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# Wide-Angle Phase-Slowness Approximations in VTI Media

O. Pedersen* (Norwegian University of Science & Technology), B. Ursin (Norwegian University of Science & Technology) & A. Stovas (Norwegian University of Science & Technology)

## SUMMARY

Several phase-shift migration methods depend on the vertical slowness, which in general can be represented as a nonlinear function of the horizontal slowness. In a VTI media, the dispersion relations relating the vertical and horizontal slowness, are complex expressions. Simple and accurate approximations of the exact slowness for both qP and qSV waves are desired for computationally fast and accurate migration algorithms. We describe new wide-angle phase slowness approximations for a VTI media.

## Abstract

Several phase-shift migration methods depend on the vertical slowness, which in general can be represented as a nonlinear function of the horizontal slowness. In a VTI media, the dispersion relations relating the vertical and horizontal slowness, are complex expressions. Simple and accurate approximations of the exact slowness for both $qP$ and $qSV$ waves are desired for computationally fast and accurate migration algorithms. We describe new wide-angle phase-slowness approximations for a VTI media.

## Introduction

If we consider wave propagation in a VTI medium with no cusps and kinks, we obtain two uncoupled dispersion relations (Aki and Richards, 1980). These relations will in turn provide us with the phase-slowness expressions for the $qP$ and the $qSV$ waves. The exact expression for the vertical slowness contains four independent parameters. By observations from Alkhalifah and Tsvankin (1995), only three parameters influence wave propagation and are of interest in seismic methods. Further, the exact expression of the vertical slowness is on a complex form, hence a simplified approximation of the vertical slowness with reduced number of parameters is desired.

## Phase-slowness approximations

Let $p$ denote the horizontal slowness. Let $\alpha_0$ and $\beta_0$ be the $qP$ and $qSV$ phase velocity in the vertical direction, and let $q_{\alpha_0}$ and $q_{\beta_0}$ be the corresponding slowness, respectively. Then the vertical slowness for $qP$ and $qSV$ waves in a VTI medium can be expressed by (Stovas and Ursin, 2003)

$$q_{\alpha,\beta}^2 = -\frac{1}{2}\Bigg[ -q_{\alpha_0}^2 - q_{\beta_0}^2 + 2p^2(\sigma+\delta) \pm \sqrt{\left(q_{\beta_0}^2 - q_{\alpha_0}^2\right)^2 - 4\frac{p^2}{\alpha_0^2}\left(\gamma_0^2-1\right)(\sigma-\delta) + 4p^4\left(2\frac{\left(\gamma_0^2-1\right)}{\gamma_0^2}\sigma + (\sigma+\delta)^2\right)}\Bigg], \tag{1}$$

where $\gamma_0$ denotes the ratio between the vertical $qP$ and $qSV$ phase velocity, $\varepsilon$ and $\delta$ are the Thomsen parameters (Thomsen, 1986), and $\sigma = \gamma_0^2(\varepsilon-\delta)$. In our further derivations it will be convenient to use the parameter $\eta = \varepsilon - \delta$. After algebraic manipulations with equation (1), and a Taylor series expansion of the square root one obtains

$$q_\alpha^2 = \frac{1}{\alpha_0^2} - p^2 H_\alpha, \qquad q_\beta^2 = \frac{1}{\beta_0^2} - p^2 H_\beta, \tag{2}$$

where

$$H_\alpha = 1 + 2\delta + \sum_{j=1}^{\infty} a_j (p\alpha_0)^{2j}, \qquad H_\beta = 1 + 2\sigma - \sum_{j=1}^{\infty} a_j (p\alpha_0)^{2j}, \tag{3}$$

with

$$a_1 = \frac{2\sigma}{\gamma_0^2}\left(1 + \frac{2\gamma_0^2\delta}{\gamma_0^2-1}\right), \qquad a_2 = \frac{-4\sigma}{\gamma_0^2\left(\gamma_0^2-1\right)}(\delta-\sigma)\left(1+\frac{2\gamma_0^2\delta}{\gamma_0^2-1}\right), \tag{4}$$

and higher-order coefficients are given in Stovas and Ursin (2006). A continued fraction approximation (Stovas and Ursin, 2004) of the Taylor series in equation (2) gives for a $qP$ wave

$$q_\alpha^2 = \frac{1}{\alpha_0^2}\left(1 - a_0 (p\alpha_0)^2 - \frac{a_1 (p\alpha_0)^4}{1 - \frac{a_2}{a_1}(p\alpha_0)^2}\right) \tag{5}$$

where $a_0 = 1 + 2\delta$. This equation can be simplified to

$$q_\alpha^2 = \frac{1}{\alpha_0^2}\left(\frac{1-(1+2\varepsilon)(p\alpha_0)^2}{1-2\eta(p\alpha_0)^2}\right). \tag{6}$$

These approximation introduces a pole for $q_\alpha$ and $q_\beta$ determined by the parameters, so our expression is valid for values bounded by this pole. The next approximation will give us the exact vertical slowness expression in both vertical and horizontal direction of propagation (Schoenberg and de Hoop, 2000; Douma et al., 2005). It is found by taking advantage of that the vertical slowness is zero when $p^2 = 1/v_h^2$, where $v_h$ is the horizontal phase velocity, for each wavemode. Factoring out the zero in (2) yield the following representation of the vertical slowness,

$$q_\alpha^2 = \frac{1}{\alpha_0^2}\left(1-\left(\frac{\alpha_h}{\alpha_0}\right)^2 (p\alpha_0)^2\right)\left(1-\sum_{j=0}^{\infty} b_j\,(p\alpha_0)^{2j+2}\right), \tag{7}$$

where

$$b_0 = a_0 - \left(\frac{\alpha_h}{\alpha_0}\right)^2, \qquad b_j = a_j + \left(\frac{\alpha_h}{\alpha_0}\right)^2 b_{j-1},\ j = 1, 2, \ldots \tag{8}$$

and $\alpha_h = \alpha_0\sqrt{1+2\varepsilon}$. The approximations of the sum in equation (7) ensure the interpolation of the approximated and exact slowness both in the horizontal and vertical direction. We denote these approximations the wide-angle approximations. Similar as for the continued fraction approximation of the equations in (2), a continued fraction approximation of the wide-angle representation in equation (7) is thus provided by

$$q_\alpha^2 = \frac{1}{\alpha_0^2}\left(1-\left(\frac{\alpha_h}{\alpha_0}\right)^2 (p\alpha_0)^2\right)\left(1 - b_0\,(\alpha_0 p)^2 - \frac{b_1\,(p\alpha_0)^4}{1-\frac{b_2}{b_1}\,(p\alpha_0)^2}\right). \tag{9}$$

This can be further simplified to

$$q_\alpha^2 = \frac{1}{\alpha_0^2}\left(1-\left(\frac{\alpha_h}{\alpha_0}\right)^2 (p\alpha_0)^2\right)\frac{1+\eta\,(p\alpha_0)^2}{1-\eta\,(p\alpha_0)^2}. \tag{10}$$

For a $qSV$ wave, equation (2) gives

$$q_\beta^2 = \frac{1}{\beta_0^2}\left(1 - c_0\,(p\beta_0)^2 - \frac{c_1\,(p\beta_0)^4}{1-\frac{c_2}{c_1}\,(p\beta_0)^2}\right) \tag{11}$$

where

$$c_0 = 1+2\sigma, \qquad c_j = -a_j\gamma_0^{2j}. \tag{12}$$

This equation can be simplified to

$$q_\beta^2 = \frac{1}{\beta_0^2}\left(1 - (1+2\sigma)\,(p\beta_0)^2 + \frac{2\sigma\,(p\beta_0)^4}{1-2\sigma\,(p\beta_0)^2}\right). \tag{13}$$

A wide-angle continued fraction approximation for the $qSV$-slowness squared is

$$q_\beta^2 = \frac{1}{\beta_0^2}\left(1-(p\beta_0)^2\right)\left(1 - d_0\,(p\beta_0)^2 - \frac{d_1\,(p\beta_0)^4}{1-\frac{d_2}{d_1}\,(p\beta_0)^2}\right). \tag{14}$$

with

$$d_0 = c_0 - 1 = 2\sigma, \qquad d_j = c_j - d_{j-1}, \tag{15}$$

since $\beta_h = \beta_0$. This can be further simplified to

$$q_\beta^2 = \frac{1}{\beta_0^2}\left(1-(p\beta_0)^2\right)\frac{1-\sigma\,(p\beta_0)^2}{1+\sigma\,(p\beta_0)^2}. \tag{16}$$

## Numerical results

The approximations are illustrated using two models. In both models we let $\alpha_0 = 2.0[km/s]$ and $\beta_0 = 1.0[km/s]$. We use two set of parameters to characterize two distinct VTI models. For each model we compute the approximate $qP$ and $qSV$ slowness curves. In model 1, we assume $\varepsilon = 0.1$ and $\delta = 0.05$. In model 2, we assume $\varepsilon = 0.1$ and $\delta = 0.15$. The figures show the difference between the exact slowness (equation (1)) and the continued fraction (equations (5) and (11), denoted CF), the second order wide-angle (equation (7), denoted WA), and the wide-angle continued fraction (equations (9) and (14), denoted WACF) approximations as functions of horizontal slowness $p$. In addition, the simplified continued fraction (equations (6) and (13), denoted SCF) and the simplified wide-angle continued fraction approximations (equations (10) and (16), denoted SWACF) are illustrated. The second order Taylor expansion (equation (2), denoted T) is included for reference. The slowness approximations are constructed to be exact in the vertical direction, and the wide-angle approximations are in addition exact at the horizontal directions. These interpolation points are determined by the values $q_{\alpha_0}$ and $\alpha_h$ for the $qP$ slowness, and $q_{\beta_0}$ for the $qSV$ slowness curve. In the models we have defined, these wide-angel interpolation points are $p = 0.46$ for the $qP$ and $p = 1.00$ for the $qSV$ curves. In the second media, the difference between the wide-angle continued fraction approximation of the $qP$ slowness curve and the exact $qP$ slowness curve is of the order $10^{-6}$.

## Conclusions

We have derived new simple and accurate wide-angle approximations for the $qP$ and $qSV$ phase-slowness in a VTI media. For our model parameters, the wide-angle approximations are the most accurate for the $qP$ and $qSV$ slowness in model 1, and for the $qSV$ slowness in model 2. The wide-angle continued fraction approximation gives the best fit for the $qSV$ slowness in model 2. For the simplified approximations, the simplified wide-angle approximations are the most accurate.

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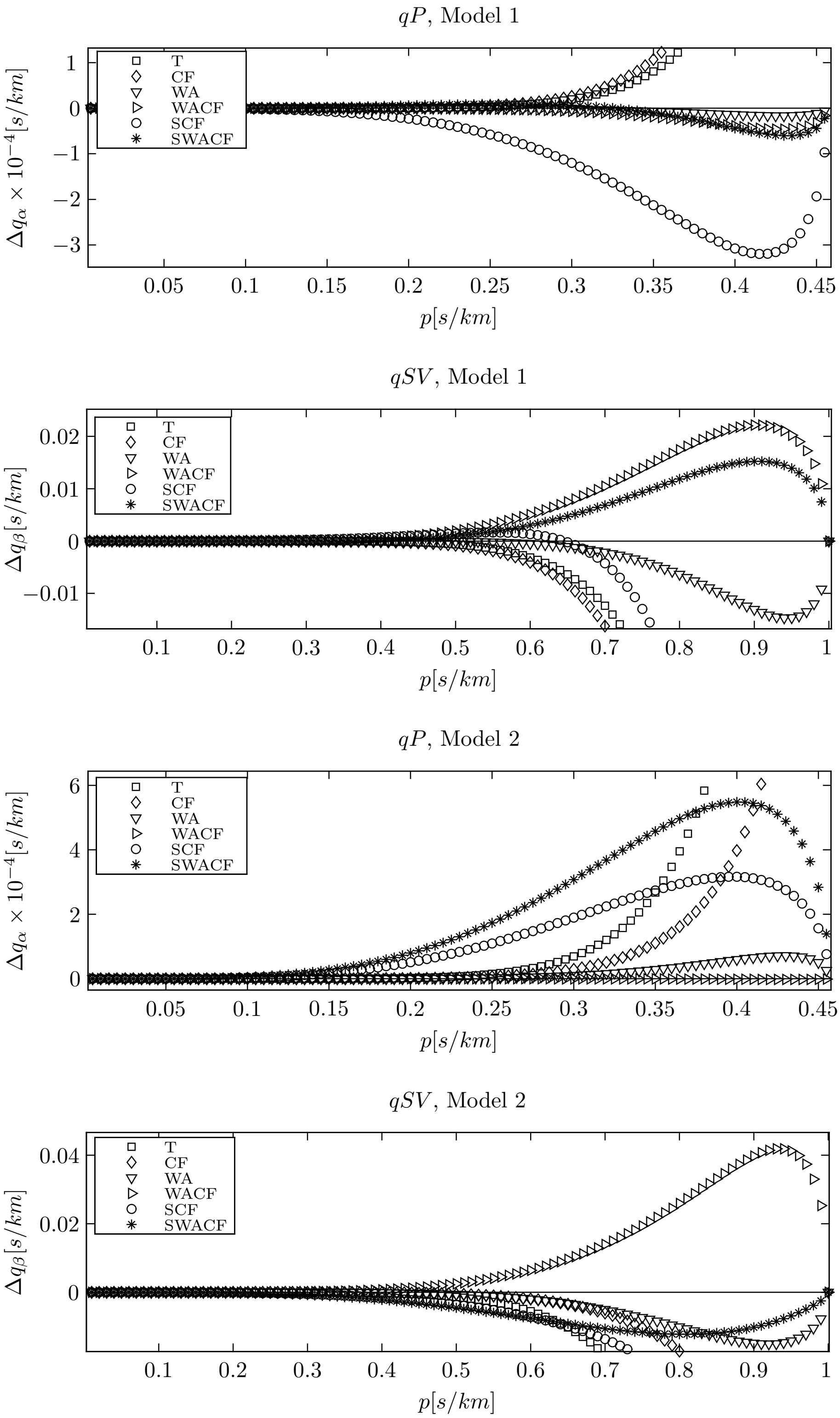


Figure 1: Difference between the exact and approximated $qP$ and $qSV$ slowness for model 1 and model 2. The curves are the second order Taylor (T), continued fraction (CF), the wide-angle (WA), the wide-angle continued fraction (WACF), the simplified wide-angle continued fraction (SWACF), and the simplified continued fraction (SCF) approximation.