Initial velocity models for Full Waveform Inversion

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SUMMARY

Full Waveform Inversion of seismic data requires the solution of a strongly non-linear optimization problem. Usually iterative gradient based methods are employed for the solution. These methods is based on the Born approximation and assumes that the derivative of the error function with respect to the velocity can be adequately represented by a linear approximation in each iteration step. This usually requires that the initial velocity model is close to the initial model to avoid cycle-skipping which cannot be described by the Born approximation. A common approach has been to use an initial velocity model based on ray tomography and depth migration which produces smooth models with kinematic properties similar to the true model. We suggest to replace ray tomography with Wave Equation Migration Velocity Analysis (WEMVA) based on a differential semblance error function. This method yields smooth velocity models comparable with models obtained from ray tomography, and in many cases requires only relatively simple one-dimensional models as a starting point. The advantage of this approach is that models obtained with WEMVA can then be used as initial models for Full Waveform Inversion to obtain a potentially automatic two-stage work flow for estimating accurate velocity models.

INTRODUCTION

Full-waveform inversion of seismic data based on least-squares model fitting is an old technique (Tarantola, 1984) but has recently successfully been used to estimate velocity models from field data (Sirgue et al., 2009). The limitation with this technique is that the initial model must be sufficiently close to the true model to prevent cycle-skipping (Virieux and Operto, 2009). A common approach to solve this problem is to rely on velocity models estimated using ray tomography (Sirgue et al., 2009) in combination with depth migration. This usually yields a smooth velocity model with kinematic properties similar enough to the true model to prevent the cycle-skipping problem. We propose an alternative approach based on wave theory only. By performing model fitting in the image space based on differential semblance Biondi and Sava (1999); Shen and Symes (2008); Weibull and Arntsen (2011) a low resolution velocity model with good kinematic properties can be obtained. Cycle-skipping can be avoided by using this model as a starting model for full-waveform inversion in the data space. In the next sections we show synthetic examples where an object function based on differential semblance is used to estimate an initial model and a least-squares object function is then minimized to refine the initial model.

FULL WAVEFORM INVERSION

Full-waveform inversion in the acoustic approximation is based on minimizing the error e_l between observed seismic data p^{obs} and simulated seismic data p with respect to the unknown seismic velocity c using the least-squares object function

$$e_l = \sum_{r,s}\sum_{t}\left[p^{obs}(x_r,t) - p(x_r,t)\right]^2, \tag{1}$$

where x_r is the receiver position, t is the time and s is the source index. Minimization of e_l with respect to the velocity c and linearization leads to an iterative scheme for estimating the velocity where the normal equations must be solved for each iteration step n (Tarantola, 1984):

$$J_n^T r_n = J_n^T J_n \Delta c_n, \tag{2}$$

where J_n is the Jacobi integral operator defined by

$$\int d^3x' J_n(x,x')\Delta c_n(x') = \int d^3x' g(x,x',t) * \frac{p_{n-1}(x',t)}{c_{n-1}^3}\Delta c_n(x'). \tag{3}$$

Here c_{n-1} is the known velocity in iteration no $n-1$ and Δc_n is the update of the velocity. g is the acoustic Green's function. Equation 3 assumes that the Born approximation is valid and will lead to an incorrect solution if this is not the case.

A condition for the the validity of the Born approximation was given by Jian-Bing et al. (2009) for electromagnetic waves. For acoustic waves this condition can be expressed in the frequency domain as

$$(\omega/c_{n-1})^2 I \ll 1, \tag{4}$$

where ω is the angular frequency and I is given by

$$I = \max|\int g(x_r,\omega,x)\frac{\Delta c_n(x)}{c_{n-1}(x)}d^3x|. \tag{5}$$

We see from equation 4 that the Born approximation is valid if either the frequency is very low or the deviation Δc_n between the true velocity model and the current background model is small. For a simple case of constant c_{n-1}, constant Δc_n and a spherical scattering volume of radius R, we see that the combination $4\pi(\omega/c_{n-1})^2(\frac{\Delta c}{c_{n-1}})R^2$ must be small. This requires very low frequencies even for a moderate value of $\Delta c/c_{n-1}$ at 10 percent. The use of low frequencies is standard procedure in solving this problem, but is difficult in practice because of lack of high-quality low frequency data. To be able to utilize also higher frequencies, we must then make sure that our initial model for the inversion is sufficiently close (i.e Δc_n sufficiently small) to the true model to fulfill equation 4. A smooth model with kinematic properties close to the true velocity model is likely to satisfy equation 4 even for moderately high frequencies, and can be used as an initial model.

WAVE EQUATION MIGRATION VELOCITY ANALYSIS

Migration Velocity Analysis using reverse-time migration (Weibull and Arntsen, 2011) yields smooth velocity models which are kinetically correct and are consequently potentially useful initial models for full waveform inversion. An object function is defined in the migrated image space to estimate velocity models and a good choice is the differential semblance object function

$$e_s = \sum_x \sum_h h^2 \left[\frac{\partial R(x,h)}{\partial z} \right]^2 , \tag{6}$$

where R is the migrated image and h is the subsurface offset. The velocity is updated with a gradient based optimization where the update in iteration n is given by

$$\begin{aligned} \Delta c_n = & - \sum_{s,t} \frac{2}{c_{n-1}^3} \partial_t^2 p_n(x,t) \xi_n(x,t) \\ & - \sum_{s,t} \frac{2}{c_{n-1}^3} \partial_t^2 u_n(x,t) q_n(x,t). \end{aligned} \tag{7}$$

Here ξ is a time-reversed field driven by the source

$$\sum_{h,s} h^2 \frac{\partial R(x+h,h)}{\partial z} u(x+2h,t,s), \tag{8}$$

where u_n is the backward extrapolated data and q_n is a wavefield due to the source

$$\sum_{h,s} h^2 \frac{\partial R(x-h,h)}{\partial z} p(x-2h,t,s). \tag{9}$$

The velocity-estimation method outlined above is essentially the wave-equivalent of ray tomography methods based on flattening common image point gathers.

JOINT INVERSION IN THE IMAGE AND DATA SPACE

To overcome the cycle-skipping problem we suggest to combine WEMVA in the image space and full-waveform inversion in the data space. Ideally we want to use an error function equal to $e = w_l e_l + w_s e_s$, where w_l and w_s are appropriate weights. This is difficult in practice due to the different resolution properties of the two error functions; a more pragmatic approach is to first minimize e_s to obtain a kinetically correct low-resolution velocity model, and then minimize e_l obtain a velocity model with high-resolution. This approach makes it possible to start with a simple initial model far from the true model.

NUMERICAL EXAMPLES

Figure 1 shows the initial one-dimensional velocity model of a synthetic numerical example of full-waveform inversion in the data space minimizing the least squares error e_l.

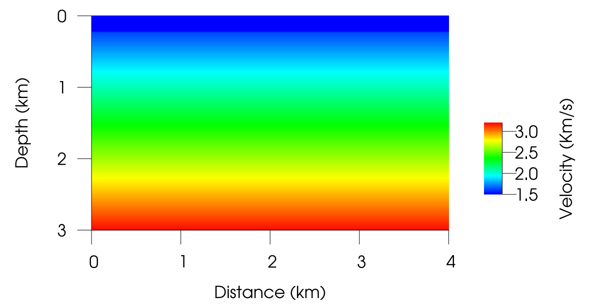

Figure 1: One-dimensional initial velocity model with linear increase

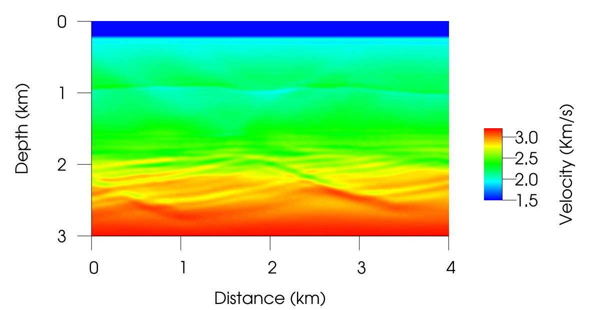

Figure 2: Velocity model estimated with full-waveform inversion in the data space using the model in Figure 1 as the initial velocity field. Note the discrepancy with the true model shown in Figure 3.

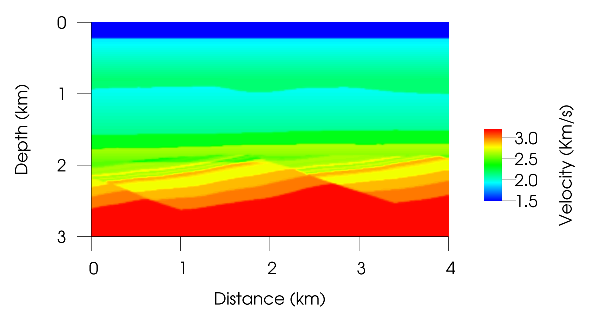

Figure 3: The true velocity model. Compare with Figures 2 and 5.

The estimated model is shown in Figure 2 and by comparing with the true model shown in Figure 3 it is clear that the result is quite far from the true model.

Figure 4 shows the estimated velocity model obtained by minimizing the differential semblance error function e_s given by equation 6. The resolution of this model is low, but is kinetically close to the true model.

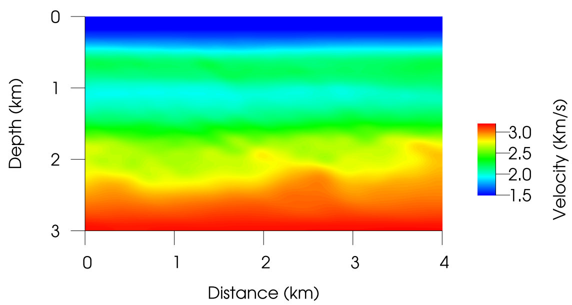

Figure 4: Velocity model estimated with WEMVA in the image space using the model in Figure 1 as the initial velocity field.

Figure 5 shows the result of minimizing the least squares error function e_l with the model shown in figure 4 as the initial model. The result is close the true model and significantly better than the model obtained by using the simple one-dimensional velocity function.

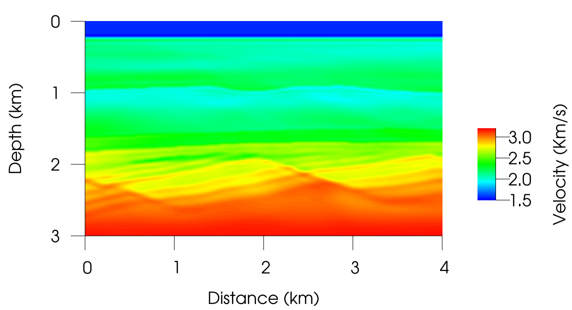

Figure 5: Velocity model estimated with full-waveform inversion in the data space using the model in Figure 4 as the initial velocity field.

From the above examples it is clear that it is advantageous to combine migration velocity analysis and full waveform inversion, since it is possible to use a relatively simple initial one-dimensional velocity and still avoid the cycle-skipping problem. The Born approximation is still used in the algorithm for the migration velocity analysis, but the differential semblance error function has a wider basin of attraction, making simple initial models possible.

CONCLUSIONS

By combining wave equation migration velocity analysis in the image space and full waveform inversion in the data space the problem of cycle-skipping can be reduced, and a synthetic numerical example shows that the velocity field of a realistic model can be accurately estimated using a simple one-dimensional initial model.

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