SEISMIC REFLECTION ESTIMATION WITH COLOURED NOISE

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(Received March 14, 1995; revised version accepted November 21, 1995)

ABSTRACT

Ursin, B., de Matos Neto, J.B. and Porsani, M.J., 1996. Seismic reflection estimation with coloured noise. *Journal of Seismic Exploration*, 5: 51-62.

The convolutional model of a seismic trace consists of a seismic pulse convolved with the reflectivity series plus noise. Assuming coloured noise with known or estimated auto-correlation functions, two approaches to inverse filtering are compared. A maximum-likelihood estimate of the reflectivity series is obtained by filtering the data with a whitening filter and using a least-squares inversion technique. Alternatively, a pulse-shaping filter is applied to the data. The design of the filter is based on the known pulse and the known auto-correlation function of the noise.

The different approaches are compared on synthetic and real seismic data. Maximum-likelihood estimation and pulse-shaping filter give almost identical results, and both methods give clearly superior results compared to assuming white noise. Optimal pulse-shaping filter is the best technique, since it is computationally fast and numerically more stable. (Maximum-likelihood estimation requires a matrix inversion for each trace).

KEY WORDS: deconvolution, inverse filtering, whitening filter, pulse-shaping, maximum-likelihood, least-squares, coloured noise, reflectivity series.

0963-0651/96/$5.00

INTRODUCTION

The theory and practical application of least-squares inverse filtering of seismic data is well established (Robinson and Treitel, 1980). The convolutional model of a seismic trace consists of a seismic pulse convolved with a reflectivity series plus noise. Although the theory for inverse filtering with coloured noise is well known (Treitel and Robinson, 1966; Berkhout, 1977), there seem to be few examples of data processing taking into account the shape of the noise auto-correlation function. Belykh (1990) designed an inverse filter in the frequency domain and applied it to real seismic data.

Chi et al. (1984) derived a maximum-likelihood algorithm to estimate the seismic pulse and sparse reflectors,assuming white noise. Ursin and Holberg (1985) estimated both the reflectivity series and the coefficients in a moving-average noise filter from seismic data and the seismic pulse. Further practical examples of the use of this method were given by Brevik et al. (1990).

Here, we study a simpler form of maximum-likelihood estimation assuming that the seismic pulse and the noise auto-correlation function are known. The noise auto-correlation function may be estimated from separate noise records. These should be recorded at the same time as the data, but without activating the source. An alternative approach is to design a pulse-shaping filter taking into account the seismic pulse and the noise auto-correlation function. The different algorithms will be compared on synthetic and real seismic data.

CONVOLUTIONAL MODEL

A seismic trace is assumed to consist of a known seismic pulse convolved with a reflection coefficient series plus a moving average noise process (coloured noise)(Berkhout, 1977; Ursin and Holberg, 1985). An observed seismic trace may then be modelled as

$$y(t) = s(t) + w(t) \quad , \qquad t = 0, 1, \ldots, N_y \tag{1}$$

where s(t) is the noise-free signal and w(t) is the noise. The signal consists of the seismic pulse p(t) convolved with the reflectivity series r(t):

$$s(t) = \sum_{j=0}^{N_p} p(j)r(t-j) \quad , \tag{2}$$

and the noise consists of a white-noise process e(t) convolved with a noise filter with impulse response c(t):

$$w(t) = \sum_{j=0}^{N_c} c(j)e(t-j) \quad . \tag{3}$$

In a matrix-vector notation (Robinson and Treitel, 1980) equations (1) to (3) may be written as

$$\mathbf{y} = \mathbf{s} + \mathbf{w} = \mathbf{Pr} + \mathbf{Ce} \quad . \tag{4}$$

The noise process {e(t)} is a zero-mean Gaussian white-noise series with variance σ_e^2. It is assumed that {r(t)} and {e(t)} are uncorrelated.

First, the reflectivity series is assumed to consist of unknown deterministic parameters which are estimated by a maximum-likelihood method. Alternatively, the reflectivity series is assumed to be a zero-mean white-noise process with variance σ_r^2. In this case, a Wiener shaping filter is applied to the data. The filter design is based on the knowledge of the signal pulse and the noise auto-correlation function.

MAXIMUM-LIKELIHOOD ESTIMATION

Ursin and Holberg (1985) and Brevik et al. (1990) estimated the reflection coefficients series and the coefficients of the noise filter from the data and a known seismic pulse, using a nonlinear optimization algorithm. The expression

$$\Phi = \|\mathbf{e}\|^2 = \|\mathbf{C}^{-1}\mathbf{y} - \mathbf{C}^{-1}\mathbf{Pr}\|^2$$

was minimized with respect to **r** and the noise filter **c**. $\|\mathbf{e}\|^2 = \mathbf{e}^T\mathbf{e}$ (T denotes transpose) is the squared Euclidian norm. The estimated noise filter c is chosen to be minimum phase, so that it has a stable inverse filter. The matrix $\mathbf{C}^{-1}$ will then exist, at least asymptotically. The practical implementation of $\mathbf{C}^{-1}$ was actually done by applying this inverse filter to the residual $y - p * r$.

When the noise filter or the noise auto-correlation function $\mathbf{R}_w = \sigma_e^2\mathbf{CC}^T$ is known, the maximum-likelihood estimate of the reflection coefficient vector is

$$\hat{\mathbf{r}} = [\mathbf{P}^T(\mathbf{CC}^T)^{-1}\mathbf{P}]^{-1}\mathbf{P}^T(\mathbf{CC}^T)^{-1}\mathbf{y}$$
$$= [\mathbf{P}^T(\mathbf{R}_w)^{-1}\mathbf{P}]^{-1}\mathbf{P}^T(\mathbf{R}_w)^{-1}\mathbf{y} \quad . \tag{5}$$

When the additive noise is white (corresponding to $\mathbf{C} = \mathbf{I}$, the identity matrix), this gives the least-squares estimate

$$\hat{\mathbf{r}}_{LS} = (\mathbf{R}_p + \lambda^2\mathbf{I})^{-1}\mathbf{r}_{yp} \quad . \tag{6}$$

Here $\mathbf{R}_p$ is the pulse auto-correlation matrix, $\mathbf{r}_{yp}$ is the cross-correlation vector between the trace and the pulse, and λ^2 is a stabilizing factor which is given in percent of the zero-lag pulse auto-correlation function.

In order to obtain the maximum-likelihood estimate in equation (5) it is necessary to compute $\mathbf{R}_w^{-1}$ and apply it to each trace. We have used a computationally more efficient method consisting of applying a whitening filter a(t) to the data and to the pulse. The filtered data are then

$$y'(t) = a(t) * y(t) = p'(t) * r(t) + e(t) \quad ,$$

where the filtered seismic pulse is $p'(t) = a(t) * p(t)$. The maximum-likelihood estimate of the reflection coefficient vector is now

$$\hat{\mathbf{r}}_{ML} = (\mathbf{R}_{p'} + \lambda^2\mathbf{I})^{-1}\mathbf{r}_{y'p'} \quad , \qquad (7)$$

where again a stabilizing factor has been included.

The whitening filter a(t) is computed from the equation

$$(\mathbf{R}_w + \lambda^2\mathbf{I}) \begin{bmatrix} 1 \\ a(1) \\ \vdots \\ a(N_a) \end{bmatrix} = \begin{bmatrix} \sigma_e^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (8)$$

where the stabilizing factor λ^2 is chosen as in equation (6).

LEAST-SQUARES PULSE-SHAPING FILTER

Using the least-squares method one can obtain the filter h(t) that, convolved with the trace y(t), gives an optimum approximation of the desired output $d(t) * r(t)$, where d(t) is the desired pulse. In this case we choose a shaping filter h(t) which minimizes the expected error energy (Treitel and Robinson, 1966)

$$\Phi = E\{[h(t) * y(t) - d(t) * r(t)]^2\}$$

$$= \sigma_r^2 \sum_t [h(t)*p(t) - d(t)]^2 + \sigma_e^2 \sum_t [h(t) * c(t)]^2$$

$$= \sigma_r^2 \| \mathbf{Ph} - \mathbf{d} \|^2 + \sigma_e^2 \| \mathbf{Ch} \|^2 \quad . \qquad (9)$$

The optimal filter is then given by

$$\mathbf{h} = (\mathbf{R}_p + \lambda^2\mathbf{R}_w)^{-1}\mathbf{r}_{dp} \quad , \qquad (10)$$

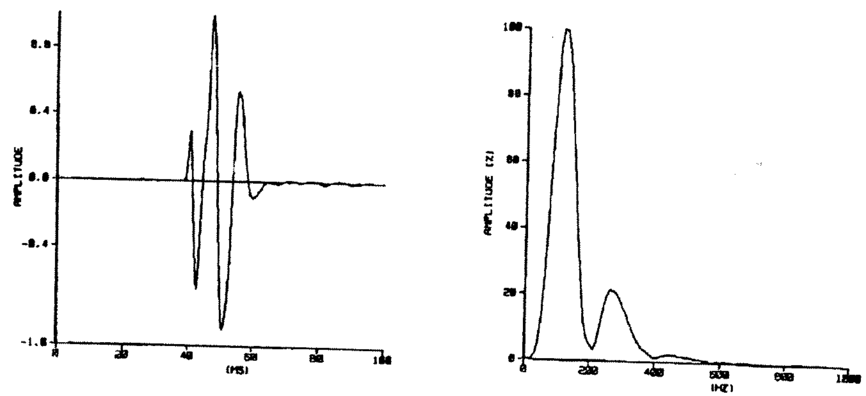
where $\lambda^2 = 1/\sigma_r^2$ is a weighting factor and $\mathbf{r}_{dp}$ is the cross-correlation vector between the desired pulse and the pulse. Belykh (1990) designed a similar filter in the frequency domain with the desired pulse being a unit impulse (a spike). When the noise is assumed to be white, we put $\mathbf{R}_w = \mathbf{I}$.

SYNTHETIC SEISMIC DATA

In the following data examples, equations (6), (7), (8) and (10) were all stabilized by choosing a suitable λ^2 (which has a slightly different interpretation in the different cases). They were all chosen after testing a range of values and choosing the smallest value which gave a stable result.

Synthetic seismic data were generated according to the model in equations (1)-(3) using the mixed-phase pulse shown in Fig. 1. The reflection response corresponds to a wedge or a pinch-out with reflection coefficients of equal magnitude and opposite polarity. The noise-free synthetic data are shown in Fig. 2. Coloured noise was generated by passing white Gaussian noise through a band-pass filter. The noise was added to the signal traces with different scaling factors. In Fig. 3a the signal-to-noise ratio is 2 (S/N = 2), while in Fig. 3b there is much more noise, corresponding to S/N = 0.2.

The least-squares estimate of the reflection coefficients is obtained using equation (6), ignoring the fact that the noise is coloured. The results are shown in Figs. 4a and 4b. Even though a high stabilizing factor has been used, the result is unstable for low S/N.



(a) (b)

Fig. 1. (a) The seismic pulse. (b) The amplitude spectrum of the seismic pulse.

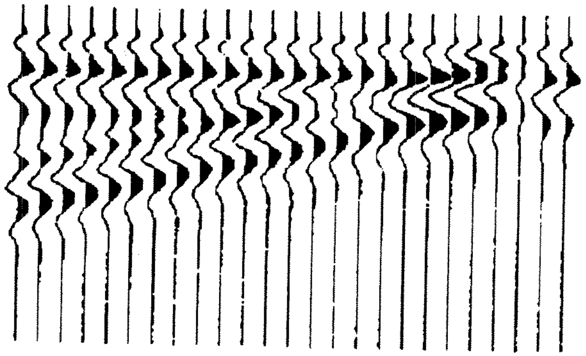


Fig. 2. The noise-free synthetic seismic data.

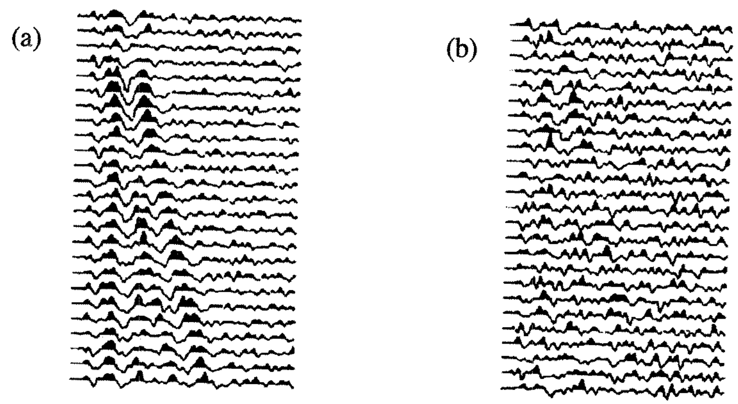


Fig. 3. (a) Synthetic seismic data with S/N = 2.0. (b) Synthetic seismic data with S/N = 0.2.

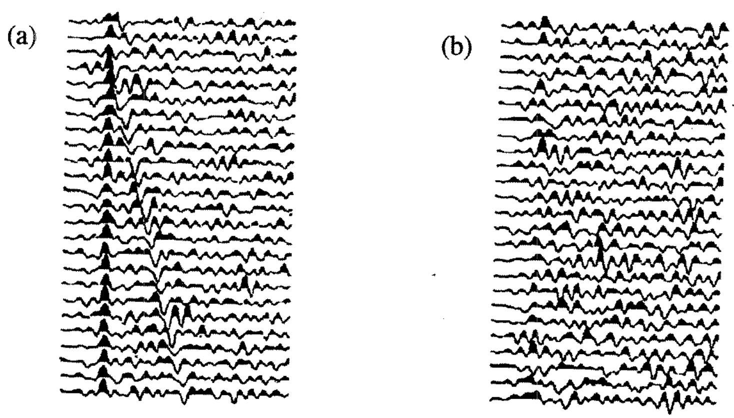


Fig. 4. (a) Least-squares estimation of the reflection coefficients, S/N = 2.0. (b) Least-squares estimation of the reflection coefficients, S/N = 0.2.

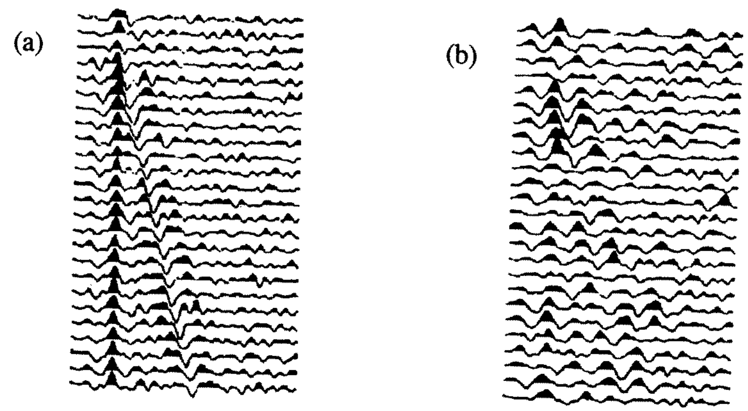


Fig. 5. (a) Maximum-likelihood estimation of the reflection coefficients, S/N = 2.0. (b) Maximum-likelihood estimation of the reflection coefficients, S/N = 0.2.

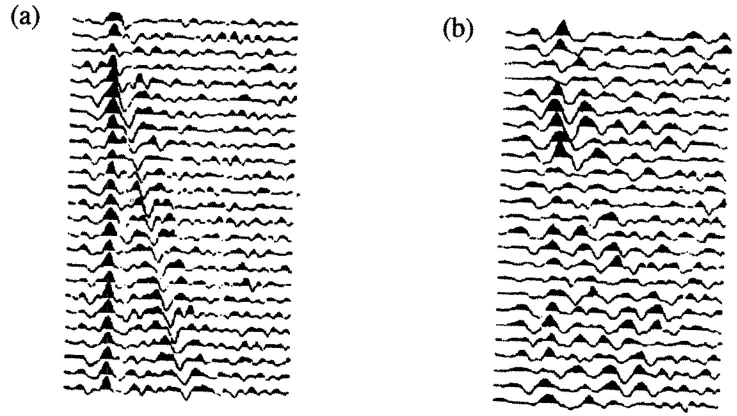


Fig. 6. (a) Least-squares pulse-shaping to spike, S/N = 2.0. (b) Least-squares pulse-shaping to spike, S/N = 0.2.

Figs. 5a and 5b show the maximum-likelihood estimates of the reflection coefficients. These were obtained by filtering the data with a whitening filter and applying the modified least-squares equation (7). The result is a drastic improvement compared to the least-squares estimates in Fig. 4.

In Fig. 6 an optimal pulse-shaping filter has been applied to the data. The filter was designed to produce a desired pulse equal to an impulse (a spike). The result is almost identical to the maximum-likelihood estimation shown in Fig.5.

REAL SEISMIC DATA

The different processing techniques were also applied to the seismic data described by Belykh (1990). A sparker source was placed at 3.75 m depth, and the seismic response was recorded by a hydrophone at about 500 m depth. The total water depth was about 3500 m. While the ship was drifting, the hydrophone recorded the composite downgoing pulse, followed by noise, and then the reflections from the sediments. The seismic pulse and its amplitude spectrum are shown in Fig. 1. The notches in the spectrum due to the reflection from the sea/air interface can clearly be seen at about 200 Hz and 400 Hz.

The recorded single-trace, zero-offset section is shown in Fig. 7a. The least-squares estimate of the seismic reflectivity series is shown in Fig. 7b, and the maximum-likelihood estimate is shown in Fig. 7c. The latter section is more stable and has more resolution than the former. Fig. 7d shows the result of applying a pulse-shaping filter assuming white noise, while Fig. 7e shows the result of applying a shaping filter designed using an estimate of the noise auto-correlation function. Again the latter section is superior to the former. By comparing the maximum-likelihood estimate in Fig. 7c and the optimally filtered section in Fig. 7e, it is seen that the results are comparable, as they were for the synthetic data.

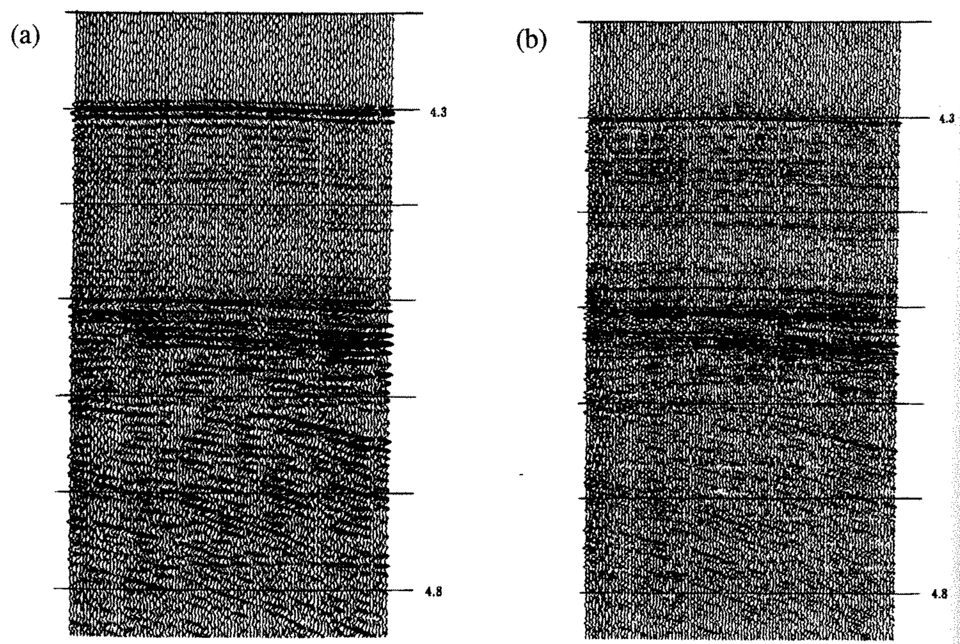


Fig. 7. (a) Initial seismic section. (b) Least-squares estimate.

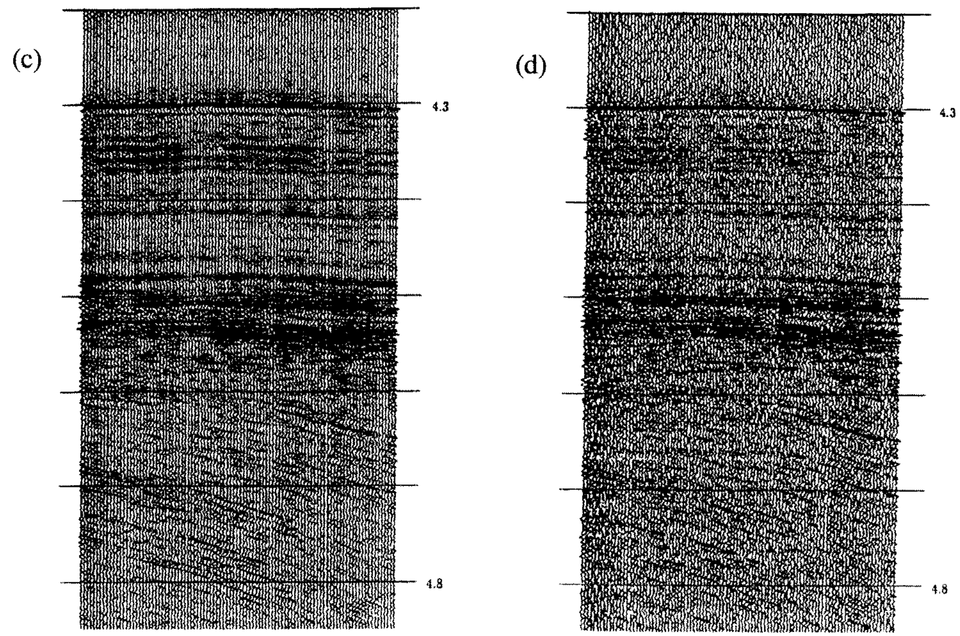


Fig. 7. (c) Maximum-likelihood estimate. (d) Pulse-shaping filtering, assuming white noise.

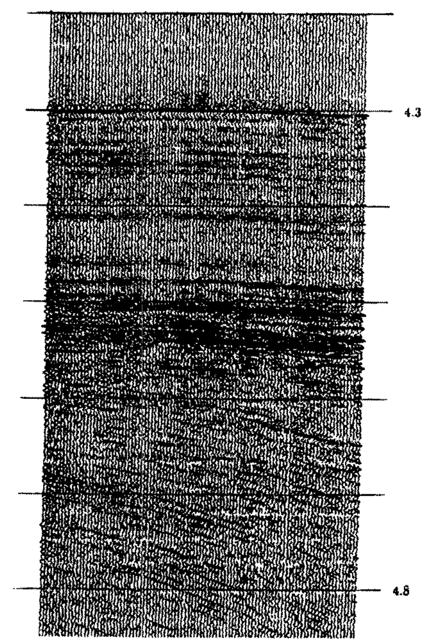
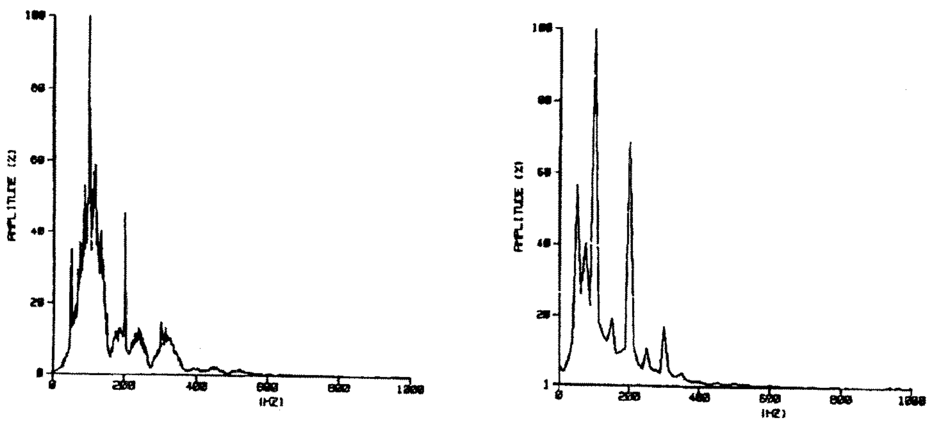


Fig. 7. (e) Pulse-shaping filtering, assuming coloured noise.



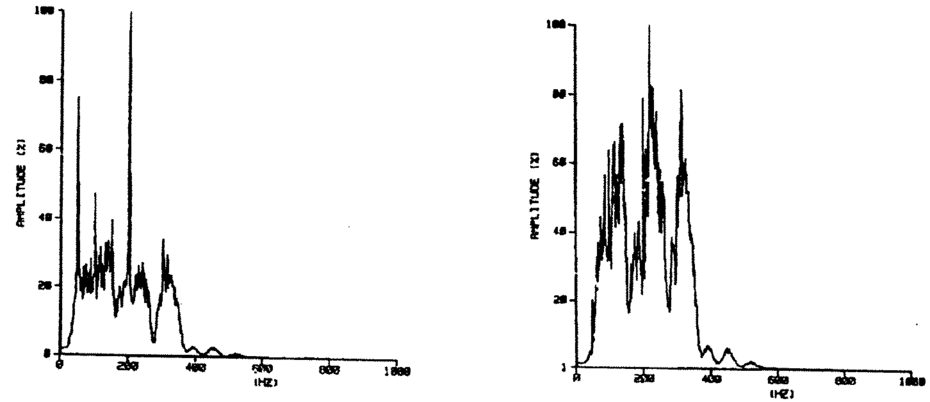
(a) (b)

Fig. 8. (a) Amplitude spectrum of initial section. (b) Amplitude spectrum of the noise.

To further analyze the processed data, the normalized average amplitude spectra of the different sections were computed. The normalized average amplitude spectrum of the recorded seismic data shown in Fig. 7a is shown in Fig. 8a, and the estimated noise amplitude spectrum is shown in Fig. 8b. The noise is dominated by peaks in the amplitude spectrum at 50 Hz, 100 Hz and 200 Hz. These are almost certainly due to problems with the electrical power-distribution system. The same noise is also clearly seen in the spectrum of the recorded seismic data.

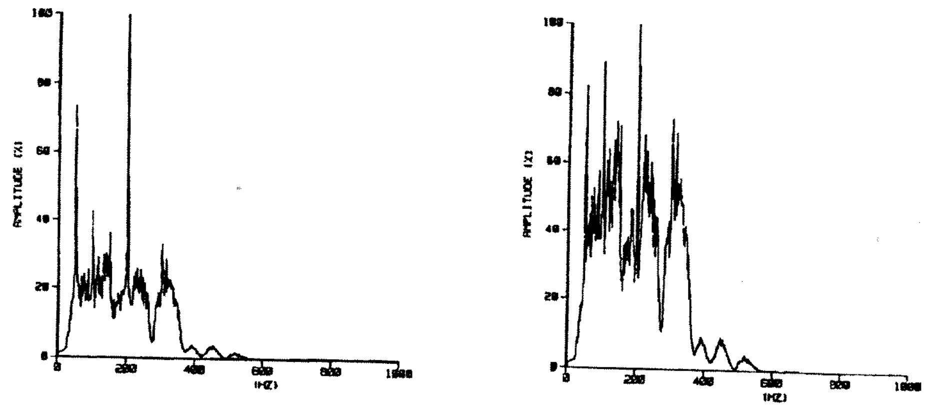
The normalized average amplitude spectrum of the least-squares estimate in Fig. 7b is shown in Fig. 9a, while the spectrum of the maximum-likelihood estimate in Fig. 7c is shown in Fig. 9b. The maximum-likelihood estimate clearly has less of the dominant noise than the least-squares estimate. The normalized average amplitude spectrum of the filtered section in Fig. 7d is shown in Fig. 10a, while the spectrum of the optimally filtered section in Fig. 7e is shown in Fig. 10b. Again it is seen that taking into account the noise auto-correlation function reduces the noise level.

Finally, by comparing the spectrum of the maximum-likelihood estimate in Fig. 9b and the spectrum of the optimally filtered section in Fig. 10b, it appears that maximum-likelihood estimation gives the best result. However, the processed sections in Figs. 7c and 7e are very similar.



(a) (b)

Fig. 9. (a) Amplitude spectrum of the least-squares section. (b) Amplitude spectrum of the maximum-likelihood section.



(a) (b)

Fig. 10. (a) Amplitude spectrum of the filtered section, assuming white noise. (b) Amplitude spectrum of the filtered section, assuming coloured noise.

CONCLUSIONS

The theory of seismic reflection coefficient estimation with coloured noise was reviewed. From the examples it can be concluded that improved results are obtained by taking into account the shape of the noise auto-correlation function. The results are very similar for maximum-likelihood estimation using a whitening filter and pulse-shaping filtering to a spike. The computational effort for pulse-shaping filtering is much less than for maximum-likelihood estimation.

In the first case, a set of normal equations is solved only once to compute the filter, and then each trace is convolved with the filter. In the case of maximum-likelihood estimation, the design and application of the whitening filter corresponds to a similar computational effort. In addition, the matrix inversion in equation (7) must be done for each trace.

Optimal pulse-shaping filtering is therefore the preferred technique, since it is computationally fast and numerically more stable (no matrix inversion is required).

ACKNOWLEDGEMENTS

A preliminary version of this paper was presented at the 62nd SEG meeting in New Orleans, 1992. We wish to thank I.N. Belykh for providing the seismic data. B. Ursin wishes to thank the Conselho Nacional de Desenvolvimento Científico e Tecnológico, CNPq, and Petrobrás for the opportunity to participate in the PPPG/UFBA program. J.B. de Matos Neto wishes to thank Petrobrás for the opportunity to study in the PPPG/UFBA program. M.J. Porsani wishes to thank CNPq for their kind support.

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