ROME 2008
LEVERAGING TECHNOLOGY

P052

# 3D Modeling of Acoustic Green's Function in Layered Media with Diffracting Edges

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## SUMMARY

We present the results of 3D acoustic Green's function modeling using the multiple tip wave superposition method (MTWSM) in a layered medium with interfaces containing diffracting edges. The MTWSM algorithm is based on an explicit representation of the scattered wavefield as the superposition of events multiply reflected/transmitted in accordance to their wavecodes. Each event is computed using high-frequency uniform asymptotic approximations of the Kirchhoff-type surface integral by superposition of tip-diffracted waves and convolution-type reflection and transmission operators in the form of effective coefficients. We show that the analytical technique is capable of reproducing complex wave phenomena such as caustics, edge diffractions and head waves at curved interfaces. We also compare the wavefield modeled by the MTWSM algorithm with that of asymptotic ray theory, diffraction theory and finite-differences

**Introduction.** Current practice shows that analytical seismic modeling based on the Kirchhoff-type surface integral, and a heuristic description of its boundary values using the plane-wave reflection/transmission coefficients or their various regularizations is the best option for preserving the seismic resolution, regarded as a tradeoff between accuracy and computational speed (Ursin, 2004). However such a technique fails to reproduce caustic fields, head waves and near- and post-critical effects caused by the curvature of interfaces and diffracted waves generated by their edges and vertices. Therefore the quality of synthetic seismograms significantly decreases for models with strong-contrast and steeply dipping interfaces. To improve the Kirchhoff approach for piecewise smooth interfaces, we introduced a rigorous description of the boundary values using new reflection/transmission operators of the convolution type. The technique reproduces the complete scattered wavefield in multi-layered media in the form of the superposition of wave events reflected/transmitted at piecewise smooth curved interfaces in accordance to their wavecodes. Moreover, the kinematics and dynamic features of a realistic wavefield are preserved.

**Theory.** We consider scattering of acoustic waves in a 3D layered medium with piecewise smooth curved interfaces. Each layer is described by a velocity $c$ and a density $\rho$. Assume a source of acoustic pressure wave positioned at a point $\mathrm{x}^s$. The total pressure wavefield represents a solution of the acoustic transmission problem, which is described by the acoustic wave equations inside the layers, radiation conditions at the infinity, and the conditions of the continuity of the pressure and normal acceleration at the interfaces. To solve this problem analytically, we rewrite the interface conditions in an equivalent convolution form with the reflection and transmission operators and obtain a modified statement of the acoustic transmission problem in terms of the mathematical wave theory (Aizenberg et al., 2007c).

The direct boundary integral equation method reduces this problem to a modified system of boundary integral equations of the second type. The Neumann iterative method generates its exact analytical solution as a branching set of events multiply reflected/transmitted in accordance with their wavecode (Klem-Musatov et al., 2005). Therefore, the total scattered acoustic pressure wavefield $p\left(\mathrm{x}^r\right)$ recorded at a point $\mathrm{x}^r$ can be represented by the sum,

$$p\left(\mathrm{x}^r\right)=\sum_{i\in\{wavecodes\}} p_i\left(\mathrm{x}^r\right) ,$$

where $p_i\left(\mathrm{x}^r\right)$ is a multiply reflected/transmitted wavefield in accordance with the wavecode $i$, which belongs to the set of all possible wavecodes for the model. Each reflected/transmitted wavefield is represented by a composition of surface integral propagators $\mathrm{P}$ inside layers and reflection/transmission operators $\mathrm{R/T}$ at interface, which acts over the wavefield impinging on a preceding interface (Figure 1). It is essential that such a representation does not experience the problems of convergence and uniqueness, since each event describes its own wavecode and can not be modified by other ones.

The surface integral propagator from an intermediate interface $S$ to any point $\mathrm{x}$, which either belongs to a neighboring interface or coincides with the receiver point $\mathrm{x}^r$, is,

$$\mathrm{P}\left(\mathrm{x},\mathrm{x}'\right)=\iint_S \frac{1}{\rho\left(\mathrm{x}'\right)}\left[\frac{\partial g\left(\mathrm{x},\mathrm{x}'\right)}{\partial n\left(\mathrm{x}'\right)}-g\left(\mathrm{x},\mathrm{x}'\right)\frac{\partial}{\partial n\left(\mathrm{x}'\right)}\right]dS\left(\mathrm{x}'\right) ,$$

where $g\left(\mathrm{x},\mathrm{x}'\right)$ is the Green's function in the layer, and $n\left(\mathrm{x}'\right)$ is the internal normal to $S$. For numerical simulation, a high-frequency uniform asymptotic approximation of the propagators is realized in the form of a multiple tip wave superposition method (MTWSM). The approximation is based on the integrated Green's functions in terms of superposition of tip-diffracted waves (Ayzenberg et al., 2007). The method has proved to be capable of correct reproducing such complex wave phenomena as head waves, caustics and diffractions.

The reflection and transmission operators are,

$$\mathrm{R}(\mathrm{x},\mathrm{s}) = \mathrm{F}^{-1}(\mathrm{x},\mathrm{k})\,\hat{R}(k)\,\mathrm{F}(\mathrm{k},\mathrm{s}), \quad \mathrm{T}(\mathrm{x},\mathrm{s}) = \mathrm{F}^{-1}(\mathrm{x},\mathrm{k})\,\hat{T}(k)\,\mathrm{F}(\mathrm{k},\mathrm{s}) \ ,$$

where $\hat{R}(k)$ and $\hat{T}(k)$ are the plane-wave reflection and transmission coefficients, $\mathrm{F}(\mathrm{k},\mathrm{s})$ is the Fourier transform from the Chebyshev-type curvilinear coordinates $\mathrm{s} = (s^1, s^2)$ on a smooth piece of interface $S$ to the wavenumber plane $\mathrm{k} = (k_1, k_2)$, $\mathrm{F}^{-1}(\mathrm{x},\mathrm{k})$ is the inverse Fourier transform to the point $\mathrm{x}$, and $k = |\mathrm{k}|$ (Aizenberg et al., 2007a). For numerical simulation, a high-frequency uniform asymptotic approximation of the reflection and transmission operators is realized as effective reflection (ERC) and transmission (ETC) coefficients depending on the incidence angle $\gamma$ and additional second argument accounting for the interface and wavefront curvatures and the dominant frequency. Introduction of the ERC and ETC provides correct description of head waves and near- and post-critical phenomena (Ayzenberg et al., 2007). Therefore, they generalize the plane-wave coefficients to arbitrary interface curvature, non-planar waves and frequency content of the wavelet. The ERC for various frequencies $f$ is compared with the plane-wave reflection coefficient (PWRC) in Figure 2.

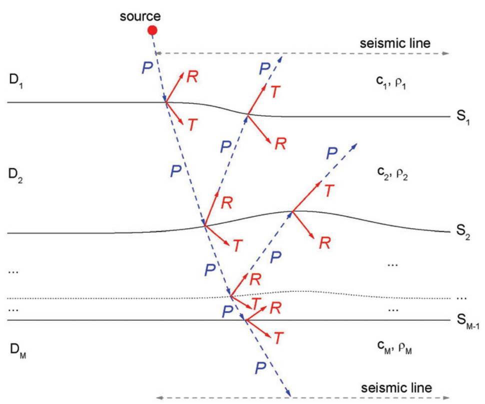


Figure 1: Schematic action of propagators and reflection/transmission operators.

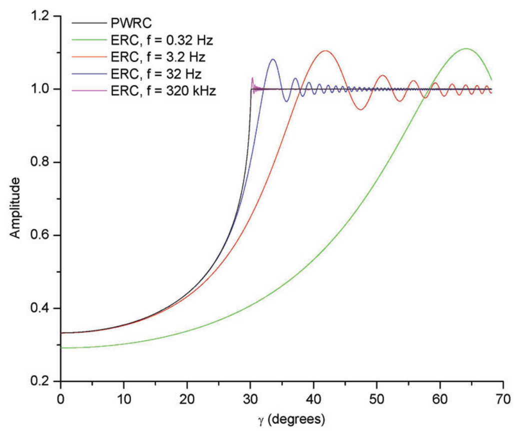


Figure 2: Effective reflection coefficient.

**Modeling.** We ran modeling for a three-layer acoustic model known as the French model (Aizenberg et al., 2007b). The model and its vertical in-line cross-section are shown in Figures 3 and 4. Because of a non-smooth geometry of the top reflector, modeling reproduces strong diffraction events thus allowing testing the ability of a MTWSM algorithm to model complex wave phenomena. Additionally modeling results from a finite-difference (FD) code are available for comparison.

The wavefield C1 in Figure 5 is the superposition of the primary reflections from 5 smooth pieces of the top reflector and 8 single diffractions from two straight and two circular edges of the top reflector. The primary reflections can be described by the asymptotic ray theory, and the single diffractions by the theory of edge and tip diffractions (Klem-Musatov et al., 2008). For example, the amplitude at zero offset estimated using the asymptotic ray theory and the diffraction theory is about 4% higher that the one obtained with the MTWSM code. The wavefield C2 in Figure 6 is the complex superposition of the wavefields reflected from the bottom reflector with intermediate transmission through 5 smooth pieces of the top interface and diffracted wavefield which includes 16 single diffractions on 4 edges and 64 double diffractions on 16 pairs of edges. The wavefield C3 in Figure 7 is the first-order per-leg which is mainly formed by the superposition of the single, double and triple diffractions inside the low-velocity layer, and therefore has quite weak amplitude. Because analytic methods can not handle the complexity of the wavefields C2 and C3, we tried to justify the correctness of the MTWSM algorithm using a FD code. The comparison of the complete MTWSM seismogram (Figure 8) to the FD seismogram showed a good fit, both kinematical and dynamic. Accounting for architectural differences between the supercomputers where the MTWSM and FD codes were run for the French model, the MTWSM code is approximately 3 times faster than the FD code.

**Conclusions.** We presented an analytical technique for the description and modeling of the pressure wavefields in 3D layered acoustic media. The technique describes the total scattered wavefield as the branching sequence of multiply transmitted and diffracted wavefields, which can de directly identified with the reflectors generating them. It was showed that modeling using the MTWSM algorithm with the effective coefficients enables preserving the kinematic and dynamic properties of reflected and diffracted waves.

**Acknowledgements.** The authors acknowledge StatoilHydro (Bergen, Norway) and Russian Foundation for Basic Research (grant 07-05-00671) for support of this study. We thank Lisbeth Engell-Sørensen for the possibility of comparing our results with the FD seismogram.

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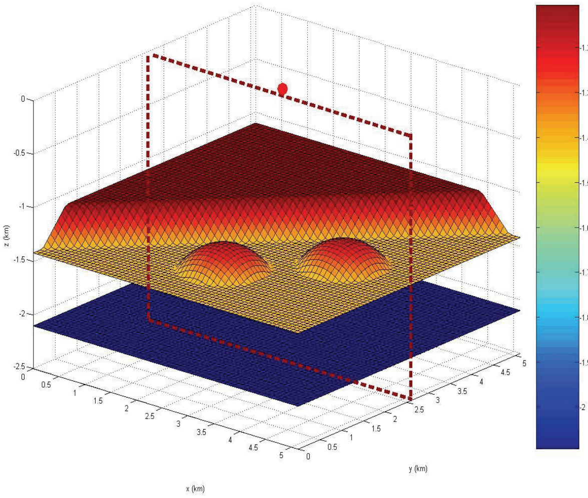


Figure 3: 3D French model.

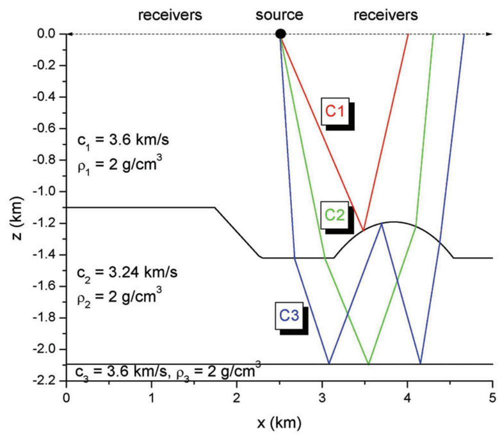


Figure 4: Vertical in-line cross-section.

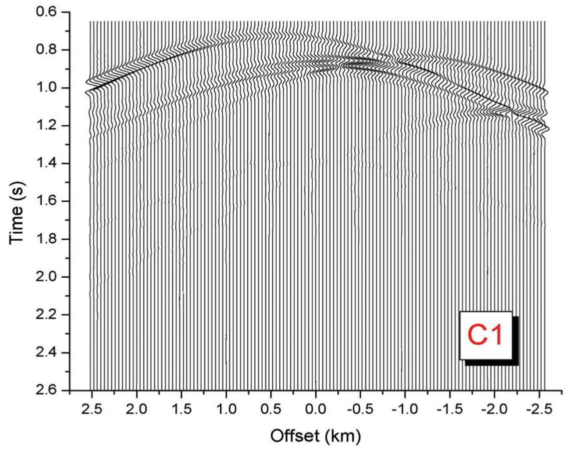


Figure 5: Seismogram for wavecode C1.

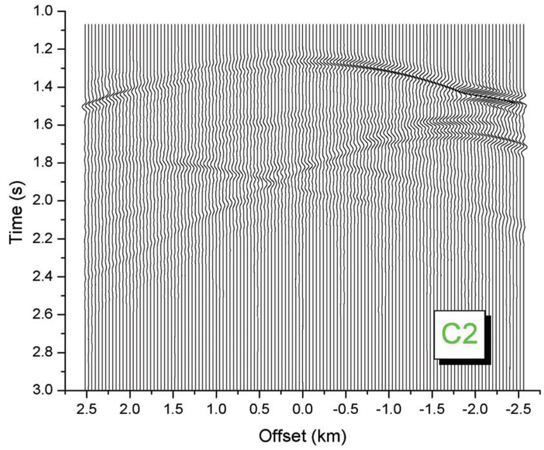


Figure 6: Seismogram for wavecode C2.

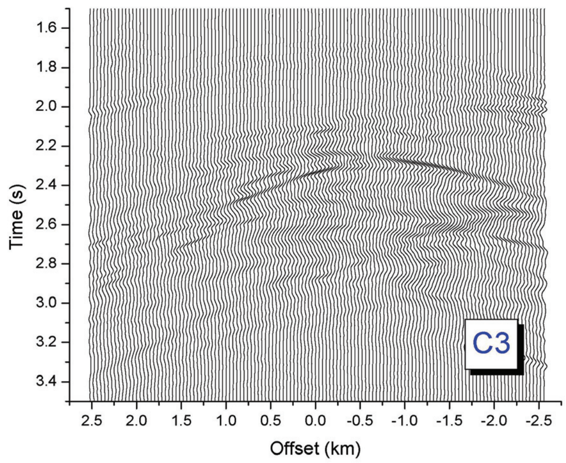


Figure 7: Seismogram for wavecode C3.

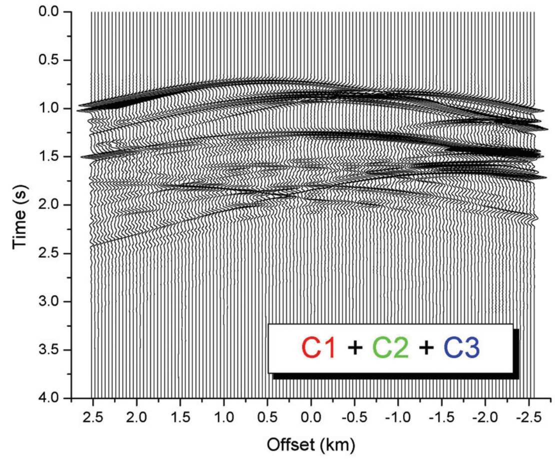


Figure 8: Complete seismogram.