Multicomponent OBS and VC acquisition for wavefield reconstruction

Lasse Amundsen, Harald Westerdahl, Mark Thompson, Jon A Haugen, Arne Reitan, Martin Landrø and Bjørn Ursin*
Statoil Research Center, and NTNU

SUMMARY

In ocean-bottom seismic (OBS) and vertical-cable (VC) surveying, receiver stations are stationary on the sea floor while a source vessel shoots on a predetermined $x-y$ grid on the sea surface. To reduce exploration cost, the shot point interval often is so coarse that the data recorded at a given receiver station are undersampled and thus irrecoverably aliased. However, when the pressure field and its *x*- and *y*-derivatives are measured in the water column, the non-aliased pressure field can be reconstructed by interpolation. Likewise, if the vertical component of the particle velocity (or acceleration) and its *x*- and *y*-derivatives are measured, then also this component can be reconstructed by interpolation. The interpolation scheme can be any scheme that reconstructs the field from its sampled values and sampled derivatives. In the case that the two field's first-order derivatives are recorded the number of components are six. When also their second-order derivatives are measured, the number of components is ten. The properly interpolated measurements of pressure and vertical component of particle velocity from the multicomponent measurements allow proper up/down wavefield decomposition, or deghosting. New wavefield reconstruction methods as those suggested here are of significant interest since, presently, the seismic industry is in the process of developing multicomponent cables or streamers, and is in the process of carrying out research on new multicomponent sensors.

INTRODUCTION

To reduce 3-D marine seismic acquisition cost the receiver spacing is often made larger than desirable. As a consequence, the recorded wavefield is spatially aliased. Specifically, in towed streamer acquisition, the sampling challenge is the large streamer separation, typically 50-100 m. In ocean-bottom seismic (OBS) or vertical cable (VC) acquisition, where data can be processed as common-receiver gathers, it is the coarse shot interval spacing, typically chosen 50 m by 50 m or more, that leads to undersampling. Here, OBS refers to acquisition with either nodes or cables. The undersampling of the wavefield causes challenges for 3-D up/down decomposition or deghosting of the recorded wavefield, which is one of the data preprocessing steps applied before seismic imaging.

In this paper we introduce the concept of multicomponent (multi-C) wavefield measurements in the water column while the source vessel, just like in OBS and VC surveying, traverses the surface shooting on a predetermined grid. Six wavefield components — the pressure and the vertical component of the particle velocity and their horizontal first-order derivatives in *x*- and *y*-directions — are required for proper reconstruction of the undersampled pressure and vertical component of particle velocity. When the second-order derivatives are recorded, the number of components is ten. This reconstruction allows the step of 3-D up/down decomposition or deghosting of common receiver station recordings to be achieved in the frequency-wavenumber domain (Amundsen, 1993). New wavefield reconstruction methods as those presented in the present paper are of interest since, presently, the seismic industry is in the process of developing multicomponent cables or streamers (Robertsson, 2006; Singh el al., 2009). Further, the industry is actively carrying out research on and testing new multicomponent sensors. Here, multicomponent refers to a combination of sensors that includes two or more closely-spaced sensors such as a hydrophone, a geophone, an accelerometer, a rotational seismometer, a pressure derivative configuration of hydrophones, or a vertical particle velocity derivative configuration of hydrophones or geophones. The derivatives can be a first order derivative, a second order derivative or a higher order derivative.

Before we demonstrate the use of 6-C and 10-C common-receiver recordings for reconstruction of pressure and vertical component of particle velocity we briefly review possible new trends in marine seismic acquisition.

Marine seismic acquisition: new proposals

Robertsson et al. (2006, 2008) state that 3-C geophone measurements would bring significant benefits to towed-marine seismic data if recorded and processed in conjunction with the pressure data. They show that particle velocity measurements can increase the effective Nyquist wavenumber by a factor of two or three, depending on how they are used. A true multi-component streamer would enable accurate pressure data reconstruction in the crossline direction with cable separations for which pressure-only data would be irrecoverably aliased.

The major purpose of having a hydrophone/3-C geophone streamer is thus to achieve crossline pressure field reconstruction by interpolation using pressure and its crossline derivative. But without introducing assumptions such a streamer will not enable the reconstruction of the vertical component of the particle velocty in the crossline direction that is needed to achieve the 3-D up/down decomposition objective.

Singh et al (2009) propose seismic acquisition using a plurality of streamers, with a streamer having a plurality of compact clusters of hydrophones and/or particle motion sensors. Cluster means a plurality of sensors of the same type that are used together. The streamer is adapted to provide gradient measurements of pressure with the objective to provide improved methods of interpolating seismic data between adjacent streamers.

Wavefield reconstruction

MARINE MULTI-C WAVEFIELD RECORDING

In marine OBS or VC surveying, the shot grid interval is seldom less than 50 m by 50 m to avoid excessive exploration cost. The 50 m by 50 shot grid implies that any recorded pressure and vertical component of particle velocity data alone at a receiver station will be undersampled even for moderate frequencies of the source signal.

With the purpose to achieve a proper 3-D up/down wavefield decomposition of undersampled seismic data, we suggest to record six components or more of the wavefield in OBS or VC surveying configurations. The six-components are the pressure and its horizontal first-order derivatives in x- and y-directions, and the vertical component of the particle velocity and this component's horizontal first-order derivatives in x- and y-directions. The additional recording of second-order derivatives gives a ten-component measurement.

There are many ways to measure the first-order x- and y-derivatives of the pressure wavefield in the water column. One way is to use horizontally oriented geophones since the equation of motion relates measured particle velocities (v_x, v_y) to spatial derivatives of the pressure p. In particular, in the frequency (ω) domain, for a fluid

$$\partial_x p = \mathrm{i}\omega\rho v_x\,, \qquad \partial_y p = \mathrm{i}\omega\rho v_y\,, \tag{1}$$

where ρ is density. Accelerations are related to particle velocities as $(a_x, a_y) = -\mathrm{i}\omega(v_x, v_y)$.

Likewise, there are several ways to measure the x and y first-order derivatives of the vertical component of the particle velocity v_z in the water column. One possibility is to construct a cluster of vertically oriented geophones with known separation between each geophone so that the spatial derivatives of the particle velocity can be derived by velocity field differencing.

Marine seismic acquisition with hydrophone cluster

Another example of a receiver system that could provide ten components of the wavefield required for wavefield reconstruction and proper up/down wavefield decomposition would be fifteen clustered hydrophones as illustrated in Figure 1 where three are staggered in the vertical direction at depths z and $z \pm \Delta z$ at horizontal positions (x, y), $(x \pm \Delta x, y)$, and $(x, y \pm \Delta y)$.

Then, the pressure wavefield would be recorded in fifteen closeby points in space, allowing all the ten sought-after components of the field to be derived by simple field differencing operations. Such a system can be designed and installed in a receiver station deployed on the sea floor.

The art of numerical differentiation is well known in the field of mathematics and is described in standard mathematical textbooks (e.g., Abramowitz and Stegun, 1972).

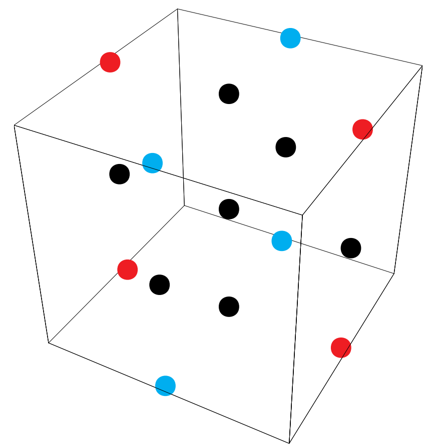

Figure 1: Fifteen hydrophones (represented by dots) in a cluster. The seven black dots are oriented along the axes of a Cartesian coordinate system at locations $(0,0,0)$, $(\pm 1,0,0)$, $(0,\pm 1,0)$ and $(0,0,\pm 1)$. The four red dots are at locations $(\pm 1,0,\pm 1)$. The four blue dots are at locations $(0,\pm 1,\pm 1)$. The hydrophone spacing along axes is unity.

NUMERICAL EXAMPLES

We generate a simple synthetic shot gather of pressure and its horizontal derivatives and vertical component of particle velocity and its horizontal derivatives to illustrate the significance of multicomponent recordings for the reconstruction of pressure and vertical component of particle velocity data between recording locations. Any interpolation technique that uses and benefits from field and field derivative measurements can be applied. In Appendix A one class of such reconstruction methods based on the extended sampling theorem is outlined. In the case that only the field is measured, the sampling theorem reduces to the well-known sinc interpolation. When the field and its first derivatives are measured and used in the extended sampling theorem, we call the method for sinc^2 interpolation. In the case that the field and its first and second derivatives are measured and used, we call the method for sinc^3 interpolation.

We consider a homogeneous halfspace of water below a free surface. A point source is located at postion $(x_s, y_s, z_s) = (0,0,300)$ m. The receivers are located over a horizontal plane at depth $z_r = 100$ m. The offset range is ± 3 km in both horizontal directions. In the numerical example, we select the receiver spacing interval $\Delta x = \Delta y$ but every second line is staggered by $\Delta x/2$ (hexagonal grid). The data at the receiver plane simply consist of an upgoing wave from the source and a downgoing wave reflected at the free surface.

sinc^2 interpolation

In this numerical example the benefit of sinc^2 interpolation is demonstrated. We generate a 6-C component data gather that consists of pressure and its horizontal derivatives, $(p, \partial_x p, \partial_y p)$, and vertical component of particle velocity and its horizontal derivatives, $(v_z, \partial_x v_z, \partial_y v_z)$. The source wavelet has dominant frequency of 14 Hz. Its amplitude spectrum is tapered to zero above 30 Hz.

Figure 2 shows the results of two interpolation and reconstruction tests for p. The results for v_z are not shown here but are similar. The upper and lower parts show selected 2-D gathers in time-offset (t-x) and frequency-wavenumber (f-k) domains, respectively. The t-x gather is that for $y = y_s$. Figure 2a show modeled reference data sampled at 25 m that would be the ideal result from any reconstruction technique. These data are now decimated spatially by a factor of two so that the sampling interval is 50 m, see Figure 2b. Aliasing is clearly visible in the f-k domain.

In the subsequent tests, these data are now interpolated.

First, we apply traditional sinc interpolation using as input p data to reconstruct p data. Sinc interpolation of aliased data makes no attempts to de-alias the data before interpolation. Thus, when aliasing is present in single component data acquisition, it is not possible to identify the correct waveforms from the acquired samples, unless assumptions are introduced. Therefore, not surprisingly, the aliased components of the events are interpolated incorrectly as seen in Figure 2c where data are band-limited in the spatial sampling bandwidth.

Second, we apply sinc^2 interpolation band-limited up to twice the spatial Nyquist frequency, as introduced in Appendix A. Input data $(p, \partial_x p, \partial_y p)$ are used to reconstruct p, and input data $(v_z, \partial_x v_z, \partial_y v_z)$ are used to reconstruct v_z (not shown). Figure 2d show that the data are well reconstructed. The data derivative information effectively has doubled the spatial Nyquist frequency, so that the data are not aliased.

Figures 2e and f present the difference between the reference data in a and the sinc and sinc^2 interpolated results, respectively.

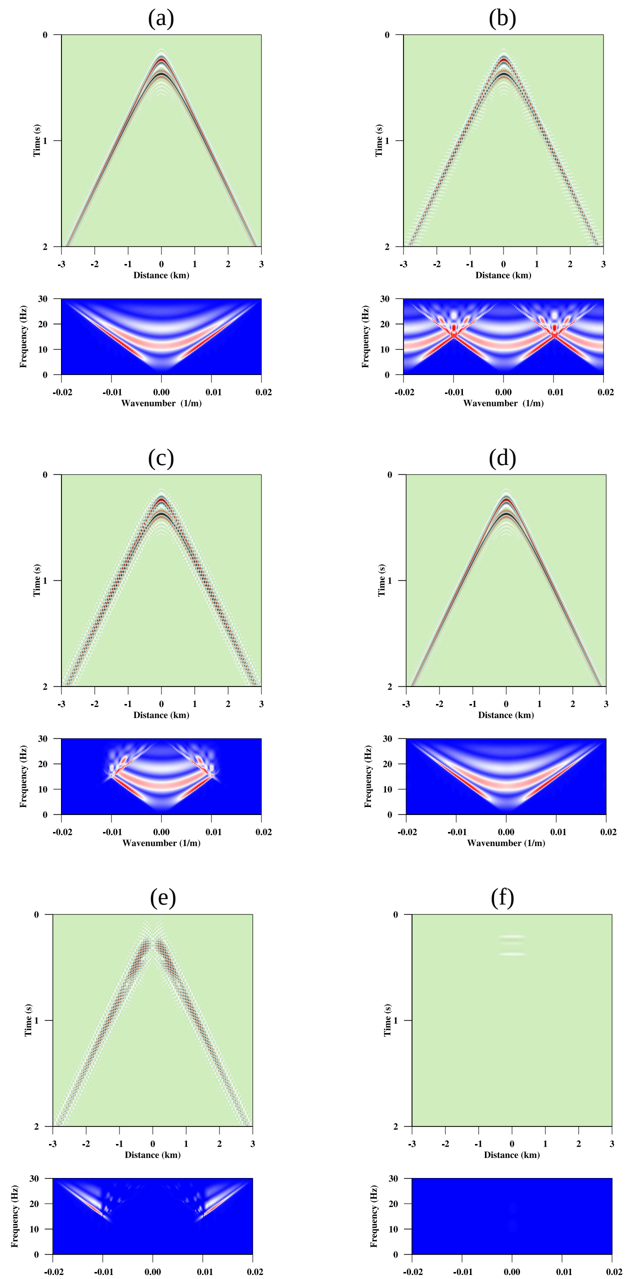

Figure 2: Comparison of sinc and sinc^2 interpolation on simple synthetic pressure data. (a) Reference data (ideal result), (b) data after 2:1 decimation, (c) sinc interpolation, (d) sinc^2 interpolation, (e) and (f) difference plots of (a)-(c) and (a)-(d), respectively. $t - x$ data are displayed above their $f - k$ spectra. See the text for details.

CONCLUSION

We have showed that recordings of the horizontal derivatives of pressure and vertical component of particle velocity in OBS or VC surveying have the potential to reduce aliasing by a factor of at least two and three compared to recording only pressure and vertical component of particle velocity data alone. Using a simple synthetic data set, we demonstrated the potential that these new measurements have to reconstruct data at desired locations in between the original shot grid.

ACKNOWLEDGMENT

We thank Statoil for allowing us to present this paper and their proprietary acquisition concept.

APPENDIX A

THE EXTENDED SAMPLING THEOREM

Let Δ denote a sampling interval and $\kappa_N = \frac{1}{2\Delta}$ the Nyquist wavenumber. Let $p(x)$ be a continuous function with finite Fourier transform $F(k)$ [$F(k)$=0 for $|k| > 2\pi\kappa_N$]. Introduce the points

$$x_m = m\Delta, \qquad m = 0, \pm 1, \pm 2, ... \tag{A-1}$$

and define

$$h = (R+1)\Delta \tag{A-2}$$

The extended sampling theorem (Poularikas, 1996) shows how the function can be reconstructed from itself and its derivatives $p^{(R)}$ up to order R at the points $mh = (R+1)x_m$ via the formula

$$p(x) = \sum_{m=-\infty}^{\infty} \Big[p(mh) + (x-mh)g^{(1)}(mh) + .. + \frac{(x-mh)^R}{R!} g^{(R)}(mh) \Big] \operatorname{sinc}^{(R+1)} \left[\frac{1}{h}(x-mh) \right] \tag{A-3}$$

where

$$g^{(j)}(mh) = \sum_{i=0}^{j} \begin{pmatrix} j \\ i \end{pmatrix} \left(\frac{\pi}{h} \right)^{j-i} \Gamma_{R+1}^{(j-i)} p^{(i)}(mh) \tag{A-4}$$

$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t} \tag{A-5}$$

and

$$\Gamma_\alpha^{(0)} = 1, \quad \Gamma_\alpha^{(2)} = \frac{\alpha}{3}, \quad \Gamma_\alpha^{(4)} = \frac{\alpha(5\alpha+2)}{15}, \quad \ldots, \quad \Gamma_\alpha^{(\beta)} = 0 \quad \text{for odd } \beta \tag{A-6}$$

In the case only the function is sampled, use $R = 0$ in equation A-4. Then $h = \Delta$ and Shannon's sampling theorem is obtained (Ikelle and Amundsen, 2005):

$$p(x) = \sum_{m=-\infty}^{\infty} p(x_m) \operatorname{sinc} \left[\frac{1}{\Delta}(x - x_m) \right] \tag{A-7}$$

This result is known also as sinc interpolation.

When the function and its first-order derivative are available, use $R = 1$ in equation A-4. Then $h = 2\Delta$, $g^{(1)} = p^{(1)}$, and we obtain the multichannel sampling theorem (Linden, 1959; Robertsson et al., 2008)

$$p(x) = \sum_{m=-\infty}^{\infty} \left[p(2x_m) + (x-2x_m)p^{(1)}(2x_m) \right] \times \operatorname{sinc}^2 \left[\frac{1}{2\Delta}(x-2x_m) \right] \tag{A-8}$$

From equation A-8, we note that when the function and its derivative is sampled, we can reconstruct functions sampled twice as coarsely as those reconstructed when only the function is available. Observe that the sinc function in the multichannel sampling theorem is squared. Therefore, for brief, we call this result for sinc^2 interpolation.

In the case that the function and its first and second order derivatives are sampled, use $R = 2$. Then $h = 3\Delta$. Further, $\Gamma_3^{(2)} = 1$, $g^{(2)} = ap + p^{(2)}$ where $a = \left(\frac{\pi}{3\Delta}\right)^2$. The function then can be reconstructed via the formula

$$p(x) = \sum_{m=-\infty}^{\infty} \Big[p(3x_m) + (x-3x_m)p^{(1)}(3x_m) + \frac{(x-3x_m)^2}{2} \Big(ap(3x_m) + p^{(2)}(3x_m) \Big) \Big] \times \operatorname{sinc}^3 \left[\frac{1}{3\Delta}(x-3x_m) \right] \tag{A-9}$$

From equation A-9, we observe that when the function and its first and second-order derivatives are known, we can reconstruct functions sampled three times as coarsely as those reconstructed when only the function is available. In this paper, we refer to this result as sinc^3 interpolation.

sinc, sinc^2 and sinc^3 interpolation in data processing

In data processing, data are given at sampling interval Δ. sinc interpolation is applied according to equation A-7. When sinc^2 interpolation is performed, Δ in equation A-8 must be set to $\Delta/2$. Compared with equation A-7, the resulting equation doubles the effective Nyquist wavenumber.

Likewise, when sinc^3 interpolation is performed, Δ in equation A-9 must be set to $\Delta/3$. Compared with equation A-7, the resulting equation triples the effective Nyquist wavenumber.

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2010 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Abramowitz, M., and I. A. Stegun, 1972, Handbook of mathematical functions: Dover Publ.

Amundsen, L., 1993, Wavenumber-based filt ering of marine point source data: Geophysics, **58**, 1335–1348, doi:10.1190/1.1443516.

Amundsen, L., 2001, Elimination of free-surface related multiples without need of the source wavelet: Geophysics, **66**, 327–341, doi:10.1190/1.1444912.

Ikelle, L. T. and L. Amundsen, 2005, Introduction to petroleum seismology: SEG

Linden, D. A., 1959, A discussion of sampling theorems: Proceedings of the Institute of Radio Engineers, **47**, 1219–1226.

Poularikas, A. D., 1996, The transforms and applications handbook: CRC Press.

Robertsson, J. O. A., 2006, Interpolation and extrapolation method for seismic recordings: G. B. Patent 2,414,299.

Robertsson, J. O. A., I. Moore, M. Vassallo, A. K. Ozdemir, D. J. van Manen, and A. Ozbek, 2008, On the use of multicomponent streamer recordings for reconstruction of pressure wavefields in the crossline direction: Geophysics, **73**, no. 5, A45–A49, doi:10.1190/1.2953338.

Singh, R., J. O. A. Robertsson, and O. Kristiansen, 2009, Seismic streamer system and method: G. B. Patent 2,425,596.