AUTOMATIC NONHYPERBOLIC VELOCITY ANALYSIS

By

Brahim Abbad¹, Bjørn Ursin¹, and Didier Rappin²

¹Norwegian University of Science and Technology,
Department of Petroleum Engineering and Applied Geophysics,
S.P. Andersens vei 15A, NO-7491 Trondheim, Norway,
Email: abbad@ntnu.no; bjornu@ipt.ntnu.no

²Total Exploration & Production, CSTJF, Avenue Larribau
F-64018 Pau Cedex, France,
Email: didier.rappin@total.com

Shortened title for running-head: Automatic velocity analysis

August, 29 2008
ABSTRACT

The stacking of long-offset seismic data requires a nonhyperbolic traveltime function which depends on two-way traveltime, normal moveout velocity and effective anellipticity. Based on a standard fractional approximation, a new parameterization in slowness squared parameters provides optimal sampling of the normal moveout velocity and anellipticity. The automatic velocity analysis is performed with a normalized bootstrapped differential semblance coherency estimator which works on stochastically scrambled traces within a time window to detect small time shifts thus increasing resolution in velocity spectra. Reflections are identified at the peaks of the maximum bootstrapped differential semblance curve. Centering the reflection pulse within its corresponding time gate results in improved estimate of the two-way time and reduced bias in the estimates of the normal moveout velocity and anellipticity. Generalized Dix equations give estimates of apparent interval thickness, velocity and anellipticity. The interval parameters will fit a homogeneous anisotropic VTI medium or an isotropic layer with a linear velocity gradient. The algorithm outputs an automatic stack and laterally varying moveout velocity and anellipticity maps which can be used for subsequent time processing. The algorithm is implemented in a two-step strategy. A coarse hyperbolic velocity analysis identifies events in the gather and estimates a velocity law for truncation, is followed by a dense nonhyperbolic search to infer the physical parameters required for time processing of PP data.

Automatic nonhyperbolic velocity analysis was tested on a synthetic gather and a real data set from North Sea. Nonhyperbolic parameter search shows enhanced estimate of the processing parameters, velocity and anellipticity, and improved quality of the stacked section compared to that from hyperbolic search. The interval moveout
velocity maps show a great correlation when compared to the results of more advanced processing.
INTRODUCTION

Algorithms for conventional velocity analysis are based on the computation of coherency estimators (measures) on collections of traces sorted by common mid-point gathers along hyperbolic trajectories described by tentative values of zero-offset times and stacking velocities (Taner and Koehler, 1969; Neidell and Taner, 1971; Key and Smithson, 1990; Biondi and Kostov, 1989 among others). The purpose is to extract the normal moveout velocity as a function of the two-way zero-offset travelt ime at selected CDP locations along the seismic line. The algorithms use search methods along trajectories described by the hyperbolic equation (Dix, 1955).

The validity of the hyperbolic traveltime equation is restricted to layered isotropic media with apertures (or equivalently offset-to-depth ratios) smaller than 1 (Thore et al., 1994). However, the increased use of large offset recordings, especially in marine seismics, and the evidence of anisotropy in most subsurface sediments (Thomsen, 1986; Alkhalifah et al., 1996; Toldi et al., 1999) reduces significantly the resolution of this approach. It generally over-estimates the normal moveout velocities and hence the deduced depths of the key reflectors.

Reflection moveouts can be more accurately described by traveltime approximations with three parameters that account for large offsets as well as for anisotropy inside the layers. Several nonhyperbolic traveltime approximations have been proposed to describe the behavior of reflection curves in stratified isotropic and anisotropic media (de Bazelaire, 1988; Castle, 1994; Thore et al., 1994; Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995; Siliqi and Bousquié, 2000; Ursin and Stovas, 2006). These equations share the fact that they are expressed using three
parameters: the two-way zero offset traveltime, the short-spread normal moveout velocity and a heterogeneity factor or, alternatively, a related parameter called the effective anellipticity. The value of the anellipticity combines the effects of isotropic layering (always positive) and the intrinsic anisotropy in the accumulated layers above the considered interface. This parameter defines the nonhyperbolic contribution in the reflection curve and affects significantly the reflection moveouts at the far offsets. In analogy with optics, Thore et al. (1994) presented a parameterization defined by a time focusing parameter and a tuning velocity which showed robustness and efficiency even for large apertures (Rappin et al., 2002).

In the application of a particular algorithm for nonhyperbolic velocity analysis, a number of choices are considered to obtain accurate velocity and anellipticity fields. First, the traveltime equation in use should be sufficiently accurate to describe the reflection behavior over isotropic bedding or anisotropic subsurface for long spreads. The traveltime equations based on the shifted hyperbola (de Bazelaire, 1988; Castle, 1994; Siliqi and Bousquié, 2000) as well as the continued-fraction traveltime approximations (Alkhalifah and Tsvankin, 1995; Ursin and Stovas, 2006) can be equally used for such a purpose. AVO based velocity and anisotropy was also introduced using a wavelet correlation versus offset technique (Swan, 2001). This is not part of the scope of the present work since the kinematical behavior is not explicit.

Second, different parameterizations can be used to run the search. While the zero-offset traveltime stands as a permanent axis in the analysis, different transformations can be made to search for the optimal moveout parameters. Alkhalifah (1997a), as well as Grechka and Tsvankin (1998), considered a search method in the moveout velocity-horizontal velocity domain, whereas Siliqi and Bousquié (2000) performed a similar scan in a domain involving two time parameters.
The adopted strategy to estimate the reflection parameters from moveout curves is also an important issue. Alkhalifah (1997a) proposed a two-step iterative technique which begins by performing a hyperbolic velocity analysis on a truncated gather to estimate the moveout velocity. Next, data with the full-offset range (without truncation) are used to deduce the effective anellipticity. This is achieved by applying hyperbolic normal moveout corrections using the obtained velocity function and picking the residual moveout of the corrected wavelets at each offset to estimate the anellipticity parameter. It was assumed that the traveltime equation in use fits perfectly the reflection curve in the full offset range. We propose a modified version where an initial estimate of the normal moveout velocity is obtained using seismic data truncated using an aperture value equal to 0.9. Then the large-offset data are used to estimate a refined velocity and anellipticity at an aperture value of 1.9.

Automatic velocity analysis was formulated in the $\tau - p$ domain to recover the interval velocities using elliptical moveout corrections with a set of tentative velocities (Calderon-Macias et al., 1998). The method combines feedforward neural networks and very fast simulating annealing. The fitting of the goodness of event flattening after these corrections were estimated using the $l_1$-norm harmonic measure. The method is time consuming due to the need to transform seismic data to the $\tau - p$ domain and to apply the elliptical time corrections. Moreover, the number of training gathers should increase when the lateral velocity changes are significant. Automatic nonhyperbolic parameter search was also proposed by several authors based on a time parameterization and using the shifted hyperbola traveltime approximation, in addition to geostatistical filtering to obtain dense parameter fields for the velocity and the anellipticity parameters (Siliqi et al., 2003, Siliqi and Le Meur, 2004). The method suffers from poor anellipticity sampling when this quantity describing anisotropy is
van der Baan and Kendall (2002) presented another inversion approach in the $\tau-p$ domain and outlined the same problems related to non-uniqueness in the solution and the existence of a family of kinematically equivalent models that exhibit identical moveout curves.

The objective of this paper is to present a workflow for the implementation of automatic nonhyperbolic velocity analysis to infer the physical parameters characterizing a reflection moveout, and to solve for limitations of similar automatic approaches (Calderon-Macias et al., 1998). To avoid sampling problems related to popular parameterizations, we propose a new search in the slowness squared domain that leads to optimal track of the time processing parameters, the moveout velocity and the effective anellipticity. Parameter search is implemented using a bootstrapped differential semblance estimator having a better resolution than conventional differential semblance. The method is implemented in a two-step strategy. First, a coarse hyperbolic moveout search is made to identify events in the gather, while the second search is nonhyperbolic to update the estimated normal moveout velocity and track the effective anellipticity. The errors on the picked parameters are reduced using an iterative event centering procedure that enhances parameter estimates and removes errors due to coarse time sampling. The algorithm was tested on a synthetic example and a real data set from North Sea to prove its applicability. The resulting velocity and anellipticity parameters can be used to invert for interval parameters through Dix-type equations for VTI media or isotropic media with linear velocity gradients. The estimated parameters can be applied for anisotropic time processing and lithology discrimination.

TRAVELTIME PARAMETER SAMPLING
We shall consider reflections where the traveltime is approximated by the fractional approximation (Alkhalifah and Tsvankin, 1995)

\[
T(x)^2 = T(0)^2 + \frac{x^2}{V_{NMO}^2} \left( \frac{2\overline{\eta} x^4}{V_{NMO}^2 T(0)^2 (1 + 2\overline{\eta}) x^2} \right),
\]

where \(x\) denotes the offset, \(V_{NMO}\) is the normal moveout velocity, and \(\overline{\eta}\) is an effective anellipticity parameter which is related to the coefficient \(S_2\) (Siliqi and Bousquié, 2000; Ursin and Stovas, 2006) by

\[
S_2 = 1 + 8\overline{\eta}.
\]

\(S_2\) is a parameter known as the heterogeneity coefficient and is related to the shifted hyperbola traveltime representation (de Bazelaire, 1988; Castle, 1994). For \(\overline{\eta} = 0\), equation 1 reduces to the standard hyperbolic traveltime approximation.

Several papers discussed the nonhyperbolic behavior of moveout curves and investigated possibilities and limitations in inverting traveltime curves for the moveout parameters, velocity and anellipticity. Grechka and Tsvankin (1998) introduced a correction factor in the denominator of equation 1 to increase the equation accuracy at intermediate offsets. They showed that the inverted anellipticity remains a sensitive parameter to errors in reflection traveltimes, even small. Thus, the method assumes that static corrections have been accurately applied. When the anellipticity is important, the nonhyperbolic moveout inversion is highly biased. This non-uniqueness in the inversion is explained by the existence of a set of kinematically equivalent models having close reflection traveltimes in a wide offset range. This family of models leads to comparable results on post-stack time migration but may fail when applied to anisotropic DMO. With this high sensitivity, the DMO process can stand as an attractive domain to invert for the moveout parameters.
Nonhyperbolic velocity analysis is implemented by scanning for the optimal reflection parameters inside predefined limits (known as corridors) for the moveout velocity and the effective anellipticity. A set of tentative velocities and anellipticities are tested in a given traveltime approximation, such as equation 1, to construct different time windows. The reflection is perfectly centered inside the window when the true moveout parameters are used in the traveltime equation. A coherency estimator is computed for all trial parameter pairs (velocities and anellipticities) to measure their fit to the moveout curve, and, it is displayed as a coherency slice for each trial $T(0)$-value. A direct search in the moveout velocity and the anellipticity parameter is impractical because they are different quantities. These can be handled easier with better subspace strategies.

The most familiar parameterization in use is to perform a regular sampling over the moveout velocity, $V_{NMO}$, and the horizontal velocity $V_H$ (Alkhalifah and Tsvankin, 1995; Alkhalifah, 1997a; Grechka and Tsvankin, 1998)

$$V_H = V_{NMO} \sqrt{1 + 2\eta}. \quad (3)$$

From the two velocities, the anellipticity is easily computed through

$$\eta = \frac{1}{2} \left( \frac{V_H^2}{V_{NMO}^2} - 1 \right). \quad (4)$$

This parameterization samples the anellipticity very well, but undersamples the moveout curvature at low velocities (large moveouts).

Siliqi and Bousquié (2000) proposed a sampling domain for the moveout time orthogonal (uncorrelated) variables. They defined a regular search along the nonhyperbolic moveout $dT_n$ at the maximum absolute offset $x_{\text{max}}$ and another time parameter $\tau_0$ which is related to $T(0)$ and $\eta$. 

9
This parameterization is sensitive to the employed traveltime equation defined by $T(x)$ at $x_{\text{max}}$. The time processing parameters are then expressed by first order relation with velocity and anisotropy as

$$
\bar{\eta} = \frac{T(0) - \tau_0}{8 \tau_0},
$$

(6)

where $f$ and $g$ are some functions that depend on the traveltime equation in use. The major shortcoming of this domain is that the moveout velocity depends on the two variables $dT_n$ and $\tau_0$ while the anellipticity depends on $\tau_0$ only (Siliqi and Le Meur, 2004). Thus, the anellipticity is obtained before the moveout velocity. This corresponds to the fact that different $dT_n$ values are scanned for the same $\tau_0$ value. In moveout parameters, this implies that several moveout velocities are tested for the same trial anellipticity. However, the moveout velocity represents the dominant component in the moveout equation while the anellipticity is a secondary parameter whose effect can only be seen at far-offsets. Thus an appropriate parameterization should consider this fact when choosing the search variables.

We define alternatively two variables $q_1$ and $q_2$ to sample the moveout velocity and the effective anellipticity according to the simple relations

$$
q_1 = \frac{1}{V_{\text{NMO}}^2},
$$

$$
q_2 = \frac{2 \bar{\eta}}{V_{\text{NMO}}^2} = 2 \bar{\eta} q_1.
$$

(7)

Both variables are in units of slowness squared. The variable $q_1$ is responsible for the sampling of the moveout velocity and $q_2$ is governing the sampling of the effective
an ellipticity given a tentative moveout velocity. It is linearly related to $q_i$ through $\bar{\eta}$. The processing parameters are deduced from the optimal $q_1 - q_2$ couple through simple relations too

\[ V_{\text{NMO}} = \frac{1}{\sqrt{q_1}}, \]  
\[ \bar{\eta} = \frac{q_2}{2q_1}. \]  

(8)

With this new parameterization, the traveltime approximation in equation 1 can be compactly written as

\[ T(x)^2 = T(0)^2 + q_i x^2 \left[ 1 - \frac{q_2 x^2}{T(0)^2 + (q_1 + q_2) x^2} \right]. \]  

(9)

Figure 1 depicts the sampling results for the three different parameterizations discussed above. Figures 1a, b, and c show the sampling of the moveout range at a maximum offset equal to 4 km and a velocity parameter scanned for in the range [1.5-2.5 km/s] and a zero-offset time range [0-4 s]. The decrease in the moveout range with increasing zero-offset time for a time-invariant velocity corridor is an obvious fact from these figures. Figures 1d, e, and f show the sampling of the moveout velocity and an ellipticity for the same moveout velocity range and an an ellipticity varying in the range [0-0.4] at a zero-offset traveltime $T(0) = 0.5$ s. These figures represent a slice taken in Figures 1a, b, and c and highlight the sampling for the moveout parameters $V_{\text{NMO}}$ and $\bar{\eta}$ at a given $T(0)$ value. The result for the $V_{\text{NMO}} - V_{\text{H}}$ parameterization is shown in Figures 1a and 1d. It is seen that this parameterization undersamples large moveouts (corresponding to low velocities) as illustrated in Figure 1a, but offers a good sampling of the an ellipticity coefficient (Figure 1d). The result for the $dT_n - \tau_0$ parameterization is shown in Figures 1b and 1e. The $dT_n - \tau_0$ domain samples regularly the moveout range (Figure 1b), but the an ellipticity parameter is badly handled (Figure
1e). Small anellipticities are densely sampled while large values are poorly sampled leading to increased inaccuracy in estimating accurate $\bar{\eta}$ when this parameter is large ($\bar{\eta} > 0.2$ in this example). The results for the proposed $q_1 - q_2$ parameterization are shown in Figures 1c and 1f. The moveout sampling for this domain (Figure 1c) is quite similar to that allowed by the $dT_n - \tau_0$ domain. This almost regular moveout sampling treats the moveout velocities in an optimal way. Thus, low velocities are densely sampled according to their large curvatures, while high velocities corresponding to small curvatures are coarsely sampled. The anellipticity sampling (Figure 1f) is almost regular and looks similar to the one offered by the $V_{NMO} - V_H$ domain. Enhanced velocity sampling for the $V_{NMO} - V_H$ domain can be achieved by increasing the number of velocities, while the anellipticity sampling for the $dT_n - \tau_0$ domain can be refined by using more tentative values for the anellipticities which naturally increases the computing time. Thus the proposed sampling method in the $q_1 - q_2$ domain is optimal and solves the limitations of the previous parameterizations. This is important for enhanced parameter tracking which affects directly the estimated parameters from the nonhyperbolic search.

**BOOTSTRAPPED DIFFERENTIAL SEMBLANCE**

Coherency estimators serve as criteria for the fitting of a given reflection moveout using a combination of the moveout parameters $T(0), V_{NMO}$, and $\bar{\eta}$ in a traveltime approximation. These are used to construct a time window sufficiently narrow to contain at most one event. Several estimators based on spatial wavelet semblance and coherency have been proposed in the industry with varying levels of success and computing requirements such as summation methods (Taner and Koehler,
1969; Neidell and Taner, 1971; Douze and Laster, 1979), cross-correlation-based methods (Schneider and Backus, 1968; Neidell and Taner, 1971; Sherwood and Poe, 1972; Fuller and Kirlin, 1992; Larner and Celis, 2007), differential semblance (Symes and Carazzone, 1991; Brandsberg-Dahl et al., 2003) and eigenstructure methods (Biondi and Kostov, 1989; Key and Smithson, 1990, Sacchi, 1998). The last category results in well-resolved velocity spectra but requires heavy computing due to the need for the eigenvalue decomposition of the covariance matrix of seismic data inside the time window. Their use in nonhyperbolic velocity analysis is impractical due to the computing charges they require for a three dimensional parameter search.

We propose a new coherency estimator to discriminate the moveout parameters in a reasonable computing time comparable to that of summation methods but offering a higher resolution in velocity spectra. Differential semblance (DS) is defined as a summation along the vertical axis (time or depth) of squared spatial derivatives of seismic data with respect to the horizontal axis (offset or CDP). For time-offset domain data \(d(t,x)\), this is expressed as (Brandsberg-Dahl et al., 2003)

\[
DS = \int_x \int_t \left( \frac{\partial d(t,x)}{\partial x} \right)^2 dt \, dx.
\]  

(10)

All adjacent traces inside the window are differentiated with respect to offset and the results are squared and summed to give a coefficient measuring the lateral semblance between the seismic wavelets inside the window. In absence of noise, amplitude and phase changes inside the data, this coefficient will be exactly zero if the monochromatic event is perfectly centered inside the window. Thus this estimator will be rather minimized than maximized for a good fit of the moveout curve. To show up the fitting of weak reflections, a normalization factor is introduced in the differential semblance operator to scale the coefficient with respect to the total energy in the window.
We define bootstrapping in the seismic context as a random reordering of seismic traces so that these initially arranged in increased offset order will be reorganized in an unpredictable manner using random number generation algorithms (Sacchi, 1998). This is of great importance when dealing with time windows. Bootstrapping is applied here to the differential semblance to form what we define as bootstrapped differential semblance (called BDS for brevity). This allows for a better discrimination of the reflection parameters. When the tentative moveout parameters are close to the true velocity and anellipticity, they result in a nearly horizontal time window and differential semblance will give a broad region with values approaching zero for several possible combinations of tentative velocities and anellipticities. The application of bootstrapping will increase the sensitivity to slight deviations of the event from being horizontal, and it detects minor time shifts between the wavelets inside the window.

For discrete seismic data in space and time, the bootstrapped differential semblance (BDS) coefficient is defined as

\[
BDS = 1 - \frac{N \sum_{i=2}^{N} \sum_{t=\lambda_{e}/2}^{\lambda_{e}+\lambda/2} \left[ d(t, x_{i}) - d(t, x_{i+1}) \right]^2}{2(N-1) \sum_{i=1}^{N} \sum_{t=\lambda_{e}/2}^{\lambda_{e}+\lambda/2} d(t, x_{i})^2} \cdot \frac{\sum_{i=1}^{N} \sum_{t=\lambda_{e}/2}^{\lambda_{e}+\lambda/2} d(t, x_{i})^2}{\sum_{i=1}^{N} \sum_{t=\lambda_{e}/2}^{\lambda_{e}+\lambda/2} d(t, x_{i})^2},
\]

where \( x_{i} \{i = 1, N \} \) is the bootstrapped series of the offset array \( x_{i} \{i = 1, N \} \) obtained by random sorting of the traces inside the window. \( N \) is the total number of traces inside the window after applying the aperture mute, and \( \lambda \) is the width of the time window. Under this new formulation, the BDS coefficient is positive and ranges in the interval [0-1], and the coefficient is maximized when an event is horizontally centered inside the time window. Care should be taken when seismic data exhibit significant
amplitude variations with offset. This may harm the efficiency of the BDS coefficient more drastically than for the conventional differential semblance. In practice, data equalization using $t^\alpha$-compensation or automatic gain control reduces temporal and spatial amplitude changes and leads to better results. One can also use scaled differences to account for important amplitudes changes of the traces being differentiated in the BDS equation. Alternatively, automatic gain functions can be computed on local space-time sliding windows to account of the amplitude effects in the original gather (before bootstrapping). Later, these variations are accounted for by weighting the seismic data with the inverse gain function.

Figure 2a shows a time window for a reflection with a good guess of the moveout parameters. The wavelets inside the window are nearly in phase, and the conventional differential semblance would give a very small value, because the time shifts are minor between traces sorted in increasing offset order. When bootstrapping is applied, the differences are shown up more clearly than before (Figure 2b). Thus, the BDS coefficient is more sensitive to wavelet time shifts than the standard differential semblance. To show the benefits of trace random sorting on velocity spectra, we consider the gather in Figure 3a which contains five primary reflections with four interfering multiples and random noise. Building the velocity spectrum using the estimator in equation 11 without bootstrapping is illustrated in Figure 3b. Most of the reflections, primaries and multiples, are mapped into the velocity spectrum despite the level of noise. A significant increase in resolution is gained by constructing the velocity spectrum using equation 11 combined with bootstrapping as shown in Figure 3c. In particular, the two events at 2.7 s have a small moveout difference and are difficult to resolve on Figure 2a while they can be clearly distinguished on Figure 3c.
The goal of nonhyperbolic velocity analysis is to extract the moveout parameters $T(0)$, $V_{NMO}$, and $\eta$ for possible reflections unknown a priori in common mid-point gathers. This is implemented by scanning for the velocity and the anellipticity fields inside predefined corridors for selected $T(0)$-values. The time axis is scanned for possible reflections by moving toward larger times using a time step that is usually equal to half the window width thus allowing for 50%-window overlapping to detect reflections from thin beds in the model. The output is a curve for the maximum coherency as a function of the window central time $T(0)$ which is filtered to select the peaks that correspond to primary reflections inside the gather. This is an event detection algorithm which is fundamental in nonhyperbolic velocity analysis to avoid manual interpretation due to the intensive search and the large number of time slices to display and interpret.

Here, the filtering process is based on the maximum coherency field to remove insignificant velocity and anellipticity values and keep only those corresponding to existing reflections. The derivative of this field at each CDP analysis location is used to identify the peaks on the maximum coherency curve. The need for a normalized coherency estimator is evident here as it gives a similar response for both strong and weak reflections. A relative threshold is set to filter out the low coherency values which correspond naturally to noise.

The time increment between successive time windows defines the temporal resolution of the search. However, this large time sampling leads to increased errors in the estimated moveout velocity and effective anellipticity. This is due to the fact that
two parallel reflection moveouts with different zero-offset traveltimes will have different velocities and anellipticities. To overcome this artifact and allow for more accurate parameter estimation, we consider a time window containing an event inside. The picked zero-offset time is the center of the window and is assumed to be the true zero-offset time for the event inside. This is true only when the event is exactly centered inside the window. Thus, the time window should be updated until the event is centered inside the window. This is achieved by computing a pilot (stacked) trace of the window and evaluating the time shift of its maximum peak with respect to the window center time to update the zero-offset time value. Later, a time slice is constructed to find the optimal moveout parameters $V_{\text{NMO}}, \bar{\eta}$ at the newly estimated $T(0)$-value using a two-dimensional parameter search via the proposed parameterization. A new time window is built with the updated parameters $(T(0), V_{\text{NMO}}, \bar{\eta})$, until the maximum peak of the pilot trace coincides with the window center time or when a maximum number of iterations is achieved. The procedure is further illustrated with the following example. Figure 4a shows a time window with an event inside taken from the gather in Figure 6a with a good guess of the moveout parameters. The true parameters for the event in the gate are $T(0) = 2.1\, s$, $V_{\text{NMO}} = 3068\, m/s$, $\bar{\eta} = 0.32$. The estimated parameters from automatic nonhyperbolic parameter search without event centering are $T(0) = 2.11\, s$, $V_{\text{NMO}} = 3084\, m/s$, $\bar{\eta} = 0.38$. By applying the described approach these parameters are updated to reduce the inaccuracies. After two iterations, the new parameters are $T(0) = 2.1\, s$, $V_{\text{NMO}} = 3052\, m/s$, $\bar{\eta} = 0.32$. The resulted time window is given in Figure 4b. The event is now centered inside the gate and the parameters are in a good match with the true parameters (see Table 2).

To accurately track the moveout parameters $V_{\text{NMO}}, \bar{\eta}$, the data should be truncated at far-offsets using an optimal aperture value. The aperture is defined as the
ratio of offset to approximate depth at a given two-way time value using an estimate of the moveout velocity. It depends on the employed traveltime equation and affects highly the estimated anellipticity. Trace samples corresponding to offset-to-depth ratios larger than the defined aperture are muted. For $PP$-waves in VTI media, the continued-fraction traveltime equation (Alkhalifah and Tsvankin, 1995), given in equation 1, has the highest accuracy at large offsets (Ursin and Stovas, 2006) and is used here to find the moveout parameters in the truncated gather for each reflection. Several values for the truncation limit have been tested on synthetic examples and the aperture of 1.9 provided the smallest errors in the estimated moveout parameters for small to moderate anellipticity values. This value is used for the synthetic data as well as for the real dataset.

DIX-TYPE INVERSION FOR LAYER PARAMETERS

The moveout velocity and the anellipticity that characterize a reflection moveout from a given interface inside the model are effective parameters, thus including wave propagation inside the upper layers. Assuming horizontal layers, the familiar Dix (1955) inversion formula allows inverting for interval moveout velocities and apparent layer thicknesses. Other methods exist for recovering interval parameters such the layer-stripping approach (van der Baan and Kendall, 2002, Dewangan and Tsvankin, 2006). Nonhyperbolic moveout curves from horizontal layers can be fitted using either a homogeneous transversely isotropic layer with a vertical symmetry axis (a VTI layer) or an isotropic layer with a linear velocity gradient in depth. For a homogeneous VTI layer this gives the interval moveout velocity and the apparent layer thickness
Here \( \Delta \) denotes the difference in parameters between the reflections from the base and top of the layer of interest. We also estimate an interval anellipticity parameter by

\[
\eta = \frac{1}{8} \left[ \frac{\Delta T(0) \nu_{\text{NMO}}^2 (1 + 8\eta)}{\Delta T(0) \nu_{\text{NMO}}^2} - 1 \right] = \frac{\varepsilon - \delta}{1 + 2\delta} \left[ 1 + \frac{2\gamma_0^2}{\gamma_0^2 - 1} \right].
\]  

In the equations above \( \Delta z \) is the true layer thickness, \( \varepsilon \) and \( \delta \) are the Thomsen (1986) parameters and \( \gamma_0^2 = \alpha_o^2 / \beta_0^2 \) with \( \alpha_o \) and \( \beta_0 \) being the vertical \( P \)- and \( S \)-wave velocities, respectively. When we assume that \( \gamma_0^2 >> 1 \), the so-called acoustic approximation, the interval anellipticity parameter is approximated by (Alkhalifah, 1997b)

\[
\eta = \frac{\varepsilon - \delta}{1 + 2\delta}.
\]  

Note that it is more natural to estimate \( \delta \) and \( \varepsilon - \delta \) than \( \delta \) and \( \varepsilon \), since only the first combinations appear in the traveltime expressions. This has also been observed in joint \( PP \) and \( PS \) traveltime tomography (Foss et al., 2005). Xiao et al. (2004) concluded that the anellipticity estimates are good when the absolute value of the difference \( \varepsilon - \delta \) is smaller than 0.2. When this term exceeds this value, there is non-uniqueness in the solution for the moveout parameter fitting. Equations 12-14 show that PP moveout data alone cannot invert for the five layer parameters in a VTI medium: \( \Delta z, \alpha_o, \beta_o, \delta, \) and \( \varepsilon \).

Additional information in the form of PS moveout traveltimes or well log information (van der Baan and Kendall, 2002) is required to estimate all the parameters in the VTI layers as outlined by Ursin and Stovas (2005).
The nonhyperbolic behavior of reflection moveouts can be equally explained by an isotropic model with a velocity which depends on depth only. For a layer with a constant $P$-wave velocity gradient, the estimated traveltime parameters from the top and bottom of a layer can be used to estimate the layer thickness, the velocity at the top of the layer and the velocity gradient from PP moveout data only (Stovas and Ursin, 2006, 2007).

The flowchart in Figure 5 summarizes the main steps in implementing automatic nonhyperbolic velocity analysis as applied below. The aim is to invert for the moveout parameters characterizing reflection curves in seismic data, and recover the interval velocity and anellipticity models required for subsequent time processing in anisotropic media. The approach outputs a raw stack after applying nonhyperbolic moveout corrections using the estimated parameters.

SYNTHEtic DATA EXAMPLE

A model containing five VTI layers with parameters given in Table 1 was used to test the algorithm. Figure 6a shows five primary reflections computed using raytracing in the model with Gaussian noise being added. Figure 6b shows the truncated gather at an aperture value equal to 1.9 using the exact velocity law of the reflections. Note that the spread length is not large enough to achieve the desired aperture at the two deepest interfaces. Figure 6c depicts the truncated data after applying nonhyperbolic moveout corrections with parameters estimated from velocity analysis.
Velocity analysis was run to detect the reflections in the time range [0.1 - 3 s] and the obtained BDS curve is shown on Figure 7c with its derivative on Figure 7d. The latter is used to identify the peaks on the BDS curve which correspond to zero-crossings in the derivative curve when this is changing sign from positive to negative. A smoothness factor and a threshold limit can act additionally as filters to impose more constraints on the selected peaks and to reject possible high-amplitude picks corresponding to noise (as those preceding the first event in Figure 7c). The reflections in the gather can be identified by projecting the picks from the BDS curve or its derivative on the raw curves of velocity and anellipticity in Figures 7a and 7b respectively. The identified picks from the nonhyperbolic search correspond in time to the reflections in the VTI model. The raw curves show high-frequency oscillations of the moveout parameters in the defined corridors. The theoretical values of the effective moveout parameters were computed using equations from Stovas and Ursin (2005) are listed in Table 2 together with the estimated parameters. The match in Figures 7a and 7b between the estimated parameters (shown as large empty circles) and the true parameters (small filled circles) is very good. The absolute errors in the moveout velocity range in the interval [4-24 m/s] for the five reflections, while the errors in anellipticity are in the interval [0-0.04] as summarized in Table 2. Both errors are sufficiently small to prove the accuracy of the traveltime equation 1 at the aperture limit 1.9. Note also that the errors are more significant when the effective anellipticity is large (> 0.3) proving the non-uniqueness of the model fitting the traveltime curve as outlined by several authors (Grechka and Tsvankin, 1998, van der Baan and Kendall, 2002, Douma and Calvert, 2006). The BDS curve shows that the event identification can be applied in an automatic way, the five maxima corresponding to the reflections in the gather are well resolved in the curve despite the presence of noise. Additionally, the estimated velocities from the hyperbolic search using the aperture value of 0.9 are shown on Figure 7a by the inverted triangle (\(\nabla\)), while the results from using the full-
spread length (equivalent to infinite aperture) are represented by the upward triangles \( \Delta \). These new results prove that hyperbolic velocity analysis using the aperture value 0.9 or using the full-spread length over-estimates the moveout velocity.

The obtained parameters were used to apply nonhyperbolic NMO corrections by means of equation 1 to flatten the reflections inside the gather and to enhance the stack quality over conventional hyperbolic velocity analysis. The corrected gather (Figure 6c) demonstrates that all the reflections in the truncated gather (Figure 6b) were corrected for the traveltime difference with respect to zero-offset. As the aperture decreases with depth for the same spread length, deep reflections are well-corrected within the spread length. For instance, the deepest event on the gather at 2.5 s is located at depth equal to 3.73 km. The maximum offset corresponds to an aperture value of only 1.34. Such a reflection can be well corrected using hyperbolic moveout corrections assuming zero anellipticity or even with an inaccurate velocity function. Thus, a rule of thumb, to obtain significant moveout parameters for a given reflector the maximum offset with respect to the expected reflector depth should be at least two.

The estimated effective traveltime parameters were used in the Dix-type inversion formulas 12 and 13 to obtain the interval parameters \( \Delta z_{\text{Dix}}, v_p \) and \( \eta \). These are listed in Table 3 together with the theoretical values computed from the model parameters in Table 1. There is a good agreement between the estimated values and the theoretical values computed from the model parameters.

NORTH SEA DATA EXAMPLE

The presented methodology was applied to a marine seismic dataset from the North Sea. The dataset has long spread length with maximum offset approaching 5 km.
The CDP interval is 25 m and the recording time is 7.5 s. Basic preprocessing was applied to the data prior to velocity analysis: amplitude scaling, tidal static corrections, shaping deconvolution and frequency filtering. Multiples were mostly attenuated by means of tau-p deconvolution. Tests have been performed on high-fold CDP gathers of the survey to choose the analysis parameters, in particular the design of the velocity corridor and the setting of the threshold limit to automatically detect the reflected energy inside the gathers. The algorithm was applied to all CDP gathers with fold coverage greater than or equal to 30. The guide velocity used for the truncation was derived by applying hyperbolic velocity analysis to few CDP gathers in the full-spread range (without truncation). The obtained velocity law was later used to apply the aperture truncation and to design the time-varying velocity corridor. The anellipticity was scanned for in the interval [0.0-0.4]. The velocity function obtained from running the above analysis on a given CDP gather is used to apply the truncation at the next analysis location to better constrain the estimated parameters. Additionally, the limits of the velocity corridor were updated using the velocity information obtained at the previously processed CDP gather. To compensate statistically for spatial and temporal amplitude variations, gain corrections were applied to the data. Figure 8 shows the gather at CDP 330. Note the strong reflectors around zero-offset times 2.4 and 2.7 s.

Both automatic hyperbolic and nonhyperbolic velocity analysis were applied to this gather to pick the moveout parameters of the reflectors inside. We used equation 1, combined with the proposed parameterization $q_1 - q_2$, to perform the nonhyperbolic parameter search, while the standard hyperbolic traveltime equation was used for the hyperbolic search. A truncation at an aperture equal to 0.9 was applied to the gather to perform hyperbolic velocity analysis and the mute limit is shown by the black curve on Figure 8, while an aperture value equal to 1.9 was used for the nonhyperbolic search, and is illustrated by the white curve in Figure 8.
The curve for the bootstrapped differential semblance computed for the time range [0.5-4 s] is depicted in Figure 9a with its derivative in Figure 9b. The window width used in the search was 32 ms, and a 50% overlap was applied to sample the time axis for all possible events in the gather. The corrected gathers for both the hyperbolic and the nonhyperbolic searches are shown on Figures 9c and 9d respectively. Most of the events inside the gathers have been flattened automatically using the estimated parameters from velocity analysis. Note in particular better results using the nonhyperbolic moveout search. Events at zero-offset times 1.8 and 2.4 s have some residual moveouts at far offsets from the hyperbolic moveout corrections while the two events are perfectly flattened using moveout corrections with parameters estimated from nonhyperbolic velocity analysis. To avoid noise magnification at shallow times where the truncation is generally severe, a minimum of 6 traces per time window are required to compute the BDS coefficient, otherwise the coefficient is set to zero.

To accelerate the nonhyperbolic moveout search and avoid the heavy computing required for a two-dimensional search along the time axis, we first run hyperbolic velocity analysis, using the aperture value of 0.9, to detect the reflectors in the gather. Later, the nonhyperbolic search is limited only to the identified events with the aperture value of 1.9 (see the Flowchart in Figure 5). In the second run, a refined search is done for both velocity and anellipticity at a larger aperture applied using the velocity obtained from the hyperbolic search. To make dense parameter search, the number of tentative parameters is increased by the respective multipliers for the two parameters. The value for the velocity multiplier depends on the maximum width of the velocity corridor and on the initial number of tentative parameters, while the anellipticity multiplier is chosen such that an accuracy of 0.01 is ensured for estimating the parameter $\bar{\eta}$. 
The algorithm was later applied to the whole seismic line to produce several attribute maps. The obtained picks from the nonhyperbolic search are displayed in Figure 10. They show a high spatial correlation proving the success of the automatic picking. The key reflectors can be easily identified on this plot especially the two events about zero-offset times 2.4 and 2.7 s and the reflector in between. In the shallow part, some correlation can also be seen between the picks, but it is less important than in the deep area. Some lateral correlation can also be observed at the event at about 3 s which corresponds to a slat body. The BDS curve and its derivative at each CDP location are indicators of the reflected energy inside the gathers and are less corrupted by noise in the data. These two attributes contain the fingerprints of the key reflectors in the stacked section. While the maxima correspond to event arrival in the BDS curve, the zero-crossings of the derivative map are, to a small static shift that is equal to half the time window width, indicators for the events. Figure 11a shows the BDS curve along the line showing the main reflectors. The two strong reflectors on CDP 330 are clearly visible on this map, and the thickness of the layer in between is decreasing toward the right to form a geologic discordance around CDP 550. The derivative map, Figure 11b, illustrates the same features. Another reflector at about zero-offset time 1 s is also visible on both maps.

The stacked sections from automatic hyperbolic and nonhyperbolic velocity analyses are shown in Figures 12a and 12b, respectively. They contain the same structures already predicted by the attributes in Figure 11. Note the significant energy enhancement and signal strength in the stack from the nonhyperbolic search over the stack from the hyperbolic search. This was proven in Figures 9c and 9d, where the hyperbolic NMO suffers from residual moveout at far offsets which reduce the signal amplitude in the stack. The details of the section and the continuity of the reflectors are
clearly enhanced using the nonhyperbolic search. The diffractions resulting from the seabed topography are better defined in the stack in Figure 12b.

The moveout parameters obtained from the automatic search are interpolated, in space and time, to provide continuous and smooth parameter fields. Since the automatic picking is performed bin after bin, the lateral coherency of velocity and anellipticity values is not always achieved. We apply geostatistical tools based on variographic analysis to remove strong lateral variation between adjacent pickings (nugget effect according to the variographic analysis of velocity and $\eta$ variations). We used also the kriging interpolation method to fill missing values according to variographic description. In some areas, this way is not sufficient to achieve the lateral coherency and a trend filtering maybe required.

The moveout velocity field obtained using the hyperbolic search is shown in Figure 13a, while the results from nonhyperbolic velocity analysis are displayed in Figures 13b and 13c for the normal moveout velocity and the effective anellipticity, respectively. The difference between the moveout velocity maps is due to the fact that the isotropic assumption compensates for the anellipticity which is assumed to be zero. The anellipticity field demonstrates a slight to average anisotropy strength with most values in the range [0-0.2]. Some correlation can be observed between the velocity and the anellipticity fields. The velocity models demonstrate important heterogeneity and variations and are in a good match with velocity model estimated from anisotropic depth imaging combining both PP and PS data.

Using Dix-type inversions and the obtained picks from velocity analysis, we computed interval parameter fields for the moveout velocity and the anellipticity. These are shown in Figure 14. The velocity models show a slight parameter increase to about
2 s, and a sudden jump in the interval velocity after due to the reflector at 2.5s. Another
jump occurs at the interface at about 3 s and corresponding to the salt body. Despite the
fact that this reflector was not highlighted in the seismic stacks (Figures 12a and b), but
the inverted velocity models show clearly this interface corresponding to a significant
velocity increase with respect to the overburden. With the VTI assumption, this interval
velocity map is only apparent and further information is needed to solve for vertical
velocity. The interval anellipticity map is shown on Figure 14c. It contains a large zone
between 1 and 2 s with a $\eta$ value exceeding 0.10. A negative value for $\eta$ is observed
also in the time range 2-3 s. The remaining field is characterized by weak $\eta$ values
($\eta < 0.10$). This map is highly sensitive to errors in the effective anellipticity values.
Dix-type equation (equation 13) magnifies errors occurring on effective parameters
when converted to interval parameters. Thus, the accuracy of the interval anellipticity
map which is a direct anisotropy indicator is always discussable.
CONCLUSIONS

We proposed a new methodology for the automatic implementation of nonhyperbolic velocity analysis which combines efficient sampling algorithms and automatic event detection. The approach aims at producing accurate moveout parameters essential for successful time processing (nonhyperbolic moveout corrections, accurate compensation for geometrical spreading, anisotropic DMO and time migration) of PP data. The algorithm also produces an automatic stack and parameter maps for the coherency estimator and its derivative. The proposed parameterization allows for an optimal moveout sampling leading to a better estimation of the moveout parameters. The developed approach showed efficiency on a real long-spread marine dataset from the North Sea and produced parameter maps with a good accordance with more advanced processing. The obtained velocity and anellipticity fields were used to invert for apparent interval parameters through generalized Dix equations. Both heterogeneous isotropic layers and homogeneous VTI layers will give the same effective moveout parameters. When the interval anellipticity is zero, the layer is VTI with elliptic anisotropy or isotropic. The estimated parameters are sufficiently accurate for small-to-moderate anellipticity values. When the latter is important ($\bar{\eta} > 0.2$) the parameters are biased as outlined by several authors (Grechka and Tsvankin, 1998; van der Baan and Kendall, 2002). The estimated interval velocity maps have a great correlation with the events in seismic section and can be applied in lithology identification as well as in subsequent imaging. The interval anellipticity map is highly sensitive to errors in the effective anellipticity values. Dix-type equation magnifies errors occurring on effective parameters when converted to interval parameters. Thus, the accuracy of the interval anellipticity map which is a direct anisotropy indicator is always discussable.
The presented methodology has some shortcomings. Vertical stacking of common offsets in adjacent CDP traces is a problem when the local dip is important. In complex geology the method has limitations but may serve to build a background model to more sophisticated approaches. The application of the presented method to land seismic data remains a challenge due to their more complicated nature (high noise level, static problems, more distortions on amplitudes and signal frequencies).
ACKNOWLEDGEMENTS

The authors are grateful to financial support from Total R&D in Pau (France), the Norwegian Research Council (NFR) through Sintef Petroleum Research and the ROSE project. We thank StatoilHydro for providing the real dataset and allowing the publication of the results. We thank also Jean-Luc Boëlle from Total R&D and Børge Arntsen from StatoilHydro for constructive comments. We thank also Tamas Nemeth, the associate editor of Geophysics, and two anonymous reviewers for fruitful discussions that helped improve the paper.
REFERENCES


Foss, S.-K., B. Ursin, and M. V. de Hoop, 2005, Depth-consistent reflection tomography using PP and PS seismic data: Geophysics, 70, U51-U65.


Neidell, N. S., and M. T. Taner, 1971, Semblance and other coherency measures for multichannel data: Geophysics, 36, 482-497.


FIGURE CAPTIONS

Figure 1. Sampling of the moveout parameters for the different parameterizations. Moveout range sampling for the $V_{NMO} - V_H$, $dT_n - \tau_0$, and $q_1 - q_2$ domains in Figures (a, b, and c), respectively. Velocity and anellipticity sampling for the respective domains in Figures (d, e, and f).

Figure 2. Effect of bootstrapping on a time window. a) The time window with traces arranged in increasing offset order. b) The same time window with random trace ordering by applying bootstrapping.

Figure 3. Effects of bootstrapping on the velocity spectrum. a) A synthetic gather containing five primary reflections with five interfering multiples and random noise. b) Velocity spectrum using differential semblance. c) Velocity spectrum using bootstrapped differential semblance.

Figure 4. The process of event centering inside time windows. a) An initial time window with a positive error on the zero-offset time. b) The same time window after iterative event centering.

Figure 5. A flowchart summarizing the main steps in the algorithm of nonhyperbolic velocity analysis.

Figure 6. a) Synthetic data from a VTI model consisting of five primary reflections corrupted with Gaussian noise. b) The data truncated according to an aperture equal to 1.9. c) The truncated data after applying nonhyperbolic NMO corrections.

Figure 7. Results of automatic nonhyperbolic velocity analysis on the synthetic gather in Figure 6b. a) The raw moveout velocity field inside the time-varying corridor with true and estimated parameters superposed for comparison. b) The raw effective anellipticity field within the time-invariant corridor [0-0.5] with true and estimated
parameters. c) The BDS maximum coherency curve with the reflections indicated by filled circles. d) The BDS derivative with the picks corresponding to zero-crossings.

Figure 8. Far-offset truncation of a seismic gather from the North Sea. Original CDP gather with the truncation limits at an aperture equal to 0.9 for the hyperbolic search (black line) and an aperture equal to 1.9 used for the nonhyperbolic search (white line).

Figure 9. Results of velocity analysis on the CDP gather shown in Figure 8. a) The BDS curve obtained by hyperbolic search on the gather in Figure 8 with the truncation highlighted by the black curve. b) The BDS derivative curve used to identify coherency maxima. c) NMO-corrected gather using parameters from the hyperbolic search. d) NMO-corrected gather using nonhyperbolic search parameters.

Figure 10. Positions of the picked moveout parameters from the automatic search.

Figure 11. Attribute maps obtained from automatic velocity analysis.

a) The BDS map along the seismic line, b) The BDS derivative map.

Figure 12. Raw stacks produced from a) hyperbolic and b) nonhyperbolic velocity analysis.

Figure 13. Smoothed moveout velocity and effective anellipticity fields obtained by the method. a) Moveout velocity model from hyperbolic search. b) Moveout velocity field using nonhyperbolic search. c) Effective anellipticity field from nonhyperbolic search.

Figure 14. Smoothed interval parameter maps obtained through Dix-type inversions.

a) Interval moveout velocity model from hyperbolic search. b) Interval moveout velocity field using nonhyperbolic search. c) Interval anellipticity map from nonhyperbolic search.

LIST OF TABLES

Table 1. Parameters of the VTI model.

Table 2. True and estimated effective traveltime parameters.

Table 3. True and estimated interval parameters derived from the effective parameters.
a) Moveout range (s)
   T(0) (s)
   0  0.4  0.8  1.2
   0  1  2  3  4

b) Moveout range (s)
   T(0) (s)
   0  0.4  0.8  1.2
   0  1  2  3  4

c) Moveout range (s)
   T(0) (s)
   0  0.4  0.8  1.2
   0  1  2  3  4

d) $V_{NMO}$ (km/s)
   $I = 0.2$


e) $V_{NMO}$ (km/s)
   $I = 0.2$

f) $V_{NMO}$ (km/s)
   $I = 0.2$
Figure 1. Sampling of the moveout parameters for the different parameterizations. Moveout range sampling for the $V_{NMO} - V_h$, $dT_n - T_0$, and $q_1 - q_2$ domains in Figures (a, b, and c), respectively. Velocity and anellipticity sampling for the respective domains in Figures (d, e, and f).

Figure 2. Effect of bootstrapping on a time window. a) The time window with traces arranged in increasing offset order. b) The same time window with random trace ordering by applying bootstrapping.
Figure 3. Effects of bootstrapping on the velocity spectrum. a) A synthetic gather containing five primary reflections with four interfering multiples and random noise. b) Velocity spectrum using differential semblance. c) Velocity spectrum using bootstrapped differential semblance.
Figure 4. The process of event centering inside time windows. a) An initial time window with a negative error on the zero-offset time. b) The same time window after iterative event centering.
Figure 5. A flowchart summarizing the main steps in the algorithm of nonhyperbolic velocity analysis.
Figure 6. a) Synthetic data from a VTI model consisting of five primary reflections corrupted with Gaussian noise. b) The data truncated according to an aperture equal to 1.9. c) The truncated data after applying nonhyperbolic NMO corrections.
Figure 7. Results of automatic nonhyperbolic velocity analysis on the synthetic gather in Figure 6b. a) The raw moveout velocity field inside the time-varying corridor with true and estimated parameters superposed for comparison. b) The raw effective anellipticity field within the time-invariant corridor [0-0.5] with true and estimated parameters. c) The BDS maximum coherency curve with the reflections indicated by filled circles. d) The BDS derivative with the picks corresponding to zero-crossings.
Figure 8. Far-offset truncation of a seismic gather from the North Sea. Original CDP gather with the truncation limits at an aperture equal to 0.9 for the hyperbolic search (black line) and an aperture equal to 1.9 used for the nonhyperbolic search (white line).
Figure 9. Results of velocity analysis on the CDP gather shown in Figure 8. a) The BDS curve obtained by hyperbolic search on the gather in Figure 8 with the truncation highlighted by the black curve. b) The BDS derivative curve used to identify coherency maxima. c) NMO-corrected gather using parameters from the hyperbolic search. d) NMO-corrected gather using nonhyperbolic search parameters.
Figure 10. Positions of the picked moveout parameters from the automatic search.
Figure 11. Attribute maps obtained from automatic velocity analysis.

a) The BDS map along the seismic line, b) The BDS derivative map.
Figure 12. Raw stacks produced from a) hyperbolic and b) nonhyperbolic velocity analysis.
Figure 13. Smoothed moveout velocity and effective anellipticity fields obtained by the method. a) Moveout velocity model from hyperbolic search. b) Moveout velocity field using nonhyperbolic search. c) Effective anellipticity field from nonhyperbolic search.
Figure 14. Smoothed interval parameter maps obtained through Dix-type inversions. a) Interval moveout velocity model from hyperbolic search. b) Interval moveout velocity...
field using nonhyperbolic search. c) Interval anellipticity map from nonhyperbolic search.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\Delta z$ (km)</th>
<th>$\alpha_0$ (km/s)</th>
<th>$\beta_0$ (km/s)</th>
<th>$\delta$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.66</td>
<td>2.2</td>
<td>0.7</td>
<td>0.02</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>0.81</td>
<td>2.7</td>
<td>0.9</td>
<td>0.04</td>
<td>0.09</td>
</tr>
<tr>
<td>3</td>
<td>0.64</td>
<td>3.2</td>
<td>1.3</td>
<td>0.03</td>
<td>0.23</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>4.0</td>
<td>1.8</td>
<td>-0.02</td>
<td>0.34</td>
</tr>
<tr>
<td>5</td>
<td>0.94</td>
<td>4.7</td>
<td>2.5</td>
<td>0.05</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 1. Parameters of the VTI model.

<table>
<thead>
<tr>
<th>Interface</th>
<th>$T(0)$ (s)</th>
<th>True parameters</th>
<th>Estimated for aperture=1.9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$V_{NMO}$ (km/s)</td>
<td>$\bar{\eta}$</td>
</tr>
<tr>
<td>1</td>
<td>0.6</td>
<td>2.244</td>
<td>0.11</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
<td>2.540</td>
<td>0.07</td>
</tr>
<tr>
<td>3</td>
<td>1.6</td>
<td>2.748</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>2.1</td>
<td>3.068</td>
<td>0.32</td>
</tr>
<tr>
<td>5</td>
<td>2.5</td>
<td>3.434</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 2. True and estimated effective traveltime parameters.
<table>
<thead>
<tr>
<th>Layer</th>
<th>Theoretical parameters</th>
<th>Estimated values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta z_{Dix}$ (km)</td>
<td>$v_p$ (km/s)</td>
</tr>
<tr>
<td>1</td>
<td>0.673</td>
<td>2.244</td>
</tr>
<tr>
<td>2</td>
<td>0.842</td>
<td>2.806</td>
</tr>
<tr>
<td>3</td>
<td>0.659</td>
<td>3.295</td>
</tr>
<tr>
<td>4</td>
<td>0.980</td>
<td>3.919</td>
</tr>
<tr>
<td>5</td>
<td>0.986</td>
<td>4.929</td>
</tr>
</tbody>
</table>

Table 3. True and estimated interval parameters derived from the effective parameters.