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GENERALIZED DIX EQUATIONS FOR A LAYERED TRANSVERSELY ISOTROPIC MEDIUM

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Abstract

In the standard Dix (1955) equations the normal-incidence traveltimes and the RMS or NMO velocities for the PP reflections are used to estimate the interval velocity and thickness for homogeneous isotropic horizontal layers. For a stack of homogeneous horizontal layers which are transversely isotropic with a vertical symmetry axis the qPqP traveltimes (and the qPqSV traveltimes) depend on four parameters and thickness of each layer. Traveltime parameters can be estimated from the traveltimes of the qPqP and qPqSV reflected waves. Using semblance analysis (taner and Koehler, 1969), the estimated qPqP traveltime parameters are the P-wave normal-incidence traveltime, the P-wave NMO velocity and a heterogeneity factor entering in the shifted hyperbolic traveltime approximation or in a continued fraction traveltime approximation. The estimated qPqSV traveltime parameters are the PS normal-incidence traveltime and the PS NMO velocity combined with the qPqP traveltime parameters these are used to compute the S-wave normal-incidence traveltime and the SV-wave NMO velocity. The estimated traveltime parameters enter in explicit, layer-recursive expressions for the four layer parameters and thickness of each layer.

Traveltime approximations

The shifted hyperbola approximation is (Castle, 1994; de Bazelaire, 1988; Ursin and Stovas, 2004)

$$T(x) \approx T(0) + \frac{T(0)}{S_2}\left[\sqrt{1+\frac{x^2 S_2}{T(0)^2 v_{NMO}^2}} - 1\right]. \tag{1}$$

A continued fraction approximation which is accurate for a qPqP reflection is (Ursin and Stovas, 2004)

$$T(x)^2 = T(0)^2 + \frac{x^2}{v_{NMO}^2} - \frac{(S_2-1)x^4}{4v_{NMO}^4\left[T(0)^2 + \frac{S_2}{2}\frac{x^2}{v_{NMO}^2}\right]}. \tag{2}$$

We now consider qPqP and qPqSV reflected from the bottom of N homogeneous layers with the source and receiver at the top of layer 1. From the qPqP traveltime we estimate three parameters: the P-wave normal-incidence traveltime, the P-wave NMO velocity and a heterogeneity factor,

$$\begin{aligned}
T_{PP,N}(0) &= \sum_{k=1}^{N} \Delta T_{PP,k} = 2\sum_{k=1}^{N}\frac{\Delta z_k}{\alpha_{0,k}} \\
v_{PP,N}^2 &= \frac{2}{T_{PP,N}(0)}\sum_{k=1}^{N}\alpha_{0,k}(1+2\delta_k)\Delta z_k \\
S_{PP,N} &= \frac{2}{v_{PP,N}^4 T_{PP,N}(0)}\sum_{k=1}^{N}\alpha_{0,k}^3\left[(1+2\delta_k)^2 + 8(\varepsilon_k-\delta_k)\left(1+\frac{2\delta_k\gamma_{0,k}^2}{\gamma_{0,k}^2-1}\right)\right]\Delta z_k
\end{aligned} \tag{3}$$

From the qPqSV wave, reflected and converted at the bottom of layer N, we estimate two parameters: the PS normal-incidence traveltime and the PS NMO velocity,

$$\begin{aligned} T_{PS,N}(0) &= \frac{1}{2}\left[T_{PP,N}(0) + T_{SS,N}(0)\right] \\ v_{PS,N}^2 &= \frac{v_{PP,N}^2 T_{PP,N}(0) + v_{SS,N}^2 T_{SS,N}(0)}{2T_{PS,N}(0)}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} T_{SS,N}(0) &= \sum_{k=1}^{N} \Delta T_{SS,k} = 2\sum_{k=1}^{N} \frac{\Delta z_k}{\beta_{0,k}} \\ v_{SS,N}^2 &= \frac{2}{T_{SS,N}(0)} \sum_{k=1}^{N} \beta_{0,k}\left[1 + 2\gamma_{0,k}^2(\varepsilon_k - \delta_k)\right]\Delta z_k \end{aligned}. \tag{5}$$

Equation (4) gives

$$\begin{aligned} T_{SS,N}(0) &= 2T_{PS,N}(0) - T_{PP,N}(0) \\ T_{SS,N}(0)v_{SS,N}^2 &= 2T_{PS,N}(0)v_{PS,N}^2 - T_{PP,N}(0)v_{PP,N}^2 \end{aligned}. \tag{6}$$

In layer number *k* of thickness Δz_k, $\alpha_{0,k}$ and $\beta_{0,k}$ are the vertical P- and S-wave velocities, $\gamma_{0,k} = \alpha_{0,k}/\beta_{0,k}$ their ratio, and δ_k and ε_k are the Thomsen (1986) parameters.

For a homogeneous medium we shall express all quantities in terms of the Thomsen (1986) parameters The estimated traveltime parameters from layer N and N-1 give the difference expressions from which we compute five equations for the parameters defining layer N (for simplicity, the subscript N is omitted):

$$\begin{aligned} \Delta T_{PP}(0) &= \frac{2\Delta z}{\alpha_0} \\ \frac{\Delta\left(T_{PP}(0)v_{PP}^2\right)}{\Delta T_{PP}(0)} &= \alpha_0^2(1+2\delta) \\ \frac{\Delta\left(T_{PP}(0)v_{PP}^4 S_{PP}\right)}{\Delta T_{PP}(0)} &= \alpha_0^4\left[(1+2\delta)^2 + 8(\varepsilon-\delta)\left(1 + \frac{2\gamma_0^2\delta}{\gamma_0^2 - 1}\right)\right] \\ \Delta T_{SS}(0) &= \frac{2\Delta z}{\beta_0} \\ \frac{\Delta\left(T_{SS}(0)v_{SS}^2\right)}{\Delta T_{SS}(0)} &= \beta_0^2\left[1 + 2\gamma_0^2(\varepsilon-\delta)\right]. \end{aligned} \tag{7}$$

From these we compute the vertical v_P/v_S ratio

$$\gamma_0 = \frac{\Delta T_{SS}(0)}{\Delta T_{PP}(0)} = \frac{\alpha_0}{\beta_0}, \tag{8}$$

and the v_S/v_P NMO velocity ratio squared

$$\gamma_1^{-2} = \frac{\Delta\left(T_{SS}(0)v_{SS}^2\right)}{\Delta T_{SS}(0)} \frac{\Delta T_{PP}(0)}{\Delta\left(T_{PP}(0)v_{PP}^2\right)} = \gamma_0^{-2}\frac{1+2\gamma_0^2(\varepsilon-\delta)}{1+2\delta} \tag{9}$$

Combining equations (7), (8) and (9), yields

$$\alpha_0^2 = \left\{\frac{\Delta\left(T_{PP}(0)v_{PP}^2\right)}{\Delta T_{PP}(0)}\right\}\left\{\frac{\gamma_0^2}{2}\right\}\left\{1 + \gamma_1^{-2} - \sqrt{\left(1-\gamma_1^{-2}\right)^2 + \phi}\right\}, \tag{10}$$

where

$$\phi = \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2}\left[\frac{\Delta\left(T_{PP}\left(0\right)v_{PP}^4 S_{PP}\right)\Delta T_{PP}\left(0\right)}{\left[\Delta\left(T_{PP}\left(0\right)v_{PP}^2\right)\right]^2} - 1\right]. \tag{11}$$

The other layer parameters are computed from

$$\Delta z = \frac{\alpha_0 \Delta T_{PP}\left(0\right)}{2}$$

$$\beta_0 = \frac{\alpha_0}{\gamma_0}$$

$$\delta = \frac{1}{2}\left[\frac{1}{\alpha_0^2}\left(\frac{\Delta\left(T_{PP}\left(0\right)v_{PP}^2\right)}{\Delta T_{PP}\left(0\right)}\right) - 1\right] \tag{12}$$

$$\varepsilon = \frac{1}{2}\left[\frac{1}{\alpha_0^2}\left(\frac{\Delta\left(T_{PP}\left(0\right)v_{PP}^2\right)}{\Delta T_{PP}\left(0\right)} + \frac{\Delta\left(T_{SS}\left(0\right)v_{SS}^2\right)}{\Delta T_{SS}\left(0\right)}\right) - \left(1 + \frac{1}{\gamma_0^2}\right)\right].$$

The difference expressions in equation (7) are computed in a layer-recursive fashion starting with N=1 and $T_{PP,0}\left(0\right) = T_{SS,0}\left(0\right) = 0$. The layer parameters are computed next using equation (10) - (12).

Sensitivity analysis

In order to estimate the heterogeneity coefficient S_{PP} it is necessary to have large-offset data. For a simple sensitivity analysis we shall assume that T_{P0}, v_{PP}, T_{S0} and v_{SS} are accurately determined, but S_{PP} (or really $\Delta\left(T_{PP}\left(0\right)v_{PP}^4 S_{PP}\right)$) is poorly determined and study the sensitivity of the estimated parameters with respect to S_{PP}. Note that this heterogeneity coefficient is determined for single layer, not for a stack of layers. For that purpose we use the model from Grechka and Tsvankin (2002) with $\Delta z = 1.0\,\mathrm{km}$, $\alpha_0 = 2.5\,\mathrm{km/s}$, $\beta_0 = 1.0\,\mathrm{km/s}$, $\varepsilon = 0.2$ and $\delta = 0.05$. The v_P / v_S NMO velocity ratio is $\gamma_1 = 1.546$. The correct value for the heterogeneity coefficient is $S_{PP} = 2.11$.

In the Figure we show how the estimated parameters: α_0, β_0, ε, δ and Δz vary as functions of S_{PP}. One can see that for the relatively small errors in S_{PP}, the errors in the estimated parameters are not very large.

Conclusions

For a transversely isotropic medium with vertical symmetry axis consisting of homogeneous horizontal layers there exists explicit formulas for four layer parameters and layer thickness computed from three qPqP traveltime parameters and two qPqSV traveltime parameters. For a layer at depth it is necessary to estimate the traveltime parameters for reflections from the top and bottom of the layer and use differences in traveltime parameter combinations.

Acknowledgements

This research has received financial support from the Norwegian Research Council via the ROSE project. We also thank the reviewers for many instructive comments.

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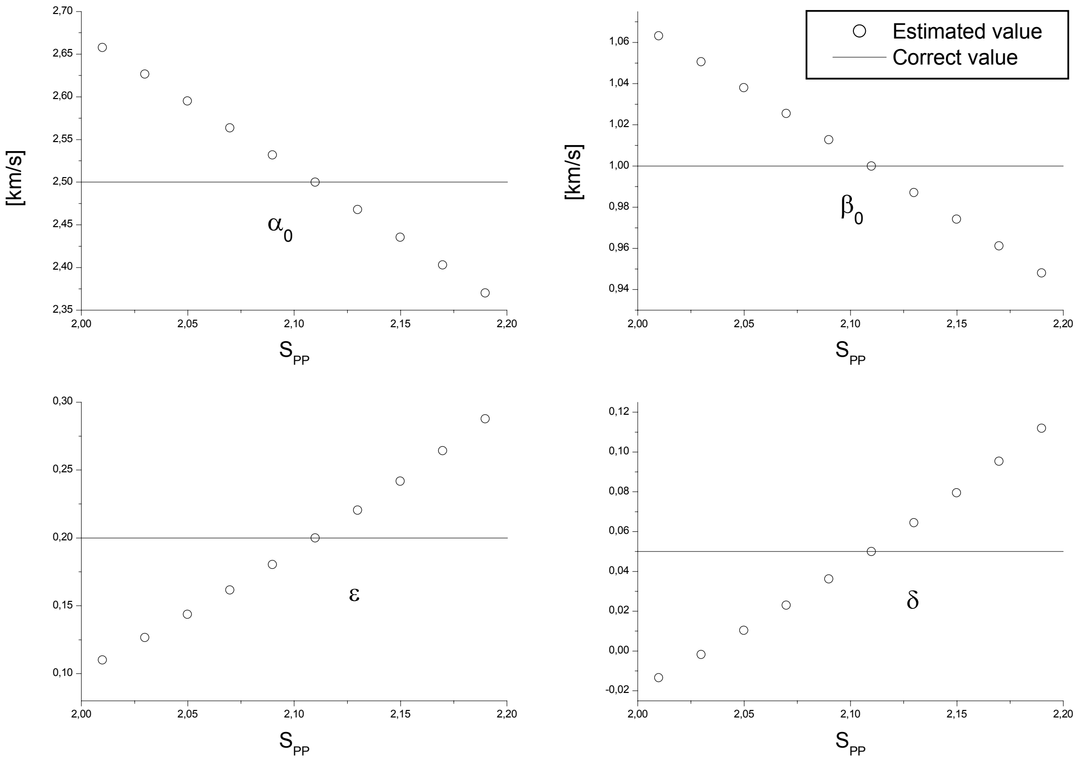

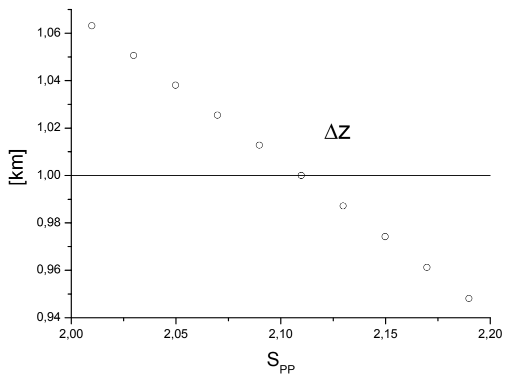

Figure. Error in the estimated parameters due to error in S_{PP} .