Seismic-Velocity / Electrical-Conductivity Relations

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Summary

Cross-property relations are useful when some rock properties can be more easily measured than other properties. Relations between electrical conductivity and seismic velocity, stiffness moduli and density can be obtained by expressing the porosity in terms of those properties. There are many possibilities to combine the constitutive equations to obtain a relation, each one representing a given type of rock, depending on the assumptions to obtain the involved constitutive equations. Mainly, these involve Archie's law and its modifications for a conducting frame, the Hashin-Shtrikman bounds, the self-similar and CRIM (complex refraction index) models in the electromagnetic case, and the time-average, the Hashin-Shtrikman bounds, and Gassmann equation in the elastic case. Also, expressions for dry rocks and for anisotropic media, using Backus averaging, are analyzed. The relations are applied to the a shale saturated with brine (overburden) and to a sandstone saturated with oil (reservoir).

Introduction

Electrical, seismic and electromagnetic methods can be used for non-invasive determination of subsurface physical and chemical properties. Seismic measurements provide wave velocities and attenuations, which can be translated to stiffness and quality factor, while electromagnetic data provide electromagnetic velocity and attenuation, which can be translated to dielectric constant and electrical conductivity. Most of the relevant properties for the oil-exploration problem are represented by electrical conductivity and wave velocity. Hence, it is important to obtain relationships between these physical quantities and overburden and reservoir composition and fluid type.

The use of mixture theories is essential to obtain the conductivities and the velocities (e.g., Schön, 1996; Carcione, 2007). We first establish the different constitutive equations. Mainly, the electromagnetic theories involve Archie's law (Archie, 1942) and its modifications for a conducting frame, the Hashin-Shtrikman bounds, and the self-similar and CRIM models; the main elastic models are the time average equation, the Hashin-Shtrikman bounds and Gassmann equation. Thus, we develop new theories relating the electrical conductivity and the seismic velocity, i.e, knowing the conductivity, the P-wave velocity can be obtained, and vice versa. This is important in the sense that if one property, e.g., electrical conductivity, can be more easily measured than seismic velocity, the latter can be obtained by using a cross-property relation. The importance of such relations has been pointed out by Berryman and Milton (1988) and Gibiansky and Torquato (1995).

Relations between various effective properties have been investigated in several works, starting from the classical paper of Bristow (1960), in which an explicit connection between the conductivity and the elastic moduli of a solid with cracks is derived, to the works by Berryman and Milton (1988), Gibiansky and Torquato (1995, 1996a,b) and Kachanov et al. (2001). The problem of electrical conductivity is mathematically equivalent to the ones of thermal conductivity, dielectric and magnetic permeabilities or diffusion coefficients; therefore, the approach can be applied to the mentioned physical properties as well.

Examples of relations to interpret logging data are given in Brito Dos Santos et al. (1988), who use a self-similar model for the conductivity and the time-average equation for the seismic velocity, and in

Hacikoylu et al. (2006), who use the lower Hashin-Shtrikman bound for the resistivity and Raymer's equation for the seismic velocity.

The general approach to establish various cross-property relations is outlined by Milton (1997); see also the recent review of Markov (1999). We consider the electrical conductivity/seismic velocity relations for the overburden (shales) and the reservoir (sandstone). The common property allowing us to establish the relations is the porosity. One may use different mixtures theories to obtain the electromagnetic and seismic properties, and then combine these theories in differents ways; for instance, Archie's law with the time-average equation, or the CRIM with the time-average equation are two choices. Or one may combine Gassmann equation with the different electromagnetic constitutive equations. Other possibilites involve the HS bounds, the self-similar equation, etc.

Methodology

The key property to relate the electrical conductivity to the P-wave velocity (or to the stiffness) is the porosity. Assume that the conductivity and velocity have the form

$$\sigma = f(\phi), \quad \text{and} \quad v = g(\phi),$$

where ϕ is the porosity. Then, the relation is given by

$$\sigma = f[g^{-1}(v)].$$

This simple 1D concept is quite general and can be applied to higher spatial dimensions and the case of anisotropy.

Examples

The following examples show combinations of electromagnetic and elastic constitutive equations. Archie denotes Archie’s law, HS(-)(+) the Hashin-Shtrikmann lower and upper bounds, CRIM the complex refraction index equation, etc. For more details, see Carcione and Ursin (2006).

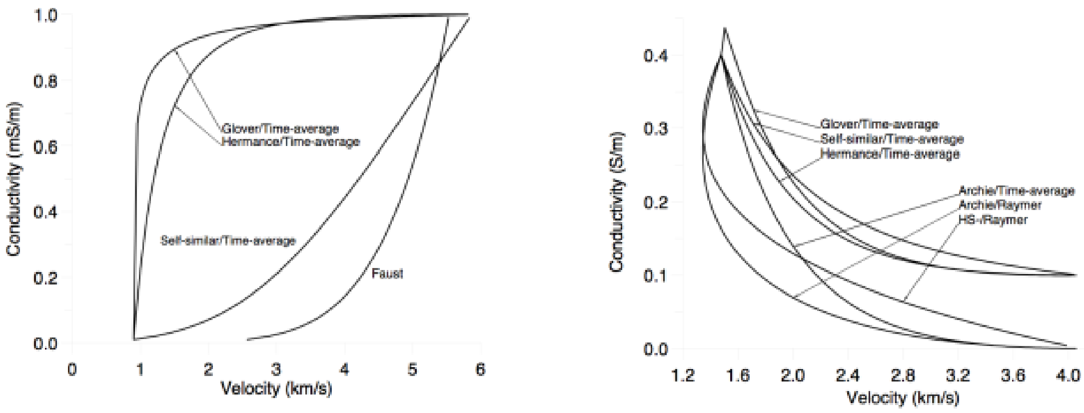

Figure 1. Left: Cross-property relations for different models of the overburden (shale saturated with brine). Right: Cross-property relations for different models of the reservoir (sandstone saturated with oil). The Faust curve corresponds to 2 km depth.

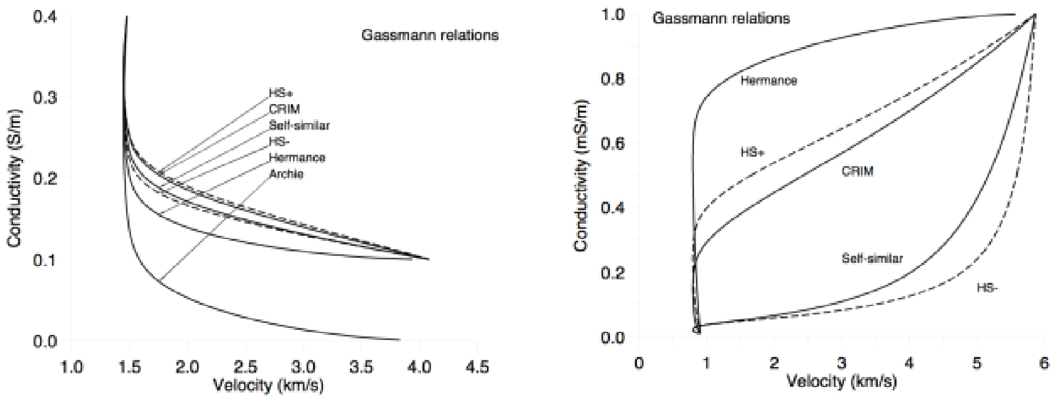

Figure 2. Left: Cross-property relations for different conductivity models of the overburden (shale saturated with brine), combined with Gassmann equation. The dashed lines correspond to the HS bounds. Right: Cross-property relations for different conductivity models of the reservoir (sandstone saturated with oil), combined with Gassmann equation. The dashed lines correspond to the HS bounds.

Conclusions

Cross-property relations are obtained between the electrical conductivity and seismic velocity using different combinations of the electromagnetic and elastic models. If the Hashin-Shtrikman bounds represent realistic bounds, then, the CRIM and self-similar electromagnetic models are the best choice to obtain reliable cross-property relations. The elastic models are within the bounds generally. The Krief model is used to obtain Gassmann (wet-rock) modulus and calculate the seismic velocity. Gassmann's based relations give tight curves for the overburden, while the curves are more dissimilar for the reservoir. The trend is that as the velocity in the overburden (shale saturated with high-conductivity brine) increases, the conductivity decreases. The opposite behavior is obtained for the sandstone (saturated with high-resistivity oil).

The different relations have to be tested with controlled real data, such as laboratory experiments on synthetic and real rocks. To our knowledge, there seems to be a complete lack of these type of data for rocks. The next stage of the research is to perform such tests and determine the more reliable cross-property relations.

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