Nonlinear Bayesian joint inversion of seismic reflection coefficients

Tor Erik Rabben, Håkon Tjelmeland, and Bjørn Ursin

February 21, 2007

Abstract

Inversion of the seismic reflection coefficients are formulated in a Bayesian framework. Measured reflection coefficients and model parameters are assigned statistical distributions based on information known prior to the inversion, and together with the forward model uncertainties can be propagated into the final result. A quadratic approximation to the Zoeppritz equations is used as the forward model and compared with the linear approximation the bias is reduced. The differences when using the quadratic approximations and the exact expressions are minor. Joint inversion using information from both reflected PP-waves and converted PS-waves yield smaller bias compared to using only reflected PP-waves. The solution algorithm is sampling based and because of the nonlinear forward model the Metropolis-Hastings algorithm is used.

Introduction

The seismic reflection coefficients contain information about elastic parameters in the subsurface. In an amplitude versus angle (AVA) inversion, the main objective is to estimate elastic parameters from the reflection coefficients. Our forward model linking the elastic parameters to the reflection seismic are the quadratic approximations to the Zoeppritz equations Stovas and Ursin (2003). We formulate the inversion in a Bayesian framework following Buland and Omre (2003) and test both PP and joint PP and PS inversion, and because of the nonlinear model we use the Metropolis-Hastings algorithm from Tjelmeland and Eidsvik (2005).

Model

The parametrization we are using for the reflection coefficients is in P-wave impedance, S-wave impedance, and density. Stovas and Ursin (2003) derived implicit second order expressions for reflections between two transversely isotropic media. Explicit expressions for PP and PS-reflections simplified for two isotropic media, read

$$
\begin{aligned}
r_{\mathrm{PP}} = {} & \frac{1}{2\cos^2\theta_p}\frac{\Delta I_\alpha}{\bar{I}_\alpha} - 4\sin^2\theta_s\frac{\Delta I_\beta}{\bar{I}_\beta} - \frac{1}{2}\tan^2\theta_p\left(1-4\gamma^2\cos^2\theta_p\right)\frac{\Delta\rho}{\bar{\rho}} \\
& + \tan\theta_p\tan\theta_s\Bigg[4\gamma^2(1-(1+\gamma^2)\sin^2\theta_p)\left(\frac{\Delta I_\beta}{\bar{I}_\beta}\right)^2 \\
& - 4\gamma^2(1-(\tfrac{3}{2}+\gamma^2)\sin^2\theta_p)\left(\frac{\Delta I_\beta}{\bar{I}_\beta}\frac{\Delta\rho}{\bar{\rho}}\right) \\
& + \left(\gamma^2(1-(2+\gamma^2)\sin^2\theta_p) - \frac{1}{4}\right)\left(\frac{\Delta\rho}{\bar{\rho}}\right)^2\Bigg]
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
r_{\mathrm{PS}} = {} & \sqrt{\tan\theta_p\tan\theta_s}\Bigg\{\left[(1-\cos\theta_s(\cos\theta_s+\gamma\cos\theta_p))\left(2\frac{\Delta I_\beta}{\bar{I}_\beta} - \frac{\Delta\rho}{\bar{\rho}}\right) - \frac{1}{2}\frac{\Delta\rho}{\bar{\rho}}\right] \\
& + \frac{1}{2}\left[(1-\cos\theta_s(\cos\theta_s-\gamma\cos\theta_p))\left(2\frac{\Delta I_\beta}{\bar{I}_\beta} - \frac{\Delta\rho}{\bar{\rho}}\right) - \frac{1}{2}\frac{\Delta\rho}{\bar{\rho}}\right] \\
& \times\Bigg[\frac{1}{2\cos^2\theta_p}\frac{\Delta I_\alpha}{\bar{I}_\alpha} + \left(\frac{1}{2\cos^2\theta_s} - 8\sin^2\theta_s\right)\frac{\Delta I_\beta}{\bar{I}_\beta} \\
& + \left(4\sin^2\theta_s - \frac{1}{2}(\tan^2\theta_p + \tan^2\theta_s)\right)\frac{\Delta\rho}{\bar{\rho}}\Bigg]\Bigg\}
\end{aligned}
\tag{2}
$$

where $\gamma = \bar{\beta}/\bar{\alpha}$ is the background v_S/v_P-ratio, θ_p is the angle of the incoming P-wave (and also the reflected P-wave because of isotropic medium) and θ_s is the angle of the reflected S-wave. I_α is P-wave impedance, I_β is S-wave impedance and ρ is density. The parameters are collected in $\mathbf{m}$ and the measured reflection coefficients in $\mathbf{d}$, both defined over a two dimensional grid

$$\mathbf{m} = \{m_{ij} \in \mathbb{R}^{D_m}; \quad i = 1..n_y, j = 1..n_x\} \tag{3}$$

$$\mathbf{d} = \{d_{ij} \in \mathbb{R}^{D_d}; \quad i = 1..n_y, j = 1..n_x\}. \tag{4}$$

The forward model is the link from $\mathbf{m}$ to $\mathbf{d}$. Our focus will be on the quadratic approximations written

$$\mathbf{d} = f(\mathbf{m}) + \mathbf{e}, \tag{5}$$

where f is (1) and (2) in the case of both PP and PS reflections, but we will use both the exact and linearised Zoeppritz equations as references.

In our Bayesian formulation the prior and likelihood are assumed to be multivariate normal distributed

$$\begin{aligned}\pi(\mathbf{m}|\sigma_m^2) &= \mathcal{N}(\mathbf{m};\boldsymbol{\mu}_m,\sigma_m^2\boldsymbol{\Sigma}_m)\\ \pi(\mathbf{d}|\mathbf{m},\sigma_e^2) &= \mathcal{N}(\mathbf{d};f(\mathbf{m}),\sigma_e^2\boldsymbol{\Sigma}_e)\end{aligned} \tag{6}$$

The latter is a result of the assumption that the noise is multivariate normal distributed with expectation 0 and variance $\sigma_e^2\boldsymbol{\Sigma}_e$. Using Bayes rule we find the posterior distribution

$$\pi(\mathbf{m}|\mathbf{d},\sigma_e^2,\sigma_m^2) \propto \pi(\mathbf{d}|\mathbf{m},\sigma_e^2)\,\pi(\mathbf{m}|\sigma_m^2). \tag{7}$$

Inversion algorithm

To sample from the posterior is impossible because of the nonlinear forward model. We will therefore use the Metropolis-Hastings algorithm which consists of two steps: (i) propose a new sample, and (ii) accept the sample with a probability p. The proposal distribution in the first step will be using the posterior with the linearised forward model. Together with $\mathbf{m}$ we also need to generate samples from σ_e^2 and σ_m^2, the details can be found in Buland and Omre (2003).

Numerical example

We will use a synthetic model to test the inversion algorithm. From a chosen true $\mathbf{m}$ we can use the exact Zoeppritz equations to generate synthetic measurements $\mathbf{d}$ by assuming the P-wave velocity in the upper medium and the background v_P/v_S ratio to be known. Our true $\mathbf{m}$ is defined on the grid in (3) and are ranging from 0.2 to 0.5. In Figs 1 and 2 we show the synthetic data $\mathbf{d}$ generated from the true $\mathbf{m}$ together with the bias in the linear and quadratic approximations for PP and PS data respectively. It is clear how superior the nonlinear approximations is.

In Fig. 3 we see the bias in the posterior $\mathbf{m}$ when using $\mathbf{d}$ containing only the PP data. For the P-wave impedance parameter the bias is very low and there are almost no differences between the three forward models, but for the two other parameters we see that the two nonlinear models yield slightly better results than the linear model. However, for the joint PP and PS inversion in Fig. 4 we see that using a nonlinear model greatly reduces the bias for S-wave impedance and density. For the P-wave impedance the bias is as in the PP case.

Conclusion

In our approach to inversion of reflection coefficients we have formulated the problem in a Bayesian framework. We have also used new quadratic approximations to the Zoeppritz equations and compared them with both the linearised and exact equations. The quadratic approximations yield lower bias compared with the linear ones, especially when performing joint inversion, while the difference between the quadratic and exact are minor.

Acknowledgement

We wish to thank BP, Hydro, Schlumberger, Statoil, and The Research Council of Norway for their support through the Uncertainty in Reservoir Evaluation (URE) project.

References

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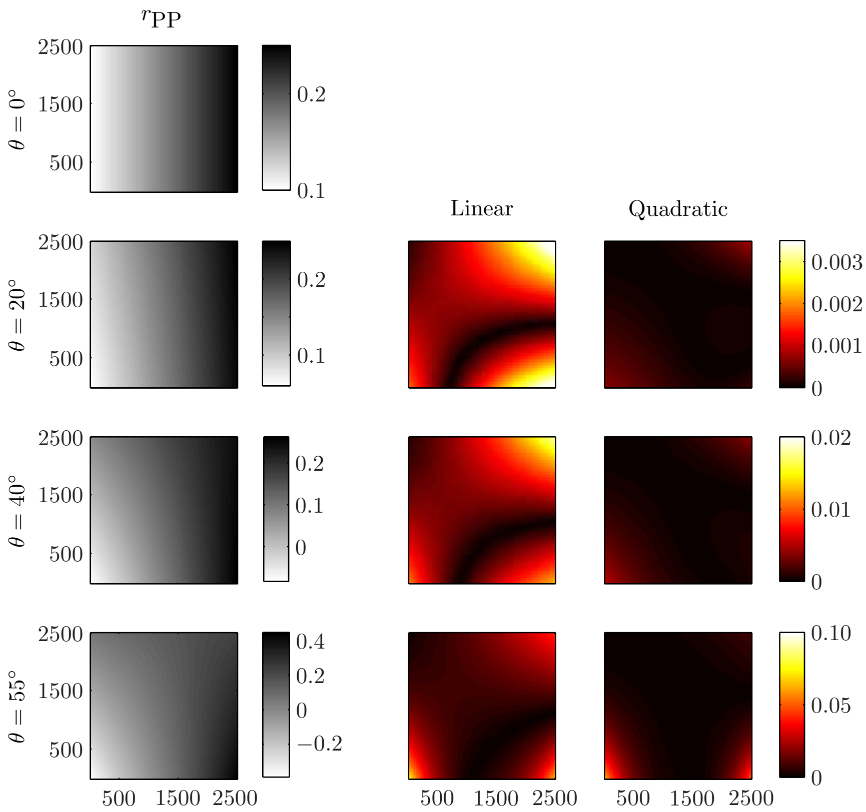

Figure 1: To the left is PP reflection coefficients from the Zoeppritz model for 4 different incidence angles. The two right columns show the bias in the linear and quadratic approximations, relative to the Zoeppritz model, for the nonzero angles. For $\theta = 0°$ the bias is zero.

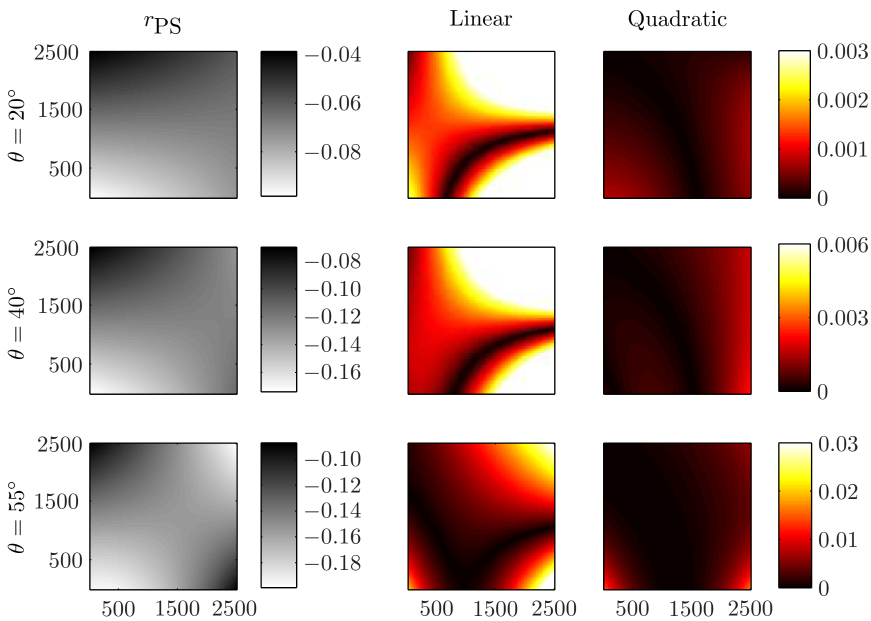

Figure 2: To the left is PS reflection coefficients from the Zoeppritz model for 3 nonzero incidence angles. The two right columns show the bias in the linear and quadratic approximations, relative to the Zoeppritz model.

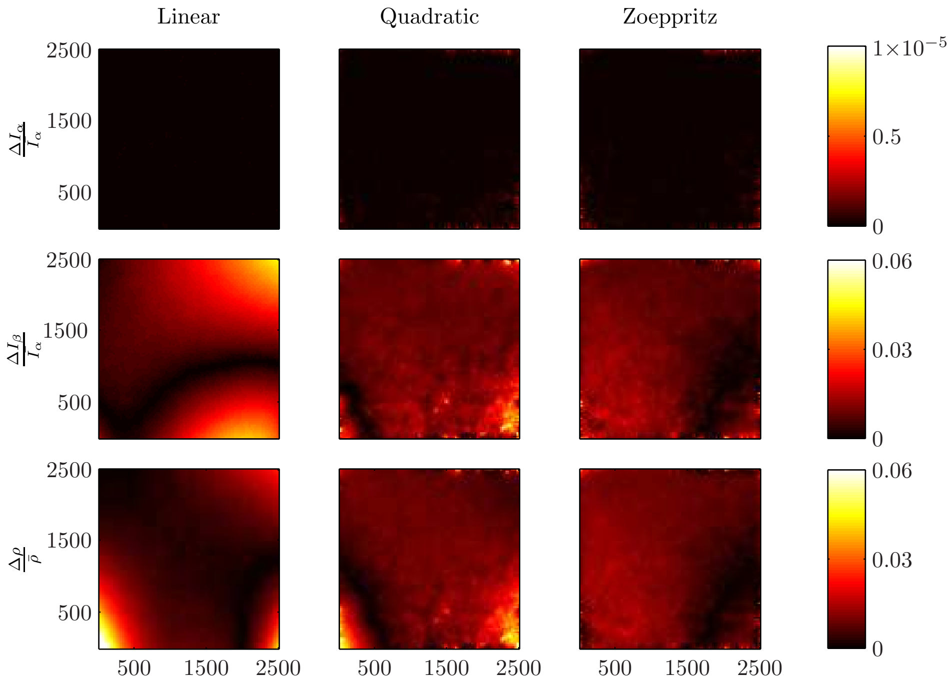

Figure 3: Bias in the posterior distribution of $\mathbf{m}$ from PP inversion. Each column is the result of three inversions using three different models; linear, quadratic, and exact Zoeppritz. The rows displays the three different parameters of $\mathbf{m}$; contrasts in P-wave impedance, S-wave impedance, and density.

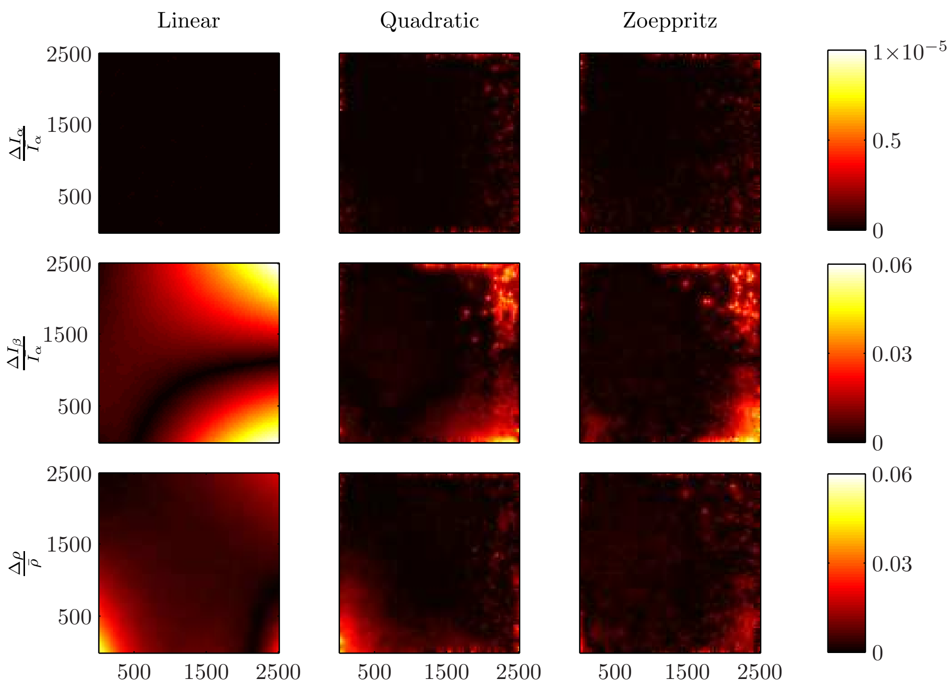

Figure 4: Bias in the posterior distribution of $\mathbf{m}$ from joint PP and PS inversion.