Extraction of the normal component of the particle velocity from marine pressure data

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ABSTRACT

We present a general wave theoretical method for extracting the normal component of the particle velocity from marine pressure data. A possible use of the normal component of the particle velocity and the pressure is the separation of upgoing and downgoing waves at the receivers. For one special acquisition geometry, the source wavelet can also be estimated. The method in principle is exact. No information about the properties of the elastic earth is required.

When the pressure data are recorded on a *single* surface, it is necessary to know the source signatures if the source array location is above the receiver surface. If the sources are located below, the signatures need not be known. The locations of the individual receivers must be specified, and the reflecting properties of the sea surface must be known. When the receiver surface is plane and horizontal, the extraction process can be performed in the frequency-horizontal wavenumber domain.

The normal component of the particle velocity can furthermore be extracted from pressure data recorded at two *surfaces* at different depths. In this case the reflectivity of the sea surface does not come into play; it is only the medium properties between the two receiver surfaces that enter the problem. The actual depths of the receivers need not be known, only their relative distances. If the sources are located above the uppermost receiver surface, the source signatures can also be estimated.

A simple synthetic data example demonstrates the extraction of the normal component of the pressure from the pressure field recorded along a dipping receiver line below a free surface.

INTRODUCTION

In conventional marine seismic acquisition only the pressure wavefield is recorded. However, several seismic processing algorithms need information about the normal derivative of the pressure, or the normal component of the particle velocity, to extract the optimum information about the subsurface from the data. The pressure and its normal derivative can, for instance, be used to separate upgoing and downgoing waves, and for certain acquisition geometries to estimate the seismic source signatures. The two wavefields are also the boundary conditions necessary for most two-way acoustic wavefield extrapolation schemes.

Weglein and Secrest (1990,1992) and Secrest and MacBain (1992) have shown that the source wavelets, or the source array radiation pattern, can be estimated from the pressure and its normal derivative recorded in a marine environment along an arbitrary surface when the sources are located above the receiver array. Their algorithm, which is independent of the subsurface geology, requires that the receiver depths and the medium above the receivers be known.

In this paper we take advantage of their result and show that from the pressure along an arbitrary surface, we can extract its normal derivative when we know the source wavelets. We also show that the source functions need not be known when the sources are located below the receiver surface. An assumption in the derivation is that the reflecting properties of the air/water surface are known. A sketch of these two configurations related to marine acquisition is shown in Figures 1a and 1b.

Furthermore, we show how to extract the normal derivative of the pressure using pressure measurements at two different depth levels (Figure 1c). Such dual streamer data have been used to separate upgoing and downgoing waves [Sønneland et al. (1986), and Monk (1990)]. This process is independent of the reflectivity of the air/water surface.

Our method requires no information about the properties of the subsurface. The relationship between the normal derivative of the pressure and the pressure itself is derived from an integral equation for the pressure by using Green's second identity. The normal component of the particle velocity can readily be computed from the normal derivative of the pressure.

In the case that the receiver surface(s) is (are) planer and horizontal, the extraction process may be performed in the frequency-horizontal wavenumber domain. The derived equation for sources below a single streamer is consistent with the equation given by Filho (1992) when the air/water surface is free. Assuming cylindrical symmetry, our equations are consistent with the equations derived by Amundsen (1993).

Rigsby et al. (1987) used the bottom-cable technique to collect pressure data in a shallow water area. One possible application of our proposed method may be to extract the normal derivative of the pressure from such bottom-cable pressure data. The normal derivative can then be used to solve the problem of the receiver ghost reflection giving multiple notches in the data spectrum. Of course, another
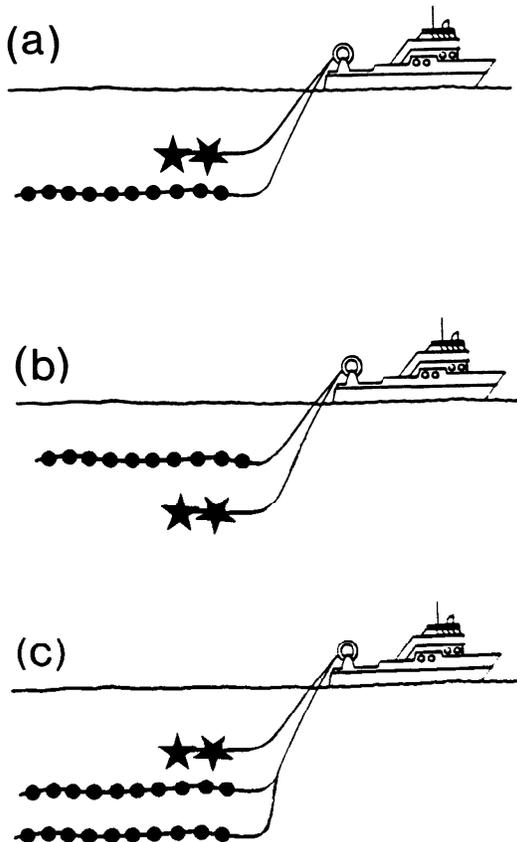

FIG. 1. Sketches of various configurations of sources and receivers pertinent to the theoretical calculations. The stars represent sources, and the bullets represent receivers. To extract the normal derivative of the pressure for configuration (a), the source functions must be known. Configuration (b), however, does not need source function information. In configuration (c), the normal derivative of the pressure is extracted from dual streamer data.

solution to the notch problem is to record the normal component of the particle velocity. This solution was introduced by Barr and Sanders (1989). When such dual-sensor data are acquired, our method could be used to verify the consistency between the pressure and velocity detector data.

In the following section we present the theory underlying our extraction scheme. We then consider the situation with one receiver surface and give special attention to the location of the sources. Next, the situation of two independent measurements of the pressure is treated. Finally, a numerical example is given.

THEORY

In the frequency-space domain the constant-density acoustic wave equation for the pressure field P caused by a sequence of m localized point sources at spatial positions $\mathbf{r}_{s_j}$ reads

$$\left(\nabla^2 + \frac{\omega^2}{c^2(\mathbf{r})}\right) P(\mathbf{r}, \mathbf{r}_{s_1}, \cdots, \mathbf{r}_{s_m}, \omega)$$

$$= \sum_{j=1}^{m} A_j(\omega)\delta(\mathbf{r} - \mathbf{r}_{s_j}), \quad (1)$$

where ∇^2 is the Laplacian, ω is the circular frequency, c is the propagation velocity, r is a shorthand notation for the Cartesian coordinates, A_j is the Fourier transform of the source time function for the source at position $\mathbf{r}_{s_j}$, and $\delta(\mathbf{r})$ represents a 3-D spatial Dirac delta function. The pressure is recorded in a marine environment at receiver coordinates $\mathbf{r}_r$ along a surface S_r below the air/water surface S_0 of the earth. For notational convenience in the following equations we drop the dependence of the fields on the source points $\mathbf{r}_{s_1}, \cdots, \mathbf{r}_{s_m}$.

We characterize the velocity c(r) in terms of a reference value c_0 and a variation in the index of refraction α,

$$\frac{1}{c^2(\mathbf{r})} = \frac{1}{c_0^2}[1 - \alpha(\mathbf{r})], \quad (2)$$

and define the causal Green's function G_{k_0} in the reference medium by

$$(\nabla^2 + k_0^2)G_{k_0}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}'), \quad (3)$$

where $k_0 = \omega/c_0$. Substituting equation (2) into (1) we may write

$$(\nabla^2 + k_0^2)P(\mathbf{r}, \omega) = k_0^2 \alpha(\mathbf{r})P(\mathbf{r}, \omega)$$

$$+ \sum_{j=1}^{m} A_j(\omega)\delta(\mathbf{r} - \mathbf{r}_{s_j}). \quad (4)$$

Using Green's second identity

$$\int_V d\mathbf{r} \, [B\nabla^2 C - C\nabla^2 B] = \int_S dS \, \mathbf{n} \cdot [B\nabla C - C\nabla B], \quad (5)$$

where B and C are two twice differentiable scalar fields in a volume V bounded by a closed surface S with outward

pointing normal vector **n,** an integral equation for P can be derived. Setting $B = P$ and $C = G_{k_0}$, we get

$$\int_V d\mathbf{r}' \, [P(\mathbf{r}', \omega)\nabla'^2 G_{k_0}(\mathbf{r}', \mathbf{r}) - G_{k_0}(\mathbf{r}', \mathbf{r})\nabla'^2 P(\mathbf{r}', \omega)]$$

$$= \int_S dS \, \mathbf{n} \cdot [P(\mathbf{r}', \omega)\nabla' G_{k_0}(\mathbf{r}', \mathbf{r})$$

$$- G_{k_0}(\mathbf{r}', \mathbf{r})\nabla' P(\mathbf{r}', \omega)], \qquad (6)$$

where ∇' operates on the $\mathbf{r}'$ coordinates. Using equations (3) and (4) in equation (6), i.e.,

$$\nabla'^2 G_{k_0}(\mathbf{r}', \mathbf{r}) = -k_0^2 G_{k_0}(\mathbf{r}', \mathbf{r}) + \delta(\mathbf{r}' - \mathbf{r}), \qquad (7)$$

$$\nabla'^2 P(\mathbf{r}', \omega) = -k_0^2 P(\mathbf{r}', \omega) + k_0^2 \alpha(\mathbf{r}')P(\mathbf{r}', \omega)$$

$$+ \sum_{j=1}^m A_j(\omega)\delta(\mathbf{r}' - \mathbf{r}_{s_j}), \qquad (8)$$

we find the general equation

$$\int_V d\mathbf{r}' \, [P(\mathbf{r}', \omega)\delta(\mathbf{r}' - \mathbf{r}) - G_{k_0}(\mathbf{r}', \mathbf{r})$$

$$\times \sum_{j=1}^m A_j(\omega)\delta(\mathbf{r}' - \mathbf{r}_{s_j}) - G_{k_0}(\mathbf{r}', \mathbf{r})k_0^2\alpha(\mathbf{r}')P(\mathbf{r}', \omega)]$$

$$= \int_S dS \, \mathbf{n} \cdot [P(\mathbf{r}', \omega)\nabla' G_{k_0}(\mathbf{r}', \mathbf{r})$$

$$- G_{k_0}(\mathbf{r}', \mathbf{r})\nabla' P(\mathbf{r}', \omega)]. \qquad (9)$$

We have not yet specified the volume V enclosed by S. In the following we will use two different geometries to extract the normal derivative of the pressure, first from single streamer pressure data and then from dual streamer pressure data.

The normal component of the particle velocity V_n, follows from the equation of motion as

$$V_n = -\frac{\iota}{\rho\omega}\mathbf{n} \cdot \nabla P, \qquad (10)$$

where ρ is the density.

SINGLE STREAMER DATA

Consider the geometry drawn in Figure 2. Let the closed surface S be composed of the recording surface S_r and a hemispherical cap S_R of radius $R = |\mathbf{r}'|$, that is, $S = S_r + S_R$. The air/water surface S_0 is inside V, and above S_r. Letting R go to infinity, S_R approaches an infinite hemispherical shell, and its contribution to the surface integral in equation (9) becomes zero because of the Sommerfeld radiation condition (Sommerfeld, 1954). The field $P(\mathbf{r}', \omega)$ fulfills the radiation condition on S_R since

$$R[\hat{\mathbf{r}} \cdot \nabla' P(\mathbf{r}', \omega) - ik_0 P(\mathbf{r}', \omega)] \to 0, \quad R \to \infty, \qquad (11)$$

uniformly for all directions $\hat{\mathbf{r}} = \mathbf{r}'/|\mathbf{r}'|$. We demand that the reference medium agrees with the actual medium above the receiver surface S,, so that the reference medium contains the air/water surface. Hence, the reference medium consists of two homogeneous halfspaces: one with air and one with water. The Green's function G_{k_0}, propagating in the spatially variant reference medium, takes into account the reflectivity properties of the air/water surface S_0. When the air/water surface is free, the Green's function is zero on S_0. In Appendix A we give the frequency-wavenumber expansion of the Green's function when S_0 is horizontal.

Now, choosing the source point r of the Green's function below S_r, the first term in the volume integral in equation (9) vanishes. Since α has no support inside V, the third term in the volume integral is also zero. Setting r' = $\mathbf{r}_r$ in the remaining surface integral, equation (9) becomes

$$\sum_{j=1}^m A_j(\omega) \int_V d\mathbf{r}' \, G_{k_0}(\mathbf{r}', \mathbf{r})\delta(\mathbf{r}' - \mathbf{r}_{s_j})$$

$$= -\int_{S_r} dS_r \, \mathbf{n} \cdot [P(\mathbf{r}_r, \omega)\nabla_r G_{k_0}(\mathbf{r}_r, \mathbf{r})$$

$$- G_{k_0}(\mathbf{r}_r, \mathbf{r})\nabla_r P(\mathbf{r}_r, \omega)], \qquad (12)$$

where ∇_r operates on the $\mathbf{r}_r$ coordinates.

Equation (12) constitutes a functional relationship between the pressure field and its normal derivative on S_r, i.e., the fields cannot be prescribed independently. The influence of possible sources below S,, as well as the properties of the medium outside S, are implicitly expressed in terms of the fields P and **n** l V,.P on S_r. In the following subsections we will use this functional relationship to extract the normal derivative of the pressure on S_r when the pressure field is known.
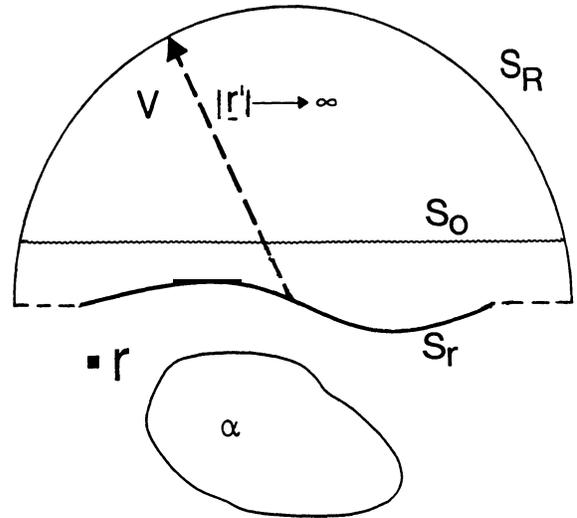

FIG. 2. Model geometry for single streamer data: $S = S_r + S_R$, where S_r is the receiver surface. S_0 is the air/water surface. The source point r of the Green's function and the scattering region α is located below S,. In the text, **two** source locations are considered: either above or below S_r.

Sources above the receiver surface

We let the sources be above the receiver surface S_r inside the volume under consideration. Then equation (12) becomes

$$\sum_{j=1}^{m} A_j(\omega) G_{k_0}(\mathbf{r}_{s_j}, \mathbf{r})$$

$$= - \int_{S_r} dS_r \; \mathbf{n} \cdot [P(\mathbf{r}_r, \omega)\nabla_r G_{k_0}(\mathbf{r}_r, \mathbf{r})$$

$$- G_{k_0}(\mathbf{r}_r, \mathbf{r})\nabla_r P(\mathbf{r}_r, \omega)]. \qquad (13)$$

Equation (13) has been used by Weglein and Secrest (1990, 1992) for wavelet estimation. Equation (13) can, however, be rearranged into a Fredholm integral equation of the first kind for the normal derivative of the pressure,

$$\int_{S_r} dS_r \; \mathbf{n} \cdot G_{k_0}(\mathbf{r}_r, \mathbf{r})\nabla_r P(\mathbf{r}_r, \omega)$$

$$= \sum_{j=1}^{m} A_j(\omega) G_{k_0}(\mathbf{r}_{s_j}, \mathbf{r})$$

$$+ \int_{S_r} dS_r \; \mathbf{n} \cdot P(\mathbf{r}_r, \omega)\nabla_r G_{k_0}(\mathbf{r}_r, \mathbf{r}), \qquad (14)$$

where the right-hand side contains only known fields if the source wavelets are known. Note that the Green's functions in principle can be evaluated with any source point r located below the receiver surface. The reader is referred to Tricomi (1957) or Antia (1991) for a discussion of numerical solution techniques (such as quadrature or expansion methods) for Fredholm integral equations of the first kind. Such integral equations are, in general, ill-conditioned and their accurate solution may be difficult to obtain. In quadrature schemes, the integral is approximated by a quadrature formula, and the resulting system of algebraic equations is solved. In expansion methods, the solution is approximated by an expansion in terms of some convenient basis functions. The coefficients of expansion may be determined by minimizing some error norm.

For most marine acquisition geometries, however, we may assume that the receiver surface is plane and horizontal. The integral equation (14) then can be transformed to the wavenumber domain where it is easier to handle. This solution technique is discussed below.

Also note that the locations of the sources and the individual receivers must be known to solve equation (14).

Sources below the receiver surface

When the sources are located below the recording surface, equation (12) gives the following integral equation

$$\int_{S_r} dS_r \; \mathbf{n} \cdot G_{k_0}(\mathbf{r}_r, \mathbf{r})\nabla_r P(\mathbf{r}_r, \omega)$$

$$= \int_{S_r} dS_r \; \mathbf{n} \cdot P(\mathbf{r}_r, \omega)\nabla_r G_{k_0}(\mathbf{r}_r, \mathbf{r}) \qquad (15)$$

for the normal derivative of the pressure. Note that in this case the source wavelets A_j need not be known to extract the normal derivative of the pressure. The locations of the individual receivers must, however, be specified.

Wavenumber domain extraction of V_z

For simplicity, in the rest of this section we will assume that the air/water surface S_0 is plane and horizontal, with vanishing pressure, implying that the reflection coefficient is - 1. In the case that the receiver array is also plane and horizontal, equations (14) and (15) can be transformed to the horizontal wavenumber domain for finding the vertical component of the particle velocity, V_z. In Appendix A [see equation (A-lo)], we show that the 3-D version of equation (14), which is valid when the source locations $r_{s_j} = (x_{s_j}, y_{s_j}, z_{s_j})$, j = 1, . . . , m, are above the receiver debth level z_r, becomes

$$V_z(k_x, k_y, z_r, \omega)$$

$$= -\frac{i}{\rho\omega} [\exp(-ik_z z_r) - \exp(ik_z z_r)]^{-1}$$

$$\times \left(\sum_{j=1}^{m} A_j(\omega) \times \exp(-ik_x x_{s_j} - ik_y y_{s_j}) \right.$$

$$\left. \times [\exp(-ik_z z_{s_j}) - \exp(ik_z z_{s_j})] \right)$$

$$- \frac{k_z}{\rho\omega} \left(\frac{\exp(-ik_z z_r) + \exp(ik_z z_r)}{\exp(-ik_z z_r) - \exp(ik_z z_r)} \right)$$

$$\times P(k_x, k_y, z_r, \omega),$$

where k_x, k_y and k_z are the two horizontal and vertical wavenumbers, respectively.

When the source depths z_{s_j}, j = 1, . . . , m, are below the receiver depth level z_r, equation (15) gives the following relation

$$V_z(k_x, k_y, z_r, \omega) = -\frac{k_z}{\rho\omega} \left(\frac{\exp(-ik_z z_r) + \exp(ik_z z_r)}{\exp(-ik_z z_r) - \exp(ik_z z_r)} \right)$$

$$\times P(k_x, k_y, z_r, \omega), \qquad (17)$$

between V_z and P. No knowledge of the source wavelets is required.

In Appendix A we also interpret equations (16) and (17) in terms of ghost operators and direct and reflected parts of the pressure.

Equations (16) and (17) have been derived by Amundsen (1993) assuming cylindrical symmetry. Equation (17) has

earlier been derived by Filho (1992) who introduced differential equations for upgoing and downgoing waves. Filho (1992) also demonstrated the applicability of the extraction method both on synthetic and real data.

Finally, note that the wavenumber domain extraction algorithms for V_z are independent of the source point r of the Green's function which enters equations (14) and (15).

DUAL STREAMER DATA

In this section, we show how to obtain the normal derivative of the pressure by measuring the pressure field at two depths. We assume that neither the sources nor any of the scattering body is located between two receiver surfaces S_1 and S_2 of infinite extension. Letting S_1 be above S_2, and bringing the source point r of the Green's function below S_2 (Figure 3), equation (9) becomes

$$\int_{S_2} dS_2\, \mathbf{n} \cdot [P(\mathbf{r}_2, \omega)\nabla_2 G_{k_0}(\mathbf{r}_2, \mathbf{r})$$

$$- G_{k_0}(\mathbf{r}_2\ \mathbf{r})\nabla_2 P(\mathbf{r}_2, \omega)] \qquad (18)$$

$$= -\int_{S_1} dS_1\, \mathbf{n} \cdot [P(\mathbf{r}_1, \omega)\nabla_1 G_{k_0}(\mathbf{r}_1, \mathbf{r})$$

$$- G_{k_0}(\mathbf{r}_1, \mathbf{r})\nabla_1 P(\mathbf{r}_1, \omega)].$$

Equation (18) constitutes a functional relationship between the pressure field and its normal derivative on $S = S_1 + S_2$.

We now demonstrate how to find the normal derivative of the pressure at the lowermost receiver surface S_2. To eliminate the unknown contribution from the normal derivative of the pressure at the receiver surface S_1, we choose the Green's function G_{k_0} zero on this surface. Equation (18) then becomes an integral equation for the normal derivative of the pressure at the receiver surface S_2,
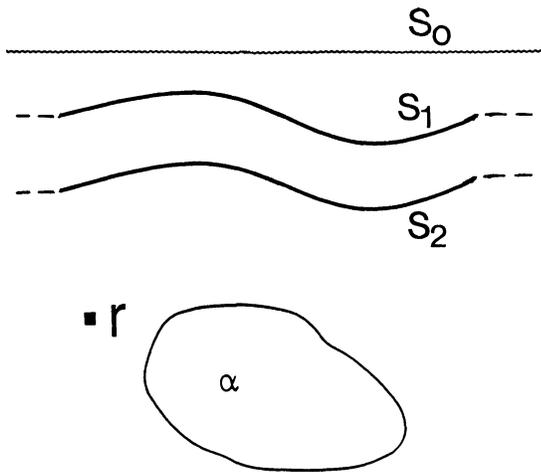

FIG. 3. Model geometry for dual streamer data: $S = S_1 + S_2$, where S_1 and S_2 are the receiver surfaces. S_0 is the air/water surface. The source Point r of the Green's function and the scattering region α are located below S_2. The sources are assumed to be located either above S_1 or below S_2.

$$\int_{S_2} dS_2\, \mathbf{n} \cdot G_{k_0}(\mathbf{r}_2, \mathbf{r})\nabla_2 P(\mathbf{r}_2, \omega)$$

$$= \int_{S_2} dS_2\, \mathbf{n} \cdot P(\mathbf{r}_2, \omega)\nabla_2 G_{k_0}(\mathbf{r}_2, \mathbf{r})$$

$$+ \int_{S_1} dS_1\, \mathbf{n} \cdot P(\mathbf{r}_1, \omega)\nabla_1 G_{k_0}(\mathbf{r}_1, \mathbf{r}). \qquad (19)$$

The right-hand side contains only known fields. In the derivation of equation (19) we have not made any assumption on the air/water surface. This equation is therefore independent of the properties of the reflecting sea surface.

An equation for the normal derivative of the pressure at the receiver surface S_1 can be derived from equation (18) by choosing the Green's function zero on S_2. It then is convenient to locate the source point r of the Green's function above S_1.

Note that the measured pressure and the extracted normal derivative of the pressure from dual streamer data can be used to estimate the source signatures by the method of Weglein and Secrest (1990) when the source array is located above the dual receiver array.

Wavenumber domain extraction of V_z

For practical purposes, the receiver arrays would be assumed to be horizontal. For this case equation (19) can be transformed to the horizontal wavenumber domain for finding V_z. In Appendix B [see equation (B-6)], we show that the equation in 3-D becomes

$$V_z(k_x, k_y, z_2, \omega)$$

$$= \frac{k_z}{\rho\omega}\left[\exp\left(-ik_z\Delta z\right) - \exp\left(ik_z\Delta z\right)\right]^{-1}$$

$$\times \{2P(k_x, k_y, z_1, \omega) - [\exp\left(-ik_z\Delta z\right)$$

$$+ \exp\left(ik_z\Delta z\right)]P(k_x, k_y, z_2, \omega)\}, \qquad (20)$$

where $\Delta z = z_2 - z_1 > 0$ is the relative distance between the receiver arrays. Note that the actual streamer depths z_1 and z_2 need not necessarily be known; it is the relative distance Δz that enters equation (20). This equation has earlier implicitly been used by Sonneland et al. (1986) and Amundsen (1993) to find the upgoing waves from dual streamer pressure data. The equation for the upgoing wave components is given in Appendix B, equation (B-8).

Finally, note that the wavenumber domain extraction algorithm for V_z is independent of the source point r of the Green's function which enters equation (19).

NUMERICAL EXAMPLE

In the numerical experiment we use a 2-D synthetic data set to test and investigate the performance of the spatial filtering process described by the Fredholm integral equation (14) of first kind for one source *(m = 1)* located above a dipping receiver line. In Appendix C we approximate the integral by a simple quadrature formula. The discretized version of the integral equation is given in Appendix C,

equation (C-4). The corresponding matrix equation of the form $\underline{K}f = g$ is defined in equation (C-5).

We choose a very simple model, a homogeneous medium bounded by a free surface. In this case the normal derivative of the pressure can be calculated from analytical expressions in the frequency domain; thus, we have the possibility of checking the extracted normal derivative with the true normal derivative.

The source is located at the horizontal coordinate $x_s = 0$ m at a depth $z_s = 5$ m (Figure 4). Its frequency content is approximately 60 Hz. The receiver line has a quite large dip angle, $\phi = 30$ degrees, relative to the free surface. The number of receivers is $N_r = 201$, with the first receiver at position (-433 m, 25 m) and the last receiver at position (433 m, 525 m). The receiver spacing along the recording line is 5 m. The recording time is .5 s with a time sampling interval of 8 ms. The evaluation source points r of the Green's functions are chosen to follow a line parallel to the receiver line, located 2.5 m below. The number of evaluation points in the example is $N_c = N_r$, which means that we have an even-determined problem.

The reference data (modeled pressure data) in the numerical experiment are shown in Figure 5a This pressure record is transformed to the frequency domain, and processed by the algorithm developed on the basis of equation (C-4). The building block of the process is a subroutine that solves a complex system of linear equations. The output of the process is the extracted normal derivative of the reference pressure recording; this data set is displayed in Figure 5b. To validate the extraction process, we have modeled the normal derivative of the pressure, shown in Figure 5c, from analytical expressions. Figure 5d shows the difference of the modeled and the extracted normal pressure derivative. The difference is very small, showing that the extraction process has worked satisfactorily.

We have, however, observed numerical instabilities of the extraction process for some choices of the evaluation points $r = (\chi, \zeta)$. Choosing the depth coordinates ζ too far from the receiver line turns out to give an inaccurate solution with numerical artifacts. This inaccuracy is related to several
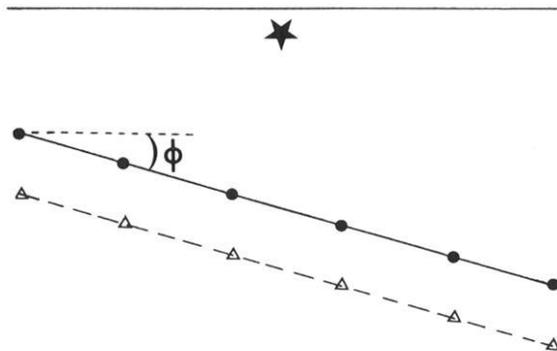
eigenvalues with very small magnitudes in the matrix $\underline{K}$, leading to an ill-conditioned system of equations. It is well known that quadrature methods are not well suited for solving Fredholm equations of the first kind. Several synthetic data tests have, however, shown that the extraction algorithm outlined in Appendix C gives acceptable results when the source points of the Green's function are chosen relatively close to the pressure receiver points. Prudent choices are found to be in the range from 2-25 m.
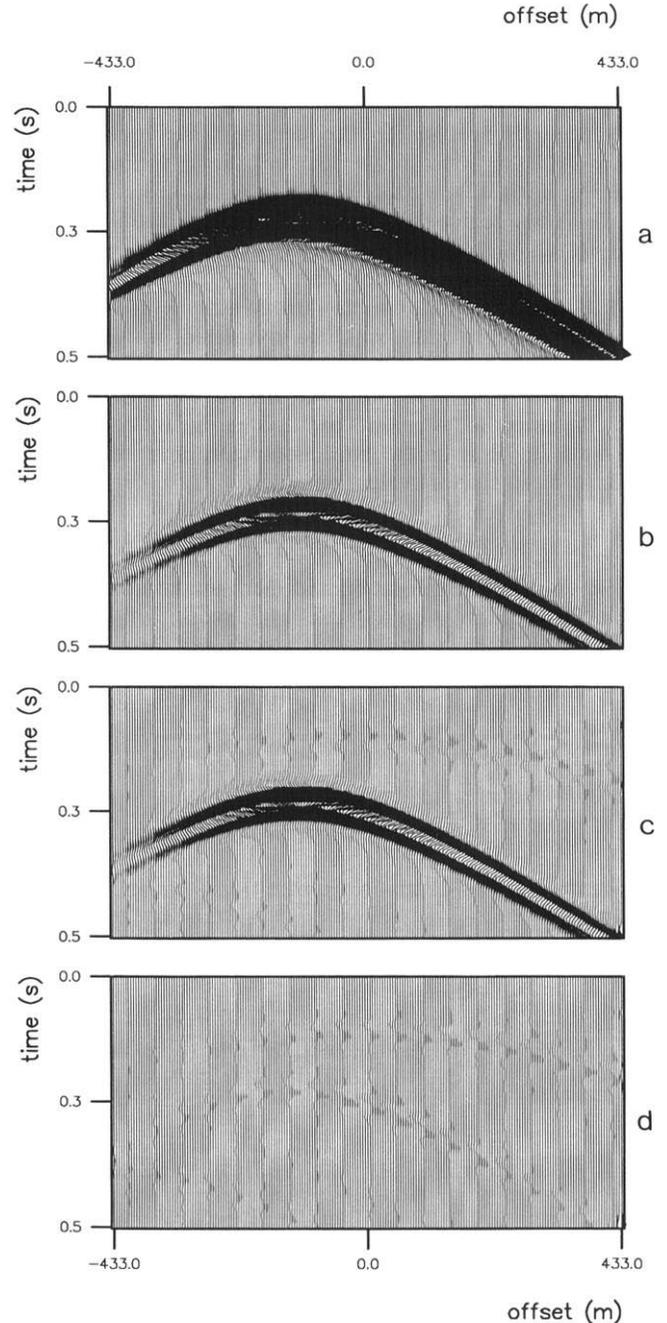

FIG. 5. (a Reference pressure data. (b) Extracted normal pressure derivative. (c) Modeled normal pressure derivative. (d) Difference between modeled and extracted normal pressure derivative.

FIG. 4. Model geometry in numerical example. The star (*) denotes the source position. The solid line with dip angle ϕ = 30 degrees represents the receiver line where the solid dots (·) denote receiver positions. The source points of the Green's function are located along the dashed line with triangle symbols ($\triangle$).

CONCLUSIONS

We have derived a general wave theoretical method for extracting the normal component of the particle velocity from marine pressure data.

When the pressure data are recorded on a single surface, the source signatures must be known if the source array location is above the receiver surface. If the sources are located below, the signatures need not be known. The reflecting properties of the sea surface, the source array depth, and the depths of the individual receivers must be known.

When the pressure data are recorded at two surfaces at different depths, only the medium properties between the two receiver surfaces enter the problem. The true depths of the receivers need not be known, only their relative distances. If the sources are located above the uppermost receiver surface, the source signatures can be estimated.

When the receiver surface(s) is (are) plane and horizontal, the extraction process can be performed in the frequency-horizontal wavenumber domain.

The synthetic data example showed that the normal derivative of the pressure could be extracted by solving an even-determined set of equations. The extraction algorithm in the space domain is, however, sensitive to the choice of the evaluation source points of the Green's functions. In the wavenumber domain the extraction algorithms are independent of these spatial evaluation points.

ACKNOWLEDGMENTS

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APPENDIX A

WAVENUMBER DOMAIN EXTRACTION OF V_z FROM SINGLE STREAMER PRESSURE DATA

We define the 2-D spatial Fourier transform as

$$F(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx_r \, dy_r$$
$$\times \exp(-ik_x x_r - ik_y y_r) f(x_r, y_r) \quad \text{(A-1)}$$

with inverse

$$f(x_r, y_r) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x \, dk_y$$
$$\times \exp(ik_x x_r + ik_y y_r) F(k_x, k_y), \quad \text{(A-2)}$$

where k_x and k_y are the horizontal wavenumbers corresponding to the spatial coordinates x_r and y_r.

In this Appendix we derive the 3-D wavenumber version of the integral equations (14) and (15) for the vertical component of the particle velocity, V_z. The pressure is recorded on a horizontal receiver surface at depth z_r located below the horizontal air/water surface at depth $z_0 = 0$. We assume that the air/water surface is a free surface with vanishing pressure, which implies a reflection coefficient of -1. We set $\mathbf{r}_{s_j} = (x_{s_j}, y_{.,}, z_{s_j})$, $\mathbf{r}_r = (x_r, y_r, z_r)$, $\mathbf{r} = (\chi, \gamma, \zeta)$, and let the depth axis be positive downwards with $\zeta > z_r > z_{s_j} > 0$. The 3-D Green's function G_{k_0} of the reference medium consisting of a water halfspace below the free surface then has the wavenumber expansion (Morse and Feshbach, 1953; Weglein and Secrest, 1990)

$$G_{k_0}(\mathbf{r}_r, \mathbf{r}) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x \, dk_y$$
$$\times \exp[ik_x(\chi - x_r) + ik_y(\gamma - y_r)]$$
$$\times \left(\frac{\exp[ik_z|\zeta - z_r|] - \exp[ik_z(\zeta + z_r)]}{2ik_z} \right),$$
$$\text{(A-3)}$$

with normal derivative

$$\frac{\partial G_{k_0}(\mathbf{r}_r, \mathbf{r})}{\partial z_r} = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x \, dk_y$$

$$\times \exp\left[ik_x(\chi - x_r) + ik_y(\gamma - y_r)\right] \times (ik_z)$$

$$\times \left(\frac{-\text{sign}\,(\zeta - z_r)\exp[ik_z|\zeta - z_r|] - \exp[ik_z(\zeta + z_r)]}{2ik_z}\right),$$

(A-4)

where the vertical wavenumbers in the water is $k_z = \sqrt{(\omega/c_0)^2 - k_x^2 - k_y^2}$, and sign (z) is the signum function.

Sources above the receiver surface

We assume that $z_r > z_{s_j}$ $(j = 1, \ldots, m)$. Using $\nabla_r \cdot \mathbf{n} = d/dz_r$, equation (14) may be written

$$\sum_{j=1}^{m} A_j(\omega)G_{k_0}(\mathbf{r}_{s_j}, \mathbf{r}) = -\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx_r \, dy_r$$

$$\times \left[P(\mathbf{r}_r, \omega)\frac{dG_{k_0}(\mathbf{r}_r, \mathbf{r})}{dz_r} - G_{k_0}(\mathbf{r}_r, \mathbf{r})\frac{dP(\mathbf{r}_r, \omega)}{dz_r}\right].$$

(A-5)

Inserting the expansions for the Green's functions in equation (A-5) we obtain

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x \, dk_y \frac{\exp(ik_x\chi + ik_y\gamma + ik_z\zeta)}{2ik_z}$$

$$\times \sum_{j=1}^{m} A_j(\omega)\exp(-ik_xx_{s_j} - ik_yy_{s_j})[\exp(-ik_zz_{s_j})$$

$$- \exp(ik_zz_{s_j})]$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx_r \, dy_r \left\{P(\mathbf{r}_r, \omega) \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x \, dk_y\right.$$

$$\times \exp\left[ik_x(\chi - x_r) + ik_y(\gamma - y_r)\right] \times (ik_z)$$

$$\times \left(\frac{\exp[ik_z(\zeta - z_r)] + \exp[ik_z(\zeta + z_r)]}{2ik_z}\right)$$

$$+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x \, dk_y \, \exp\left[ik_x(\chi - x_r) + ik_y(\gamma - y_r)\right]$$

$$\times \left.\left(\frac{\exp[ik_z(\zeta - z_r)] - \exp[ik_z(\zeta + z_r)]}{2ik_z}\right)\frac{\partial P(\mathbf{r}_r, \omega)}{\partial z_r}\right\}.$$

(A-6)

Transforming the pressure field and its normal derivative to the wavenumber domain using equation (A-1) and gathering common factors, we find

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x \, dk_y \frac{\exp[i(k_x\chi + k_y\gamma + k_z\zeta)]}{2ik_z}$$

$$\times \sum_{j=1}^{m} A_j(\omega)\exp(-ik_xx_{s_j} - ik_yy_{s_j})$$

$$\times [\exp(-ik_zz_{s_j}) - \exp(ik_zz_{s_j})]$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x \, dk_y \frac{\exp[i(k_x\chi + k_y\gamma + k_z\zeta)]}{2ik_z}$$

$$\times \left\{(ik_z)[\exp(-ik_zz_r)\right.$$

$$+ \exp(ik_zz_r)]P(k_x, k_y, z_r, \omega)$$

$$+ [\exp(-ik_zz_r)$$

$$\left.- \exp(ik_zz_r)]\frac{\partial P(k_x, k_y, z_r, \omega)}{\partial z_r}\right\}.$$

(A-7)

Equating integrands, we obtain

$$\sum_{j=1}^{m} A_j(\omega)\exp(-ik_xx_{s_j} - ik_yy_{s_j})[\exp(-ik_zz_{s_j})$$

$$- \exp(ik_zz_{s_j})] = ik_z[\exp(-ik_zz_r)$$

$$+ \exp(ik_zz_r)]P(k_x, k_y, z_r, \omega)$$

$$+ [\exp(-ik_zz_r) - \exp(ik_zz_r)]\frac{\partial P(k_x, k_y, z_r, \omega)}{\partial z_r}.$$

(A-8)

Note that this equation is independent of the evaluation source point (x, y, ζ) of the Green's function.

Solving equation (A-8) for the vertical derivative, and furthermore for the vertical component of the particle velocity,

$$V_z(k_x, k_y, z_r, \omega) = -\frac{i}{\rho\omega}\frac{\partial P(k_x, k_y, z_r, \omega)}{\partial z_r},$$

(A-9)

we find

$$V_z(k_x, k_y, z_r, \omega)$$

$$= -\frac{i}{\rho\omega}[\exp(-ik_zz_r) - \exp(ik_zz_r)]^{-1}$$

$$\times \left(\sum_{j=1}^{m} A_j(\omega)\exp(-ik_xx_{s_j} - ik_yy_{s_j})\right.$$

$$\times [\exp(-ik_zz_{s_j}) - \exp(ik_zz_{s_j})]\Bigg)$$

$$- \frac{k_z}{\rho \omega} \left(\frac{\exp(-ik_z z_r) + \exp(ik_z z_r)}{\exp(-ik_z z_r) - \exp(ik_z z_r)} \right)$$

$$\times P(k_x, k_y, z_r, \omega). \qquad (A\text{-}10)$$

Introducing the ghost operators

$$G_{\pm}(k_x, k_y, z, \omega) = 1 \pm \exp(2ik_z z), \qquad (A\text{-}11)$$

and the direct wave contribution P_d in the water layer

$$P_d(k_x, k_y, z_r, \omega) = \sum_{j=1}^{m} \frac{A_j(\omega)}{2ik_z} \exp(-ik_x x_{s_j} - ik_y y_{s_j})$$

$$\times \exp[ik_z(z_r - z_{s_j})] G_-(k_x, k_y, z_{s_j}, \omega), \qquad (A\text{-}12)$$

equation (A-9) may be written

$$V_z(k_x, k_y, z_r, \omega)$$

$$= \frac{k_z}{\rho \omega} \left[\frac{2}{G_-(k_x, k_y, z_r, \omega)} P_d(k_x, k_y, z_r, \omega) \right.$$

$$\left. - \frac{G_+(k_x, k_y, z_r, \omega)}{G_-(k_x, k_y, z_r, \omega)} P(k_x, k_y, z_r, \omega) \right]. \qquad (A\text{-}13)$$

Here, $G + (z_r)$ and $G_- (z_r)$ are receiver ghost operators that would be experienced by geophones and hydrophones, respectively, and $G_- (z_{s_j})$ is a source ghost operator. The total pressure P is the sum of the direct part P_d, and the reflected (scattered) part P_r, that is, $P = P_d + P_r$. Equation (A-13) thus can be written

$$V_z(k_x, k_y, z_r, \omega) = -\frac{k_z}{\rho \omega} \left[-P_d(k_x, k_y, z_r, \omega) \right.$$

$$\left. + \frac{G_+(k_x, k_y, z_r, \omega)}{G_-(k_x, k_y, z_r, \omega)} P_r(k_x, k_y, z_r, \omega) \right]. \qquad (A\text{-}14)$$

Equation (A-14) demonstrates that the pressure receiver ghost operator must be filtered from the reflected part of the pressure, $P_r = P - P_d$, before the conversion to the vertical particle velocity component. The direct waves contain source ghosts only, and therefore are transformed by multiplication with the factor $k_z/\rho \omega$. The source signatures must of course be known to compute P_d.

Sources below the receiver surface

In this case $z_r < z_{s,}$ ($j = 1, \ldots, m$). Equation (15) gives the following relation'

$$'_z(k_x, k_y, z_r, \omega)$$

$$= -\frac{k_z}{\rho \omega} \left(\frac{\exp(-ik_z z_r) + \exp(ik_z z_r)}{\exp(-ik_z z_r) - \exp(ik_z z_r)} \right) P(k_x, k_y, z_r, \omega) \qquad (A\text{-}15)$$

between V_z and P. In terms of receiver ghost operators equation (A-15) reads

$$V_z(k_x, k_y, z_r, \omega)$$

$$= -\frac{k_z}{\rho \omega} \frac{G_+(k_x, k_y, z_r, \omega)}{G_-(k_x, k_y, z_r, \omega)} P(k_x, k_y, z_r, \omega). \qquad (A\text{-}16)$$

In this case both the reflected part of the pressure and the direct wave contain a receiver ghost, which gives the simpler extraction equation (A-16) as compared to equation (A-14).

APPENDIX B
WAVENUMBER DOMAIN EXTRACTION OF V_z FROM DUAL STREAMER PRESSURE DATA

In this Appendix we derive the 3-D wavenumber version of the integral equation (19) for the vertical component of the particle velocity. The depth axis is positive downwards, and the pressure is recorded at two horizontal receiver surfaces at depths z_1 and z_2 with $\zeta > z_2 > z_1$. Using $\mathbf{n} \cdot \nabla_1 = -d/dz_1$ and $\mathbf{n} \cdot \nabla_2 = d/dz_2$, equation (19) becomes

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx_r\, dy_r\, G_{k_0}(x_r, y_r, z_2, \mathbf{r}) \frac{dP(x_r, y_r, z_2, \omega)}{dz_2}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx_r\, dy_r\, P(x_r, y_r, z_2, \omega) \frac{dG_{k_0}(x_r, y_r, z_2, \mathbf{r})}{dz_2}$$

$$- \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx_r\, dy_r\, P(x_r, y_r, z_1, \omega) \frac{dG_{k_0}(x_r, y_r, z_1, \mathbf{r})}{dz_1}. \qquad (B\text{-}1)$$

In this case the 3-D Green's function G_{k_0}, which is chosen zero at the uppermost recording level z_1, has the wavenumber expansion

$$G_{k_0}(x_r, y_r, z_\alpha, \mathbf{r})$$

$$= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x\, dk_y\, \exp[ik_x(\chi - x_r) + ik_y(\gamma - y_r)]$$

$$\times \left(\frac{\exp[ik_z|\zeta - z_\alpha|] - \exp[ik_z(\zeta + z_\alpha - 2z_1)]}{2ik_z} \right) \qquad (B\text{-}2)$$

with normal derivative

$$\frac{\partial G_{k_0}(x_r, y_r, z_\alpha, \mathbf{r})}{\partial z_\alpha} = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x\, dk_y$$

$$\times \exp[ik_x(\chi - x_r) + ik_y(\gamma - y_r)] \times (ik_z)$$

$$\times \left(\frac{-\text{sign}(\zeta - z_\alpha)\exp[ik_z|\zeta - z_\alpha|] - \exp[ik_z(\zeta + z_\alpha - 2z_1)]}{2ik_z} \right), \qquad (B\text{-}3)$$

where the index a = 1, 2, and $\mathbf{r} = (\chi, \gamma, \zeta)$. Inserting the expansions for the Green's functions into equation (B-l), and Fourier transforming the pressure and its derivative to the wavenumber domain, equation (B-l) becomes

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x\, dk_y\, \frac{\exp(ik_x\chi + ik_y\gamma + ik_z\zeta)}{2ik_z}$$

$$\times [\exp(-ik_z z_2) - \exp(ik_z(z_2 - 2z_1))]$$

$$\times \frac{dP(k_x, k_y, z_2, \omega)}{dz_2}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x\, dk_y\, \frac{\exp(ik_x\chi + ik_y\gamma + ik_z\zeta)}{2ik_z}$$

$$\times \{2ik_z \exp(-ik_z z_1)P(k_x, k_y, z_1, \omega) - ik_z$$

$$\times [\exp(-ik_z z_2) + \exp(ik_z(z_2 - 2z_1))]P(k_x, k_y, z_2, \omega)\}.$$

$$\text{(B-4)}$$

Equating integrands, multiplying by $\exp(ik_z z_1)$, and introducing $\Delta z = z_2 - z_1$, we obtain

$$[\exp(-ik_z\Delta z) - \exp(ik_z\Delta z)] \frac{dP(k_x, k_y, z_2, \omega)}{dz_2}$$

$$= 2ik_z P(k_x, k_y, z_1, \omega) - ik_z[\exp(-ik_z\Delta z)$$

$$+ \exp(ik_z\Delta z)]P(k_x, k_y, z_2, \omega). \quad \text{(B-5)}$$

Note that this equation is independent of the source point (χ, γ, ζ) of the Green's function.

By use of equation (10) the vertical component of the particle velocity becomes

$$V_z(k_x, k_y, z_2, \omega) = \frac{k_z}{\rho\omega} [\exp(-ik_z\Delta z) - \exp(ik_z\Delta z)]^{-1}$$

$$\times \{2P(k_x, k_y, z_1, \omega)$$

$$- [\exp(-ik_z\Delta z)$$

$$+ \exp(ik_z\Delta z)]P(k_x, k_y, z_2, \omega)\}.$$

$$\text{(B-6)}$$

Equation (B-6) may for instance be used to find the upgoing wavefield at the lower surface. Using

$$U = \frac{1}{2}[P - (\rho\omega/k_z)V_z], \quad \text{(B-7)}$$

it immediately follows

$$U(k_x, k_y, z_2, \omega)$$

$$= \frac{P(k_x, k_y, z_2, \omega) - \exp(ik_z\Delta z)P(k_x, k_y, z_1, \omega)}{1 - \exp(2ik_z\Delta z)}.$$

$$\text{(B-8)}$$

This equation has earlier been derived by Sønneland et al. (1986) and reviewed by Amundsen (1993).

APPENDIX C
DISCRETIZATION OF INTEGRAL EQUATION (14)

We here consider integral equation (14) in the 2-D case for one source $(m = 1)$ with $r = (\chi, \zeta)$, $\mathbf{r}_r = (x_r, z_r)$, and $\mathbf{r}_{s_1} = (x_s, z_s)$. The depth axis is positive downward. Assume that the receivers are not necessarily flat, but follow the curve C: $x_r = x,(s)$, $z_r = z,(s)$, where s is the distance along C (see Figure C-l). The normal vector has the components

$$\mathbf{n} = (n_x, n_z) = (-\sin\phi, \cos\phi) = \left(-\frac{dz_r}{ds}, \frac{dx_r}{ds}\right), \quad \text{(C-1)}$$
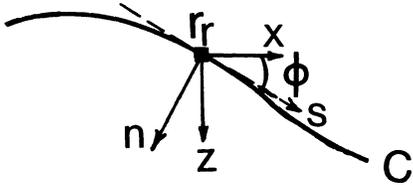

FIG. C-l. The receivers are not necessarily flat, but follow the curve C: $x_r = x,(s)$, $z_r = z,(s)$, where s is the distance along the curve. The normal vector at the point $\mathbf{r}_r = (x_r, z_r)$ has the components n = (-sin ϕ, cos ϕ) = $(-dz_r/ds, dx_r/ds)$, where ϕ be the local dip of the line.

where ϕ is the local dip angle of the receiver line at the point (x_r, z_r). Substituting equation (C-l) into equation (14) we find

$$I_C\, ds\, G_{k_0}(\mathbf{r}_r, \mathbf{r}) \frac{\partial P(\mathbf{r}_r, \omega)}{\partial n} = A\,1\,(\omega)G_{k_0}(\mathbf{r}_{s_1}, \mathbf{r})$$

$$+ \int_C ds\, P(\mathbf{r}_r, \omega)\left[-\frac{\partial G_{k_0}(\mathbf{r}_r, \mathbf{r})}{\partial x_r}\cdot\frac{dz_r}{ds}\right.$$

$$\left.+ \frac{\partial G_{k_0}(\mathbf{r}_r, \mathbf{r})}{\partial z_r}\cdot\frac{dx_r}{ds}\right]. \quad \text{(C-2)}$$

The simplest technique for solving an integral equation numerically is by the quadrature method. The use of equidistant abscissas and unity weights in the quadrature formula correspond to replacing the integrals with sums over the horizontal receiver coordinates, hence,

$$\sum_{\ell=1}^{N_r} G_{k_0}(\mathbf{r}_{r\ell}, \mathbf{r}) \frac{\partial P(\mathbf{r}_{r\ell}, \omega)}{\partial n} = \frac{A_1(\omega)}{\Delta s} G_{k_0}(\mathbf{r}_{s_1}, \mathbf{r})$$

$$+ \sum_{\ell=1}^{N_r} P(\mathbf{r}_{r\ell}, \omega) \left[- \frac{\partial G_{k_0}(\mathbf{r}_{r\ell}, \mathbf{r})}{\partial x_{r\ell}} \cdot \frac{dz_{r\ell}}{ds} \right.$$

$$\left. + \frac{\partial G_{k_0}(\mathbf{r}_{r\ell}, \mathbf{r})}{\partial z_{r\ell}} \cdot \frac{dx_{r\ell}}{ds} \right], \qquad \text{(C-3)}$$

where N_r is the number of receivers, $\mathbf{r}_{r\ell} = (x_{r\ell}, z_{r\ell})$, and As is the receiver spacing. We can now solve for $\partial P/\partial n$ by evaluating the Green's functions at N_c points $\mathbf{r}_j = (\chi_j, \zeta_j)$, where $N_c \geq N_r$, using a least-squares method. Equation (C-3) then becomes

$$\sum_{\ell=1}^{N_r} G_{k_0}(x_{r\ell}, z_{r\ell}, \chi_j, \zeta_j) \frac{\partial P(x_{r\ell}, z_{r\ell}, \omega)}{\partial n}$$

$$= \frac{A_1(\omega)}{\Delta s} G_{k_0}(x_s, z_s, \chi_j, \zeta_j) + \sum_{\ell=1}^{N_r} P(x_{r\ell}, z_{r\ell}, \omega)$$

$$\times \left[- \frac{\partial G_{k_0}(x_{r\ell}, z_{r\ell}, \chi_j, \zeta_j)}{\partial x_{r\ell}} \cdot \frac{dz_{r\ell}}{ds} \right.$$

$$\left. + \frac{\partial G_{k_0}(x_{r\ell}, z_{r\ell}, \chi_j, \zeta_j)}{\partial z_{r\ell}} \cdot \frac{dx_{r\ell}}{ds} \right] \qquad \text{(C-4)}$$

$$j = 1, \ldots, N_c.$$

For a receiver line with constant dip angle ϕ, we have $dz_{r\ell}/ds = \sin \phi$ and $dx_{r\ell}/ds = \cos \phi$.

The discretized equation (C-4) can conveniently be written as a matrix equation

$$\underset{\sim}{\mathbf{K}} \mathbf{f} = \mathbf{g}, \qquad \text{(C-5)}$$

where $\underset{\sim}{\mathbf{K}}$ is a matrix with kernel elements

$$K_{j\ell} = G_{k_0}(x_{r\ell}, z_{r\ell}, \chi_j, \zeta_j),$$

and f is the unknown vector containing the normal derivative elements

$$f_\ell = \frac{\partial P(x_{r\ell}, z_{r\ell}, \omega)}{\partial n},$$

which are to be determined, and g is a known vector with elements

$$g_j = \frac{A_1(\omega)}{\Delta s} G_{k_0}(x_s, z_s, \chi_j, \zeta_j) + \sum_{\ell=1}^{N_r} P(x_{r\ell}, z_{r\ell}, \omega)$$

$$\times \left[- \frac{\partial G_{k_0}(x_{r\ell}, z_{r\ell}, \chi_j, \zeta_j)}{\partial x_{r\ell}} \cdot \frac{dz_{r\ell}}{ds} \right.$$

$$\left. + \frac{\partial G_{k_0}(x_{r\ell}, z_{r\ell}, \chi_j, \zeta_j)}{\partial z_{r\ell}} \cdot \frac{dx_{r\ell}}{ds} \right].$$

For the 2-D case the Green's function G_{k_0}, being zero at the free surface, is

$$G_{k_0}(x_{r\ell}, z_{r\ell}, \chi_j, \zeta_j) = -\frac{i}{4} H_0^{(1)}(k_0 \rho^-) + \frac{i}{4} H_0^{(1)}(k_0 \rho^+), \qquad \text{(C-6)}$$

where $H_0^{(1)}$ is the Hankel function of the first kind, order zero, and

$$\rho^- = \sqrt{(\chi_j - x_{r\ell})^2 + (\zeta_j - z_{r\ell})^2} \qquad \text{(C-7)}$$

$$\rho^+ = \sqrt{(\chi_j - x_{r\ell})^2 + (\zeta_j + z_{r\ell})^2}. \qquad \text{(C-8)}$$

The partial derivatives of G_{k_0} becomes

$$\frac{\partial G_{k_0}}{\partial x_{r\ell}} = - \left(\frac{ik_0(\chi_j - x_{r\ell})}{4\rho^-} \right) H_1^{(1)}(k_0 \rho^-)$$

$$+ \left(\frac{ik_0(\chi_j - x_{r\ell})}{4\rho^+} \right) H_1^{(1)}(k_0 \rho^+) \qquad \text{(C-9)}$$

$$\frac{\partial G_{k_0}}{\partial z_{r\ell}} = - \left(\frac{ik_0(\zeta_j - z_{r\ell})}{4\rho^-} \right) H_1^{(1)}(k_0 \rho^-)$$

$$- \left(\frac{ik_0(\zeta_j + z_{r\ell})}{4\rho^+} \right) H_1^{(1)}(k_0 \rho^+), \qquad \text{(C-10)}$$

where *we* have used $dH_0^{(1)}(\xi)/d\xi = -H_1^{(1)}(\xi)$.

Note that the Green's functions are only a function of the relative horizontal distances. When the receiver surface is plane and horizontal (i.e., $\phi = 0$), $z_{r\ell} = z_r$, $\zeta_j = \zeta$, and $\underset{\sim}{\mathbf{K}}$ is a Toeplitz matrix.