P216 LOW FREQUENCY WAVES IN FINELY LAYERED MEDIA

A.STOVAS[1] and B.ARNTSEN[2]
[1]NTNU Dept.of Petroleum Engineering and Applied Geophysics, Trondheim, Norway
[2]Statoil ASA

Abstract
The low frequency limit of waves propagating through a layered medium is investigated, and we find that the typical ratio of wavelength to layer thickness (λ/d) at which the plane layered stack can be regarded as a homogeneous medium is strongly dependent on the strength of reflection coefficients and can not be characterized by a single value.

Introduction
The effect of multiple scattering in finely layered sediments is of importance for stratigraphic interpretation, matching of well log-data with seismic data and seismic modelling. This problem was first studied in the now classical paper by O'Doherty and Anstey (1971). In the limit of infinite wavelength finely layered media can be regarded as an effective homogeneous medium (Backus, 1962). Folstad and Schoenberg (1992) investigated models with different layer thickness and concluded that fine layering of the order of 1/10 of the smallest wavelength could effectively be regarded as a homogeneous medium. In this paper we show that the transition to an effective medium depends not only on the ratio between wavelength and layer thickness, but also on the strength of the reflection coefficients.

Finely layered medium with variable contrast
To examine the influence of the layer thickness we have constructed a set of models where the first four are shown in figure 1. Model M1 is a stack of fine layers taken from a real well log. Layer no i is characterized by layer thickness d_i, velocity V_i and density ρ_i. Models M2,M4 and M8 are constructed from model M1 by dividing the layer spacing by a factor of 2,4 and 8, respectively. The resulting model is then duplicated 2,4 or 8 times preserving the total depth interval of 500m. Such repeated lithological sequences can be found in, for example, turbidite systems. In the numerical examples below we will also study models duplicated 16 and 32 times.
Each of the models described above is further modified by scaling the reflection coefficients $r^{\lambda}=\gamma r$ with a factor equal to γ. A new density profile is then computed but velocities are not changed. The scaling of the reflection coefficients allow us to study models with the same velocity profile but different contrast between layers.

The weak contrast approximation
The transmission $t_D^{(N)}$ and reflection $r^{(N)}{}_D$ responses of a normal-incident plane wave from a stack of N layers can be computed approximately by including only second order multiples (Shapiro and Treitel ,1997), also referred to as the weak contrast approximation

$$t_D^{(N)}=\frac{e^{i\vartheta_N}\prod_{k=1}^{N}\left(1-r_k\right)}{\left[1+\Phi\right]}=\left|t_D^{(N)}\right|e^{i\varphi_N}, \qquad r_D^{(N)}=\frac{e^{2i\vartheta_N}\left(\sum_{j=1}^{N}r_j e^{-2i\left(\vartheta_N-\vartheta_j\right)}+...\right)}{\left[1+\Phi\right]}. \tag{1}$$

Here the phases are given by $\vartheta_i=\omega\tau_i=\omega d_i/V_i$, and ω is the frequency. The reflection coefficient correlation function is given by

$\Phi=\sum_{k=1}^{N-1}\sum_{j=k+1}^{N}r_k r_j e^{2i\left(\vartheta_j-\vartheta_k\right)}+...$ The velocity in the zero-frequency limit is obtained as

$$\frac{1}{V_0}=\lim_{\omega\to 0}\frac{1}{V(\omega)}=\frac{1}{V_{TA}}-\frac{2}{D}\frac{\sum_{k=1}^{N-1}\sum_{j=k+1}^{N} r_k r_j\left(\tau_j-\tau_k\right)+...}{1+\sum_{k=1}^{N-1}\sum_{j=k+1}^{N} r_k r_j+...} \quad , \qquad (2)$$

where $V_{TA}=\omega D/\vartheta_N$ is the time-average velocity. D is here the total depth of the stack. Note that for a real medium the zero-frequency limit and the Backus limit (Backus, 1962) are different.

O'Doherty-Anstey (ODA) approximation

The weak-contrast approximation means that we neglect the higher order terms in Φ. The O'Doherty-Anstey approximation ($1+\Phi\approx e^{\Phi}$) reconstructs to a certain degree the neglected terms. The transmission amplitude reduces to the well-known O'Doherty-Anstey formula:

$\left|t_D\right|=e^{-\sum_{k=1}^{N-1}\sum_{j=k+1}^{N} r_k r_j \cos 2\left(\vartheta_j-\vartheta_k\right)}\prod_{k=1}^{N}\left(1-r_k\right)$, and the zero-frequency limit (2) reduces to

$$V_0=V_{TA}e^{\frac{2}{\tau_N}\sum_{k=1}^{N-1}\sum_{j=k+1}^{N} r_k r_j\left(\tau_j-\tau_k\right)} . \qquad (3)$$

Numerical results

For the numerical examples we use 500m of the real log sampled at 0.125m (M1) and the duplicated models Mn, n=2,4,…32. (Figure 1). The comparison between the Backus limit, zero-frequency limit and zero-frequency limit in ODA approximation versus contrast is shown in Figure 2 for model M1. One can see that all these limits are different, but for the relatively weak contrast the zero-frequency limit in ODA gives a good fit with the zero-frequency one. The comparison between the exact, weak-contrast and ODA velocities and transmission amplitudes is given in Figure 3. The ODA approximation reconstructs both amplitudes and velocities, and this effect is more pronounced in the low frequency domain. In Figure 4 the transmission and reflection responses are shown for the wavelet with maximum spectrum on 15 Hz for models Mn, n=1,2,…,32 and the contrast factor $\gamma=1,4$.

The typical layer thickness of model M32 is 1/32 of the typical layer thickness in model M1 and from the comparison of the transmission and reflection responses we can conclude that there is a transition zone between the effective medium and time-average medium, and the typical layer thickness (d) this occurs for depends on the contrast in reflectivity. The effective medium parameters also depend on the contrast (the reflections from the bottom of effective medium have different polarity for $\gamma=1$ and $\gamma=4$). Denoting the dominant wavelength by λ in figure 5 the λ/d ratios are plotted against the contrast in reflectivity. With increase of reflectivity contrast the transition zone becomes larger. It means that the transition between the effective medium and time-average medium is defined not by a given constant λ/d ratio but also depends on the contrast in reflectivity.

Conclusions

The ODA approximation reconstructs not only amplitudes, but also the dispersion equation. There is a transition zone between the effective medium and time-average medium, and transition frequencies are reflectivity dependent. The effective medium velocity is also reflectivity dependent, the larger reflectivity the smaller velocity.

References

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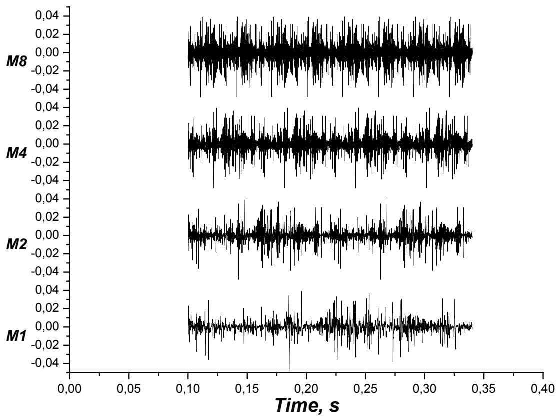

Figure 1. Reflection coefficient series for models M1,M2,M4 and M8.

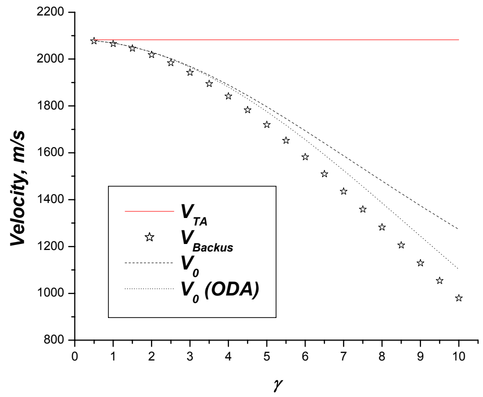

Figure 2. Time-average velocity, Backus limit, zero-frequency limit and zero-frequency limit in ODA approximation versus reflectivity computed for model M1.

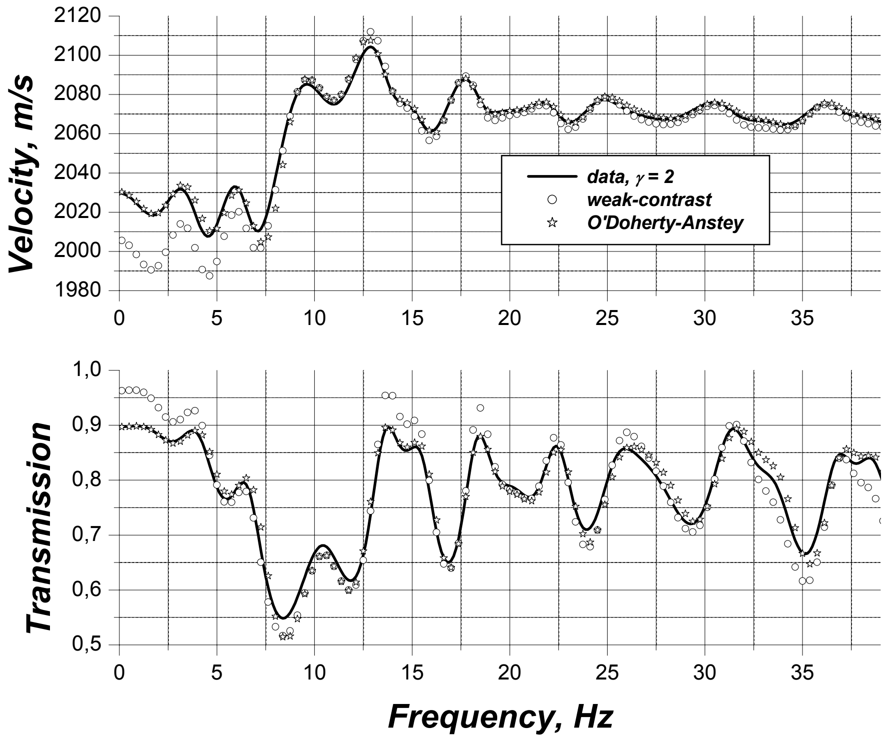

Figure 3. Dispersion equation and transmission amplitudes with weak-contrast and O'Doherty-Anstey approximation for model M1 with $\gamma = 2$.

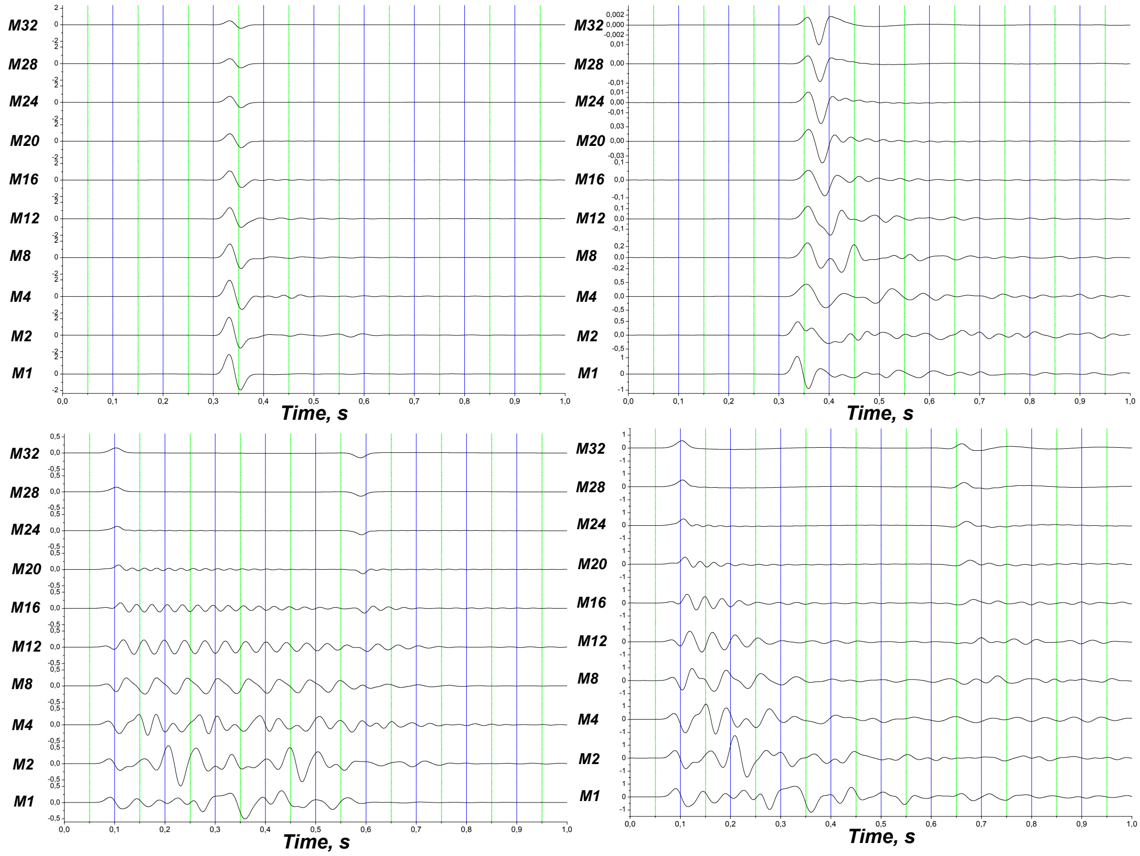

Figure 4. Transmission (to the top) and reflection (to the bottom) responses for $\gamma = 1$ (to the left) and $\gamma = 4$ (to the right).

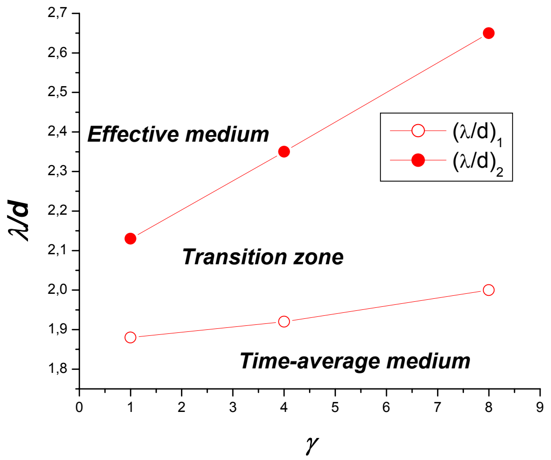

Figure 5. λ/d ratio versus reflectivity contrast.