GEOMETRIC ANALYSIS OF DATA-DRIVEN INVERSION/DEPTH IMAGING

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ABSTRACT

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Based on simple geometric analysis of a one-dimensional acoustic piecewise-constant layered medium, this paper presents a methodology that provides a new perspective on how the actual velocity potential (actual velocity and layer interfaces) can be estimated from single scattering data. The non-linear inversion method requires no information of the velocities of the subsurface except for the velocity of the uppermost layer which is the reference medium where the source and receiver are situated at finite distance above the scattering medium.

The inversion scheme consists of three steps. In step one the "Born potential" depth profile is computed by constant-velocity imaging of the single scattering data. Generally, interfaces in the Born potential are severely mislocated in depth compared to those of the true velocity potential. In step two, a "squeezed" velocity potential is estimated by amplitude-scaling the Born potential profile. Step three estimates the actual depth-dependent velocity potential by stretching the squeezed potential so that its interfaces are moved towards the correct depth. Contrary to conventional, velocity-dependent depth migration, which requires an accurate estimate of the velocities of the actual medium to obtain a proper image, the depth imaging in step three requires the squeezed velocity potential with interfaces matching those of the Born potential depth profile. This is exactly the velocity potential that is estimated in step two.

The step of forward modeling that is generally encountered in non-linear inversion schemes is not required. Furthermore, the three inversion steps require no information of the medium other than that provided by the Born potential. In this respect, the inverse solution for the actual velocity potential is fully data-driven.

In a series of papers related to the inverse scattering series Weglein and coworkers describe a general approach to the problem of inversely reconstructing the medium ("scattering potential") from reflection seismic data without introducing a priori information about the subsurface [Weglein et al. (2000, 2002, 2003), Innanen (2003), Innanen and Weglein (2003), Shaw et al. (2004), Shaw (2005), Zhang and Weglein (2005), Zhang et al. (2005), and Liu et al. (2005a, 2005b)]. The scattering potential to be recovered is expanded in a series, each term of which is determined in terms of the scattering data and a reference Green's function. Further, Weglein and coworkers show the validity of the concept of "subseries" within the expansion of the potential, where each subseries is associated with a specific inversion task that can be carried out separately. The four subseries associated with the inversion process are: (1) Free-surface multiple removal, (2) internal multiple attenuation or elimination, (3) spatial location of reflectors in the subsurface, and (4) identification of changes in medium properties across reflectors. The two last subseries work on primaries (multiple-free data). These two subseries have been cast by Innanen (2003) and Shaw (2005) as algorithms that, through specific non-linear operations acting on the constant-velocity migrated data, create a map to the imaged and/or inverted output. The primaries thus are those reflection events that are used to determine the locations and properties (contrast in wave velocities) of the reflector that caused their observation (recording at the surface).

The seismic theories based on the inverse scattering series hold a promising potential to revitalize among others amplitude versus angle (AVA) analysis and depth imaging. During the last few years, other significant work with the same objective has been published. Berkhout and Verschuur (2001) introduce operator-driven common-focus-point (CFP) migration which is considered to be a generalized approach to seismic depth imaging. Another new approach to automated seismic imaging is that based on a modified Lippmann-Schwinger equation proposed by Ikelle (2005), which combines the demultiple process, velocity estimation and imaging in one single algorithm.

Inverse scattering problems are concerned with differential equations in unbounded media. The full inverse scattering problem of seismic, viewed as a three-dimensional (3D) problem, is one of the most challenging problems of geophysics and mathematical physics. In this paper, therefore, influenced by the leading-order closed-form one-dimensional (1D) depth imaging algorithm presented in this journal by Shaw et al. (2003), and the 1D imaging/amplitude inversion algorithm described by Innanen (2003), we shall be concerned with 1D inverse acoustic single scattering from layered media. Applying simple analysis to the geometry of a piecewise-constant medium (which is straight lines parallel to the depth axis in the coordinates depth and velocity), our objective is to demonstrate how the actual velocity potential of the medium can be precisely estimated, both with respect to layer velocities and depth location of interfaces, from the Born potential associated with constant-velocity imaging of single-scattered data.

In a broad sense, the process by which an object is excited to radiate an outgoing field due to the incoming field of an external source of acoustic, elastic or electromagnetic radiation is called scattering. By this definition, reflection, refraction, and diffraction of acoustic, elastic, and electromagnetic waves are subsumed under scattering. For the 1D acoustic piecewise-constant medium that we consider here, the terms "primary reflection" and "single scattering" are synonymous. Primary reflection is an expression used in the field of reflection seismic to indicate a reflection event which has been reflected only once and hence is not a multiple. In the field of scattering the same process is most often referred to as single scattering, but is sometimes also referred to as primary scattering.

Note that the inverse scattering problem to be analyzed in the present paper is limited to that of processing single scattering events. As any data from a layered medium will contain both primaries and multiples, data have to go through a preprocessing step to remove all types of multiples before applying the proposed inversion/depth imaging steps to be presented. The preprocessing is in agreement with the standard practice to seek to attenuate all multiples from the seismic data before using primaries for imaging changes in the medium's properties. On its own, the single scattering model in a layered medium is an idealization realized only when the incident wave from the distant source is scattered only once at each interface in the medium. One therefore could be lead to think that the single scattering model should only be used when the medium changes slowly with depth with no strong discontinuities of the medium parameters. However, the realization is within practical reach with the use of new methods which have been developed to attenuate and eliminate internal multiples [Weglein et al. (1997), and Ramirez and Weglein (2005)].

We make four remarks. First, the complete mathematics related to our new inverse solution is published elsewhere (Amundsen et al., 2005). In the present paper we derive the three-step inverse solution in a painless manner by studying the relationship between the actual velocity potential (which here characterizes the difference between a constant-velocity reference medium and the actual medium) and the Born potential which is obtained by imaging in the constant-velocity reference medium. When the actual medium is piecewise-constant, the relationship is determined by its simple geometry, which is a set of straight lines in the depth-velocity coordinates. To distinguish the procedure proposed in this paper could become a benchmark algorithm for new methods to be developed in the field of non-iterative data-driven or velocity-independent acoustic (seismic) imaging of primaries from 1D layered media. Any new methodology can straightforwardly be benchmarked towards the one proposed here.

Second, even though we restrict the inverse scattering analysis to 1D acoustic scattering, many of the elements of the inverse scattering theory are general, and extend to higher dimensions as well as to elastic media. Work is in progress to study inverse acoustic and elastic multidimensional single scattering. For stratified media, relative to a constant background, a 3D multi-offset scheme has the potential to both estimate the velocities and density of the subsurface and move reflectors towards their correct depth location. In this respect, the generalized scheme holds a potential to become a lithology-fluid prediction tool. Results will be reported as they are obtained. Research related to if and how the proposed inverse scattering concept can be extended to depth image data from 3D media which vary both vertically and laterally is a topic of future research. The reader is referred to Liu et al. (2005a, 2005b) for the application of the inverse scattering series for vertically and laterally varying media to velocity-independent depth imaging.

Third, scattering is an important tool in studying the structure and dynamics of matter and is used in many disciplines of the physical, mathematical and engineering sciences as an investigative probe. The method we present in this paper and its multidimensional extensions can find applications, in particular, in the fields of acoustics, electromagnetics, and medical imaging. Forth, we refer the reader to Gladwell (1993) for an introduction to some of the mathematics that has been developed in an attempt to solve the one-dimensional inverse scattering problem for layered media.

In the discussion to follow we shall first introduce the Born potential associated with constant-velocity imaging of the primary reflection response. We shall then show that the actual velocity potential of the medium can be "squeezed" by non-linearly compressing the depth axis in such a way that the interface locations of the squeezed potential matches the interface locations of the Born potential. The inverse transform, which stretches the squeezed potential back to the actual velocity potential is then given. In the inverse scattering problem, of course, neither the actual velocity potential nor the squeezed velocity potential are known. Only the Born potential can directly be computed. However, we show that the squeezed potential can be precisely estimated from the Born potential by simple amplitude-scaling of the Born potential. Thus, the scaled Born potential is the key to solve our inverse scattering problem since it mimics the squeezed velocity potential which we, as we have just stated, know how to stretch to obtain an estimate of the actual velocity potential. This defines the three basic elements of data-driven inversion/depth imaging: (1) constant-velocity migration of the primary reflection response yields the Born potential, (2) amplitude-scaling of the Born potential yields an estimate of the squeezed velocity potential, and (3) stretching of the estimated squeezed potential yields an estimate of the actual velocity potential. Step (2) is a simple form of amplitude-inversion, whereas step (3) is a migration. Therefore, the full inversion process for the medium parameters in 1D can be described as constant-velocity (partial) migration - inversion - residual migration.

THE INVERSE SCATTERING SOLUTION

We consider the high-velocity contrast piecewise-constant 1D acoustic ten-layer medium with velocity c(z) shown in Fig. 1a and listed in Table 1. The depth axis is positive downwards, and the layers, numbered from zero to ten, have thicknesses h_n and velocities c_n. The source and receiver are both located at depth $z = 0$ in layer zero. The number of interfaces is $N = 9$, where the depth to interface n is

$$z_n = \sum_{j=0}^{n-1} h_j \quad . \tag{1}$$

The reflection coefficient for a downgoing wave at depth z_n is

$$R_n = (c_n - c_{n-1})/(c_n + c_{n-1}) \quad .$$

Table 1. Ten-layer model with reference velocity $c_0 = 1500$ m/s. Here, z_n is the actual layer depth, z_{nB} is the layer depth obtained from Born constant-velocity imaging, c_n is the actual layer velocity, $\hat{c}_n$ is the estimated layer velocity, α_{c_n} is the actual velocity potential, and R_n is the reflection coefficient.

n	z_n[m]	z_{nB}[m]	c_n[m/s]	$\hat{c}_n$[m/s]	α_{c_n}	R_n
0			1500	1500	0	
1	300	300	1900	1900	0.38	0.118
2	400	378.9	2000	2000	0.44	0.026
3	500	453.9	2100	2101	0.49	0.024
4	600	525.4	2200	2203	0.54	0.023
5	700	593.6	2600	2620	0.67	0.083
6	800	651.3	2300	2307	0.57	-0.061
7	1000	781.7	2200	2206	0.54	-0.022

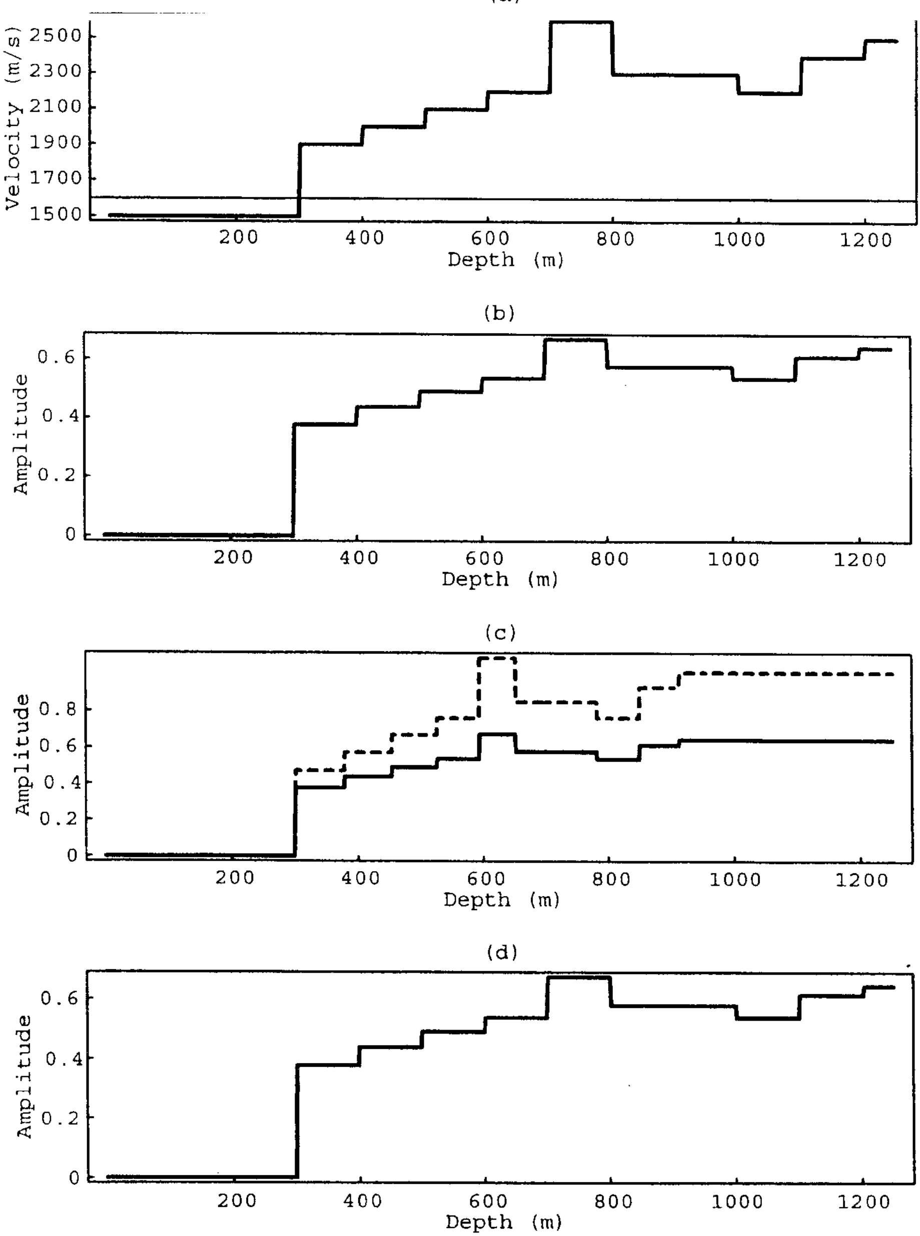


Fig. 1. "Squeezing" and "stretching" of velocity potentials. (a) The actual velocity model, c(z). (b) The actual velocity potential, $\alpha(z)$. (c) Squeezing of the actual velocity potential according to equation (8). Comparison of the squeezed velocity potential $\hat{\alpha}$ (solid line) and the Born potential α_B (dashed line) shows that the layer boundaries are at identical depths. Note that $\hat{\alpha}$ can be estimated from α_B according to equation (10). (d) Stretching of the squeezed velocity potential $\hat{\alpha}$ according to equation (9) restores the actual velocity potential, α.

Taking into account transmission losses, the primary reflection from interface n is measured at depth z = 0 with amplitude

$$\hat{R}_1 = R_1 \quad , \quad \hat{R}_n = R_n \prod_{j=1}^{n-1} (1 - R_j^2) \quad , \qquad n = 2,3,\ldots,N \ . \tag{3}$$

The normalized primary reflection response then can be represented as

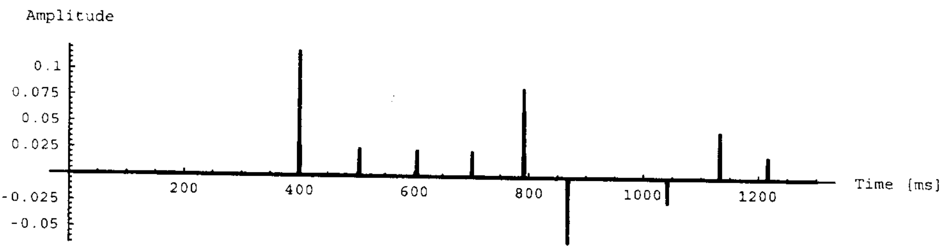
$$\Phi(t) = \sum_{n=1}^{N} \hat{R}_n \delta(t - t_n) \quad , \tag{4}$$

where $\delta(t)$ is the Dirac delta-function, and $t_n = 2 \sum_{i=0}^{n} h_i/c_i$ is the arrival time of primary n. The primary reflection response from the ten-layer model is displayed in Fig. 2. The medium is conveniently described by a scattering "velocity potential"

$$\alpha(z) = 1 - [c_0/c(z)]^2 \quad , \tag{5}$$

that characterizes the difference between the reference medium with known velocity c_0 and the actual medium with velocity c(z). The scattering velocity potential is plotted in Fig. 1b. The Born potential $\alpha_B(z)$ shown by the dashed line in Fig. 1c is obtained by data trace integration, also known as linear migration-inversion (Weglein et al., 2003), according to

$$\alpha_B(z) = 4 \int_{-\infty}^{2z/c_0} dt' \Phi(t') = 4 \sum_{n=1}^{N} \hat{R}_n H(z - z_{nB}) \quad , \tag{6}$$



where H(z) is the Heaviside function. Primary reflection events are placed at depths

$$z_{nB} = c_0 \sum_{j=0}^{n-1} h_j/c_j \quad , \tag{7}$$

computed linearly using their traveltimes together with the constant reference velocity. Since $\alpha(z) = 0$ for $z < z_1$, the first reflector is always imaged at its correct depth, $z_{1B} = z_1$. The deeper interfaces are severely mislocated, as shown in Table 1 for the ten-layer model. Given the Born potential $\alpha_B(z)$ associated with constant-velocity imaging of the single scattering data, our goal is to estimate the actual velocity potential $\alpha(z)$.

Before we proceed on the inverse problem, however, assume that the actual velocity potential $\alpha(z)$ is known. Observe then that it is possible to non-linearly shift the interfaces of the actual velocity potential onto the interfaces of the Born potential by the transformation

$$\alpha(z) = \hat{\alpha}(z - \int_{-\infty}^{z} dz'[1 - \{1 - \alpha(z')\}^{1/2}]) \quad . \tag{8}$$

Equation (8) shows that traversing along the depth axis of the actual velocity potential α, say from depth z_{n-1} to z_n, corresponds to traversing along the depth axis of the shifted velocity potential $\hat{\alpha}$ from depth $z_{n-1,B}$ to z_{nB}. For the 1D acoustic piecewise-constant medium, one may verify this on a piece of paper by evaluating by hand the integral inside equation (8). In particular, for the ten-layer model in Fig. 1b, it is straightforward to show that

$$\alpha(z_1) = \hat{\alpha}(z_1) \quad ,$$

$$\alpha(z_n) = \hat{\alpha}(z_{nB}) \quad \text{for } n = 2,\ldots,N.$$

Thus, the actual velocity potential α can be shifted in such a way that its layer interfaces coincide with the layer interfaces of the Born potential α_B. The shifted velocity potential $\hat{\alpha}$ is called the "squeezed" velocity potential since it appears like the actual velocity potential when the depth axis is squeezed. Note that it is only the layer interface positions that are affected by the squeeze operation. For the ten-layer medium, $\hat{\alpha}(z)$ is displayed in solid line in Fig. 1c. Observe that the interfaces of $\hat{\alpha}(z)$ and $\alpha_B(z)$ match. Inside layer n, the amplitude of $\hat{\alpha}$ equals that of α. The number of interfaces is preserved, of course.

We are now led to consider if there is an inverse operation. In the case we know the squeezed potential $\hat{\alpha}(z)$, can we restore the actual velocity potential

$\alpha(z)$? The answer is: yes, it is possible to "stretch" the layer interfaces of the squeezed velocity potential onto the layer interfaces of the actual velocity potential by the transformation

$$\hat{\alpha}(z) = \alpha(z + \int_{-\infty}^{z} dz'[\{1 - \hat{\alpha}(z')\}^{-1/2} - 1]) \quad . \tag{9}$$

Fig. 1d shows that the stretch procedure restores the actual velocity potential. The number of interfaces is unaffected. Stated differently, equation (9) shows that traversing along the depth axis of $\hat{\alpha}$, say from depth $z_{n-1,B}$ to z_{nB}, corresponds to traversing along the depth axis of α from depth z_{n-1} to z_n. In particular, for the ten-layer model, you do not need a computer to perform the integral inside equation (9) to verify that

$$\hat{\alpha}(z_1) = \alpha(z_1) \quad ,$$

$$\hat{\alpha}(z_{nB}) = \alpha(z_n) \quad \text{for } n = 2,\ldots,N.$$

Note that no approximations have been made as yet.

Let us return to the inverse acoustic scattering problem, where we have at our disposal the primary reflection events from which the Born potential $\alpha_B(z)$ can be computed by trace integration. The important question that develops from the above squeeze and stretch discussion is the following: can one find an amplitude scaling factor $A(z)$ that approximately corrects the Born amplitude onto the squeezed velocity potential amplitude, according to

$$\hat{\alpha}(z) \approx A(z)\alpha_B(z) \quad . \tag{10}$$

If there is such a scaling, then the scaled Born potential $\hat{\alpha} = A\alpha_B$ is the quantity that is required for finding the actual velocity potential, by stretching the depth axis of $\hat{\alpha}$. For layer one, the exact amplitude scaling function can be found. First, invert the reflection coefficient R_1 to give $c_0/c_1 = (1 - R_1)/(1 + R_1)$. Second, insert this expression into the definition (5) of the velocity potential α, and use that $\alpha_{1B} = 4R_1$. This yields the exact layer-one amplitude scaling function

$$A_1 = \hat{\alpha}_1/\alpha_{1B} = (1 + \alpha_{1B}/4)^{-2} \quad .$$

It turns out that this form of amplitude scaling is an excellent

$$A(z) = [1 + \alpha_B(z)/4]^{-} , \tag{11}$$

into equation (10). It is straightforward but somewhat tedious to show that the amplitude scaling function is precise for those acoustic layered media where the product of reflection coefficients from any three interfaces is negligible compared to the reflection coefficients themselves. Note that A(z) depends on the Born amplitude $\alpha_B(z)$ only. This implies that the squeezed velocity potential $\hat{\alpha}(z)$ can be precisely estimated from the Born potential $\alpha_B(z)$ only. For the ten-layer model in Fig. 1a, there is no visual difference between the exact squeezed velocity potential $\hat{\alpha}(z)$ and the scaled Born potential, $A(z)\alpha_B(z)$. Therefore, such a display is omitted.

We summarize the discussion to this point. The analysis of the simple geometry of a piecewise-constant 1D layered medium has shown that two operations are required to estimate the actual velocity potential from the Born potential. First, the squeezed velocity potential $\hat{\alpha}$ is estimated by scaling the Born potential according to equation (10). Next, the actual velocity potential is found by stretching the depth axis of the scaled Born potential (or equivalently, the estimated squeezed velocity potential) according to equation (9). The scaling is a simple inversion that estimates the actual velocities, not within the actual layer interfaces, but within the layer interfaces provided by the Born potential derived by the constant velocity imaging. These are the velocities that are required for depth imaging, which is aimed to move interfaces towards their correct location in depth. Since the scaling (inversion) and stretching (imaging) only depend on the Born profile information, this inverse scattering method is fully data-driven.

We remark that from the computation of $\hat{\alpha}$ according to equation (10) the layer-velocities are readily estimated by use of equation (5). In Table 1 we present the layer-velocity estimates $\hat{c}$ along with other quantities related to the model in Fig. 1. Observe that the layer-velocity estimates are very precise.

ON MULTI-OFFSET DATA-DRIVEN INVERSION/DEPTH IMAGING IN STRATIFIED 3D ACOUSTIC AND ELASTIC MEDIA

The present paper gives an inverse solution to the one-dimensional acoustic scattering problem in piecewise-constant layered media. We here want to indicate that this solution can be extended to invert the multi-offset primary reflection response of a 3D acoustic or elastic piecewise-constant stratified medium. The main difference between the 1D scheme presented here and the 3D scheme is related to the Born potential. In 3D stratified elastic media the Born potential is offset-dependent, or angle-dependent when the data are processed in the τ-p domain. The amplitude and angle-dependency of the Born potential then can be used to estimate squeezed velocity and density potentials in the stratified subsurface. For PP single scattering, both the squeezed velocity and density potentials can be depth imaged (stretched) by using information of the estimated squeezed P-wave velocity potential.

CONCLUSIONS

The ultimate objective of inverse scattering is to determine the medium and its nature from measurements external to the object under investigation. From the primary reflection response associated with a one-dimensional acoustic piecewise-constant layered medium we have derived from geometric analysis of the medium a data-driven three-step procedure for reconstructing the medium. First, the Born potential is obtained by constant-velocity depth migration of single scattering data. Second, the Born potential amplitude is scaled by multiplying the Born amplitude by an amplitude function of itself. The scaling which is a simple form of inversion, gives a squeezed velocity potential estimate, which in step three is non-linearly stretched (migrated) with respect to the depth axis. The non-linear stretch function is a function of the squeezed velocity potential, which again is a function of the Born potential. Thus, the solution for the actual velocity potential is obtained by amplitude and shift correcting the Born potential. No information other than the Born potential is required. In the nomenclature of seismic processing the three steps can be described by the sequence constant-velocity (partial) migration – inversion – residual migration.

Work is in progress to extend the concept presented here to acoustic and elastic multidimensional media. For stratified media, relative to a constant background, an extension of the current scheme has the potential to both estimate the velocities and density of the subsurface and move reflectors towards their correct depth location. In this case, the method has the potential to become a lithology-fluid prediction tool. Imaging of mislocated reflectors (obtained from constant-velocity migration) in laterally varying media is a topic of future research.

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