P314 NEW TRAVELTIME APPROXIMATIONS FOR A TRANSVERSELY ISOTROPIC MEDIUM

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Abstract

The basic idea of this paper is to derive approximate equations for vertical slowness for P- and SV-waves and PP, SS and PS traveltimes in TIV media not using a weak-anisotropy approximation. Comparison with the weak anisotropy approximation shows that the first and second terms in Taylor series are the same as were used in weak anisotropy approximation, but the third term is non-linear with respect to anisotropy parameters and differs from one in weak anisotropy approximation. This fact results in different definitions of heterogeneity coefficient in the traveltime approximation. In derivation of approximate equation for the traveltimes we use the continued fraction approximation which gives more accurate approximations than the standard weak anisotropy approximation.

P-wave traveltime approximations

We consider P- and SV-waves in a TIV-medium characterized by the Thomsen (1986) parameters which in terms of elastic constants are: vertical P-wave velocity, $\alpha_0 = \sqrt{c_{33}/\rho}$; vertical S-wave velocity, $\beta_0 = \sqrt{c_{44}/\rho}$; vertical P-wave to S-wave velocity ratio, $\gamma_0 = \alpha_0/\beta_0 = \sqrt{c_{33}/c_{44}}$; P-wave moveout parameter, δ ; SV-wave moveout parameter, σ .

The P-wave traveltime and offset squared can be written

$$T_P^2 = T_{P0}^2 \frac{\left(1+p^2\alpha_0^2 H_\alpha\right)^2}{1-p^2\alpha_0^2\left(1+S_\alpha\right)}$$
$$\bar{x}_P^2 = p^2\alpha_0^2 \frac{\left(1+S_\alpha+H_\alpha\right)^2}{1-p^2\alpha_0^2\left(1+S_\alpha\right)} \quad (1)$$

where p is horizontal slowness, T_{P0} is vertical traveltime, $\bar{x}_P = x_P/\alpha_0 T_{P0}$ is normalized offset, S_α is defined following Stovas and Ursin (2003)

$$\frac{1}{\alpha^2} = \frac{1}{\alpha_0^2} - p^2 S_\alpha \quad (2)$$

and $H_\alpha = pS'_\alpha/2$. The function S_α can be expressed in Taylor series

$$S_\alpha = a_0 + a_1 p^2\alpha_0^2 + ... \quad (3)$$

with

$$a_0 = 2\delta \qquad a_1 = 2\sigma\frac{\gamma_0^2 + 2\delta\gamma_0^2 - 1}{\gamma_0^2\left(\gamma_0^2-1\right)} \quad (4)$$

From the last equation in (1) we can write

$$p^2\alpha_0^2 = \frac{\bar{x}_P^2}{\left(1+S_\alpha+H_\alpha\right)^2 + \bar{x}_P^2\left(1+S_\alpha\right)} \quad (5)$$

Substituting this into the first equation in (1) gives

$$T_P^2 = T_{P0}^2\frac{\left(1+\bar{x}_P^2+S_\alpha+H_\alpha\right)^2}{\left(1+S_\alpha+H_\alpha\right)^2+\bar{x}_P^2\left(1+S_\alpha\right)} \quad (6)$$

By using Taylor expressions for S_α and H_α, and the continued fraction approximation for $p^2\alpha_0^2$ we derive

$$T_{P1}^2 = T_{P0}^2\left[1+\tilde{x}_P^2 - \Phi_P \tilde{x}_P^2 \frac{1+4\Phi_P+\tilde{x}_P^2}{\left(1+2\Phi_P\right)^2+\tilde{x}_P^2\left(1+\Phi_P\right)}\right], \tag{7}$$

where the normalized offset now is

$$\tilde{x}_P^2 = \overline{x}_P^2 \big/ \left(1+a_0\right) = x_P^2 \big/ v_P^2 T_{P0}^2 \tag{8}$$

with $v_P^2 = \alpha_0^2\left(1+a_0\right) = \alpha_0^2\left(1+2\delta\right)$ being the moveout velocity (Thomsen, 1986) and

$$\Phi_P = \frac{G_P \tilde{x}_P^2}{1+\left(1+4G_P\right)\tilde{x}_P^2} \tag{9}$$

The other traveltime parameter (similar to the heterogeneity parameter introduced in Fomel and Grechka (2001)) is

$$G_P = a_1 \big/ \left(1+a_0\right)^2 \tag{10}$$

A much simpler traveltime approximation is obtained by neglecting the H_α^2 - term in equation (6) and using the continued fraction approximation

$$T_{P2}^2 = T_{P0}^2\left[1+\tilde{x}_P^2 - G_P \tilde{x}_P^4 \frac{1+\tilde{x}_P^2\left(8+G_P\right)}{\left[1+\tilde{x}_P^2\left(6+G_P\right)\right]^2}\right], \tag{11}$$

Tsvankin and Thomsen (1994) introduced the approximation which can be written in the notations introduced above

$$T_{P3}^2 = T_{P0}^2\left[1+\tilde{x}_P^2 - \frac{G_P \tilde{x}_P^4}{1+\tilde{x}_P^2\left(1+G_P\right)}\right], \tag{12}$$

with $G_P = 2\left(\varepsilon-\delta\right)$ in this case. A similar approximation can be obtained by neglecting higher- order terms in equation (7), resulting in

$$T_{P4}^2 = T_{P0}^2\left[1+\tilde{x}_P^2 - \Phi_P \tilde{x}_P^2\right] = T_{P0}^2\left[1+\tilde{x}_P^2 - \frac{G_P \tilde{x}_P^4}{1+\tilde{x}_P^2\left(1+4G_P\right)}\right], \tag{13}$$

where G_P now is defined in equation (10).

SV-wave traveltime approximations

For an SV-wave the phase velocity can be computed from (Stovas and Ursin, 2003)

$$\frac{1}{\beta^2} = \frac{1}{\beta_0^2} - p^2 S_\beta, \tag{14}$$

where S_β can be expanded in the Taylor series

$$S_\beta = b_0 + b_1 p^2 \beta_0^2 + \ldots \tag{15}$$

with

$$b_0 = 2\sigma \qquad\qquad b_1 = -2\sigma \frac{\gamma_0^2 + 2\delta\gamma_0^2 - 1}{\gamma_0^2 - 1} \tag{16}$$

All results and derivations in the previous section can be used by replacing P with S. The SV-wave moveout parameters are

$$\begin{aligned} G_S &= b_1 \big/ \left(1+b_0\right)^2 \\ v_S^2 &= \beta_0^2\left(1+2\sigma\right) \end{aligned} \tag{17}$$

Converted-wave traveltime approximations

The similar traveltime approximations are obtained for a converted wave which has travelled a horizontal distance x_P as a P-wave and a horizontal distance x_S as an SV-wave. Using the Taylor series for P- and SV-wave traveltime and offset we define the following parameters

$$v_C^2 = \frac{v_S^2 T_{S0} + v_P^2 T_{P0}}{T_{S0} + T_{P0}}$$
$$G_C = \frac{4\left(v_S^4 T_{S0} G_S + v_P^4 T_{P0} G_P\right) + \left(v_P^2 - v_S^2\right) T_{S0} T_{P0}}{4\left(v_S^2 T_{S0} + v_P^2 T_{P0}\right)} \qquad (18)$$

Now we may use the traveltime approximations similar to equation (7), (11)-(13).

Numerical examples

To compare our approximations for the vertical slowness and traveltimes we consider two single layer models with parameters given in Table. The parameter σ is positive for the model I and is negative for the model II. For both single layer models the Taylor series for S_α (coefficients a_j) converges very fast, but the Taylor series for S_β (coefficients b_j) converges very slow. In practice it means that all weak-anisotropy approximations are more accurate for PP reflection and less accurate for SS and PS reflections. The negative value of σ also creates problems for SS traveltimes: $G_S = -0.23129$ for model I and $G_S = 1.55556$ for model II. In Figure 1 and 2 we show the deviation from the exact traveltime for PP reflection (equation (7), (11)-(13)), SS reflection and PS reflection computed for model I and model II, respectively. One can see that Tsvankin-Thomsen approximation has serious problems for far offset both for model I and model II. With the proposed approximations we can improve the accuracy of the traveltime computation for far offset region.

Conclusions

We have derived new approximations for the P- and SV-wave vertical slowness and PP, SS and PS reflection traveltimes for a TIV media not using the weak-anisotropy assumption. The comparison with the Tsvankin-Thomsen (1994) nonhyperbolic approximation made for single layer models shows that our approximations are more accurate.

References

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Stovas, A., and Ursin, B., 2003, Reflection and transmission responses of layered transversely isotropic visco-elastic media, *Geophysical Prospecting,* **51**, 447-477.

Thomsen L. 1986, Weak elastic anisotropy. *Geophysics,* **51**, 1954-1966.

Tsvankin, I., and Thomsen, L., 1994, Nonhyperbolic reflection moveout in anisotropic media, *Geophysics*, **59**, 1290-1304.

Parameters	*Model I*	*Model II*
$\alpha_0\ [km/s]$	2.0	2.0
$\beta_0\ [km/s]$	1.0	1.0
ε	0.1	0.1
δ	0.05	0.15
σ	0.2	-0.2
$\mathrm{v_P}\ [\mathrm{km/s}]$	2.098	2.28
G_P	0.09366	-0.08284
$\mathrm{v_S}\ [\mathrm{km/s}]$	1.18322	0.7746
G_S	-0.23129	1.55556
$\mathrm{v_C}\ [\mathrm{km/s}]$	1.549	1.46
G_C	0.13927	0.17627

Table. Parameters for single layer model I and II.

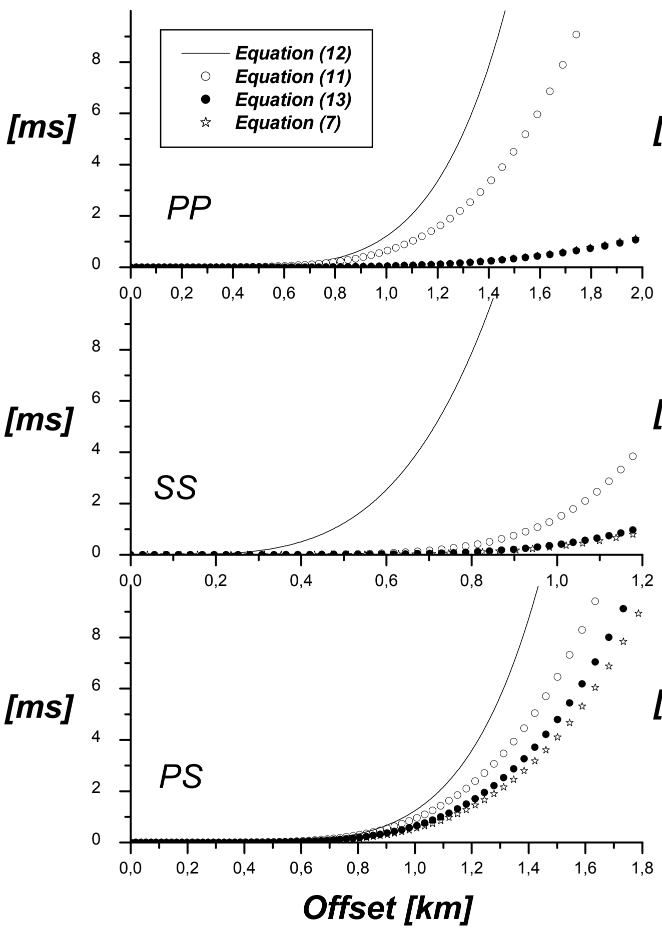

Figure 1. The deviation in ms from the exact traveltime computed from model I.

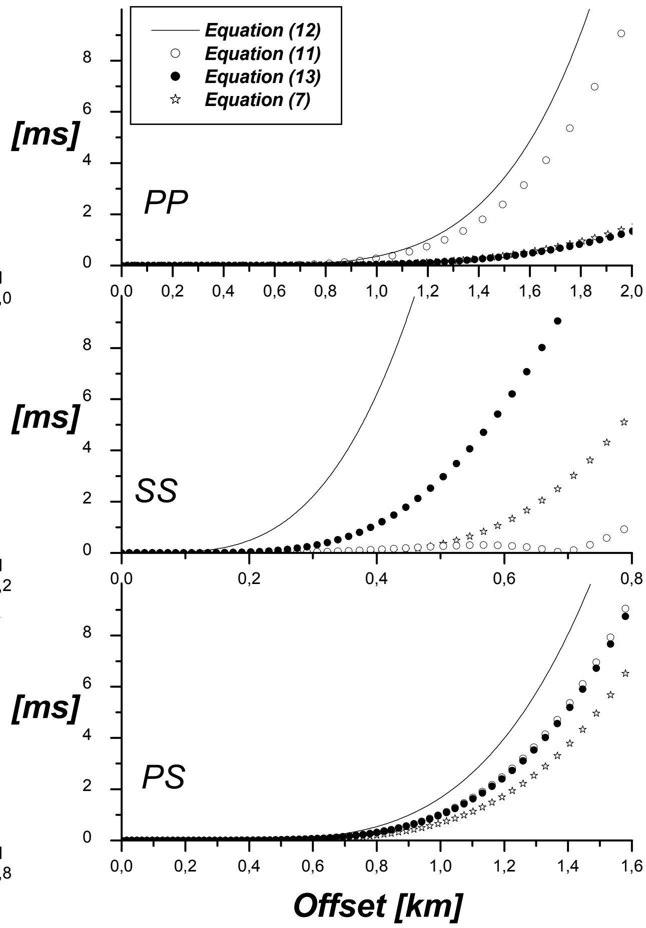

Figure 2. The deviation in ms from the exact traveltime computed from model II.