Initial velocity models for Full Waveform Inversion
W. Weibull, B. Arntsen* and E. Nilsen, Norwegian University of Science and Technology.

FULL WAVEFORM INVERSION
Full-waveform inversion in the acoustic approximation is based on minimizing the error \( e_l \) between observed seismic data \( p^{obs} \) and simulated seismic data \( p \) with respect to the unknown seismic velocity \( c \) using the least-squares object function

\[
e_l = \sum_{e} \sum_{l} \left[ p^{obs}(x_r, t) - p(x_r, t) \right]^2,
\]

where \( x_r \) is the receiver position, \( t \) is the time and \( s \) is the source index. Minimization of \( e_l \) with respect to the velocity \( c \) and linearization leads to an iterative scheme for estimating the velocity where the normal equations must be solved for each iteration step \( n \) (Tarantola, 1984):

\[
J_n^T r_n = J_n^T J_n \Delta c_n,
\]

where \( J_n \) is the Jacobi integral operator defined by

\[
\int d^3x' J_n(x, x') \Delta c_n(x') = \int d^3x' g(x, x', t) \cdot \frac{p_n(x')}{c_{n-1}} \Delta c_n(x').
\]

Here \( c_{n-1} \) is the known velocity in iteration \( n-1 \) and \( \Delta c_n \) is the update of the velocity. \( g \) is the acoustic Green’s function. Equation 3 assumes that the Born approximation is valid and will lead to an incorrect solution if this is not the case.

A condition for the validity of the Born approximation was given by Jian-Bing et al. (2009) for electromagnetic waves. For acoustic waves this condition can be expressed in the frequency domain as

\[
(\omega/c_{n-1})^2 I \ll 1,
\]

where \( \omega \) is the angular frequency and \( I \) is given by

\[
I = \max | \int g(x_r, \omega, x) \frac{\Delta c_{n}(x)}{c_{n-1}}(x) d^3x |.
\]

We see from equation 4 that the Born approximation is valid if either the frequency is very low or the deviation \( \Delta c_n \) between the true velocity model and the current background model is small. For a simple case of constant \( c_{n-1} \), constant \( \Delta c_n \) and a spherical scattering volume of radius \( R \), we see that the combination \( 4\pi(\omega/c_{n-1})^2(\Delta c_n/R^2) \) must be small. This requires very low frequencies even for a moderate value of \( \Delta c/c_{n-1} \) at 10 percent. The use of low frequencies is standard procedure in solving this problem, but is difficult in practice because of lack of high-quality low frequency data. To be able to utilize also higher frequencies, we must then make sure that our initial model for the inversion is sufficiently close (i.e. \( \Delta c_n \) sufficiently small) to the true model to fulfill equation 4. A smooth model with kinematic properties close to the true velocity model is likely to satisfy equation 4 even for moderately high frequencies, and can be used as an initial model.

INTRODUCTION
Full-waveform inversion of seismic data requires the solution of a strongly non-linear optimization problem. Usually iterative gradient based methods are employed for the solution. These methods are based on the Born approximation and assumes that the derivative of the error function with respect to the velocity can be adequately represented by a linear approximation in each iteration step. This usually requires that the initial velocity model is close to the initial model to avoid cycle-skipping which cannot be described by the Born approximation. A common approach has been to use an initial velocity model based on ray tomography and depth migration which produces smooth models with kinematic properties similar to the true model. We suggest to replace ray tomography with Wave Equation Migration Velocity Analysis (WEMVA) based on a differential semblance error function. This method yields smooth velocity models comparable with models obtained from ray tomography, and in many cases requires only relatively simple one-dimensional models as a starting point. The advantage of this approach is that models obtained with WEMVA can then be used as initial models for Full Waveform Inversion to obtain a potentially automatic two-stage workflow for estimating accurate velocity models.

SUMMARY
Full Waveform Inversion of seismic data requires the solution of an old technique (Tarantola, 1984) but has recently successfully been used to estimate velocity models from field data (Sirgue et al., 2009). The limitation with this technique is that the initial model must be sufficiently close to the true model to prevent cycle-skipping (Virieux and Operto, 2009). A common approach to solve this problem is to rely on velocity models estimated using ray tomography (Sirgue et al., 2009) in combination with depth migration. This usually yields a smooth velocity model with kinematic properties similar enough to the true model to prevent the cycle-skipping problem. We propose an alternative approach based on wave theory only. By performing model fitting in the image space and using the least-squares object function based on differential semblance error function, we can then obtain a velocity model with good kinematic properties similar to the true model. We suggest to replace ray tomography with WEMVA in combination 4.

*Corresponding author.
INITIAL MODELS FOR FWI

WAVE EQUATION MIGRATION VELOCITY ANALYSIS

Migration Velocity Analysis using reverse-time migration (Weibull and Arntsen, 2011) yields smooth velocity models which are kinetically correct and are consequently potentially useful initial models for full waveform inversion. An object function is defined in the migrated image space to estimate velocity models and a good choice is the differential semblance object function

\[ e_s = \sum_x \sum_h h^2 \left( \frac{\partial R(x,h)}{\partial z} \right)^2, \]  

where \( R \) is the migrated image and \( h \) is the subsurface offset. The velocity is updated with a gradient based optimization where the update in iteration \( n \) is given by

\[ \Delta c_n = - \sum_{s,t} 2 c^2_n \frac{\partial^2 p_n(x,t)}{\partial^2 t} \xi_n(x,t) - \sum_{s,t} 2 c^2_n \frac{\partial^2 u_n(x,t)}{\partial^2 t} q_n(x,t). \]  

Here \( \xi \) is a time-reversed field driven by the source

\[ \sum_{h,s} h^2 \frac{\partial R(x+h,h)}{\partial z} u(x+2h,t,s), \]  

where \( u_n \) is the backward extrapolated data and \( q_n \) is a wavefield due to the source

\[ \sum_{h,s} h^2 \frac{\partial R(x-h,h)}{\partial z} p(x-2h,t,s). \]  

The velocity-estimation method outlined above is essentially the wave-equivalent of ray tomography methods based on flattening common image point gathers.

JOINT INVERSION IN THE IMAGE AND DATA SPACE

To overcome the cycle-skipping problem we suggest to combine WEMVA in the image space and full-waveform inversion in the data space. Ideally we want to use an error function equal to \( e = w_l e_l + w_s e_s \), where \( w_l \) and \( w_s \) are appropriate weights. This is difficult in practice due to the different resolution properties of the two error functions; a more pragmatic approach is to first minimize \( e_s \) to obtain a kinetically correct low-resolution velocity model, and then minimize \( e_l \) obtain a velocity model with high-resolution. This approach makes it possible to start with a simple initial model far from the true model.

NUMERICAL EXAMPLES

Figure 1 shows the initial one-dimensional velocity model of a synthetic numerical example of full-waveform inversion in the data space minimizing the least squares error \( e_l \). Figure 2: Velocity model estimated with full-waveform inversion in the data space using the model in Figure 1 as the initial velocity field. Note the discrepancy with the true model shown in Figure 3.

Figure 3: The true velocity model. Compare with Figures 2 and 5.
Initial models for FWI

The estimated model is shown in Figure 2 and by comparing with the true model shown in Figure 3 it is clear that the result is quite far from the true model.

Figure 4 shows the estimated velocity model obtained by minimizing the differential semblance error function $e_s$ given by equation 6. The resolution of this model is low, but is kinetically close to the true model.

Figure 4: Velocity model estimated with WEMVA in the image space using the model in Figure 1 as the initial velocity field.

Figure 5 shows the result of minimizing the least squares error function $e_l$ with the model shown in figure 4 as the initial model. The result is close to the true model and significantly better than the model obtained by using the simple one-dimensional velocity function.

Figure 5: Velocity model estimated with full-waveform inversion in the data space using the model in Figure 4 as the initial velocity field.

From the above examples it is clear that it is advantageous to combine migration velocity analysis and full waveform inversion, since it is possible to use a relatively simple initial one-dimensional velocity and still avoid the cycle-skipping problem. The Born approximation is still used in the algorithm for the migration velocity analysis, but the differential semblance error function has a wider basin of attraction, making simple initial models possible.

CONCLUSIONS

By combining wave equation migration velocity analysis in the image space and full waveform inversion in the data space the problem of cycle-skipping can be reduced, and a synthetic numerical example shows that the velocity field of a realistic model can be accurately estimated using a simple one-dimensional initial model.

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