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3D Anisotropic Depth Migration Operators for Marine Controlled–Source Electromagnetic Data

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SUMMARY

We present frequency-wavenumber (FK) and explicit finite-difference (FD) operators for 3-D anisotropic depth migration of controlled-source electromagnetic (CSEM) data in transversely isotropic (TI) media. A requirement for the applicability of the proposed one-way migration operators is separation of the CSEM data into up-going and down-going, transverse electric (TE) and transverse magnetic (TM) modes.

Introduction

CSEM data are now widely accepted as a useful tool for hydrocarbon exploration. In general, the measured (total-field) CSEM data can be separated into transverse electric (TE) and transverse magnetic (TM) modes, which carry complementary information about geophysical and geological properties of the earth. The TE mode is mainly sensitive to near-surface and overburden properties, whereas the TM mode carries information about high-resistive anomalies (e.g. hydrocarbon accumulations). Also, the TE and TM modes react differently to anisotropic conductivity which is frequently observed in a stratified subsurface (Løseth and Ursin, 2007).

Based on well-known ideas from seismic imaging (Claerbout, 1985), Zhdanov et al. (1996) introduced frequency-wavenumber (FK) and finite-difference (FD) depth migration methods for CSEM data. Hokstad and Røsten (2006) extended their work for 2D migration in transversely isotropic (TI) media. Røsten et al. (2006) presented isotropic explicit one-way 3-D FD migration operators for CSEM data, using the approach applied to seismic depth migration by Holberg (1988). Here, we extend this scheme to include anisotropy. We present 3-D FK and FD depth migration operators for up-going and down-going, TE and TM fields in TI background conductivity models.

Two-way diffusive wave equations

In the ultra-low frequency approximation, neglecting displacement currents, the Faraday and Ampere equations in the frequency domain can be approximated by

$$\nabla \times \mathbf{E} \;=\; i\omega\mu_0 \mathbf{H}, \text{ and } \nabla \times \mathbf{H} = \mathbf{J}^E, \tag{1}$$

where $\mathbf{E}$ is the electric field, $\mathbf{H}$ is the magnetic field, μ_0 is the vacuum permeability, and ω is the angular frequency. Without external sources, the current density $\mathbf{J}^E$ in a linear anisotropic medium is given by Ohm's law

$$J_i^E = \sigma_{ij} E_j, \tag{2}$$

where σ_{ij} is the conductivity tensor. Combining the equations above to eliminate the magnetic field, we obtain the two-way diffusive wave equation

$$\partial_j \partial_j E_i - \partial_i \partial_j E_j + i\omega\mu_0 \sigma_{ij} E_j = 0. \tag{3}$$

We assume that the medium is smooth, such that the spatial derivatives of the conductivity can be neglected. This is justified by the fact that our goal is to perform migration, where we propagate the electric field in a smooth background medium.

In the isotropic case $\partial_j E_j = 0$, and equation (3) reduces to the scalar diffusive Helmholz equation for each component separately. This does not hold in the anisotropic case. However, from Ampere's law (with zero external source current), we obtain

$$\partial_i J_i^E = \partial_i \sigma_{ij} E_j = 0, \tag{4}$$

since the divergence of a curl is identically zero. The conductivity tensor in a TI medium can be written as

$$\sigma = \begin{bmatrix} \sigma_1 & & \\ & \sigma_1 & \\ & & \sigma_3 \end{bmatrix}, \tag{5}$$

where σ_1 and σ_3 are the horizontal and vertical conductivities, respectively. Substituting the TI conductivity tensor above in equation (4), the divergence of the electric field can be expressed in two alternative ways which both will be used below,

$$\partial_i E_i = -2\eta(\partial_x E_x + \partial_y E_y), \quad (6)$$

$$\partial_i E_i = \frac{2\eta}{1+2\eta}\partial_z E_z, \quad (7)$$

where the electromagnetic anisotropy parameter

$$\eta = \frac{\sigma_1 - \sigma_3}{2\sigma_3}, \quad (8)$$

is analogous to the Thomsen parameters ϵ and γ in the seismic case. Substituting equation (6) for the divergence in equation (3), we obtain two coupled two-way equations for the horizontal components of the electric field

$$(1+2\eta)\partial_x^2 E_x + \partial_y^2 E_x + \partial_z^2 E_x + 2\eta\partial_x\partial_y E_y + \kappa_1^2 E_x = 0, \quad (9)$$

$$\partial_x^2 E_y + (1+2\eta)\partial_y^2 E_y + \partial_z^2 E_y + 2\eta\partial_x\partial_y E_x + \kappa_1^2 E_y = 0, \quad (10)$$

where $\kappa_1 = \sqrt{i\omega\mu_0\sigma_1}$ is the complex wavenumber. Returning to equation (3), and using equation (7) to eliminate the divergence of the electric field, we obtain a separate two-way equation for the vertical component

$$(1+2\eta)(\partial_x^2 E_z + \partial_y^2 E_z) + \partial_z^2 E_z + \kappa_1^2 E_z = 0. \quad (11)$$

Plane-wave analysis

In the Fourier domain, equations (9) to (11) can be expressed as a 3×3 eigenvalue problem,

$$\begin{bmatrix} q^2 - 2\eta k_x^2 & -2\eta k_x k_y & 0 \\ -2\eta k_x k_y & q^2 - 2\eta k_y^2 & 0 \\ 0 & 0 & q^2 - 2\eta(k_x^2 + k_y^2) \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = k_z^2 \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}, \quad (12)$$

where $q^2 = \kappa_1^2 - k_x^2 - k_y^2$. The sign convention in the Fourier transform is such that $\partial_t \leftrightarrow -i\omega$ and $\partial_j \leftrightarrow ik_j$. Solving the characteristic equation, we find three pairs of eigenvalues

$$k_z^{(1)} \pm \sqrt{\kappa_1^2 - (k_x^2 + k_y^2)}, \quad \text{and} \quad k_z^{(2)} = k_z^{(3)} = \pm\sqrt{\kappa_1^2 - (1+2\eta)(k_x^2 + k_y^2)}, \quad (13)$$

where the positive and negative signs correspond to down-going and up-going plane waves, respectively. The three corresponding orthogonal and normalized eigenvectors can be written as

$$\boldsymbol{x}^{(1)} = \frac{1}{k_r}\begin{bmatrix} -k_y \\ k_x \\ 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{x}^{(2)} = \frac{1}{k_r}\begin{bmatrix} k_x \\ k_y \\ 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{x}^{(3)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad (14)$$

where $k_r = \sqrt{k_x^2 + k_y^2}$ is the radial horizontal wavenumber.

Consider a plane wave propagating with wavenumber $\boldsymbol{k} = (k_x, k_y, k_z)$. From equation 14 it is clear that $\boldsymbol{x}^{(1)}$ is confined to the horizontal plane and that $\boldsymbol{k} \cdot \boldsymbol{x}^{(1)} = 0$. Hence, the eigenvector $\boldsymbol{x}^{(1)}$ corresponds to the TE mode with vertical wavenumber $k_z^{(1)}$. The eigenvectors $\boldsymbol{x}^{(2)}$ and $\boldsymbol{x}^{(3)}$ belong to the degenerate eigenvalues $k_z^{(2)} = k_z^{(3)}$. Then, any linear combination $\boldsymbol{y} = \alpha\boldsymbol{x}^{(2)} + \beta\boldsymbol{x}^{(3)}$ is also an eigenvector with the same pair of eigenvalues. Since $\boldsymbol{y} \cdot \boldsymbol{x}^{(1)} = 0$ we conclude that $\boldsymbol{x}^{(2)}$ and $\boldsymbol{x}^{(3)}$ form an orthogonal basis for the TM mode with vertical wavenumber $k_z^{(2)} = k_z^{(3)}$.

3-D anisotropic depth migration operators

In a 1-D background medium, standard one-way equations in the FK domain for the down-going field E_i^D and migrated field E_i^M (Zhdanov et al., 1996) are obtained as

$$\partial_z E_i^\nu = \gamma k_z E_i^\nu, \tag{15}$$

where the vertical wavenumber k_z for the TE and TM modes is obtained from equation (13) as

$$k_z^{TE} = \sqrt{\kappa_1^2 + \gamma^2(k_x^2 + k_y^2)}, \quad \text{and} \quad k_z^{TM} = \sqrt{\kappa_1^2 + \gamma^2(1+2\eta)(k_x^2 + k_y^2)}, \tag{16}$$

where $\gamma = i$ for $\nu = D$ and $\gamma = -1$ for $\nu = M$. The solution to equation (15) is given by

$$E_i^\nu(k_x, k_y, z + \Delta z, \omega) = \mathrm{e}^{\gamma \Delta z k_z} E_i^\nu(k_x, k_y, z, \omega), \tag{17}$$

which is the basis for the wavefield extrapolation step of (Gazdag) FK migration.

The depth-stepping equation for E_i^M is numerically stable with exponential decay (like E_i^D) and backward phase rotation (like E_i^U).

The FK migration operators are accurate up to 90 degrees from the vertical, but limited to 1-D background media. To relax the 1-D background assumption, we replace the phase-shift operator in equation (17) by discrete convolution filters in the FX domain, as proposed by Holberg (1988). The extrapolation of the electric field can be written as

$$E_i^\nu(x, y, z + \Delta z, \omega) = \sum_{m,n=-L}^{L} [W^\nu(m\Delta x, n\Delta y, \hat{\kappa}_1, \eta) E_i^\nu(x - m\Delta x, y - n\Delta y, z, \omega)]. \tag{18}$$

The convolution operators W depend only on the normalized wavenumber $\hat{\kappa}_0 = \kappa_0 \Delta x$, the anisotropy parameter η and the ratio $\Delta z / \Delta x$. Hence, for a given $\Delta z/\Delta x$-ratio, the operator coefficients can be precomputed for all relevant values of $\hat{\kappa}_0$ and η and stored in a look-up table. Computation of the finite impulse-response filter with complex-valued coefficients $W^\nu(m\Delta x, n\Delta y, \hat{\kappa}_1, \eta)$, is posed as an inverse problem, minimizing the objective function in the L_4 norm

$$J = ||W^\nu(i\Delta k_x, j\Delta k_y, \hat{\kappa}_0, \eta) - \mathrm{e}^{\gamma \Delta z k_z}||^4, \tag{19}$$

where $W^\nu(i\Delta k_x, j\Delta k_y, \hat{\kappa}_0, \eta)$ is the discrete Fourier transform of $W^\nu(m\Delta x, n\Delta y, \hat{\kappa}_0, \eta)$ for a discrete set of horizontal wavenumbers $i\Delta k_x$ and $j\Delta k_y$. The dispersion relation for diffusive EM fields is smooth and continuous for all wavenumbers. Hence, different from the seismic case, we do not need to introduce a dip-limitation on the corresponding 3-D filter operators to get a stable depth migration scheme for CSEM data. The optimization is generally performed for all wavenumbers, and the real and imaginary part of the filter operator are optimized separately. In practice, we need to compute tabulated filter coefficients only for W_i^D. Then the operator for W_i^M can be obtained by complex conjugation.

Numerical example

To demonstrate the 3-D anisotropic FK and FD migration operators presented above, we propagate the x-component of an electric Hertz dipole oriented in the x-direction. The frequency of the dipole is 0.75 Hz. The background medium is homogeneous with horizontal conductivity $\sigma_1 = 1.0\,\mathrm{S/m}$ and anisotropy parameter $\eta = 0.5$. The dipole field was computed analytically at 500 m depth below the source, and then propagated numerically from 500 m to 5000 m. The grid spacing was $\Delta x = \Delta y = 100$ m and $\Delta z = 50$ m. Figure 1 shows the phase of the isotropic 3-D FK operator and the anisotropic 3-D FK and FD operators for the TM mode. The effect of anisotropy can be noticed from the lateral stretch of the constant-phase surfaces.

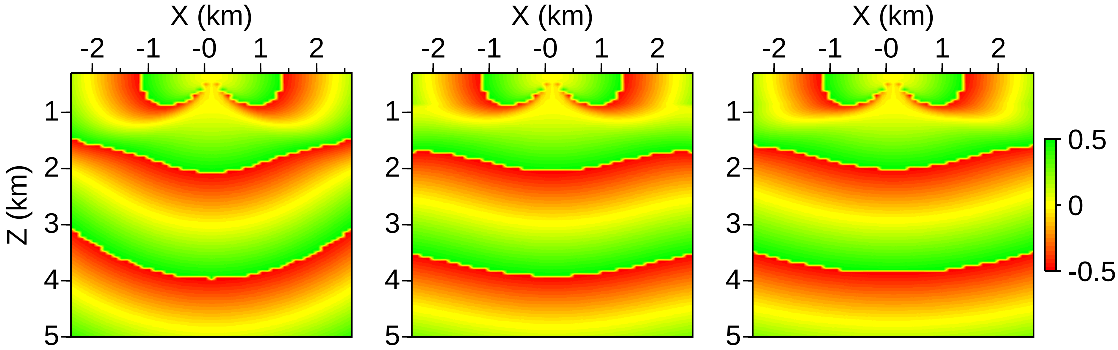

Figure 1: Phase (in units of π) of the impulse responses of the isotropic FK operator (left) anisotropic FK operator (center) and anisotropic explicit FD operator (right).

Conclusions

We have presented 3-D FK and explicit 3-D FD operators for depth migration of TE and TM modes in TI background conductivity models. The migration operators were demonstrated by numerical downward extrapolation of the field of a horizontal electric dipole. The TE and TM modes react differently to anisotropic conductivity, and should be propagated with different operators.

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