C046 WAVEFIELD EXTRAPOLATORS FOR COST-EFFECTIVE 3D DEPTH MIGRATION

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INTRODUCTION

It is well known that stable and cost-effective explicit finite-difference 2D wavefield extrapolators can be designed using least-squares filters (Holberg, 1988). The method handles lateral velocity variations and high angle energy that cannot be properly migrated is suppressed. This energy is often smeared and dispersed when conventional methods are used. The immediate extension of Holberg's operators to the 3D case leads to numerically expensive 2D convolutional filters (Blacquière et al, 1989). Hale (1991b) introduced 3D wave extrapolation using McClellan transformations reducing the computational cost significantly; but spatial oversampling is needed to avoid numerical distortion. Soubaras (1992) proposed a more accurate scheme using a series expansion of the 3D extrapolation operator in terms of second derivatives. He applied the Remez exchange algorithm for optimization. Our approach is based on an expansion of second derivatives and the use of Chevbyshev recursion. Furthermore, we apply tabulated filters of variable order approximating second derivatives up to pre-specified spatial frequencies. All optimization is done by using least-squares. The resulting extrapolation algorithm for 3D migration is computationally more efficient than comparable methods and eminently suited for high dip imaging.

3D WAVE EXTRAPOLATOR DESIGN

In explicit finite-difference 3D migration each frequency of the wavefield is downward continued by convolution with a 2D circularly symmetric spatial filter, which has a Fourier transform that can be written:

$$F(\omega/c, k_x, k_y, \Delta x, \Delta z) = 2.0 \sum_{n=0}^{N} f_n \cos(nk) \approx \exp\left\{ i\frac{\Delta z}{\Delta x}\left[\left(\frac{\omega \Delta x}{c}\right)^2 - k_x^2 - k_y^2\right]^{\frac{1}{2}}\right\}. \tag{1}$$

The variable k is given by $k^2 = \frac{1}{2}(k_x^2 + k_y^2)$ with $k, k_x, k_y \in [0, \pi]$. The temporal frequency is denoted ω and c is the velocity.

The Chevbyshev recursion identity $\cos(nk) = 2\cos k \cos[(n-1)k] - \cos[(n-2)k]$ enables us to compute the $\cos(nk)$ terms in (1) from $\cos k$. Using this formula, the 2D filter design approach consists of solving two 1D filter problems: *i)* We design a spatial filter with a frequency response $H(k)$ that approximates $\cos k$ by using the series expansion $H(k) = \beta_0 + \beta_1 D(k)$, where $D(k)$ is a real one-dimensional filter approximating k^2 (Soubaras,1992). *ii)* The complex coefficents $f_n, n = 0, N$ in (1) are optimized by using least-squares and $H(k)$ instead of $\cos k$ in the Chevbyshev recursion formula.

The fact that $D(k) = \frac{1}{2}[D(k_x) + D(k_y)]$ implies that $h(x, y)$, which corresponds to $H(k)$ in the spatial domain, consists of two one-dimensional filters operating in the $x-$ and $y-$direction respectively. The 2D convolution is carried out by applying the spatial filter $h(x, y)$ and the coefficents $f_n, n = 0, N$ in a Chevbychev recursion scheme.

Soubaras (1992) used a fixed second derivative operator which is accurate up to 70% of the spatial Nyquist frequency. We suggest to precompute and tabulate optimally designed operators for each ratio $\omega\Delta x/c$, leading to frequency responses of the form:

$$D(\frac{\omega \Delta x}{c}, k) = 2.0 \sum_{l=0}^{L} \beta_l(\frac{\omega \Delta x}{c}) \cos(kl), \tag{2}$$

approximating k^2 for $| k | \leq | \frac{\omega \Delta x}{c} \sin\theta_{max} |$. The design parameter θ_{max} denotes the maximum angle of dips to be migrated. We also vary the filter length L, using shorter filters for low frequencies to reduce the number of computations, and longer filters for higher frequencies to meet the demand for accuracy.

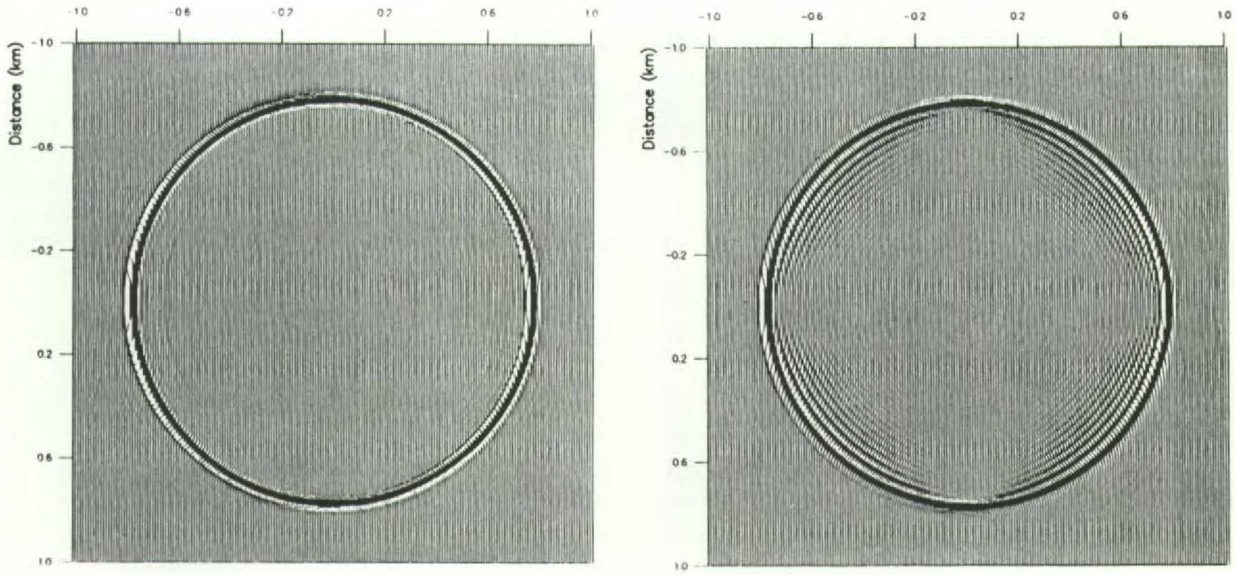

Figure 1: Horizontal slice of 3D image of a spherical reflector, cutting through the reflector at a depth where the reflector dip angle is 60°. Proposed scheme (left) and Hale's improved McClellan transform (right).

COMPUTATIONAL COST AND NUMERICAL EXAMPLE

The presented method is very flexible and an example illustrates its potential in terms of efficiency and accuracy. We have utilized a set of second derivative operators with half-lengths $L = 1, 3, 5$. In this case the mean number of coefficients used in $h(x, y)$ is ≈ 12.6. Hale's (1991b) improved McClellan transform and Soubaras' (1992) scheme require 17 coefficients. In addition, Hale (1991a,b) used sub-optimal 2D extrapolators and Soubaras' method needs more expansion terms (N) than the proposed method, both resulting in more recursion steps than the proposed scheme requires when high dip imaging is desired.

A wavelet with a bandwith of 50Hz has been migrated to compare the proposed scheme to the Hale-McClellan method. Depth slices are shown in Figure 1. In the Hale-McClellan migration we are not using Hale's original migration operators, but the more optimal operators of Holberg. The disturbing dispersion artefacts are only due to the sub-circular McClellan transform. The number of coefficents is $N = 12$ for both methods. The grid spacing is $\Delta x = \Delta y = \Delta z = 10$m, the velocity $c = 2000$ m/s and $\theta_{max} = 60°$.

The proposed scheme is limited by the gridspacing, requiring only of two gridpoint per wavelength. In comparsion, the improved McClellan transform needs three to four gridpoints per shortest wavelength to get rid of the dispersion in the azimuth direction.

The conclusion is that in situations where high accuracy is required, the proposed scheme is five to ten times faster that the Hale-McClellan scheme, decreasing the computing time for post-stack 3D depth migration on a state of the art vector computer from days to hours.

REFERENCES

HOLBERG, O. 1988.Towards Optimum One-way Wave Propagation. *Geophysical Prospecting* 36, 99-114.

BLACQUIÈRE, G., DEBEYE, H.W.J., WAPENAAR, C.P.A., BERKHOUT, A.J. 1989. 3D Table-driven Migration. *Geophysical Prospecting* 37, 925-958.

HALE, D. 1991. Stable Explict Depth Extrapolation of Seismic Wavefields. *Geophysics* 56. 1770-1778.

HALE, D. 1991. 3-D depth migration via McClellan transformations. *Geophysics* 56. 1778-1785.

SOUBARAS, R. 1992. Explict 3D Migration Using Equiripple Polynomal Expansion and Laplacian Synthesis, Expanded abstracts of the 62th Annual International S.E.G Meeting, New Orleans.