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New travel-time approximations for a transversely isotropic medium

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Abstract
In seismic data processing it is common to use a non-hyperbolic travel-time approximation assuming weak anisotropy in a transversely isotropic medium with vertical symmetry axis. It has the correct short-spread (normal) moveout velocity, but higher order terms in offset squared are only approximations. Using Taylor expansions for the squared vertical slowness for P- and SV-waves result in new approximations for the phase velocities which may be used in pre-stack migration algorithms. These approximations are combined with expressions for travel time and offset as functions of horizontal slowness, giving three new approximations for travel time squared as functions of offset squared, without the assumption of weak anisotropy. Numerical examples show that these new approximations all have better accuracy than the standard weak-anisotropy approximation. Based on the numerical results, we propose a new travel-time approximation for reflected PP-, SS- and PS-waves to be used in seismic data processing. They have almost the same functional form as the weak-anisotropy approximations, with the same normal moveout velocities but with a different heterogeneity factor which is non-linear in the anisotropy parameters.

Keywords: travel time, anisotropy, seismic waves

1. Introduction

In anisotropic media the seismic velocities depend on direction, and the simple hyperbolic formula for the travel times is no longer valid. For a transversely isotropic (VTI) medium with a vertical symmetry axis, Tsvankin and Thomsen (1994) proposed a non-hyperbolic travel-time approximation by assuming weak anisotropy. This approximation has the correct normal moveout velocity, and is useful for relatively short offsets. It is also correct for very long offsets, that is, for a horizontally travelling wave. Our goal is to derive approximations for travel time squared as a function of offset for a reflected, possibly mode-converted wave, which are valid for short and medium offsets. Such approximations can more easily be extended to the multi-layer case, and they can be directly used in seismic time processing (Alkhalifah 1997, Ursin and Hokstad 2003). An alternative approach is to perform velocity analysis and seismic data processing in the τ–p domain (Van der Baan and Kendall 2002, 2003, Sen and Mukherjee 2003).

We derive Taylor series approximations for the squared vertical slownesses and phase velocities for (quasi-)P- and (quasi-)SV-waves in a VTI medium without using a weak-anisotropy approximation. Assuming weak anisotropy our approximation for the P-wave phase velocity reduces to that given by Sen and Mukherjee (2003), and by setting the S-wave velocity equal to zero our approximation is close to that given by Alkhalifah (1998). Fowler (2003) has compiled a large number of approximations for the phase velocities, group velocities and travel times in a homogeneous VTI medium, many of these are based on perturbations from the elliptical transverse isotropy (Schoenberg and de Hoop 2000). Combining the Taylor series approximations for the squared vertical slownesses and the phase velocities (Stovas and Ursin 2003) with expressions from Ursin and Hokstad (2003) gives travel time and offset as functions of horizontal slowness. Further, combining these with a continued fraction approximation gives three new approximations of travel time squared as a function of offset. Comparison with the weak-anisotropy approximation shows that the first and second terms in the Taylor series are the same, but the third term is non-linear with respect to anisotropy parameters and differs from that used in the weak-anisotropy approximation. This fact results in different definitions of the heterogeneity factors (Fomel and Grechka 2001) used in the travel-time approximations.

Numerical examples show that the new approximations are more accurate than the weak-anisotropy approximation. By comparing the numerical performance of these approximations, we propose a simple travel-time approximation to be used in seismic data processing. It is of almost the same functional form as the standard weak-anisotropy approximation, with the same normal moveout velocity but with a different heterogeneity factor.

2. P-wave travel-time approximations

We consider P- and SV-waves in a VTI medium characterized by the Thomsen (1986) parameters which, in terms of elastic constants, are

vertical P-wave velocity,

$$\alpha_0 = \sqrt{\frac{c_{33}}{\rho}} \tag{1a}$$

vertical S-wave velocity,

$$\beta_0 = \sqrt{\frac{c_{44}}{\rho}} \tag{1b}$$

vertical P-wave to S-wave velocity ratio,

$$\gamma_0 = \frac{\alpha_0}{\beta_0} = \sqrt{\frac{c_{33}}{c_{44}}} \tag{1c}$$

P-wave moveout parameter,

$$\delta = \frac{(c_{13}+c_{44})^2 - (c_{33}-c_{44})^2}{2c_{33}(c_{33}-c_{44})} \tag{1d}$$

SV-wave moveout parameter,

$$\sigma = \gamma_0^2(\varepsilon - \delta) = \frac{(c_{11}-c_{44})(c_{33}-c_{44}) - (c_{13}+c_{44})^2}{2c_{44}(c_{33}-c_{44})}. \tag{1e}$$

For a P-wave travelling along a ray which makes an angle θ_{P} with the symmetry axis, being the vertical z-axis, the travel time is

$$T_{\mathrm{P}} = \frac{z_{\mathrm{P}}}{V_{\mathrm{P}} \cos\theta_{\mathrm{P}}} \tag{2}$$

and the horizontal distance (offset) is

$$x_{\mathrm{P}} = z_{\mathrm{P}} \tan\theta_{\mathrm{P}}. \tag{3}$$

Here z_{P} is the total vertical distance measured along the ray (up and down for a PP reflection), V_{P} is the P-wave group velocity and θ_{P} is the angle the ray (and group velocity) makes with the vertical axis. In terms of the phase velocity α and the phase angle θ, these expressions become (Ursin and Hokstad 2003)

$$\begin{aligned} T_{\mathrm{P}} &= \frac{z_{\mathrm{P}}}{\alpha\cos\theta}(1 + p\alpha') \\ x_{\mathrm{P}} &= p\frac{z_{\mathrm{P}}\alpha}{\cos\theta}\left(1 + \frac{\alpha'}{p\alpha^3}\right), \end{aligned} \tag{4}$$

where p is horizontal slowness or ray parameter

$$p = \frac{\sin\theta}{\alpha} \tag{5}$$

and $\alpha' = \mathrm{d}\alpha/\mathrm{d}p$. Following Stovas and Ursin (2003) we can write

$$\frac{1}{\alpha^2} = \frac{1}{\alpha_0^2} - p^2 S_\alpha. \tag{6}$$

The function S_α can be expressed in Taylor series

$$S_\alpha = a_0 + a_1 p^2 \alpha_0^2 + \cdots \tag{7}$$

with

$$\begin{aligned} a_0 &= 2\delta \\ a_1 &= 2\sigma\frac{\gamma_0^2 + 2\delta\gamma_0^2 - 1}{\gamma_0^2(\gamma_0^2 - 1)} = 2(\varepsilon - \delta)\left[1 + \frac{2\gamma_0^2\delta}{\gamma_0^2 - 1}\right]. \end{aligned} \tag{8}$$

With equations (5) and (6), equation (4) can be written as

$$\begin{aligned} T_{\mathrm{P}} &= \frac{z_{\mathrm{P}}}{\alpha_0}\frac{\left(1 + \frac{1}{2}p^3\alpha_0^2 S'_\alpha\right)}{\sqrt{1 - p^2\alpha_0^2(1 + S_\alpha)}} \\ x_{\mathrm{P}} &= p z_{\mathrm{P}}\alpha_0\frac{\left(1 + S_\alpha + \frac{1}{2}pS'_\alpha\right)}{\sqrt{1 - p^2\alpha_0^2(1 + S_\alpha)}}. \end{aligned} \tag{9}$$

We now introduce the variables

$$\begin{aligned} T_{\mathrm{P0}} &= \frac{z_{\mathrm{P}}}{\alpha_0} \\ \bar{x}_{\mathrm{P}} &= \frac{x_{\mathrm{P}}}{\alpha_0 T_{\mathrm{P0}}} = \frac{x_{\mathrm{P}}}{z_{\mathrm{P}}} \\ H_\alpha &= \frac{1}{2}pS'_\alpha \end{aligned} \tag{10}$$

and write the travel time and offset squared as

$$\begin{aligned} T_{\mathrm{P}}^2 &= T_{\mathrm{P0}}^2\frac{\left(1 + p^2\alpha_0^2 H_\alpha\right)^2}{1 - p^2\alpha_0^2(1 + S_\alpha)} \\ \bar{x}_{\mathrm{P}}^2 &= p^2\alpha_0^2\frac{(1 + S_\alpha + H_\alpha)^2}{1 - p^2\alpha_0^2(1 + S_\alpha)}. \end{aligned} \tag{11}$$

From the last equation we can write

$$p^2\alpha_0^2 = \frac{\bar{x}_{\mathrm{P}}^2}{(1 + S_\alpha + H_\alpha)^2 + \bar{x}_{\mathrm{P}}^2(1 + S_\alpha)}. \tag{12}$$

Substituting this into the first equation gives

$$\begin{aligned} T_{\mathrm{P}}^2 = T_{\mathrm{P0}}^2\Bigg[1 &+ \bar{x}_{\mathrm{P}}^2\frac{1 + S_\alpha + 2H_\alpha}{(1 + S_\alpha + H_\alpha)^2} \\ &+ \bar{x}_{\mathrm{P}}^4\frac{H_\alpha^2}{(1 + S_\alpha + H_\alpha)^2[(1 + S_\alpha + H_\alpha)^2 + \bar{x}_{\mathrm{P}}^2(1 + S_\alpha)]}\Bigg]. \end{aligned} \tag{13}$$

By using Taylor expressions for S_α and H_α, and substituting for $p^2\alpha_0^2$ with equation (A6) gives the approximation

$$T_{\mathrm{P1}}^2 = T_{\mathrm{P0}}^2\left[1 + \tilde{x}_{\mathrm{P}}^2 - \Phi_{\mathrm{P}}\tilde{x}_{\mathrm{P}}^2\frac{1 + 4\Phi_{\mathrm{P}} + \tilde{x}_{\mathrm{P}}^2}{(1 + 2\Phi_{\mathrm{P}})^2 + \tilde{x}_{\mathrm{P}}^2(1 + \Phi_{\mathrm{P}})}\right] \tag{14}$$

where the normalized offset now is

$$\tilde{x}_{\mathrm{P}}^2 = \frac{\bar{x}_{\mathrm{P}}^2}{1 + a_0} = \frac{x_{\mathrm{P}}^2}{v_{\mathrm{P}}^2 T_{\mathrm{P0}}^2} \tag{15}$$

with

$$v_{\rm P}^2 = \alpha_0^2(1+a_0) = \alpha_0^2(1+2\delta) \quad (16)$$

being the normal moveout velocity (Thomsen 1986) and

$$\Phi_{\rm P} = \frac{G_{\rm P}\tilde{x}_{\rm P}^2}{1+(1+4G_{\rm P})\tilde{x}_{\rm P}^2}. \quad (17)$$

The other travel-time parameter (similar to the heterogeneity parameter introduced in Fomel and Grechka (2001)) is

$$G_{\rm P} = \frac{a_1}{(1+a_0)^2} = \frac{2(\varepsilon-\delta)}{(1+2\delta)^2}\left[1+\frac{2\gamma_0^2\delta}{\gamma_0^2-1}\right]. \quad (18)$$

A much simpler travel-time approximation is obtained by neglecting the H_α^2 term in equation (13). This gives, with Taylor expressions for S_α and H_α and using equation (A6),

$$T_{\rm P2}^2 = T_{\rm P0}^2\left[1+\tilde{x}_{\rm P}^2 - G_{\rm P}\tilde{x}_{\rm P}^4\frac{1+\tilde{x}_{\rm P}^2(8+G_{\rm P})}{\left[1+\tilde{x}_{\rm P}^2(6+G_{\rm P})\right]^2}\right]. \quad (19)$$

Tsvankin and Thomsen (1994) introduced the weak-anisotropy approximation

$$T_{\rm P3}^2 = T_{\rm P0}^2\left[1+\tilde{x}_{\rm P}^2 - \frac{G_{\rm P}\tilde{x}_{\rm P}^4}{1+\tilde{x}_{\rm P}^2(1+G_{\rm P})}\right], \quad (20)$$

where $G_{\rm P} = 2(\varepsilon-\delta)$ in this case. A similar approximation can be obtained by neglecting higher order terms in equation (14), resulting in

$$\begin{aligned} T_{\rm P4}^2 &= T_{\rm P0}^2\left[1+\tilde{x}_{\rm P}^2 - \Phi_{\rm P}\tilde{x}_{\rm P}^2\right] \\ &= T_{\rm P0}^2\left[1+\tilde{x}_{\rm P}^2 - \frac{G_{\rm P}\tilde{x}_{\rm P}^4}{1+\tilde{x}_{\rm P}^2(1+4G_{\rm P})}\right], \end{aligned} \quad (21)$$

where $G_{\rm P}$ is now defined in equation (18).

Dellinger *et al* (1993) have proposed an anelliptic travel-time approximation which in our notation can be written as

$$T_{\rm P5}^2 = T_{\rm P0}^2\left[1+\tilde{x}_{\rm P}^2 - \frac{F(F-1)\tilde{x}_{\rm P}^4}{1+F\tilde{x}_{\rm P}^2}\right], \quad (22)$$

where F is an anellipticity factor. This approximation is consistent with equation (20) or (21) only in the case $F = 1$ and $G_{\rm P} = 0$, corresponding to the standard hyperbolic approximation.

3. SV-wave travel-time approximations

For an SV-wave the phase velocity can be computed from (Stovas and Ursin 2003)

$$\frac{1}{\beta^2} = \frac{1}{\beta_0^2} - p^2 S_\beta, \quad (23)$$

where S_β can be expanded in the Taylor series

$$S_\beta = b_0 + b_1 p^2\beta_0^2 + \cdots \quad (24)$$

with

$$\begin{aligned} b_0 &= 2\sigma \\ b_1 &= -2\sigma\frac{\gamma_0^2+2\delta\gamma_0^2-1}{\gamma_0^2-1}. \end{aligned} \quad (25)$$

All results and derivations in the previous section can be used by replacing P with S.

This gives the approximation

$$T_{\rm S1}^2 = T_{\rm S0}^2\left[1+\tilde{x}_{\rm S}^2 - \Phi_{\rm S}\tilde{x}_{\rm S}^2\frac{1+4\Phi_{\rm S}+\tilde{x}_{\rm S}^2}{(1+2\Phi_{\rm S})^2+\tilde{x}_{\rm S}^2(1+\Phi_{\rm S})}\right] \quad (26)$$

with

$$\Phi_{\rm S} = \frac{G_{\rm S}\tilde{x}_{\rm S}^2}{1+(1+4G_{\rm S})\tilde{x}_{\rm S}^2}, \quad (27)$$

and

$$\begin{aligned} T_{\rm S0} &= \frac{z_{\rm S}}{\beta_0} \qquad \tilde{x}_{\rm S}^2 = \frac{x_{\rm S}^2}{v_{\rm S}^2 T_{\rm S0}^2} \\ G_{\rm S} &= \frac{b_1}{(1+b_0)^2} = -\frac{2\sigma}{(1+2\sigma)^2}\left[1+\frac{2\gamma_0^2\delta}{\gamma_0^2-1}\right] \\ v_{\rm S}^2 &= \beta_0^2(1+2\sigma). \end{aligned} \quad (28)$$

The next approximation is directly adapted from equation (19):

$$T_{\rm S2}^2 = T_{\rm S0}^2\left[1+\tilde{x}_{\rm S}^2 - G_{\rm S}\tilde{x}_{\rm S}^4\frac{1+\tilde{x}_{\rm S}^2(8+G_{\rm S})}{\left[1+\tilde{x}_{\rm S}^2(6+G_{\rm S})\right]^2}\right]. \quad (29)$$

And similarly, the Tsvankin–Thomsen approximation

$$T_{\rm S3}^2 = T_{\rm S0}^2\left[1+\tilde{x}_{\rm S}^2 - \frac{G_{\rm S}\tilde{x}_{\rm S}^4}{1+\tilde{x}_{\rm S}^2(1+G_{\rm S})}\right], \quad (30)$$

where the linearized $G_{\rm S} = -2\sigma$. Finally, from equation (21)

$$T_{\rm S4}^2 = T_{\rm S0}^2\left[1+\tilde{x}_{\rm S}^2 - \frac{G_{\rm S}\tilde{x}_{\rm S}^4}{1+\tilde{x}_{\rm S}^2(1+4G_{\rm S})}\right]. \quad (31)$$

4. Converted-wave travel-time approximations

In order to obtain similar travel-time approximations for a converted wave which has travelled a horizontal distance $x_{\rm P}$ as a P-wave and a horizontal distance $x_{\rm S}$ as an SV-wave, we first expand equation (9) in Taylor series. The result is

$$\begin{aligned} T_{\rm P} &\approx T_{\rm P0}\left[1+\tfrac{1}{2}p^2v_{\rm P}^2+\tfrac{3}{8}(1+4G_{\rm P})p^4v_{\rm P}^4\right] \\ x_{\rm P} &\approx p v_{\rm P}^2 T_{\rm P0}\left[1+\tfrac{1}{2}(1+4G_{\rm P})p^2v_{\rm P}^2\right]. \end{aligned} \quad (32)$$

Similarly, for the SV-wave

$$\begin{aligned} T_{\rm S} &\approx T_{\rm S0}\left[1+\tfrac{1}{2}p^2v_{\rm S}^2+\tfrac{3}{8}(1+4G_{\rm S})p^4v_{\rm S}^4\right] \\ x_{\rm S} &\approx p v_{\rm S}^2 T_{\rm S0}\left[1+\tfrac{1}{2}(1+4G_{\rm S})p^2v_{\rm S}^2\right]. \end{aligned} \quad (33)$$

Combining these equations gives

$$\begin{aligned} T_{\rm C} &= T_{\rm S}+T_{\rm P} \approx T_{\rm C0}\left[1+\tfrac{1}{2}p^2v_{\rm C}^2+\tfrac{3}{8}(1+4G_{\rm C})p^4v_{\rm C}^4\right] \\ x_{\rm C} &= x_{\rm S}+x_{\rm P} \approx p v_{\rm C}^2 T_{\rm C0}\left[1+\tfrac{1}{2}(1+4G_{\rm C})p^2v_{\rm C}^2\right] \end{aligned} \quad (34)$$

with

$$\begin{aligned} T_{\rm C0} &= T_{\rm S0}+T_{\rm P0} \\ v_{\rm C}^2 &= \frac{v_{\rm S}^2T_{\rm S0}+v_{\rm P}^2T_{\rm P0}}{T_{\rm S0}+T_{\rm P0}} \\ G_{\rm C} &= \frac{4\left(v_{\rm S}^4T_{\rm S0}G_{\rm S}+v_{\rm P}^4T_{\rm P0}G_{\rm P}\right)(T_{\rm S0}+T_{\rm P0})+\left(v_{\rm P}^2-v_{\rm S}^2\right)T_{\rm S0}T_{\rm P0}}{4\left(v_{\rm S}^2T_{\rm S0}+v_{\rm P}^2T_{\rm P0}\right)^2}. \end{aligned} \quad (35)$$

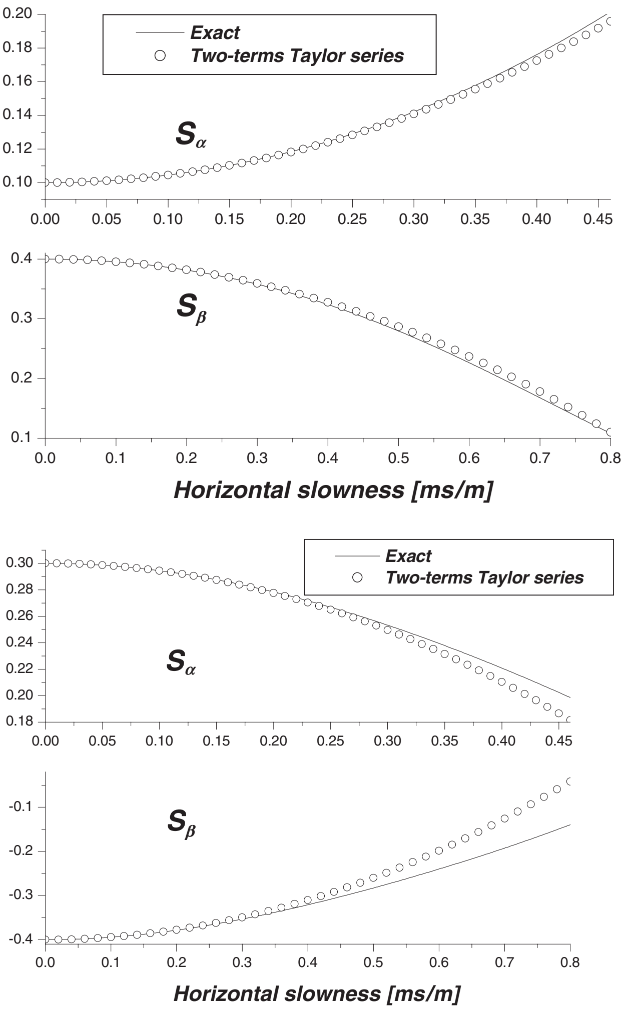

Figure 1. P- and SV-wave-anisotropy terms versus horizontal slowness (model I is at the top, model II is at the bottom).

Consequently, we may use the following expressions for a converted wave,

$$T_{C1}^2 = T_{C0}^2 \left[1 + \tilde{x}_C^2 - \Phi_C \tilde{x}_C^2 \frac{1 + 4\Phi_C + \tilde{x}_C^2}{(1+2\Phi_C)^2 + \tilde{x}_C^2(1+\Phi_C)} \right], \tag{36}$$

$$T_{C2}^2 = T_{C0}^2 \left[1 + \tilde{x}_C^2 - G_C \tilde{x}_C^4 \frac{1 + \tilde{x}_C^2(8+G_C)}{\left[1 + \tilde{x}_C^2(6+G_C)\right]^2} \right], \tag{37}$$

$$T_{C3}^2 = T_{C0}^2 \left[1 + \tilde{x}_C^2 - \frac{G_C \tilde{x}_C^4}{1 + \tilde{x}_C^2(1+G_C)} \right], \tag{38}$$

$$T_{C4}^2 = T_{C0}^2 \left[1 + \tilde{x}_C^2 - \frac{G_C \tilde{x}_C^4}{1 + \tilde{x}_C^2(1+4G_C)} \right], \tag{39}$$

with

$$\tilde{x}_C^2 = \frac{x_C^2}{v_C^2 T_{C0}^2} \qquad \Phi_C = \frac{G_C \tilde{x}_C^2}{1 + (1+4G_C)\tilde{x}_C^2}. \tag{40}$$

Note that G_C in the Tsvankin–Thomsen approximation (37) is approximate and can be computed from equation (34) by substituting $G_P = 2(\varepsilon - \delta)$ and $G_S = -2\sigma$.

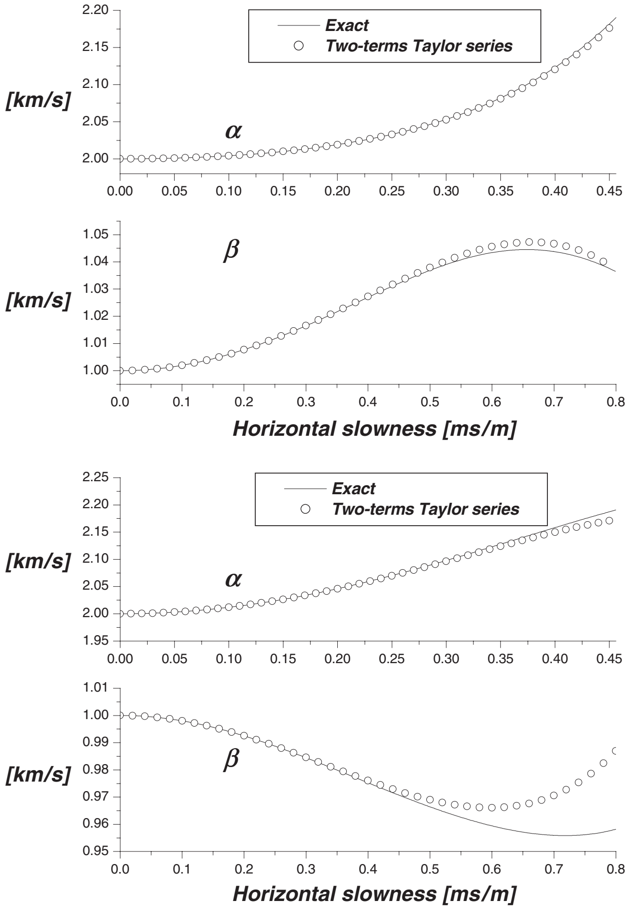

Figure 2. The P- and SV-wave phase velocities versus horizontal slowness (model I is at the top, model II is at the bottom).

5. Discussion and numerical results

By comparing the Taylor series for the P-wave phase velocity in equations (6)–(8) and the SV-wave phase velocity in equations (23)–(25) we note that the terms a_0 and a_1 for the P-wave phase velocity are small for weak anisotropy while the terms b_0 and b_1 for the SV-wave phase velocity contain terms of the same order multiplied by $\gamma_0^2 = (\alpha_0/\beta_0)^2$ which may be large. The Taylor series for S_β converges much more slowly than the series for S_α. Both these facts explain why all approximations for the SV-waves are valid for much smaller slowness (and offset) ranges than the corresponding P-wave approximations. All travel-time approximations have the same normal moveout velocity in the offset-squared term and the higher order offset terms are multiplied by the heterogeneity factor G. For the Tsvankin–Thomsen (1994) approximation G is given by a weak-anisotropy approximation. For SV-waves, G will be large when σ is negative. This corresponds to $\varepsilon - \delta < 0$ which is impossible for an effective medium corresponding to a stack of isotropic layers (Berryman 1999).

We compare all approximations for two single-layer models given in table 1. Model I is a weak-anisotropy model ($\delta = 0.05$) with $\varepsilon - \delta = 0.05 > 0$, $G_P = 0.09$ and $G_S = -0.23$. Model II has larger anisotropy ($\delta = 0.15$) with

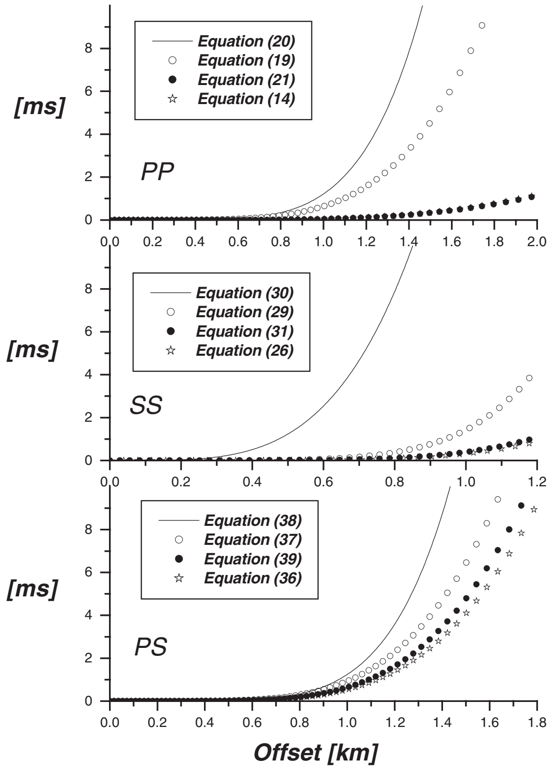

Figure 3. The deviation in milliseconds from the exact travel time computed from model I.

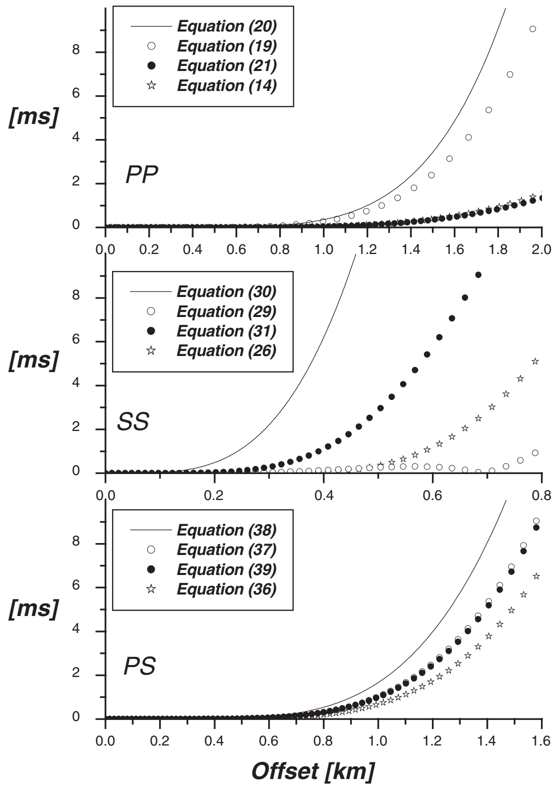

Figure 4. The deviation in milliseconds from the exact travel time computed from model II.

Table 1. Parameters for single-layer models I and II.

Parameters	Model I	Model II
α_0 (km s^{-1})	2.0	2.0
β_0 (km s^{-1})	1.0	1.0
ε	0.1	0.1
δ	0.05	0.15
σ	0.2	-0.2
a_0	0.1	0.3
a_1	0.11333	-0.14
b_0	0.4	-0.4
b_1	-0.45333	0.56
T_{P0} (s)	1.0	1.0
v_P (km s^{-1})	2.098	2.28
G_P	0.09366	-0.08284
T_{S0} (s)	2.0	2.0
v_S (km s^{-1})	1.18322	0.7746
G_S	-0.23129	1.55556
T_{C0} (s)	1.5	1.5
v_C (km s^{-1})	1.549	1.46
G_C	0.13927	0.17627

$\varepsilon - \delta = -0.05 < 0$, $G_P = -0.08$ and $G_S = 1.56$. The results for the anisotropy terms S_α and S_β are given in figure 1 for the P-wave, and for SV-wave phase velocities in figure 2. It is seen that the approximations for the P-wave phase velocities and the SV-wave phase velocity in model I are good up to nearly horizontal propagation. The approximation for the SV-wave phase velocity in model II is good up to only 30° due to the negative value of $\varepsilon - \delta$.

Figures 3 and 4 show the absolute value of the travel-time errors for the different approximations for models I and II, respectively. The reflector depth is 1 km in both cases. It is seen that in all cases, the weak-anisotropy approximation has the poorest performance. With the exception of the SVSV reflection for model II, T_1 and T_4 have similar performance and are better than T_2. The behaviour of the SVSV reflection time approximations for model II is due to the large value of G_S and may not be typical because of $\varepsilon - \delta < 0$ for this model.

Based on these numerical results, we propose to use the following simple travel-time approximation,

$$T(x)^2 = T_0^2 + \frac{x^2}{v_{NMO}^2} - \frac{Gx^4}{v_{NMO}^4\left[T_0^2 + \frac{x^2}{v_{NMO}^2}(1+4G)\right]}, \qquad (41)$$

where v_{NMO}^2 and G are defined in

- equations (16) and (18) for PP reflection,
- equation (28) for SVSV reflection,
- equation (35) for PSV and SVP reflection.

These approximations differ from the standard Tsvankin–Thomsen (1994) weak-anisotropy approximation only in the factor 4, and the more accurate definition of the heterogeneity factor G.

6. Conclusions

We have derived new approximations for the P- and SV-wave phase velocities and PP, SS and PS (SP) reflection travel times for a TI media not using the weak-anisotropy assumption. The comparison with the Tsvankin–Thomsen

(1994) non-hyperbolic approximation made for single layer models shows that our approximations are more accurate. For seismic data processing, we propose to use new simple travel-time approximations which are almost of the same form as the Tsvankin–Thomsen approximation, with the same normal moveout velocities but different heterogeneity factors.

Appendix. Continued fraction approximation

We consider the second-order equation

$$ax^2 + bx + c = 0. \tag{A1}$$

A continued fraction approximation of the solution is

$$x = \frac{bc}{ac - b^2}. \tag{A2}$$

For small values of a this gives the correct solution $x = -c/b$.

With the approximation in equation (7) and $H_\alpha = a_1 p^2 \alpha_0^2$, equation (11) gives

$$\bar{x}_{\mathrm{P}}^2 = p^2\alpha_0^2 \frac{\left(1 + a_0 + 2a_1 p^2\alpha_0^2\right)^2}{1 - (1 + a_0)p^2\alpha_0^2 - a_1 p^4\alpha_0^4}. \tag{A3}$$

Neglecting a term with a_1^2 gives

$$a_1\left(4 + \tilde{x}_{\mathrm{P}}^2\right)p^4\alpha_0^4 + \left(1 + \tilde{x}_{\mathrm{P}}^2\right)(1 + a_0)p^2\alpha_0^2 - \tilde{x}_{\mathrm{P}}^2 = 0, \tag{A4}$$

where

$$\tilde{x}_{\mathrm{P}}^2 = \frac{\bar{x}_{\mathrm{P}}^2}{1 + a_0}. \tag{A5}$$

Using the approximation in (A2) gives

$$p^2\alpha_0^2 = \frac{\tilde{x}_{\mathrm{P}}^2}{(1 + a_0)\left[1 + \left(1 + \frac{4a_1}{(1+a_0)^2}\right)\tilde{x}_{\mathrm{P}}^2\right]}. \tag{A6}$$

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