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Wave-equation Depth Migration in Elastic Media - New Results

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SUMMARY

One-way differential equations and associated integral equations for seismic migration in laterally varying elastic media are derived by wavenumber coupling techniques in the frequency-wavenumber domain. Their solutions involve propagators determined by horizontally stratified reference velocities. Successive approximations of the integral equations according to the Neumann series lead to numerically efficient wavefield extrapolation algorithms for migration. Numerical examples are given.

Introduction

A variety of different methods for seismic migration have been developed by plane-wave analysis where the governing differential equations are transformed to the horizontal wavenumber domain. Well-known examples are f-k migration (Stolt 1978), phase-shift migration (Gazdag 1978) and split-step Fourier migration (Stoffa et al. 1990). A perhaps less known example is migration based on the WKBJ approximation model brought to prominence in seismic imaging by Robinson (1982, 1986) and Ursin (1984, 1987). The main reason that this model, which can be traced back to Bremmer (1950), was not much explored during the 1980-90's is that the model was presented under the assumption of a vertically layered, laterally homogeneous earth, thereby leading to a slight generalization of the phase-shift migration method.

However, the WKBJ approximation model lately has received new attention. De Hoop (1996) extended the Bremmer coupling series to multidimensional varying media and developed a ray "picture" scattering theory. Zhang et al. (2005) by using pseudo-differential calculus published one-way wave equations with WKBJ type solutions by modifying the Bremmer-Robinson-Ursin (BRU) model to heterogeneous media. Amundsen et al. (2008a) published papers using the BRU model with zero-order and first-order interactions between downgoing and upgoing elastic fields to derive solutions to the inverse scattering problem for layered media.

Plane-wave decomposition of the seismic wavefield can also be used to study wave propagation in laterally heterogeneous media by allowing for coupling between different horizontal wavenumbers in the field vector (Kennett 1972, 1986; Aki and Richards 1980). By representing the inhomogeneous medium as a laterally varying part superimposed on a horizontal stratification and transforming the differential system to the horizontal wavenumber domain, the differential system has a well-known integral equation solution that is expressed in terms of the propagator matrix for the horizontal stratification. Whereas the integral equation solution has been investigated for modeling synthetic seismograms in laterally varying media (Haines 1988) it has, to the best of our knowledge, until recently attracted no attention as a possible solution for the seismic migration problem (Amundsen et al. 2008, 2009).

The objective of the current work is to extend the results on migration in acoustic media presented by Amundsen et al. (2008, 2009) to migration in elastic media. Our migration model can be considered a generalization of the BRU model referenced above to heterogeneous media where the lateral and vertical medium variations introduce the possibility of coupling between different wavenumbers for the same wavetype as well as conversion between wavetypes by altering of polarization.

The down-up wave interaction model for elastic media

Amundsen et al. (2008, 2009) have given the recipe how to derive the integral equation solutions for acoustic media. The steps for elastic media are:

1. Represent the medium as a horizontally stratified reference part with superimposed lateral variation.
2. Write the set of ordinary differential equations that governs elastic wave propagation in terms of particle velocity-vertical traction field vectors that are coupled in the horizontal wavenumber variables through convolutions between the Fourier transform of the medium and the fields.
3. Derive the equivalent integral equation, expressed in terms of a propagator that is determined from the reference velocities of the horizontal stratification. The effect of

the laterally heterogeneous component of the velocity is then taken account of by inter-wavenumber coupling.

4. Introduce new wave variables that relate to downgoing and upgoing P- and S-wave components in the laterally uniform reference medium (Ikelle and Amundsen, 2005). Substitute the wave variables into the integral equation solution for the field vector to obtain the integral solution for the downgoing and upgoing P- and S-wave components. This is a two-way wavefield solution that fully describes all interactions between the components. This interaction model is denoted the down-up wave interaction (DUWI) model. The DUWI model generalizes the BRU model to arbitrary complex velocity media, but importantly, does not invoke any WKBJ type solutions.
5. Derive one-way and pseudo two-way differential equations and wavefield extrapolation solutions by fully neglecting or partially including wave interactions.

A major merit of the model that we have introduced is that it gives a systematic treatment of multiple interactions and inter-conversion between wavetypes. The main differerence between the acoustic and the elastic approach is that in the latter case one can decompose the field into three independent wavetypes: P-waves, SV-waves and SH-waves.

The differential equation for the downgoing P-wave is derived in the zero-order DUWI model, which neglects the interactions with the upgoing field as well as wavetype conversion, resulting in a pure one-way wave equation for the downgoing P-wave. Similarly, the zero-order DUWI model yields one way wave equations for the upgoing P-wave and the upgoing SV-wave. By combining the downgoing P-wave solution with the upgoing SV-wave solution mode-converted waves in ocean bottom seismic recordings can be migrated. In the first-order DUWI model, the downgoing P-wave from the zero-order DUWI model is used as a source for the upgoing P-wave. This solution gives a pseudo two-way wave equation which may be used to migrate overturning waves.

The DUWI solutions are Fredholm integral equations of the second kind for the upgoing and downgoing wavefields. Successive approximations according to the Neumann series lead to numerically fast wavefield extrapolation schemes for migration (Amundsen 2009). In the wavenumber (k_x) domain, the zero-order DUWI wavefield extrapolation solution for the downgoing P-wave in the acoustic limit, expressed in terms of the Neumann series, becomes

$$D_0^{(0)}(k_x, z) = \Phi_0^{(D)}(k_x, z)\,,$$

and for $n \geq 1$,

$$D_0^{(n)}(k_x, z) = \Phi_0^{(D)}(k_x, z) + \lambda(z) R(k_x, z) \cdot T(k_x, z) * D_0^{(n-1)}(k_x, z)$$

where the known data function $\Phi_0^{(D)}$ is determined from the downgoing P-wave at z-Δz, λ is a parameter depending on the wavenumber in the reference medium times the depth step Δz , *R* is the ratio of the wavenumber and its vertical component, *T* is the Fourier transform of the velocity potential which measures the lateral velocity inhomogeneity at depth *z* relative to the reference velocity, and * denotes convolution. The zero-order DUWI wavefield extrapolation solution for the upgoing P-wave in the acoustic limit has a similar representation.

Numerical example

We now show the performance of the zero-order DUWI migration method for P-waves by comparing it to migration with the split-step Fourier method (Stoffa et al., 1990) on the IFP Marmousi model shown in Figure 1, a benchmark of testing the accuracy of prestack migration methods.

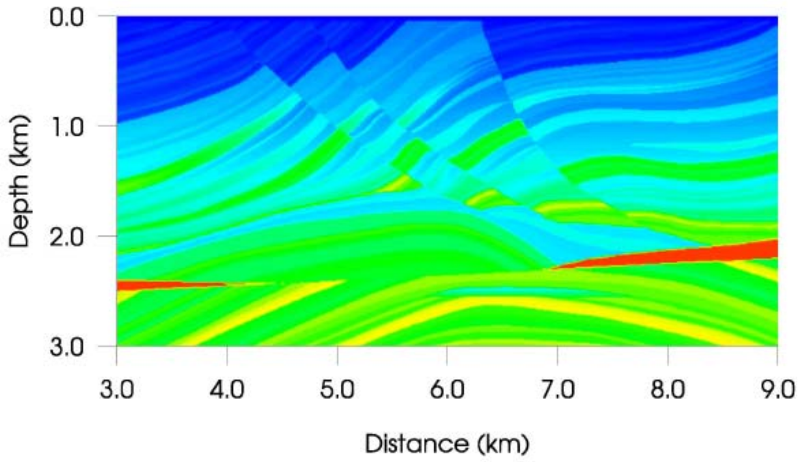

Figure 1. The Marmousi velocity model. The velocity varies from 1.5 to 5.5 km/s.

The Marmousi model has complex structures with significant lateral velocity variations. The data set consists of 240 shot gathers with 96 receivers for each shot. The shot point interval and receiver interval are both 25 m. The minimum offset is 200 m. The time record length is 1.9 s with sampling interval 4 ms.

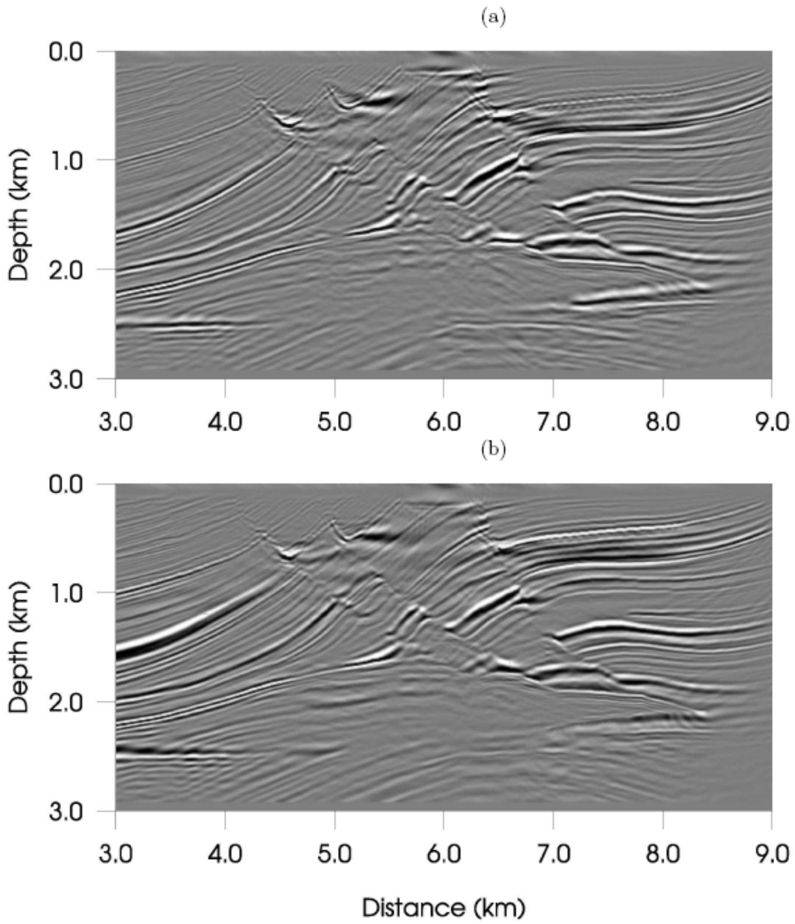

Figure 2. Prestack depth migration of the Marmousi data (a) with the split-step Fourier method, and (b) with the zero-order DUWI method.

Figure 2 compares the migrated sections obtained with the split-step Fourier method and the zero-order DUWI method. The Neumann series is truncated at n=2 for the extrapolation of the downgoing and upgoing waves.

We observe that dipping events are more accurately positioned and better focused by the DUWI migration approach. The DUWI method better accommodates larger propagation angles than does the split-step Fourier method and therefore yields more continuity in, and less deformation of, the reflectors.

Conclusions

We have shown that seismic data acquired over laterally varying media can be extrapolated (and migrated) by wavenumber coupling techniques, in the frequency-wavenumber domain. The down-up wave interaction model allows the selective extraction of wavefield extrapolation solutions for the purely downgoing and upgoing P-, SV-, and SH-waves.

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