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3D Seismic Diffraction Modeling in Multilayered Media in Terms of Surface Integrals

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SUMMARY

We present an improved multiple tip wave superposition method (MTWSM) for the 3-D seismic modeling in multilayered media in terms of the Kirchhoff type integrals. The main improvement consists in correct description of their boundary values by the reflection/transmission operators. The high-frequency approximation of the operators (effective coefficients) generalizes the plane-wave coefficients used in the conventional seismics and allows adequate reproduction of multiple reflections and transmissions accounting for the nearcritical effects. To show the potential of the MTWSM we give results of the Green's function modeling for a strong-contrast curvilinear interface in two-layered acoustic medium.

Introduction. One of the widely used analytical methods for seismic modeling, inversion and migration of 3-D wave fields in multilayered inhomogeneous media is the Kirchhoff integral method. It is well-known that in any high-frequency realization of the Kirchhoff integral there are two problems: description of the Green's function for inhomogeneous medium and the boundary values of the wave field for curvilinear interfaces. To overcome the first problem, the TWSM was introduced as a high-frequency approximation of the Kirchhoff integral in a form invariant to inhomogeneity of the medium (Aizenberg et al., 1996). Regarding the second problem, it is common to represent the boundary values as multiplication of incident wavefield and the plane-wave coefficient which have irregularities at critical angles. However such an approach causes artifacts in the form of edge waves diffracted at the contours of critical points on the interface (Kampfmann, 1988; Wenzel et al., 1990; Sen and Frazer, 1991). To overcome this problem, the exact description of the boundary values in terms of integral reflection and transmission operators was introduced (Klem-Musatov et al., 2004; Klem-Musatov et al., 2005). A high-frequency approximation of these operators in the form of effective coefficients (Aizenberg et al., 2004; Aizenberg et al., 2005) allows elimination of the diffraction artifacts and accounting for the head waves.

Kirchhoff integral propagators. Assume the source wavefield be m times scattered at the system of interfaces in multilayered medium. Let the interfaces, sequentially be intersected by the wave in accordance to the wave code, be numbered $S_1,...,S_m$. Note that the same interface may be intersected several times. The scattered wavefield can be expressed by the Kirchhoff integral (Klem-Musatov et al., 2005)

$$p^{(m)}(\mathrm{x}) = \iint_{S_m} \left[\frac{1}{\rho_m(\mathrm{x}')} \frac{\partial g_m(\mathrm{x},\mathrm{x}')}{\partial n_m} b^{(m)}(\mathrm{x}') - g_m(\mathrm{x},\mathrm{x}') \frac{1}{\rho_m(\mathrm{x}')} \frac{\partial b^{(m)}(\mathrm{x}')}{\partial n_m} \right] dS_m \ ,$$

where $g_m(\mathrm{x},\mathrm{x}')$ is the Green's function for the m-th layer with the absolutely absorbing interfaces (Klem-Musatov et al., 2004) and $\rho_m(\mathrm{x}')$ is the density. The boundary values $b^{(m)}(\mathrm{x}')$ and $\partial b^{(m)}(\mathrm{x}')/\partial n_m$ describe the wave field locally reflected/transmitted at the point x' of the interface S_m and its derivative. The wavefield $p^{(m)}(\mathrm{x})$ represents superposition of the wave fields coming from various reflection/transmission points. The limiting value of integral $p^{(m)}(\mathrm{x}')$ does not in general coincide with the boundary value $b^{(m)}(\mathrm{x}')$.

Integral reflection and transmission operators. It was shown in (Klem-Musatov et al., 2005) that the boundary values $b^{(m)}(\mathrm{x}')$ and $\partial b^{(m)}(\mathrm{x}')/\partial n_m$ at any chosen point x' of the interface S_m can be exactly expressed through the traces of the wavefield $p^{(m-1)}(\mathrm{x}'')$ and its normal derivative $\partial p^{(m-1)}(\mathrm{x}'')/\partial n_m$ as the reflection/transmission transform

$$b^{(m)}(\mathrm{x}') = K_{m,m-1}(\mathrm{x}',\mathrm{x}'')\, p^{(m-1)}(\mathrm{x}''), \qquad \frac{\partial b^{(m)}(\mathrm{x}')}{\partial n_m} = -\,K_{m,m-1}(\mathrm{x}',\mathrm{x}'') \frac{\rho_m(\mathrm{x}')}{\rho_{m-1}(\mathrm{x}'')} \frac{\partial p^{(m-1)}(\mathrm{x}'')}{\partial n_m},$$

using the reflection and transmission operators

$$K_{m,m-1}(\mathrm{x}',\mathrm{x}'') = \mathbb{F}^{-1}(\mathrm{x}',\zeta)\, k_{m,m-1}(\zeta)\, \mathbb{F}(\zeta,\mathrm{x}'').$$

The space Fourier operator $\mathbb{F}(\zeta,\mathrm{x}'')$ transforms the traces $p^{(m-1)}(\mathrm{x}'')$ and $\partial p^{(m-1)}(\mathrm{x}'')/\partial n_m$ from the curvilinear interface coordinates $\mathrm{x}'' = (x_1'', x_2'')$ into the wave number domain $\zeta = (\zeta_1, \zeta_2)$. Coefficient $k_{m,m-1}(\zeta)$ is similar to the plane-wave reflection or transmission coefficient.

Effective reflection and transmission coefficients. In the high-frequency approximation the action of the reflection/transmission operators $K_{m,m-1}(\mathrm{x}',\mathrm{x}'')$ can be substituted by the multiplication with the corresponding effective coefficients $\chi_{m,m-1}(\mathrm{x}')$ (Aizenberg et al., 2004; Aizenberg et al., 2005). Therefore the reflection/transmission transform can be represented as

$$b^{(m)}(\mathrm{x}') \simeq \chi_{m,m-1}(\mathrm{x}')\, p^{(m-1)}(\mathrm{x}'), \quad \frac{\partial b^{(m)}(\mathrm{x}')}{\partial n_m} \simeq -\chi_{m,m-1}(\mathrm{x}')\frac{\rho_m(\mathrm{x}')}{\rho_{m-1}(\mathrm{x}')}\frac{\partial p^{(m-1)}(\mathrm{x}')}{\partial n_m}.$$

The effective coefficients represent a generalization of the plane-wave coefficients to non-planar waves, arbitrary interface curvature and finite dominant frequency of the wavelet. In the exceptional case of a plane incident wave and plane interface the effective and plane-wave coefficients coincide.

Modeling of the Green's function for two-layered medium. The Green's function is the basic analytical tool used in various algorithms of solving inverse problems such as parameter inversion, angle migration etc. (Ursin, 2004). It is usually assumed that multilayered overburden could be approximated by some smooth inhomogeneous medium. In the case of strong-contrast internal interfaces and/or steeply dipping interfaces this approach may lead to decreasing of the seismic image resolution. To account for such type of interfaces it is natural to use the Green's function for multilayered medium instead of the free-space Green's function. Using the Kirchhoff integral propagators together with the reflection/transmission operators allows modeling of the Green's function for a multilayered overburden. In the numerical example given below we demonstrate modeling of the Green's function for two-layered overburden using the single transmission approximation. Numerical simulation is done with help of the MTWSM algorithm. For comparison we show synthetic seismograms obtained both with the effective and the plane-wave transmission coefficients.

The 2-D section of the two-layered 3-D overburden, source and data window are shown in Figure 1a. Figure 1b displays modulus of the effective transmission coefficient (bold line) for the dominant linear frequency $30\,Hz$ as a function of the horizontal coordinate. Modulus of the plane-wave transmission coefficient (normal line) is given on the same figure. The seismograms (Figures 1c and 1e) present the Green's function computed using the plane-wave and effective coefficients respectively. On the seismograms we highlight the traces corresponding to the offset $2.0\,km$ and show them separately in Figures 1d and 1f respectively. In comparison to the second seismogram (Figure 1e), the first seismogram (Figure 1c) is distorted by the artificial diffraction caused by the curves of the critical transmission points with radii $0.192\,km, 0.288\,km, 0.577\,km$ (Figure 1b). These critical curves coincide with the discontinuities of the first derivatives of the plane-wave coefficient. The artifacts are clearly visible on the zero-offset traces. To demonstrate that the amplitudes and forms of the wavelet are also essentially different for the other traces, we give the highlighted traces in Figures 1d and 1f. From the figures it is seen that the plane-wave coefficient produces the phase-shift of $\pi/2$ and the three-fold increase of the amplitude. These differences can be explained by the fact that the Fresnel zone strongly influencing the data window and belonging to the anticline part of the interface contains the local perturbation of the plane-wave coefficient in comparison to the effective coefficient between $0.192\,km$ and $0.288\,km$. As a result this perturbation is spread over all the seismogram in Figure 1c. Similar amplitude effect can be expected when modeling of the Green's function using the asymptotic ray theory. The effect can be even stronger due to absence of the diffraction smoothing of the wavefield in comparison to the Kirchhoff integral approach.

Conclusions. We have presented the improved MTWSM algorithm for the 3-D seismic diffraction modeling in multilayered media. On the numerical example we have demonstrated that the algorithm provides an adequate representation of the acoustic Green's function for the two-layered medium.

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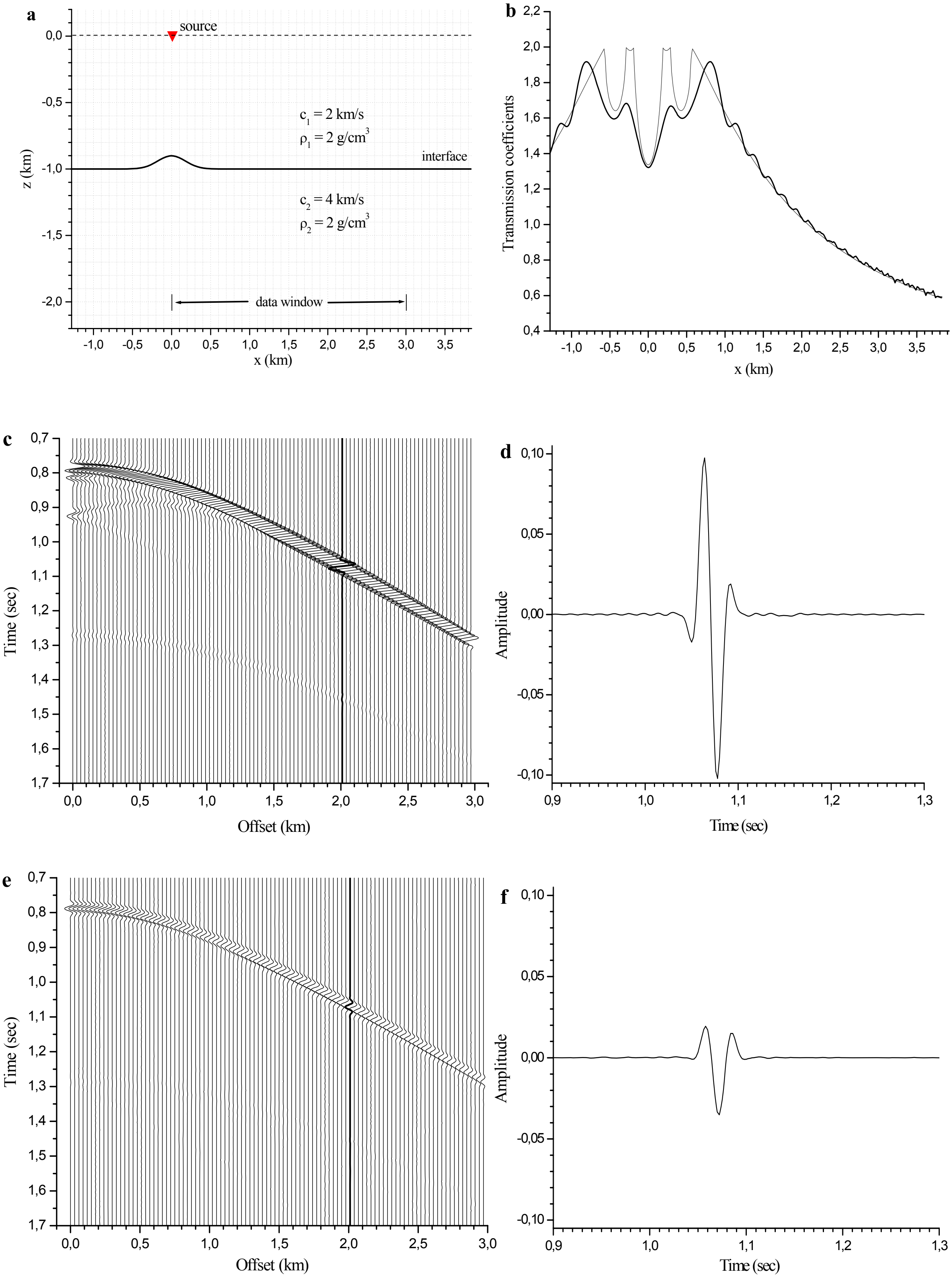

Figure 1: *Green's function for two-layered medium.*