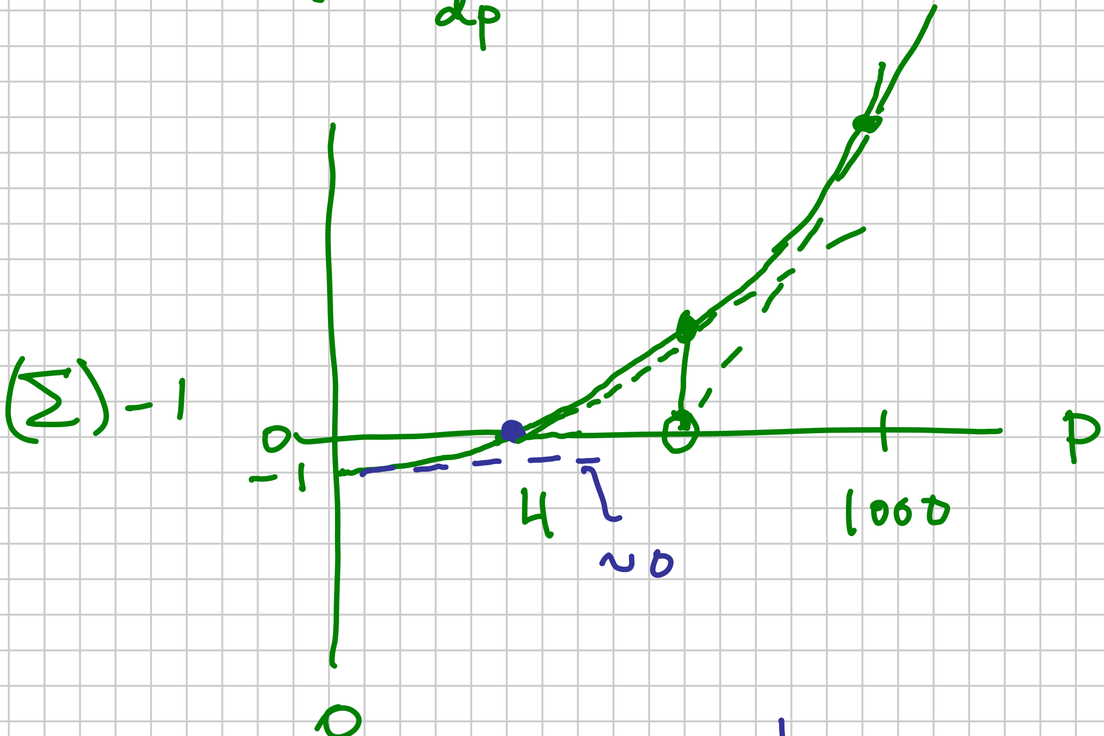


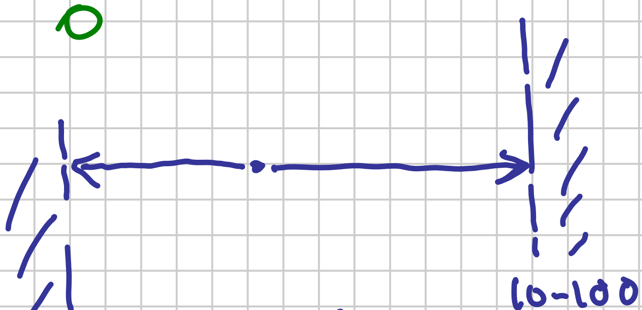
- Start with a guess ✓
- Need a bound ✓

Solver $f = (1-\Sigma)$

$$p^n = p^0 - \frac{f}{f'}$$

$$f' = \frac{df}{dp}$$





Radius of Convergence

SOLVING FOR SATURATION PRESSURE

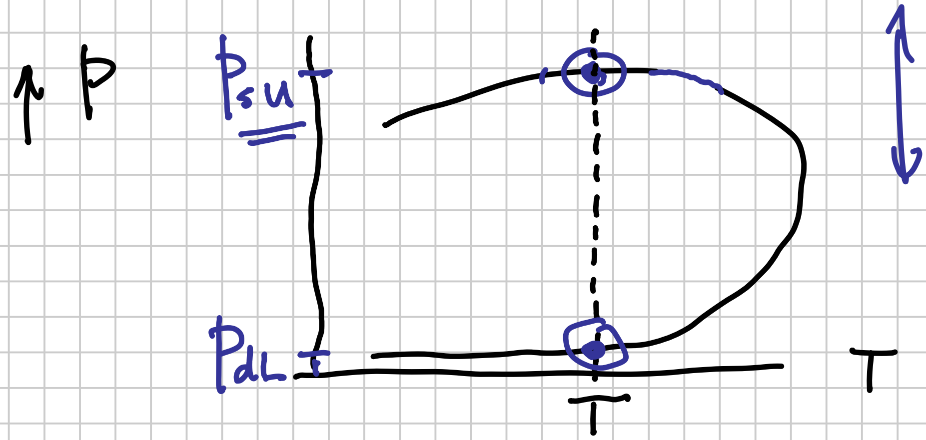
Lower DP : $g_{LDP} = 1 - \sum x_{iL} = 1 - \sum z_i / K_{iL} = 0$

Upper DP : $g_{UDP} = 1 - \sum y_{iL} = 1 - \sum z_i / K_{iU} = 0$

(Upper) BP : $g_{BP} = 1 - \sum x_{iU} = 1 - \sum z_i K_{iU} = 0$

$$K_i = y_i / x_i \Rightarrow y_i = K_i x_i \,,\; x_i = y_i / K_i$$

SIMULTANEOUS LDP & USP SOLUTION?



Need $\boxed{g^*} = 0$ by changing $\boxed{P_{dL}}, \boxed{P_{su}}$

$P_{LDP} > P_{lower\ bound}$

$P_{LDP} < P_{su}$

$P_{su} < P_K$

Constraints?

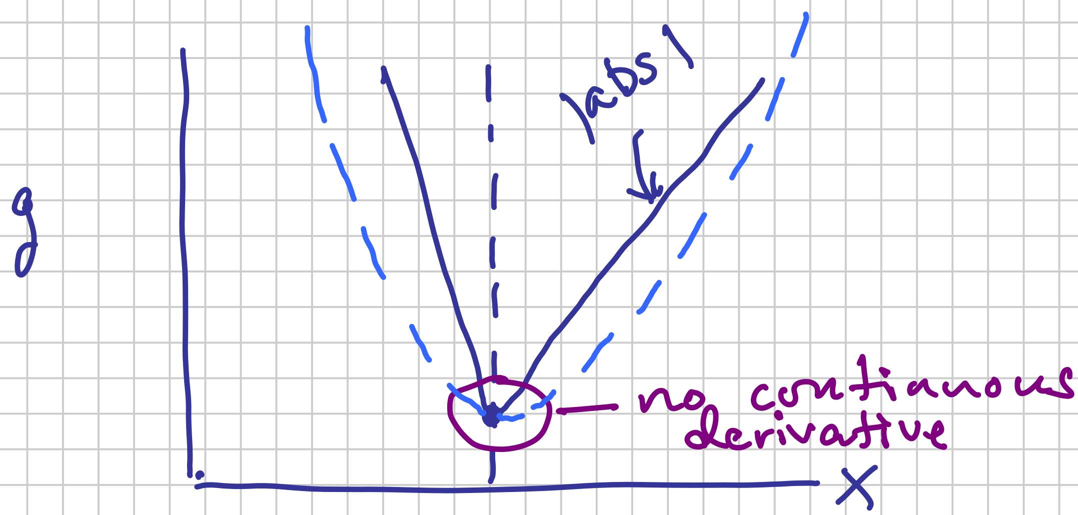
Ideas:

$$\underset{\to 0}{g_{LDP}} + g_{UDP} + g_{BP} \neq 0$$

$$? \neq 0$$

$$\underbrace{\underset{\to 0}{g_{LDP}} * g_{UDP} * \underset{\to 0}{g_{BP}}}_{} = 0$$

$$\underset{\substack{\to 0 \\ (3)}}{g_{LDP}^2} + \left(\underset{\substack{.0.342 \\ (6)}}{g_{UDP}} \cdot \underset{\substack{\to 0 \\ (6)}}{g_{BP}}\right)^2 = \underset{\substack{\to 0 \\ (6)}}{0}$$



ESTIMATING p_K FOR A RESERVOIR FLUID SYSTEM

$$K_i(p, T, z_i) \sim K_i(p, T, p_K)$$

Modified Wilson Eq.
"represents" z_i dependence of K-values using Convergence Pressure p_K.

Use a measured data for our fluid system.

- The data should be very dependent on K-values, and particularly high-p K-values.
  - Namely, <u>Upper</u> Sat. Pressure

$$P_{su} \begin{cases} P_{du} \\ P_b \end{cases} \Bigg\} \text{ Measured at } T_R$$

$$g^* = g_{DPW} \cdot g_{BP} = 0$$

z_i: ✓ (Known)

$K_i(p_{sw}, T_R, p_K)$

Know: measured

Also know DP (w) BP

When p_K found, we have a general relation (mod-Wilson eq.) to get $K_i(\underbrace{p, T}_{\text{Vary}}, p_K = \text{const})$

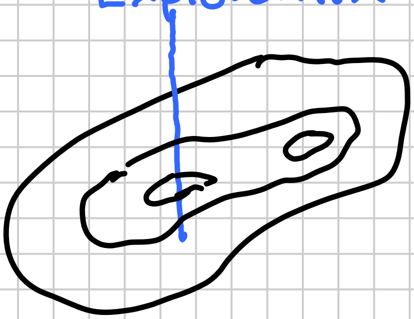
Vary Reservoir, Production, Surface Process

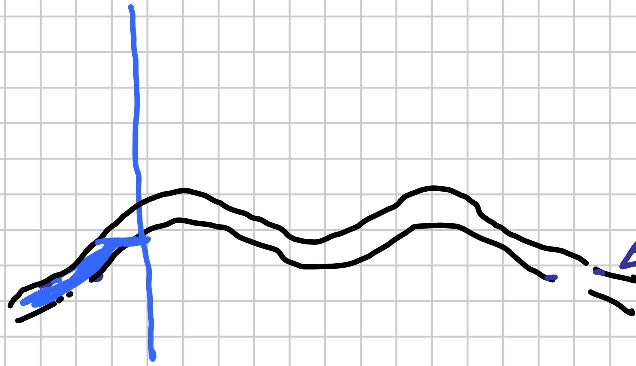
PROBLEM 5: Excercises 15 & 16 SPEPBM (extra credit, Ex. 17)

using a North Sea Fluid.

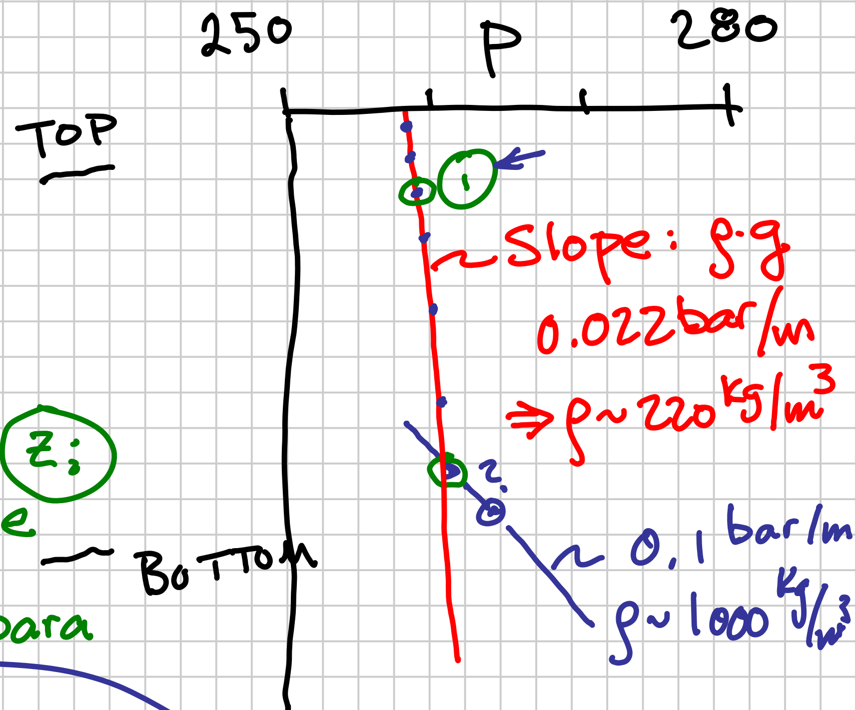
Seismic

Exploration





Structure
(top view)



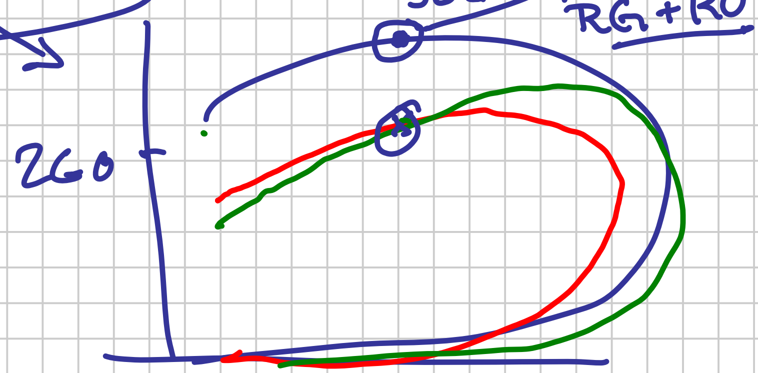
⊙ Fluid Samples

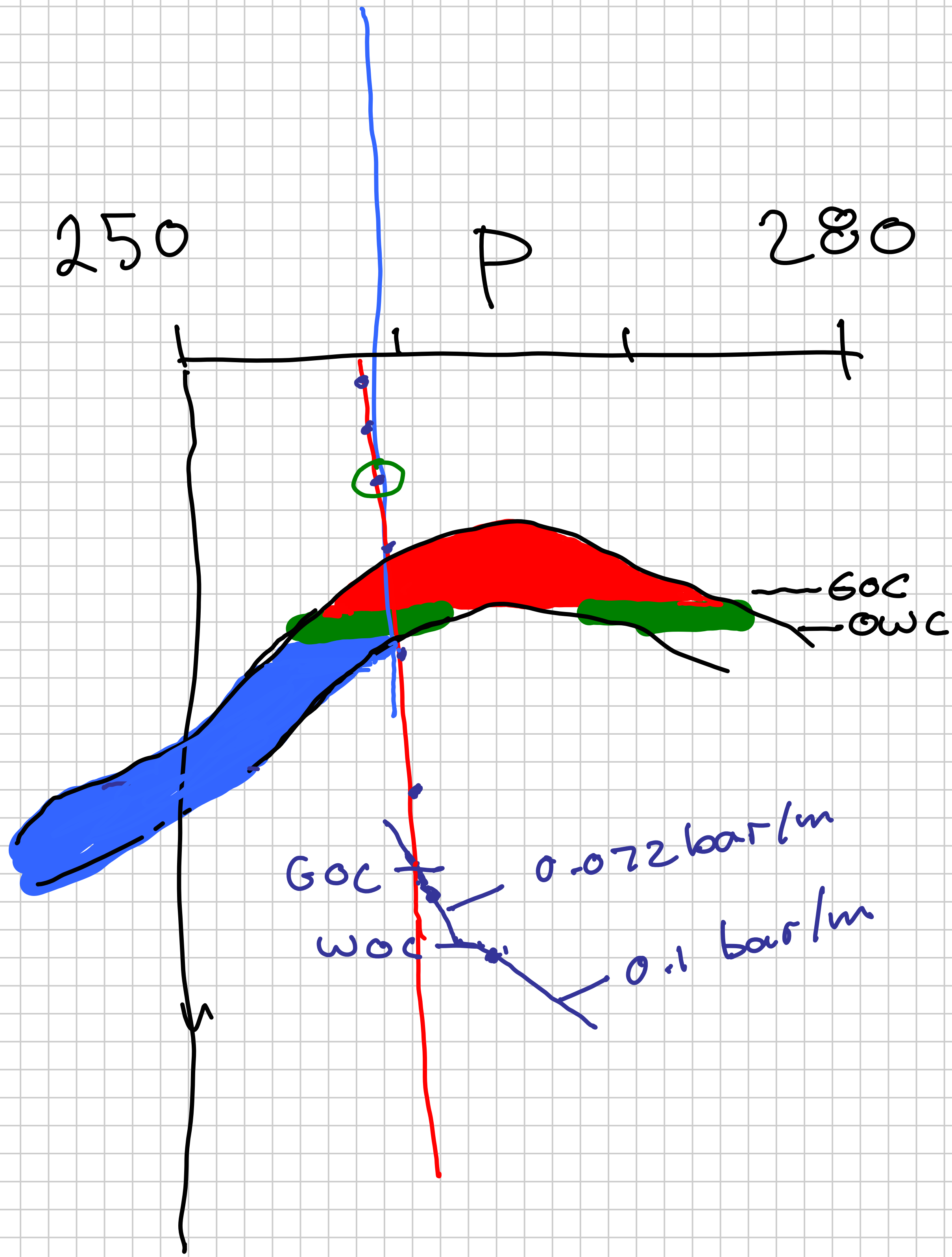
→ Lab

- Composition z_i
- Sat. Pressure

Explain ?

- p_d (1) = 258 bara
- p_d (2) = 382 bara





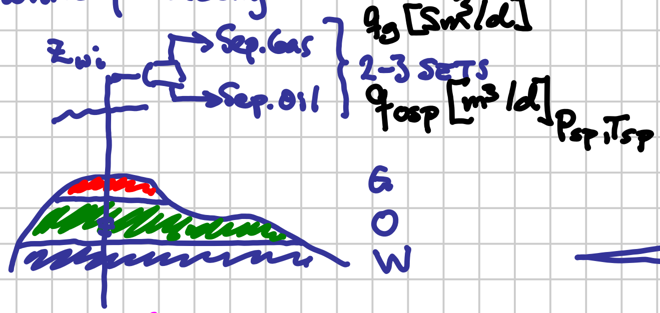
250
P
280
GOC
OWC
GOC
WOC
0.072 bar/m
0.1 bar/m

PVT Laboratory Data
(Ch. 6)

* Collect Samples

SURFACE

- Separator Gas & Oil while producing
  - Gas: 20 L
  - Oil: ~1 L

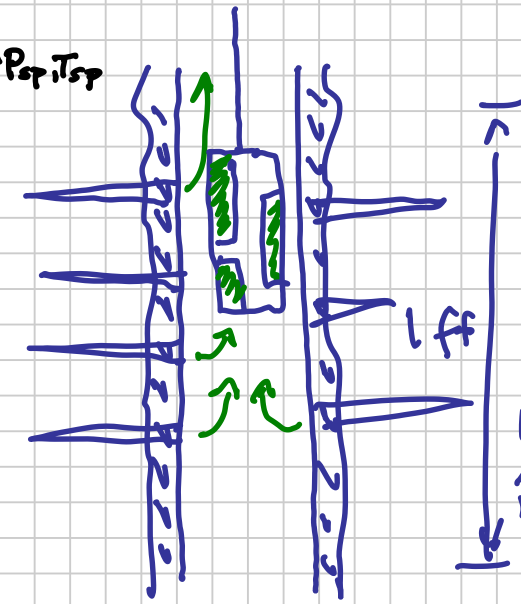


RO & RG

Larger (more) Sample

BOTTOMHOLE

Cased Hole

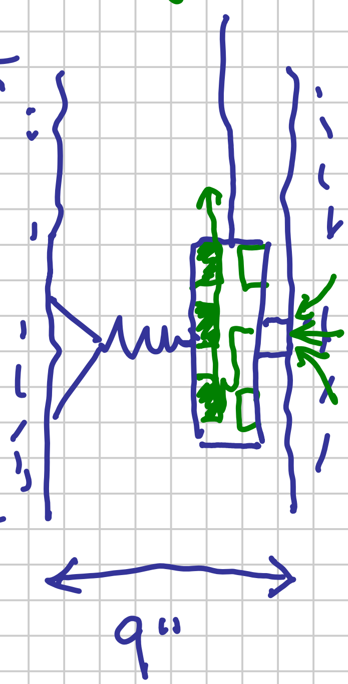


During Production Test

ONLY RESERVOIR OILS

Open Hole

(MDT; RCI)



RO or RG

Open Hole
Well Log

→ Open Hole Formation Tests (onsite flow indicators)

G ✓
O ✓

Logs indicating HC type (G vs O) are not always reliable (Neutron / Density)

What do we do with Samples?

(1) Inspection of Stock-tank oil @ 1atm, 20ish °C
- look, smelling, feeling; density*

(2) Onsite chemical analysis (seldom)
- gas composition (CO_2, H_2S, C_1, N_2 ...)

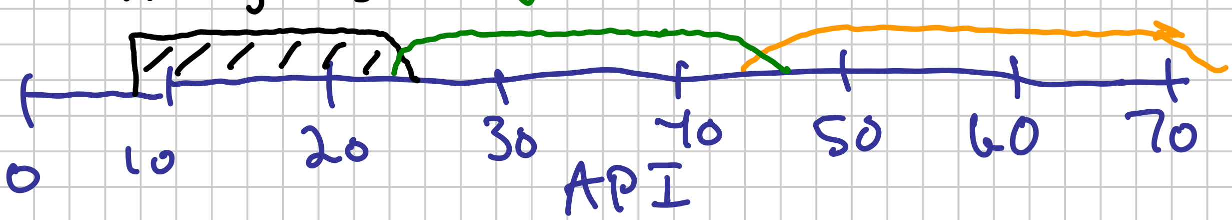
* Unit API for density ("gravity")

Liquid Specific Gravity γ_o

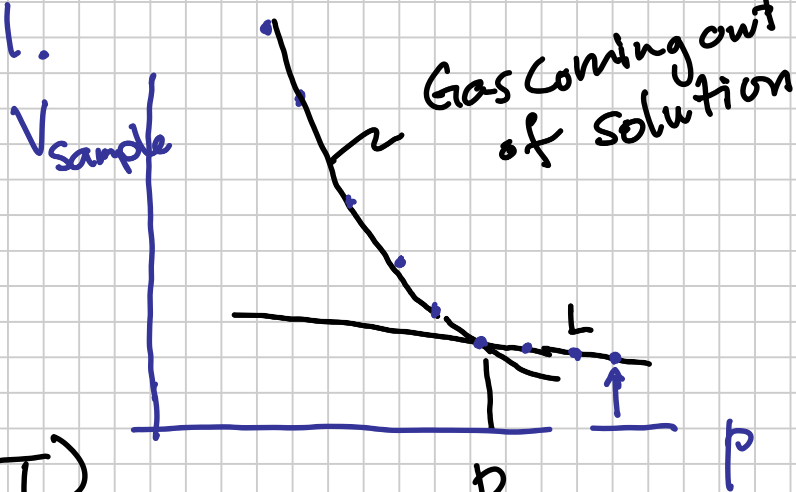
$$\gamma_o \equiv \frac{\rho_o(1\ \text{atm}, 15.56^\circ C)}{\rho_w(1\ \text{atm}, 15.56^\circ C)}$$

$$\gamma_{API} \equiv \frac{141.5}{\gamma_o} - 131.5$$

$\gamma_{API} = 10$: water



(3) On-site bubblepoint measurement of a BH oil.



P_b @ 10°C --> 35°C

Compare P_b Field (T) vs P_b Lab (T) =

Check for LEAKS

(4) SEND to a lab (several labs)

- Storage (short-term & long-term)
- Compositional Analysis z_i
- Conduct experiments on the "reservoir" (wellstream) samples, at T_R.

- DEPLETION (EXPANSION) TESTS
- MULTI-STAGE SEPARATOR
- GAS INJECTION (Specialty) tests
- SOLIDS PRECIPITATION
  - Waxes
  - Asphaltenes

PVT Laboratories

1. Reslab (Stavanger & UK & Middle East)
   Egil Linjord
2. Core Laboratories
   - Generally Quite Good
3. Oil Phase (Schlumberger)
   - Don't recommend !

PVT EXPERIMENTS

Depletion-Type Tests

- Constant Composition (Mass) Expansion "CCE"

* – Determine p_s < DP / BP (Windowed PVT Cell)

Volumetric Data

– Single Phase Density, $\rho(p > p_s)$

– Oils: ρ_o, $c_o = \frac{1}{\rho_o}\left(\frac{d\rho_o}{dp}\right)$

– Gases: Z_g ; $\rho_g = \frac{p M_g}{Z_g R T_R}$ (M_g = const)

– High GOR mixtures ($> 300^+$ Sm^3/Sm^3)

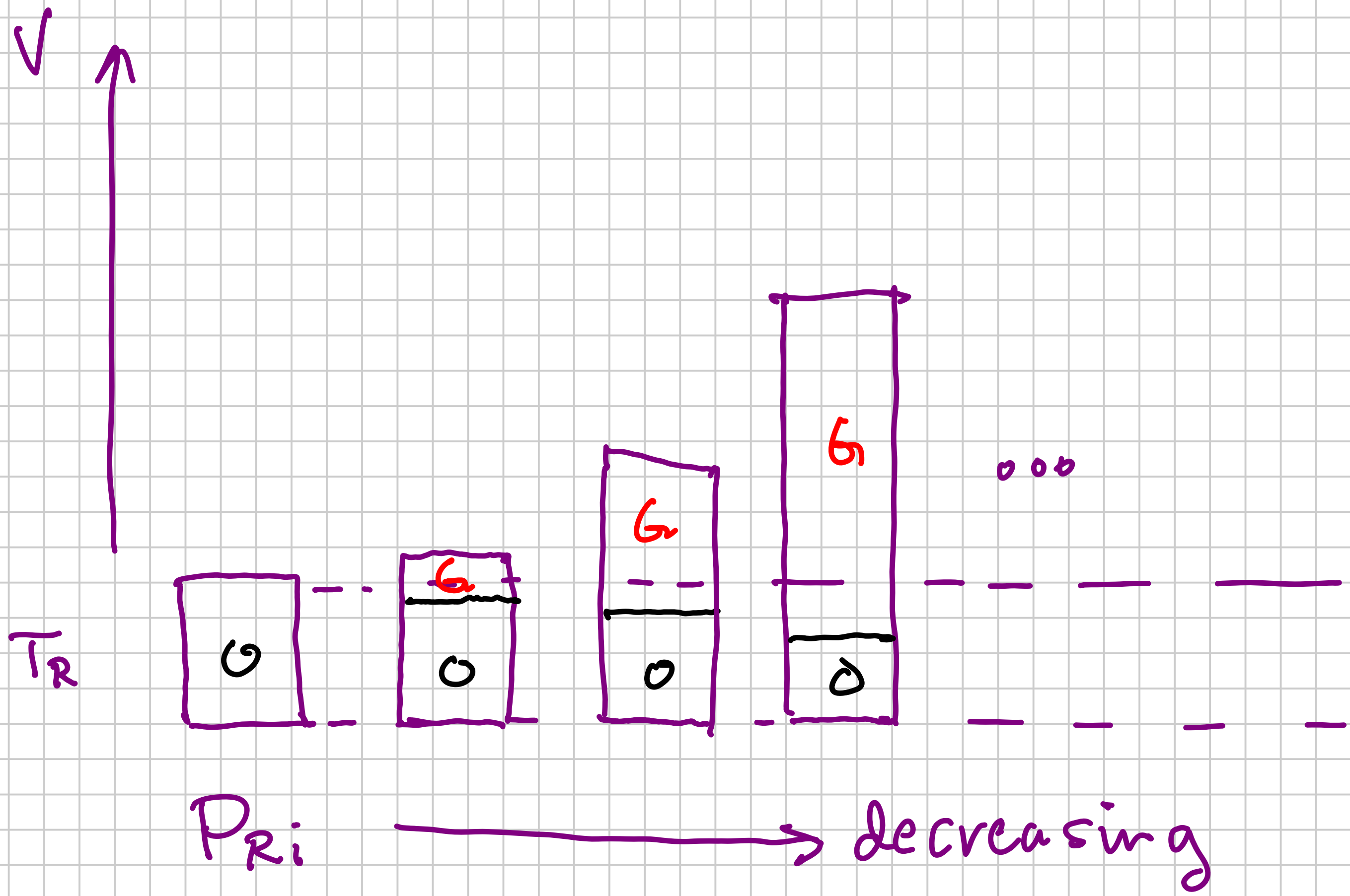
* Phase Equilibrium type data

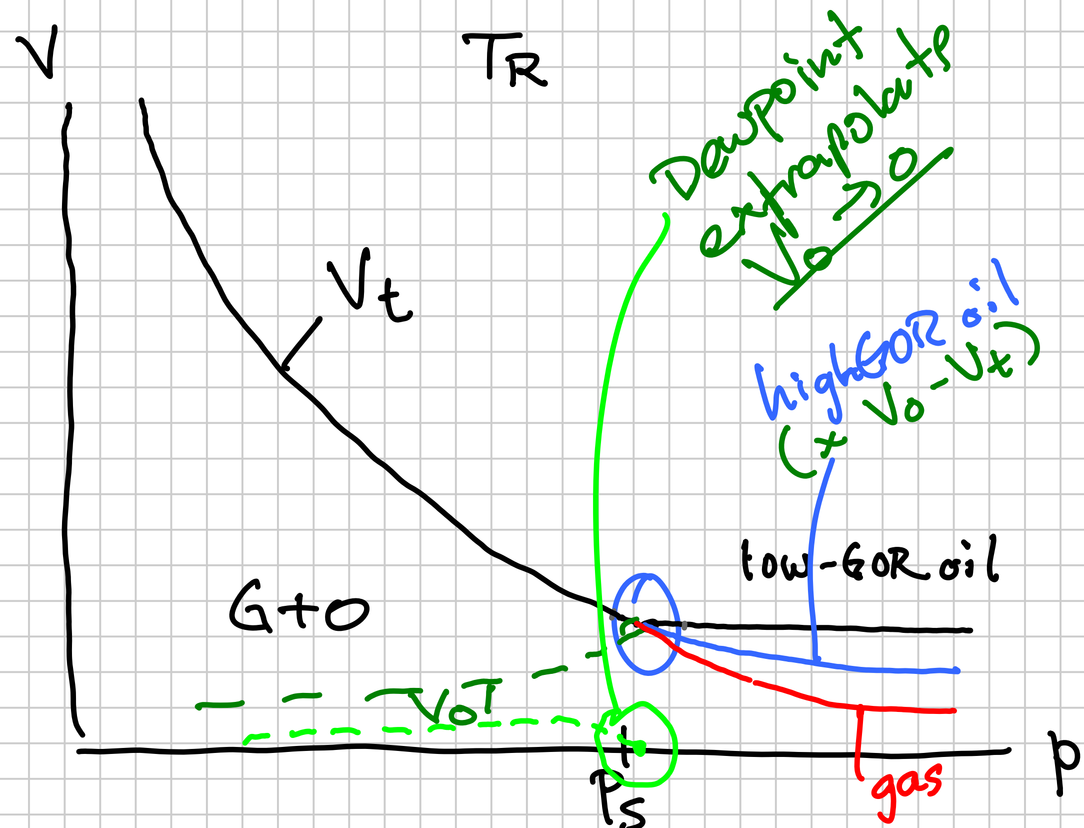
– Oil Relative Volume (visual cell), $p < p_s$

$V_o/(V_o + V_g)$ || CLOSELY LINKED to f_g (f_v) RR flash calc

phase molar | $f_g = \frac{n_g}{n_g + n_o}$

ratio $\quad f_o = 1 - f_g = \dfrac{n_o}{n_g + n_o} \sim \dfrac{V_o}{V_o + V_g}$





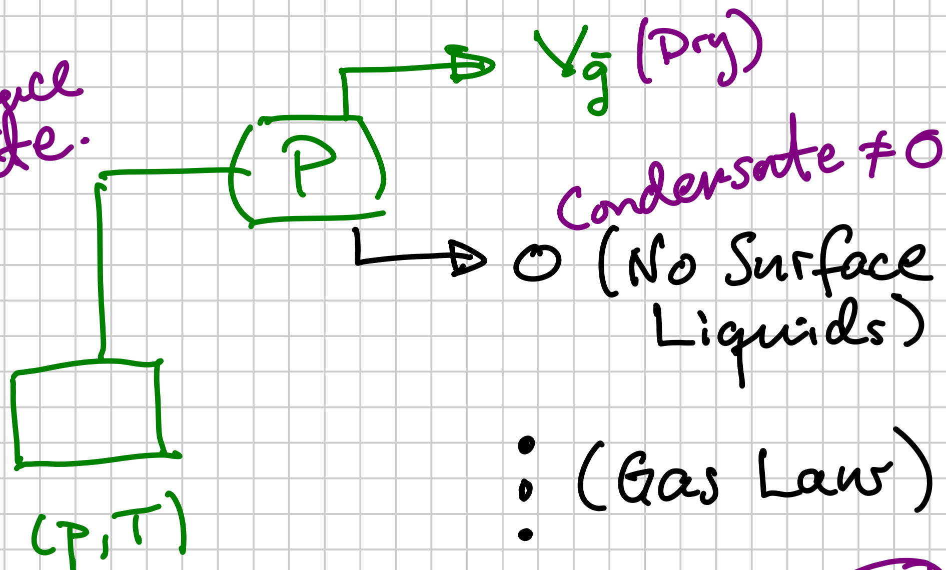
B_{gd} ?

Traditionally B_g, gas FVF

$$B_{gd} = \frac{V_g(p,T)}{V_{\bar{g}}(p_{sc},T_{sc})}$$

Assumes all reservoir gas becomes Surface Gas

We do NOT assume zero surface condensate.



∴ (Gas Law)

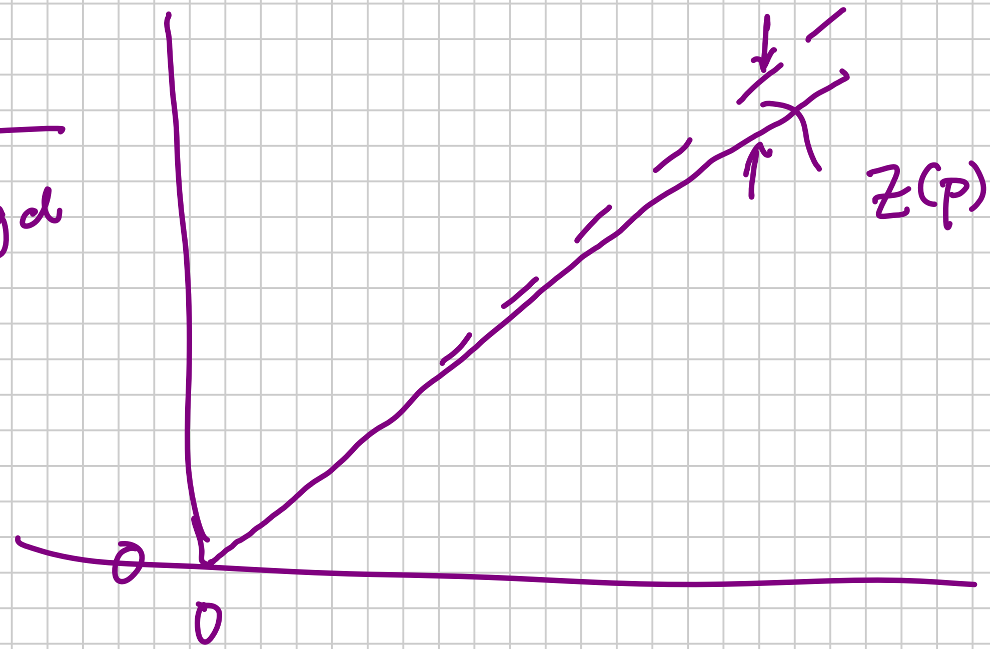
Traditional "Wet Gas"

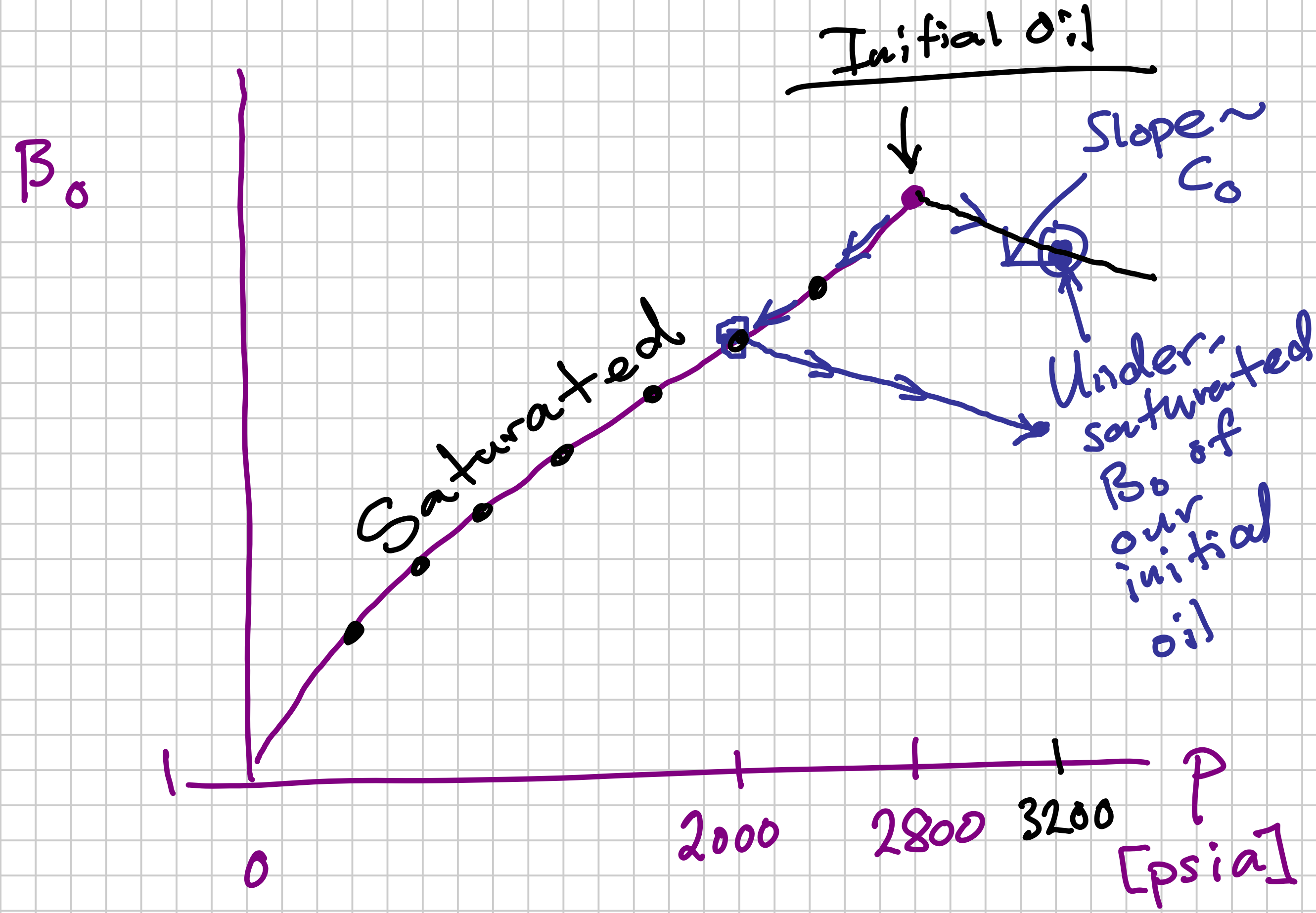
$$B_g = \frac{p_{sc}}{T_{sc}} \cdot \frac{T_R Z_R}{p_R}$$

Why plot $\left(\frac{1}{B_{gd}}\right) \tilde{\propto} p$ instead B_{gd}

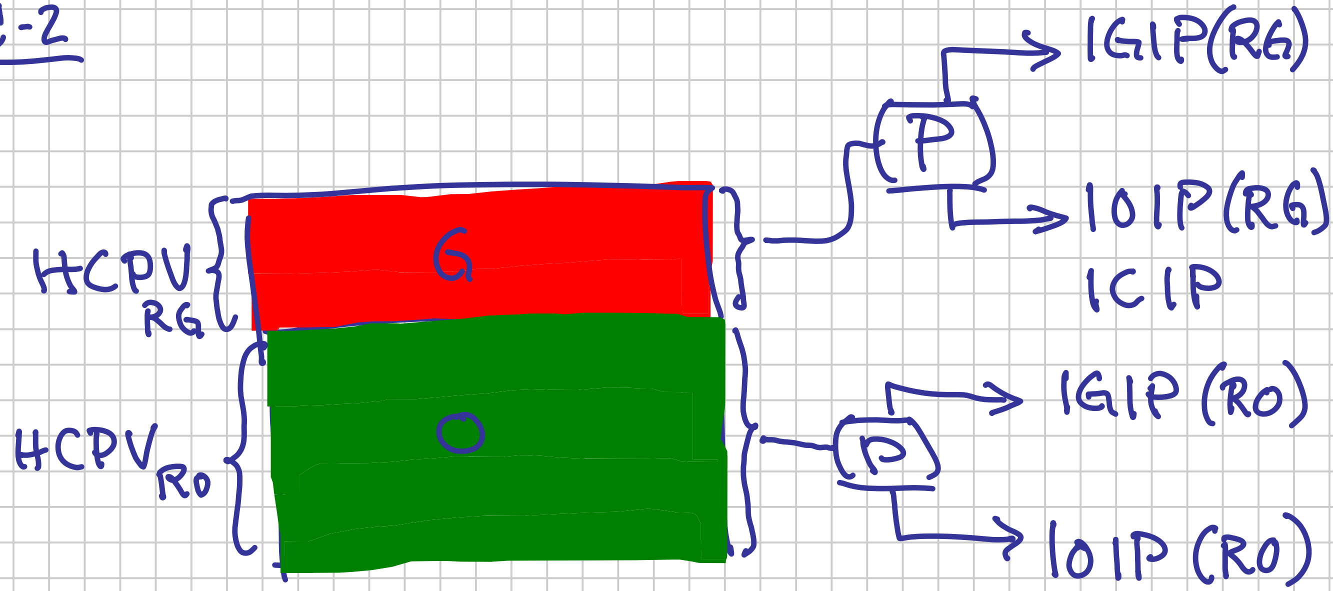
vs p

$$b_{gd} = \frac{1}{B_{gd}}$$





C-2



$$HCPV = \bar{A} \cdot \bar{h} \cdot \bar{\phi}(1-\bar{S}_w)$$

[redacted] $= \dfrac{\text{[redacted]}}{B_{gd,i}}$

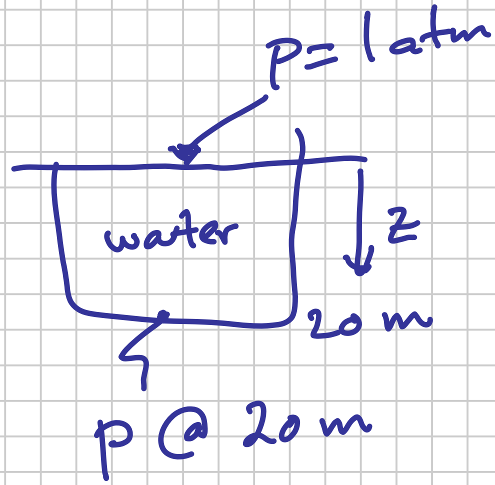
[redacted] $= ICIP = IGIP_{RG} \cdot r_s$

$= HCPV_{RG} \cdot \dfrac{r_s}{B_{gd}}$

[redacted] $= IOIP_{Ro} \cdot R_s =$ [redacted] $\cdot \dfrac{R_s}{B_o}$

[redacted] $= \dfrac{HCPV_{Ro}}{B_{o,i}}$

$$G = \frac{dp}{dz} \qquad \frac{bar}{m} , \quad \frac{psi}{ft}$$



$$\frac{dp}{dz} = \rho g$$

lb_m "g_c" lb_f

$$\frac{dp}{dz} = \rho \left[\frac{lb_m}{ft^3}\right] \frac{g}{g_c}$$

$$\boxed{p(z) = p_{ref} + \rho g \cdot (z - z_{ref})}$$

$$\frac{lbf}{in^2}$$

$g = 32. \; ft/s^2$

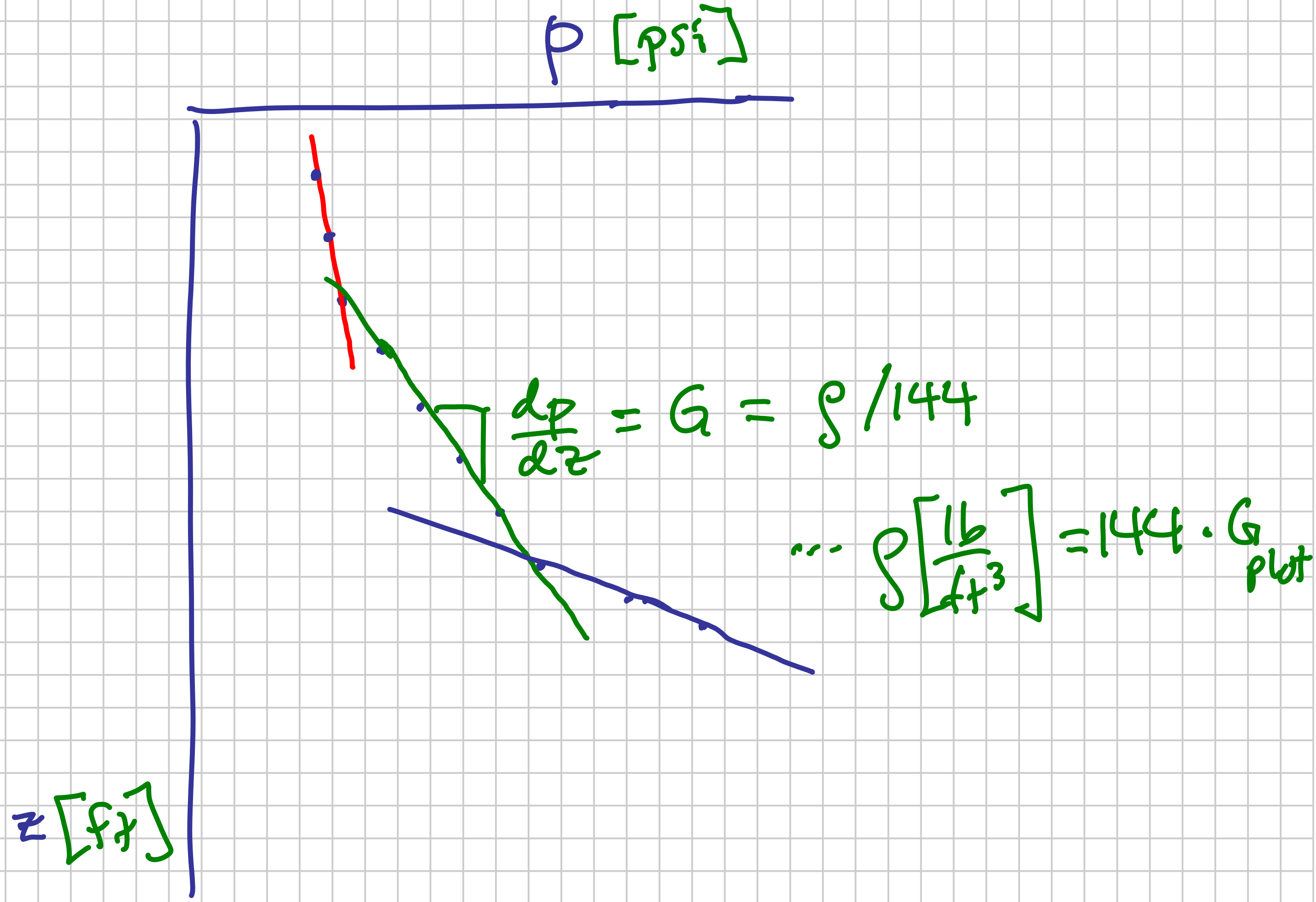
$$= 1 \text{ bar} + \underbrace{1030(9.8)(20)}_{Pa} \cdot 10^{-5} \frac{bar}{Pa}$$

$g_c = 32 \ldots$

$$= 1.2 \text{ bar}$$

$$G\left[\frac{psi}{ft}\right] = \rho \left[\frac{lbm}{ft^3}\right] \cdot \frac{1}{144 (in^2/ft^2)}$$

$$G\left[\frac{bar}{m}\right] = \rho \left[\frac{kg}{m^3}\right] \cdot 9.8 \left[\frac{m}{s^2}\right] \cdot 10^{-5}$$



Compare $\rho_{(G)}$ vs ρ_{BOPVT}

BO Model

$$\rho_o = \frac{\rho_{\bar{o}} + \rho_{\bar{g}} R_s}{B_o}$$

$$\rho_g = \frac{\rho_{\bar{g}} + \rho_{\bar{o}} \cdot r_s}{B_{gd}}$$

BEWARE

Units

R_s [scf/STB]

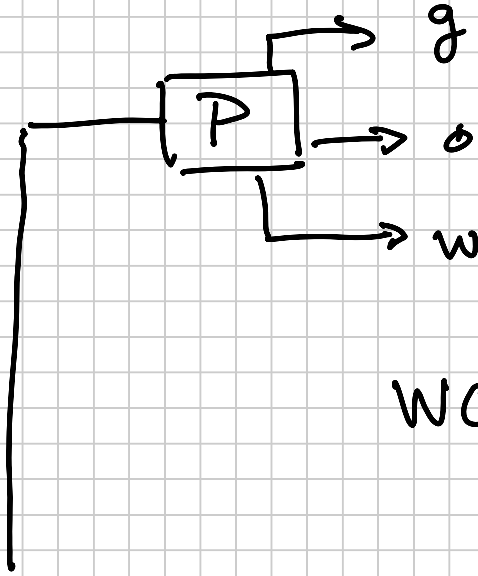
Liquid Sp. Grav. Gas Sp. Grav.

$$\rho_{\bar{o}} = \gamma_{\bar{o}} \cdot \rho_{\bar{w}} \; ; \; \rho_{\bar{g}} = \gamma_{\bar{g}} \cdot \rho_{air,sc}$$

Water Cut

Water-Oil Ratio

$$WOR = F_{wo} = \frac{q_{\bar{w}}}{q_{\bar{o}}}$$



Total Liquid

Water Cut, WC,

$$WC = f_{\bar{w}} \equiv \frac{q_{\bar{w}}}{q_{\bar{w}} + q_{\bar{o}}} = \frac{q_{\bar{w}}}{q_{\bar{l}}}$$

Gas-Oil Ratio : $R = R_p = \dfrac{q_{\bar{g}}}{q_{\bar{o}}}$

Oil-Gas Ratio

Condensate-Gas Ratio

$$r = r_p = \frac{q_{\bar{o}}}{q_{\bar{g}}}$$

$$R_p = \frac{1}{r_p}$$

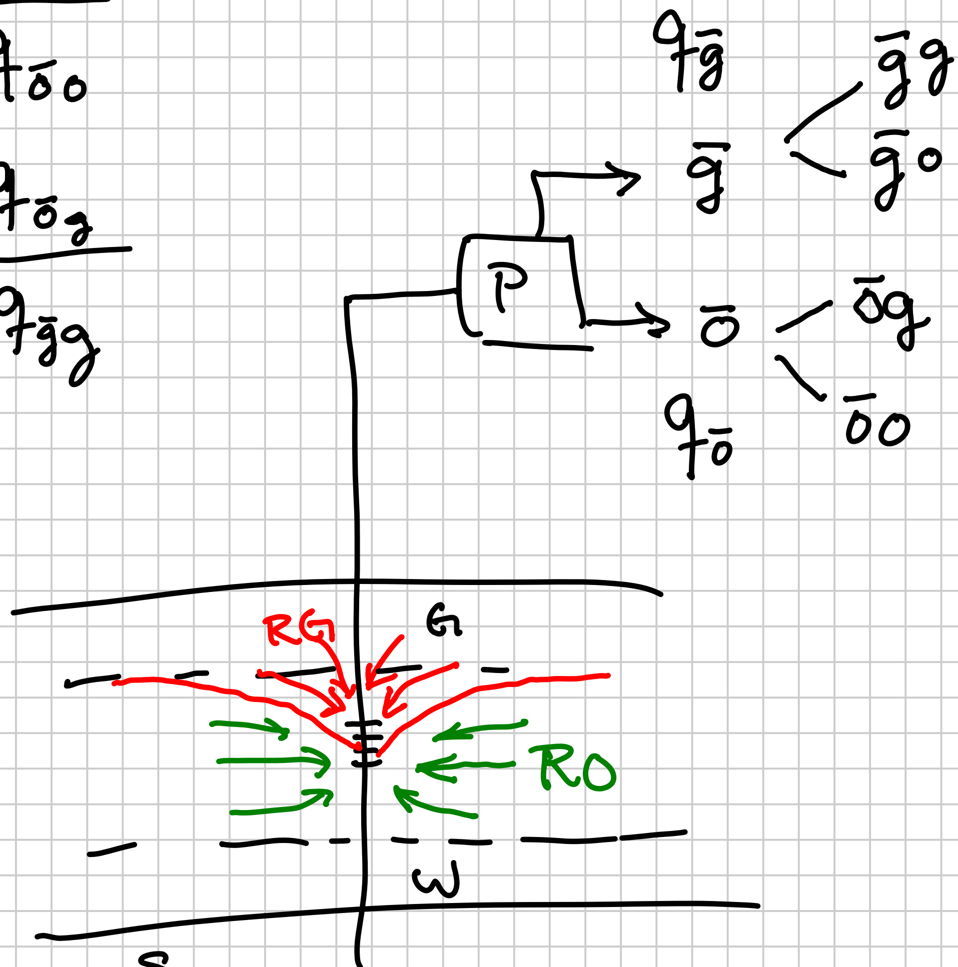
Reservoir Recovery Methods

$$f_w = \frac{q_w}{q_w + q_o}$$

Analyzing Water Injection

$$R_s = \frac{q_{\bar{g}o}}{q_{\bar{o}o}}$$

$$r_s = \frac{q_{\bar{o}g}}{q_{\bar{g}g}}$$



R	S
q_g	$q_{\bar{g}} = q_{\bar{g}g} + q_{\bar{g}o}$
q_o	$q_{\bar{o}} = q_{\bar{o}o} + q_{\bar{o}g}$

Give: q_o, q_g, p_R (p_{wf})

B_o, R_s, r_s, B_{gd}

Calc: $\underbrace{q_{\bar{o}o}, q_{\bar{o}g}}_{q_{\bar{o}}}$; $\underbrace{q_{\bar{g}g}, q_{\bar{g}o}}_{q_{\bar{g}}}$?

$$q_{\bar{o}o} = \frac{q_o}{B_o} \qquad q_{\bar{g}o} = q_{\bar{o}o} \cdot R_s$$

$$q_{\bar{g}g} = \frac{q_g}{B_{gd}} \qquad q_{\bar{o}g} = q_{\bar{g}g} \cdot r_s$$

Discount Rate

Economic Evaluation

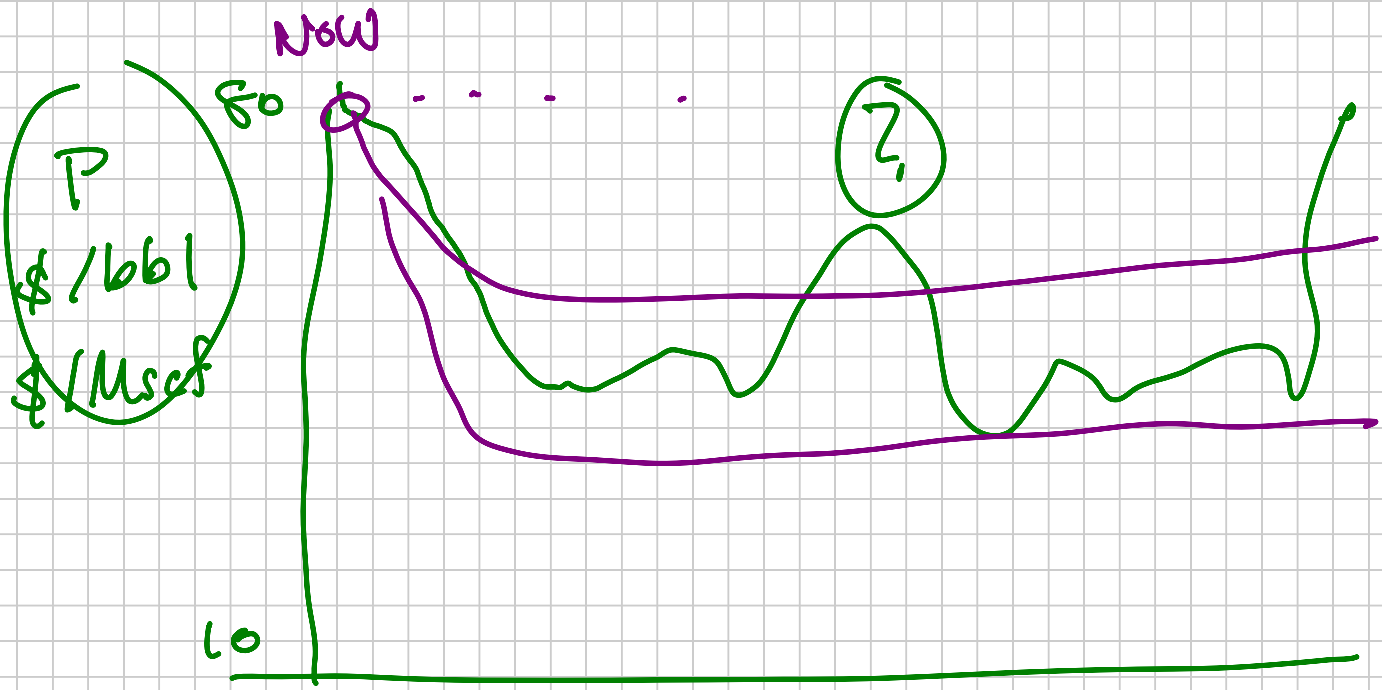
1) Cost Items
2) Revenues
3) Time value of money
   - Present value of "future" money
     Net Present Value (NPV)
4) Time dependence of Prices

5% interest rate

2005	100
2006	$100(1+0.05)$
2007	$100(1+0.05)(1+0.05)$
2008	$100(1+0.05)(1+0.05)(1+0.05)$
200n	$100(1+0.05)^y$

i : "interest"

10
1
%
?
t



NOW
80
60
P
$/bbl
$/Mscf
?

NPV for Project

Eimund

Sensor MODEL

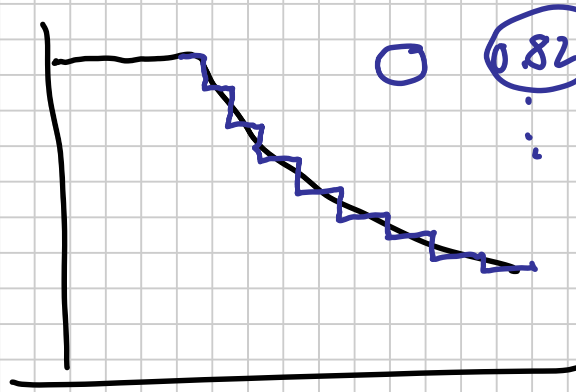
OPEX

t	$\bar{q}_{g}$	$\bar{q}_{o}$	$\bar{q}_{w}$	$P_{\bar{g}}$	$P_{\bar{o}}$	$P_{\bar{w}}$	I	DF	NPI
⋮								1	
								0.82 \$	
								⋮	

$$P_{\bar{g}} = 5 \ \frac{USD}{Mscf}$$

$$P_{\bar{o}} = 50 \ \frac{USD}{STB}$$

$$P_{\bar{w}} = -2 \ \frac{USD}{STB}$$



0

$\boxed{\text{Sum}}$

$$I = \bar{q}_{\bar{g}} \Delta t \, P_{\bar{g}} + \bar{q}_{\bar{o}} \Delta t \, P_{\bar{o}} + \bar{q}_{\bar{w}} \Delta t \, P_{\bar{w}}$$

$$DF = \frac{1}{(1+i)^{(t_d/365)}}$$

$\pm$ | $i(t)$ 2–10%
| $P(t)$ 10–100 USD/bbl

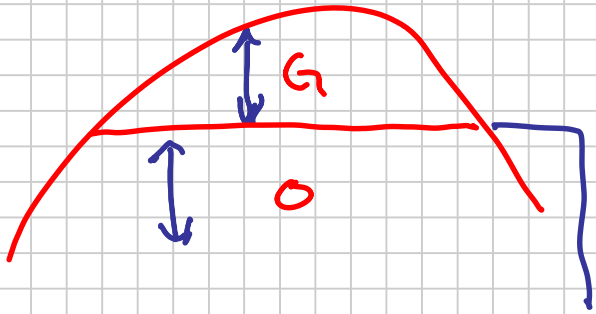
$$NPI = I \cdot DF$$

$$NPV = \left(\sum NPI\right) - \left(\text{INITIAL COSTS}\right)$$

$$= \left(\sum (NPI - OPEX)\right) - \left(\text{INIT COSTS}\right)$$

Exam 2002

Prob. 2d



2 "components"

SG SO

C_1 C_{10}

(1) Saturated

$$K_i = \frac{y_i}{x_i} \qquad i \in \{SG, SO\}$$
$$\{C_1, C_{10}\}$$

$$x_{C_1} = 0.6$$
$$K_{C_1} = 1.662$$

$y_{C_1} = 0.997$ $x_{C_1} = 0.6$

$y_{C_{10}} = 0.003$ $x_{C_{10}} = 0.4$

$K_{C_{10}} = 0.0075$

ANSWER:

$$x_{C_1} + x_{C_{10}} = 1$$
$$x_{C_{10}} = 1 - 0.6$$
$$\underline{x_{C_{10}} = 0.4}$$

$$K_{C_1} = \frac{y_{C_1}}{x_{C_1}}$$

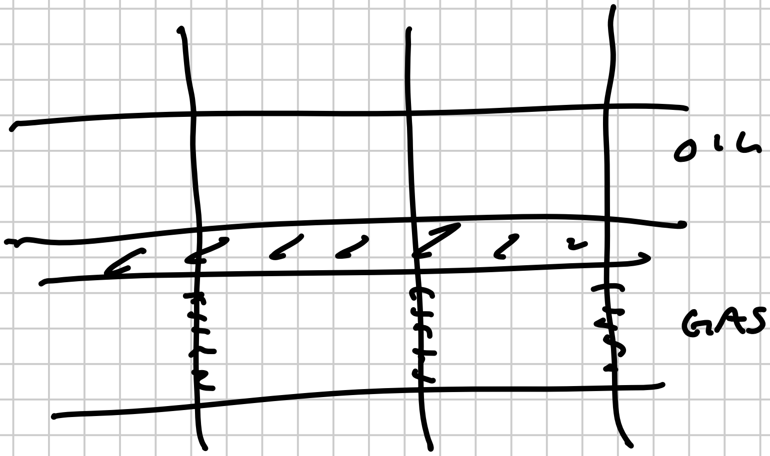
$$y_{C_1} = (1.662)(0.6)$$
$$= 0.997$$

$$y_{C_{10}} = 0.003$$
$$\underline{y_{C_{10}} = 1 - 0.997}$$

$$K_{C_{15}} = \frac{y_{C_{10}}}{x_{C_{10}}} = \frac{0.003}{0.4}$$

$$K_{C_{10}} = 0075$$

2001



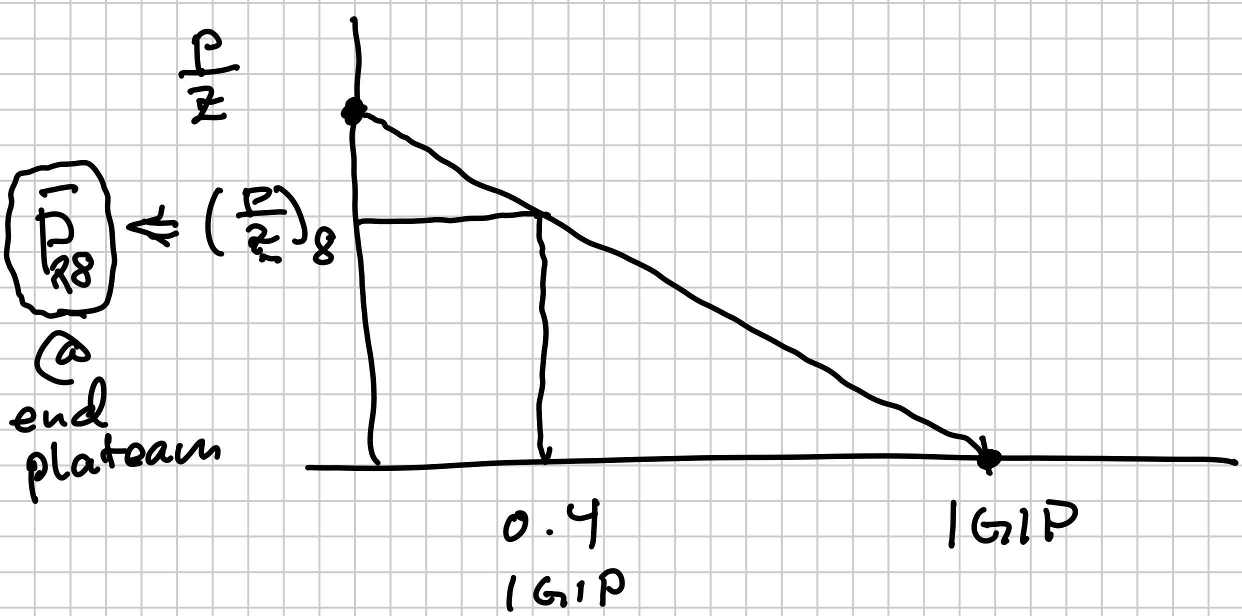
1D.

5% IGIP /yr for 8 yr = 40% RF

$Z = 1$ @ P_R when 8 yr

– Calc. $IGIP_{RG} = 10^9$ scf

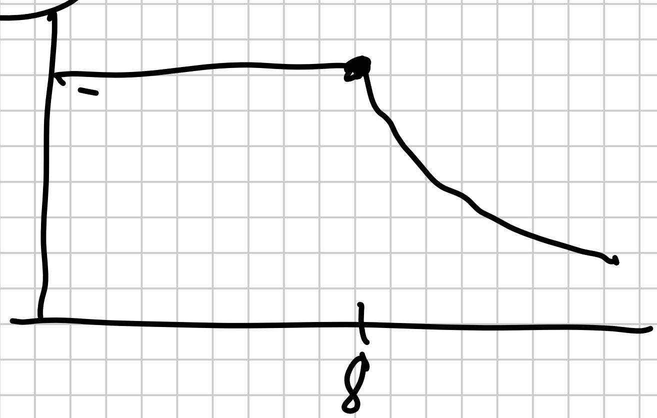
– Kh, s, T_R, μ_g, r_w, $P_{wf\,min}$



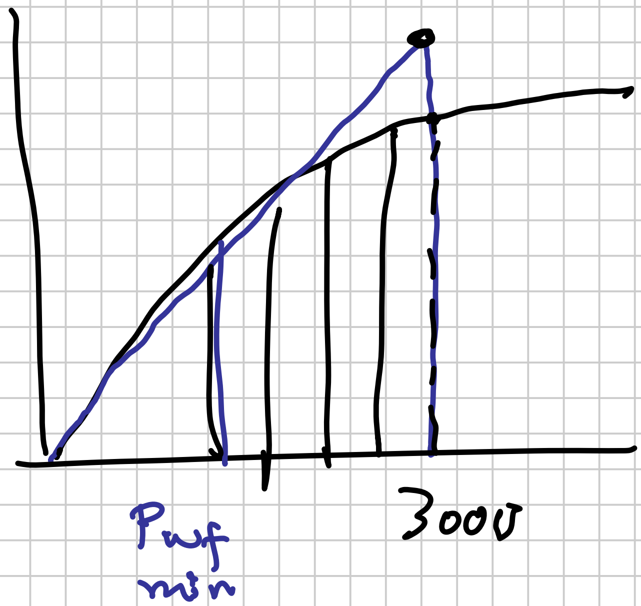
Know @ 8 yr, $\bar{P}_R$ ~ 3000 psia

$$\bar{q}_{gw} = \frac{Kh}{T_R\left[\ln \frac{\quad}{r_w} + s\right]} \int^{\bar{P}_{R,8}} \frac{P}{\mu Z}\, dp$$

$$q_{gF} = \frac{0.05\ GIIP}{365\ d} = N_w\,\bar{q}_{gw}$$



$\frac{P}{\mu Z}$



$$\cancel{N_w (\pi r_e^2) = A_{Field}}$$

~~well~~

$$\cancel{r_e = \left(\frac{A_F}{\pi N_w}\right)^{1/2}}$$

$$\boxed{\bar{r}_e = 1000\ m}$$

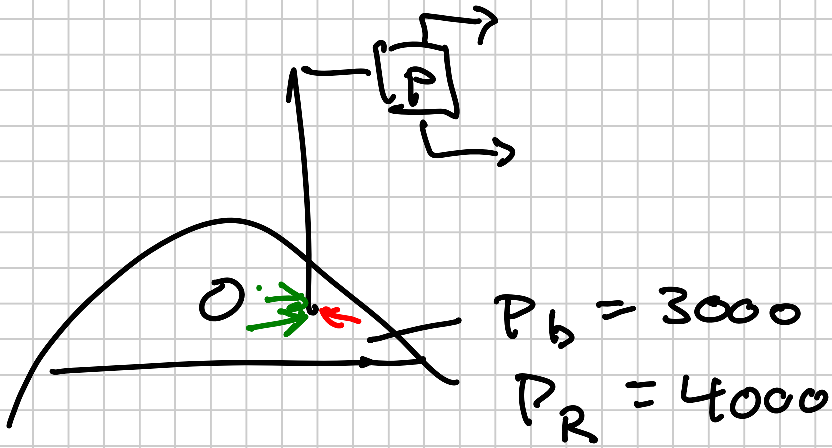
$\ln r_e$

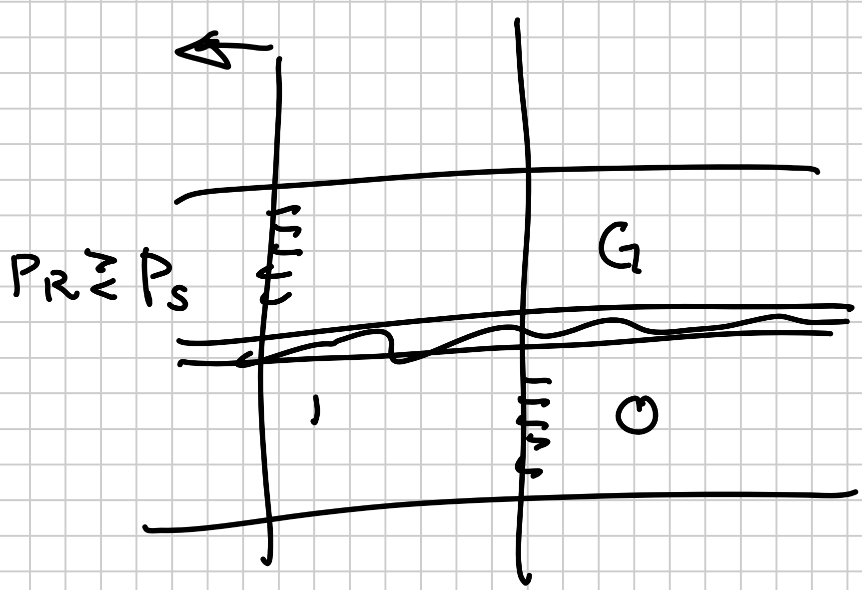
Test

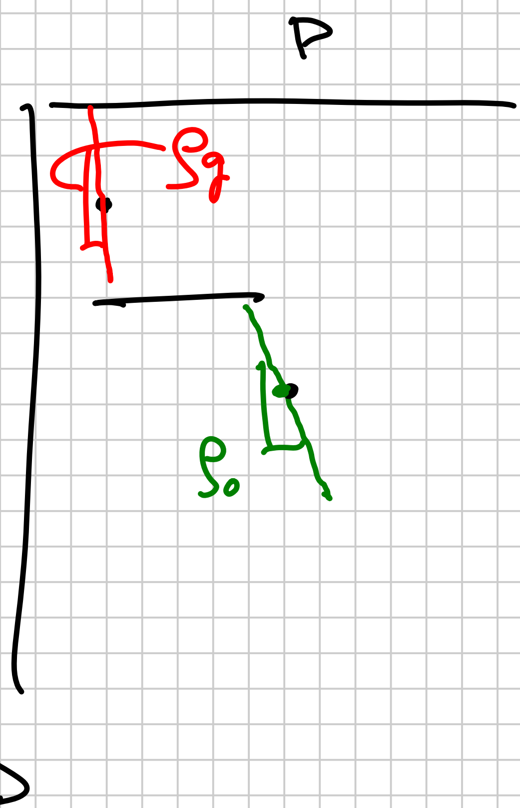
$q_{\bar{g}}$

$q_{\bar{o}}$

Calc. $R_p = GOR = \dfrac{q_{\bar{g}}}{q_{\bar{o}}}$



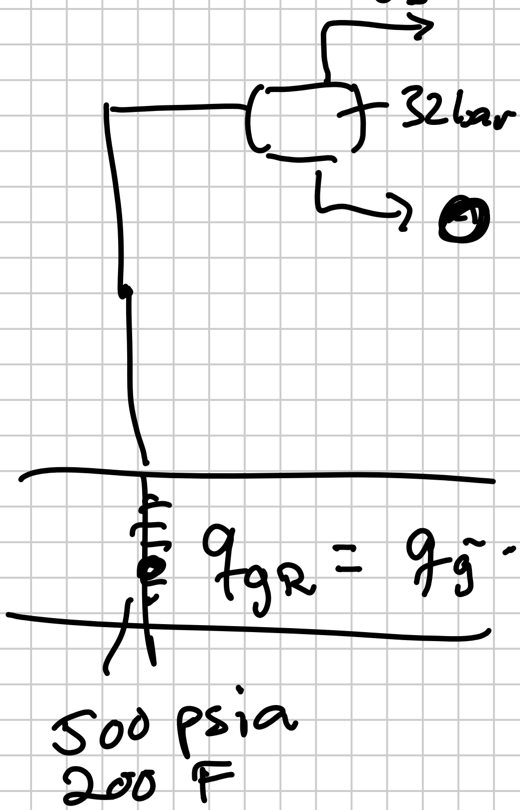




$\gamma_{\bar{o}}$ $\gamma_{\bar{g}}$ always @ 1 atm 60 F

$$\gamma_g = \frac{\rho_{gsc}}{\rho_{air\,sc}} = \frac{M_g}{M_{air}}$$

$$q_{\bar{g}} = 10^6 \frac{scf}{D}$$



DRY GAS

$$\boxed{B_g = \frac{p_{sc}}{T_{sc}} \frac{z T_R}{p}}$$

B_{gd} if $q_o > 0$

2001

Table (Eclipse 100)

Surface Gas Density $= 0.913 \ kg/m^3$

$$= \frac{p_{sc} \cdot M_g}{R \, T_{sc}}$$

$$= \rho_{air,sc} \cdot \gamma_g$$

$$= 1.22 \cdot \gamma_g$$

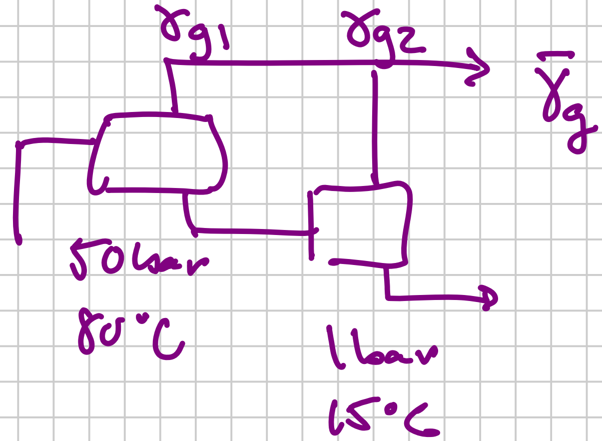
$$\Rightarrow \gamma_g = 0.75$$

Table 1

$\gamma_g = 0.71$

$p_{sp} = 15$ barg

$T_{sp} = 70\,°C$



R_{sb} = solution GOR of an oil at its (<u>initial</u>) bubblepoint

---

2003 Prob 1

Derive the straight line gas material balance.

$$\left\{ \begin{array}{l} p_i V = n_i R T_R Z_i \\ \quad \updownarrow = \\ p V = n_r R T_R Z \end{array} \right.$$

Initial

Later

$n_r = n_i - n_p$

$$G_p = n_p \cdot \frac{R T_{sc}}{p_{sc}}$$

$$G = n_i \, \frac{R T_{sc}}{p_{sc}}$$

$$\frac{p}{Z} = \left(\frac{p}{Z}\right)_i \left[1 - \frac{G_p}{G}\right]$$

$V = HCPV = \text{constant}$

$c_w = 0$

$c_f = 0$

No aquifer

2. Z_i

γ_g, T_R, P_{Ri}

Correlation Chart $\gamma_g \rightarrow T_{pc}, P_{pc}$ Fig. 3.7

$$T_{pr} = \frac{T_R}{T_{pc}}$$

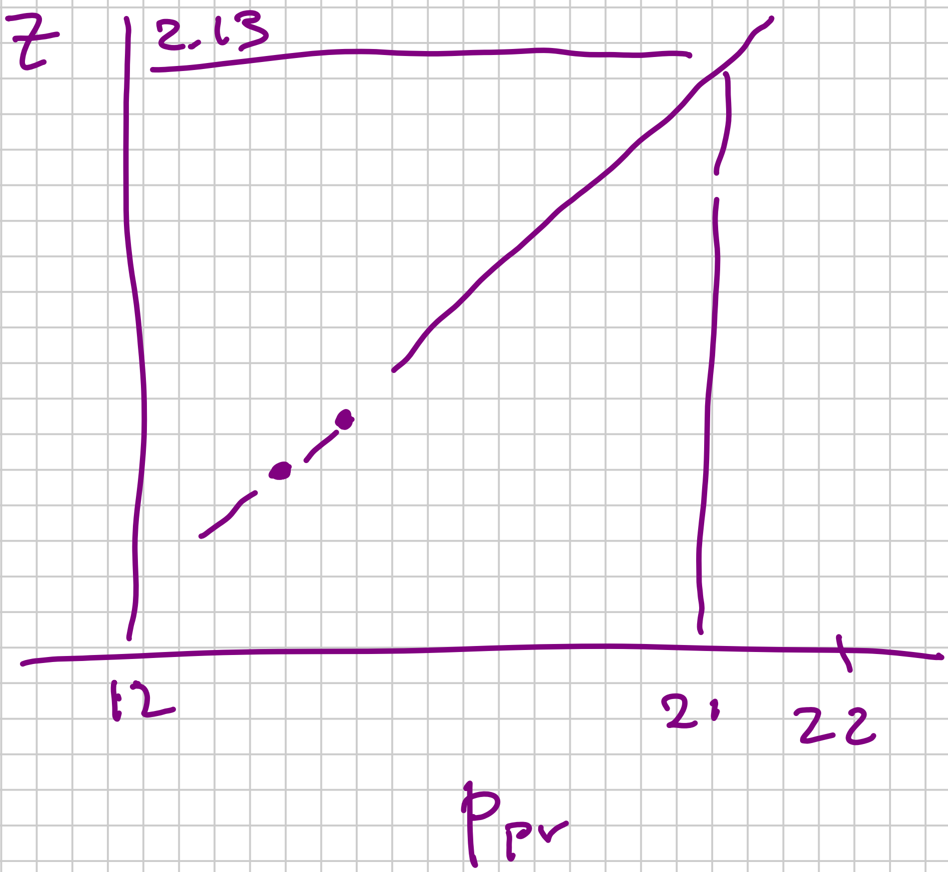
$$P_{pr} = \frac{P_{Ri}}{P_{pc}} < 15$$

Standing-Katz Z Chart Fig. 3.6

$Z(T_{pr}, P_{pr})$

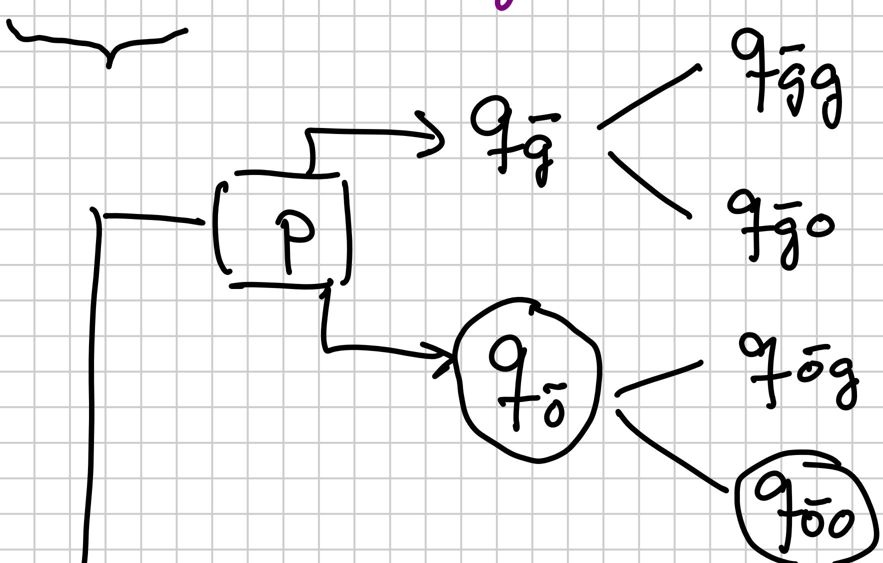
$P_{pr} = 21$

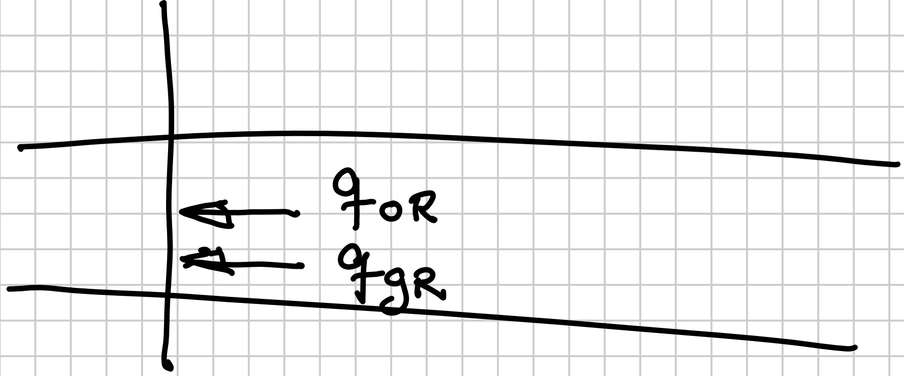
P_{pr}	Z
14	1.923
15	1.931



2003

(3) $\dfrac{q_{t\bar{o}o}}{q_{t\bar{o}}}$ or $\dfrac{q_{t\bar{g}g}}{q_{t\bar{g}}}$





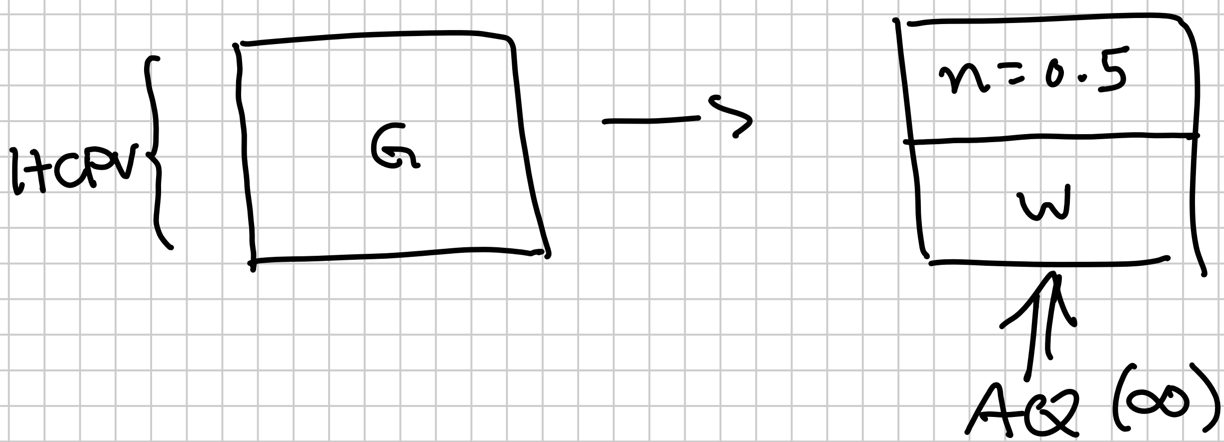
$$q_{\bar{o}} = \underbrace{q_{oR}\frac{1}{B_o}}_{\bar{o}o} + \underbrace{q_{gR}\cdot\frac{1}{B_{gd}}\cdot r_s}_{\bar{o}g} =$$

$$\boxed{\frac{q_{\bar{o}o}}{q_{\bar{o}}} = \frac{q_{oR}\frac{1}{B_o}}{q_{oR}\frac{1}{B_o} + q_{gR}\frac{r_s}{B_{gd}}}}$$

$$\boxed{\frac{q_{\bar{o}o}}{q_{\bar{o}}} = \frac{1}{1+\left(\frac{q_{gR}}{q_{oR}}\right)B_o\frac{r_s}{B_{gd}}}}$$

Eq. 7.38

Prob. 3



$n_i = 1$ mole

@ $\frac{G_P}{G_i} = 0.5$

$P_i V_i = n_i R T Z_i$

$p(\frac{1}{2} V_i) = (\frac{1}{2} n_i) R T Z$

$\Rightarrow \frac{P_i}{Z_i} = \frac{n_i R T}{V_i}$ solve for $\frac{P}{Z}$

$$\left(\frac{P}{Z}\right) = \frac{n_i R T}{V_i} = \frac{P_i}{Z_i}$$

$\left(\frac{P}{Z}\right)$ given

how to find P?

SK chart