P274

The Geometrical Spreading in VTI Medium Revisited

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SUMMARY

We derived the new approximations for the relative geometrical spreading in a layered transversely isotropic medium using three parameters: the two-way vertical traveltime, the normal moveout velocity and the heterogeneity coefficient. In derivation we use the Taylor series expansion for the geometrical spreading and its asymptotic behaviour at infinite offset

Introduction

We propose a new method to compute the relative geometrical spreading in a layered transversely isotropic medium based on the Taylor series and its asymptotical behavior.

Geometrical spreading from traveltime approximations

The standard approach to compute the relative geometrical spreading is to use the traveltime approximations (Ursin and Hokstad, 2003). The relative geometrical spreading is defined as

$$L = L^{\parallel} L^{\perp} \tag{1}$$

with in-plane and out-of-plane geometrical spreading factors are given by

$$L^{\parallel} = \left(\cos\alpha^{r}\cos\alpha^{s}\right)^{1/2}\left(\frac{d^2t}{dx^2}\right)^{-1/2}, \quad L^{\perp} = \left(\frac{1}{x}\frac{dt}{dx}\right)^{-1/2}, \tag{2}$$

where α^r and α^s are the angles the ray makes with the vertical axis at the receiver and source, respectively.

There are two major types of traveltime approximations for non-hyperbolic moveout. A continuous fraction approximation is given by

$$t^2\left(\tilde{x}\right) = t_0^2\left[1+\tilde{x}^2+\frac{A\tilde{x}^4}{\left(1+B\tilde{x}^2\right)}\right], \tag{3}$$

with the normalized offset $\tilde{x} = x/\left(v_{NMO}t_0\right)$, t_0 and v_{NMO} are the vertical traveltime and normal moveout velocity, respectively. The coefficient A is defined as follows

$$A = \frac{1}{4}\left(1-S_2\right), \tag{4}$$

where S_2 is heterogeneous coefficient of second order. We can choose the coefficient B such that to preserve the sixth order coefficient in the Taylor expansion of the traveltime squared (Ursin and Stovas, 2006):

$$B_1 = \frac{2S_2^2-S_2-S_3}{2\left(S_2-1\right)}, \tag{5}$$

with S_3 is heterogeneous coefficient of third order, or by preserving the horizontal velocity in a single VTI layer (Alkhalifah and Tsvankin, 1995):

$$B_2 = 1+2\eta = \frac{3+S_2}{4}. \tag{6}$$

Another type of traveltime approximations is a shifted hyperbola approach (Castle, 1994)

$$t\left(\tilde{x}\right) = t_0+\frac{t_0}{S_2}\left[\sqrt{1+\tilde{x}^2S_2}-1\right]. \tag{7}$$

The Taylor series for geometrical spreading

We can define the relative geometrical spreading in p-domain

$$L^{\parallel} = \left(\cos\alpha^{r}\cos\alpha^{s}\right)^{1/2}\left(\frac{dx}{dp}\right)^{1/2}, \quad L^{\perp} = \left(\frac{x}{p}\right)^{1/2}. \tag{8}$$

Then by using the Taylor series expansion for offset from Ursin and Stovas (2006) we can compute the Taylor series for relative geometrical spreading in the form

$$L = t_0v_{NMO}^2\left(\cos\alpha^{r}\cos\alpha^{s}\right)^{1/2} \left[1+S_2\tilde{x}^2+\frac{9}{8}\left(S_3-S_2^2\right)\tilde{x}^4+\frac{1}{8}\left(10S_4-27S_2S_3+17S_2^3\right)\tilde{x}^6+\dots\right], \tag{9}$$

with S_4 is heterogeneous coefficient of fourth order. We are going to use the continued fraction approximation of the form

$$L = t_0 v_{NMO}^2 \left(\cos\alpha^r \cos\alpha^s\right)^{1/2} \left[1 + S_2 \tilde{x}^2 + \frac{C\tilde{x}^4}{\left(1 + D\tilde{x}^2\right)}\right]. \quad (10)$$

The coefficient C is the forth order coefficient from the Taylor series (9) in the acoustic approximation (Alkhalifah, 1998) is given by

$$C = \frac{9}{8}\left(S_3 - S_2^2\right) \approx -\frac{9}{16}\left(S_2^2 - 1\right). \quad (11)$$

To define coefficient D we have similar choices as for the continued fraction traveltime approximation. To preserve the sixth order term in (9) we have to choose (in the acoustic approximation)

$$D_1 = -\frac{1}{9}\frac{10S_4 - 27S_2S_3 + 17S_2^3}{S_3 - S_2^2} \approx 1 + \frac{4}{3}\left(S_2 - 1\right). \quad (12)$$

Another type approximation we have to define the asymptotical behaviour of geometrical spreading at the infinite offset limit. Expression for D we obtain from asymptotic behaviour of Taylor series (9)

$$D_2 \approx -\frac{S_3 - S_2^2}{S_2 - 1} = \frac{S_2 + 1}{2}. \quad (13)$$

Numerical examples

We test the traveltime based approximations (from derivatives of approximation (3) with B_1 (equation (5)) and B_2 (equation (6)) and the shifted hyperbola approximation (7)) and direct approximations (equation (10) with D_1 (equation (12)) and D_2 (equation (13))) on two single layer VTI models (Table 1) and one multilayered VTI model (Ursin and Hokstad, 2003) shown in Table 2.

In Figure 1 we show the relative error in L versus offset/depth for single VTI layer from model I and model II. One can see that the direct approximation from equations (10), (11) and (13) performs the best, especially for the model I. For short offset the direct approximation from equations (10), (11) and (12) behaves better than the traveltime based approximation with Alkhalifah-Tsvankin definition of parameter B (equation (6)). One of the worse results gives the traveltime based approximation using the shifted hyperbola approach.

In Figure 2 we show the relative geometrical spreading versus offset/depth for a layered VTI medium from model III. We do computations for 13 layers and 9 layers. For both cases the direct approximation from equations (10), (11) and (12) is superior.

Conclusions

We derived new approximations for the relative geometrical spreading in the layered VTI medium based on three parameters only: the two-way vertical traveltime, the normal moveout velocity and heterogeneity coefficient. The new approximations tested on the single layer and multilayered VTI models show better performance compared with the standard approach which is based on the traveltime approximations.

Acknowledgements

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References

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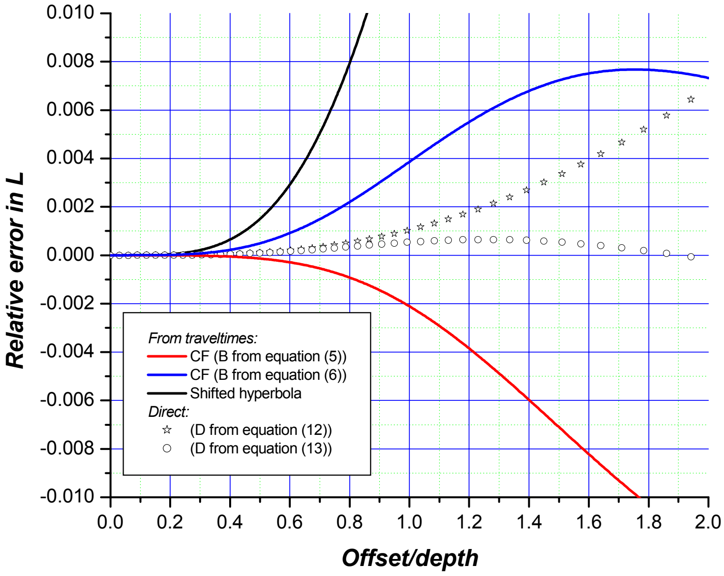
	$\alpha_0,[km/s]$	$\beta_0,[km/s]$	ε	δ	$v_{NMO}[km/s]$	S_2
Model I	2.000	1.000	0.10	0.05	2.098	1.364
Model II	2.000	1.000	0.05	0.10	2.191	0.667

Table. Single layer VTI model parameters (Model I and II).



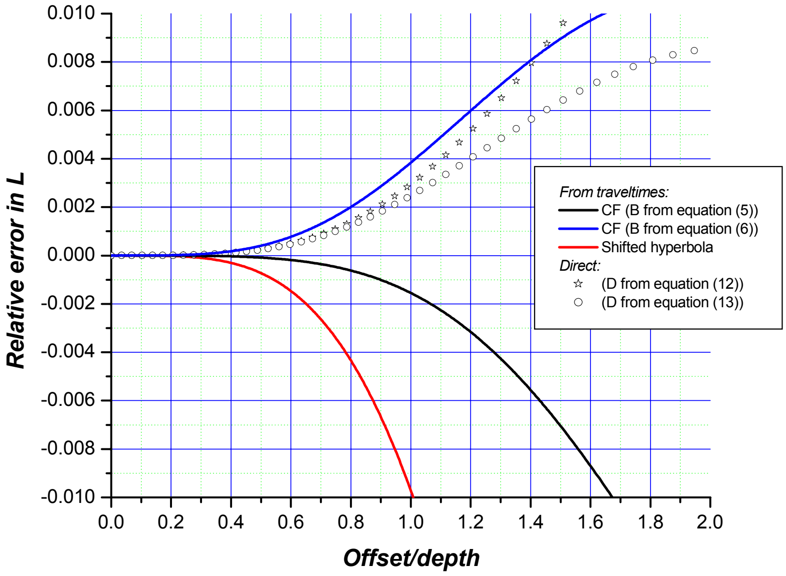


Figure 1. Relative error in L versus offset/depth for single VTI layer: model I (top) and model II (bottom).

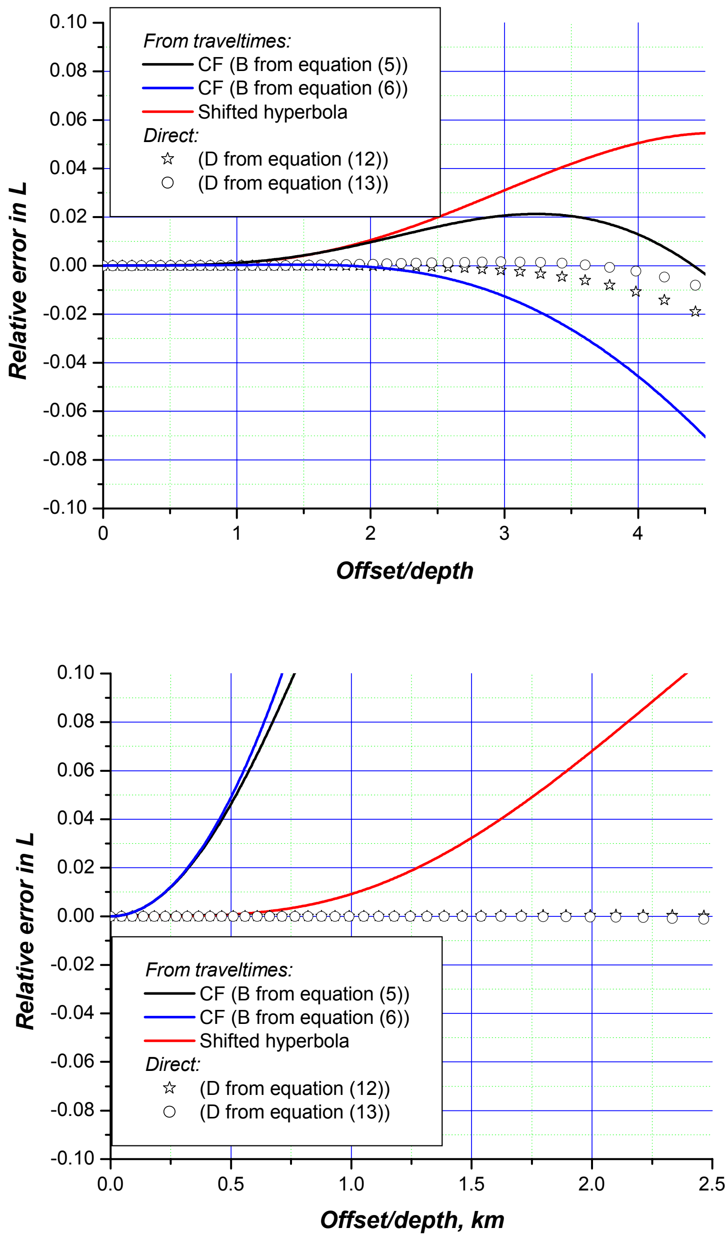


Figure 2. Relative error in L versus offset/depth for the layering VTI model (13 layers is to the top and 9 layers is to the bottom).