A 3D ELASTIC HYBRID MODELING SCHEME

Rune Mittet[1] and Børge Arntsen[2]

[1]Sting Research, Rosenborg gt 17, N-7014 Trondheim, Norway
[2]Statoil Research Centre, Postuttak, N-7005 Trondheim, Norway

SUMMARY

Full scale 3D elastic finite-difference modeling requires very large CPU times. A hybrid modeling scheme which combines an elastic cylinder-symmetric finite-difference modeling scheme with a 3D elastic finite-difference modeling schemes can reduce the CPU time considerably. The main limitation is that the overburden must be close to a plane-layer geology. The coupling of the two schemes is performed using an elastic representation theorem. Calibration of the hybrid scheme by comparison with the solution from a frequency-wavenumber algorithm give good results.

3D ELASTIC HYBRID MODELING

If a realistic response from a model which has a typically 3D geometry is required, then some type of 3D modeling scheme must be used. All wave modes and conversions should be included. Explicit finite-difference schemes may include these effects. The problem, however, is that such schemes are computationally expensive. For the purely elastic problem, neglecting absorption and anisotropy, the CPU time at 1 Gflops is at least 13 hours for a realistic size experiment. This may be acceptable for experiments with a few shots, but not for simulation of a survey with several hundreds of shots or more. The solution is in this work limited to typically North-Sea problems where there is a plane-layered overburden over a structurally more complicated target volume. The basic idea is to use two finite-difference schemes to solve the 3D elastic modeling problem. If a plane-layered overburden is assumed, then a cylindrical-symmetric scheme can be used here. This scheme has 3D elastic wave propagation. Normally, a cylindrical-symmetric scheme gives the solution as a function of radial coordinate r and depth coordinate z, but the symmetries of the elastic field and its invariance with respect to translation can be used to calculate the Greens tensor connecting any positions in a plane layer 3D model. In the deeper part of the model a full 3D elastic finite-difference scheme is used. The size of the 3D modeling problem is reduced in three ways. Firstly, the actual size of the 3D grid is reduced since the 3D elastic finite-difference scheme is used in the deeper part of the model only. Secondly, the spatial sampling intervals may be increased since the S-wave velocities generally are higher at larger depths. Thirdly, the 3D elastic finite-difference scheme can be solved for a reduced number of timesteps. The initial time of the 3D simulation is given by the time the field requires to reach the coupling depth, z_c ($\boldsymbol{x}_c$=(x_c,y_c,z_c)), where the 3D elastic finite-difference scheme is used and the final time is given by the desired time for the shot record minus the time required to propagate the solution from the coupling depth z_c up to the receivers at depth z_r, ($\boldsymbol{x}_r$=(x_r,y_r,z_r)). The CPU time required to perform the cylinder-symmetric modeling in the overburden is negligible compared to the CPU time of the full 3D elastic finite-difference scheme. The reduction factor in CPU time using the hybrid scheme as compared to the full 3D elastic scheme is model dependent, but is typically 5-15 for a full size model.

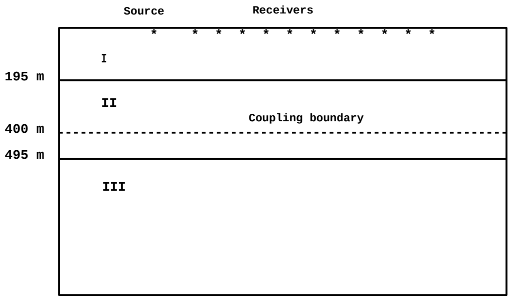

Figure 1: Cross section of plane layer model.

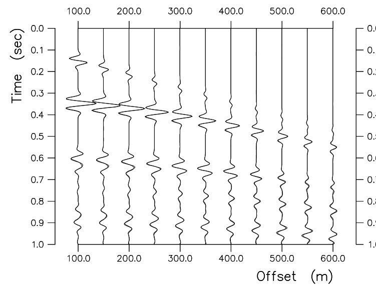

Figure 2: OSIRIS and hybrid modeling response.

The elastic Greens tensor G_{ijnm} is given by the equation,

$$\partial_t^2 G_{ijmn}(\boldsymbol{x}, t|\boldsymbol{x}', t') - c_{ijpq}(\boldsymbol{x}) \left[\partial_p \rho^{-1}(\boldsymbol{x}) \partial_k G_{qkmn}(\boldsymbol{x}, t|\boldsymbol{x}', t')\right] = c_{ijmn}(\boldsymbol{x}) \delta(\boldsymbol{x} - \boldsymbol{x}') \delta(t - t'), \qquad (1)$$

where the elastic Hooke's tensor reduces to $c_{ijmn}(\boldsymbol{x}) = \lambda(\boldsymbol{x})\delta_{ij}\delta_{mn} + \mu(\boldsymbol{x})(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm})$, in the isotropic case. Here λ and μ are the Lamé parameters and ρ is the density. The elastic representation theorem is,

$$\sigma_{mn}(\boldsymbol{x}, t) = \int_0^{t+} dt' \oint_S dS(\boldsymbol{x}') \{G_{mnij}(\boldsymbol{x}, t - t'|\boldsymbol{x}', 0) n_i a_j(\boldsymbol{x}', t') - [\partial_i' G_{mnij}(\boldsymbol{x}, t - t'|\boldsymbol{x}', 0)] t_j(\boldsymbol{x}', t')\}. \quad (2)$$

where a_j is an acceleration component and t_j is a traction component divided by density. Equation (2) is used to couple the field exited by the source and calculated with the cylinder-symmetric finite-difference scheme with the full 3D scheme at depth z_c and to couple the response from the full 3D scheme with the cylinder-symmetric Greens tensor connecting $\boldsymbol{x}_c$ to $\boldsymbol{x}_r$.

A cross section of a plane-layer 3D model is shown in Figure 1. Layer I is a waterlayer, whereas layer II and layer III are fully elastic. This model is used for calibration of the modeling scheme. As a reference, the response for this model is also calculated using the OSIRIS modeling package. The OSIRIS modeling package is based on a direct global matrix method (Schmidt and Jensen, 1985). The OSIRIS modeling response and the hybrid response are shown in Figure 2. The coupling depth for the hybrid modeling is at approximately 400 m. A good fit between the modeling responses are obtained. The internal multiple in layer II, expected to arrive at 0.75 s at near offsets, is not included in the hybrid modeling response. The amplitude of this event seems to be negligible since the number of events appear to be similar in the two shotgathers. The fit between the two shotgathers is somewhat poorer at high offsets. This may be an aperture effect since the boundary surface at depth z_c have a limited aperture in order to minimize CPU time. For models with complicated 3D geometries below a given depth $\boldsymbol{x}_c$ we find a good fit between the 3D elastic finite-difference solution and the hybrid modeling solution, as will be demonstrated during the presentation.

ACKNOWLEDGMENTS

This work has been financially supported by Statoil Research. We thank Statoil Research for the permission to publish this paper.

REFERENCES

Schmidt, H., and Jensen, F. B., 1985, Efficient numerical solution techniques for wave propagation in horizontally stratified environments Computers and Mathematics with Applications, **11**,699–715.